Unit 3: Mathematics for Engineering Technicians

Level: 3

Unit type: Mandatory

Assessment type: Internal

Guided learning: 60

Unit introduction

One of the main responsibilities of engineers is to solve problems quickly and effectively. This unit will enable learners to solve mathematical, scientific and associated engineering problems at technician level. It will also act as a basis for progression to study other units both within the qualification, such as the Unit 6: Further Engineering Mathematics for, and at BTEC Higher National level.

This unit enables learners to build on knowledge gained at GCSE and use it in a more practical context for their chosen discipline. Learning outcome 1 will develop learners' knowledge and understanding of algebraic methods, from a look at the use of indices in engineering to the use of the algebraic formula for solving quadratic equations. Learning outcome 2 involves the introduction of the radian as another method of angular measurement, the shape of the trigonometric ratios and the use of standard formulae to solve problems involving surface areas and volumes of regular solids. Learning outcome 3 requires learners to be able to represent statistical data in a variety of ways and calculate the mean, median and mode. Finally, learning outcome 4 is intended as a basic introduction to the arithmetic of elementary calculus.

Note that the use of 'e.g.' in the content is to give an indication and illustration of the breadth and depth of the area or topic. As such, not all content that follows an 'e.g.' needs to be taught or assessed.

Learning outcomes

On completion of this unit a learner should:

1. Be able to use algebraic methods
2. Be able to use trigonometric methods and standard formulae to determine areas and volumes
3. Be able to use statistical methods to display data
4. Be able to use elementary calculus techniques.

Unit content

1. Be able to use algebraic methods

Indices and logarithms: laws of indices ($a^m \, x \, a^n = a^{m+n}, \frac{a^m}{a^n} = a^{m-n}, (a^m)^n = a^{mn}$) laws of logarithms ($\log A + \log B = \log AB, \log A^n = n \log A, \log A - \log B = \log \frac{A}{B}$) e.g. common logarithms (base 10), natural logarithms (base e), exponential growth and decay.

Equations and graphs: linear equations e.g. $y = mx + c$; straight line graph (coordinates on a pair of labelled Cartesian axes, positive or negative gradient, intercept, plot of a straight line); quadratic graph y= $ax^2 + bx + c$; experimental data e.g. Ohm's law; pair of simultaneous equations in two unknowns (two linear or one linear and one quadratic).

Factorisation and quadratics: multiply expressions in brackets by a number, symbol or by another expression in a bracket; by extraction of a common factor e.g. $ax + ay, a(x + 2) + b(x + 2)$; by grouping e.g. $ax - ay + bx - by$; quadratic expressions e.g. $a^2 + 2ab + b^2$; roots of an equation e.g. quadratic equations with real roots by factorisation, and by the use of formula.

2. Be able to use trigonometric methods and standard formulae to determine areas and volumes

Circular measure: radian; degree measure to radians and vice versa; angular rotations (multiples of π radians); problems involving areas and angles measured in radians; length of arc of a circle ($s = r\theta$); area of a sector ($A = ½ r^2 \theta$)

Triangular measurement: functions (sine, cosine and tangent); sine/cosine wave over one complete cycle; graph of tan A as A varies from 0° and 360° ($tanA = sin A/cos A$); values of the trigonometric ratios for angles between 0° and 360°; periodic properties of the trigonometric functions; the sine and cosine rule; practical problems e.g. calculation of the phasor sum of two alternating currents, resolution of forces for a vector diagram

Mensuration: standard formulae to solve surface areas and volumes of regular solids e.g. volume of a cylinder = $\pi r^2 h$, total surface area of a cylinder = $2\pi rh + 2\pi r^2$, volume of sphere = $\frac{4}{3} \pi r^3$, surface area of a sphere = $4 \pi r^2$, volume of a cone = $\frac{1}{3} \pi r^2 h$, curved surface area of cone = $(\pi r) \, x \, \sqrt{(r^2 + h^2)}$.

3. Be able to use statistical methods to display data

Data handling: data represented by statistical diagrams e.g. bar charts, pie charts, frequency distributions, class boundaries and class width, frequency table; variables (discrete and continuous); histogram (continuous and discrete variants); cumulative frequency curves.

Statistical measurement: arithmetic mean; median; mode; discrete and grouped data.

4. Be able to use elementary calculus techniques

Differentiation: differential coefficient; gradient of a curve $y = f(x)$; rate of change; Leibniz notation ($\frac{dy}{dx}$); differentiation of simple polynomial functions, exponential functions and sinusoidal functions; problems involving evaluation e.g. gradient at a point.

Integration: integration as reverse of differentiating basic rules for simple polynomial functions, exponential functions and sinusoidal functions; indefinite integrals; constant of integration; definite integrals; limits; evaluation of simple polynomial functions; area under a curve e.g. $y = x(x - 3)$, $y = x^2 + x + 4$

Assessment and grading criteria

In order to pass this unit, the evidence that the learner presents for assessment needs to demonstrate that they can meet all the learning outcomes for the unit. The assessment criteria for a pass grade describe the level of achievement required to pass this unit.

Assessment and grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**	**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**
P1 manipulate and simplify three algebraic expressions using the laws of indices and two using the laws of logarithms		
P2 solve a linear equation by plotting a straight-line graph using experimental data and use it to deduce the gradient, intercept and equation of the line	M1 solve a pair of simultaneous linear equations in two unknowns	D1 solve a pair of simultaneous equations, one linear and one quadratic, in two unknowns
P3 factorise by extraction and grouping of a common factor from expressions with two, three and four terms respectively	M2 solve one quadratic equation by factorisation and one by the formula method	
P4 solve circular and triangular measurement problems involving the use of radian, sine, cosine and tangent functions		
P5 sketch each of the three trigonometric functions over a complete cycle		
P6 produce answers to two practical engineering problems		

involving the sine and cosine rule		
P7 use standard formulae to find surface areas and volumes of regular solids for three different examples respectively		
P8 collect data and produce statistical diagrams, histograms and frequency curves		
P9 determine the mean, median and mode for two statistical problems		
P10 apply the basic rules of calculus arithmetic to solve three different types of function by differentiation and two different types of function by integration.	M3 apply the rules for definite integration to two engineering problems that involve summation	D2 apply graphical methods to the solution of two engineering problems involving exponential growth and decay, analysing the solutions using calculus

Essential guidance for tutors

Assessment

The centre's assessment strategy used will need to cover all the learning outcomes and associated pass criteria but not necessarily all the topics included in the unit content.

Criterion P1 could be assessed in the form of a short written test and could possibly also include criterion P3.

P2 could be assessed through an assignment using data from *Unit: Electrical and Electronic Principles*, which ideally would be delivered concurrently with this unit. If this is not possible, learners should be given a range of data sufficient for them to plot the graph and work out the gradient, intercept and the equation. Data forcing them to draw the line of best fit, as opposed to a set of points directly on the graphical line, might be most appropriate.

For P4, learners could be given a range of different values and assessed by an assignment or a short formal test. The problems given should collectively cover radian, sine, cosine and tangent functions. When considering the content part of this learning outcome it is important that these problems give the learner the opportunity to convert multiples of π radians to degrees and vice versa. The circular measurement problems also need to cover the length of an arc and area of a sector as well as areas and angles measured in radians. Obviously the triangular measurement problems are more basic and only expect application of the three functions.

P5 requires learners to sketch each of the three trigonometric ratios and this is probably best done as a classroom exercise. Similarly, P6 could take the form of a written assignment where learners must produce answers to two practical engineering problems involving the sine and cosine rule (for example calculate the phasor sum of two alternating currents and evaluate the resultant and the angle between two forces).

Criterion P7 requires learners to calculate the surface areas and volumes for three different regular solids. This could be achieved through an assignment or perhaps by combining it with other criteria in a short formal test.

An assignment could be used for P8 where learners collect meaningful data (for example classification of workers within their company) and display this information using different graphical methods (for example bar charts). They also need to produce a histogram and plot frequency curves (for example resistance values of 100 resistors or external diameter of pins).

For P9, learners must provide evidence that they are able to determine and then explain the relevance of the mean, median and mode for a set of discrete and grouped data (for example time taken to produce components on a machine rounded to the nearest ten seconds and the 100 resistor values or diameters of pins from P8). This could be done by an assignment. P10 may be assessed through a short formal test, with learners being given a list of the standard differential coefficients and integrals to use.

For M1, learners will need to provide evidence that they can solve a pair of simultaneous linear equations in two unknowns (for example equations formed after the application of Kirchhoff's laws, power transmitted for different belt tensions in a mechanical system). This could be extended to D1 by the introduction of a quadratic equation to be solved simultaneously with a linear equation.

It would be appropriate to use the same assessment method and instrument as P2, possibly combining these two criteria as one assessment activity.

M2 could also be assessed by assignment as it requires learners to evaluate the roots of a quadratic equation by factorisation and by the formula method (for example evaluation of an equation formed after the realisation of a practical situation).

Both the criteria required to achieve D2 could be assessed through a written assignment. Learners need to apply the graphical methods to the solution of two engineering problems involving exponential growth and decay (for example growth of voltage in a capacitor, radioactive decay, application of Taylor's tool life equation $C = VT^n$) and then analyse the results by applying the appropriate method of differential calculus to check the results.

M3 requires learners to demonstrate that they can accurately evaluate two engineering problems involving definite integration (for example area under a velocity-time graph, area under a voltage-current graph).

Programme of suggested assignments

The table below shows a programme of suggested assignments that cover the pass, merit and distinction criteria in the assessment and grading grid. This is for guidance and it is recommended that centres either write their own assignments or adapt any Edexcel assignments to meet local needs and resources.

P1, P2, P3, M1, M2	Algebraic Methods	A written activity/test requiring learners to complete five tasks, one for each of the criteria.	A report containing written solutions to each of the five tasks carried out under controlled conditions.
P4, P5, P6, P7	Trigonometric Methods and Standard Formulae	A written activity requiring learners to use trigonometric methods and standard formula to determine areas and volumes.	A report containing the results of calculations, and graphic evidence to support the use of trigonometric methods and standard formula for the determination of areas and volumes

P8, P9	Statistical Methods	A written activity requiring learners to collect and display data using different graphical methods, also evaluate the mean, median and mode for a set of discrete and grouped data.	A report containing bar charts, pie charts and the results of calculations to determine the mean, median and mode for a set of discrete and grouped data
P10, M3, D1,	Calculus Techniques	A written activity requiring learners to produce calculations, graphical solutions and analysis to demonstrate use of calculus techniques.	A report containing the solutions to calculations, graphs and analysis of several calculus techniques. Carried out under controlled conditions.

Essential resources

Learners will need to possess an electronic scientific calculator and have access to software packages that support understanding of the principles and their application to engineering.

Indicative reading for learners

Textbooks

Boyce A, Cooke E, Jones R and Weatherill B – *BTEC Level 3 National Engineering Student Book* (Pearson, 2010) ISBN 9781846907241

Boyce A, Cooke E, Jones R and Weatherill B – *BTEC Level 3 National Engineering Teaching Resource Pack* (Pearson, 2010) ISBN 9781846907265

Bird J – *Engineering Mathematics* (Routledge, 2014) ISBN 9780415662802

Fuller A, Greer A, Taylor G W – *BTEC National Mathematics for Technicians* (Nelson Thornes, 2004) ISBN 9780748779499

Tooley M and Dingle L – *BTEC National Engineering* (Routledge, 2010) ISBN 9780123822024

Websites

Science, Technology, Engineering and Mathematics Network – www.stemnet.org.uk