**Pearson Edexcel Level 3 Certificate in Mathematics in Context**

**PRACTICE TASK: EBOLA**

***The Ebola virus is MUTATING, and 'could become more contagious', warn scientists who first identified the outbreak.***

A team of researchers from the Institut Pasteur in France first identified the outbreak of Ebola in Guinea, in March 2014. Patient zero - the first person to be infected - was two-year-old Emile Ouamouno from the rural village of Meliandou. He died in December 2013, four days after he fell ill, with a sky-high fever and vomiting.

Just weeks later his sister succumbed, followed by their mother and grandmother.

From there the virus spread. Three months later, the scientists at Institut Pasteur identified it as Ebola, after it was reported to the health authorities.

The team have since been tracing the virus' spread through Guinea, to establish if the disease could become more contagious. They examined hundreds of blood samples from Ebola patients in the West African nation where the first cases struck..

Human geneticist, Dr Anavaj Sakuntabhai, told the BBC: 'We know the virus is changing quite a lot. That's important for diagnosing and for treatment. We need to know how the virus (is changing) to keep up with our enemy.'

He told Radio 4's *Today* programme that viruses have to 'fight a balance' between infecting people and spreading. 'We have seen several cases that don't have any symptoms at all when infected,' he said. 'These people may be the ones who could spread the virus better, we do not know yet. A virus can change from more deadly into less deadly but more contagious and that is something we are afraid of.'

Viruses do change over a period of time.

Read more: <http://www.dailymail.co.uk/health/article-2931410/The-Ebola-virus-MUTATING-contagious-warn-scientists-identified-outbreak.html#ixzz3U5eZavYM>

**How disease spreads**

The typical behaviour of a new disease as it spreads through a population is shown in Graph 1.

Number of new infections per week

*t*

**Graph 1**

*t* is the time (usually in days) measured from the outbreak of the disease.

Typically, new diseases are very virulent, even deadly. This is because the local population have little resistance to the newly arrived bacterium or virus. It is estimated that about 50% of the native population of Honduras were killed off in the late 16th century by measles brought over by Spanish colonists. Even if resistance is built up in a population, this can be to no avail, as the virus responsible for a disease may mutate, making the body defences inadequate. The virus that causes the common cold is notorious for this, as is the flu virus.

There are various national and international bodies which are responsible for monitoring the progress of the outbreak of diseases. In England it is the responsibility of Public Health, a government department. The USA has a similar structure - the Centers for Disease Control and Prevention, commonly known as CDC. Internationally, the World Health Organisation (WHO), a branch of the United Nations, monitors issues on a worldwide basis.

**Data source A**

**Table 1** charts the outbreak of influenza in an English city (City A).

**Table 1: Number of new cases of influenza per week in the winter of 2004/2005 (City A)**

|  |  |
| --- | --- |
| **Number of new cases** | **Number of weeks** |
| 1 - 40 | 2 |
| 41 - 100 | 5 |
| 101 - 200 | 7 |
| 201 - 500 | 6 |
| 501 + | 4 |

In another city the data about an influenza outbreak was collected in a different way.

This is shown in **Table 2.**

**Table 2: Number of new cases of influenza per week in the winter of 2004/2005 (City B)**

|  |  |
| --- | --- |
| **Number of new cases** | **Number of weeks** |
| 1 - 20 | 3 |
| 21 - 40 | 8 |
| 41 - 60 | 10 |
| 61 - 80 | 5 |
| 81 -100 | 2 |
| 101 - 120 | 1 |
| 121 - 140 | 1 |

**Data source B**

Using data from WHO, scientists have plotted a graph to show the progression of Ebola**.**

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**Graph 2: The cumulative number of deaths in countries in West Africa, April - August 2014**

Mathematical modelling of the spread of diseases is well established and has had some success in predicting such outcomes as the likely number of people who will be infected and the length of time the disease will affect a significant number of people.

A simple model of the spread of a disease is an exponential one which describes the number of people, *In* with the disease on day *n*.

This has the form 

where *b* is a measure of the rate of infection.

However, the most common model, the SIR model, deals with Susceptibles (*S*), those who are not immune and have yet to catch the disease, Infectives (*I*), who have caught the disease and can pass it on to others and Recovered (*R*) or more morbidly, 'removed', who are no longer infectious.

This leads to the recurrence relations  
 **** (1)

**** (2)

**** (3)

where *a* and *b* are constants.

These equations can be used to study the progression of a disease in a population that has a constant size.

**Refer to Data source A**

**1** (a) In what fraction of weeks were there more than 200 new cases of influenza reported in city A? **(1)**

(b)Assuming the width of the 501+ interval is the same as the width of the previous interval, calculate an estimate of the mean. **(4)**

(c)Explain why an estimate of the median may be a better average to use for the information in **Table 1** than an estimate for the mean. **(1)**

**2** (a) Draw a cumulative frequency diagram for the information for city B. **(3)**

(b) Use your cumulative frequency diagram to draw a box plot for this data. State any assumptions you make about the ends of the box plot. **(3)**

This box plot shows the information about an outbreak in city C.

20

40

60

80

1000

120

140

0

0

City C

Number of new cases per week

(c) Compare the two distributions. **(3)**

**3** In the early stages of a disease the number of infected people on day *n*, *In*, is given by



(a)When *I*1 = 10 and *b* = 1.05, find the value of *I*15. **(2)**

(b) Write down an estimate for the doubling time of this disease. **(1)**

Suppose that the rate of infection is such that the number with the disease on any day after the first is 4% more than with the disease on the previous day.

(c) Given that there were 100 people infected on day 1, work out the total number of people infected on day 41. **(3)**

The exponential model is not useful when *n* gets large.

(d) Explain why. **(1)**

**Refer to Data source B**

**4** Assume that the total number of deaths from Ebola can be modelled by exponential growth and

* the number of deaths on day 1 is 75
* the rate of increase in the total number of deaths is 1.98% per day.

Compare the prediction for August 10th of the model with the data shown on **Graph 2. (3)**

The most well-known mathematical model of the spread of infection through time is the SIR model.

The SIR model is a set of 3 recurrence relations

**** (1)

**** (2)

**** (3)

where *a* and *b* are constants.

Key  is the number of susceptibles on day *n*  
 is the number of infectives on day *n*  
 is the number recovered (or removed) on day *n*  
For a new disease it is normal to have as a small number with *R*1 = 0, (often  
 ) and − 1 where *P* is the total population that is susceptible to the disease. In practice, as *P* is so large, it is usual to put 

**5** (a) Comment on these initial values. **(2)**

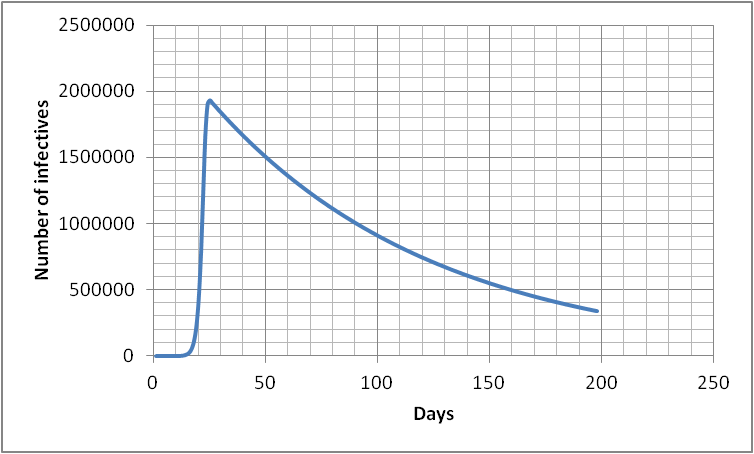
(b) Show that the percentage change per day of the number of susceptibles is −100*aIn*and make a comment on how the value of *a* affects this percentage change. **(3)**

(c) Give an interpretation of equation (3) and comment on the effect different values of *b* will have. **(2)**

(d) (i) Use one of the recurrence relations with *I*20  = 400 000, *S*20 = 1 500 000, *a* = 5 × 10-7, to work out *S*21, **(3)**

(ii) Given that *S*1 = 2 million, write down the value of *R*20. **(1)**

**6** Here are two graphs showing the variation of *In* and of *Rn* with *n.* In each case, *I*1= 1, *a* = 0.0000005, *S*1 = 2 million



**Variation of *In* with *n***

**Variation of *Rn* with *n***

(a) (i) Estimate the value of *I*100 and the value of *R*100. **(2)**

(ii) Comment on the value of S100. **(1)**

(iii) Interpret each graph. **(3)**

(b) On graph paper, draw a sketch of *Sn* against *n*, for 0 ≤ *Sn* ≤ 2.5 million and 0 ≤ *n* ≤ 200. **(2)**

Near the maximum point of the *In* graph , *In* changes very slowly**.**

(c) By putting *In*+1 = *In* = *I*max **,** show that at this value of *n,*  *Sn = * **(2)**

**Total 47 marks**

**Source information**

Extract from ‘The Ebola virus is MUTATING, and ‘could become more contagious’, warn scientists who first identified the outbreak’, *The Daily Mail*, 29/01/2015 (Lizzie Parry), copyright © Solo Syndication, 2015; Table 1 published in "Infuenza modeling with a Discrete SIR model" by Joseph M. Mahaffy-SDSU, Modified for R by Jack Dockery, 20/02/2013, SOURCE Center of Disease Control; Table 2 'Number of new cases of influenza per week in the winter of 2004/2005 (City B)', reprinted from *Global Health Atlas 2004/2005,* <http://apps.who.int/globalatlas/dataQuery/reportData.asp?rptType=1>, copyright © 2005, World Health Organisation; and Graph from data in 'Ebola Situation Reports', <http://apps.who.int/ebola/ebola-situation-reports>, copyright © 2005, World Health Organisation and Centers for Disease Control & Prevention.