**Pearson Edexcel Level 3 Certificate in Mathematics in Context**

**Practice questions:**

**Sequences**

**Topic practice questions**

These materials have been gathered together to help provide opportunities for skills practice on some of the mathematics topics within the content of the Mathematics in Context specification. The materials comprise four sets of questions organised by topic area as follows:

A Statistics

B Probability and Venn Diagrams

C Linear Programming

**D Sequences**

The majority of the questions have been taken from past exam papers in GCE Mathematics Core 1 and Core 2 and the GCSE Mathematics Linked Pair Pilot. Some questions in the Sequences strand have been written afresh for this purpose to broaden the range of topics covered.

None of the questions is intentionally written in the style of Mathematics in Context exam questions. You and your students may however find them useful for classroom discussion, group work and/or individual practice on some of the mathematics skills within the specification.

**Contents**

[Arithmetic progressions 3](#_Toc468110328)

[Exercise 1 3](#_Toc468110329)

[The Fibonacci sequence 6](#_Toc468110330)

[Exercise 2 6](#_Toc468110331)

[Geometric series 8](#_Toc468110332)

[Exercise 3 8](#_Toc468110333)

[The doubling time graph 11](#_Toc468110334)

[Exercise 4 12](#_Toc468110335)

[More complex sequences and series 13](#_Toc468110336)

[Exercise 5 13](#_Toc468110337)

[The logistic curve for populations 17](#_Toc468110338)

[Exercise 6 18](#_Toc468110339)

[Quadratic sequences 21](#_Toc468110340)

[Exercise 7 22](#_Toc468110341)

[Finding the nth term of a quadratic sequence: 24](#_Toc468110342)

[Exercise 8 25](#_Toc468110343)

[Past GCE Mathematics examination questions 27](#_Toc468110344)

[Exercise 9 27](#_Toc468110345)

**D SEQUENCES**

# Arithmetic progressions

## Exercise 1

**1**. Each month Billy saves £100 more than he did the month before.

For the first month he saves £200

(a) How much does he save in the 12th month?

(b) How much has he saved in the total at the end of 12 months?

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**2.** There are 500 gallons of oil in a tank. Each day the farmer uses 15 gallons of oil.

(a) Work out how many gallons there are in the tank at the end of 16 days.

(b) How long will it take to empty the tank?

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**3.** Each year a family spends more on its house. The first year it spends £1000.

For each subsequent year it spends £200 more than it did the year before.

(a) Write this as a recurrence relation.

(b) Work out how much the family spends in the 6th year.

(c) Work out how much the family spends altogether after 10 years.

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**4.** The amount of money a bank has after *n* days, *An*, obeys the recurrence relationship

*An* = *An* – 1 – 20 000.

Initially the bank had £10 000 000.

(a) Work out the amount in the bank after *n* days.

(b) After how many days will there be no money in the bank?

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**5.** One possible model of the stock market is that each month it increases by a constant amount until it reaches a maximum and then crashes to a low value.

This can be modelled by  where  is the value of the stock market.

The initial value was 3000.

(a) Work out the value after 2 years,

The stock market then crashed to a value of 2000. The behaviour of the stock market after the crash can be modelled by  where *k* is a constant.

The stock market reached 3000 in 10 months.

(b) Work out how long the model predicts that the stock market will take to reach the value it crashed at.

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**6.** Jim wants to save for a car.

He plans to start with £100 in the first month and then increase the amount he saves each month by £10.

Work out how much he will save in total at the end of 12 months.

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**7.** Alice wants to save for a holiday. She has to pay for the holiday in 30 weeks time.

She starts by putting £30 in a special account.

Each week she puts in £1 less than she did the week before.

How much, in total will she have saved at the end of 30 weeks?

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**8.** A farmer wants to establish his own farm. Initially he owns no farmland.

Each 5 years the farmer buys 10 more hectares of farmland than he did in the previous 5 years.

How much farmland will the farmer own after 50 years have passed?

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**9.** The diagram shows the cross section of a set of pipes.

In each row there is one more pipe than there is in the row above.

In each row one pipe is blocked off (shown as shaded in the diagram).

Find an expression, in terms of *n* for the total number of **unblocked** pipes when there are *n* rows.

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# The Fibonacci sequence

The Fibonacci sequence starts 1 1 2 3 5 8

The sequence is defined by

*un*+2 = *un*+1 + *un*and *u*1 = 1, *u*2 = 1

There are variations on this where the recurrence relation is the same but the two start values are not 1 and 1.

For example 2 1 3 4 7 11 18

follows the same rule but *u*1 = 2 and *u*2 = 1.

When you form fractions of the form  you get , , , ,  which are approximations to the *golden ratio*, a special number that people have claimed has extensive use in the proportions of ancient buildings.

The exact value of the golden ratio is  and is approximately 1.6.

## Exercise 2

**1** (a) Find the 10th term of the Fibonacci sequence

(b) *u*n + 2 = *un* + 1 + *un*and *u*1 = 1, *u*2 = 3

Write down the first 5 terms.

(c) *un* + 2 = *un* + 1 + *un*and *u*1 = 2, *u*2 = 2

Write down the first 5 terms.

(d) *un* + 2 = *un* + 1 + *un*and *u*1 = 0, *u*2 = 4

Write down the first 5 terms.

(e) The 12th term of a Fibonacci-like sequence is 18. The 13th term is 25. Find the 14th term and the 11th term

(f) The 7th term of a Fibonacci-like sequence is 18. The 9th term is 38. Find the 8th term and the 10th term.

(g) The 4th and 5th term of a Fibonacci-like sequence are 16 and 25. Find the first term.

(h) The 4th and 6th term of a Fibonacci-like sequence are 7 and 18. Find the first term.

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**2.** For the Fibonacci sequence, show that *un* + 3 = 2*un* + 2 – *un*and that *un* + 3 = 2*un* + 1 + *un*

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**3.**

1 cm by 2 cm

2 cm by 2 cm

3 cm by 2 cm

These rectangles are 1 cm by 2 cm.

(a) Show that there are 3 ways of making a larger rectangle that is 3 cm by 2 cm (two are already shown in the diagram)

(b) Show all the ways of making a 4 cm by 2 cm rectangle.

(c) Show how to find the number of 5 cm by 2 cm rectangles without having to draw them.

(d) Find, by calculation, the number of ways there are of making a 9 cm by 2 cm rectangle.

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**4.** You have an unlimited number of £1 and £2 coins.

1. Show that you can make £3 in 3 different ways.

(b) How many different ways can you make £4?

(c) How many different ways can you make £10?

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**5.** (a)Use a spreadsheet to produce values of the Fibonacci sequence.

(b) Use your spreadsheet to find the value of the 20th term.

(c) Adapt your spreadsheet to work out values of  and confirm that the values get closer and closer to the golden ratio.

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# Geometric series

## Exercise 3

**1.** Jim invests £1000 at 3% per annum compound interest.

(a) Write down an expression, in terms of *n*, for the value of the investment at the end of the *n*th year.

(b) Write down an expression, in terms of *n* for the interest earned by the investment at the end of the *n*th year

(c) Find the value of the investment at the start of the 11th year and find the total interest earned.

(d) Use your calculator, or a spreadsheet to find after how many years the value of the investment has doubled.

(e) Make a spreadsheet which will allow you to calculate the value of an investment after any number of years. The key variables will be

(i) the initial investment,

(ii) the constant interest rate,

(iii) the number of years of the investment.

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**2.** Billy invests £10000 at 4% per annum compound interest. Each year he pays tax at the rate of 20% calculated on the interest earned for that year.

(a) Work out the value of the investment after 3 years.

(b) Use a spreadsheet to work out the value of an investment when tax at 20% has to be paid each year. Your spreadsheet should allow you to input the initial investment and the interest rate.

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**3.** Hannah invests £1000 in an account paying compound interest.

For the first year the interest rate is 2% per annum.

For the second year the interest rate is 2.5% per annum.

For the third year the interest rate is 4% per annum.

Work out the AER (Correct to 3 significant figures) for the investment.

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**4.** Megan invests £1000 each year in an account paying 4% compound interest.

She starts on 1st January 2015.

1. Explain why the first £1000 is worth £1000 × 1.045 on 1st Jan 2020
2. What is the value of the second £1000 (invested on 1st Jan 2016) on 1st Jan 2020?
3. Work out the total value of the investment on 1st Jan 2020.

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**5.** Each year, Alice invests £*P* at 3% compound interest per annum.

(a) Show that the total value of her investment after *n* years is

.

(b) Use the sum of a geometric progression formula to find the total value of the investment when *P* = 1000 and *n*= 10.

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**6.** Andrew invests £1200 every year in an account paying 3.2% per annum compound interest. Find the total value of the investment after 6 years.

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**7.** Suresh borrows £50000 at a rate of 5% per annum.

(a) How much would he have to repay each year so that the amount he owes remains at £50000?

At the end of the first year, he repays £10000.

(b) How much will he owe at the end of the second year before any further repayments?

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**8.** When £*P* is borrowed at a rate of *r*% per annum and yearly repayments of £*R* are made, the amount owing £*A* at the end of *n* years after the *n*th repayment has been made is



(a) Interpret each of the two terms on the right hand side of the formula.

Magda borrows £120000 at a rate of 4% per annum. She repays £6000 per annum.

(b) Use the formula to work out the amount owing after the 5th repayment.

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**9.** Emeka borrows £100000 at a rate of 5% per annum for 10 years. He repays £*R* per year.

(a) Use the amount owed formula to show that after the 10th repayment, Emeka owes

.

(b) Explain why .

(c) Find the value of *R* so that Emeka has paid off the loan after 10 years.

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**10.** The population at year *n* of a country is related to the population at year *n* – 1 by

, where *k* is a constant.

(a) Interpret the value of *k* and the affect on the population for different values of *k*.

The population at year zero is *N*.

(b) Write down an expression for  in terms of *n*, *N* and *k.*

(c) Sketch a graph of  against *n* in the cases where *k* = 1, *k* < 1 and *k* > 1.

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**11.** The population of Liberia grows at a rate of 4.5% per annum. The current population is 4 million.

Work out an estimate of the population in 3 years time.

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**12.** The population of a country was 60 million. Ten years later the population of the country was 73 million.

Work out the growth rate per annum of the population.

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# The doubling time graph

This graph shows the relationship between the doubling time, *T* years, and the growth factor, *k* , for a population which experiences exponential growth.

2

4

6

8

10

12

14

16

1

1.2

1.4

1.6

1.8

*T*

*k*

The doubling time is independent of the initial population. The growth factor is the fraction



so the doubling time is in years.

For some populations of bacteria the doubling time could be in days, or even hours.

## Exercise 4

**1.** In this question, *T* is measured in years.

(i) (a) Find an estimate for the doubling time for a growth factor of 1.1.

(b) A population doubles in size after 4 years. Work out an estimate for the value of *k*.

(ii) A population increases in size exponentially by 20% each year. Find an estimate for

(a) the time it takes the population to double,

(b) the time it takes the population to increase by a factor of 4

(iii) An island has a population of 80000 in 2016. The growth rate is 5%

1. By what year will the population have become 160000?

(b) By what year will the population have become 320000?

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**2.** The bacterium *Escherichia coli*(*e. coli*) lives in the gut of humans. Most strains are harmless.

The growth rate of the bacteria colony is about 40% per day. A culture of *e. coli* is grown. The initial size of the culture is 1 million.

(a) Find the size of the *e. coli* population the following day.

(b) Use the graph to find an estimate of the doubling time of the population.

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**3**. The parasite which causes many cases of malaria is called *Plasmodium falciparum.* Cultures of the parasite typically have a doubling time of 1 hour.

(a) Work out the growth factor.

(b) Using the fact that 210 is 1024 work out how long approximately it takes a population of *Plasmodium falciparum* to increase by a factor of 1000.

(c) Work out an estimate for how long a population will take to become one million times its initial size.

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# More complex sequences and series

## Exercise 5

**1.** The natural exponential increase in a population is 2% per year. There is a constant immigration into the population of *k* people per year.

The population in year *n* is .

(a) Explain why  for *n* > 0.

Given that  and that *k* = 1000,

(b) find the values of  and . Comment on the value of  that you have found.

Let .

(c) Find an expression for  in terms of .

(d) (i) Write down the value of .

(ii) Hence, find the values of  and of in terms of *n*.

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**2.** Here are two models of saving.

Model 1

Start with £*a* and add £*d* to the account every year.

Let £*Sn* be the total savings up to and including year *n.*

(a) Express *Sn*in terms of *Sn*-1 and *d*.

(b) Find *Sn* in terms of *a* and *d*.

Model 2

Start with £*a* and add £*nd* to the account at the end of year *n.*

Let £*Sn* be the total savings up to and including year *n.*

(c) Express *Sn* in terms of *Sn –* 1 and *d*.

(d) Find *Sn* in terms of *a* and *d*.

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**3.** Here is a set of patterns made of tiles.

Pattern 1

Pattern 2

Pattern 3

Let *Tn* be the number of tiles in pattern *n*

(a) Show that *Tn*+1*= Tn* + 2*n* + 1

(b) Use the formula for the sum of an arithmetic series to show that *Tn* = (*n* + 1)2*.*

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**4.** A recurrence relation is

****,with *S*0 = 0.

(a) Find the value of *S*5.

(b) Find the value of .

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**5.** The recurrence relation  is used to model the total number of cans in a stack of cans (*n +* 1) layers high.

Stack 1

Stack 2

Stack 3

(a) Write down the number of cans in the bottom layer of stack 10.

(b) Use the formula  to work out the total number of cans in stack 10.

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**6.** Jim throws a coin *n* times.

The outcomes can be described by a probability tree diagram.

*n* = 1

*B*1 = 2

*n* = 2

*B*2 = 6

*n* = 3

*B*2 = 14

H

H

H

H

H

H

H

H

H

H

H

T

T

T

T

T

T

T

T

T

T

T

Consider a sequence of probability tree diagrams.

Let *Bn* be the total number of branches of the probability tree diagram for *n* throws*.*

(a) Write down the value of *B*4.

(b) Show that  for *n* > 1.

(c) Use the formula for the sum of a geometric series to find an expression, in terms of *n,* for *Bn*.

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**7.** The greatest number of pieces a pizza can be cut into with 1 straight cut is 2.

The greatest number of pieces a pizza can be cut into with 2 straight cuts is 4.

*n* = 1

*P*1 = 2

*n* = 2

*P*2 = 4

(a) Show that when 3 straight cuts are used the maximum number of pieces is 7 and when 4 straight cuts are used is 11.

Let *Pn* be the maximum number of piecesfor *n* cuts.

(b) Show that .

(c) Find a formula, in terms of *n*, for *Pn*.

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**8.**  Produce a spreadsheet which will enable you to find

(i) the sum of the first *n* square numbers, for *n* = 1 to 20,

(ii) the sum of the first *n* cube numbers for *n* = 1 to 20,

(iii) the sum of the first *n* triangular numbers from *n* = 1 to 20.

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# The logistic curve for populations

The exponential model for populations follows the recurrence relation

,

where *Pn* is the population on the *n*th day (or minute, or year).

For values of *k* > 1 this predicts indefinite growth with.

A better model would be one which includes a term which prevents such exponential behaviour (eventually the population will starve, for example).

An alternative model is

,

where *k* and *N* are constants.

*N* is the steady state (or equilibrium) population.

This excel print out shows how the population behaves when *k* = 0.0001 and *N* = 1000 for an initial population of 200:

This is known as the logistic equation or the Verhulst model. In this case the 1000 is the maximum size of the population.

If the population starts off at a value greater than *N* then the value of *k* gives an estimate of how quickly the population increases/decreases at the outset, i.e. it is the rate of growth of the population.

## Exercise 6

**1.** (a) Show that *Pn = Pn-1 = N* is a solution of the recurrence relation

(b) For *N* = 1 million, *k* = 0.0000001 and

(i) *P*1 = 200 000, work out the values of *P*2 and *P*3,

(ii) *P*1 = 2 million, work out the values of *P*2 and *P*3.

(c) Show that the percentage change in population from time *n* – 1 to time *n* is .

(d) Show that the change in population from time *n* − 1 to time *n* such that  is  (i.e. the rate of change of the population is  when the population has reached half the equilibrium population).

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**2.**[](http://en.wikipedia.org/wiki/File:Paramecium_caudatum_Ehrenberg,_1833.jpg)Experimental data collected on a colony of   
*Paramecium caudatum* showed that   
 *N* = 20 000 and *k* = 3.3 × 10-5 (per day).

Find the change in population from day *n* – 1 to day *n*,

(i) when *Pn* – 1 = 10 000,

(ii) when *Pn –* 1  = 15 000.

*Paramecium caudatum*

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**3.** The graph shows how the population of a colony of *Paramecium aurelia* changed over time.

The initial size of the population was 100.

Use the graph and the recurrence relation to find the values of *N* and of *k.*

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**4.** Barnacles are sea creatures which stick   
to rocks and also to the hulls of ships.

The table gives information about  
the number of barnacles in a new port.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Day | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 |
| Number  per cm2 | 2 | 3 | 6 | 10 | 11 | 28 | 48 | 68 | 70 | 73 | 70 | 74 | 76 | 80 | 81 | 78 | 77 | 76 | 78 | 78 |

(a) Plot a graph of density against time.

(b) Draw a curve of best fit through the data.

(c) Use your smooth curve to estimate the value of *N* and the value of *k.*

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**5.** Use an excel spreadsheet to show how a population changes over time using the logistic equation.

Your spreadsheet should allow you to input the initial population, the value of *N* and the value of *k* and should provide values for *n* from 2 to 30.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Quadratic sequences

A *linear sequence* is one where the *n*th term *un* = *an* + *b* where *a* and *b* are constants.

Another name for a linear sequence is an *arithmetic sequence.*

A *quadratic sequence* is one where the *n*th term *un* = *an*2 + *bn* + *c* where *a*, *b* and *c* are constants.

You can tell whether a sequence is linear by checking that the differences between successive terms are constant.

You can tell whether a sequence is quadratic by first of all working out the differences between successive terms and working out the differences of the differences (the 2nd differences).

The working should be set out in a difference table.

For example, the square numbers, *un* = *n*2 (below).

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 3 | 5 | 7 | 9 | 11 | 13 | 15 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 2 | 2 | 2 | 2 | 2 | 2 |

1st differences

2nd differences are constant

e.g. *un* = 2*n*2 + 3*n* + 5

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 10 | 19 | 32 | 49 | 70 | 95 | 124 | 157 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 9 | 13 | 17 | 21 | 25 | 29 | 33 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 4 | 4 | 4 | 4 | 4 | 4 |

1st differences

2nd differences are constant

You can extend a quadratic sequence by using a difference table.

e.g. The first five terms of a quadratic sequence are

3 6 11 18 27

Work out the next 2 terms.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 3 | 6 | 11 | 18 | 27 | *27+11 = 38* | *38+13 = 51* |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 3 | 5 | 7 | 9 | *9+2 = 11* | *11+2 = 13* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 2 | 2 | 2 | *2* | *2* |

Work out the missing 1st differences

All 2 so complete the 2nd differences row

From the table, the next two terms are 38 and 51.

## Exercise 7

**1.** Here are the first 5 terms of some sequences.

The sequences are either are linear, quadratic or neither. State which.

(a) 3 7 11 15 19

(b) 20 18 16 14 12

(c) 2 5 10 17 26

(d) 1 2 4 8 16

(e) 1 2 3 5 8

(f) 3 7 13 21 31

(g) 4 9 18 31 48

(h) 90 85 75 60 40

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**2.** In each case, find the next two terms of these quadratic sequences.

(a) 2 6 12 20 30

(b) 1 3 7 13 21

(c) 3 9 19 33 51

(d) 100 99 96 91 84

(e) 45 48 49 48 45

(f) 80 78 74 68 60

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Finding the nth term of a quadratic sequence:

***Method 1***

This uses the fact that for *un* = *an*2 + *bn* + *c* the second differences are equal to 2*a*.

e .g. Find the nth term of the quadratic sequence which starts

6 13 24 39 58

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 6 | 13 | 24 | 39 | 58 |

|  |  |  |  |
| --- | --- | --- | --- |
| 7 | 11 | 15 | 19 |

|  |  |  |
| --- | --- | --- |
| 4 | 4 | 4 |

All 4 so *a* = 2

Write the values of 2*n*2 under the original sequence and find the differences between these values and the original sequence.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *un* | 6 | 13 | 24 | 39 | 58 |
| 2*n*2 | 2 | 8 | 18 | 32 | 50 |
| *un* −2*n*2 | 4 | 5 | 6 | 7 | 8 |

*un* −2*n*2 = *n* + 3 so *un*= 2*n*2 + *n* + 3

***Method 2***

This uses the fact that any quadratic sequence can be written in the form

*un* = *p*(*n* - 1)(*n* - 2) + *q*(*n*-1) + *r* where *p*, *q* and *r* are constants

e .g. Find the nth term of the quadratic sequence which starts

6 13 24 39 58

Put *n* = 1 *u*1 = 6 = *p* × 0 + *q* × 0 + *r* so *r* = 6.

Put *n* = 2 *u*2 = 13 = *p* × 0 + *q* × 1 + *r* so 13 = *q* + 6 and so *q* = 7

Put *n* = 3, *u*3 = 24 = *p*×2×1 + *q*× 2 + 5 so 24 = 2*p* +2*q* + r and so *p* =  = 2

*un* = 2(*n* - 1)(*n* - 2) + 7(*n*-1) + 6 = 2(*n*2 - 3*n* + 2) + 7*n* - 7 + 6 = 2*n*2 - 6*n* + 4 +7*n* - 7 + 6

*un* = 2*n*2 +*n* + 3

## Exercise 8

**1.** Find the nth term of these quadratic sequences.

(a) 2 5 10 17 26

(b) 2 6 12 20 30

(c) 5 11 21 35 53

(d) 7 18 33 52 75

(e) 10 17 26 37 50

(f) 80 78 74 68 60

(g) 45 48 49 48 45

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**2.** Here is a sequence of patterns made from half centimetre squares.

The number of half centimetre squares in each pattern forms a quadratic sequence.

Find an expression, in terms of n, for the number of half centimetre squares in the nth pattern.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**3.** When two coins are thrown there is 1 way of getting two heads, HH. When three coins are thrown there are 3 ways of getting two heads HHT, HTH and THH.

(a) When four coins thrown show there are 6 ways of getting two heads.

(b) Find the number of ways of getting two heads when five coins are thrown.

(c) Find an expression, in terms of n, for the number of ways of getting two heads when *n* coins are thrown (*n* > 1).

(d) Ada throws n fair coins. Give an expression, in terms of n, for the probability she throws

(i) No heads, (ii) Exactly 1 head, (iii) Exactly 2 heads.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Past GCE Mathematics examination questions

## Exercise 9

**1.** A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £*P*.

Salary increases by £(2*T*) each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £(*P* + 1800).

Salary increases by £*T* each year, forming an arithmetic sequence.

(a) Show that the **total** earned under Salary Scheme 1 for the 10-year period is

£(10*P* + 90*T*)

**(2)**

For the 10-year period, the **total** earned is the same for both salary schemes.

(b) Find the value of *T*.

**(4)**

For this value of *T*, the salary in Year 10 under Salary Scheme 2 is £29 850

(c) Find the value of *P*.

**(3)**

**(Total 9 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**2.** A sequence of numbers *a*1, *a*2, *a*3 … is defined by



where *c* is a constant.

(a) Write down an expression, in terms of *c*, for *a*2

**(1)**

(b) Show that *a*3 = 12 − 3*c*

**(2)**

Given that 

(c) find the range of values of *c*

.

**(4)**

**(Total 7 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**3.** A boy saves some money over a period of 60 weeks.

He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60.

His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15

**(2)**

(b) Calculate the total amount he saves over the 60 week period.

**(3)**

The boy's sister also saves some money each week over a period of *m* weeks.

She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence.

She saves a total of £63 in the *m* weeks.

(c) Show that

*m*(*m* + 1) = 35 × 36

**(4)**

(d) Hence write down the value of *m*.

**(1)**

**(Total 10 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**4.** A sequence of positive numbers is defined by



(a) Find *a*2 and *a*3, leaving your answers in surd form.

**(2)**

(b) Show that *a*5 = 4

**(2)**

**(Total 4 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**5.** A 40-year building programme for new houses began in Oldtown in the year   
1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term *a* and common difference *d*.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of *d*,

**(3)**

(b) the value of *a*,

**(2)**

(c) the total number of houses built in Oldtown over the 40-year period.

**(3)**

**(Total 8 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**6.** A sequence *u1*, *u2*, *u3*, ... satisfies

*un* + 1 = 2*un* − 1, *n* ≥ 1

Given that *u*2 = 9,

(a) find the value of *u*3 and the value of *u*4,

**(2)**

(b) evaluate 

**(3)**

**(Total 5 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**7.** A sequence of numbers *a*1, *a*2, *a*3... is defined by

*an* + 1 = 5*an* − 3,     *n* ≥ 1

Given that *a*2 = 7,

(a) find the value of *a*1

**(2)**

(b) Find the value of 

**(3)**

**(Total 5 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**8.** In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001,   
170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007.

**(2)**

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

**(3)**

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred.

**(4)**

**(Total 9 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**9.** Xin has been given a 14 day training schedule by her coach.

Xin will run for *A* minutes on day 1, where *A* is a constant.

She will then increase her running time by (*d* + 1) minutes each day, where *d* is a constant.

(a) Show that on day 14, Xin will run for

(*A* + 13*d* + 13) minutes.

**(2)**

Yi has also been given a 14 day training schedule by her coach.

Yi will run for (*A* – 13) minutes on day 1.

She will then increase her running time by (2*d* – 1) minutes each day.

Given that Yi and Xin will run for the same length of time on day 14,

(b) find the value of *d*.

**(3)**

Given that Xin runs for a total time of 784 minutes over the 14 days,

(c) find the value of *A*.

**(3)**

**(Total 8 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**10.**   A geometric series has first term *a* = 360 and common ratio *r* = .

Giving your answers to 3 significant figures where appropriate, find

(a) the 20th term of the series,

**(2)**

(b) the sum of the first 20 terms of the series,

**(2)**

(c) the sum to infinity of the series.

**(2)**

**(Total 6 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**11.**   The second and fifth terms of a geometric series are 750 and −6 respectively. Find

(a) the common ratio of the series,

**(3)**

(b) the first term of the series,

**(2)**

(c) the sum to infinity of the series.

**(2)**

**(Total 7 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**12.** The third term of a geometric sequence is 324 and the sixth term is 96.

(a)  Show that the common ratio of the sequence is .

**(2)**

(b)  Find the first term of the sequence.

**(2)**

(c)  Find the sum of the first 15 terms of the sequence.

**(3)**

(d)  Find the sum to infinity of the sequence.

**(2)**

**(Total 9 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**13.**   A car was purchased for £18 000 on 1st January. On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.

**(1)**

The value of the car falls below £1000 for the first time *n* years after it was purchased.

(b) Find the value of *n*.

**(3)**

An insurance company has a scheme to cover the maintenance of the car. The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88.

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.

**(2)**

(d) Find the total cost of the insurance scheme for the first 15 years.

**(3)**

**(Total 9 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**14.**    A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05.

(a)  Show that the predicted profit in the year 2016 is £138 915.

**(1)**

(b)  Find the first year in which the yearly predicted profit exceeds £200 000.

**(5)**

(c)   Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

**(3)**

**(Total 9 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**15.**   The first three terms of a geometric series are 18, 12 and *p* respectively, where *p* is a constant.

Find

(a)    the value of the common ratio of the series,

**(1)**

(b)    the value of *p*,

**(1)**

(c)    the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

**(2)**

**(Total 4 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**16**.  The second and third terms of a geometric series are 192 and 144 respectively. For this series, find

(a)  the common ratio,

**(2)**

(b)  the first term,

**(2)**

(c)   the sum to infinity,

**(2)**

(d)  the smallest value of *n* for which the sum of the first *n* terms of the series exceeds 1000.

**(4)**

**(Total 10 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**17**. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a)  Show that the predicted adult population at the end of Year 2 is 25 750.

**(1)**

(b)  Write down the common ratio of the geometric sequence.

**(1)**

The model predicts that Year *N* will be the first year in which the adult population of the town exceeds 40 000.

(c)  Show that .

**(3)**

(d)  Find the value of *N*.

**(2)**

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e)  find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

**(2)**

**(Total 9 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_