Unit 7: Calculus to Solve Engineering Problems

Delivery guidance

This unit builds on mathematical techniques covered in the mandatory unit Engineering Principles, and hence it is suggested that it is delivered after or at the same time as *Unit 1: Mechanical Principles and/or Unit 57: Electrical and Electronic Principles*. Learners will investigate the rules and manipulation techniques of calculus and apply them to the solution of engineering problems, making them aware of the importance of understanding advanced mathematical techniques. You should encourage learners to develop their pure mathematical skills by using them in an applied environment.

You should enable learners to appreciate why correctly modelling of an engineering system using differential and/or integral calculus techniques is important: working with numbers is much more cost effective than building hardware that does not perform to specification. Your learners' algebraic manipulation, numerical accuracy and presentation skills will develop so that assignments can be presented to an agreed standard.

A large part of this unit involves the teaching and learning of mathematical techniques that are transferable and not just specific to engineering. For the first two learning aims, the unit content is structured so that your learners will initially investigate generic calculus techniques and then develop them to produce the solutions to engineering problems. The third learning aim focuses on the solution of a complex, defined, specialist engineering problem by applying thinking skills (eg reductionism) and modelling techniques (analytical and numerical).

To complete this unit, your learners will need access to a spreadsheet package. You can use a range of delivery methods in this unit, such as:

- formal teaching
- individual and small-group investigation
- structured worksheets
- case studies.

Learners will benefit from access to web-based mathematics support.

This unit, like the other mandatory units, could be delivered in a specialist context such as aeronautical, manufacturing or electrical and electronic engineering. For example, a centre wanting to deliver the mandatory units in an electrical/electronic context could explore a range of problems linked to electrical applications. However, care must be taken to ensure learners are prepared before they attempt the internal assessment activities.

You can involve local employers in the delivery of this unit if there are local opportunities to do so.

Approaching the unit

You should make your learners aware that there are two strands to the teaching and assessment of this unit. The first strand is to teach learners how to apply the principle of calculus to a range of polynomial, trigonometric, logarithmic and exponential functions. To do this will require formal classroom delivery supported by significant amounts of learner practice in the form of graded worksheets. You should work with the universally accepted variables '*x*' and '*y*' but, as the unit develops, replace them with those used by engineers, for example '*s*' (displacement) and '*t*' (time). This first strand is, in effect, pure mathematics with an engineering 'flavour'.

The second strand is the use of calculus to solve real engineering problems and much of your teaching will be in form of supporting learners as they carry out self-directed study done on an individual or small-group basis. You should guide learners to use calculus techniques when studying other units within their programme, for example, to determine the maximum bending moment in a beam or the energy discharge from an electronic capacitor. Throughout your teaching of this unit, do reinforce the notion that mathematics is a modelling process that is much more interesting and valuable than simply a set of routines to be learnt by rote. Guide your learners to correctly present what they set out on paper, be it hand-written or computer-generated; mathematics is a communicative language.

Delivering the learning aims

For learning aim A, introduce the topic by showing your learners a displacement– time graph for an object that is accelerating in a straight line with constant acceleration. Discuss how you might determine the object's instantaneous velocity and acceleration at a given time point on the graph. Remind them of the concept of gradient, which they will have already investigated when they studied *Unit 1: Mechanical Principles*. Through discussion, develop the concept of rate of change and think about other engineering situations where there are time dependent processes happening, for example heat transfer, flow of electrical charge, mechanical energy and power.

Explain to your learners that, in the initial stages of delivering this learning aim, they will learn about differential calculus manipulation techniques, the starting point being different types of mathematical function (building on what they learnt in *Unit 1: Mechanical Principles*). Follow this up with a graphical explanation of gradient, small change and limiting value (derivative) of a simple power function such as $y = x^2$. Once your learners understand the concepts of variables and rates of change, you can go on to work through the various rules of differential calculus as set out in the unit content. This will have to be done in a formal way – with initial classroom delivery followed by worksheets. Until learners are confident in applying the various rules of differentiation, for example the product and quotient rules, your learners will not be able to move on.

To make the learning aim more accessible, it is important to incorporate spreadsheet mathematics, particularly when investigating the turning points and second derivative. You will need to explain to learners how to find the maximum and minimum points of a function (eg finding the maximum and minimum bending moments for a beam) by using differential calculus (analytical method) and plotting (graphical, spreadsheet method) and then use the second derivative to confirm that the point is a maximum or minimum.

Learners can find the concept of variables tending to zero (limiting value/Leibniz notation) very difficult to accept; for a lot of the content in learning aim A (and also B) they can just mechanically apply the rules of differentiation and work with the table of standard differential coefficients. They need to be convinced of what (dy/dx or ds/dt) means – this is easily done by setting out functions in spreadsheets, plotting, and following up with 'what if' repetitive calculation. You should use animations sourced from on-line mathematics support packages to reinforce your teaching of learning aim A.

Assessment of this learning aim will be through the use of a time-constrained, controlled assignment; your learners should be given a formulae sheet and the table of derivatives. Assignment tasks are based on the application of differential calculus techniques and not just simple recall of techniques (i.e. they are not memory tests).

For learning aim B, introduce the topic by telling your learners that, in simplistic terms, integration is the 'reverse' of differentiation. Get them to have a look at the table of standard integrals and compare with the table of derivatives. Before moving on to teach the rules of integral calculus, do introduce the concept of area measurement and summation. Start by discussing how to measure the areas of regular shapes such as rectangles and triangles. Then pose the question: 'How do we measure the area of an irregular shape?'. Lead the discussion to consider splitting the shape up into a number of smaller, regular pieces or, if the outline of the shape can be defined by a mathematical function, using a summating technique based on integral calculus and/or a numerical method such as the trapezium rule. Then explain to your learners that, as with the first learning aim, before they can apply calculus techniques, they must first learn the rules. This will have to be done in a formal way, that is, by initial classroom delivery followed by individual activities using worksheets.

The types of function to be investigated will be the same as those in learning aim A; do ensure that your learners are fully competent in applying the rule of integration eg integration by parts. Having mastered the rules of integration, your learners can move on to investigate how integration is used as a summating tool; the concept of strip theory can be difficult to grasp and you may find it useful to support your delivery with animations sourced from on-line mathematics support packages. Understanding the significance of 'tending to zero' can be difficult for some learners.

As with learning aim A, in learning aim B learners can just mechanically apply the rules of integration. What you should be doing is encouraging them to investigate engineering uses for integral calculus, in particular, as a summation tool for finding the defined areas of plotted functions. To 'convince' your learners that summation using analytical calculus is valid, you will need to show them how to carry out numerical integration using spreadsheet mathematics. Summation techniques link very well to content within other units in their programme, for example mechanical/electrical energy transfer.

Assessment of this learning aim will be through the use of a time-constrained, controlled assignment for which your learners should be given a formulae sheet and the table of integrals. Assignment tasks are based on the application of integral calculus techniques and not just simple recall of techniques (i.e. they are not memory tests).

For learning aim C, you will be developing the application of thinking methods to the solution of a complex problems, that is, breaking a problem down into a set of linked manageable steps, each of which is solvable through the use of calculus (differential and integral). Your learners will have been doing this already when they were working with non-routine functions (learning aims A and B) and in other programme units. For this learning aim, you will probably only have to provide a small amount of formal input. Most of your support for learners will take the form of providing guidance as they investigate a complex engineering problem and mathematically model it.

Start by reviewing the use of reductionism, synectics (idea connection) and logical thinking; this does not have to be directly related to mathematics as the principles are generic. A group discussion led by you is a good way to cover this topic. In preparation for assessment of this learning aim, you should provide your learners with one or two well defined case studies and ask them to come up with solutions to be presented to the group and evaluated.

Assessment of this learning aim will be through the use of a time-constrained, controlled assignment for which your learners should be given a formulae sheet, the table of integrals and a pre-release case study to set the scene.

Learning aim	Key content areas	Recommended assessment approach
A Examine how differential calculus can be used to solve engineering problems	**A1** Functions, rate of change, gradient **A2** Methods of differentiation **A3** Numerical value of a derivative **A4** Second derivative and turning points	A report containing the results of learners' analysis and calculation, carried out under controlled conditions.
B Examine how integral calculus can be used to solve engineering problems	**B1** Integration as the reverse/inverse of differentiation **B2** Integration as a summating tool **B3** Numerical integration	A report containing the results of learners' analysis and calculation, carried out under controlled conditions.
C Investigate the application of calculus to solve a defined specialist engineering problem	**C1** Thinking methods **C2** Mathematical modelling of engineering problems **C3** Problem definition and proposed solution **C4** Solution implementation	A report containing the results of learners' analysis, planning and calculation, carried out under controlled conditions.

Assessment guidance

This unit is internally assessed and you should use three time-constrained assignments. There are authorised assignments for this unit, each covering one learning aim. If you choose to use your own assignment briefs, it is essential that each one covers a complete learning aim and is not split into sub-tasks for each criterion.

Each learner must independently generate their own evidence presented as a portfolio. There is no requirement for them to word process their mathematical manipulations; for most learners, hand-written will be the most time efficient method of presentation. Repetitive numerical evidence (eg spreadsheets and graphs) is better when presented in printed form. It is important that learner evidence is fully authenticated. For learning aims A and B, evidence should be based on fixed tasks; for learning aim C, learners may wish to discuss with you their solution strategy for an identified engineering problem and this could involve the use of an observation record to support their evidence.

Getting started

This gives you a starting place for one way of delivering the unit, based around the recommended assessment approach in the specification.

Unit 7: Calculus to Solve Engineering problems

Introduction

Begin by introducing the unit to your learners through a group discussion exploring the reasons why mathematics is such an important tool in supporting all aspects of engineering. Move your learners away from the simplistic reasons for learning mathematical skills and talk to them about the wider and much more exciting aspects of mathematical modelling. Tell them that mathematical modelling is something we all do every day, for example money budgeting, and then open out the discussion to think about why it is crucial to design engineering products so that they perform to specification.

Explain to your learners that calculus is a subset of mathematics that is of particular use to engineers because it allows them to investigate time-based systems, for example aircraft performance. As an example, you could talk in general terms about the design and development of a typical new commercial aircraft – everything is worked out using mathematical modelling so that, when the first one rolls off the production line, the pilot knows it will handle exactly as predicated in the simulator. Mathematics, as an engineering tool, has the same significance as the most complex CNC machining centre.

It is important to motivate learners particularly in the early stages of this unit which can seem dry some of the time.

Learning aim A – Examine how differential calculus can be used to solve engineering problems

- Using a formal presentation you could review types of function, making reference to *Unit 1: Mechanical Principles and Unit 57: Electrical and Electronic Principles*.
- You could run a short quiz about types of function to identify learners who may require additional support.
- You could demonstrate access to the ‘mathcentre’ website (see Resources) and ask learners to pick a level 2/3 topic. For some topics there are four options to choose from: ‘teach yourself’, ‘video’, ‘practice and revision’ and ‘test yourself’. As a fun group exercise, do a couple of the algebra tests.
- You should now carry out formal teaching of unit content A1 and A2, supporting your teaching with self-directed learner activities, such as worksheets.
- When your learners have mastered the basic concepts of differential calculus, you can move on the more interesting applications of differentiation in engineering (content A3 and A4). This will require formal instruction from yourself but there should also be an emphasis on learner-centred activities.
- In preparation for the assessment activity, you could guide your learners through:
  - plotting the graphs of given functions either by hand or by using a spreadsheet and measuring their gradients at different points
  - using the first derivative for each of the functions to calculate the gradient at different points and comparing these with the graphically found values

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    - finding the first and second derivatives for given functions
    - finding the turning points of polynomial and trigonometric functions
    - investigating the use of maximum and minimum theory to solve engineering problems, for example optimising the dimensions of a container so that its internal volume is at a maximum but its surface area is a minimum.
- Once your learners have established the most effective dimensions for the container, it is worth repeating the exercise using numerical analysis. Using a spreadsheet, set up formulae for the volume and surface area, step through a range of dimensions, plot out the figures and find where the maximum occurs – does it agree with the value found by maximum/minimum theory?

Learning aim B – Examine how integral calculus can be used to solve engineering problems

- Using a formal presentation, you could review types of function, making reference to *Unit 1: Mechanical Principles, Unit 57: Electrical and Electronic Principles* and learning aim A.
- You could talk through the table of standard integrals and compare its layout with the table of differentials.
- You could give your learners a brief worksheet of polynomial functions, ask them to differentiate and then do the reverse action of integration; then talk about the constant of integration.
- Your learners could be reminded about the mathcentre website and the link to integral calculus support.
- When your learners have mastered the basic concepts of integral calculus, you can move on the more interesting applications of it to engineering (content B2 and B3). This will require formal instruction from yourself but there should also be an emphasis on learner-centred activities.
- In preparation for the assessment activity you could guide your learners through:
    - finding indefinite and definite integrals for a range of given functions
    - investigating the use of numerical and analytical integration methods to solve engineering problems
    - finding the properties of periodic functions using integral calculus
    - finding the areas of shapes with outlines defined by mathematical functions and applying this to engineering situations (eg determination of work done/energy transfer).

Learning aim C – Investigate the application of calculus to solve a defined specialist engineering problem
● This learning aim builds on the knowledge and understanding from learning aims A and B. You could use formal teaching to introduce thinking methods but most of the topic content is best covered as self-directed investigation. ● In preparation for the assessment activity, you could guide your learners through: o using thinking methods to analyse an identified engineering problem (eg breaking the problem down into a series of manageable elements, and producing a specification) o preparing a valid proposal for solving the problem and presenting the proposal to an audience o producing mathematical models for the identified elements of the problem solving the problem using both differential and integral calculus methods to produce answers for each of the elements of the problem and critically analyse the results o presenting the solution as a formal presentation. ● The problem to be solved could be taken from another unit within the BTEC programme to suit the learner's specialist area of study, for example: o the stress analysis of a load carrying structure where differentiation is applied to determine maximum and minimum bending moments and integration is used to determine the '*I*' value for a beam (i.e. the moment of inertia) with a non- regular cross section. o analysis of the signal frequency from op-amp differentiator and integrator circuits. o Learners should be encouraged to incorporate spreadsheet maths into the presentation.

Details of links to other BTEC units and qualifications, and to other relevant units/qualifications

Pearson BTEC International Level 3 Qualifications in Engineering:

- *Unit 1: Mechanical Principles*
- *Unit 15: Electrical Machines*
- *Unit 17: Power and Energy Electronics*
- *Unit 20: Analogue Electronic Circuits*
- *Unit 27: Static Mechanical Principles in Practice*
- *Unit 28: Dynamic Mechanical Principles in Practice*
- *Unit 29: Principles and Applications of Fluid Mechanics*
- *Unit 31: Thermodynamic Principles and Practice*
- *Unit 48: Aircraft Flight Principles and Practice*
- *Unit 57: Electrical and Electronic Principles*

Resources

There are no special resources required for this unit but your learners will benefit from access to maths support websites.

In addition to the resources listed below, publishers are likely to produce Pearson-endorsed textbooks that support this unit of the BTEC International Level 3 Qualifications in Engineering. Check the Pearson website (http://qualifications.pearson.com/en/support/published-resources.html) for more information as titles achieve endorsement.

Textbooks

- Boyce A, Cooke E, Jones R, Mantovani B, Roberts D and Weatherill B – *BTEC Level 3 National Engineering Student Book* (Pearson, 2010)
  ISBN 9781846907241.
  This contains learner support activities.
- Bird J – *Engineering Mathematics* (Elsevier Science & Technology, 2007) ISBN 9780750685559.
  Useful exercises to complement teaching.
- Fuller A, Greer A and Taylor G W – *BTEC National Mathematics for Technicians, 3rd Edition* (Nelson Thornes, 2004) ISBN 9780748779499. Useful additional resource.
- Tooley M and Dingle L– *BTEC National Engineering, 2nd Edition* (Elsevier Science & Technology, 2007) ISBN 9780750685214.
  A good course textbook.
- Greer A and Taylor G W – *BTEC National NIII Mathematics for Technicians* (Nelson Thornes, 1991) ISBN 9780859509329.
  Maths activities.
- Greer A and Taylor G W – *BTEC National Further Mathematics for Technicians* (Nelson Thornes, 2005) ISBN 9780748794102. Higher-level maths activities.

Websites

- www.mathcentre.ac.uk/students/topics
  The mathcentre is a free and robust resource. It provides easy access to topic reviews, revision worksheets, tests and animations.
- www.ncetm.org.uk
  The National Centre for Excellence in the Teaching of Mathematics website – this site provides ideas and support for teachers to enhance mathematics teaching.

Pearson is not responsible for the content of any external internet sites. It is essential for tutors to preview each website before using it in class so as to ensure that the URL is still accurate, relevant and appropriate. We suggest that tutors bookmark useful websites and consider enabling learners to access them through the school/college intranet.