Mathematics Scheme of Work Year 9 – Exemplar Unit 11 (Algebra 5)

Unit Objectives

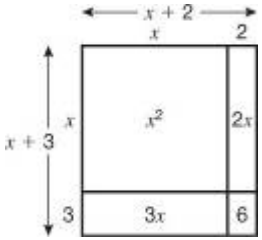
Pupils will learn to:

- Simplify expressions by multiplying out single brackets and collecting like terms
- Factorise expressions involving a single bracket
- Multiply out double brackets
- Use the difference of two squares to do mental calculations
- Factorise a quadratic expression
- Use factorisation to simplify algebraic fractions
- Substitute integers into formulae and use the correct order of operations
- Write expressions and construct formulae
- Use a formula and find the value of a letter which is not the subject of a formula
- Change the subject of a formula
- Understand inequality signs
- Identify and represent inequalities on a number line
- Solve inequalities
- Show inequalities graphically and link to practical problems

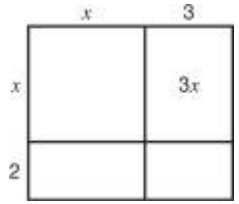
Teaching Resources
Level Up Maths (6–8), Longman Active Maths 8 Pupil Book (available from http://pearson.vrvbookshop.com/)
Mathematical language expanding, factorising, highest common factor, identity, like terms, square, expanding, factor, factorising, quadratic expression, generalise, in terms of, order of operations, subject, substitute, equivalent, greater than or equal to, inequality, less than, less than or equal to

Objectives	Teaching	Resources
Simplify expressions by multiplying out single brackets and collecting like terms Factorise expressions involving a single bracket	If necessary, remind pupils of the meaning of lowest common multiple (LCM) and highest common factor (HCF). Ask pupils to find the LCM of each of these pairs: • 3, 7 • 7, 12 • 8, 12 Emphasise its relationship to the HCF: $\text{LCM}(a, b) = \frac{ab}{\text{HCF}(a, b)}$ Now apply the same principles algebraically to the following pairs: a, b; a^2, b; a^2, ab; and x^2y, xy^2. Explain that pupils will be brushing up on some key skills: expanding brackets, factorising and adding algebraic fractions. Review multiplying out brackets. Display these expressions and ask pupils to work out each one, noting what is the same and what is different as they move from one to the other. Pupils may need to be reminded that a negative sign in front of a bracket switches the sign of each term within it. • $3(x + 2) + 2(2x + 1)$ • $3(x + 2) - 2(2x + 1)$ • $3(x + 2) - 2(2x - 1)$ • $3(x + 2) - 2(1 - 2x)$ Once the brackets have been expanded, emphasise the need to look at the sign in front of each term carefully when collecting like terms. Ask pupils to multiply out a bracket of their choice and to put the answer on the back a piece of paper. Encourage pupils to extend to examples such as $6x^5y^6(3x^2z + 2y^3)$, which require use of the laws of indices in order to expand. Display a mixture of the unfactorised and factorised sides of the paper at the front of the class. *How can you work out what is on the other side of the paper?*	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.1* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*

	Emphasise that the processes of expanding and factorising are inverse (opposite) processes, for example expand $7(2x - 3)$ to $14x - 21$ and then factorise back to $7(2x - 3)$. Work through the full factorisation of an expression such as $18x^2 + 27y$, emphasising the need to find the highest common factor of each number. Give pupils more examples to practise as appropriate.	
Factorise expressions involving a single bracket	Display the expression $x^2y^5 + x^3y^2$. *How can you factorise this expression fully?* (Find the highest power of each letter that appears in every term and put it in front of the bracket.) Work through the full factorisation of an expression which has numeric and algebraic factors, such as $18x^5y^{11} + 24x^8y^8$. Draw out the process of finding the highest numeric factor and then the highest common factor (highest power) of each letter. Ask pupils to write as many expressions as they can that simplify to $4x + 3y$. This is a good opportunity for pupils to build up skills from basic collection of like terms to using more than one bracket. Encourage pupils to experiment with negative signs by prompting them to change a sign and then to work out how to compensate for this in order to give the same final answer, for example $4(x - y) + 7(y - 2) + 14$. Write $5 \times 2y + 3 + 3 \times y + 5 \times y$. Ask pupils to insert one or two pairs of brackets anywhere to form different expressions and to simplify each expression. After a few minutes, draw findings together as a class.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.1* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*
Multiply out double brackets	Ask pupils to think of a whole number, then to add or subtract another whole number and to square the answer. Model how to form an equation (eg starting with x, add 7 to get 30, square it to get 900, giving $(x + 7)^2 = 900$). Pupils work in teams, exchanging questions for the other team to solve for x. Recall that 900 has both positive and negative square roots, leading to two solutions for x. Ask pupils to draw a rectangle with dimensions $x + 3$ and $x + 2$. *How can you work out the area of the rectangle?* (Split it vertically and horizontally and work out the four separate areas to get $x^2 + 3x + 2x + 6$.)	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.2* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*



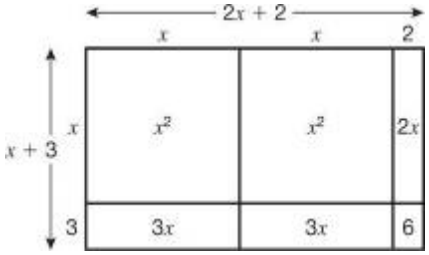
Ask pupils to complete this diagram:



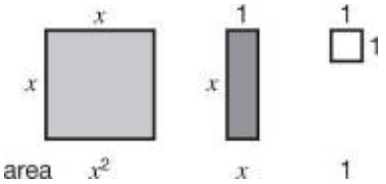
Repeat for $(x + 3)(2x + 2)$

$= (2x^2 + 2x + 6x + 6)$

$= 2x^2 + 8x + 6$



Multiply out double brackets Use the difference of two squares to do mental calculations	Draw a square with side length $x + 4$ and find its area ($x^2 + 8x + 16$). Ask pupils to make up further examples and describe what they notice (i.e. the 4 is doubled to give the 8 and squared to give the 16). Generalise to $(x + y)(x + y) = x^2 + 2xy + y^2$, noting that y is doubled to give $2y$ and squared to give y^2. *Can you predict what $(x - y)(x - y)$ will be? $(x - y)(x + y)$?* Discuss using the difference of two squares to work out the product of two odd or two even numbers mentally, eg for 38×42: ● Find the middle number (40). ● Find the gap between the middle number and the numbers to be multiplied (2). ● So $38 \times 42 = (40 + 2)(40 - 2)$ $= 40^2 - 2^2$ $= 1600 - 4 = 1596$ Reinforce the result $(x - y)(x + y) = x^2 - y^2$ as the difference between two squares. Try out further examples as a class (eg $22 \times 28 = (x + y)(x - y) = (25 + y)(25 - y) = (25 + 3)(25 - 3) = 25^2 - 3^2 = 625 - 9 = 616$). Give pupils further examples to practise as necessary.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.2* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*
Factorise a quadratic expression Use factorisation to simplify algebraic fractions	Draw a regular octagon. *The perimeter of this shape is 16 + 8x, with all measurements in centimetres. What is the length of each side?* (2 + x cm) Write up $24x + 16$. *What regular shapes have this perimeter? Draw them and label one side length*. (Limiting to whole number coefficients: square, side $6x + 4$; octagon, side $3x + 2$) Draw out that each side length multiplied by the number of sides represents a factorisation of $24x + 16$, but that only the expression for the maximum number of sides is fully factorised. Repeat for examples involving two unknowns, for example $24x + 30y + 12$ (hexagon, side length $4x + 5y + 2$). Use templates to introduce factorising quadratics. Provide pupils with pieces that have areas of x^2, x and 1. Pupils need access to a pile of each type.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.3* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*



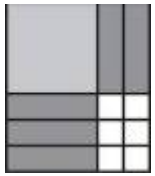
These are used to make and rearrange expressions; in this case to factorise expressions. The area must be conserved since the pieces remain the same; this prevents pupils from interchanging $2x$ with x^2, etc.

Hold up x^2 and x. *What is the expression for the total area?* ($x^2 + x$) *How can you write this using brackets*? ($x(x + 1)$) Hold the large square against the rectangle and emphasise that the new rectangle has dimensions x and $x + 1$ and so has area $x(x + 1)$.

Ask pupils to hold up pieces with a total area of $x^2 + 2x + 1$ and then to arrange them into a square.

What is the side length of the square? ($x + 1$) *How can you write its area using brackets?* ($(x + 1)^2$)

Ask pupils to hold up template pieces with a total area of $x^2 + 5x + 6$, and then to arrange them to form a rectangle:

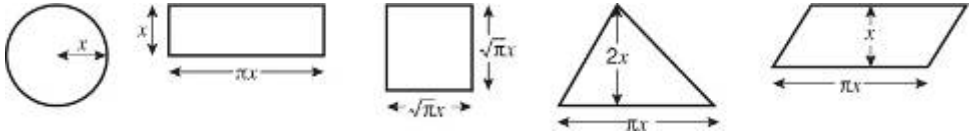


What are the side lengths of the rectangle? ($x + 2$ and $x + 3$) *How can you write its area using brackets?* ($(x + 2)(x + 3)$)

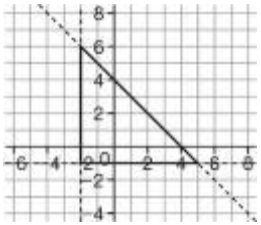
Ask pupils to create further quadratic expressions to factorise by forming rectangles from the templates. Pupils then swap pieces, write down the unfactorised expression and rearrange into a rectangle to factorise.

Write up the following from the previous lesson.

$$(x + a)(x + a) = x^2 + 2ax + a^2$$

	$(x + a)(x - a) = x^2 - a^2$ Discuss how to use these identities to factorise expressions such as $x^2 + 8x + 16$. Work informally to draw out that $(x + a)(x + b) = x^2 + (a + b)x + ab$ and that this works irrespective of signs.	
Substitute integers into formulae and use the correct order of operations Write expressions and construct formulae	Draw a circle and label the radius x. *What is the circle's area?* (πx^2) Pupils work as a class or in pairs to draw as many shapes as possible with the same area as the circle.  Draw a rectangle on the board, label the sides x and y. Ask the class to write down its area and perimeter. ($A = xy$ and $P = 2(x + y)$) *What is x if $A = 156$ and $y = 13$?* ($156 = x \times 13$, so $x = 156 \div 13 = 12$) *What is y if $P = 46$ and $x = 7$?* ($46 = 2(7 + y)$, so $46 \div 2 = 7 + y$, $46 \div 2 - 7 = y$, $y = 16$) Draw another rectangle. *If the length is four more than the width, how many letters are needed to describe the perimeter or area?* (one letter, because once one of the dimensions is known, the other can be worked out from it). Write down the area and perimeter. ($A = x(x + 4)$, $P = 4x + 8$) Draw a square with a perimeter equal to that of this rectangle. *What is its side length and area? ($P = 4x + 8 = 4(x + 2)$, so the side length is $x + 2$ and the area $A = (x + 2)^2$)* Ask pupils to find x for a square with a given area. Repeat as required.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.4* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*
Write expressions and construct formulae Use a formula and find the value of a letter which is not the subject of a formula	Explain that a rectangular shape is to be enclosed by fencing. A firm charges INR100 per metre of fencing, INR60 per post and a planning cost of INR65. Ask pupils to suggest total lengths of fencing. Work as a class to find the total price and then begin to generalise. *What is the formula for the price C in rupees of fencing around a rectangle of dimensions a metres and b metres, requiring p posts?* ($C = 200(a + b) + 60p + 65$) Ask pupils to replace the prices with letters; for example f for the price per metre, t for the price per post and l for the planning price, to give $C = 2f(a + b) + pt + l$.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.4* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*

	Ask pupils to write down their own values for a, b, p, t and l and to work out C. Ask some pupils to give all but one of their values for the class to find the remaining one. This practises finding a value which is not the subject in a familiar context.	
Change the subject of a formula	Display the following table three times. In the top-left hand corner, put $\times$ in the first table, $\div$ in the second table, and + in the third table. Complete as a class or in pairs. Once complete, ask pupils to explain why the results in the second and third columns are reversed in the multiplication and division grids. $\begin{array}{\|c\|c\|c\|c\|}\hline & \frac{a}{2} & \frac{b}{3} & \frac{3}{b} \\ \hline \frac{a}{2} & & & \\ \hline \frac{b}{3} & & & \\ \hline \end{array}$ Write an equation on the board, such as $5x + 2 = 17$. Solve it as a class. Make explicit the operation that is performed on both sides at each stage. Now replace one of the numbers with a letter and confirm the operation each time. Repeat for an equation with x on both sides, such as $5x + y = 39 + 2x$, and then replace the coefficients of x with letters. Draw out that pupils need to factorise in order to express the number of xs they have. Ask the class for the formula for the volume of a cylinder ($V = \pi r^2 h$). Now ask them to work in pairs to choose values for r and h and work out the volume. Working in pairs or as a class, ask pupils to make each variable the subject of the formula and substitute the other two variables to check the result.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.5* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*

Understand inequality signs Identify and represent inequalities on a number line Solve inequalities Show inequalities graphically and link to practical problems	Draw these three straight lines enclosing a triangle on a grid:  *What are the coordinates of the vertices?* ((–2, –1), (5, –1), (–2, 6)) *How many units are the base and height of the triangle?* (both 7) *What is the perimeter to the nearest unit?* (hypotenuse is $\sqrt{98} \approx 10$ units, so perimeter is 24 units) *What are the equations of the lines which enclose the triangle?* ($y = -1$; $x = -2$; $y + x = 4$) Draw a number line on the board. Use coloured pens to show different inequalities. Ask the class to describe what has been drawn. Make sure they know that a filled-in circle includes the number and corresponds to 'or equal to'. Display $-4 \leq x < 1$ on a number line. Ask the class to work out what the inequality is. *What integer (whole number) solutions are there?* (–4, –3, –2, –1, 0) Now write down $3x + 2 < 23$. *How can you work backwards from this inequality to the number line?* Draw out that this is called solving the inequality and that the steps are similar to those involved in solving an equation.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.5* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*

Show inequalities graphically and link to practical problems	Ask a pupil to sketch the graph of $y = x^2$. *What values of x satisfy $x^2 \leq 16$?* Highlight -4 to 4 on the x-axis. Explain that pupils are now going to look at inequalities that use two variables. Ask pupils to sketch the lines $y = x$, $y = 5$ and $x = 3$. Discuss as a class how to describe the triangle that these lines enclose and write up the inequalities $y > x$, $y \leq 5$, $x \geq 3$. Look at points within the triangle and confirm that the inequalities are satisfied. Point out that inequalities which do not include points on the line, such as $y \geq x$, are shown by a dashed rather than a solid line. Ask a pupil to draw three straight lines which enclose a right-angled triangle on a coordinate grid. Give the class a few minutes to write down the equation of each line and then to write down the inequalities that describe the enclosed area.	*Level Up Maths (Levels 6–8) Unit 11* *Teaching and Assessment Pack 11.6* *Pupil Book 6–8* *LiveText CD-ROM* *Longman Active Maths 8 Pupil Book*

End of Unit Test

Algebra 5

(Year 9 Unit 11)

Answer the questions below.

1. Expand each bracket and simplify each expression. (4 marks)

   a) $3a^2(a^4 1\ b^2) - 5b^2(a^2 - 1)$

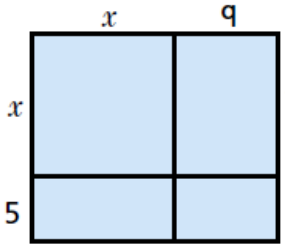
   b) $x^4(2x - 5) + 2x^2(7 + 3x^2 - 2x^3)$

   c) $7s^3(2s - t) - st(3s + 5s^2)$

   d) $3x^2(x\ \ 2x^2) - 2x(x^2 - 4x^3)$

2. Look at the diagram of the blue rectangle below.

   a) Write down the side lengths of the blue rectangle. (1 mark)



   b) Write down the area of the blue rectangle using brackets. (1 mark)

   c) Write down the area of the blue rectangle without using brackets. (1 mark)

3. Multiply out the brackets for each expression and then simplify. (4 marks)

   a) $(2x + 15)(2x + 15)$

   b) $(x + y)(x - y)$

   c) $\frac{(x + y)^2 - (x - y)^2}{4}$

   d) $(x + 8)(x - 8)$

4. Factorise each of these expressions. (4 marks)

a) $x^2 + 8x + 15$

b) $x^2 + 8x + 16$

c) $x^2 - 10x + 21$

d) $x^2 - 10x + 25$

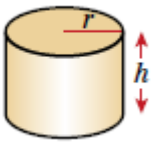
5. This fraction is made up of two different quadratic expressions.

$$\frac{x^2 + 3x}{x^2 + 5x + 6}$$

a) Factorise the top and bottom of the fraction separately. (1 mark)

b) Simplify by dividing the top and bottom by the common factor. (1 mark)

6. The surface area S of a cylinder is given by the formula $S = 2\pi r(r + h)$, where r is its radius and h is its height.



a) Work out the surface area when r = 5cm and h = 12cm. (1 mark)

b) Work out the height if S = 192cm^2 and r = 3cm. (1 mark)

c) Work out the radius to the nearest centimetre if the cylinder has a height of 7cm and a surface area of 1000cm^2. (1 mark)

7. The diagram shows a semicircle.

a) Write a formula for its perimeter P in terms of its radius r. (1 mark)

b) Write a formula for its area A in terms of its radius r. (1 mark)

c) If the area is 50cm^2, use both your formulae to find the perimeter. (1 mark)

8. Solve the equation $5x + 7 = 2x + 22$. (1 mark)

9. Rearrange this formula by filling in the gaps. (1 mark)

$6x + 7 = 4x + y$

$2x + 7 =$ _____

$2x =$ _____

$x =$ _____

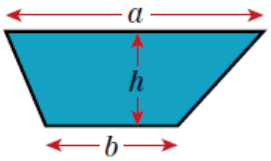
10. Make *r* the subject of each formula. (2 marks)

a) $A = \pi r^2$

b) $V = \pi r^2 h$

11. This is the formula for the area of a trapezium.

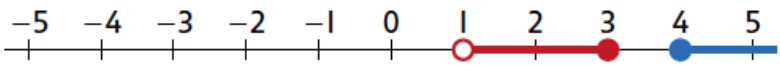
$$A = \frac{1}{2}(a + b)h$$



a) Make *h* the subject of the formula. (1 mark)

b) Make *a* the subject of the formula. (1 mark)

12. Look at this number line.



a) Write down the inequality shown by the blue line. (1 mark)

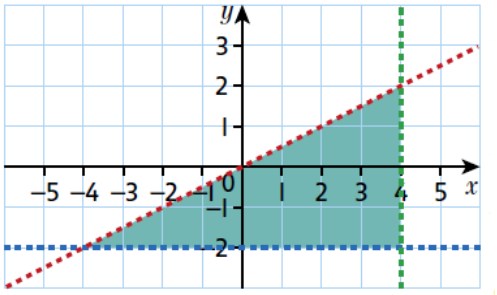
b) Write down the inequality shown by the red line. (1 mark)

c) Draw a number line to show the inequality $x < -4$. (1 mark)

d) Draw a number line to show the inequality $-3 \leq x < 0$. (1 mark)

13. Look at the graph below.

a) Write down the equation of each dotted line. (1 mark)



b) Write down three inequalities which are satisfied in the shaded region. (1 mark)

[Total 35 marks]

Mark Scheme for End of Unit Test

Algebra 5

(Year 9 Unit 11)

This test should be set as soon as possible after pupils have finished their work on the unit. They should work independently and should need about 30 minutes to complete the test.

The aim of the test is to find out what pupils have learnt and understood as a result of their experience of studying the unit. The test is based on information and experiences they should have met during their work on the unit, so they should approach the test with confidence.

The teacher should be looking in their marking to award marks, not withhold them. The purpose at this stage is to give pupils the confidence that they can recall and understand mathematical knowledge and concepts.

Marking Guide

1. Expand each bracket and simplify each expression. (4 marks)

a) $3a^2(a^4\ 1\ b^2) - 5b^2(a^2 - 1)$ $3a^6 - 2a^2b^2 + 5b^2$

b) $x^4(2x - 5) + 2x^2(7 + 3x^2 - 2x^3)$ $14x^2 + x^4 - 2x^5$

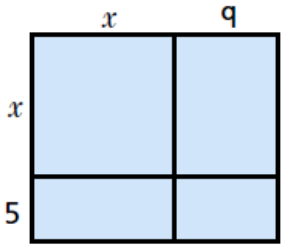
c) $7s^3(2s - t) - st(3s + 5s^2)$ $14s^4 - 12s^3t - 3s^2t$

d) $3x^2(x\ \ 2x^2) - 2x(x^2 - 4x^3)$ $x^3 + 2x^4$

2. Look at the diagram of the blue rectangle below.

a) Write down the side lengths of the blue rectangle. (1 mark)

$x + 5, x + 9$



b) Write down the area of the blue rectangle using brackets. (1 mark)

$(x + 5)(x + 9)$

c) Write down the area of the blue rectangle without using brackets. (1 mark)

$x^2 + 14x + 45$

3. Multiply out the brackets for each expression and then simplify. (4 marks)

a) $(2x + 15)(2x + 15)$ $4x^2 + 60x + 225$

b) $(x + y)(x - y)$ $x2 - y2$

c) $\frac{(x + y)^2 - (x - y)^2}{4}$ xy

d) $(x + 8)(x - 8)$ $x^2 - 64$

4. Factorise each of these expressions. (4 marks)

a) $x^2 + 8x + 15$ $(x + 5)(x + 3)$

b) $x^2 + 8x + 16$ $(x + 4)(x + 4)$

c) $x^2 - 10x + 21$ $(x + 7)(x + 3)$

d) $x^2 - 10x + 25$ $(x + 5)(x + 5)$

5. This fraction is made up of two different quadratic expressions.

$$\frac{x^2 + 3x}{x^2 + 5x + 6}$$

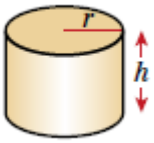
a) Factorise the top and bottom of the fraction separately. (1 mark)

$$\frac{x(x + 3)}{(x + 3)(x + 2)}$$

b) Simplify by dividing the top and bottom by the common factor. (1 mark)

$$\frac{x}{x + 2}$$

6. The surface area S of a cylinder is given by the formula $S = 2\pi r(r + h)$, where r is its radius and h is its height.



a) Work out the surface area when r = 5cm and h = 12cm. (1 mark)

$S = 170\pi cm^2 = 534cm^2$

b) Work out the height if S = 192cm^2 and r = 3cm. (1 mark)

$\underline{h = 32 - 3\pi}$ cm = 7.2cm
π

c) Work out the radius to the nearest centimetre if the cylinder has a height of 7cm and a surface area of 1000cm^2. (1 mark)

$r = 9.6cm$

7. The diagram shows a semicircle.

a) Write a formula for its perimeter P in terms of its radius r. (1 mark)

$P = \pi r + 2r$

b) Write a formula for its area A in terms of its radius r. (1 mark)

$\frac{1}{2}\pi r^2$

c) If the area is 50cm^2, use both your formulae to find the perimeter. (1 mark)

r = 5.64cm, so P = 29cm

8. Solve the equation $5x + 7 = 2x + 22$. (1 mark)

$x = 5$

9. Rearrange this formula by filling in the gaps. (1 mark)

$6x + 7 = 4x + y$

$2x + 7 = y$

$2x = y - 7$

$x = \underline{y - 7}$
2

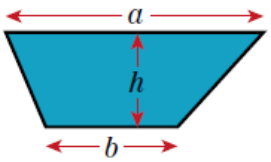
10. Make *r* the subject of each formula. (2 marks)

a) $A = \pi r^2$ $\quad r = \sqrt{\frac{A}{\pi}}$

b) $V = \pi r^2 h$ $\quad r = \sqrt{\frac{V}{\pi h}}$

11. This is the formula for the area of a trapezium.

$$A = \frac{1}{2}(a + b)h$$



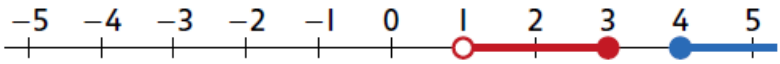
a) Make *h* the subject of the formula. (1 mark)

$h = \frac{2A}{a + b}$

b) Make *a* the subject of the formula. (1 mark)

$a = \frac{2A}{h} - b$

12. Look at this number line.



a) Write down the inequality shown by the blue line. (1 mark)

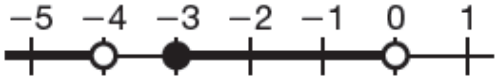
$x \geq 4$

b) Write down the inequality shown by the red line. (1 mark)

$1 < x \leq 3$

c) Draw a number line to show the inequality $x < -4$. (1 mark)
(See below)

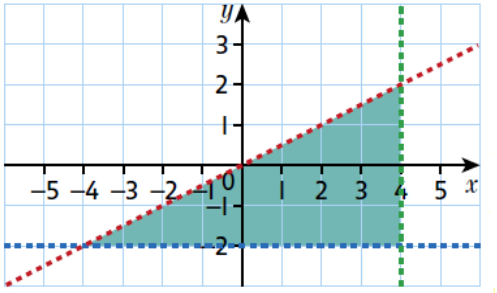
d) Draw a number line to show the inequality $-3 \leq x < 0$. (1 mark)
(See below)



13. Look at the graph below.

a) Write down the equation of each dotted line. (1 mark)

$y = \frac{1}{2}x,\ x = 4,\ y = -2$



b) Write down three inequalities which are satisfied in the shaded region. (1 mark)

$y < \frac{1}{2}x,\ x < 4,\ y > -2$

[Total 35 marks]