Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number

Candidate Number

Time 2 hours

Paper reference **4PM1/01**

Further Pure Mathematics

PAPER 1

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \dfrac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \dfrac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The quadratic equation

$$3(k + 2)x^2 + (k + 5)x + k = 0$$

has real roots.

Find the set of possible values of k.

(6)

(Total for Question 1 is 6 marks)

2 Angle α is acute such that $\cos\alpha = \frac{3}{5}$

Angle β is obtuse such that $\sin\beta = \frac{1}{2}$

(a) Find the exact value of

(i) $\tan\alpha$

(ii) $\tan\beta$

(3)

(b) Hence show that

$$\tan(\alpha + \beta) = \frac{m\sqrt{3} - n}{n\sqrt{3} + m}$$

where m and n are positive integers whose values are to be found.

(3)

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Question 2 continued

(Total for Question 2 is 6 marks)

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3 A curve C has equation $y = \dfrac{ax - 3}{x + 5}$ where a is a constant and $x \neq -5$

The gradient of C at the point on the curve where $x = 2$ is $\dfrac{18}{49}$

(a) Show that $a = 3$

(3)

Hence

(b) write down an equation of the asymptote to C that is

(i) parallel to the x-axis,

(ii) parallel to the y-axis,

(2)

(c) find the coordinates of the point where C crosses

(i) the x-axis,

(ii) the y-axis.

(2)

(d) Sketch the curve C, showing clearly its asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

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Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4 The nth term of an arithmetic series is u_n where

$$u_n = (n + 1)\ln 4$$

Given that the sum of the first n terms of the series is S_n

show that $S_n = \ln 2^{(n^2 + an)}$ where a is an integer whose value is to be found.

(5)

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Question 4 continued

(Total for Question 4 is 5 marks)

5 (a) Expand $(1 + ax)^n$ in ascending powers of x up to and including the term in x^3

Express each coefficient of x in terms of a and n where a and n are constants and $n > 2$

(2)

The coefficient of x is 15 and the coefficient of x^2 is equal to the coefficient of x^3

(b) Find the value of a and the value of n.

(6)

(c) Find the coefficient of x^3

(2)

Question 5 continued

(Total for Question 5 is 10 marks)

6 (a) Show that $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(3)

The quadratic equation $x^2 - 7kx + k^2 = 0$, where k is a positive constant, has roots α and β where $\alpha > \beta$

(b) Show that $\alpha - \beta = 3k\sqrt{5}$

(3)

(c) Hence form a quadratic equation with roots $\alpha + 1$ and $\beta - 1$

Give your equation in the form $x^2 + px + q = 0$ where p and q should be given in terms of k.

(4)

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Question 6 continued

Turn over

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7 The curve C has equation $y = \dfrac{x}{x^2 + 4}$

(a) Using calculus, find the coordinates of the stationary points on C.

(5)

(b) Show that $\dfrac{d^2y}{dx^2} = \dfrac{2x(x^2 - 12)}{(x^2 + 4)^3}$

(4)

(c) Hence, or otherwise, determine the nature of each of these stationary points.

(2)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

Turn over

8 Given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

(a) find the value of n.

(3)

Given that $\log_p x = 3$ and $\log_p y - 3\log_p 2 = 4$

(b) (i) express x in terms of p,

(1)

(ii) express xy in terms of p.

(4)

Question 8 continued

Turn over

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9 Find an equation of the normal to the curve with equation

$$y = (x^3 - 2x)e^{(1-x)}$$

at the point on the curve with coordinates $(1, -1)$

(5)

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Question 9 continued

(Total for Question 9 is 5 marks)

10

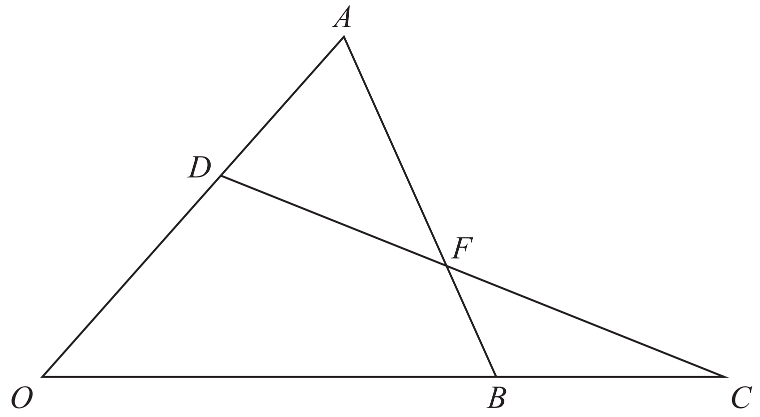

Figure 1

Figure 1 shows triangle *OAB* and triangle *OCD*.

$$\overrightarrow{OA} = 5\mathbf{p} \quad \overrightarrow{AB} = 3\mathbf{q} \quad \overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB} \quad \overrightarrow{OD} = \frac{3}{5}\overrightarrow{OA}$$

(a) Find $\overrightarrow{DC}$ as a simplified expression in terms of **p** and **q**.

(3)

The line *DC* meets the line *AB* at *F*.

(b) Using a vector method, find $\overrightarrow{OF}$ as a simplified expression in terms of **p** and **q**.

(7)

The point *G* lies on *OB* such that *FG* is parallel to *AO*.

(c) Using a vector method, find $\overrightarrow{OG}$ as a simplified expression in terms of **p** and **q**.

(4)

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Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 14 marks)

11 (a) Using a formula from page 2, show that $\cos 2x = 1 - 2\sin^2 x$

(3)

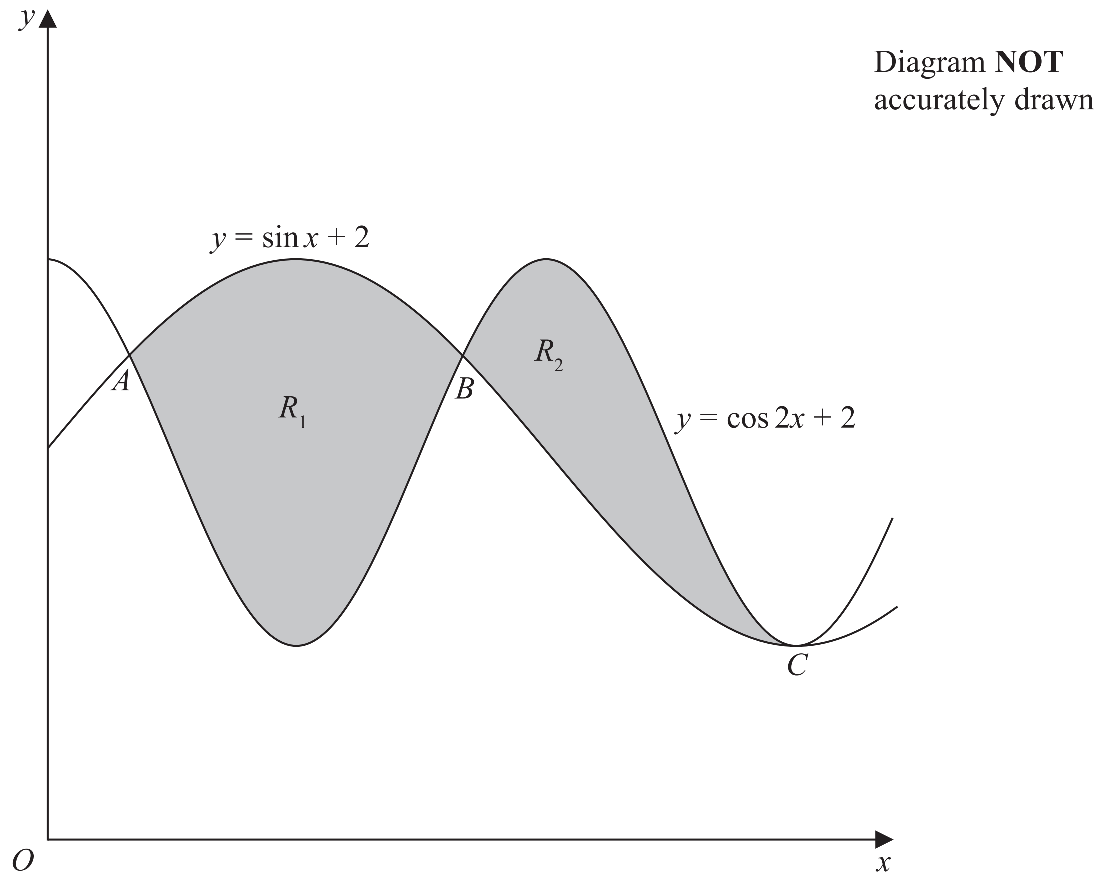

Figure 2

Figure 2 shows a sketch of part of the curves with equations $y = \sin x + 2$ and $y = \cos 2x + 2$

The points A, B and C, shown in Figure 2, are three points that are common to both curves.

(b) Find the coordinates of each of these points.

(4)

R_1 and R_2, shown shaded in Figure 2, are two regions enclosed by the two curves.

(c) Use calculus to find, in its simplest form, the ratio

$$\text{area of } R_1 : \text{area of } R_2$$

(8)

Question 11 continued

Turn over

Question 11 continued

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Question 11 continued

Question 11 continued

(Total for Question 11 is 15 marks)

TOTAL FOR PAPER IS 100 MARKS