Paper Reference 4PM1/01
Pearson Edexcel
International GCSE

# Further Pure Mathematics

## PAPER 1
## (Calculator)

**Time: 2 hours plus your additional time allowance.**

**ITEMS INCLUDED WITH QUESTION PAPER**
Diagram Book
Answer Book
Formulae Pages



Pearson

# Calculators may be used.

## INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions.

Without sufficient working, correct answers may be awarded no marks.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

Do NOT write on the Question Paper.

You must NOT write anything on the Formulae Pages. Anything you write on the Formulae Pages will gain NO credit.

## INFORMATION

The total mark for this paper is **100**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

## ADVICE

Read each question carefully before you start to answer it.

Check your answers if you have time at the end.

Good luck with your examination.

**Answer all ELEVEN questions.**

**Write your answers in the Answer Book.**

**You must write down all the stages in your working.**

1. The quadratic equation

$$3(k + 2)x^2 + (k + 5)x + k = 0$$

has real roots.

Find the set of possible values of $k$

(Total for Question 1 is 6 marks)

2. Angle $\alpha$ is acute such that $\cos\alpha = \frac{3}{5}$

Angle $\beta$ is obtuse such that $\sin\beta = \frac{1}{2}$

(a) Find the exact value of

(i) $\tan\alpha$

(ii) $\tan\beta$

(3 marks)

(b) Hence show that

$$\tan(\alpha + \beta) = \frac{m\sqrt{3} - n}{n\sqrt{3} + m}$$

where $m$ and $n$ are positive integers whose values are to be found.

(3 marks)

**(Total for Question 2 is 6 marks)**

3. A curve $C$ has equation $y = \dfrac{ax - 3}{x + 5}$ where $a$ is a constant and $x \neq -5$

The gradient of $C$ at the point on the curve where $x = 2$ is $\dfrac{18}{49}$

(a) Show that $a = 3$

(3 marks)

Hence

(b) write down an equation of the asymptote to $C$ that is

(i) parallel to the $x$–axis,

(ii) parallel to the $y$–axis,

(2 marks)

(continued on the next page)

3. continued.

(c) find the coordinates of the point where $C$ crosses

(i) the $x$–axis,

(ii) the $y$–axis.

(2 marks)

(d) Sketch the curve $C$, showing clearly its asymptotes and the coordinates of the points where $C$ crosses the coordinate axes.
There are blank axes on pages 34 to 45 in the Answer Book if you wish to use them.
(3 marks)

**(Total for Question 3 is 10 marks)**

4. The $n$th term of an arithmetic series is $u_n$ where

$$u_n = (n + 1)\ln 4$$

Given that the sum of the first $n$ terms of the series is $S_n$

show that $S_n = \ln 2^{(n^2+an)}$ where $a$ is an integer whose value is to be found.

**(Total for Question 4 is 5 marks)**

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5. (a) Expand $(1 + ax)^n$ in ascending powers of $x$ up to and including the term in $x^3$

Express each coefficient of $x$ in terms of $a$ and $n$ where $a$ and $n$ are constants and $n > 2$

(2 marks)

The coefficient of $x$ is 15 and the coefficient of $x^2$ is equal to the coefficient of $x^3$

(b) Find the value of $a$ and the value of $n$

(6 marks)

(c) Find the coefficient of $x^3$

(2 marks)

**(Total for Question 5 is 10 marks)**

6. (a) Show that $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(3 marks)

The quadratic equation $x^2 - 7kx + k^2 = 0$, where $k$ is a positive constant, has roots $\alpha$ and $\beta$ where $\alpha > \beta$

(b) Show that $\alpha - \beta = 3k\sqrt{5}$

(3 marks)

(c) Hence form a quadratic equation with roots $\alpha + 1$ and $\beta - 1$

Give your equation in the form $x^2 + px + q = 0$ where $p$ and $q$ should be given in terms of $k$

(4 marks)

**(Total for Question 6 is 10 marks)**

7. The curve C has equation $y = \dfrac{x}{x^2 + 4}$

(a) Using calculus, find the coordinates of the stationary points on C

(5 marks)

(b) Show that $\dfrac{d^2y}{dx^2} = \dfrac{2x(x^2 - 12)}{(x^2 + 4)^3}$

(4 marks)

(c) Hence, or otherwise, determine the nature of each of these stationary points.

(2 marks)

**(Total for Question 7 is 11 marks)**

8. Given that $n$ satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

(a) find the value of $n$

(3 marks)

Given that $\log_p x = 3$ and $\log_p y - 3\log_p 2 = 4$

(b) (i) express $x$ in terms of $p$,

(1 mark)

(ii) express $xy$ in terms of $p$

(4 marks)

(Total for Question 8 is 8 marks)

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9. Find an equation of the normal to the curve with equation

$$y = (x^3 - 2x)e^{(1-x)}$$

at the point on the curve with coordinates $(1, -1)$

**(Total for Question 9 is 5 marks)**

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10. Look at the diagram for Question 10 in the Diagram Book.

It is NOT accurately drawn.

It shows triangle OAB and triangle OCD

$\overrightarrow{OA} = 5\underline{p}$

$\overrightarrow{AB} = 3\underline{q}$

$\overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB}$

$\overrightarrow{OD} = \frac{3}{5}\overrightarrow{OA}$

(a) Find $\overrightarrow{DC}$ as a simplified expression in terms of $\underline{p}$ and $\underline{q}$

(3 marks)

(continued on the next page)

10. continued.

Remember:

$\overrightarrow{OA} = 5\underline{p}$

$\overrightarrow{AB} = 3\underline{q}$

$\overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB}$

$\overrightarrow{OD} = \frac{3}{5}\overrightarrow{OA}$

The line DC meets the line AB at F

(b) Using a vector method, find $\overrightarrow{OF}$ as a simplified expression in terms of $\underline{p}$ and $\underline{q}$
(7 marks)

(continued on the next page)

**10. continued.**

**Remember:**

$\overrightarrow{OA} = 5\underline{p}$

$\overrightarrow{AB} = 3\underline{q}$

$\overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB}$

$\overrightarrow{OD} = \frac{3}{5}\overrightarrow{OA}$

**The point G lies on OB such that FG is parallel to AO**

**(c) Using a vector method, find $\overrightarrow{OG}$ as a simplified expression in terms of $\underline{p}$ and $\underline{q}$**

**(4 marks)**

**(Total for Question 10 is 14 marks)**

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**11. (a) Using a formula from the Formulae Pages, show that $\cos 2x = 1 - 2\sin^2 x$**

**(3 marks)**

**Look at the diagram for Questions 11(b) and (c) in the Diagram Book.**
**It is NOT accurately drawn.**
**It shows a sketch of part of the curves with equations $y = \sin x + 2$ and $y = \cos 2x + 2$**

**The points A, B and C, shown in the diagram, are three points that are common to both curves.**

**(b) Find the coordinates of each of these points.**

**(4 marks)**

**(continued on the next page)**

**11. continued.**

**$R_1$ and $R_2$, shown shaded in the diagram, are two regions enclosed by the two curves.**

**(c) Use calculus to find, in its simplest form, the ratio**

**area of $R_1$ : area of $R_2$**

**(8 marks)**

**(Total for Question 11 is 15 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**

**END OF PAPER**

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