Paper Reference 4PM1/01
Pearson Edexcel
International GCSE

Further Pure Mathematics

Paper 1

Monday 17 June 2019 − Afternoon

Time: 2 hours plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION

Calculators may be used.

ITEMS INCLUDED WITH QUESTION PAPERS

Diagram Book
Answer Book
Formulae Pages

X58286A

INSTRUCTIONS

Answer ALL questions.

Without sufficient working, correct answers may be awarded no marks.

Answer the questions in the Answer Book or on the separate diagrams – there may be more space than you need.

You must NOT write anything on the Formulae Pages. Anything you write on the Formulae Pages will gain NO credit.

INFORMATION

The total mark for this paper is 100

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams.

ADVICE

Read each question carefully before you start to answer it.

Check your answers if you have time at the end.

Turn over

Answer all ELEVEN questions.

Write your answers in the Answer Book.

You must write down all the stages in your working.

1.

$$f(x) = x^3 + 2x^2 - 5x - 6$$

(a) Factorise

$x^2 - x - 2$

(1 mark)

(b) Hence, or otherwise, show that

$(x^2 - x - 2)$ is a factor of $f(x)$

(3 marks)

(Total for Question 1 is 4 marks)

2. Given that

$$\frac{4+2\sqrt{3}}{5-2\sqrt{3}}$$ can be written in the form

$$\frac{a+b\sqrt{3}}{c}$$ where a and b are integers and c is prime, find the value of a, the value of b and the value of c

Show your working clearly.

(Total for Question 2 is 3 marks)

3. In triangle ABC,

AC = 7 cm

BC = 10 cm

angle BAC = 65°

(a) Find, to the nearest 0·1°, the size of angle ABC

(3 marks)

(b) Find, in cm^2 to 3 significant figures, the area of triangle ABC

(3 marks)

(Total for Question 3 is 6 marks)

4. Look at the diagram for Question 4 in the Diagram Book.

It is NOT accurately drawn.

It shows a sector OAB of a circle where angle $AOB = \theta$ radians.

The circle has centre O and radius 15 cm

The point C divides OA in the ratio $2:1$ and the point D divides OB in the ratio $2:1$

The area of the region $ABDC$, shown shaded in the diagram, is $100\,\text{cm}^2$

Find

(a) the value of θ,

(3 marks)

(b) the perimeter of the region $ABDC$

(3 marks)

(Total for Question 4 is 6 marks)

5.

$$f(x) = 3x^2 - 9x + 5$$

Given that $f(x)$ can be written in the form

$a(x - b)^2 + c$,

where a, b and c are constants,

find

(a) the value of a, the value of b and the value of c

(3 marks)

(b) Hence write down

(i) the minimum value of $f(x)$,

(ii) the value of x at which this minimum occurs.

(2 marks)

(Total for Question 5 is 5 marks)

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6. Look at the diagram for Question 6 in the Diagram Book.

It is NOT accurately drawn.
It shows a lawn $ABCDEF$, where $ABDE$ is a rectangle of length y metres and width $2x$ metres.
Each end of the lawn is a semicircle of radius x metres.
The lawn has perimeter 90 metres and area S m^2

(a) Show that S can be written in the form
$$S = kx - \pi x^2$$
where k is a constant.

State the value of k

(4 marks)

(continued on the next page)

6. continued.

(b) Use calculus to find, to 4 significant figures, the value of x for which S is a maximum, justifying that this value of x gives a maximum value of S

(5 marks)

(c) Find, to the nearest whole number, the maximum value of S

(2 marks)

(Total for Question 6 is 11 marks)

7. (a) Solve, in degrees to one decimal place,

$$(3\cos\theta + 5)(5\sin\theta - 3) = 0$$

for $0 \leq \theta < 180°$

(2 marks)

(b) Show that the equation

$$8\sin(x - \alpha) = 3\sin(x + \alpha)$$

can be written in the form

$$5\tan x = 11\tan\alpha$$

(5 marks)

(c) Hence solve, to one decimal place,

$$8\sin(2y - 30°) = 3\sin(2y + 30°)$$

for $0 \leq y < 180°$

(5 marks)

(Total for Question 7 is 12 marks)

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8. (a) Solve

$$5p^2 - 9p + 4 = 0$$

(2 marks)

(b) Hence solve

$$5^{2x + 1} - 9(5^x) + 4 = 0$$

Give your answers to 3 significant figures where appropriate.

(4 marks)

The curve with equation $y = 5^{2x + 1} + 5^x$ intersects the curve with equation $y = 2(5^{x + 1}) - 4$ at two points.

(c) Find the coordinates of each of these two points.

Give your answers to 3 significant figures where appropriate.

(4 marks)

(Total for Question 8 is 10 marks)

9. (a) Solve the equation

$$2\log_p 9 + 3\log_3 p = 8$$

(6 marks)

Given that

$$\log_2 3 = \log_4 3^k$$

(b) find the value of k

(2 marks)

(c) Show that

$$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3$$

$$= \log_4\left(\frac{x^{6x-5}}{3^{6x-20}}\right)$$

(4 marks)

(Total for Question 9 is 12 marks)

10. (a) Expand

$(1 + 2x^2)^{-\frac{1}{3}}$ in ascending powers of x up to and including the term in x^6, expressing each coefficient as an exact fraction in its lowest terms.

(3 marks)

(b) State the range of values of x for which your expansion is valid.

(1 mark)

$$f(x) = \frac{2 + kx^2}{\left(1 + 2x^2\right)^{\frac{1}{3}}} \qquad \text{where } k \neq 0$$

(c) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^6

Give each coefficient in terms of k where appropriate.

(3 marks)

(continued on the next page)

10. continued.

Given that the coefficient of x^4 in the series expansion of $f(x)$ is zero

(d) find the value of k

(2 marks)

(e) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0\cdot5} f(x)dx$$

(5 marks)

(Total for Question 10 is 14 marks)

11. The curve C has equation

$$3y = x^2 + 2$$

The point P lies on C and has x coordinate 4

The line k is the tangent to C at P

(a) Find an equation for k, giving your answer in the form

$$ay = bx + c$$

where a, b and c are integers.

(6 marks)

The line L is the normal to C at P

(b) Find an equation for L, giving your answer in the form

$$dy = ex + f$$

where d, e and f are integers.

(2 marks)

(continued on the next page)

11. continued.

(c) Find the area of the triangle bounded by the line k, the line L and the x–axis.

(3 marks)

The finite region bounded by C, the line L, the x–axis and the y–axis is rotated through $360°$ about the x–axis.

(d) Use algebraic integration to find, to the nearest whole number, the volume of the solid generated.

(6 marks)

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER
