Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 02R

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Introduction

A detailed report on every question follows, but in general, candidates would benefit from,

- Reading the question carefully for;
  - the instruction 'show that'
  - the instruction to use algebraic methods
  - looking carefully at angle ranges given in questions involving trigonometrical questions.
- Checking work carefully – impossible answers indicate there is an error.
- Drawing sketches in questions involving coordinate geometry.
- Showing full working, particularly when evaluating integrals or solving quadratics.

Question 1

This proved to be a very challenging question for most candidates. Although virtually all knew they had to apply $b^2 - 4ac$ to the given f(x) almost nobody knew how to proceed from there. The most obvious approach was to complete the square on the resulting three term quadratic from applying the discriminant, and then complete the square to show that it had to be always > 0. We only observed this however, in a small minority of solutions.

The other possible approaches were to differentiate and show that the y coordinate of the minimum point is 48, to draw a sketch or to find the discriminant again and show that as the value is negative (-192) then the functions does not have real roots, but sight of any of these were very rare.

Question 2

(a)

It was very rare to see an erroneous solution here. Very occasionally, the inequality was the wrong way around.

(b)

This was another very well answered question. Virtually every candidate found the critical values and proceeded to give the range of the inside region correctly.

There was a little carelessness in some cases as we saw $(3x+5)(x-2)=0 \Rightarrow x=\frac{5}{3},-2$ but we still gave credit for specifying an inside region by following through erroneous critical values. Those candidates who use calculators to solve equations, need to do so very carefully, and incorrect values without sight of an acceptable method will not score the M mark.

(c)

Most candidates managed to combine the inequalities together correctly and we allowed follow through from earlier erroneous work.

Question 3

This question was generally answered better than in previous years with most candidates appearing to know how to approach this question and the steps required; however, it was evident that candidates did not always read the given question, particularly in part b) and c) where it was surprisingly common for candidates to complete work for c)

(a)

A very high percentage of the candidates were able to write out the Area equation successfully; $675=\frac{\theta r^2}{2}$ although some did not process this any further into the required $\theta = ...$ or $\theta r = ...$ for the first mark. An alternative method of using the formula $A=\frac{1}{2}rS$ was seen quite often in candidates' responses and was used effectively. To score both method marks candidates were able to derive the correct expression for *P* and to score the final A mark we needed to see $P=2r+\frac{1350}{r}$ in full.

A very popular oversight by candidates was to ensure they had *P* =…… at the end of the 'show that'.

(b)

The first two marks of this part was answers well by candidates with many converting to index form before differentiating. Most candidates got the first method mark by differentiating successfully to at least the minimally acceptable standard. The majority of candidates also knew to equate their derivate to zero and attempted to solve. It was uncommon but worth noting that some candidates were mistakenly finding the second derivative and set that equal to zero in error. It is also worth mentioning that a very small number of candidates gave r as a negative value not realising the implications of r being a radius and thus a length so must be positive.

It was very popular for candidates to omit moving on to find P even after successfully finding r from equating $\frac{dP}{dr} = 0$ thus dropping the last MI and A1 in this part of the question. It was also common for candidates to mistakenly find the second differential in this part and P in part (c) which is not answering the given question and thus losing the final 4 marks of question 3. Candidates are reminded to answer the question as given not what they think should be next.

(c)

Most candidates successfully used the second derivative to determine the nature of P. A small number didn't mention that "since ……..> 0; hence it is minimum"; they went on straight to just write, 'it is minimum'; thus losing the A mark for an incomplete conclusion. A small minority of candidates, tried to use calculus to find the gradient before and after $r = 15\sqrt{3}$ thus showing a minimum by testing around the point, which with enough correct working shown was a correct method but it is worth reminding candidates that using this approach they must show their working rather than stating " \ _ / therefore minimum". Some candidates also tested using the expression for P instead of the gradient.

Question 4

(a)

This was a well answered question overall with many achieving full marks.

$\overrightarrow{OB}$ was usually found correctly in terms of a, although there were some who subtracted rather than added, the vectors. Pythagoras was used mainly correctly to achieve a quadratic in a by using $\left|\overrightarrow{OB}\right| = \left(5\sqrt{29}\right)^2$. Most were able to solve these although the negative solution of -19 was sometimes omitted.

(b)

Although some candidates clearly did not know the definition of a unit vector, the majority of candidates were able to find $\left|\overrightarrow{AB}\right| = \sqrt{377}$ and put together a correct unit vector.

<u>Question 5</u>

This kinematics question concerned the relationship between distance, velocity and acceleration. Giving candidates information about velocity thus requiring both differentiation to find acceleration and integration to find distance.

It was very rare to see errors in either parts (a) or (b).

(a)
Almost all candidates achieved the marks in this part which involved differentiating a quadratic equation to give a linear one, and then substituting a given value for the linear term to find the acceleration at a given point in time.

(b)
This part asked them to find the times at which instantaneous rest occurred, a process requiring them to solve the given quadratic equation for the velocity, having recognised that this must necessarily be zero. The equation easily factorised $(2t-5)(t-7)=0$ leading to both values of $t = \frac{5}{2}, 7$ which was seen in the vast majority of scripts.

(c)

Candidates were asked to find the distance between the two pints of instantaneous rest, having cautioned them to use calculus in order to find the value.

Some candidates chose to use other methods, some correctly and others not, but in any event did not earn marks as they had not followed the clear instruction. The instruction use calculus, does not mean that candidates can use their calculators to integrate and write down the final answer, it means that we **require** to see every step: integration and then substitution.

Those recognising the need for calculus chose, mainly, to integrate the equation for velocity and evaluate the integrated expression between the two values they found in part (b). Marks were available for making a reasonable attempt at integrating the various terms in the equation correctly and for substituting the candidate's own values from part (b) even if incorrect.

An occasional error seen was for the candidate to integrate correctly but reverse the values for the times, so that they gave a negative value for the distance between the points. No merit is given for an answer for a distance which is not a positive number, the negative value being a displacement for which we did not ask.

This question overall did not challenge candidates as much as others on the paper, but those who did go astray either integrated in (a) and differentiated in (c), leaving them somewhat lost, or else they attempted to find the acceleration in (a) by direct substitution into the constant velocity formula.

While this paper is not concerned with Newtonian equations of motion, candidates should have an understanding of the relationship between distance, velocity and acceleration in order to be able to correctly choose the calculus approach required in each circumstance.

Question 6

(a)

The majority of candidates correctly stated $\alpha+\beta=-\frac{5}{2}$ $\alpha\beta=-\frac{p}{2}$ for the B mark. The most common error was usually not including the minus sign for $\alpha\beta$.

Candidates who were successful in using $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ usually continued to be successful in part b). Other correct forms were used so that values could be substituted correctly, common ones were $(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ or $\alpha^3+\beta^3=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ having found that $\alpha^2+\beta^2=\frac{25}{4}+p$. Some candidates did not gain the first M mark for correct algebra where the most common mistakes was writing $(-2\alpha\beta)$ in all three expressions, although most did go on to gain the second M mark for correct substitution of their sum and product.

The majority of candidates were able to rearrange their linear equation to find a value for *p*. Some candidates due to erroneous algebra earlier in the question did not arrive at a linear equation and thus could not score the M mark. There were the usual issues with negative signs that caused candidates some confusion/issues when manipulating and there were still candidates at this level that wrote $\left(-\frac{5}{2}\right)^2 = -\frac{25}{4}$ and thus did not obtain a correct value of *p*.

(b)

For this part the majority of candidates used their value of *p* with their for $\alpha\beta$. A common incorrect value for *p* seen was $\frac{9}{4}$ following erroneous work in part (a) and there were a number of candidates who did not divide their *p* value by 2 in this part.

Most candidates were able to find a correct expression for the product, however, a number of candidates expanded the brackets in the numerator which made the working more involved instead of stating and using $\frac{(\alpha+\beta)^2}{\alpha^2\beta^2}$. It was not uncommon for candidates to incorrectly expand their correct numerator and spoil what could have been a correct product with incorrect algebraic manipulation.

The sum expression proved more challenging for candidates, with a number abandoning their attempts at this question, or other having multiple pages of erroneous work, which was worth little and would have undoubtedly cost candidates time towards the end of the paper. Quite a few candidates stated the denominator as $\alpha^2+\beta^2$ instead of $(\alpha\beta)^2$ and others had difficulty in expressing the numerator in a form which allowed substitutions to be made. There was some interesting errors seen throughout this part of the questions, which was not credit worthy.

Of the two approaches to the algebra presented in the mark scheme, neither was favoured more by candidates but the far more successful method was those that approached it using $\alpha^3+\beta^3$ from part (a). Those that attempted to use the approach which resulted in a substitution for $(\alpha+\beta)^2$ where on the whole successful but this did lead to more errors in algebraic manipulation than the direct approach using $\alpha^3 + \beta^3$. It also was not uncommon for candidates for realise that $\alpha^2 + \beta^2 = \frac{25}{4} + p$ and use this substitution instead; again, on the whole was successful but could lead to algebraic errors.

The correct answer of $-\frac{185}{18}$ was frequently seen and correctly used along with the product value of $\frac{25}{9}$ in the final equation, remembering to negate the sum value. A number of candidates failed to change the sign for the sum, losing the final A mark. Others did not express the equation with integer coefficients and/or including = 0 for the final A mark. It was a frequent [and commonly seen over the years] mistake to see "equation =…".

Question 7

(a)

Candidates were required to find the exact coordinates of the points where a given curve intersected with the $\theta -$axis.

In order to do so, it was necessary to solve and equation of the form $A\cos3\theta + B\sin3\theta = 0$ where A and B were constants.

The majority of candidates were able to rearrange this equation into the form $\tan3\theta = C$. Infrequently the 3θ became θ; similarly, there were some efforts to expand the trigonometric functions of 3θ but these made no progress as they were not required and only served to complicate a straightforward equation.

Although clearly the question required answers in radians there were many instances of answers being given in degrees which could not receive the final A marks in both parts (a) and (b) beyond marks for a correct method.

The technique of proceeding from an initial value of 3θ to generate another solution within the required range and hence to achieve the values of m and n was well understood.

(b)

This part required the definite integration of the given function to find the volume of a solid of revolution.

The formula for the required volume of the solid of revolution was well known with only a small minority either omitting the multiplicative constant π or multiplying by 2π.

The integrals of the two trigonometric functions were also well known and it was rare to see these integrated incorrectly.

The mechanism of applying the limits and subtracting was also well applied with many fully correct solutions being seen. It is crucially important that centres stress the need to their students to show full substitution of values. We will only score the M mark for substitution if candidates either, use the correct limits and obtain the correct final value, or if they do not have the correct limits [following an error in (a)], or do not obtain the correct final value and if each case show us full substitution.

A small minority of candidates added rather than subtracted. A similarly small minority made more work for themselves by splitting the definite integral into two parts – from *m* to 0 and from 0 to *n* – but most still then proceeded to the correct answer.

<u>Question 8</u>

This was a multipart question, with the majority of candidates scoring full marks, with the remaining candidates usually scoring 1-12 marks. It was not uncommon for candidates to score all marks in parts (a), (b) and (c) then make no more mark worthy progress.

(a)

This was a good source of marks for candidates with most successfully able to use the given information to form the equation of a straight line, with just a few candidates failing to cast their answer into the required form. The most successful and common approach was finding the equation of the line using $y - y_1 = m(x - x_1)$. A good portion of candidates still use the $y = mx + c$ approach, and although usually successful, the most common wrong answers involved minor slips with signs, which resulted in only gaining the method mark in this part. Some students could not gain the second A1 as they did not change the final equation into the required general form with integer coefficients; students should remember to read the question carefully and give answers in the required form.

(b)

This again was generally well answered with most candidates able to successfully obtain the coordinate of point *X*, take the negative reciprocal of their gradient in (a) and then use whatever method they preferred to reach the printed answer, usually the same method as used in part (a). A few candidates took a long route to find the coordinates of *X* but most reached the correct ones and proceeded to set up the required equation. The major issue for those who did not score full marks was finding *X*, many thought it was simply the midpoint but did gain the B and subsequent M mark. It was feasible to see only the B mark scored, for the negative of their gradient in part (a), for those candidates who struggled with this question.

(c)

This was a very easy mark and almost all candidates who attempted the question were able to score this. They usually stated $y = 10$ or $p = 10$. A few gave their answer as coordinates, but this was also acceptable.

(d)

This was a problematic part for candidates where many could not work beyond part (c). A variety of approaches seen here with some candidates reaching the correct co-ordinates in a couple of lines, others taking a couple of pages, with a lot more working out than really required by what was a straight forward calculation.

The vector method in the marking scheme was the least popular despite being by far the easiest. Some worked successfully via vertical and horizontal translation. There were a few cases where no workings were shown but the coordinates of point *D* were correct students should be reminded that answers without working might score no marks. In this case, sight of $\left(-2, 6\right)$ with or without working scored M1A1A1. If errors were made using this approach it was due to some elementary slips with subtraction which a quick check would have identified.

Lengthy solutions involving simultaneous equations, which again, when attempted were usually successful although there were significantly more opportunities for mistakes. Some candidates who attempted the method outlined in **ALT 3** had difficulties in substituting the equation of straight line into the equation for the side length. Their working and algebra were usually very lengthy. As a result, the algebraic work looked messy or unorganized and candidates using this approach rarely found the correct coordinates. Those candidates who methodically constructed their answers using this approach usually had a greater success rate.

For those who attempted the lengthier approaches, the method given in **ALT 2**, solving the equations of line *AD* and *CD*, was generally successful except for candidates who either made mistakes when they tried to write the two equations, or formed one of the equations based on the wrong coordinates.

The most common unsuccessful method was to equate the lengths or gradients of DC and AB or *AD* and *CB*. These produced equations in x and y that the candidates could not solve.

(e)

This part very much depended upon the success of candidates at part (d) in finding the coordinates of *D* but it was clear in this part of the question that some candidates did not know how to calculate the area of a trapezium. Correct answers using the point *X* were rare with many just calculating the area by multiplying *AB* and *BC*. Those who realised the perpendicular height was required were usually successful in multiplying the correct values together.

Using determinants was as usual now, the most popular method with those that had answered part (d) correctly able to set up a correct array and for those that had the correct array it was rare to see incorrect processing and generally students got the correct result. For many whose determinant contained incorrect point(s), usually the point *D*, they lost their M1 for not showing detailed working of how they evaluated the determinant. It was not uncommon for candidates to set up the array and score no further marks due to lack of working or issues correctly evaluating their array. It was also evident that a number of candidates knew how to set up their array but simply left blanks where *D* should have been, as they did not calculate a value of *D*.

A few candidates used the “rectangle – 4 triangles” and produced a slick solution or candidates found the area of triangle *ABC* using determinant and multiplied its result by 2 which avoided using point *D*.

Question 9

(a) and (b)

This question proved a good source of marks for the majority of candidates, although incorrect labelling in parts (a) and (b) cost some candidates marks.

Parts (a) and (b) were successfully completed by most candidates, although some transposed their answers to (a) (i) and (ii). This was also seen in (b), although less commonly.

When no labelling was present, then the answers were marked in the order in which they appeared. There again, incorrect ordering led to the loss of marks.

(c)

Most reasonable attempts at parts (a) and (b) lead to a good sketch with two asymptotic branches the correct way around. Many correct solutions were seen, with a graph of the correct shape in the correct position, with clearly labelled and correct asymptotes and intersections with the axes seen. There was unfortunately also some poor sketching seen, most notably where some candidates had wobbled one section of the graph to get it through the intersections on their axes. It might be a better strategy to teach candidates to draw the curve first and mark the intersections afterwards. Some solutions did not show the asymptotic nature as well as they could with a significant number allowing the ends of their curve to drift away from their asymptotes, although in most cases where the intention was clear we condoned such attempts.

(d)

In this part, most candidates realised differentiation was required and many candidates correctly applied the Quotient Rule to differentiate the function and most then expressed this in the form seen in the mark scheme, with a squared term in the denominator. However, a high proportion of candidates failed to give a full justification of why this was always negative. Most commonly, having obtained an expression of the form required, they simply stated that it was always negative, gave a partial explanation or attempted to use numerical values which can never score, as we required an explanation why the gradient was ALWAYS negative.

(e)

This part proved the most challenging part of this question. Success in part (d) was required to make significant progress here, with many candidates failing even to attempt it. Others equated $-\frac{3}{5}$ to the value of the function rather than the derivative. Of those candidates who correctly formed an equation equating $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{5}$, most went on to produce a fully correct solution, although a small proportion only gave one value of *x* as a solution to the quadratic.

It was also quite common to see the expression $-\frac{15}{(x+6)^2} = \frac{3}{5}x + k$ formed, leading to no significant further progress. If $-\frac{15}{(x+6)^2} = \frac{3}{5}x + k$ was formed, then most candidates went on to find . $x = -11, -1$ Many then went on to find the associated *y* values, formed equations of straight lines and correctly selected the positive value of $k = \frac{2}{5}$.

Question 10

This question posed a deceptively simple logarithmic equation which was to be solved. Seven marks were available for finding solutions to the equation which had a term in $\log_4$ and another in $\log_x$.

In order to progress, candidates needed to find a common base for their logarithms. Most chose base 4, recognising that the term in base *x* involved $\log_x 64$ and that 64 = 4^3. It therefore offered smoother routes to simplification.

However, a significant number of candidates preferred to convert the base 4 term to base *x*. A small number chose to convert both logarithms, typically to base 2, a familiar base to many from other studies.

The first two marks were awarded for the technique of converting a logarithm from one base to another, and for dealing correctly with powers such as the 4^3.

Perhaps because the change of base was more obvious, more candidates did this first and more got it correct. A minority of candidates failed to properly resolve the powers.

Where the two logarithmic terms were correctly reduced in powers and converted to the same base, the majority of candidates were then able to make further progress. A minority became confused at this point and recorded no further useful work, ending up with only 2 marks on the question. In part, I think, this is down to finding a fraction confusing where the denominator contained a log.

The route that most candidates took out of this quandary was to substitute *x* (or sometimes *y*) in place of the log. This allowed them to recognise the need to multiply through by the substituted *x* and to therefore gain a three-term quadratic. From there they were on safer ground and could solve the quadratic, and then convert their substitution back to a log. There are 25 letters in the alphabet other than *x*, and so using *x* in the substitution can only lead to errors.

One error seen a few times involved those candidates who had not performed any manipulation on the first term in the given equation, $\log_x 4^3$. Having achieved a fraction with the other term, candidates then attempted to multiply through by their denominator, usually $\log_4 x$. However, they incorrectly concluded that $\log_x 4^3$ multiplied by $\log_4 x$ resulted in $\log_4 x^4$, demonstrating a lack of full understanding of the way in which logarithms interact.

The quadratic equation yielded values of 6 and $\frac{4}{3}$ when working in base 4, values 12 and $\frac{8}{3}$ when working in base 2, and values $\frac{3}{4}$ and $\frac{1}{6}$ when working in base x.

These logarithmic values then had to be converted back to values of *x*. In all cases the correct values of *x* are 4096 and approximately 6.35

There were few errors seen in the conversion back to *x* values from candidate's values for the logs.

Overall, this question tested knowledge and facility with logs and those who could not prove themselves lost marks quite quickly, and were unable to redeem themselves. However, the majority of candidates handled the question well and a good understanding of the different possible approaches was widely seen.

negative solution, could not gain full marks for the question. Candidates who reached this stage usually knew how to find *x* from the equation in $\log x$ and were very successful at doing this, no matter which base they had chosen to use. We gave credit to those candidates who reached an incorrect equation, and awarded the final M mark by solving it correctly by knowing how to find *x*.

We saw some solutions involving natural logs and log base 10, but these were very rare indeed, but usually correct.

Question 11

Candidates found this question to be the most challenging on the paper, particularly part (b).

The structure of the question meant that candidates could actually attempt part (a), (b) and (c) independently.

(a)

Part (a) is a standard proof requiring candidates to start from a summation formula for $\cos 2A$ to derive a formula for $\sin^2 A$. A relatively popular approach saw candidates start with the right-hand side of the required result and then manipulate it to reach $\sin^2 A$. It did not matter if candidates started from the LHS or RHS, but it was disappointing to note how many candidates could not apply $\cos 2A = \cos^2 A - \sin^2 A$ followed by $\cos^2 A = 1 - \sin^2 A$.

(b)

Fully correct solutions here were very rare. There are three different routes in the mark scheme, but what we frequently observed were pages of erroneous trigonometrical manipulations leading nowhere. Attempts often used the result from part (a) on the $\sin^4 A$ term but failed to use a similar formula to write the $\cos^4 A$ term. Much expansion of this term into various forms involving $\sin A$ and $\cos A$ often ensued with no progress being made.

(c)

Unsuccessful attempts at (a) and (b) often meant that some candidates left part (c) unattempted. However, the structure of the question did mean that it was possible to complete part (c) by using the results from parts (a) and (b) to transform the given equation.

Candidates attempting part (c) often made progress and although this question was not answered well, it was not uncommon to see no marks awarded in parts (a) and (b) but obtain the correct angles in part (c).

However, errors seen included slips in the functions of θ, for example θ instead of 2θ. Once the correct equation had been obtained, the method of transforming it into an equation involving $\tan 2\theta$ was well understood. A common error here was $\frac{\sin 2\theta}{\cos 2\theta} = \tan \theta$ and careless slips such as these resulted in the loss of the last three marks.

The method used to generate solutions from an initial solution was well known.

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