Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 02

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Question 1

Although most candidates realised that real unequal roots were determined by the value of the discriminant, a significant minority failed to state that $b^2 - 4ac > 0$. Nonetheless, many continued to form and solve a quadratic equation in *k*. Some candidates did not realise that they needed two roots to their equation. As it was a simple two term quadratic, it was perhaps less obvious. Many having obtained two roots to their equation simply either;

- gave the roots as the answer
- gave the outside region
- gave the region as $k > -\frac{4\sqrt{3}}{3} \quad k > \frac{4\sqrt{3}}{3}$

The correct required **inside** region $-\frac{4\sqrt{3}}{3} < k < \frac{4\sqrt{3}}{3}$ was unfortunately a rare sight.

Question 2

It is a pre-requisite for a question like this, that a reasonable diagram should be drawn. It was noticeable that those candidates who answered the question well, drew a diagram, and those who did not answer it well, had either an incorrect diagram, or no diagram at all.

Part (a)

Virtually every candidate was able to apply cosine rule correctly for either AC or AC^2. Subsequent processing errors abounded however, and only a minority of candidates found the correct length, or alternatively, an expression for the correct length. We observed errors such as $\sqrt{34x^2 - 30x^2 \cos 110} = 2x\sqrt{\cos 110}$.

Although candidates then realised that sine rule was needed to find the required angle. We also saw the following too frequently: $\frac{\sin BCA}{3x} = \frac{\sin 110}{6.65}$ leaving a loose *x* which candidates knew not what to do with. Quite a few candidates seemed to have difficulty with dealing with lengths in terms of x, not realising that they would cancel out.

There were only a handful of candidates attempted to use the ALT method to find the required angle. The alternative method is a good and efficient method to solve this question, however, it was less successful than the main method, candidates who attempted this way, could usually only score the first M for setting up the equation correctly via sine rule with use of angle $(70-\theta)$.

We saw quite a lot of premature rounding leading to the loss of the final M mark. Centres should remind their students to use greater accuracy in intermediate calculations, preferably leaving values within their calculators.

<u>Part (b)</u>

This part of the question was tackled much better, the formula for the area of a triangle given 2 sides and the included angle was well known and most candidates proceeded to the correct answer gaining all 3 marks with many correct solutions. The most common error here was to forget to square root the final answer, and a value of $x = 3.41$ was seen too frequently.

Question 3

Part (a) (i) and (ii)

The majority of candidates appreciated that an expression for the velocity could be obtained by integrating the given expression for the acceleration $a = 6t - 16$. Generally, integration was completed accurately, though a number of candidates did not add *c*, the constant of integration, resulting in them only achieving the method marks. Finding the correct complete expression for the velocity was generally more successful than finding an expression for the displacement, possibly due to the constant of integration in the latter being = 0.

Part (b)

Many candidates did not understand that their expression for the displacement was required here, with a significant minority of those who attempted this part of the question setting their expression for $v = 0$. Of those who did use the required expression for s, managed to solve their equation = 0, to find the three values of *t* of 0, **2**, and 6.

Only a small minority scored all the marks here.

Question 4

Parts (a) and (b)

Overall this was a very good source of marks for virtually every candidate and it was very rare to see lines incorrectly or inaccurately drawn and then the region incorrectly specified.

Part (c)

This was less successful, some candidates not even attempting it. Linear programming is tested quite regularly and this topic should be a good source of marks to students.

There were two ways of finding the coordinates of intersection of the lines; either by using the graphs [and we allowed a generous tolerance on the coordinates] or by using simultaneous equations. The given function was applied efficiently and because we generously allowed the first A mark here for just one pair of coordinates given correctly, the most common marking pattern here was M1A1dM1A0 because both Maximum **AND** Minimum was not seen that often.

Question 5

Part (a)

The number of attempts using the most efficient method was seen a handful of times.

Using the intersections with the *x*-axis first - $y = Q(x-6)(x+2)$ **M1**

Using the coordinates $(4, -6)$ $\Rightarrow -6 = Q(4-6)(4+2) \Rightarrow Q = \frac{-6}{-12} = \frac{1}{2}$ **M1**

And so $y = \frac{1}{2}(x-6)(x+2) \Rightarrow y = \frac{x^2}{2} - 2x - 6$ **A1**

The most popular method used by those candidates who made any attempt at all, was to set up three simultaneous equation with the three variables *p*, *q* and *r* and to solve them. This took generally, at least two sheets of the answer book for a total of 3 marks. Those who attempted it this way [and did not give up halfway], were generally successful. The allocation of marks for each part of the question is given in the paper and centres should teach students to look at this to give an indication of the amount of work required.

Part (b)

Candidates showed that they were very well versed in these procedures, and the correct line was found accurately and most importantly, by showing full working. Candidates were clearly comfortable with this part of the question; most proceeding to differentiate *S* and substitute $x = 4$ to obtain the gradient of the curve at *P* equal to 2 and hence the gradient of the normal equal to $-\frac{1}{2}$. From here many were able to establish that the equation of the line was $2y + x + 8 = 0$. Just a few candidates that formed their equation of line *L* used a gradient of normal that did not come from the use of calculus.

Part (c)

This part of the question was not well answered at all. Many candidates failed to appreciate that in order to find the coordinates of intersection of the curve and the line they needed to equate L with C, bearing in mind that the sketch was given in the question. The most common limits seen used were $x = 4$ and -2 [which was part of the information given in the question and are the coordinates of the points of intersection of the curve with the x-axis]. This resulted in a loss of 5 marks! 3 marks for not finding the correct coordinates, the M mark for setting up the correct expression, and the final A mark for the final correct area. The integration was well done in virtually every case, but centres must impress on their students, that full substitution should always be shown. Where the final area is incorrect and the limits are incorrect, we will only award the mark for substitution when it is explicitly seen.

Question 6

This question on connected rates of change was more accessible than some we set, candidates responding well to this question with a significant minority scoring full marks.

It is important in this topic to identify the rates that are required carefully. We could only award marks for $\frac{\mathrm{d}V}{\mathrm{d}t} = 12$ and $\frac{\mathrm{d}V}{\mathrm{d}h} = 9h^2$ seen explicitly identified, unless seen later used correctly. More than a few did not use the formula given but used the formula for the volume of a cylinder which inevitably lead to the loss of many marks.

It was noted that there were a few misreads of $V = 3h^3$ as $V = 3h^2$ leading to an oft seen h value of $16\sqrt{2}$.

Candidates who introduced their own "lettering" system (rather than using the given letters ie v for volume, h for height) tended to get their algebraic expressions mixed up and were generally unsuccessful.

However, this question was overall well answered and a good source of marks for many candidates, although some wasted the final mark by giving a rounded decimal value – the question clearly stated exact value.

Question 7

Part (a)

The two marks available here were the easiest marks in the whole paper, and the majority of candidates managed to gain these very easily. Surprisingly, there were still a few sign errors from some candidates thus losing the A mark. A very small minority, presumably being unable to realise the simplicity of the question, wrote some complicated and irrelevant algebra without success.

Part (b)

Virtually every candidate scored the next three marks for differentiating the **given** expression for *S*, and then setting = 0 to find the two values of *x*. What was less successful was establishing which was the minimum [and which the maximum, although the latter is not required]. Whilst $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ was often found correctly, the conclusion was either missing or incomplete in many candidates work. It is insufficient to state $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 30x + 18 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 30\left(\frac{4}{5}\right) + 18 = 42$. We must see 42 > 0 and therefore $\frac{4}{5}$ is a minimum value.

We also occasionally saw candidates confusing finding the minimum value with finding the minimum value of *S*.

Part (c)

A few candidates thought that finding the minimum of the expression *S* was down to finding the second derivative and concluding on the nature of the stationary points solving this in part (c) and losing 2 marks that could not be awarded retrospectively. From those who successfully progressed to (c) the majority found the correct minimum value of *S*. However, a few of those who failed to decide/conclude the minimum between the two *x*-solutions in part (b) could not score the two marks awarded in part (c) even though the correct answer of $\frac{128}{25}$ was among their options.

Question 8

This question on Arithmetic Series was not answered well at all.

Part (a)

There was much confusion in this part about the information given to candidates. A significant minority misread the 5th term = 23 as the sum of the first 5 terms. It is always important to read the information given in questions carefully. Unfortunately, this resulted in simultaneous equations where the coefficients of a were the same and as this oversimplified the problem no marks could be awarded for solving their simultaneous equations.

Moreover, solving the two simultaneous equations arising from the one correct equation and one incorrect equation lead to such improbable values such as $d = -\frac{69}{5}$ and $a = \frac{156}{5}$. The sight of such awkward values should have been sufficient to ring alarm bells and encourage a revisit of earlier work, but virtually every candidate who made this error carried these forward to the next stage.

In order to find the required expression, candidates were required to use the *n*th term, but the majority of candidates used the formula for the sum to *n* terms and inevitably lost the two marks here.

Part (b)

Although it was pleasing to note that many candidates worked accurately and carefully in this part, the vast majority of errors arose from the incorrect use of the summation formula. It was necessary here to use $S_n = \frac{2n}{2}\left[2a + (\mathbf{2n} - 1)d\right]$ but many candidates did not use $2n - 1$ but $n - 1$ and thus lost the M mark.

It is reported on nearly every similar report, that it is necessary to show all working when solving a quadratic equation. However, with the use of modern calculators, many candidates are showing no working, but just writing down the roots using their calculators. If the equation and the roots are correct, we will always, award full credit, but if the 3TQ is incorrect (which was often the case her – arising from incorrect values of a and d found in part (a)) then we will not award the M mark for solving a quadratic if the roots just ‘appear’. Candidates must be encouraged to always show their working or lose valuable marks.

Question 9

This was without a doubt the most challenging question in this paper. The majority of responses presented completely blank pages.

Part (a)

This was a question that posed multiple challenges to the candidates. Although many candidates were able to state that angle AOB = 1.8, there were many errors in finding the diameter AC and many candidates appeared confused as to how to proceed. A few successful candidates were able to show that the diameter was $2x\sin 0.9 + 2x$, although we also allowed 3.57x for two M marks, obtain the perimeter of the semi-circle and sector ADC to obtain the given perimeter of the logo $P = x\left(\pi + \pi\sin 0.9 + 3.8\right)$, with a = 1 and b = 3.8. This was an unsuccessful question for the vast majority of candidates as they did not know how to proceed. It was also unfortunate that, to for candidates, the stem to part (b) Given x = 10 cm was used in part (a). Many candidates assumed that B was the midpoint of AC so their areas were incorrect. Use of the cosine rule in part (a) was more common, but difficult to resolve to sine. Given that the given answer involved sine, this should have been a hint on how to proceed.

Part (b)

Some carried through the misconception from part a) that the radius = 2x = 20. Having obtained a diameter or radius of the semi-circle in part (a), just a few candidates were able to obtain an area of the semi-circle. However, two marks were frequently scored by finding the correct areas of the sector or /and triangle/segment obtaining the two B marks and even the final M mark if they found an expression for the area of the logo irrespective of their erroneous expression of the area of the semicircle. Most candidates used area of sector - area of triangle OAB to calculate the area of the segment, which was then subtracted from the area of the semi-circle to give the final area of the logo of 458. This part of the question was generally slightly more successfully attempted than part (a).

However, the general marking pattern in this question [for those who even attempted it at all] was (a) B1M0M0A0M0M0A0   (b) M0M0B1B1M0A0

Question 10

This generally a very well answered question and a good source of marks for many candidates.

Part (a)

Most candidates know the two expansions for $\alpha^3+\beta^3$ in the form of $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ most commonly or else $(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$, and also knew how to apply them correctly. As a result, there was very little ‘fudging’ taking place and most candidates scored the three marks here.

Part (b)

This was an accessible question on this topic and so many candidates knew they had to find the sum, the product and then put these values into a fresh equation.

Apart from some quite unnecessary algebraic manipulation seen in just a few scripts, most candidates who attempted this part of the question realised that for the SUM the required values were given to them in the sum. Some needlessly applied $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ to $\alpha^3+\beta^3$ despite that value being given.

There was a slight twist in the tail in finding the PRODUCT as it was necessary to apply $(\alpha+\beta)^2-2\alpha\beta$, but this was generally carried out correctly and accurately.

There was the occasional confusion as usual with the use of the sum and product in forming the final equation, but it was mostly very well answered. Even if candidates' values of the product and sum were incorrect, but used correctly to form an equation the

M mark was available. Some candidates could not double 61 and we saw 112 in place of 122 more than a few times.

It is pleasing to note, that it is rare to see any of the following incorrect final answers these days;

$32x^2 - 95x + 122$ [missing = 0] or $y = 32x^2 - 95x + 122$

which are of course all A0, although we do award the M mark for them.

<u>Question 11</u>

<u>Part (a)</u>

Most candidates were able to find both the vector $\overrightarrow{OM}$ and use it to find the required vector $\overrightarrow{MA}$. Whilst we will always give both M1A1 for the correct vector seen $\overrightarrow{MA} = 3\mathbf{p} + 7\mathbf{q}$ with or without working, it is worth reminding centres that we award the M mark for a correct vector path seen, for example $\overrightarrow{MA} = -\overrightarrow{OM} + \overrightarrow{OA}$. Candidates who omit this vital step and then make a minor sign or direction error, will lose both marks if we do not see a correct path. Directions are all important in vector work, and so credit could not be given for those candidates who found $\overrightarrow{AM}$ even though their $\overrightarrow{AM}$ was often correct.

<u>Part (b)</u>

Candidates either knew how to tackle this part or they did not, and this was completed by far fewer candidates than part(a). Those who did, produced accurate and correct solutions using any two parameters.

This part needed a combination of strong algebraic skills and proper use of properties of collinear points. Some candidates gain no marks on this part by failing to use parameters to express vectors $\overrightarrow{MN}$ or $\overrightarrow{AN}$. Some of the candidates that correctly formed two different expressions for vectors $\overrightarrow{MN}$ (or $\overrightarrow{AN}$) failed to equate them properly so could not produce simultaneous equations. Those candidates who managed to produce two equations, had to deal with an extra challenge and use proper algebra to solve them. Almost all candidates who ended up with the correct values of parameters $\left(\frac{7}{18} \text{ or } \frac{11}{18}\right)$ managed to give the correct answer for the ratio. Occasionally errors were made by getting to $\frac{7}{18}$ and not realising this meant the two line-sections would be $\frac{7}{18}$ and $\frac{11}{18}$ so losing the final A mark for giving 7:18. A few candidates who had established their constants could not directly use this to find the required ratio and went on to do considerably more unnecessary vector algebra before finding a ratio.

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