Pearson Edexcel

Examiners' Report

Principal Examiner Feedback

Summer 2023

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 01

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Introduction

A detailed report on every question follows, but in general, candidates would benefit from,

- Reading the question carefully for;
  - the instruction 'show that'
  - the instruction to use algebraic methods
  - looking carefully at angle ranges given in questions involving trigonometrical questions .
- Checking work carefully – impossible answers indicate there is an error.
- Drawing sketches in questions involving coordinate geometry.

Question 1

(a)

This part of the question was answered very well indeed, and it was a rare sight to see a candidate not achieving all three marks available here. The two popular approaches were to either find *a* and *d* and then to use the summation formula, or otherwise find *a* and then use the first + last formula with the last term being $3n + 2$.

We occasionally saw candidates using $\sum_{r=1}^{n}(3r+2) = 3\sum_{r=1}^{n} r + 2\sum_{r=1}^{n} 1$ with standard results, but these were very rare and in any case the algebra was a little more involved.

(b)

The same cannot be said for part (b) where finding the correct sum for $\sum_{r=10}^{40}(3r+2) = 2387$ with only something like half of the cohort. We allowed use of $n = 10$ for this mark, but even so many candidates just summed from $n = 1$ to $n = 40$ and left that as the final answer.

Question 2

This question was generally answered well. Candidates were able to rewrite the square root as a power of $\frac{1}{2}$ and then used the product rule for differentiation. The majority gained the first 3 marks for this differentiation being correct. If mistakes came they came mainly in the second half of the question when they were manipulating the derivative to be over $\sqrt{3+2x}$ At this point many candidates either lost the 2 from $2\cos 2x$ within their working or they just forgot it was there to multiply the bracket out after and therefore ended up with $A = 3$ and $B = 2$ as their answer.

Most candidates identified the question as a product rule differentiation, and quoted or applied the formula accurately. Using the chain rule to correctly differentiate the square root term was more problematic, with errors in the constant multiplier or problems with the index.

It was interesting that some students converted this to quotient rule, which was probably from trying to anticipate the required answer. Students lost all marks if they did not put in permissible differentiable form, so lost marks if they did not apply chain rule properly in particular to the $\sqrt{3+2x}$.

The most common marking pattern in this question was M1A1A1M0A0.

Question 3

This was a 5-mark question, with no scaffolding although the question itself did require two distinct sections; finding the equation of the line and then, drawing and using it. Overall, we observed mixed responses, where some candidates where able to effectively and efficiently present their working for the correct line to be drawn or otherwise it was clear that some candidates made use of a permissible calculator with no justification as to where their answers came from. As usual, here is a timely reminder that answers with insufficient working, as per the rubric on the front of the paper will score no marks. It is clear some candidates are using a calculator to solve questions with no evidence they know how to algebraically solve the question presented. Candidates should not be relying on calculators to do all the work from them. Candidates should note that they needed to show sufficient evidence of working to match the number of marks to be allocated and therefore drawn lines [plotted from the two points of intersection obtained by using a calculator] only scored no marks.

It was necessary to show some working to justify the line they drew, the most popular approach being to divide by 2 and then x^2 separately, rather than in one step. Some candidates did divide by x^2 or 2 but not always both, and thus could not gain the first M mark. If step one was completed correctly some then followed this with incorrect simplification which meant they did not add 6 and/or subtract x as required for the next mark.

Only a few candidates followed the expected method of equating the given equation of the curve to $Ax + B$, and when attempted in this manner, it usually scored all of the marks as it is the most logical approach. Although it is possible to solve this question using simultaneous equations, seen by subtracting $x^3 - 12x^2 = 2x^2 y$ from the equation $3x^3 - 12x^2 + 8 = 0$ which when rearranged gave you the desired equation of the straight line, it was very rarely successful.

Once the equation of the straight line was established, on the whole there were no errors with the remainder of the question. It is worth pointing out however, that too many straight lines were drawn not very accurately and some were even roughly/freehand drawn – for example, not exactly passing through 6 on either the x or y axis as they should. This was accurate enough at the points of intersection to have answers showing $x = 0.9$ and 3.8, luckily for some candidates the marks were also given to 1.0 for the first intersection, which gave those less accurate candidates more chance of scoring well.

Question 4

This was a good source of marks to most candidates with the most common marking pattern being (a) M1A1 (b) M1A1A1M1A1M1A1A0

(a)

It was rare for a candidate not to score both available marks for part (a).

Virtually every student knew they had to find the derivative for acceleration then this was answered well with nearly all then reaching an answer of 4.

(b)

This was also well answered by a large majority of candidates and was familiar to many who realised what was needed in terms of the integration. That process was straightforward for most, although some candidates did not realise they needed to set the given velocity equation = 0 to find the two times at which P was at rest. The inevitable result of this was that they did not have limits for their integration.

When Values for t_1 and t_2 were found, the integration was carried out accurately and limits substituted in correctly, although errors were occasionally seen in this calculation. However, many candidates failed to recognise the distinction between displacement, which has direction, and distance which does not. Many candidates gave a positive result when they evaluated the integral but a sizeable minority just subtracted their results form evaluation of the integral the 'wrong way' around despite having their limits the correct way around in their statement of integration. Only a few came up with $-\frac{8}{3}$ and then interpreted the distance correctly as $\frac{8}{3}$.

Question 5

(a)

Most candidates were able to find formulae for *h* or *hx*. This flexibility from the mark scheme allowed the candidate to score well here. The next mark was allocated to finding an expression for the surface area, which was mostly answered well with some candidate collecting sides together in pairs or others collecting 3 sides and multiplying by 2. It is worth pointing out that a limited number of candidates, scored only this mark due to the minimally accepted form as it was, a somewhat common incorrect approached that **four** sides of the cuboid had area 4*xh* rather than **two**. Even if there was a deduction of the B and first M mark, most candidates were able to substitute in their expression for *h* even if they were unable to go any further to find *S*. The final mark was not always scored because of notation with some calling it *SA* and other just writing an expression. We are strict in this type of question as it is a formal proof and we require to see $S = 8x^2 + \frac{375}{2x}$ and not *SA* =… or Surface Area = … These did not score the final A mark.

(b) (i)

This was a fresh starting point for candidates so they needed to restart with the formula given in the question and differentiate. The first mark was almost always scored as most candidates were able to get the right form of differentiated expression. As this was a method mark, even if candidates had made a minimally acceptable mistake, they generally were able to write the correct differentiated form of expression enough to score this mark. The difficulties in the differentiation often came from them making $\frac{375}{2x} = 375 \times (2x)^{-1}$ where confusion came in as to how to deal with the 2, usually resulting in candidates obtaining 750 as the numerator in $\frac{375}{x^2}$. Setting the differentiated expression equal to 0 was also well answered by candidates even if they had the wrong expression which led them to an incorrect value of x. There were many answers of 3.61 given in error here, a direct result of incorrect though minimally acceptable differentiation.

(b) (ii)

Some candidates tested the gradient either side of their value to test whether it was a minimum, and a few very mistakenly, tested P on either side of their x, clearly misunderstanding the requirements for these two marks. Otherwise, candidates who had successfully found the first differentiated expression, as long as it was of the form $\frac{dS}{dx} = 16x - \frac{k}{x^2}$ were able to continue to gain this next mark for the second differentiated expression.

When concluding on whether their values was a minimum, candidates mostly gave the correct value of 48, but even if they did not use the correct value of x they were still able to get a positive value that led to the conclusion that it is a minimum and score the final A mark as it is a ft..

(c)

Some candidates unfortunately used the wrong equation here to substitute their value of x into, with some using the first or even the second derivatives and did not arrive at an answer for S at all. As this is the final mark in the question there is no ft, and candidates lost this final A mark if they had been using the wrong answer for x which led to a wrong value for S, although many fully correct answers were seen here.

Question 6

It is encouraging to report that nearly every candidate attempted this single part question albeit with a wide range of success from zero to full marks. Once again, a single answer for *x* appearing out of nowhere with zero work to support it and was credited with no marks. Candidates should not rely on permissible calculators to do the work for them and must provide workings to support their answers, which is reflected in the number of marks available.

It is essential that we are able to determine the base of the log in which a candidate is working and unfortunately some presentation was very poor, with it not always being clear, for example, whether a candidate meant $\log_2 x, \log 2x$ or $\log^2 x$.

The given log equation was written in 3 different bases and most candidates started off trying to rewrite it in one base. Most attempts were in base 2 but a significant number chose base 4 and a smaller number chose base *x*. A tiny minority of candidates tried base 10 or base ln, but these were very rarely solved correctly.. Candidates also demonstrated the power law for logs during this stage, usually the first thing they did when answering this question.

Those who immediately wrote all 3 terms in the same base, generally made good progress to an equation in $\log x$ to their chosen base, with substitution being a popular method, many used *a* this stage. It is worth reminding candidates that there ae 25 other letters in the alphabet so choosing the letter *x* as their choice for a substitution is unwise and for some led to unnecessary complications further into the question. If candidates did not use a substitution, some, did struggle with dealing with (log *x*)(log *x*) and would mistakenly write $\log x \log x = \log x^2 \Rightarrow 2\log x$ which resulted in a linear equation instead of a quadratic.

The correct log equation was often achieved and solved to give 2 values of $\log x$ to their chosen base. Those who only gave one value, usually as a result of disregarding the negative solution, could not gain full marks for the question. Candidates who reached this stage usually knew how to find *x* from the equation in $\log x$ and were very successful at doing this, no matter which base they had chosen to use. We gave credit to those candidates who reached an incorrect equation, and awarded the final M mark by solving it correctly by knowing how to find *x*.

Question 7

This was a relatively complex differentiation problem. Many did not understand the notation used $(x+\delta x)$ and did not attempt it at all. Those who recognised the need to use Quotient Rule, realised they could attempt to score the first 7 marks even if they did not understand how to tackle part (b).

(a)

The first step was to simply the given expression by eliminating the square root and set up a quotient rule question.

The index of $\frac{1}{2}$ within differentiation (as with Question 2) caused just a few issues for some. However, of those who attempted the question we saw many candidates who scored the first 5 marks in this part by simply applying quotient [or product] rule correctly. For those using either of these methods the subsequent simplification proved very taxing for many candidates, who struggled with the common denominator. Few were able to accurately work through to the given answer.

Some who reached $\frac{dy}{dx}=\frac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}$ failed to write the answer in the required form and needlessly lost the final A mark.

A significant minority of candidates used Chain Rule. Whilst this was a more complex approach, it did have the advantage that the subsequent algebra to simplify to the required form was slightly easier. As with the standard methods, we still observed a significant number of errors in candidate's working

In these 'show' questions it is essential that every step is seen. Skipping a few stages of working out and proceeding straight to the answer will not convince examiners who will not then be unable to award marks.

(b)

This part was very hit and miss. Either students seemed to get all the marks or no marks. Many did not calculate the increase of 0.2% of x correctly and just used 0.2% or 0.002 or 1.002 rather than finding the correct value. We also saw substitution of 0.2 into the expression instead of 2.5. More often than not if they got the value of 0.005 correct then they went on to get all 3 marks available.

Question 8

This accessible question was answered well by the majority of students.

Virtually all realised that the starting point was to integrate the given f'(x) to find an expression.   Integration was done well here. There were mixed methods to try and find the value of the constant of integration. Candidates who did not include a constant of integration at this stage were unable to score the next 2 marks, but the marks for polynomial division were available for correct work.

Many substituted $\frac{1}{2}$ in and set it equal to 0. Some made errors here that resulted in a value of c = 7 rather than $c = -7$. Quite a few candidates also used polynomial division. Most errors here came from those using polynomial division.
If they found the correct f(x) then the polynomial division that followed was mostly correct.

A significant number of students struggled with how to show there was only one intersection of the x-axis following this. They used the quadratic formula or discriminant but were unable to conclude this correctly. A few responses stated the complex roots, although without a suitable conclusion to the meaning of complex roots we could not award marks. Whilst complex numbers are well beyond this specification, any candidate using more advanced techniques must use them correctly to be assured of scoring marks.

Question 9

(a)
Most attempted this part, many successfully, for all 4 marks. Starting with the double angle formula for cos2A quickly lead to the required identity for $\cos^2 A$. Some candidates not realising they had to use the same starting point, then tried to use the double angle formulae for sin2A which did not lead to the identity for $\sin^2 A$. Many candidates, however, followed the same steps to show the identity for $\cos^2 A$ and $\sin^2 A$ to great effect. Some candidates, potentially failed to read the question and did not use the formula given on page 2 of their exam paper, or chose to use the given identities to prove the other; scoring only two marks maximum for this. The students who answered this the most successfully were the ones who showed one step or substitution at a time, rather than doing multiple conversions in one go

and making small errors. As this was a “show that” question, the only marks available were for use of the correct identities and accurate working with no errors at all.

(b)

The majority of candidates realised they should expand the brackets, although this was not always done correctly, and substitute in the equations from part (a). It was not uncommon for candidates to score M1M1M1 here and then no more. Many achieved a correct equation but there were then some contrived answers with very few able to show clearly how the $-7\sin 2x$ term was reached in the given answer, before presenting this. Some candidates who had not spotted this use of $\sin x\cos x$ tried to work backwards from the RHS of the given however without this explicit substitution of $7\sin x\cos x=\frac{7}{2}\sin 2x$ marks could not be awarded. As a proof question requires demonstration of all elements correctly this resulted in deduction of marks for lack of evidence for many candidates. Candidates should be reminded for the need to present all steps of working in show that questions, rather than marking leaps.

(c)

Few candidates were able to answer this section successfully. Those starting from the given answer (b), often differentiated correctly, wrote a correct equation in tan2x and hence worked out 2*x* and *x* for all 4 marks. Even those who completed the first step successfully often failed to get to an expression in terms of tan 2*x*. It was unfortunately very common to see $\frac{\sin 2x}{\cos 2x}=\tan x$ and with this error, none of the final 3 marks could be scored. Those who correctly arrived at tan2x, rarely did not remember to halve the solutions to their tan2x equation or gave extra solutions. Those who did not realise the connection with (b), often used product rule to differentiate but very rarely achieved the correct solution or realised that an equation in tan2x would be possible. This approach mostly resulted in zero marks scored, as the differentiation was rarely correct. Only the most able students were successful at using this approach.

Question 10

This is an unusual presentation of a common question on this specification and candidates did not attempt it very well at all. Although the expected way to solve involved finding vector $\overrightarrow{AB}$ first, most opted to find $\overrightarrow{OB}$. Some candidates failed to even score these marks as they did not write a vector statement [i.e. $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ or a correct equivalent] and opted to try, unsuccessfully, to write the vector, with many a sign error seen. It was rare for candidates to score the next two mark unless they were able to find *b* successfully and required this to substitute to find *a*.

Although most candidates can find a unit vector starting with a vector, seemingly very few can deal with extracting the length of a vector from a given unit vector and so most candidates managed to gain the first 2 marks getting as far as $\overrightarrow{OB} = (b+4)\mathbf{i} + b\mathbf{j}$, possibly also the next two for finding $3a - b = 2$ but then didn't know what to do next to continue successfully.

Candidates experienced problems with the remaining marks in this question and only the most able where able to make any further progress. The problems arose because of the $\sqrt{\frac{17}{34}}$ in the given unit vector $\overrightarrow{OB}$ it appearing that many candidates did not know the implications of this being the unit vector and not the vector $\overrightarrow{OB}$. Root $\sqrt{\frac{17}{34}}$ was often included in equations causing problems everywhere and lots of abandoned working. In some candidates work they were able to use Pythagoras on their $\overrightarrow{OB}$ or $\overrightarrow{AB}$ but then were not able to successfully form an expression in terms of *b* and/or *a*; many left in form **i** and **j** which caused a whole host of confusion for candidates.

Of those who actually managed to score the 5$^{\text{th}}$ Mark in this question, did so using $\overrightarrow{OB}$ and arriving at an equation in terms of *b* only and would generally go on to solve for *b* correctly. It was rare but possible, for those who made it past the first 2 marks, to see an allocation of M1A1M0A0M1M1A1M0A0 in this question for those attempts.

Question 11

(a)

Completing the square on a quadratic is often tested on 4MA1 so we expect this to be a well known and practiced technique. The negative coefficient of x^2 however caused all sorts of problems although virtually every candidate found the value of *A* correctly. The majority of solutions were however fully correct.

(b)

Although this is 'write down' question, the intention being to use the result from part (a), some candidates continue to differentiate $\frac{dy}{dx} = 6 - 2x = 0 \Rightarrow x = 3$ and then substituting this value into f(x) to find the value of 19. However, candidates should note the allocation of marks for this part (1) and (1) and differentiating/setting = 0/finding y would have a higher mark tariff.

(c)

Virtually every candidate equated the two curves and found the two points of intersection correctly. In fact, it was very rare so see an error here.

(d)

Integration was done well here with equal amounts integrating two expressions separately to those integrating the combined expression. If marks were lost in this part, it was mainly due to the incorrect subtraction of the limits, incorrect limits being used or in some cases added the two values that came from integrating two separate expressions rather than subtracting them.

Where candidates attempted the correct integrations of $-2x^2 + 7x - 3$ with the correct limits then there were some very good solutions. A minority of candidates lost the last mark either due to numerical errors when substituting the limits or ended up with $-\frac{125}{4}$ when attempting the integration of $2x^2 - 7x + 3$.

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