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Principal Examiner Feedback

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General

This paper proved to be a very accessible paper, and it was pleasing to see candidates were able to make attempts at all of the questions. There were many familiar types of questions, so candidates should have felt prepared had they completed past papers. The first two questions were answered most successfully and provided candidates with confidence to pursue the rest of the paper. Overall, marks were available to candidates of all abilities and the parts of questions which proved to be most challenging were 5(d), 7(b) and 9(c). Time did not appear to be an issue for candidates.

Question 1

This question was accessible to virtually all candidates. About half of all candidates scored full marks and many lost only one mark which was typically in part (b) where more than one solution was given for inequality. If errors were made in part (a) then typically they were gained as follow through marks in part (c).

In part (a) the transformation of the modulus graph was well understood with most candidates gaining both marks.

In part (b) most candidates were able to write a correct inequality to solve, usually of the form $-2(x-5)+10>6x$ or $2(5-x)+10>6x \Rightarrow x<\frac{5}{2}$ (or equivalent) or sometimes it was written as an equation to give $x=\frac{5}{2}$ which was also acceptable for the first method mark. Many candidates gave two solutions which lost them the final accuracy mark by solving the inequality $2(x-5)+10>6x$ which had no solution. About half the candidates realised that there was only one solution to this inequality and this part of the question was done well when candidates drew a line of $y=6x$ on the graph to fully understood how it intersected with the modulus graph given. Very few candidates used the squaring method to remove the modulus signs or had the less than or equal sign for the inequality in their final answer. Most candidates understood how the graph $y=\mathrm{f}(x)$ transformed to $3\mathrm{f}(x-2)$ by writing their new coordinates as $((a)+2,\ (a)\times 3)$.

Question 2

This question working with rational functions and integrating the function was a good source of marks for the majority of candidates, most of whom were able to earn full marks here.

In part (a), a mixture of approaches was seen with algebraic division being the most common, although formation of an identity followed by comparison of coefficients was seen from time to time. Occasionally slips were made resulting in an incorrect coefficient of the *x* term or sometimes errors in the constant term ($b = 1$ and $c = 10$ was a common incorrect answer). In these cases, it was possible for candidates to achieve marks in part (b) using their result from part (a) provided it was of the correct form. To achieve full marks in this part it was necessary for candidates to ensure they stated g(*x*) in full. It was not uncommon for candidates to stop once they had achieved values for '*A*', '*B*' and '*C*' although recovery was allowed, on this occasion, for those who went on to write or use g(*x*) correctly in part (b).

In part (b), the integration was, on the whole, completed very successfully. The vast majority of candidates recognised that a natural logarithm was required, and the polynomial component of the function was integrated with ease. There was occasional confusion with coefficients and, for example, $\frac{1}{6}\ln(x-2)$ was seen a number of times. Substitution for the integration was seen only in a very small number of cases but was usually successful. A small minority of candidates differentiated the function rather than integrated and others were at a loss of how to integrate the reciprocal function correctly which was costly as they could access no marks in this part. Very few candidates combined the log terms incorrectly following substitution of the limits although, perhaps to avoid being penalised for showing incorrect log work, minimal steps in working were shown in most cases. The use of brackets and/or modulus signs was sometimes inconsistent and occasionally poor bracket use led to errors. A common incorrect result seen in this part which followed from part (a) was $52 + 10\ln 3$ which was often able to earn three out of the four marks. Some candidates wrote the result of their integration as $\ln(x-2)^6$ to obtain $\ln(46656) - \ln(64)$ initially but were usually able to continue accurately to obtain the required form.

Question 3

This question was slightly more challenging than anticipated with some candidates confusing the two parts and information provided. Part (ii) was usually better answered than part (i) with many unsure how to proceed from the given equation to an appropriate logarithmic graph.

In part (i) sketching a log graph was challenging for many candidates with many not seeing the connection between a straight-line graph and the information given in the question. It was rare for candidates to gain all three marks with a significant number scoring zero. The errors were varied with many reciprocal graphs, missing intercept values and graphs with a positive gradient. The most common error was usually having graphs that stopped on the axes. Some candidates who were successful with drawing the correct shape and labelling correct points of intersection with the axes then lost the first mark for having incorrectly labelled axes.

Part (ii) was slightly more accessible and provided candidates appreciated that this was a different question to part (i), they usually proceeded to score most of the available marks. However, a significant number failed to find the correct equation in N and t. The most common error was using base 10 instead of base 3 and hence only being able to be considered for a single mark. Another frequently seen mistake was a failure to leave the answer as an equation thinking that the values of a and b were sufficient. A few made errors in their log work when proceeding to the exponential form. Answers with no working scored no marks so candidates should be reminded that where a question say "Show…" they make sure that they provide sufficient evidence of their method.

Question 4

This question was fairly routine and was attempted by nearly all candidates with the majority making good progress in parts (a) and (b). Part (c)(ii), however was very poorly answered with most candidates gaining no marks.

In part (a), there were many fully correct answers. The first method mark was awarded for a correct identity seen for $\sin 2x$ or $\cos 2x$ in terms of $\cos^2 x$. Nearly every candidate was awarded this for their knowledge of the double angle formula for $\sin 2x$ and scored the first M mark for this. A few had 16 or 8 as their coefficient with some still gaining the mark if they had a correct identity written down. Fewer knew the trig identity for $\cos 2x$, some correctly derived it, but quite a few incorrectly wrote "$\cos 2x = \cos^2 x - 1$".

There were two common approaches:

- $4\cos^2 x - 3 = 4\cos^2 x - 2 - 1 = 2(2\cos^2 x - 1) - 1$
- $4\cos^2 x - 3 = 4\left(\dfrac{\cos 2x + 1}{2}\right) - 3$

The latter approach of replacing the $\cos^2 x$ term within the expression with the exact equivalent expression for $\cos 2x$ and then simplifying was the most successful; those attempting the former often made errors from poor algebraic manipulation with the brackets.

Part (b) was generally well answered with most candidates proficient at finding *R* and α using $R = \sqrt{a^2 + b^2}$ and $\tan\alpha = \frac{b}{a}$ directly without expanding the $R\sin(2x+\alpha)$. A few did not give an exact value for *R* and did not gain the B mark. Although $\tan\alpha = \pm\frac{a}{b}$ and $\tan\alpha = \pm\frac{b}{a}$ were also allowed for the M mark these were rarely seen. Candidates almost always used radians in calculating α as required. A significant number failed to write out the expression after finding *R* and α which was necessary for the A mark, although some recovered this mark in par (c).

Most candidates realised they needed to use their "*R*" – "1" in part (c)(i) and stated the correct answer without any workings. A significant number then converted to decimals but isw generally saved them. A common incorrect answer was *R* (usually $2\sqrt{5}$) written on its own with some candidates failing to appreciate this subtle change to maybe previous past paper questions. Part (c)(ii) was poorly answered. The phrase "second smallest" seems to have thrown many with some ignoring the key word. Those who were successful usually attempted:

- $2x + \alpha = \frac{\pi}{2} + 2\pi \Rightarrow x = 3.69 \text{ (or } 3.70)$ finding the second smallest value directly.
- $2x + \alpha = \frac{\pi}{2} \Rightarrow x = 0.5536 + \pi = 3.69 \text{ (or } 3.70)$, finding the first smallest value and realising to add π to get the second smallest value.

Many of the correct attempts sketched a sine curve and indicated the 2nd maximum, using this sketch to deduce $\frac{5\pi}{2}$. Most, however, set $2x + \alpha = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ with $x = 0.5536$ being the most common incorrect answer and $x = 2.12$ was also often seen. These attempts scored 0 marks for the part.

Question 5

This question on functions tested some routine parts with many able to score a pleasing number of marks overall. However, part (d) was rarely correct and was one of the most challenging parts on the paper.

Part (a) was generally successfully attempted, though the majority of candidates found $f^{-1}(x)$ and then substituted 22, rather than simply solving the equation $f(x) = 22$. A disappointing number of candidates lost the A mark by choosing to turn to their calculator for $e^{\frac{22-2}{5}}$ rather than simplify the power to get an exact answer. A small number of candidates did confuse $f^{-1}(x)$ with $f'(x)$ and differentiated. Had candidates noticed that part (b) required differentiation, then they may have been discouraged from proceeding with more calculus in this part.

Most candidates recognised the need to use the quotient rule to differentiate the expression in part (b), and this was usually successfully done. Some bracket omissions were seen, and a few failures to subtract, but M1A1 was very common. Quite a high number of candidates did make a sign error multiplying out the second bracket resulting in an incorrect simplified form. Many candidates missed out on the third mark by not making a comment which linked their differentiation to the task of the question: proving g was an increasing function. A few candidates did get confused and claim g'(x) was increasing, though this was rare.

In part (c), the vast majority of candidates gained the first mark for rearranging g, but almost all did it in terms of y and x, and sadly very many failed to restate their final form in terms of $g^{-1}(x)$, meaning they could only get the M mark and not the A mark. The domain was very commonly omitted altogether, which was surprising given that this type of question appears frequently on WMA13. Of those who considered the need for a domain at all, most correctly found one end of the interval, but only about half of these candidates correctly identified both ends of the domain. Again, some evidence of confusion between g^{-1} and g' was evident, with a few candidates differentiating g, as well as a small number using f instead of g.

In part (d), most candidates were able to state the function $fg(x) = 2 + 5\ln\left(\frac{6x-2}{2x+1}\right)$, but often proceeded to conclude that 2 must be one end of the range. Very few candidates connected this task to the range of g they had found duing part (c) and considered what results came from the fraction = 0 and/or 3. Even among those who used 3, and found the value 2 + 5ln3, most still gave this as one end with 2 or 0 often given as the other end. Fully correct responses to this part were very rare.

Question 6

This was an accessible question which saw the full range of marks awarded and was generally well attempted by the candidates.

In part (a) candidates were asked to find the equation of a normal to a curve. This required the use of the chain rule to get the tangent gradient, knowledge of the relationship between the tangent gradient and the normal gradient and finally the use of the equation for a straight line. The overwhelming majority of candidates achieved full marks in this part of the question. There were some errors in the power of $(4x-7)$ either initially or in its differentiation and some arithmetic errors crept in. The relationship between tangent and normal gradient was well known. The form of the straight line $y-y_1=m(x-x_1)$ was used by nearly all candidates. There was little evidence of manipulating work to achieve the final printed answer.

In part (b), candidates were required to find an area formed of two parts – the area under the curve and a triangle. The form of the integral was well known but there were slips seen in the constant when integrating. A minority of candidates did not know how to integrate this function and so did not proceed further with the question. The use of the upper limit of 10 instead of the correct value 8 was seen on a number of occasions, not appreciating that the *x* coordinate of *P* was required. Some candidates sought to find the area using a single integral which received no credit as the limits were inappropriate and instead of using a simple method to find the area of the triangle, some candidates found the area by integration. No evidence was seen of using calculators to find the area required without algebraic integration being seen.

Question 7

This question was one of the more challenging ones on the paper although over a quarter were still able to score full marks. Part (a) was usually successfully answered, but it was part (b) where many candidates could not make the link between the earlier work and either proceeded to start again or had multiple attempts which became really confusing to follow.

Part (a) was answered well by most candidates, the majority of candidates scored at least three marks, with the compound angle identities being used correctly with exact trigonometric values being substituted in. These candidates went on to correctly manipulate the terms and rearrange to the given form. A significant number of candidates lost the final mark due to the use of "invisible" brackets or by making an error rearranging their terms. Other less common errors seen included, candidates expanding the brackets as if they were an algebraic bracket or using an incorrect compound angle formula, usually making a sign error for $\cos(x-60)°$.

In general, part (b) was not answered well. Many candidates did not understand that that the answer from (a) could be used, or how it could be used. This meant many candidates didn't know how to correctly start this question. Some candidates successfully used $x + 45° = 2\theta$ to proceed to $\tan(2\theta - 45)° = -2 - \sqrt{3}$ and were able to find the two required angles. Those that used $\tan(2\theta - 45)° = -2 - \sqrt{3}$ usually gained full marks. A few candidates left extra answers within the given range or only found one of the two solutions and lost the final mark. Many candidates applied compound angle formulae successfully to complete a solution but many attempting this method did not make any significant progress, with numerical and rearrangement errors being commonplace. The question also had the warning that all stages of working must be shown so any answers with no working did not score since these solutions could just be found using a graphical calculator. Candidates should also be reminded that typically questions with multiple parts will try to guide them with hints and information that should help them in later parts; this could well reduce time in trying to problem solve or make multiple attempts if greater appreciation of this is made.

Question 8

This question involved the modelling of the trajectory of a golf ball. The question differentiated the candidates well as shown by a wide spread of marks.

Part (a) required the candidates to set the given model equal to zero and solve. Many made good progress in this part, by setting $h = 0$, dividing by *x*, or *d*, and attempting to rearrange to the form $e^{0.02d} = c$. Those that got to this form were generally able to take logs correctly and find a correct value for *d*. A few left their answer in log form and so lost to last A mark. Some candidates did not heed the warning that solutions relying entirely on calculator technology are not acceptable and did not show their log work which resulted in a loss of marks. A few candidates did not always realise that *d* (or *x*) could be cancelled and so struggled to know how to proceed when they tried to rearrange to. Some took logs before rearranging, and often struggled to apply the log laws correctly or were unable to deal with *x* term correctly, and so were unable to gain any marks. Where candidates reached a correct answer almost all of them rounded to the required degree of accuracy.

Part (b) involved differentiating the given model, putting this equal to zero and rearranging to form the given equation. There were some excellent solutions to this part of the question, and usually candidates were able to apply the product rule and the log laws correctly. Most then were able to apply a correct strategy and set their $\frac{dh}{dx}$ equal to 0 and rearrange to the required form. Candidates generally found the expression easier to manipulate when they multiplied by 100 earlier on and then had integer coefficients and constant term. A minority of candidates did not apply the product rule correctly, often having only one term, so were unable to make any progress.

Part (c) involved using the given iterative model and was accessible to almost all candidates. The vast majority were able to gain at least the first 2 marks. Almost all substituted $x = 30$ into the given expression and obtained a correct answer. It was pleasing to note that the vast majority rounded to the required degree of accuracy. Some candidates thought they needed to write down a list of subsequent iterations, some even showing the substitutions, not realising that for a one-mark question a final answer was sufficient. Those that showed the working often only managed to do a few more iterations and did not reach the final limit. Although the majority of candidates stated the maximum height was 30.88, surprisingly they did not often give the units and so lost this mark. Candidate should be reminded that typically an appreciation of units is required on modelling type questions, and they should make sure to check that they have appropriate units where necessary.

Question 9

This question provided something of a challenge for most candidates and gave rise to a good spread of marks. It was pleasing to see that despite being the last question on the paper, a significant proportion of candidates were able to make good attempts and the achievement of full marks was not a rarity. Perhaps this is an indication that time pressures were not overly a problem for many and demonstrated that candidates were, on the whole, well prepared for this area of the specification. Of course, there were some candidates for whom this was not the case, and certainly incomplete and rushed solutions were also seen.

In part (a), candidates were often succinct in their working but earned the mark provided that demonstration of the substitution of $\frac{\pi}{3}$ into the expression without notational error was seen. Those who did lose the mark was often when they wrote $4\sin\left(\frac{\pi}{3}\right)^2 - 1$.

In part (b), most candidates were able to apply the chain rule correctly to differentiate the given function and the vast majority made some attempt to produce the required form. Unfortunately, working was frequently difficult to follow with side working seen on multiple pages in some cases. It was clear that many candidates were using the required answer to guide their work, and this sometimes led to errors in basic identities in an attempt to make the result ‘work’. There was a huge range in the amount of work undertaken here with some candidates achieving the required result in a few lines and others using many lines of working. The most successful candidates used Pythagorean identities to determine expressions for $\sin y$ and $\cos y$, or their squares. Others used an equivalent reference triangle. There were frequent numerical errors, particularly in finding the expression for $\cos y$ in terms of *x*. These often arose from errors in rearranging; particularly involving the square root of coefficients and/or sign errors. Some candidates took a different route and wrote $\frac{\mathrm{d}x}{\mathrm{d}y}$ as $4\sin 2y$ proceeding via Pythagorean identities and double angle for cosine to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of a single trigonometric function in a single angle. This required factorisation of the quadratic under the square root sign as a final step and was not the most efficient method but, as usual, the creativity of candidates under time pressure in an exam was pleasing to witness. Candidates would do well in similar questions to avoid being too influenced by the required answer and to stick to use of legitimate identities and valid manipulation which is far more likely to earn marks even if the required form is ultimately not reached. There is also often a temptation to jump to the answer when the link can almost be seen, but candidates should be reminded to show full working and make sure e.g. in this case that square root notation encompasses a full fraction.

Candidates who persevered into part (c) often had success here. The printed answer in part (b) gave this opportunity even if candidates had struggled with part (b). There was a not insubstantial amount of work required for each of the marks in this part and this did perhaps provide something of a time pressure so late in the paper, leading to some rushed working from some candidates. Most candidates correctly found the gradient of the tangent, the gradient of the normal and an equation for the normal without too many difficulties but work sometimes stopped here. Most candidates who did find the intersection of the normal with the x-axis and therefore the coordinates of point N were able to proceed to a correct area for triangle OPN and this was almost always given in the correct form.

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