Examiners' Report
Principal Examiner Feedback

January 2025

Pearson Edexcel International Advanced Level
In Further Pure Mathematics M3 (WFM03)
Paper 03

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2025
Publications Code WFM03_01_2501_ER

Overview

This paper provided many questions with a clear route to the answer, giving greater access than recent papers. The most challenging point on the paper was question 7(b), where few candidates even at A grade were able to discern the way forward. The presence of a proof by induction in question 8 did not perturb the higher ability candidates, whose understanding of this synoptic aspect was very good, though weaker candidates often made little progress (as is expected in WFM01 so no surprise here).

An over-reliance on calculator technology, particularly with reference to question 4, was more prevalent than previous series, despite warnings that all working must be shown and reliance on calculator technology is not acceptable. The specification aims to test candidates know, for example, what an eigenvector is and how to find them, not just be able to read them off a calculator. Marks were lost for not showing sufficient working in such cases.

All questions were attempted by the majority of candidates, with gaps left appearing to me more due to a lack of confidence in the subject matter rather than a lack of time to complete the paper.

Individual Question Report

Question 1

This question was easily accessed by most candidates with very few using incorrect identities and gaining some marks here with many being awarded all but the final mark. Most candidates used a correct identity for $\sinh^2 x$ and were able to form a correct 3 term quadratic in $\cosh x$. They then proceeded to solve it correctly, and usually rejecting the negative solution at this stage. However, some candidates failed to reject the negative root and gave one or more solutions arising from this, losing the final A mark as a result. More common, though, was for candidates to only give one solution, forgetting that $\cosh x$ is an even function and only taking the positive root.

A large number of candidates lost the final A mark as they only had one value for x rather than the two expected. Candidates who used the exponential definition to form a 3 term quadratic in e^x to find x from $\cosh x = \frac{3}{2}$, rather than using the ln form of arcosh, were marginally more successful in obtaining both possible solutions.

Only a very small number of solutions using the alternative approach were seen, where candidates formed a quartic in e^x . Again, correct exponential definitions were the norm and generally equations were solved by calculator, but algebraic mistakes were more common in this approach, so overall it was less successful.

Question 2

Although more challenging than question 1, most students were nevertheless able to make light work of most of this question, often losing only the final mark for failing to select the correct directrix and giving both. The general understanding of the equations was fine, but understanding the correspondence between a particular focus and its directrix was lacking.

Part (a) was well attempted with most students gaining at least some of the marks, though some did struggle to identify the correct expression for a^2 and lost marks as a result. A variety of methods of combining the equations seen, often requiring the correct equations to be implied, most of which were correct and gained at least the method mark. The most common errors were arithmetical, or confusion between a and a^2. Only very few lost marks because they used $b^2 = a^2\left(1-e^2\right)$ rather than the correct equation.

In part (b) the majority of students knew that the equation of the directrix was $x = \pm\frac{a}{e}$, but, as noted, many students lost the final mark because they wrote down the equations of both directrices, which was by far the most common cause of lost marks for this question. Again, confusion between a and a^2 occurred in this part, causing some to lose both marks. Very few used an incorrect equation for the directrices.

Question 3

This proved a very straightforward question for most candidates with good performances across all grade boundaries. Candidates appeared to be very well drilled in the techniques for solving these types of integrals, with the biggest problem being the instruction to "show all stages of your working", that some candidates overlooked and went to their calculators too soon. Marks may not be awarded if the necessary supporting working is not shown.

In part (i) the majority of candidates identified the need to complete the square and did this correctly (with approximately an even split between the two cases given in the scheme). Only a small number made a slip at this point, scoring M1A0, while even fewer (though there were some) failed to attempt the completion of the square – using other incorrect methods of trying to use partial fractions or rationalise the fraction instead.

Of candidates who had reached an appropriate completed square form, most spotted the standard integral leading to arctan (...) and were able to reach an answer in the correct form, though a few arcsinh or other similar incorrect functions were seen at times. There were some candidates that used the substitution $u = x + 2$ and were successful in reaching the correct answer from it.

The most common errors in this part of the questions were with the coefficients, particularly the leading coefficient with $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{16}$ all being relatively common amongst errors noted.

Again, in part (ii), almost all the candidates identified the need to complete the square and did so correctly. The integration step was also very well done, with most candidates identifying arcosh or the ln form of the integral correctly. Some candidates chose to use a substitution, typically $u = x + 2$, which was usually successful, though a small number of these made errors transposing the limits - or omitted to do so. Use of other substitutions was more prone to error, but many did successfully complete via such as $x+2=\sqrt{21}\cosh(u)$ or $x+2=\sqrt{21}\sec u$ used.

Most candidates were then able to apply the limits and give an answer in the correct form, but the last two marks were frequently lost due to insufficient working in the simplification. The question specified that all steps must be shown - candidates should be encouraged to show explicitly both the use of the ln form of the arcosh x formula and clear substitution of the limits before simplification. They should also be showing sufficient log work to justify a final answer of ln3.

Question 4

This was another well-answered question with a good demonstration of the process required shown by the majority; there were a large number of completely correct responses as well as many with just one or two inaccuracies. However, this was another question with the clear warning about showing all working and not relying on calculators in which some candidates penalised themselves by ignoring these instructions and simply writing down the eigenvalues and eigenvectors without any working shown at all – presumably from the more modern calculators – which is not acceptable for the marks.

Almost all candidates achieved the first 4 marks in parts (a) and (b). At this stage candidates were working with the unknown a and so could not resort to a calculator method, but were able to set up correct appropriate equations to find λ and show $a = 4$. A few did attempt longer approaches, attempting to set up the characteristic equation at this stage, but they usually managed to identify the correct values eventually. For some weaker candidates this was as far a they progressed, or perhaps into the first mark of part (c) where a few candidates knew to set up the characteristic equations but got in a mess with the algebra and were unable to find the values for λ, so were unable to gain any further marks.

In the main, though, the rest of the question was also well attempted with most knowing the correct methods to use. The expansion of $\det(\mathbf{M}-\lambda\mathbf{I})$ was well attempted in part (c) with only infrequent errors in the structure precluding the loss of the first M. Some candidates failed to state that they were solving $|\mathbf{M}-\lambda\mathbf{I}|=0$, but it was clear from working. As noted, a few students simply stated the values, gaining no marks for this part as no method was shown.

Following the expansion, many candidates made the equation more difficult than necessary by failing to notice the factor of $4-\lambda$ throughout. However, most managed to form a correct cubic and solved it, often by calculator – which at this stage was permitted as sufficient method to reach the cubic was shown. The majority did successfully find the remaining eigenvalues correctly, with only a few obtaining incorrect ones. Of the latter it was surprising how many did not make the connection that they already had one eigenvalue from part (a) and proceeded to their three incorrect values from part (b).

There were also occasional attempts via forming the general eigenvector equations and using these to find eigenvalues. Surprisingly such attempts were often done well, perhaps since only algebraically confident candidates attempted this method.

In part (d) again most candidates knew the process, and were able to write down the eigenvector equations, but not all were completely accurate in solving them, with errors in this part being the most common to see. Often they could attain one correct eigenvalue, but make a slip in the second, but this would lose them only one mark overall if all else was correct.

Most candidates then knew how to normalise their vectors, although some of the calculation was inaccurate. Some, however, left the vectors unnormalized, and lost four marks unless they recovered the process in part (e), which some did, but not all.

A very few candidates used the method of finding the vector product of 2 rows of $\mathbf{M}-\lambda\mathbf{I}$ to get an eigenvector; these were generally efficient and correct.

Most students gained both marks in part (e) as this followed from their eigenvalues and vectors. There were very few solutions where their **P** and **D** matrix did not match, though a small number of candidates did mix **P** and **D** up. The majority candidates knew how to write down the matrices **P** and **D** and correctly placed the columns in corresponding order. Some thought they were required to show the calculation of $\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$, and so did a lot of unnecessary work. A few candidates missed this part, leaving it empty.

Question 5

As with the preceding questions, performance was again very strong on this question. The lead in via part (a) led candidates to the correct expression to make good progress in the next two parts, where the correct formulae were usually clearly identified and many were able to secure full marks. A lack of working was again evident in places, particularly when evaluating the integrals – it would be advised to show the substitutions, but actually writing down the integrals themselves is imperative, as the question clearly instructed algebraic integration must be used (and therefore seen).

Part (a) was usually completely accurately with only very few losing marks in this part – usually for a lack of sufficient working and jumping too quickly to the answer. The derivatives were both determined very well, with virtually no students attempting them making errors. Most did then show the expansion required before factorising to achieve the correct answer.

With the needed result established and formula in the formula book (or remembered), most candidates were able to identify the correct form of integral needed in (b), but there was a significant minority who did not actually take the square root of the result from (a) (despite a correct formula stated), and this was the main reason for lost marks. The integration itself posed no problem, being a very straightforward polynomial, and applying correct limits almost always followed, though a small number of candidates misread the lower limit as $\frac{3}{2}$ instead of $-\frac{3}{2}$. Algebraic after substituting were rare, though again working was not universally shown, with many going to the answer direct from the integral. Consequently, the majority were able to score at least one mark, with most scoring all three in this part.

Part (c) followed a similar pattern to part (b), with the main reason for loss of marks down to omitting to square root the result of part (a) when applying the formula. Most had a correct formula for the surface area, with just a few using either $2\pi\int x\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$ or just $2\pi\int\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$, and again there was little trouble with the limits, though a few did try to use adapted limits for y instead of t. The 2π was seldom omitted completely, though in a couple of instances it disappeared before the final answer was reached.

Generally, the accuracy of the expansions and integration were good (even if they had forgotten to take the root – they could still score both M's), though algebraic slips were more common in this part than in part (b). However, once again some did not form the integral correctly but quoted the correct answer, obviously by using a calculator, so did not gain any credit.

Occasionally candidates tried to add on the area of the circular "ends" of the shape formed, not appreciating that the lines $x = 0$ and $x = 27$ were not part of the curve, and so it is an open-ended surface. This was only penalised in the final mark.

Question 6

This question was generally well done by most candidates, which for a question on vectors was a pleasant surprise. Perhaps a question of this ilk was well expected, and there were no twists in this question that caused issues, the processes were clear for each part and executed well.

Almost all candidates realised they needed to find the vector product for part (a) and did so correctly. In a small number of cases sign errors led to an incorrect vector, but this was not common. Very few used the simultaneous equation approach using the scalar product, but when done, again it was usually successful.

In part (b) the vast majority of candidates understood the process for finding the required angle and began by finding a correct direction vector for AB. They then went on to find the scalar product, with this mainly being seen embedded within an expression for cos β or sin α. In a small number of cases candidates went directly from a vector expression for the scalar product to a value - this could result in the loss of a method mark if their value was not correct for their vectors, and once again the need to show working must be stressed.

Most candidates proceeded to find the correct angle, many by using sin α directly, others by using cos β and subtracting from 90°. A small number of candidates made mistakes in finding the angle having obtained a negative value of cos β.

A few rounded to 60 degrees as their answer, without writing the more accurate 59.435… first, costing themselves a mark as the angle was required to the nearest degree, and although anything rounding to 59 was accepted, such a value needed to be seen to award the mark.

Some weaker candidates made little or no progress in this part as they either did not find *AB* at all, instead attempting the angle between the two direction vectors of the plane, or found *AB* but attempted the scalar product formula with one of the vectors from the plane equation, rather than with the normal vector found in part (a).

Part (c) was again answered well with a variety of methods being seen, as well as a variety of workings shown, with some responses being very brief and simply stating a formula followed by the answer. Such strategies are risky as an incorrect value could lead to the loss of all marks, whereas showing working is likely to gain method marks even if an error leads to an incorrect answer.

The main method in the scheme was most often seen, with many candidates correctly finding the distance using 53 (though the working to reach 53 was sometimes unclear). The first alternative method was also popular, which is very direct, although some candidates were at risk of not showing sufficient working when they applied this method (repeating their work from (b)). Other methods were less common but all approaches on the scheme were seen during marking, as well as some variations, but overall the majority were able to reach a solution, usually correct.

Question 7

This question was easily the most challenging question on the paper, providing a good discriminant of ability, with a wide variety of marks scored. There was access for lower grades in part (a), but part (b) challenged even the higher grade candidates, with relatively few able to find their way to a fully correct solution in comparison to performance on the rest of the paper.

The bookwork of part (a) was generally well attempted with most students making good progress here and many gaining full credit. Parametric differentiation was the most common approach to finding the gradient of at P, though implicit differentiation was also common. Errors in the process were rare, and likewise with moving from tangent to normal gradient, the majority reaching the correct normal gradient without difficulty. It was pleasing to note that attempts with x and y still in the gradient were much more rare this series than in previous ones, though with the result to aim for most were no doubt able to see what the gradient needed to be. Nevertheless, the working to establish it was clear in most case.

In finding the equation of the line, the form $y-b\sin\theta=m_N\left(x-a\cos\theta\right)$ was used by the majority, with use of $y = mx + c$ being much less common. Whichever was used, there was usually little problem moving from this equation via an intermediate step to the given answer, though a few did omit an intermediate step, losing the A mark.

Part (b) was much less well attempted, with many not progressing beyond the first M mark, and some not attempting this at all. The failure to identify the normal crosses at the point $\left(0,-b\right)$ stumbling point for many. Some did not give a coordinate at all, while others thought in error that it was $(0, b)$. The latter were often able to make some more progress but could not achieve a fully correct proof.

Of those who did attempt part (b), most were able to make a correct attempt at an expression for the area of the triangle, although it was often via using $y=\dfrac{\left(b^2-a^2\right)}{b}\sin\theta$ as the base of the triangle, not realising it is really just b. The main scheme formula $\frac{1}{2}a\cos\theta\times"b"$ was by far the most common approach, though the determinant approach was also noted. Fortunately, attempts at finding the area via more complicated formula were rare, as some of these led to much more intense algebra to reach the required result. For many, setting up the area formula was as far as they progressed, as without identifying the other key fact of crossing at $\left(0,-b\right)$ they could not set up appropriate equations to solve. That did not stop many trying to use the given result to backwards work a relationship between a and b and blur an answer, though they could not gain marks for this.

For those that did find two equations (whether both correct or not) there usually followed an attempt to solve simultaneously. However, most did this via using $\sin^2\theta + \cos^2\theta = 1$ to form an expression in *a* and *b*, making reaching the result more difficult. Some inventive approaches using a substitution $t = \frac{b}{a}$ or similar were seen, and used with some success, while others managed to reach the polynomial in *a* and *b* before stopping, or stating the relationship without showing the factorization, gaining the method marks only as they had not shown the factorization needed to justify the final result.

Those who saw they could equate b^2 from both equations and eliminate both *a* and *b* had more success in reaching the cubic in $\sin\theta$ but again generally omitted to show the factorization, and simply stated the result at the end, losing the final A. For a "show that" question a clear demonstration of the result is needed, not a reliance on a calculator providing a root. The scheme could even have been more demanding in insisting that a reason for no further roots be given, but a factorized form was deemed sufficient for the demand the question provided.

Part (c) was better completed compared to part (b), with access to it via the given result in part (b), and many gained at least the B mark. Further progress only required one of the equations in part (b) to have been identified, so those who only proceeded as far as the area could make progress here. However, it was evenly split between those who did and did not go on to find the *y* coordinate in terms of *a*. Many simply gave it in terms of *b* instead, while others offered no solution, or an incorrect guess for which it was unclear where they obtained their result.

Question 8

This question performed as expected with a wide variety of marks scored. The synoptic proof by induction content would certainly have caught out some, but was well answered by many. Those who know the induction process were able to make good progress, even in the mid-range grade band, while high grade candidates were able to produce some excellent proofs. Conversely there were also some very poor attempts seen, where candidates clearly were not comfortable with the process of proof by induction at all, and some attempted direct proofs, while others set out in attempts at integration by parts to work backwards, which proved a fruitless exercise – no fully correct approaches via integration were observed.

The start of the question was accessible to the majority, and most gained at least the first method mark for attempting the first derivative. However, a lot of candidates lost the accuracy mark in for the base case due to not showing sufficient detail to prove the result, as detailed on the mark scheme. A clear demonstration of how the derivative fitted the general formula was required. Most did work with $n = 1$ as the base case, though a few used $n = 0$ as a starting point, which did work in this case. It would have been instructive to have had $y = e^{3x} \cosh 2x + 5$ or similar as the function to screen out such cases.

The inductive step was less well attempted, however, and by and large dependent on whether candidate knew how to carry out proof by induction or not. Some just worked through the first few derivatives rather than making the inductive assumption and attempting the (k+1)-th derivative, meaning that they did not gain any further marks. Likewise, some attempt integrations that had no bearing on the result, while many others did no more than the first derivative and stopped.

For those who knew the process required for induction, good progress into at least the two method marks was usually made. The differentiation was again very good, with mistakes in the derivative being rare. Algebraic slips when simplifying was much more the cause of errors losing the A marks. Some struggled with the algebra and either made such mistakes, e.g. losing the 3 in the exponent, making sign slips, or just wrote down the result not seeing how to reach it and forfeiting the final three marks as a result.

But many were able to manipulate the derivative successfully to achieve the required result for the (k + 1)-th derivative, with suitable steps shown, and achieve the penultimate A.

However, not all of these went on to gain the final mark because they did not make an appropriate conclusion. Though many did know the key statements to make, many others did not state or convey that "if the result is true for $n = k$ then it is true for $n = k+1$". Commonly, statements similar to saying it is true for $n = 1$ and $n = k$ and $n = k + 1$ were made, or no concluding sentence at all. Identifying "true for $n = 1$" and "true for all n" were usually given correctly, but the "if then" connotation is what candidates are most likely to get wrong in the concluding remark.

Access to the final A mark was given even if the other A marks were not gained due only to lack of detail, so candidates were not over penalized if they lost early marks for not giving the demanded rigor, and some availed themselves of this, albeit only rarely the case.

Notation was generally sound in this question, though a few candidates worked with n instead of k throughout, which can make the proofs less clear. However, such candidates tended to be good mathematicians and so the arguments were often robust. But in general use of k helps candidates with their own understanding of the induction process, and should be encouraged.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom