Pearson Edexcel

# Examiners' Report
# Principal Examiner Feedback

## Summer 2024

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

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### Introduction

This paper proved to be a fair test of student knowledge and understanding. There were many accessible marks available to all students as well as some more challenging questions for higher ability students.

### Question 1

The opening question on a hyperbola was a good source of marks for almost all students.

In part (a), correct equations in *a* and *e* were invariably formed from the given foci and directrices and it was rare to see these solved incorrectly. Occasionally the value of *e* was given as $\pm\frac{13}{12}$.

Part (b) also saw very good scoring. Most were able to use a correct eccentricity formula with their *a* and *e* to find a value for $b^2$ or *b*, although there were some slips with the application of the formula. Most proceeded to find a consistent equation of a hyperbola although the equation of an ellipse was very occasionally seen. The most common error was to confuse values of $a^2$ and $b^2$ with values of *a* and *b*. Some did not give the answer in the required form and there were also a few slips obtaining the integer coefficients.

### Question 2

Many completely correct attempts were seen in part (a) although the last two marks of the three in part (b) were not widely awarded.

The method in (a) was well known although $\det(\mathbf{M}+\lambda\mathbf{I})=0$ was seen on occasion. Most achieved an appropriate cubic equation with some using Sarrus. This equation was invariably solved correctly although there were a few sign errors. A correct method to find an eigenvector usually followed, although there were a few who obtained a zero solution. Some did not go on to normalise their eigenvector. Those who did, usually performed this correctly, although a small number multiplied by the magnitude instead of dividing. A small number gave final answers which were not fully processed since they included fractions within fractions.

The first mark in (b) was widely scored, with most transforming the parametric form although occasionally two column vectors were used. Most matrix multiplications were fully correct. However, the $\mathbf{r}\times\mathbf{b}=\mathbf{c}$ form of a line was not familiar to many students and a significant number just substituted their two vectors as **b** and **c** into this equation instead of using $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}.$ Most of those who knew the method produced a correct equation although there were a few sign slips with the vector product.

**Question 3**

This question on differentiation was well-answered on the whole. There were lots of possible methods in (a) but the most popular was to differentiate explicitly although there were a substantial number of attempts that took sinh of both sides before differentiating implicitly. Slips included using not using the chain rule, errors with the derivative of $\sqrt{x^2-1}$ and use of $\frac{1}{\sqrt{1+x^2}}$ rather than $\frac{1}{\sqrt{1+\left(\sqrt{x^2-1}\right)^2}}$. An intermediate line of working was required and this was seen in most responses. Those who differentiated implicitly needed to show that $\cosh y = x$ with clear use of the identity $\cosh^2 x = 1+\sinh^2 x$ and this wasn't always the case. A small number used the logarithmic form of arsinh before differentiating. This isn't usually a sensible strategy when differentiating an inverse hyperbolic function, but the algebra this time was reasonably straightforward and most attempts via this route were correct.

The main error in part (b) was to carelessly use the derivative of artanh instead of arctan. However, a lot of completely correct derivatives were seen. It was surprising to see a significant number unable to appropriately process the resulting equation, with some students getting into difficulty by choosing to combine fractions rather than cross-multiplying. Those that reached the correct three term quadratic usually found the correct solutions for $x^2$. Occasionally these weren't square rooted or the incorrect $\pm\sqrt{2}$ and $\pm\sqrt{5}$ were given without subsequent rejection of the negative values.

**Question 4**

Question 4 was on hyperbolic functions and saw a more mixed response.

A few attempts at the proof in part (a) did not use exponential definitions. Those that sensibly started with the right-hand side usually made good progress. The exponential forms of sinh and cosh were generally well-recalled and most went on to suitably expand the exponential expressions. There were some untidy attempts which included wrong powers and/or sign errors. "Meet in the middle" or "1=1" style approaches needed a conclusion which wasn't always provided. A small number started with the left-hand side but these were usually unconvincing.

Although there were many fully correct and succinct solutions to part (b), some students were clearly not prepared for this question part. The two correct equations were fairly commonly seen but there was a widespread error in thinking $R^2 = 10^2 + 8^2$ instead of $10^2 - 8^2$. A correct method to obtain a value for $\alpha$ was often seen, although this was sometimes done unnecessarily via exponential definitions rather than using the logarithmic forms of the inverse hyperbolic functions that are provided in the formula book.

Part (c) was a "Hence" which required the use of part (b) and this wasn't always noticed by students. Those who used the logarithmic form of arsinh usually scored at least the first mark. Those attempting to evaluate $\operatorname{arsinh}\left(3\sqrt{7}\right)$ by exponential definitions often came unstuck, particularly those who formed the equation $3\left(\mathrm{e}^{x+\ln 3} - \mathrm{e}^{-x-\ln 3}\right) = 18\sqrt{7}$. A small number of otherwise correct answers lost the last mark for failing to bracket the $\sqrt{7} + \frac{8}{3}$.

**Question 5**

This integration question saw very good scoring with the first three marks but part (c) was more demanding.

In parts (a) and (b) it was very rare to see incorrect values. However, some students did obviously not appreciate how the setup in (b) was a hint for part (c). It was fairly common to see an arsinh function result from the first integral, although the coefficient of $\frac{1}{2}$ was sometimes missing. Occasionally the square root was lost, leading to an artanh function. A small number went straight to logarithmic form and were usually correct. The second integral proved much more difficult – it seems that $\frac{f'(x)}{(f(x))^n}$ is not recognised by some students as an integrable form. Some embarked upon substitutions for these integrals which led to mixed results including errors with limits. Those who had integrated correctly were usually able to proceed to the correct answer, although those who worked on the integrals separately occasionally failed to combine their results correctly.

**Question 6**

This question was on an ellipse and was a good source of marks in part (a), although part (b) proved to be fairly discriminating, with the last three marks not widely scored.

The standard "bookwork" of part (a) was generally performed well. Most sensibly chose to use parametric differentiation although implicit differentiation was also fairly popular. Most proceeded to form a correct derivative to form a consistent normal equation. An intermediate line of work following the equation was required before the final answer and this wasn't always seen. A small number did not achieve an acceptable form for the final equation – students are advised to end their attempts at questions where the answer is fully given with exactly what is printed on the question paper.

In part (b), most were able to obtain the correct coordinates of *Q* and the midpoint of *PQ*, although there were a few incorrect methods seen for the midpoint. A significant number did not then know how to proceed with many thinking that the equation of a locus was needed. Many did not appreciate the impact that the negative y-coordinate had on the area and corrected it to positive before attempting the triangle area. Some assumed the triangle was always right-angled. However, there were some with clear strategies – those who identified that the area of triangle *OPM* was half the area of triangle *OPQ* were almost always successful. A small number used the shoelace method and were usually correct. Those who had obtained the correct $\frac{20}{3}\sin\theta\cos\theta$ were usually able to deduce that the maximum area was $\frac{10}{3}$. Occasionally this wasn't subsequently justified. Those who converted the expression to $\frac{10}{3}\sin 2\theta$ usually followed this with appropriate reasoning involving the maximum value of $\sin 2\theta$ but students who differentiated were also likely to pick up all the marks.

**Question 7**

This question on arc length proved to be fairly challenging although a range of mark profiles were seen.

In part (a), most could achieve a correct form for the derivative and proceeded to use a correct formula, although a very small number used the formula for surface area. The key was to spot that $\frac{\mathrm{d}y}{\mathrm{d}x}$ could be written as sinh $x$ and those who noticed this were highly likely to score all four marks. Students who embarked upon processing $1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ with the half-argument expressions tended to get lost in the algebra, with several unconvincingly "fudging" the answer.

Despite the question explicitly requiring all stages of working to be shown, many responses to part (b) again jumped to the given answer without sufficient justification. Integration of coth $x$ was reasonably well known. A small number chose to use the substitution $u = \mathrm{e}^x - \mathrm{e}^{-x}$ and were usually correct. Substitution of limits was performed well but the next two marks required a clear demonstration on how the resulting exponential expression could be processed to become the given answer. Those who took a bit of time to manipulate the powers of e usually scored full marks. A few cleverly used the difference of two squares or deployed an appropriate multiplier. The arguably most elegant solution of using $\sinh 2 = 2\sinh 1\cosh 1$ was not at all common, although it was seen more than expected.

**Question 8**

Reduction formulae questions are usually ones which students find demanding but there were many fully correct solutions here.

In part (a), most knew to apply parts in the correct direction to get things started and although most could achieve a correct form, slips, particularly with signs, were fairly common. Some did not know how to progress but most realised the need to split $(k-x)^{\frac{3}{2}}$. Some attempts then floundered but many were able to expand and obtain an equation in $I_n$ and $I_{n-1}$. It was quite common to see attempts that did not apply limits to the first term at any stage. Many otherwise correct solutions did not clearly show how this term became zero. An intermediate line was required before the given answer and was occasionally omitted. There were also attempts that clearly worked backwards from the answer but invariably these did not form a coherent proof. Attempts at splitting first before integration were rare and not usually successful.

Part (b) was a reasonable source of marks for students, even if they had not made much progress in part (a). Most knew to apply the reduction formula twice – attempts at $I_2$ directly were not common, although some obtained $I_2$ in terms of just $I_1$ and then integrated which needlessly created extra work. Most knew how to integrate to achieve the correct form for $I_0$ but slips with the reduction formula and errors substituting limits meant that a fair number of incorrect equations in *k* were arrived at. Some who had the correct equation did not know how to deal with the fractional power to find a simplified value for *k*. However, the correct answer of $\frac{3}{4}$ was fairly widely achieved.

## Question 9

The last question on vectors was fairly discriminating as was expected, but there was plenty of partial scoring as well as many fully correct responses.

In part (a), most knew that a normal vector was required and almost all chose the two correct vectors although there were a few slips in calculating the vector product. Most went on to determine the scalar product of the given point with their normal. A small number did not go on to establish a Cartesian equation. Approaches using simultaneous equations were rare, but usually correct.

A wide range of approaches were seen in part (b) with a similarly wide range of outcomes. Some opted for a conventional algebraic method and were usually able to form an equation of a line. Slips with the algebra and errors extracting the relevant vectors from the initial equation were fairly common with this approach. Systematic errors such as obtaining coordinates of the point which were of the opposite sign were penalised. There was more success seen with attempts that found a point by inspection and then calculated the vector product of both normals. A small number of students found two points and obtained the direction by subtraction. Most proceeded to place the point and direction correctly into a vector equation although occasionally "**r** = ..." was missing or "*l* = ..." was written instead. Since correct points tended to have fractional components, some students inappropriately multiplied the position vector by a scalar in an attempt to simplify it.

Part (c) required use of the line from part (b) so students solving simultaneous equations manually or on their calculators received no credit here. The method was well known and many solved an appropriate equation to obtain the value of their parameter. There were a few slips substituting back to find the coordinates but a lot of well-organised solutions were seen that arrived at the correct (2, 3, –1).

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