Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 20 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

P75716A

H:1/1/
P 7 5 7 1 6 A 0 1 3 2

Pearson

1.

$$\frac{dy}{dx} - 3y\tan x = \sec^2 x$$

(a) Show that an integrating factor for this differential equation is given by

$$p(x) = \cos^3 x$$

(2)

Given that $y = 4$ when $x = \frac{\pi}{4}$

(b) determine the particular solution of the differential equation.

Give your answer in the form $y = f(x)$.

(4)

Question 1 continued

(Total for Question 1 is 6 marks)

2. (a) Use algebra to determine the exact x coordinates of the points of intersection of the curves with equations

$$y = \frac{2x}{x^2 + 1} \qquad \text{and} \qquad y = \frac{1}{x + 4}$$

(3)

Hence

(b) determine the values of x for which

$$\frac{2x}{x^2 + 1} < \frac{1}{x + 4}$$

(2)

(c) state the values of x for which

$$\frac{2x}{x^2 + 1} < \frac{1}{|x + 4|}$$

(2)

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Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3.

$$2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y\frac{\mathrm{d}y}{\mathrm{d}x} - 6xy = 0$$

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$ and $y = \frac{1}{2}$ at $x = 2$

(a) determine the value of $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ at $x = 2$

(6)

(b) Hence determine the series expansion for y about $x = 2$, in ascending powers of $(x - 2)$ up to and including the term in $(x - 2)^3$, giving each coefficient in simplest form.

(2)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

4. (a) Express $\dfrac{r+4}{r(r+1)(r+2)}$ in partial fractions.

(4)

(b) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{n(Pn+Q)}{2(n+R)(n+S)}$$

where P, Q, R and S are integers to be found.

(5)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5.

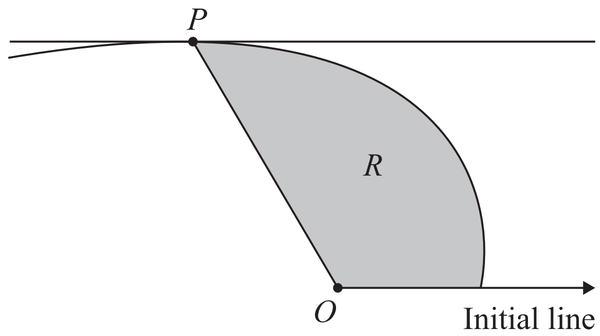

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve with polar equation

$$r = \sqrt{3} + \tan\frac{\theta}{2} \qquad 0 \leqslant \theta < \pi$$

The tangent to the curve at the point P is parallel to the initial line.

(a) Using the identity $\tan\frac{\theta}{2} \equiv \frac{1 - \cos\theta}{\sin\theta}$ or otherwise, determine the exact value of θ at P.

(4)

The region R, shown shaded in Figure 1, is bounded by the initial line, the curve and the line OP, where O is the pole.

(b) Use algebraic integration to determine the exact area of R, giving your answer in the form $p\ln 2 + q\pi + r$ where p, q and r are constants.

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. (a) Determine the general solution of the differential equation

$$4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 37y = 6e^{5x}$$

(6)

Given that $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(b) determine the particular solution for this differential equation.

(5)

Question 6 continued

Question 6 continued

P 7 5 7 1 6 A 0 2 2 3 2

Question 6 continued

(Total for Question 6 is 11 marks)

7. (a) Use De Moivre's theorem to

(i) show that

$$\sin 5\theta \equiv 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$$

(ii) determine an expression for $\cos 5\theta$ in terms of $\sin\theta$ and $\cos\theta$

(4)

(b) Hence show that, for $\cos 5\theta \neq 0$

$$\tan 5\theta \equiv \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$$

(2)

(c) Using the result of part (b) and showing all stages of your working, determine the solutions of the equation

$$2x^5 - 15x^4 - 20x^3 + 30x^2 + 10x - 3 = 0$$

giving your answers to 3 decimal places.

(5)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. A transformation T from the z-plane, where $z = x + \mathrm{i}y$ to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{(\sqrt{3} - \mathrm{i})(z - 2)}{z + 2} \qquad z \neq -2$$

(a) Show that the real axis in the z-plane is mapped by T onto the line with equation $v = -\frac{1}{\sqrt{3}}u$ in the w-plane.

(3)

(b) Show that the circle in the z-plane with equation $|z| = 2$ is mapped by T onto a line in the w-plane, stating clearly an equation for this line.

(5)

The region R in the z-plane is defined by

$$\{z \in \mathbb{C} : |z| < 2\} \cap \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$$

(c) Determine the image of R under T, giving your answer in the form

$$\{w \in \mathbb{C} : \alpha < \arg w < \beta\}$$

where α and β are rational multiples of π

(5)

Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS