Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 14 January 2025

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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P 7 6 1 9 2 A 0 1 3 2

Pearson

1. $$\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix} \qquad \text{where } p \text{ is a constant}$$

(a) Determine det $\mathbf{P}$ in simplest form in terms of p.

(2)

(b) Hence show that $\mathbf{P}$ is non-singular for all real values of p.

(2)

(c) Determine $\mathbf{P}^{-1}$ in terms of p.

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2. $$f(x) = x^2 - \frac{7x - 4\sqrt{x}}{x^3} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [0.3, 0.4]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 0.3$ as a first approximation for α, apply the Newton–Raphson procedure once to $f(x)$ to determine a second approximation for α, giving your answer to 3 decimal places.

(2)

The equation $f(x) = 0$ has another root β in the interval [1.3, 1.5]

(d) Use linear interpolation once on the interval [1.3, 1.5] to determine an approximation for β, giving your answer to 3 decimal places.

(2)

Question 2 continued

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Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. The quadratic equation

$$3x^2 - 2x + 5 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine the value of $\alpha^2 + \beta^2$

(2)

(c) determine a quadratic equation that has roots

$$\left(\alpha + \frac{1}{\alpha}\right) \text{ and } \left(\beta + \frac{1}{\beta}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(4)

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Question 3 continued

(Total for Question 3 is 7 marks)

4.
$$f(z) = 6z^3 + Az^2 + Bz + C$$

where A, B and C are integers.

Given that $z = \frac{2}{3} + \frac{\sqrt{17}}{3}\text{i}$ is a root of the equation $f(z) = 0$

(a) write down the other complex root of the equation $f(z) = 0$

(1)

Given that $z = -\frac{3}{2}$ is also a root of the equation $f(z) = 0$

(b) determine the value of A, the value of B and the value of C.

(5)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)

5. (a) Use the standard results for summations to show that for all positive integers n

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+a)(n+b)(n+c)$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the value of

$$20 \times 21 \times 25 + 21 \times 22 \times 26 + \ldots + 40 \times 41 \times 45$$

(2)

Question 5 continued

(Total for Question 5 is 7 marks)

6. The rectangular hyperbola H has equation $xy = 100$

The point $P\left(10t, \frac{10}{t}\right)$, where $t \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$t^3x - ty = 10\left(t^4 - 1\right)$$

(4)

The normal to H at P meets the y-axis at the point Q.

Given that the area of triangle OPQ, where O is the origin, is 750

(b) determine both possible pairs of coordinates of the point P.

(5)

P 7 6 1 9 2 A 0 1 6 3 2

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. (i)

$$\mathbf{A} = \begin{pmatrix} -\dfrac{1}{2} & \dfrac{\sqrt{3}}{2} \\ -\dfrac{\sqrt{3}}{2} & -\dfrac{1}{2} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a stretch scale factor 2 parallel to the x-axis.

(b) Write down the matrix **B**.

(1)

The transformation represented by matrix **B** followed by the transformation represented by matrix **A** is the transformation represented by the matrix **C**.

(c) Determine **C**.

(2)

(ii)

$$\mathbf{M} = \begin{pmatrix} k & -2 \\ -1 & 2k \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that the transformation represented by matrix **M** maps the point (k, k) onto the point $(35, 91)$

(a) determine the value of k.

(4)

A quadrilateral Q is transformed to another quadrilateral Q' by the matrix **M**.

Given that Q' has area 336

(b) use the value of k found in part (ii)(a) to determine the area of Q.

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & 9n \\ -n & 3n+1 \end{pmatrix}$$

(5)

(ii) A sequence of numbers is defined by

$$u_1 = 1 \qquad u_2 = 4$$

$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^{n-2}(n+2)$$

(5)

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. The parabola C has equation $y^2 = \frac{1}{2}x$

The point $P\left(\frac{t^2}{8}, \frac{t}{4}\right)$, where $t \neq 0$, lies on C.

(a) Use calculus to show that the tangent to C at P has equation

$$8yt - 8x = t^2$$

(3)

Given that

- the tangent to C at P meets the y-axis at the point A
- the line l_1 is the perpendicular bisector of the line segment OA where O is the origin
- the line l_2 is the perpendicular bisector of the line segment AP
- l_1 and l_2 intersect at the point Q

(b) show that, as t varies, the coordinates of Q satisfy the equation

$$y^2 = \alpha x + \beta$$

where α and β are constants to be determined.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS