

INTERNATIONAL GCSE

Mathematics (Specification B) (9-1)

EXEMPLARS WITH EXAMINER COMMENTARIES

PAPER 2

Pearson Edexcel International GCSE in Mathematics (Specification B) (4MB1)



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Introduction

1.1 About this booklet

This booklet has been produced to support teachers delivering the Pearson Edexcel International GCSE in Mathematics B specification. The Paper 2 exemplar materials will enable teachers to guide their students in the application of knowledge and skills required to successfully complete this course. The booklet looks at questions 3, 4, 5, 6, 7, 8, 9, 10 and 11(a) and (b) from the June 2019 examination series, showing real candidate responses to questions and how examiners have applied the mark schemes to demonstrate how student responses should be marked.

1.2 How to use this booklet

Each example covered in this booklet contains:

- Question
- Mark scheme
- Exemplar responses for the selected question
- Example of the marker grading decision based on the mark scheme, accompanied by examiner commentary including the rationale for the decision and where relevant, guidance on how the answer can be improved to earn more marks.

The examples highlight the achievement of the assessment objectives at lower to higher levels of candidate responses.

Centres should use this content to support their internal assessment of students and incorporate examination skills into the delivery of the specification.

1.3 Further support

A range of materials are available from the Pearson qualifications website to support you in planning and delivering this specification.

Centres may find it beneficial to review this document in conjunction with: [the specification](#), [sample assessment materials](#), [Getting Started Guide](#) and [the Principal Examiner's Report](#).

Question 3

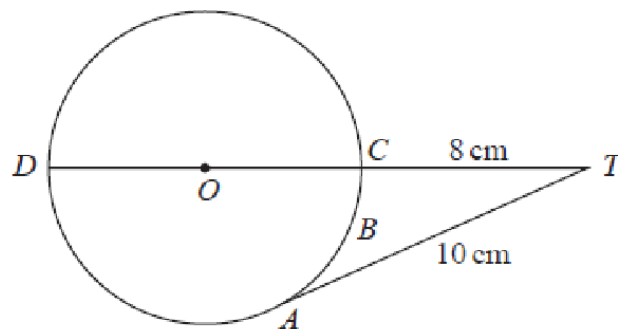


Diagram NOT accurately drawn

Figure 1

Figure 1 shows the circle $ABCD$ with centre O and diameter DC .

The point T is such that $TCOD$ is a straight line and TA is the tangent to the circle at A .

$$AT = 10 \text{ cm} \quad TC = 8 \text{ cm}$$

(a) Calculate the radius, in cm, of the circle.

(3)

(b) Calculate the length, in cm to 3 significant figures, of the arc ABC .

(3)

Mark scheme

Question	Working	Answer	Mark	Notes	
3	(a)	$8 \times (8 + CD) = 10^2$	3	M1 Allow use of x , d , $2r$ etc.. for CD Method to find DT does not gain this mark until either an expression involving CD or r is formed.	
		$(CD =) \frac{10^2}{8} - 8 (= 4.5)$ oe		M1 dep. Fully correct expression for CD or r	
				2.25 (cm) oe	A1
	(a) ALT	$10^2 + r^2 = (8 + r)^2$		M1 Correct equation involving r .	
		$100 = 64 + 16r$ oe		M1 dep. Correct equation involving only linear terms in r .	
				2.25 (cm) oe	A1
	(b)	$\tan \angle AOC =$	3	M1 Any correct expression for \sin , \cos or \tan of AOC or AOC	

		$\frac{10}{"2.25"}$ or $\sin \angle AOC =$ $\frac{10}{8+"2.25"}$ or $\cos \angle AOC =$ $\frac{"2.25"}{8+"2.25"}$ $(\Rightarrow \angle AOC = 77.3^\circ)$ Allow $10^2 = (8 + "2.25")^2 + "2.25" - 2 \times (8 + "2.25") \times 2.25$ $\square \square \square \cos \angle A$		If candidate gives an expression for ATO this mark is only gained when they use ATO to find AOC .
		$\text{arc length} = \frac{"77.3"}{360} \times 2\pi \times "2.25"$		M1 dep. Do not follow through if $64^\circ \leq AOC \leq 68^\circ$ or $AOC \varepsilon 90^\circ$
			AWR 3.03 or 3.04 (cm)	A1
Total 6 marks				

Exemplar response A

$$(a) \quad 8 \times DT = 10^2.$$

$$DT = \frac{100}{8} = 12.5.$$

$$DT - 8$$

$$12.5 - 8 = 4.5 = DC = \text{diameter}.$$

$$\frac{4.5}{2} = 2.25 = \text{radius}$$

~~cos x =~~

$$\sin x = \frac{p}{h}$$

$$(b) \quad \text{arc length} = \frac{x}{360} \times 2\pi r.$$

$$\sin x = \frac{10}{8 + 2.25}$$

$$\frac{77.3}{360} \times 2\pi(2.25) \quad x = \sin^{-1}\left(\frac{10}{10.25}\right)$$

$$= 3.04 \text{ cm}$$

$$x = 77.3^\circ$$

Examiner's comments:

This response was given 6 marks.

This candidate has given a fully correct response to both parts of the question. In part (a), the response clearly shows which value the candidate is finding at each stage, which ensures there is no ambiguity. In cases where a numerical slip prevents the award of the final accuracy mark, this ensures all method marks can be awarded.

Awarded M1M1A1.

In part (b), the correct angle of 77.3° is seen and used with working shown later in the question to justify the value. Showing working in a logical order helps ensure candidates gain full credit for their working.

Awarded M1M1A1.

Exemplar response B

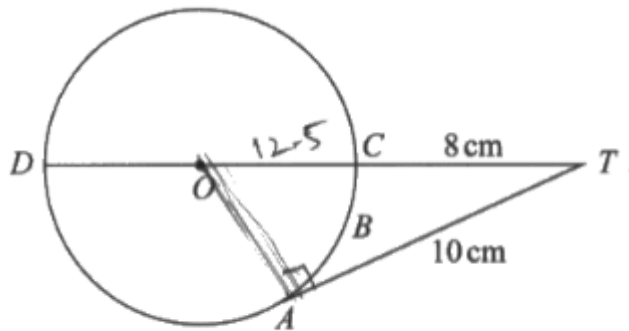


Figure 1

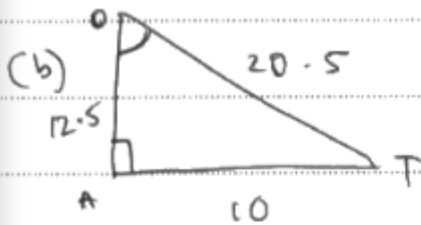
$$(a) \quad OC \times CT = 10^2$$

$$OC = \frac{10^2}{8}$$

$$OC = 12.5$$

$$OC = \text{radius}$$

$$OC = 12.5 \text{ cm}$$



$$\frac{20.5}{\sin 90} = \frac{10}{\sin \hat{\theta}}$$

$$\sin^{-1} \left(\frac{\sin 90 \times 10}{20.5} \right) = \hat{\theta}$$

$$\hat{\theta} = 29.196^\circ$$

$$\text{arclength} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{29.196}{360} \times 2\pi \times 12.5$$

$$\text{arclength} = 6.3696 \text{ cm}$$

$$\text{arclength} = 6.37 \text{ cm} \quad (3\text{sf})$$

Examiner's comments:

This response was given 2 marks.

Despite being a topic that was commonly set on the old 4MB0 specification, the intersecting chord theorem and variants such as seen here in part (a) are often poorly attempted by candidates.

Part (a) - this is a good example of a candidate who has some idea of the topic but unfortunately cannot gain any marks due to a fundamental error in the formation of their equation. Although 12.5 is a value that can be used towards the correct solution (as seen in candidate A), this candidate has clearly labeled the radius with this value, which is incorrect. Even if this candidate had correctly identified DT as 12.5, this is insufficient working for the award of the first method mark as any expression must include CD or r as outlined in the mark scheme.

Awarded M0M0A0.

Part (b) - as the answer from part (a) is incorrect, the final answer will be wrong and so it is impossible for this candidate to gain full marks. Given their result from part (a), their method is correct and so they gain the two method marks for this part. With questions requiring trigonometry it is not uncommon to see candidates use more complex techniques than required as seen here with the use of the sine rule in a right triangle. If used correctly, this will not be penalized but there is a greater chance of candidates making errors in these circumstances and they are also likely to find that these more complex techniques require more time to complete.

Awarded M1M1A0.

Question 4

Triangle A is drawn on the grid opposite.

The points $(1, 2)$, $(3, -2)$ and $(1, 4)$ are the vertices of triangle B .

(a) On the grid, draw and label triangle B . (1)

(b) Describe fully the single transformation that maps triangle A onto triangle B . (2)

Triangle C is the image of triangle B under an enlargement with scale factor 2 and centre of enlargement the origin.

(c) On the grid, draw and label triangle C . (3)

Triangle D is the image of triangle C under the transformation with matrix \mathbf{M} where

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix}$$

(d) On the grid, draw and label triangle D . (3)

Triangle C is the image of triangle D under the transformation with matrix \mathbf{N} .

(e) Find the matrix \mathbf{N} . (3)

Mark scheme

Question	Working	Answer	Mark	Notes
4	(a)	Triangle B drawn	1	B1 Do not penalise missing label across (a), (c) and (d).
	(b)	Translation $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	2	B1 B1 These 2 marks are independent. Do not award either if more than one transformations given (including two translations).
	(c)	Triangle C drawn	3	B3 $(2,4)$, $(6, -4)$, $(2,8)$ plotted and triangle drawn -1 for each error or omission B2 ft maximum for correct enlargement and centre from incorrect B , B1ft maximum for correct enlargement but incorrect centre

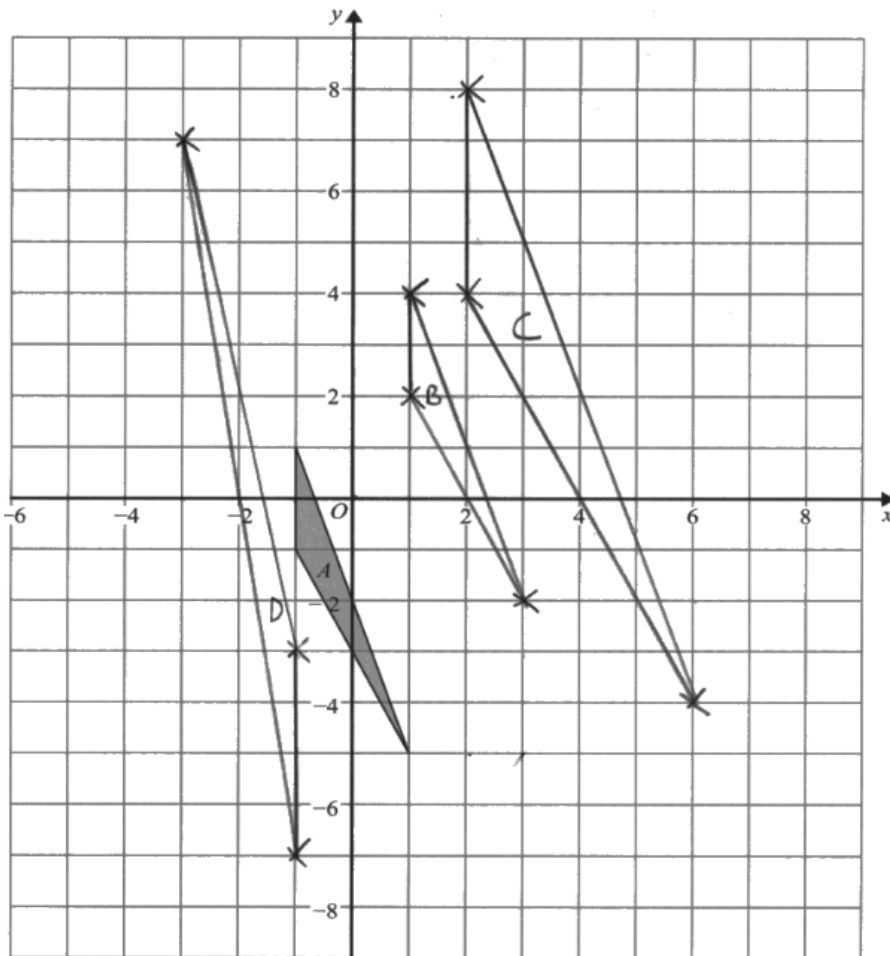
	(d)	$(-0.5 \ 0 \ 0 \ 1) \begin{pmatrix} 2 & 6 & 2 & 4 \\ 4 & 8 & & \end{pmatrix} =$ $(-1 \ -3 \ -1 \ -3 \ 7 \ -7)$		3	M2 for a matrix solution or list of coordinates containing at least 4 correct values seen or for 2 correct coordinates plotted Otherwise M1 for a correctly stated matrix product to calculate the coordinates of triangle D
			Triangle D drawn		A1 cao
4	(e)	Recognising that $\mathbf{N} = \mathbf{M}^{-1}$		3	M1 Either state $\mathbf{N} = \mathbf{M}^{-1}$ or $\mathbf{NM} = \mathbf{I}$ or $\frac{1}{-1 \times -0.5 - 0 \times 0.5}$ seen or $(-1 \ 0 \ -0.5 \ -0.5)$ seen.
			$2(-1 \ 0 \ -0.5 \ -0.5)$		A2 A1 for one correct row or column including the 2. The factor of 2 may be included in matrix. Any A marks imply M1 if not already awarded.
	(e) ALT	States $(P \ Q \ R \ S) \begin{pmatrix} -3 & -1 & -3 & 7 \end{pmatrix} =$ $(2 \ 6 \ 2 \ 4 \ -4 \ 8)$ and states at least 2 linear equations in 2 variables derived from this.			M1
			$(-2 \ 0 \ -1 \ -1)$		A2 A1 for one correct row or column or for 3 of the variables listed correctly $P = -2, Q = 0, R = -1, S = -1$
Total 12 marks					

Exemplar response A

(b) Translation ^{by} under the matrix $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$(d) \begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix} \begin{pmatrix} 6 & 2 & 2 \\ -4 & 4 & 8 \end{pmatrix} = \begin{pmatrix} -3 & -1 & -1 \\ 7 & -3 & -7 \end{pmatrix}$$

$$\begin{pmatrix} \quad & \quad & \quad \end{pmatrix} \begin{pmatrix} -3 & -1 & -1 \\ 7 & -3 & -7 \end{pmatrix} = \begin{pmatrix} 6 & 2 & 2 \\ -4 & 4 & 8 \end{pmatrix}$$



$$(d) \det M = (-0.5 \times -1) - (0 \times 0.5) = 0.5$$

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 6 & 2 & 2 \\ -4 & 4 & 8 \end{pmatrix}$$

=

Examiner's comments:

This response was given 12 marks.

A fully correct response.

Parts (a), (c) and (d) - all the shapes on the diagram are correct. For part (c), as the transformation is fairly straightforward, there is no need to show any working. For part (d), the first page shows the correct matrix multiplication clearly.

Part (b) - the correct transformation is seen. Full marks are awarded as we see the correct transformation stated along with the correct column vector.

Part (e) - despite the fact that the formula for an inverse matrix was given at the foot of this question, very few candidates picked up on this. Most who did find the correct matrix used a much more complex method than finding the inverse as seen here. The final piece of work looks like an attempt to check the answer being abandoned. This is a shame as although it does not gain any additional marks, it is good practice to build checks into methods.

Awarded B1, B1B1, B3, M2A1, M1A2.

Exemplar response B

(b) Translation ~~(2/3)~~ ← vector → $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (3)

$$(d) \quad M \begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix} \cdot C \begin{pmatrix} A'' & B'' & C'' \\ 2 & 6 & 2 \\ 4 & -4 & 8 \end{pmatrix} = D \begin{pmatrix} A''' & B''' & C''' \\ -1 & -3 & -1 \\ -3 & 7 & -7 \end{pmatrix}$$

$$N \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -1 & -3 & -1 \\ -3 & 7 & -7 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 2 \\ 4 & -4 & 8 \end{pmatrix}$$

$$-a - 3b = 2$$

$$-3a + 7b = 6$$

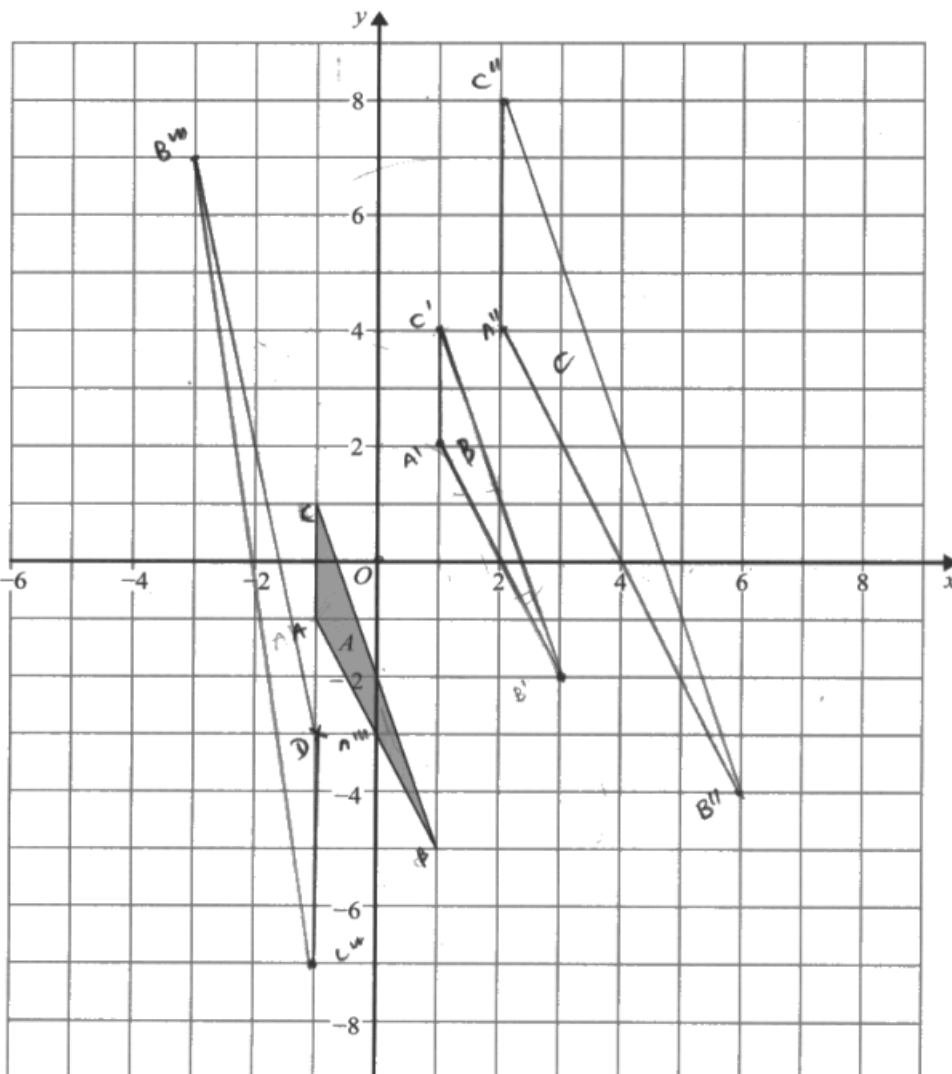
(turn to next page)

$$-c - 3d = 4$$

$$-3c + 7d = -4$$

(turn to next page)

$$N = \begin{pmatrix} -2 & 0 \\ -1 & -1 \end{pmatrix}$$



$$\begin{array}{ccc|ccc}
 A' & B' & C' & = & A'' & B'' & C'' \\
 2\uparrow & 2\downarrow & 4\uparrow & & 4\uparrow & 4\downarrow & 8\uparrow \\
 1\rightarrow & 3\rightarrow & 1\rightarrow & & 2\rightarrow & 6\rightarrow & 2\rightarrow
 \end{array}$$

$$-a - 3b = 2 \quad \times 3 \quad \rightarrow \quad -3a - 9b = 6 \quad \rightarrow \textcircled{1}$$

$$-3a + 7b = 6 \quad \rightarrow \textcircled{2}$$

$$\textcircled{2} - \textcircled{1}$$

$$\begin{array}{r}
 -3a + 7b + 3a + 9b = 6 - 6 \\
 \hline
 2b = 0
 \end{array}$$

$$2b = 0$$

$$b = 0$$

Substitute b to find a

$$-3(0) = 2 + a$$

$$0 = 2 + a$$

$$a = -2$$

$$-c - 3d = 4 \quad \times 3 \quad \rightarrow \quad -3c - 9d = 12 \quad \rightarrow \textcircled{1}$$

$$-3c + 7d = -4 \quad \rightarrow \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}$$

$$\begin{array}{r}
 -3c - 9d + 3c - 7d = 12 + 4 \\
 \hline
 -16d = 16
 \end{array}$$

$$-16d = 16$$

$$d = -1$$

Substitute d to find c

$$-3(-1) = 4 + c$$

$$3 - 4 = c$$

$$c = -1$$

$$N = \begin{pmatrix} -2 & 0 \\ -1 & -1 \end{pmatrix}$$

Examiner's comments:

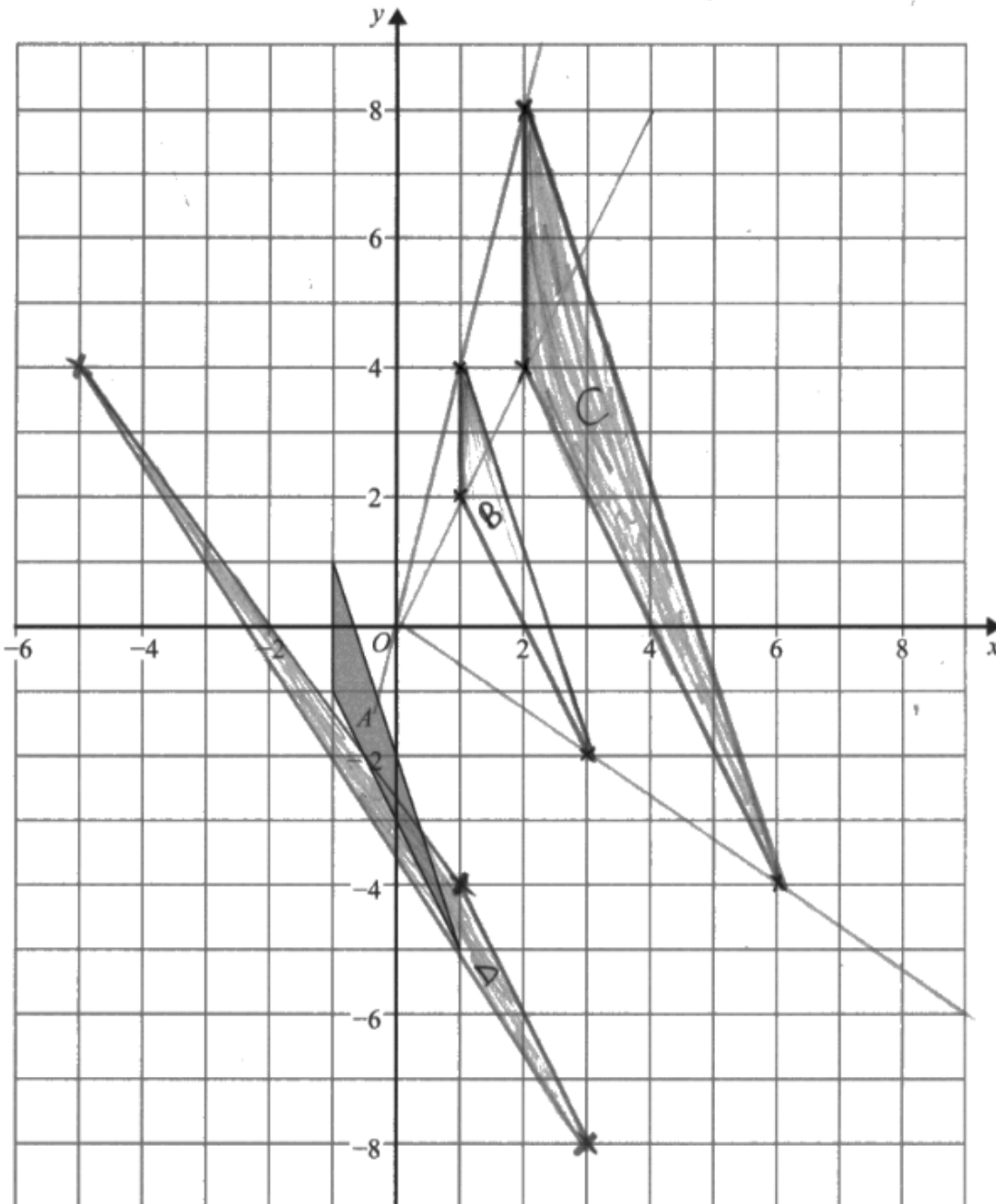
This response was given 12 marks.

The diagram indicates a completely correct response for parts (a), (c) and (d). Just after the diagram is working relevant to part (c), while not required, this is certainly not excessive. Just prior to the diagram is the correct matrix multiplication for part (d). Part (b) shows a fully correct solution, the incorrect vector seen is ignored as it is crossed through, although a single line through the working would certainly be more efficient. Part (e) is correct and so gained full marks. The start of the working and the answer are seen before the diagram but the main part of the working takes over a complete page after the diagram showing the shapes. The fact that this question is worth three marks should suggest to a candidate that there is a more efficient way of tackling this question. While they were not penalized for using an inefficient method, they almost certainly lost time that could have improved their performance on later questions.
Awarded B1, B1B1, B3, M2A1, M1A2.

Exemplar response C

b) Translation with the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$e) \frac{1}{-\frac{1}{2}-0} = -\frac{1}{\frac{1}{2}} \begin{pmatrix} -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$



d) ~~(~~2 4~~ ~~6 -4~~ ~~2 8~~)~~ $\begin{pmatrix} -1/2 & 0 \\ 1/2 & -1 \end{pmatrix}$

$$\begin{pmatrix} 2 & 4 \\ 6 & -4 \\ 2 & 8 \end{pmatrix} \begin{pmatrix} -1/2 & 0 \\ 1/2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1+2 & 0+(-4) \\ -3+(-2) & 0+4 \\ -1+4 & 0+8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -4 \\ -5 & 4 \\ 3 & -8 \end{pmatrix}$$

$$(1, -4) \quad (-5, 4) \quad (3, -8)$$

Examiner's comments:

This response was given 7 marks.

The diagram indicates a completely correct response for parts (a) and (c).

Triangle D is incorrect and so cannot score full marks.

After the diagram is the working for part (d). As the candidate has shown their matrix multiplication in the wrong order which not only produces the wrong results but shows a fundamental misunderstanding of this topic, they gain no marks for this part.

Part (b) completed before the diagram shows a fully correct solution.

Part (e) completed before the diagram is correct except for a single sign error in their determinant. Even if this was corrected, this would not have gained full marks for this response as we would expect candidates to simplify expressions such as $1/_{(1/2)}$. Therefore, this candidate gains the method mark but neither of the accuracy marks as an error in the determinant prevents either accuracy mark being gained.

Awarded B1, B1B1, B3, M0A0, M1A0.

Question 5

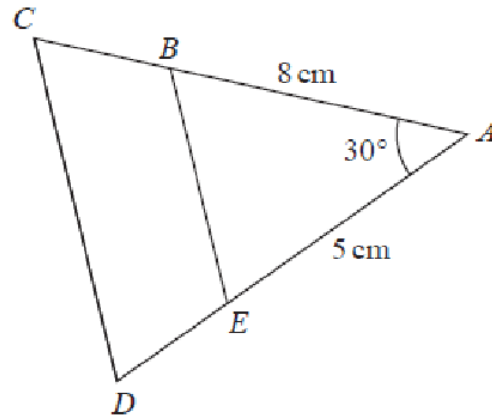


Diagram NOT
accurately drawn

Figure 2

Figure 2 shows the triangles ABE and ACD where B lies on AC and E lies on AD .
The triangles are similar with $\angle ABE = \angle ACD$.

$$AB = 8 \text{ cm} \quad AE = 5 \text{ cm} \quad \angle BAE = 30^\circ$$

- (a) Calculate the length, in cm to 3 significant figures, of BE . (3)
- (b) Calculate the size, in degrees to 3 significant figures, of $\angle ABE$. (3)

The area of quadrilateral $BCDE$ is 18.9 cm^2

- (c) Calculate the length, in cm to 3 significant figures, of CD . (4)

Mark scheme

Question	Working	Answer	Mark	Notes
5	(a)	$(BE^2 =) 5^2 + 8^2 - 2(5)(8)\cos(30)$	3	M1 Correct expression for BE^2
		$(BE =) \sqrt{5^2 + 8^2 - 40\sqrt{3}}$		M1 dep. Correct expression for BE . Must demonstrate correct order of operations. We must see $40\sqrt{3}$ or values which round to 69.3, 19.7 or 4.4 to award this mark.
			AWRT 4.44 (cm)	A1
	(b)	$\frac{\sin(\angle ABE)}{5} = \frac{\sin 30}{"4.44"}$	3	M1 Correct equation involving ABE could be labelled x, θ etc...
		$\sin(\angle ABE) = \frac{5 \sin \sin 30}{"4.44"}$		M1 dep. Correct expression for any trig ratio of ABE or correct expression for ABE
			AWRT 34.3(°)	A1
	(c)	Area of $\triangle ABE = \frac{1}{2}(8)(5)\sin 30$ (=10) or Area of $\triangle ABE = \frac{1}{2} \times 8 \times$ "4.44" $\times \sin \sin$ "34.3"	4	M1
		Length scale factor $(k =) \sqrt{\frac{18.9 + "10"}{"10"}} (= 1.7)$		M1 dep. This mark is not awarded until a square root is taken if a method involving CD^2 is used.
		$CD = "1.7" \times "4.44"$		M1 dep. Length scale factor multiplied by their value from (a)
		7.55 (cm)		A1 Allow any answer between 7.54 and 7.55 inclusive.
Total 10 marks				

Exemplar response A

$$(a) a^2 = b^2 + c^2 - 2bc \cos A.$$

$$BE = \sqrt{8^2 + 5^2 - 2(8)(5)(\cos 30)}$$

$$BE = 4.44 \text{ cm.}$$

$$(b) \frac{\sin a}{a} = \frac{\sin A}{A} = \frac{\sin B}{b}$$

$$\frac{5}{\sin B} = \frac{4.44}{\sin 30}$$

$$5 \times \sin 30 = \sin B \times 4.44.$$

$$\sin^{-1} \left(\frac{\sin 30 \times 5}{4.44} \right) = B$$

$$\hat{B} = 34.3^\circ$$

$$(c) \text{ area of } ABE = \frac{1}{2} ab \sin C$$

$$\frac{1}{2} 8 \times 5 \sin 30 = 10 \text{ cm}^2.$$

$$\text{area of } ACD = 18.9 + 10 = 28.9$$

$$\frac{\text{area of } \triangle ABE}{\text{area of } \triangle ACD} = \left(\frac{1}{1} \right)^2.$$

$$\frac{10}{28.9} = \left(\frac{4.44}{CD} \right)^2$$

$$10 CD^2 = 569.7$$

$$\sqrt{CD^2} = \sqrt{56.97}$$

$$CD = 7.55 \text{ cm.}$$

Examiner's comments:

This response was given 10 marks.

Fully correct response which scores full marks.

In part (a), this candidate has managed to show fully correct workings in a particularly efficient manner.

Awarded M1M1A1.

In part (b), again this candidate shows clear correct working and so gains full marks.

Awarded M1M1A1.

In part (c), this candidate's response does not exactly match the mark scheme. However, their working is entirely equivalent and just represents dealing with the different aspects required in a different order so gains full marks. As long as their working can clearly be followed, alternative methods will score full marks.

Awarded M1M1M1A1.

Exemplar response B

(a) BE

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 30$$

$$a^2 = 19.717$$

$$a = \cancel{4.44038} \dots \quad 4.44038 \dots$$

$$BE = 4.44 \text{ cm } \parallel$$

(b) $\frac{\sin 30^\circ}{4.4403} = \frac{\sin x}{5}$

$$\frac{\sin 30^\circ \times 5}{4.4403} = \sin x$$

$$x = \sin^{-1} \left(\frac{\sin 30^\circ \times 5}{4.4403} \right)$$

$$= 34.2652 \dots$$

$$= 34.3^\circ \parallel$$

$$(c) \text{ Area of a trapezium} = \frac{1}{2} (a+b) h$$

$$18.9 = \frac{1}{2} (4.4403 + x) \times h$$

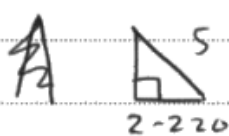
$$\text{Area of a triangle} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 8 \times 5 \times \sin 30$$

$$= 10 \text{ cm}^2$$

$$\text{Total area} = 10 + 18.9$$

$$= 28.9 \text{ cm}^2$$



$$a^2 = b^2 + c^2$$

$$5^2 = 2.22^2 + c^2$$

$$c^2 = 5^2 - 2.22^2$$

$$c^2 = 20.0716$$

$$c = 4.48013 \text{ cm}$$

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times CD \times (4.4801 + h) = 10$$

$$\frac{1}{2} (4.4403 + x) h = \frac{1}{2} \times x \times (4.8 + h) - 10$$

$$(2.22015 + \frac{x}{2}) h = \frac{1}{2} \times 4.82 \times x - 10$$

Examiner's comments:

This response was given 7 marks.

A fully correct response to the first two parts of this question.

Part (a): M1M1A1 and part (b): M1M1A1 for fully correct methods and answers clearly laid out. The inclusion of unrounded answers prior to finishing with rounded versions is a sensible precaution against a miscopy of a value. Also, with these multistage trigonometry problem use of overly rounded values from earlier in the problem may lead to answers which lose the accuracy mark.

Awarded M1M1A1, M1M1A1.

Part (c) - although this candidate clearly did not realize that this question was testing understanding of similar shapes, they did manage to pick up a mark for finding the area of triangle ABE. It is good advice for candidates to attempt to perform some related calculations if they cannot see exactly how to perform the demand of the question.

Awarded M1M0M0A0.

Exemplar response A

i) b
ii) a, b
iii) c, b
iiii) d, b

Examiner's comments:

This response was given 1 mark.

It was fairly common to see responses like this where only subsets of a specific size were listed, in this case those with only 1 or 2 elements. While responses with 4 correct subsets should have gained 2 marks, the fact that the subsets were listed without correct use of brackets meant only 1 mark was awarded. Candidates were not penalized if set brackets were not ideally formed but correct notation in set questions is important and will usually be required for marks to be awarded.

Awarded B1.

Exemplar response B

a = [a, b]
= [b]
= [b, c]
= [b, d]
= [a, b, c]
= [b, c, d]
= [a, b, d]
= [a, b, c, d]

Examiner's comments:

This response was given 4 marks.

A fully correct response. Although there is one inconsistency in the ordering of the sets when asked to list responses, candidates would be advised to work systematically in order to avoid missing responses or duplicating responses, both of which would have been penalized in this question. While the set brackets used here are not formed correctly benefit of the doubt was given, it was clear that this candidate understood the convention that listed sets are contained within brackets.

Awarded B4.

Question 7

To pass a course, Preety has to pass two tests, test **A** and test **B**.

Each test only has to be passed once.

Passing test **A** and passing test **B** are independent events.

Preety has three attempts to pass test **A**.

The probability that she passes test **A** on her first attempt is $\frac{3}{5}$

If she fails on her first attempt, the probability that she passes on her second attempt is $\frac{2}{5}$

If she fails on her second attempt, the probability that she passes on her third attempt is $\frac{3}{8}$

(a) Calculate the probability she passes test **A**. (3)

Preety has two attempts to pass test **B**.

The probability that she passes test **B** on her first attempt is $\frac{3}{5}$

The probability that she passes test **B** is $\frac{3}{4}$

(b) Calculate the probability that Preety passes test **B** on her second attempt, given that she fails test **B** on her first attempt. (3)

Preety decides to take the two tests only if the probability that she passes the course is greater than 0.5

(c) State whether or not Preety should take the two tests.
Give a reason for your answer. (2)

Mark scheme

Question	Working	Answer	Mark	Notes	
7	(a)	$\frac{3}{5} + \left(\frac{2}{5} \times \frac{2}{5}\right) + \left(\frac{2}{5} \times \frac{3}{5} \times \frac{3}{8}\right)$		3	M2, M1 for either $\frac{2}{5} \times \frac{2}{5}$ or $\frac{2}{5} \times \frac{3}{5} \times \frac{3}{8}$
			$\frac{17}{20}$ or 0.85		A1 oe.
	(a) ALT	$1 - \frac{2}{5} \times \frac{3}{5} \times \frac{5}{8}$			M2, M1 for $\frac{2}{5} \times \frac{3}{5} \times \frac{5}{8}$
			$\frac{17}{20}$ or 0.85		A1 oe
	(b)	$\frac{3}{5} + \frac{2}{5}p = \frac{3}{4}$ or $\left(\frac{3}{4} - \frac{3}{5}\right) \div \frac{2}{5}$		3	M2 or M1 for $\frac{2}{5}p$ (oe) or $\left(\frac{3}{4} - \frac{3}{5}\right)$ seen
			$\frac{3}{8}$ or 0.375		A1 oe
	(b) ALT	$\frac{1}{4} = \frac{2}{5} \times q \left(q = \frac{5}{8}\right)$			M1
		$1 - \frac{5}{8}$			M1 dep. Do not award if their value of q is not in the range $0 \leq q \leq 1$
			$\frac{3}{8}$ or 0.375		A1 oe
	(c)	$\frac{3}{4} \times \frac{17}{20}$		2	M1 fit their answer from part (a) only if it is between 0 and 1 inclusive.
			$\frac{51}{80} > \frac{1}{2}$ so yes (Preety should take the tests)		A1 cao must follow from correct part (a) $\frac{51}{80} = 0.6375$ allow any rounded value with full working seen.

Exemplar response A

$$a) \frac{3}{5} + \frac{2}{5} + \frac{3}{8} = \frac{11}{8}$$

$$\left(\frac{3}{5} \times \frac{2}{5}\right) + \left(\frac{2}{5} \times \frac{3}{5}\right) + \left(\frac{3}{8} + \frac{3}{8}\right) = \frac{1143}{1600}$$



$$\frac{3}{5} + \frac{3}{4} = \frac{27}{20}$$

$$\left(\frac{3}{5} \times \frac{2}{5}\right) \times \left(\frac{3}{4} \times \frac{3}{4}\right) = \frac{171}{400}$$

Yes the probability is greater than 0.5

Examiner's comments:

This response was given 0 marks.

This question gained no marks but it does highlight a number of common issues seen with candidates responses.

Firstly, in probability questions candidates should be able to determine that when probabilities are required answers should not be greater than 1, as is the case here with the $\frac{11}{8}$ and $\frac{27}{20}$ seen here.

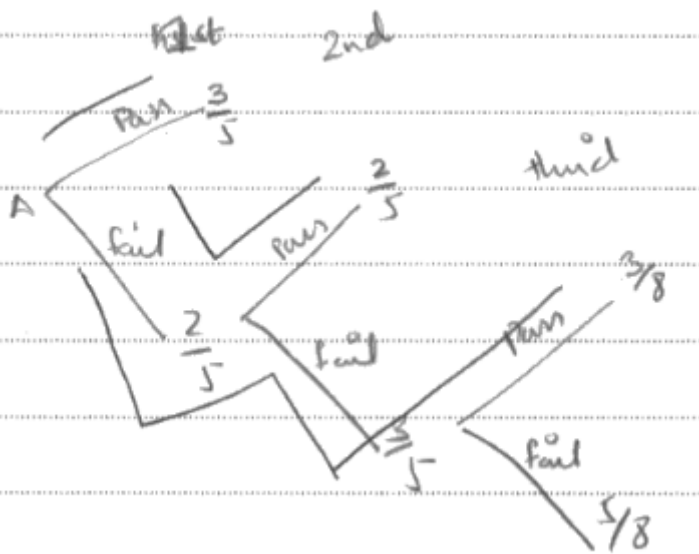
Secondly, the parts of the question are not clearly labeled while in this case it is fairly obvious which parts of the response relate to which part of the question. In some cases this can cause ambiguity and make gaining method marks more difficult.

Thirdly, for each of part (a) and (b) there are two separate pieces of working seen leading to different answers. In cases like this it is usual to award the lower of the 2 scores although this does not apply here as none of the methods seen would gain any marks.

Finally, in part (c) where a conclusion is required, it is imperative that working is seen to back up the conclusion. While this is the correct conclusion with no working seen to back this up, no marks are available.

Awarded M0A0, M0A0, M0A0.

Exemplar response B



$$a) = \frac{3}{5} + \left(\frac{2}{5} \times \frac{2}{5} \right) + \left(\frac{2}{5} \times \frac{3}{5} \times \frac{3}{8} \right)$$

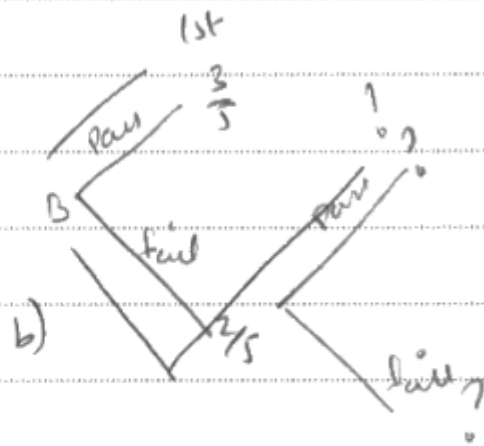
$$= \frac{3}{5} + \frac{4}{25} + \frac{18}{200}$$

$$= \frac{3}{5 \times 20} + \frac{4}{25 \times 4} + \frac{9}{100}$$

$$= \frac{60 + 16 + 9}{100}$$

$$= \frac{85}{100}$$

$$= \frac{17}{20}$$



b)

$$b) \quad \frac{3}{5} + \left(\frac{2}{5} \times x \right) = \frac{3}{4}$$

$$\frac{3+2x}{5} = \frac{3}{4}$$

$$12+8x = 15$$

$$8x = 15 - 12$$

$$\boxed{x = \frac{3}{8}}$$

c) Preety should ~~not~~ take the tests since the probabilities of passing the tests are greater than 0.5, A having $17/20$ and B $3/4$

Examiner's comments:

This response was given 6 marks.

In part (a), a fully correct response is seen. Although not required by the mark scheme, drawing a tree diagram is an excellent idea in a question such as this and will help ensure candidates organize their responses correctly.

The mark scheme will be based on the structure inherent in a tree diagram even if the drawing of one is not explicitly required.

The first line of working shown after the diagram is sufficient to gain both method marks. Candidates often failed to consider all three terms required and therefore often failed to gain the second method mark.

Awarded M2A1.

In part (b), a method using an algebraic expression for the probability of passing test B equated to $\frac{3}{4}$ is used. Candidates who used this method usually scored full marks for this part of the question.

Awarded M2A1.

In part (c), this candidate has not made any attempt to combine the two separate probabilities - this was a common issue seen in responses to this question.

Awarded M0A0.

Question 8

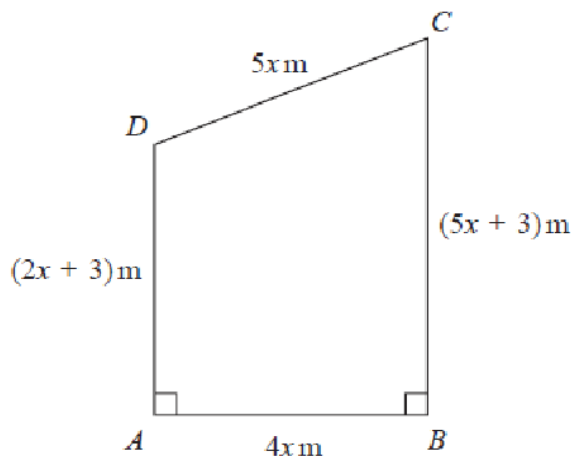


Diagram NOT
accurately drawn

Figure 3

Figure 3 shows the plan for a lawn that is in the shape of a trapezium $ABCD$ in which

$$AB = 4x \text{ metres} \quad BC = (5x + 3) \text{ metres} \quad CD = 5x \text{ metres} \quad DA = (2x + 3) \text{ metres}$$

The perimeter of the lawn is P metres.

(a) Find and simplify an expression for P in terms of x .

(2)

The area of the lawn is $A \text{ m}^2$

(b) Show that $A = 14x^2 + 12x$

(2)

The owner of the lawn wants the perimeter of the lawn to be greater than 52 m.

He also wants the area of the lawn to be at most 162 m^2

(c) Find the range of possible values of x .

Show clear algebraic working.

(6)

Mark scheme

Question	Working	Answer	Mark	Notes
8	(a)	$(P =) 4x + (5x + 3) + (2x + 3) + 5x$	2	M1 Sum of four terms in x
				A1
	(b)	$A = \frac{1}{2}((2x + 3) + (5x + 3))(4x)$ oe	2	M1 Allow $A = (2x + 3) \times 4x + \frac{1}{2} \times 3x \times 4x$ or $\frac{1}{2}(7x + 6) \times 4x$
		$A = 14x^2 + 12x$		A1 Must be clearly shown with no incorrect steps. At least one correct intermediate step must be seen.

	(c)	"16x + 6" > 52		6	M1 Their perimeter > 52, condone ε. or =
		$x > 2.875$			A1 ft their answer to part (a) as long as it is a linear expression in x . Allow as a fraction $\frac{23}{8}$ oe allow 2.88. Condone ε or =
		$14x^2 + 12x \leq 162$			M1 Accept < 162 may be inferred by an inequality for x stated in between their solutions to their quadratic.
		$(x - 3)(7x + 27) \leq 0$			M1 indep. 'equating' to zero and solving (oe) Independent of previous mark, need not be expressed as inequality, condone an equality or any inequality sign. Must be solving quadratic trinomial. By factorising brackets must expand to give 2 out of 3 terms correct or fully correct substitution into fully correct formula.
		$-\frac{27}{7} \leq x \leq 3$			A1 Correct interval accept $x \leq 3$ or $x < 3$ if no other inequality stated in relation to this inequality.
			$2.875 < x \leq 3$		A1 cao. Must follow from completely correct working. Condone only finding critical values in working as long as the working contains no ambiguous or incorrect inequalities.

Exemplar response A

$P =$ sum of all sides

a) $P = 4x + 2x + 3 + 5x + 3 + 5x$

$$P = 16x + 6$$

b) $A = \frac{1}{2}(a+b)h$

$$= \frac{1}{2}((2x+3)+(5x+3)) \times 4x$$

$$A = (7x+6) \times 2x$$

$$A = 14x^2 + 12x \Rightarrow \text{Hence proved}$$

~~$P \Rightarrow 14x^2 + 12x \geq 52$~~

~~$A \Rightarrow$~~

~~$P \Rightarrow 14x^2 + 12x$~~

~~$P \Rightarrow 16x + 6 \geq 52$~~

~~$A \Rightarrow 14x^2 + 12x \leq 162$~~

~~$2(7x^2 + 6x) \geq 52$~~

~~$7x^2 + 6x \geq 26$~~

$$P \Rightarrow x \geq \frac{52-6}{16}$$

$$x \geq \frac{46}{16}$$

$$x \geq \frac{23}{8}$$

x

$$A \Rightarrow 14x^2 + 12x \leq 162$$

$$2(7x^2 + 6x) \leq 162$$

$$7x^2 + 6x \leq 162 \times 2$$

$$7x^2 + 6x \leq 324$$

$$\frac{-3-3\sqrt{253}}{7} \leq x \leq \frac{-3+3\sqrt{253}}{7}$$

Examiner's comments:

This response was given 7 marks.

This was a strong response to what proved to be a demanding question.

Part (a) shows a fully correct solution. The inclusion of some additional annotation would have ensured the awarding of a method mark had the candidate made an error copying down one of the values as they have clearly indicated their intention.

Awarded M1A1.

Part (b) - as the demand for this question is "Show that...", it is important that no obvious steps in the working are omitted. After substituting in the lengths of the sides in terms of x , the cancelling of $\frac{1}{2}$ and $4x$ is condoned as this is a fairly standard method when simplifying an expression. In a case where something similar could lead to ambiguity, this should be avoided.

Awarded M1A1.

Part (c) - even though the inequality for the perimeter is incorrect, this was allowed for the first two marks and would have been penalized by the final accuracy mark had the candidate progressed that far. Stating the correct inequality for the area gained the candidate the third mark. After this the candidate failed to deal with the resulting quadratic inequality correctly. This is a new topic with a particularly high level of demand. Candidates expecting to achieve a high grade should be taught efficient methods for dealing with these. The earlier work which is crossed through is completely ignored when marking this response.

Awarded M1A1M1M0A0A0.

Exemplar response B

$$(a) P = 4x + 5x + 3 + 5x + 2x + 3$$

$$P = 16x + 6$$

$$(b) \text{ Area of a trapezium} = \frac{1}{2} (a+b) h$$

$$A = \frac{1}{2} (2x+3+5x+3) \times 4x$$

$$A = \frac{1}{2} (7x+6) \times 4x$$

$$A = \frac{7x+6}{2} \times \frac{4x}{1} = \frac{28x^2 + 24x}{2}$$

$$A = 14x^2 + 12x$$

$$(c) \quad 16x + 6 > 52$$

$$14x^2 + 12x \leq 162$$

~~16x + 6 > 52~~ ~~14x^2 + 12x \leq 162~~ ~~16x + 6 > 52~~ ~~14x^2 + 12x \leq 162~~

~~16x + 6 > 52~~ ~~14x^2 + 12x \leq 162~~ ~~16x + 6 > 52~~ ~~14x^2 + 12x \leq 162~~

~~16x + 6 > 52~~ ~~14x^2 + 12x \leq 162~~

$$16x - 46 > 0 \rightarrow x > \frac{46}{16} \rightarrow x > \frac{23}{8}$$

~~14x^2 + 12x \leq 162~~

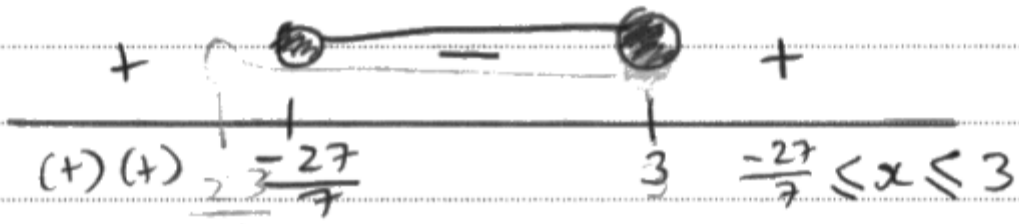
$$14x^2 + 12x - 162 \leq 0$$

$$7x^2 + 6x - 81 \leq 0$$

$$\frac{-6 \pm \sqrt{6^2 - 4 \times 7 \times -81}}{2 \times 7} = 3 \text{ or } -\frac{27}{7}$$

not possible

$$(x - 3)(7x + 27) \leq 0$$



~~16x + 6 > 52~~ ~~14x^2 + 12x \leq 162~~ $-\frac{27}{7} \leq x \leq 3$ and $x > \frac{23}{8}$

Examiner's comments:

This response was given 9 marks.

A particularly clear response with some particularly effective features.

Part (a) - a clear and fully correct response to this part.

Awarded M1A1.

Part (b) - a little more working than the minimum required but this is a sensible precaution in a question with a demand of "Show that..." as too little working would certainly be penalized.

Awarded M1A1.

Part (c) - this was an especially good response and it was a shame that the candidate did not combine their inequalities into a consistent answer. The inclusion of some corrected working is common among candidates making a good attempt at these particularly high demand questions; candidates would be better advised to just cross through with a single line rather than scribbling over the top as seen here.

This candidate has started by stating both inequalities described in the question in algebraic form thus earning two marks straight away. They have efficiently solved the linear inequality and show a fully correct solution for the quadratic inequality even annotating that the $-27/7$ solution to the critical values is "not possible" showing an appreciation of the context. The inclusion of an illustration of the solution on a number line was not required but is an effective strategy for dealing with a quadratic inequality and would have also been an effective strategy to combine this with the result from the linear inequality.

Awarded M1A1M1M1A1A0.

Question 9

$$\mathbf{p} = \begin{pmatrix} 2x - 1 \\ y \end{pmatrix} \quad \mathbf{q} = \begin{pmatrix} y + 3 \\ -y \end{pmatrix}$$

The vectors \mathbf{p} and \mathbf{q} are such that $|\mathbf{p}| = \sqrt{98}$ and $\mathbf{p} + \mathbf{q} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$

(a) Show that $x^2 - 3x - 9 = 0$ (5)

Given that $x > 0$

(b) (i) find the exact value of x , (2)

(ii) show that $y = 2 - 3\sqrt{5}$ (2)

(c) Find the exact value of $|\mathbf{q}|^2$
Show your working clearly. (3)

Mark scheme

Question	Working	Answer	Mark	Notes
9 (a)	$\sqrt{(2x - 1)^2 + y^2}$ $= \sqrt{98}$		5	M1 Correct use of modulus to form any correct equation
	$(2x - 1) + (y + 3)$ $= 7$			M1 oe e.g. $2x + y = 5$
	$(2x - 1)^2 + (5 - 2x)^2 = 98$ oe			M1 dep. on both previous M marks. Remove square-roots and substitute to gain an equation in terms of x only. May be seen in expanded form. Eg. $-4x^2 + 4x + 97 = 25 + 4x^2 - 20x$ For this and next M mark allow a maximum of 1 sign or numerical error.
	$8x^2 - 24x - 72 = 0$			M1 dep. previous mark. Expand and attempt to form 3 term quadratic For this and previous M mark allow a maximum of 1 sign or numerical error.
		$x^2 - 3x - 9 = 0$		A1 As answer given sufficient working must be shown. No incorrect work can be seen.

	(b)(i)	$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)}}{2(1)}$ or $\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 9 = 0$		2	M1 Solving quadratic using formula or completing square
			$x = \frac{3 + 3\sqrt{5}}{2}$		A1 Accept other equivalent exact forms eg $\frac{3+\sqrt{45}}{2}$ Do not accept \pm for the final answer, they must indicate positive solution. Do not isw answer given as a decimal.
	(b)(ii)	$y = 5 - 2\left(\frac{3 + 3\sqrt{5}}{2}\right)$		2	M1 Substitute their x which must be an expression involving surds into linear equation to find y If using equation for y^2 must obtain $y^2 = 98 - 49 - 12\sqrt{5}$ or simpler to gain this mark.
			$y = 2 - 3\sqrt{5}$		A1 As answer given sufficient working must be shown Allow $y^2 = 49 - 12\sqrt{5}$ from modulus equation and expansion of $(2 - 3\sqrt{5})^2 = 49 - 12\sqrt{5}$ along with an appropriate comment. Do not isw answer given as a decimal.
	(c)	$(q ^2 = (2 - 3\sqrt{5} + 3)^2 + (-(2 - 3\sqrt{5}))^2)$		3	M1 Attempt $ q ^2 = q_1^2 + q_2^2$ - allow in terms of y or x Eg. $(y + 3)^2 + y^2$ or $(8 - 2x)^2 + (5 - 2x)^2$ Allow an expression for $ q $
		$= 25 - 30\sqrt{5} + 45 + 45 - 11$			M1 dep expand brackets must involve surds. Allow square root of this.
			$119 - 42\sqrt{5} \text{ or } 7(17 - 6\sqrt{5})$		A1 cao Do not isw answer given as a decimal.
Total 12 marks					

Exemplar response A

$$(a) \begin{pmatrix} 2x-1 \\ y \end{pmatrix} + \begin{pmatrix} y+3 \\ -y \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$2x-1+y+3=7$$

$$y+(-y)=0$$

$$|p| = \sqrt{a^2+b^2}$$

$$\sqrt{98} = \sqrt{(2x-1)^2 + y^2}$$

$$2x+y+2=7$$

$$2x+y+2-7=0$$

$$2x+y-5=0$$

$$2x+y=5$$

$$y=5-2(x)$$

$$\sqrt{98} = \sqrt{4x^2 + 4x + 1 + y^2}$$

$$7\sqrt{2} = 2x + 2\sqrt{x} + 1 + y$$

$$7\sqrt{2} - 2\sqrt{x} = 2x + 1 + y$$

$$b(i) \quad x^2 - 3x - 9$$

$$\frac{-b \pm \sqrt{b^2 - 4(ac)}}{2a}$$

$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{3 + 3\sqrt{5}}{2}$$

$$\frac{3 - 3\sqrt{5}}{2}$$

$$(ii) \quad 5 - 2\left(\frac{3 + 3\sqrt{5}}{2}\right)$$

$$y = 5 - 2(x)$$

$$y = 5 - 3 + 3\sqrt{5}$$

$$y = 2 + 3\sqrt{5}$$

$$(c) |q| = \sqrt{(y+3)^2 + (-y)^2}$$

$$= \sqrt{y^2 + 6y + 9 + (-y)^2}$$

$$= (2-3\sqrt{5})^2 + 6(2-3\sqrt{5}) + 9 + (-y 2-3\sqrt{5})^2$$

$$= 49 - 12\sqrt{5} + (12 - 18\sqrt{5}) + 9 + 2 \rightarrow 49 - 12\sqrt{5}$$

$$98 - 24\sqrt{5} + 9 + 12 - 18\sqrt{5}$$

$$|q| = 25.085$$

$$|p|^2 = 629.264$$

Examiner's comments:

This response was given 7 marks.

This was a strong response to what proved to be a demanding question.

In part (a), this candidate gained one mark for having a correct equation related to the modulus of p and another mark for stating a correct equation for combining the first coefficient of the vectors.

Although these marks should be accessible to most candidates, many failed to state these. Unfortunately, this candidate then showed a fundamental lack of understanding of how square roots of sums should be handled and so gains no more marks in this part.

Awarded M1M1M0M0A0.

In part (b)(i), this candidate shows fully correct working and quotes the final answer in surd form as required by the demand to "find exact value". Many candidates lost a mark by giving their final answer as a decimal value.

Awarded M1A1.

In part (b)(ii), as the answer is given, it is imperative that no line of the working contains an error. In the line $y=5-3+3\sqrt{5}$, the candidate has missed out brackets around the last two terms. This means only 1 of the 2 marks can be awarded for this part.

Awarded M1A0.

In part (c), this candidate shows completely correct working but unfortunately cannot be awarded the final mark as they do not give their answer in exact form, as required in the question.

Awarded M1M1A0.

Exemplar response B

$$a). \quad P \begin{pmatrix} 2x-1 \\ y \end{pmatrix}$$

$$\therefore |P| = \sqrt{98}$$

$$\Rightarrow \sqrt{(2x-1)^2 + y^2} = \sqrt{98}$$

$$\Rightarrow (2x-1)^2 + y^2 = 98$$

$$\Rightarrow 4x^2 - 4x + 1 + y^2 = 98 \quad \text{————— ①}$$

$$\text{again, } P + Q = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x-1 \\ y \end{pmatrix} + \begin{pmatrix} y+3 \\ -y \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$\Rightarrow 2x-1 + y+3 = 7$$

$$\Rightarrow 2x+y = 7 - 3 + 1$$

$$\Rightarrow 2x+y = 5$$

$$\Rightarrow y = 5 - 2x \quad \text{————— ②}$$

② into ①

$$\Rightarrow 4x^2 - 4x + 1 + (5-2x)^2 = 98$$

$$\Rightarrow 4x^2 - 4x + 1 + 25 - 20x + 4x^2 = 98$$

$$\Rightarrow 8x^2 - 24x - 72 = 0 \quad \therefore \quad (\text{both side divided by 8})$$

$$\Rightarrow x^2 - 3x - 9 = 0 \quad (\text{proved}).$$

$$b) i) x^2 - 3x - 9 = 0$$

$$\Rightarrow \therefore a = 1, b = -3, c = -9$$

$$\therefore x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times (-9)}}{2 \times 1}$$

~~$x =$~~ \therefore either.

$$x = \frac{3 + 3\sqrt{5}}{2}$$

$$\text{or } x = \frac{3 - 3\sqrt{5}}{2}$$

$$\Rightarrow x = 4.85$$

$$\text{or } x = -1.85$$

(ignored, $x > 0$)

$$\therefore \text{Exact value of } x = \frac{3 + 3\sqrt{5}}{2}$$

$$c) y = 5 - 2x$$

$$\Rightarrow y = 5 - 2 \left(\frac{3 + 3\sqrt{5}}{2} \right)$$

$$\Rightarrow y = 5 - 3 + 3\sqrt{5}$$

$$\Rightarrow y = 2 + 3\sqrt{5} \quad (\text{proved}).$$

$$\begin{aligned}
 \text{d). } \therefore \mathbf{q} &= \begin{pmatrix} y + 3 \\ -y \end{pmatrix} \\
 &= \begin{pmatrix} 2 + 3\sqrt{5} + 3 \\ \cancel{2} - 2 - 3\sqrt{5} \end{pmatrix} \\
 &= \begin{pmatrix} 5 + 3\sqrt{5} \\ -2 - 3\sqrt{5} \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \therefore |\mathbf{q}|^2 &= (5 + 3\sqrt{5})^2 + (-2 - 3\sqrt{5})^2 \\
 |\mathbf{q}|^2 &= 25 + 30\sqrt{5} + 45 + 4 + 12\sqrt{5} + 45 \\
 |\mathbf{q}|^2 &= 119 + 30\sqrt{5} + 12\sqrt{5} \\
 |\mathbf{q}|^2 &= 119 + 42\sqrt{5}
 \end{aligned}$$

Examiner's comments:

This response was given 10 marks.

This was a particularly strong response to what proved to be a demanding question. In part (a), this candidate gained full marks. Their working is clear and shows clear consideration of the modulus of \mathbf{p} and the first coefficient of the vector addition. They manage to combine these together and simplify their result. As the demand for the question is "Show that...", it is crucial that no steps are missed and this is a good example of the level of detail required for full marks.

Awarded M1M1M1M1A1.

In part (b)(i), this candidate shows fully correct working and quotes their final answer in surd form as required by the demand to "find exact value". Many candidates lost a mark by giving their final answer as a decimal value. In this case, the decimal result quoted is not penalized as it is clearly being used to determine which of the solutions is greater than zero as given in the question.

Awarded M1A1.

In part (b)(ii), as the answer is given, it is imperative that no line of the working contains an error. In the line $y=5-3+3\sqrt{5}$, the candidate has missed out brackets around the last two terms. This means only 1 of the 2 marks can be awarded for this part.

Awarded M1A0.

In part (c), this candidate shows completely correct working showing an ability to handle surds which is what is required to gain full marks. Unfortunately, their incorrect value of y used prevented the awarding of the accuracy mark. As many calculators will provide the answer in surd form, it is important that candidates do show their working in these types of questions.

Awarded M1M1A0.

Question 10

The curve C has equation $y = 2x^3 + ax^2 + bx + 10$ where a and b are constants.

The point A , with coordinates $(1, 3)$, lies on C .

(a) Write down an equation in a and b .

(1)

The tangent to C at the point A has gradient -8

(b) Show that $2a + b = -14$

(3)

(c) Hence find the value of a and the value of b .

Show clear algebraic working.

(3)

Using the value of a and the value of b found in part (c),

(d) find the x coordinate of each of the points on C where the tangent to C is parallel to the line with equation $y = 7 - 4x$.

Show clear algebraic working.

(3)

Mark scheme

Question	Working	Answer	Mark	Notes
10 (a)		$2 \times 1^3 + a \times 1^2 + b \times 1 + 10 = 3$	1	B1 oe
(b)	$(f'(x) =) 6x^2 + 2ax + b$		3	M1 at least two of these terms shown
	$6 \times 1^2 + 2a \times 1 + b = -8$			M1 dep clear indication of substitution for $x = 1$ into their derivative and equates to -8
		$2a + b = -14$		A1 Note that answer given in question
(c)	Correct method for solving 2 linear simultaneous equations		3	M1 ft Correct method to form an equation in a single variable. Allow a maximum of 1 sign or numerical error. Either $a = \pm 5$ or $b = \pm 4$ with no incorrect working seen implies this.
		$a = -5$		A1 cao
		$b = -4$		A1 cao
(d)	$6x^2 - 10x - 4 = -4$		3	M1 Substitutes their a and their b into their $f'(x)$ and equate to -4

		$x(3x - 5) = 0$		M1 dep. Solve their 2 or 3 term quadratic By factorising brackets must expand to give 2 out of 3 terms correct or fully correct substitution into fully correct formula. They must have a factorised expression with 2 factors A1 cao for $x = \frac{5}{3}$ accept 1.67 or better.
			$x = 0$ and $x = \frac{5}{3}$	
				Total 10 marks

Exemplar response A

a) $(1, 3)$

$$3 = 2(1)^3 + a(1)^2 + b(1) + 10$$

$$\underline{\underline{a + b = -5}} \quad \underline{\underline{a + b = -9}}$$

b) ~~$(1, 3)$~~ mark $\frac{dy}{dx} = 6x^2 + 2ax + b$

$$\underline{\underline{3 = 1(6) + c}} \quad -8 = 6(1)^2 + 2a(1) + b$$

$$\underline{\underline{3 = 6 + c}} \quad -8 = 6 + 2a + b$$

$$\underline{\underline{c = -3}} \quad 2a + b = -8 - 6$$

$$\underline{\underline{y = -8x - 5}} \quad 2a + b = -14$$

hence shows

c)

$$\begin{aligned}2a + b &= -14 \times 2 \\ a + b &= -9 \times 2\end{aligned}$$

$$\begin{aligned}2a + b &= -14 \\ -2a + 2b &= -18 \\ \hline 0 - b &= 4 \\ & \quad \quad \quad \frac{-1}{-1} \quad \frac{-4}{-1}\end{aligned}$$

$$b = -4$$

$$2a + b = -14$$

$$2a + (-4) = -14$$

$$2a - 4 = -14$$

$$2a = -14 + 4$$

$$\frac{2a}{2} = \frac{-10}{2}$$

$$a = -5, \quad \underline{\underline{b = -4}}$$

d)

$$2x^3 - 5x^2 - 4x + 10 = 7 - 4x$$

$$2x^3 - 5x^2 + 3 = 0$$

$$x^2(2x - 5) + 3 = 0$$

$$x^2 + 3 = 0 \quad 2x - 5 = 0$$

$$\underline{\underline{x = \pm\sqrt{3}}} \quad \underline{\underline{x = \frac{5}{2}}}$$

Examiner's comments:

This response was given 7 marks.

Part (a) gains B1. Although the candidate did not need to simplify their expression here, the simplified expression will be used in later parts so it is a sensible step.

Awarded B1.

In part (b), as the answer is given, full working is required. This was a good example of a candidate who showed sufficient working to secure full marks.

Awarded M1M1A1.

In part (c), the question states "Show clear algebraic working." Even without this demand, given that many calculators are capable of solving linear simultaneous equations, candidates should show clear algebraic working in order to secure full marks. This is an excellent example of a candidate showing a sensible level of working.

Awarded M1A1A1.

Part (d) is specifically asking about gradient; this should suggest that either derivative of curves or the co-efficient of x in a " $y=mx+c$ " format line is required. This candidate has not realized this in this case.

Awarded M0M0A0.

Exemplar response B

a) $(1, 3)$ (x, y)

$$3 = 2(1)^2 + a(1)^2 + b(1) + 10$$

$$3 = 2 + a + b + 10$$

$$a + b = 3 - 12$$

$$\boxed{a + b = -9}$$

b) $y = mx + c$

$$3 = -8(1) + c$$

$$c = 3 + 8$$

$$c = 11$$

~~$$3 = 2(1)^2 + 2a(1)^2 + b(1) + 10$$~~

~~$$3 - 2 - 10 = 2a + b$$~~

~~$$2a + b = -9$$~~

c) $-1 \times (a + b = -9)$

$$2a + b = -14$$

$$-a - b = 9$$

$$+2a + b = -14$$

$$\boxed{a = -5}$$

~~$$a + b = -9$$~~
~~$$b = -9 - a$$~~
~~$$b = -9 - 5$$~~
~~$$b = -14$$~~

$$2a + b = -14$$

$$b = -14 - 2(2 - 5)$$

$$b = -14 + 10$$

$$\boxed{b = -4}$$

d) $y = 7 - 4x$

$$y = mx + c$$

$$y = -4x + 7$$

$-4 = \text{gradient}$

$$\boxed{c = 7}$$

$$y = mx + c$$

$$3 = -8(1) + c$$

$$c = 11$$

7 and 11

#

Examiner's comments:

This response was given 4 marks.

This response shows that a candidate with no knowledge of differentiation could score some marks on this question.

Part (a) is completely correct.

Awarded B1.

Part (b) showed no attempt to differentiate and hence gained no marks. The candidate's attempt to find the equation of the tangent to the curve at A was a good example of correct working that does not answer the demand of the question and so gains no credit.

Awarded M0M0A0.

Part (c) - the candidate shows clear working to solve the simultaneous equation. As the answer in part (b) is given, an inability to gain marks in that part should not prevent the award of marks in this part.

Awarded M1A1A1.

Part (d) - again this candidate fails to differentiate and so gains no marks.

Awarded M0M0A0.

Question 11(a) and (b)

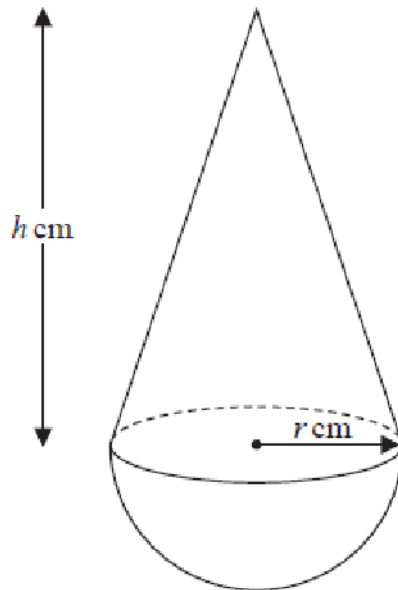


Diagram NOT
accurately drawn

Figure 4

A child's toy is made by fixing a solid right circular cone, with base radius r cm and height h cm, on the flat circular face of a solid hemisphere of radius r cm. The centre of the base of the cone coincides with the centre of the hemisphere, as shown in Figure 4

Given that $h + 6r = 15$

- (a) find the upper bound for the value of r .
Give a reason for your answer.

(2)

The volume of the toy is V cm³

- (b) Show that $V = \frac{1}{3}\pi r^2(15 - 4r)$

(3)

Mark scheme

Question		Working	Answer	Mark	Notes
11	(a)	$15 - 6r > 0$		2	M1 Consideration that $h > 0$ Alternatively award for 2.5 oe. seen in working.
			$r = 2.5$		A1 Do not isw. Allow $r < 2.5$ or $r \leq 2.5$
	(b)	$(V =) \frac{1}{3}\pi r^2 h$ $+ \frac{2}{3}\pi r^3$		3	M1
		$(V =) \frac{1}{3}\pi r^2(15 - 6r) + \frac{2}{3}\pi r^3$ or $(V =) \frac{1}{3}\pi r^2(h + 2r)$			M1 Substitutes $h = 15 - 6r$ to obtain an expression in π and r only or fully factorise expression for V . Must have V in the form $(V =) \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ to award this method mark.
			$V = \frac{1}{3}\pi r^2(15 - 4r)$		A1 Answer given must see evidence of both factorisation and substitution to award this mark. Penalise lack of $V =$ for this mark only.

Exemplar response A

(a) r has to be less than 2.5 as $h = 15 - 6r$ and the height has to be more than zero. upper bound for $r \rightarrow 2.4$

$$(b) \quad V = \frac{1}{3} \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3$$

$$\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$$

$$\frac{1}{3} \pi r^2 (15 - 6r) + \frac{2}{3} \pi r^3$$

$$\frac{1}{3} \pi r^2 (15 - 6r) + \frac{2}{3} \pi r^3 \rightarrow 3\pi r^2 + \frac{2}{3} \pi r^3$$

$$\frac{1}{3} \pi r^2 (15 - 6r) + \frac{2}{3} \pi r^3$$

$$V = \frac{1}{3} \pi r^2 (15 - 6r) + \frac{2}{3} \pi r^3$$

$$V = \frac{15\pi r^2}{3} - \frac{6\pi r^3}{3} + \frac{2}{3} \pi r^3$$

$$V = 5\pi r^2 - \frac{4}{3} \pi r^3$$

$$V = \frac{1}{3} \pi r^2 (15 - 4r)$$

Examiner's comments:

This response was given 4 marks.

Part (a) of this question proved particularly difficult for candidates, the vast majority interpreting this as a bounds question with measurements given to a specified degree of accuracy. This was unusual in showing an appreciation that the bound comes from the requirement that h needs to be positive and the given fact that $h + 6r = 15$. This candidate gives the correct answer, 2.5, within their working but then goes on to state their final answer as 2.4. This does gain them the first mark but unfortunately this sort of argument that fundamentally confuses discrete and continuous values was common and cannot gain full marks.

Awarded M1A0.

Part (b) - as the demand for this question was "Show that ...", it is crucial that the candidate shows clear working with no obvious missing steps. This is a good example of the sort of details we would expect to see for a completely correct solution.

Awarded M1M1A1.

Exemplar response B

$$a) h + 6r = 15$$

$$6r = 15 - h$$

$$r = \frac{15 - h}{6}$$

$$r = \frac{15 \cdot 5 + 1 \cdot 5h}{5 \cdot 5}$$

$$V = \frac{1}{3} \pi r^2 h + \frac{4}{3} \pi r^3$$

Eq 2 + 1

$$\frac{1}{3} \pi r^2 (15 - 6r) + \frac{4}{3} \pi r^3$$

~~15~~

~~15~~

$$\frac{1}{3} \pi r^2 (15 - 4r)$$

Examiner's comments:

This response was given 1 mark.

Part (a) of this question proved particularly difficult for candidates, the vast majority interpreting this as a bounds question with measurements given to a specified degree of accuracy - a good example of a typical response seen for this. This scored no marks. Awarded M0A0.

Part (b) - as the candidate has quoted the incorrect formula for the volume, failing to divide the volume of the sphere by 2, they fail to gain the first method mark available here. However, the next stage, i.e. substituting $h=15-6r$ into the expression, is performed correctly. Although the formula is incorrect, it is close enough to allow the award of the second mark here. In questions like this as the answer is given, a correct final answer does not imply any method marks. Awarded M0M1A0.

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