Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- Calculators may be used.

Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.
Answer ALL ELEVEN questions.
Write your answers in the spaces provided.
You must write down all the stages in your working.

1. A supermarket was open for 24 hours a day on all 365 days of 2014
   On average, the supermarket made a sale every 30 minutes of each day of 2014
   (a) Calculate the number of sales made in 2014
      \[ \text{(2)} \]
   The number of sales in 2014 was 20% more than the number of sales in 2013
   (b) Calculate the number of sales made in 2013
      \[ \text{(2)} \]
   In 2013, the supermarket was open for \( x \) hours each day, where \( x < 24 \)
   On average, the supermarket made a sale every 27 minutes of each day of 2013
   (c) Calculate the value of \( x \).
Question 1 continued

(Total for Question 1 is 6 marks)
2 Information about the marks scored by 220 candidates in an examination is shown in the incomplete table and incomplete histogram.

<table>
<thead>
<tr>
<th>Mark ((m)) range</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 &lt; m \leq 25)</td>
<td>25</td>
</tr>
<tr>
<td>(25 &lt; m \leq 50)</td>
<td>75</td>
</tr>
<tr>
<td>(50 &lt; m \leq 70)</td>
<td></td>
</tr>
<tr>
<td>(70 &lt; m \leq 80)</td>
<td>44</td>
</tr>
<tr>
<td>(80 &lt; m \leq 100)</td>
<td></td>
</tr>
</tbody>
</table>

Complete the table and the histogram.
Question 2 continued

(Total for Question 2 is 5 marks)
3 (i) Express \( \frac{3x}{x + 2} - \frac{6}{2x - 5} \) as a single fraction.

Give your answer in its simplest form.

(ii) Hence solve \( \frac{3x}{x + 2} - \frac{6}{2x - 5} = 0 \)
The mapping \( f: x \mapsto ax + b \) is represented by the diagram shown in Figure 1.

(a) Use the information in Figure 1 to write down two equations in \( a \) and \( b \).

(b) Solve your two equations to find the value of \( a \) and the value of \( b \).

(c) Using your values of \( a \) and \( b \), find the inverse function \( f^{-1} \).

Give your answer in the form \( f^{-1}: x \mapsto ... \).

(d) Hence find the value of \( x \) for which \( f(x) = f^{-1}(x) \).
Question 4 continued

(Total for Question 4 is 9 marks)
In a survey, 100 people were asked to say which of three activities they enjoyed doing. The three activities were listening to music (L), reading (R) and walking (W).

The results showed that

\[ n(L \cap R \cap W) = 7, \quad n(W \cap R \cap L') = 25, \quad n(W \cap L \cap R') = 20, \]
\[ n(R \cap L' \cap W') = 4, \quad n(L \cap [R \cup W']) = 9, \]
\[ n(R \cap L \cap W') = x = n(W \cap [R \cup L']). \]

The information from the survey is to be shown in a Venn diagram. The Venn diagram has been started below.

(a) Explain what the number 15 in the Venn diagram represents.

(b) Complete the Venn diagram.

(c) Work out the value of \( x \).

(d) Find the number of people in the survey who

(i) enjoy reading,

(ii) enjoy only one of the three activities,

(iii) enjoy reading and walking but do not enjoy listening to music.
Figure 2 shows ΔABC which is right-angled at A.

The point D lies on AC such that AD = x cm and DC = y cm.

The point E lies on AB such that AE = 4 cm and EB = 2x cm.

ED = 5 cm.

(a) Calculate the length, in cm, of AD.

Given that the area of ΔABC is 10 times the area of ΔAED,

(b) calculate the length, in cm, of DC,

(c) calculate the area, in cm², of EBCD.
Question 6 continued

(Total for Question 6 is 8 marks)
There are 50 books on a bookshelf.
These books are either textbooks or novels.
30 of these books are textbooks and the rest are novels.
Fatima takes at random a book from the bookshelf and does not return it to the bookshelf.
Fatima then takes at random another book from the bookshelf.
(a) Complete the tree diagram that represents these two events.
Question 7 continued

(b) Calculate the probability that both of the books taken from the bookshelf are textbooks.

Fatima returns both books to the bookshelf.

5 more novels are added to the bookshelf.

Fatima takes at random two books from the bookshelf, one after the other without replacement.

(c) Calculate the probability that at least one of the two books removed from the bookshelf is a novel.
Question 7 continued

(Total for Question 7 is 8 marks)
The points (2, 1), (6, 3) and (6, 1) are the vertices of triangle $A$.

(a) On the grid, draw and label triangle $A$.

Triangle $A$ is transformed to triangle $B$ under the transformation with matrix $N$ where

$$N = \begin{pmatrix} \frac{1}{2} & -2 \\ -\frac{1}{2} & 1 \end{pmatrix}$$

(b) Find the coordinates of the vertices of $B$.

(c) On the grid, draw and label $B$.

Triangle $B$ is transformed to triangle $C$ under the transformation with matrix $M$ where

$$M = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$$

(d) Find the coordinates of the vertices of $C$.

(e) On the grid, draw and label $C$.

(f) Describe fully the single transformation which maps triangle $A$ onto triangle $C$. 
Question 8 continued
Question 8 continued

(Total for Question 8 is 9 marks)
Figure 3 shows a circle $PBC$ with centre $O$ and diameter $CP$.

The point $A$ is such that $AB = 11$ cm and $AB$ is a tangent to the circle.

$APOC$ is a straight line and $\angle OAB = 15^\circ$

Calculate the length, in cm to 3 significant figures, of

(a) $OA$, 

(b) $AP$, 

(c) $BC$. 

The tangent to the circle $PBC$ at $P$ intersects $AB$ at the point $Q$.

(d) Calculate the area, in $\text{cm}^2$ to 3 significant figures, of $BCPQ$.

[Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Area of a triangle $= \frac{1}{2} \cdot bc \cdot \sin A$]
Question 9 continued
A solid $S$ is made by removing a hemisphere of radius $\frac{h}{2}$ cm from each end of a rectangular prism of length $l$ cm, depth $h$ cm and width $h$ cm, as shown in Figure 4.

The volume of the solid $S$ is $V$ cm$^3$.

(a) Find and simplify an expression for $V$ in terms of $h$, $l$ and $\pi$.

Given that $l + h = 10$

(b) show that $V = h^2 \left[ 10 - h \left( 1 + \frac{\pi}{6} \right) \right]$

\[
\left[ \text{Volume of sphere} = \frac{4}{3} \pi r^3 \right]
\]
Question 10 continued

(c) For \( V = h^2 \left[ 10 - h \left( 1 + \frac{\pi}{6} \right) \right] \), complete the table, giving the values of \( V \) to 1 decimal place.

<table>
<thead>
<tr>
<th>( h )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>0</td>
<td>27.8</td>
<td>62.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(d) On the grid, plot the points from your completed table and join them to form a smooth curve.

(e) Hence find the maximum value, to the nearest integer, of \( V \).

(f) Use your graph to find the range of values, to 1 decimal place, of \( h \) for which \( V > 60 \).
In Figure 5, $OAXB$ is a trapezium with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$ and $\overrightarrow{BQ} = k\mathbf{a}$, where $k$ is a positive constant.

The diagonals $AB$ and $OQ$ of the trapezium intersect at the point $P$.

(a) (i) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, $\overrightarrow{AB}$.

(ii) Find, in terms of $\mathbf{a}$, $\mathbf{b}$ and $k$, $\overrightarrow{OQ}$.

The point $P$ is such that $AP : AB = 1 : 3$

(b) Write down an expression for $\overrightarrow{AP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

The point $P$ is such that $OP : OQ = 1 : \mu$

(c) (i) Write down an expression for $\overrightarrow{OA}$ in terms of $\mathbf{a}$, $\mathbf{b}$, $\mu$ and $k$.

(ii) Hence find the value of $\mu$ and the value of $k$.

(d) Given that the area of $\triangle BPQ$ is $12$ cm$^2$, find the area, in cm$^2$, of $\triangle OPA$. 

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Diagram NOT accurately drawn