Pearson Edexcel

International GCSE Mathematics A

Exemplar Material for Paper 1 Higher produced from the 2018 May/June Series

4MA1/01

Paper 1H (Calculator)

Exemplar Question 1

Higher tier Paper 1 Question 8

8 $A = 3^5 \times 5 \times 7^3$

$B = 2^3 \times 3 \times 7^4$

(a) (i) Find the Highest Common Factor (HCF) of A and B.

(ii) Find the Lowest Common Multiple (LCM) of A and B.

(2)

Mean score ai (0.66/1), aii (0.46/1)

Examiner Comments

This question is within the context of Number.

This question is a little different to being asked for the HCF or LCM of 2 or 3 numbers, which is a well-rehearsed routine for many students. Instead this gives number in index form and the different presentation is testing that students really know what they are doing.

The question was done relatively well with part (i) more likely to be correct than (ii).

Mark Scheme

Question	Answer	Mark	Notes
8ai	3×7^3	B1	or for 1029
8aii	$2^3 \times 3^5 \times 5 \times 7^4$	B1	Or for 23 337 720

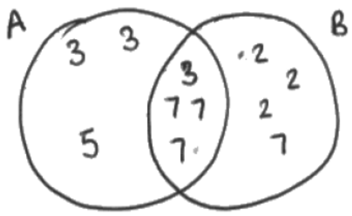
Examiner Comments

Using the calculator to find the number equivalent to the number in index form was not necessary but if students did this then they still gained full marks. Clearly it took extra time to do this.

Student Response A

$A = 3^5 \times 5 \times 7^3$
$B = 2^3 \times 3 \times 7^4$

(a) (i) Find the Highest Common Factor (HCF) of A and B.



$3 \times 7 \times 7 \times 7 = 1029$

1

(ii) Find the Lowest Common Multiple (LCM) of A and B.

$3^3 \times 7^4 \times 2^3 \times 5 =$ ~~777 9240~~

2593080

Examiner Comments

This student has used a popular method of a "Venn diagram" to find the HCF and LCM. The correct answer for part (i) is seen in two different forms and is awarded B1

For part (ii) the student has 3 to the power 3 rather than 3 to the power 5. This could be a misread but we cannot be sure – misreads are very common and students must be reminded to check their work carefully. B0 is awarded for (ii)

Student Response B

8 $A = 3^5 \times 5 \times 7^3$
$B = 2^3 \times 3 \times 7^4$

(a) (i) Find the Highest Common Factor (HCF) of A and B.

$3^5 \times 5 \times 7^3 = 416745 \div 343 = 1215$

$2^3 \times 3 \times 7^4 = 57624 \div 343 = 168$

$7^3 = 343$

343 0 Q08

(ii) Find the Lowest Common Multiple (LCM) of A and B.

$3^6 \times 7^3 \times 5 \times 3 \times 2$

7501410

(2) 0 Q08

Examiner Comments

(i) An incorrect method and wrong answer are shown. The student works with the individual factors of A and B and divides them by 7^3. B0 is awarded.

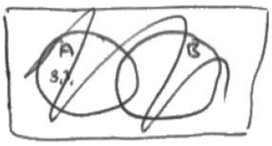
(ii) The method shown misses some of the factors. B0 is awarded.

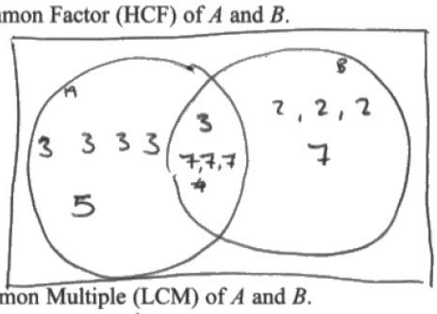
Student Response C

8 $A = 3^5 \times 5 \times 7^3$ 416745

$B = 2^3 \times 3 \times 7^4$ 57624

(a) (i) Find the Highest Common Factor (HCF) of A and B.





3×7^3

1029 .... 1 QC

(ii) Find the Lowest Common Multiple (LCM) of A and B.

$3^5 \times 5 \times 7^3 \times 2^3 \times 3 \times 7^4 = 2.4014513888 \times 10^{10}$

24014513880

24014513880 ....

(2) 0 QC

Examiner Comments

This student has used a popular method of a "Venn diagram" to find the HCF and LCM. The correct answer for part (i) is seen in two different forms and is awarded B1

For part (ii) the student has multiplied by an extra 7^3 and 3 so gains B0

Exemplar Question 2

Higher tier Paper 1 Question 13

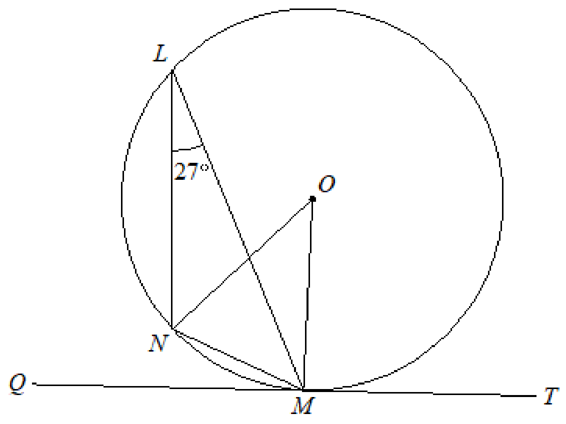


Diagram **NOT** accurately drawn

L, *M* and *N* are points on a circle, centre *O*.
QMT is the tangent to the circle at *M*.

(a) (i) Find the size of angle *NOM*.
(ii) Give a reason for your answer.

(b) (i) Find the size of angle *NMQ*.
(ii) Give a reason for your answer.

Mean score: (a)(i) (0.55/1), (a)(ii) (0.44/1), (b)(i) (0.62/1), (b)(ii) (0.33/1)

Examiner Comments

This question is within the context of shape and space.

Students often know the practical implication of circle theorems, but lack of precise vocabulary means they often gain no marks for the reasons required, e.g. stating alternate angles rather than alternate segment theorem.

Mark Scheme

Question	Answer	Mark	Notes
(a)(i)	54	1	B1 correct answer only (cao)
(ii)	Angle at centre is twice angle at circumference	1	B1 dep on B1 in (a)(i) accept alternative reasons e.g. angle at circumference is half the angle at the centre
(b)(i)	27	1	B1 follow through (ft) from (a)(i) for "54" ÷ 2
(ii)	Alternate segment theorem	1	B1 dep on B1 in (b)(i) accept alternative reason Angle between tangent and radius is 90°. If angle for (b)(i) is ft from (a)(i) then reason must be angle between tangent and radius is 90°

Examiner Comments

For the reason marks (a)(ii) and (b)(ii) the answers have to be correct or the correct follow through. For part (b)(i) we allowed a follow through mark for students who did not get the correct answer for (a)(i) which means they did not get penalised twice for the same mistake. The underlined words in the mark scheme show the minimum words that must be seen for the award of the mark.

Student Response A

13

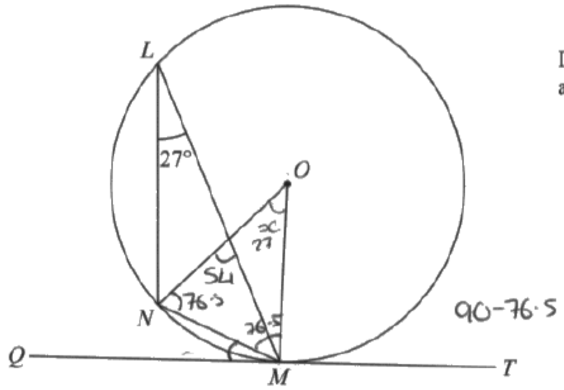


Diagram **NOT** accurately drawn

0 Q13a
0 Q13a
1 Q13b
0 Q13b

L, *M* and *N* are points on a circle, centre *O*.
QMT is the tangent to the circle at *M*.

(a) (i) Find the size of angle *NOM*.

27° °

(ii) Give a reason for your answer.

It is half a the angle in the middle

(2)

(b) (i) Find the size of angle *NMQ*.

13.5 °

(ii) Give a reason for your answer.

angle NMO is 76.5° and angle OMQ is 90° so angleNMQ is 90 - 76.5

(2)

(Total for Question 13 is 4 marks) 1

Examiner Comments

In part (a)(i) this student has the common misconception that angle *NOM* is equal to angle *NLM* – i.e. confusing angles in the same segment with the angle at the centre being twice the angle at the circumference. Their reason does not really follow from the method they have used but would be insufficient anyway. B0 B0

In part (b)(i) the student has been able to benefit from the follow through mark for an answer that is half the angle given in part (a)(i) and so gains B1. Part (b)(ii) gains no marks. We would have accepted 'the angle between a tangent and a radius is 90°' but the correct language is not seen here. B0

Student Response B

L, *M* and *N* are points on a circle, centre *O*.
QMT is the tangent to the circle at *M*.

(a) (i) Find the size of angle *NOM*.

54 °

(ii) Give a reason for your answer.

(2)

(b) (i) Find the size of angle *NMQ*.

27 °

(ii) Give a reason for your answer.

(2)

(Total for Question 13 is 4 marks) 2

Examiner Comments

This is a typical response where the student knows how to work out the angles but is unable to give the reasons.

(a)(i) gains B1 and (b)(i) gains B1

Student Response C

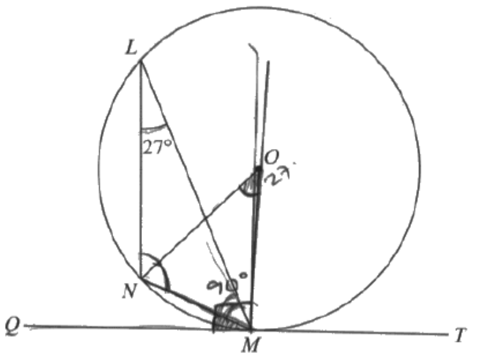


Diagram **NOT** accurately drawn

0 Q13ai
0 Q13aii
0 Q13bi
0 Q13bii

L, *M* and *N* are points on a circle, centre *O*.
QMT is the tangent to the circle at *M*.

(a) (i) Find the size of angle *NOM*.

27 °

(ii) Give a reason for your answer.

Because of the opposite angle theorem.

(2)

(b) (i) Find the size of angle *NMQ*.

~~27 + 27 = 54~~
~~90 − 27 = 63~~
180 − ~~27~~ = ~~153~~ = ~~77~~
90 − ~~77~~ = 13

13 °

(ii) Give a reason for your answer.

because the line that comes out of the tangent creates a right angle so you add all the angles you know and subtract from 90.

(2)

(Total for Question 13 is 4 marks) 0

Examiner Comments

This student is another who thinks angle *NOM* is the same as, rather than twice as much as, angle *MLN*. No marks are awarded in (a)

In part (b) the student has either miss-calculated or chosen to ignore the half that their calculator should have given them. They gain no marks for part (i).
For part (ii) they could not gain the mark as they have not gained B1 in (b)(i) [this mark is dep on that], but it shows a typical answer we see that knows a bit about the theorem, but is unable to state the precise reasons. We were looking for the 'angle between a tangent and a radius is 90°' or similar statement.

Exemplar Question 3

Higher tier Paper 1 Question 14

$$f(x) = \frac{3x - 5}{4}$$

(a) Find f(−7)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

The function g is such that

$$g(x) = \sqrt{19 - x}$$

(c) Find fg(3)

(d) Which values of x cannot be included in any domain of g?

Mean score: (a) 0.88/1, (b) 1.16/2, (c) 1.54/2, (d) 0.63/2

Examiner Comments

This question is within the context of algebra.

Part (a) was very well done but a few students solved the equation $f(x) = -7$ and a small number substituted 7 rather than −7

Those who knew how to find an inverse function generally gained full marks in part (b) but a few who made a good start, left the inverse in terms of y rather than changing it to be in terms of x. Finding the inverse is more challenging for some students.

Part (c) was well done but a few students stopped after finding g(4).

In part (d) many identified 19 as a critical value but incorrectly gave this as the answer rather than writing down the inequality to define all the required values of x. Another common error was to assume that x only took integer values and so give the answer greater than or equal to 20

Mark Scheme

Question	Answer	Mark	Notes
14 (a)	−6.5	1	B1 oe
(b)	$\frac{4x+5}{3}$	2	M1 for $4y = 3x - 5$ or $4x = 3y - 5$ A1 oe
(c)	1.75	2	M1 for $\sqrt{19-3}$ oe or f(4) = $\frac{3\sqrt{19-3}-5}{4}$ or $\frac{3\sqrt{19-x}-5}{4}$ oe A1oe for 1.75 oe (an no other solution)
(d)	$x > 19$	2	B2 for $(x) > 19$ or an equivalent statement in words. If not B2 then B1 for $(x) \geq 19$

Examiner Comments

In (a) We have given −6.5 in the mark scheme but oe (or equivalent) answers of −6½ and $-\frac{13}{2}$ would gain B1. Similarly in (c) oe answers to 1.75 would gain full marks.

Student Response A

14 The function f is such that

$$f(x) = \frac{3x - 5}{4}$$

(a) Find f(−7)

$f(-7) = \frac{3\times(-7)-5}{4}$

$= \frac{-21-5}{4}$

$= \frac{-26}{4} = -6.5$

-6.5

(1) 1 Q14a

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

$\frac{3x-5}{4} = y$

$3x-5 = 4y$

$3x = 4y+5$

$x = \frac{4y+5}{3}$

$f^{-1}(x) = \frac{4x+5}{3}$

$f^{-1}(x) = \frac{4x+5}{3}$

(2) 2 Q14b

The function g is such that

$$g(x) = \sqrt{19 - x}$$

(c) Find fg(3)

$g(3) = \sqrt{19-3}$ $f(4) = \frac{3\times 4-5}{4}$

$= \sqrt{16}$ $= \frac{12-5}{4}$

$= 4$ $= \frac{7}{4}$

$\frac{7}{4}$

(2) 2 Q14c

(d) Which values of x cannot be included in any domain of g?

$19 - x \geqslant 0$

$-x \geqslant -19$

$x \leqslant 19$

$x \leqslant 19$

(2) 0 Q14d

(Total for Question 14 is 7 marks) 5

Examiner Comments

(a) B1 for a correct answer

(b) This student shows a fully correct process to find the inverse and remembers to change y to x in their final line of working – which some failed to do. M1A1 is awarded

(c) M1A1 for a correct answer.

(d) It looks as if this student has not read the question correctly, but we cannot tell. No marks are awarded as the answer is incorrect.

Student Response B

14 The function f is such that

$$f(x) = \frac{3x - 5}{4}$$

(a) Find f(−7)

$f(x) = \frac{3(-7) - 5}{4}$ $= \frac{-21 - 5}{4}$ $= \frac{-26}{4}$ $= -6.5$

−6.5

(1) 1 Q14a

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

$f^{-1}(x) = \frac{3x - 5}{4}$

$f^{-1}(x) =$ ....

(2) 0 Q14b

The function g is such that

$$g(x) = \sqrt{19 - x}$$

(c) Find fg(3)

$g = \sqrt{19 - 3}$

$g = \sqrt{16}$

$= 4$

$f = \frac{3(3) - 5}{4}$

$= \frac{9 - 5}{4}$

$= \frac{4}{4} = 1$

4 , 1

(2) 1 Q14c

(d) Which values of x cannot be included in any domain of g?

....

(2) 0 Q14d

(Total for Question 14 is 7 marks) 2

Examiner Comments

(a) B1 for a correct answer

(b) This student does not appear to know what to do to find the inverse and just writes down the function. M0A0

(c) This student does not seem to know the meaning of a composite function and gives the 2 answers found by doing g(3) and f(3). M1 is awarded for g(3)

(d) The student does not attempt this part and gains no marks.

Student Response C

14 The function f is such that

$$f(x) = \frac{3x-5}{4}$$

(a) Find f(–7)

$$\frac{3\times(-7)-5}{4} = \frac{-21-5}{4} = \frac{-26}{4} = -6.5$$

~~-6~~ -6.5

(1) 1 Q14a

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

$$f^{-1}(x) = 3y = \frac{x-5}{4}$$

$$y = \frac{x-5}{4} \div 3$$

$$= \frac{x-5}{12}$$

$f^{-1}(x) = \frac{x-5}{12}$

(2) 0 Q14b

The function g is such that

$$g(x) = \sqrt{19-x}$$

(c) Find fg(3)

$$fg(3) = \frac{3\times(\sqrt{19-3}) - 5}{4}$$

$$= 1.75$$

1.75

(2) 2 Q14c

(d) Which values of x cannot be included in any domain of g?

(2) 0 Q14d

3

Examiner Comments

(a) B1 for a correct answer

(b) The student shows an incorrect method, it seems they falsely believe they should multiply y by 3 and remove the 3 from x. M0A0 is awarded.

(c) A correct answer is seen and M1A1 is awarded.

(d) The student does not attempt this part and gains no marks. The concept of not being able to find the square root of a negative is not always understood.

Exemplar Question 4

Higher tier Paper 1 Question 15

15 (a) Simplify fully $\left(\dfrac{256x^{20}}{y^8}\right)^{-\frac{1}{4}}$

(2)

(b) Express $\dfrac{1}{9x^2-25}-\dfrac{1}{6x+10}$ as a single fraction in its simplest form

(3)

Mean score: (a) 0.58/2 (b) 0.89/3

Examiner Comments

This question is within the context of algebra.

Students found part (a) very challenging.. Some students were able to score the first mark with a correct first step (either correctly dealing with −1 or ¼). Often, however, a mistake in their working or an incorrectly copied element of the expression prevented this from being awarded. A common error was to fail to deal with $256^{\frac{1}{4}}$ correctly, often giving this the incorrect value of 64 (1/4 of 64). A common final answer was $\dfrac{0.25x^{-5}}{y^{-2}}$, gaining one of the two available marks; those who dealt with the negative powers correctly then gained the second mark.

The main source of errors in part (b) was in expanding $-(3x - 5)$; it was disappointing to see those who factorised the quadratic and found a suitable common denominator failing in this respect. Students who worked with the lowest common denominator of $2(3x + 5)(3x - 5)$ were more successful than those who used $(9x^2 - 25)(6x + 10)$. The latter generally struggled to correctly factorise the negative quadratic that resulted in the numerator. Although some students knew how to start to combine the fractions, many did not try to simplify and made no attempt to factorise.

Mark Scheme

Question		Working	Answer	Mark	Notes
15	a	E.g. $\left(\frac{y^8}{256x^{20}}\right)^{\frac{1}{4}}$ or $\left(\frac{4x^5}{y^2}\right)^{-1}$ or $\frac{x^{-5}}{4y^{-2}}$ or $\frac{\frac{1}{4}x^{-5}}{y^{-2}}$ or $k\frac{y^a}{x^b}$ or $\frac{ky^a}{x^b}$ with 2 of $k = \frac{1}{4}$ oe , $a = 2$, $b = 5$ or $\frac{y^a}{mx^b}$ with 2 of $m = 4$, $a = 2$, $b = 5$	$\frac{y^2}{4x^5}$	2	M1 for a correct first step leading to a correct partially simplified expression A1 for $\frac{y^2}{4x^5}$ or $\frac{\frac{1}{4}y^2}{x^5}$ or $0.25\frac{y^2}{x^5}$ or $0.25y^2x^{-5}$
	b	$\frac{1}{(3x-5)(3x+5)} - \frac{1}{2(3x+5)}$ E.g. $\frac{2}{2(3x-5)(3x+5)} - \frac{1(3x-5)}{2(3x-5)(3x+5)}$ or $\frac{6x+10}{(9x^2-25)(6x+10)} - \frac{9x^2-25}{(9x^2-25)(6x+10)}$	$\frac{7-3x}{2(3x-5)(3x+5)}$	3	M1 indep for $(3x + 5)(3x - 5)$ M1 for two correct fractions with a common denominator if there is any expansion at this stage then it must be correct A1 accept equivalents eg. $\frac{7-3x}{18x^2-50}$
		Alternative scheme $\frac{6x+10}{(9x^2-25)(6x+10)} - \frac{9x^2-25}{(9x^2-25)(6x+10)}$ $\frac{(7-3x)(3x+5)}{(9x^2-25)(6x+10)}$	$\frac{7-3x}{2(3x-5)(3x+5)}$	3	M1 for two correct fractions with a common denominator M1 Numerator expanded and then factorised correctly A1 accept equivalents

Examiner Comments

(a) There were several ways of achieving 1 mark in this part and students having some knowledge of fractions or negative powers were often able to do this. Completing the question correctly was more challenging.

(b) The best way to do this part was to notice the difference of 2 squares on the denominator of the first fraction – students should be reminded to look out for this! Dealing with a negative before a bracket always has the potential to cause problems and students must have plenty of practice in doing this.

Student Response A

15 (a) Simplify fully $\left(\frac{256x^{20}}{y^8}\right)^{-\frac{1}{4}}$

$$\left(\frac{y^8}{256x^{20}}\right)^{\frac{1}{4}}$$

$$\sqrt[4]{256} = 4$$

$$\frac{y^2}{4x^5}$$

(2) 2 Q15a

(b) Express $\frac{1}{9x^2 - 25} - \frac{1}{6x + 10}$ as a single fraction in its simplest form.

$$\frac{1}{(3x+5)(3x-5)} - \frac{1}{2(3x+5)}$$

$$\frac{2-(3x-5)}{2(3x+5)(3x-5)} = \frac{7-3x}{18x^2-50}$$

$$\frac{7-3x}{18x^2-50}$$

(3) 3 Q15b

(Total for Question 15 is 5 marks) 5

Examiner Comments

(a) M1A1 for a fully correct answer. This student benefitted from working through the question by first dealing with the negative power and then understanding that the power of ¼ meant the 4th root and that the other powers needed to be multiplied by ¼
Students who tried to do everything in one go without writing down at least one intermediate step often failed.
(b) This student shows a fully correct response and is typical of students who recognised the need to use the difference of 2 squares. The working is set out logically and the student knows how to deal with the negative before the bracket.

Student Response B

15 (a) Simplify fully $\left(\dfrac{256x^{20}}{y^8}\right)^{-\frac{1}{4}}$

$$\left(\frac{256x^{20}}{y^8}\right)^{-\frac{1}{4}}$$

$$= \frac{0.25x^{-5}}{y^{-2}}$$

$\frac{0.25x^{-5}}{y^{-2}}$

(2) 1 Q15

(b) Express $\dfrac{1}{9x^2 - 25} - \dfrac{1}{6x + 10}$ as a single fraction in its simplest form.

$$\frac{1}{9x^2 - 25} - \frac{1}{6x + 10}$$

$$= \frac{6x + 10 - 9x^2 + 25}{(9x^2 - 25)(6x + 10)}$$

$$= \frac{-9x^2 + 6x + 35}{(9x^2 - 25)(6x + 10)}$$

$$= \frac{-9x^2 + 6x + 35}{54x^3 + 90x^2 - 150x - 250}$$

(3) 1 Q15

(Total for Question 15 is 5 marks) 2

Examiner Comments

(a) M1 for showing a correct first step which in this case is dealing with the power of x and the power of y and found 256 to the power of −1/4. The student has then failed to deal with the negative power

(b) The response here gained M1 for two correct fractions with a common denominator. Using this method the numerator needs to be expanded and then factorised correctly which the student failed to do – clearly the answer is not in its simplest form.

Student Response C

15 (a) Simplify fully $\left(\dfrac{256x^{20}}{y^8}\right)^{-\frac{1}{4}}$

$$\frac{256x^5}{y^8}$$

$$\frac{256x^5}{y^8}$$

(2) 0 Q15a

(b) Express $\dfrac{1}{9x^2-25} - \dfrac{1}{6x+10}$ as a single fraction in its simplest form.

$$\frac{1(6x+10)}{9x^2-25} - \frac{1(9x^2-25)}{6x+10}$$

$$= \frac{6x+10-6x+10}{9x^2-25-9x^2+25} = \frac{1}{1}$$

$\frac{1}{1}$

(3) 0 Q15b

(Total for Question 15 is 5 marks) 0

Examiner Comments

(a) M0A0 as this student seems to have ignored the negative that is part of the power and has done nothing with 256

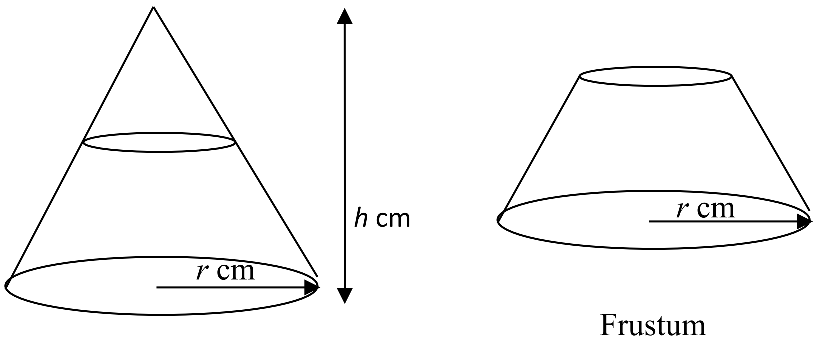
(b) M0M0A0 as the response shows no recognition of the difference of 2 squares or correct fractions over a common denominator.

Exemplar Question 5

Higher tier Paper 1 Question 16

A frustum is made by removing a small cone from a large cone.
The cones are mathematically similar.

Diagram **NOT** accurately drawn



The large cone has base radius r cm and height h cm.

Given that

$$\frac{\text{volume of frustum}}{\text{volume of large cone}} = \frac{98}{125}$$

find an expression, in terms of h, for the height of the frustum.

Examiner Comments (Mean score: 0.63/4)

This question was a challenging one on Shape, Space and Measures and was very poorly done with few students getting full marks. In order to make any real progress with this question, students had to recognise the fact that the small and large cone were similar and use the volume scale factor to find the length scale factor. Some students managed to recognise the need to work with 27 rather than 98 which gained them the first method mark. Those who introduced new variables for the height and radius of the small cone without using the relationship between the large and small cone were unable to make any progress as were those students who attempted to work with the incorrect scale factor of $\frac{98}{125}$ or $\sqrt[3]{\frac{98}{125}}$.

Mark Scheme

Question	Working	Answer	Mark	Notes
16	$1-\frac{98}{125}\left(=\frac{27}{125}\right)$ or 0.216 or 125 – 98 (=27) $\sqrt[3]{"\frac{27}{125}"}\left(=\frac{3}{5}\right)$ or $\sqrt[3]{"\frac{125}{27}"}\left(=\frac{5}{3}\right)$ $1-"\frac{3}{5}"$ or $h-"\frac{3}{5}"h$ oe	$\frac{2}{5}h$ oe	4	M1 M1 for the length scale factor may be seen as a ratio E.g. 3 : 5 M1 A1 for $\frac{2}{5}h$ oe (may not be simplified)
	Alternative scheme $\frac{1}{3}\pi r^2h-\frac{1}{3}\pi(kr)^2kh=\frac{98}{125}\times\frac{1}{3}\pi r^2h$ oe $k=\frac{3}{5}$ $1-"\frac{3}{5}"$ or $h-"\frac{3}{5}"h$ oe	$\frac{2}{5}h$ oe	4	M1 sets up an equation using scale factor M1 for the length scale factor M1 A1 for $\frac{2}{5}h$ oe (may not be simplified)

Examiner Comments:

The mark scheme shows two alternative methods of working which students tended to use; the first being the most commonly seen. Knowing to use the cube root of the scale factor is often the key to success with this type of question and students must be reminded of the relationships between the scale factors of linear, area and volume of similar figures.

Student Response A

The large cone has base radius r cm and height h cm.

Given that

$$\frac{\text{volume of frustum}}{\text{volume of large cone}} = \frac{98}{125}$$

find an expression, in terms of h, for the height of the frustum.

~~volume of cone $= \frac{1}{3}\pi r^2 h$~~

$125 - 98 = 27$

~~$125 = \frac{1}{3}\pi r^2 h$~~

~~$375 = \pi r^2 h$~~

~~$27 = \frac{1}{3}\pi r^2 h$~~

~~$81 = \pi r^2 h$~~

volume of small cone $= 27\text{cm}^3 = 3^3$

volume of large cone $= 125\text{ cm}^3 = 5^3$

ratios of small cone to large cone $3^3 : 5^3$

scale factor $= \sqrt[3]{\frac{3^3}{5^3}}$

$= \frac{3}{5}$

$\frac{2}{5}h$ cm

(Total for Question 16 is 4 marks) 4

Student Response B

Volume of cone : $\frac{1}{3}\pi r^2 h = 125$

volume of smaller cone = $(125 - 98 = 27$

$\text{vsf} = \frac{27}{125} = 0.216$

$\text{lsf} = \sqrt[3]{0.216} = 0.6$

height of smaller cone $= 0.6h$

height of frustum $= h - 0.6h$

$= 0.4h$

Examiner Comments:

These two responses, both gaining full marks, show a fully correct method.
While one worked with fractions the other worked with decimals but both of these can be marked using the first version of the mark scheme.
It must be noted that there are times when working with decimals where an inaccuracy can occur – ie when working with recurring decimals such as 1/3 and rounding to 0.3(3) etc

Student Response C

0.784

$125cm^3 - 98cm^3 = 27cm^3$

$\frac{27}{125} = 0.216$

r = 1

$height_{sp} = \frac{125}{\pi} = 39.8$ cm

$height_{sp} = 39.8cm \times 0.216 = 8.5968$ cm

$height_f = 39.8cm - 8.5968cm = 31.2$

31.2 cm

(Total for Question 16 is 4 marks) 1

Examiner Comments:

This response gains M1 for showing the scale factor of 27/125 (or 0.216) but the student failed to use the cube root of the scale factor and so gains no more marks.

Student Response D

1 Q16

16 A frustum is made by removing a small cone from a large cone.
The cones are mathematically similar.

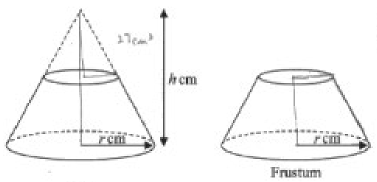


Diagram **NOT** accurately drawn

125 98

The large cone has base radius r cm and height h cm.

125 − 98

Given that

$$\frac{\text{volume of frustum}}{\text{volume of large cone}} = \frac{98}{125}$$

find an expression, in terms of h, for the height of the frustum.

.... cm

(Total for Question 16 is 4 marks) 1

Examiner Comments:

At first glance you may feel this student has done nothing, but the value of 27 gains M1 and this is seen on the diagram.

Exemplar Question 6

Higher tier Paper 1H Question 17

The diagram shows parallelogram *ABCD*.

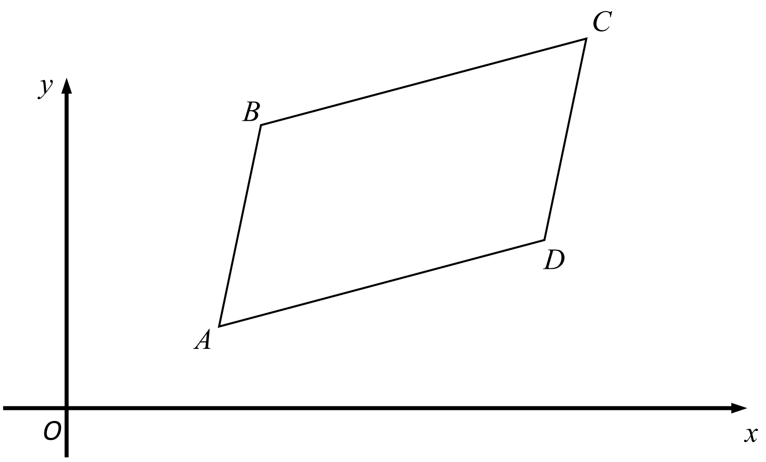


Diagram **NOT** accurately drawn

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$

The point *B* has coordinates (5, 8)

(a) Work out the coordinates of the point *C*.

(……… , ………)

(3)

The point *E* has coordinates (63, 211)

(b) Use a vector method to prove that *ABE* is a straight line.

(2)

Examiner Comments: (mean score (a) 2.14/3 (b) 0.52/2)

Part (a) was generally well done although there were some basic arithmetic errors seen that spoiled some otherwise correct solutions. The majority of students heeded the instructions in the question and used a vector method in part (b); the minority who used an algebraic method gained no marks. The common error in part (b) was to fail to give a conclusion to explain why their workings showed that *ABE* was a straight line.

Mark Scheme

Question		Working	Answer	Mark		Notes
17	a	$(\overrightarrow{BC} =)\begin{pmatrix}-2\\-7\end{pmatrix}+\begin{pmatrix}10\\11\end{pmatrix}\left(=\begin{pmatrix}8\\4\end{pmatrix}\right)$ $\begin{pmatrix}5\\8\end{pmatrix}+"\begin{pmatrix}8\\4\end{pmatrix}"$ **or** $\begin{pmatrix}10\\11\end{pmatrix}+"\begin{pmatrix}3\\1\end{pmatrix}"$	(13, 12)	3	M1 M1 A1	or coordinates (5 – 2, 8 – 7) (= (3, 1)) assigned to *A* (may be seen in vector form) **or** (13, *y*) or (*x*, 12) given as coordinates for *C* for coordinates (5 – 2 + 10, 8 – 7 + 11) assigned to *C*
	b	e.g. $\begin{pmatrix}63\\211\end{pmatrix}-\begin{pmatrix}5\\8\end{pmatrix}\left(=\begin{pmatrix}58\\203\end{pmatrix}\right)$ **with** e.g. "58" ÷ 2 (=29) **and** "203" ÷ 7 (=29) **OR** e.g. $\begin{pmatrix}63\\211\end{pmatrix}-\begin{pmatrix}3\\1\end{pmatrix}\left(=\begin{pmatrix}60\\210\end{pmatrix}\right)$ **with** e.g. "60" ÷ 2 (=30) **and** "210" ÷ 7 (=30)	Proof	2	M1 A1	may work with *A* and *E*, in which case may need to ft for method mark from (a) proof with justification eg. $\overrightarrow{BE} = 29\begin{pmatrix}2\\7\end{pmatrix}$ (or $\overrightarrow{AE} = 30\begin{pmatrix}2\\7\end{pmatrix}$) with *ABE* is a straight line or 210 ÷ 60 = 3.5 and 7 ÷ 2 = 3.5 so *ABE* is a straight line

Examiner Comments:

The biggest reminder for students with any question such as part (b) is that it is essential they show any proof clearly, and also make a final statement to make it clear to examiners that they know that what they have done has shown what was required. Note also, that a vector method was required, so any non-vector methods gained no marks.

Student Response A

The point B has coordinates (5, 8)

(a) Work out the coordinates of the point C.

$\overrightarrow{BC} = -\overrightarrow{AB} + \overrightarrow{AC}$

$= -\begin{pmatrix}2\\7\end{pmatrix} + \begin{pmatrix}10\\11\end{pmatrix}$

$= \begin{pmatrix}8\\4\end{pmatrix}$

$5+8=13$

$8+4=12$

(13 , 12)

(3) 3 Q17a

The point E has coordinates (63, 211)

(b) Use a vector method to prove that ABE is a straight line.

$\overrightarrow{AB} = \begin{pmatrix}2\\7\end{pmatrix}$

$\overrightarrow{BE} = \begin{pmatrix}63-5\\211-8\end{pmatrix} = \begin{pmatrix}58\\203\end{pmatrix}$

$\begin{pmatrix}58\\203\end{pmatrix} \div \begin{pmatrix}2\\7\end{pmatrix} = \begin{pmatrix}29\\29\end{pmatrix}$ ∴ $\overrightarrow{AB}$ is divisible by $\overrightarrow{BE}$ so they ~~are~~ ~~on t~~ have the same gradient and share a point so they are on the same line.

(2) 2 Q17b

(Total for Question 17 is 5 marks) 5

Examiner Comments:

In part (a) this student shows a fully correct response, by firstly using a method to find the coordinates of *A* and then showing the answer which comes from adding on the vector for AC to the coordinate of *A*.

(b) Uses a vector method and shows the vector *BE* and that 29 × the vector *AB* is equal to this. A correct conclusion is stated and so full marks are awarded.

Student Response B

The point B has coordinates (5, 8)

(a) Work out the coordinates of the point C.

$A = \begin{pmatrix}5\\8\end{pmatrix} - \begin{pmatrix}2\\7\end{pmatrix}$

$= \begin{pmatrix}3\\1\end{pmatrix}$ $\quad C = \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}10\\11\end{pmatrix}$

$= \begin{pmatrix}13\\12\end{pmatrix}$

(13 , 12)

(3) 3 Q17a

The point E has coordinates (63, 211)

(b) Use a vector method to prove that ABE is a straight line.

$\begin{pmatrix}63\\211\end{pmatrix} - \begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}60\\210\end{pmatrix}$

$(60, 210) \div \begin{pmatrix}2\\7\end{pmatrix} = 30$

(2) 1 Q17b

(Total for Question 17 is 5 marks) 4

Examiner Comments:

In part (a) the student shows the coordinates of A and then adds vector AC to gain a fully correct answer. 3 marks are awarded.

For part (b) the student gains M1 for subtracting the coordinates of A in vector form from the coordinates of E in vector form. The student should then have shown that $60 \div 2 = 30$ and $210 \div 7 = 30$ and then made a suitable conclusion. A0

Student Response C

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$

The point B has coordinates (5, 8)

(a) Work out the coordinates of the point C.

~~B(5,8)~~

$5-2=3$

$8-7=1$

$A(3,1)$

$3+10=13$

$1+11=12$

$C(13,12)$

(13, 12)

(3) 3 Q17a

The point E has coordinates (63, 211)

(b) Use a vector method to prove that ABE is a straight line.

$A(3,1)$

$B(5,8)$

$E(63,211)$

$y=kx+b$

$\begin{cases} 1=3k+b \\ 8=5k+b \end{cases}$

$\begin{cases} k=\frac{7}{2} \\ b=-\frac{19}{2} \end{cases}$

$211=\frac{7}{2}\times 63+\frac{19}{2}$

$=\frac{441}{2}+\frac{19}{2}$

(2) 0 Q17b

(Total for Question 17 is 5 marks) 3

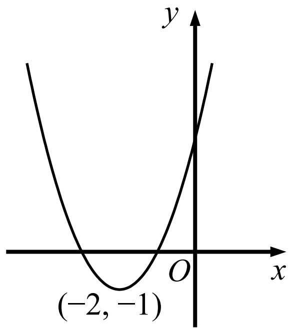
Examiner Comments:

This student shows a fully correct method and answer for part (a) so gains M1M1A1

For part (b) the student uses an algebraic method, failing to heed the statement 'Use a vector method...' and so gains no marks here. M0A0

Exemplar Question 7

Higher tier Paper 1H Question 18



The diagram shows the curve with equation $y = \mathrm{f}(x)$

The coordinates of the minimum point of the curve are $(-2, -1)$

(a) Write down the coordinates of the minimum point of the curve with equation

(i) $y = \mathrm{f}(x - 5)$

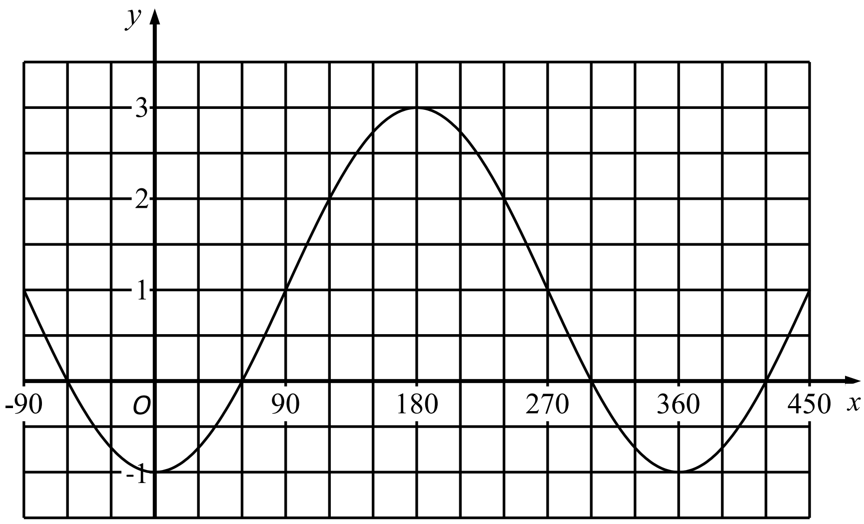
(…….. , ………)

(ii) $y = \frac{1}{2}\mathrm{f}(x)$

(…….. , ………)

(2)

The graph of $y = a\sin(x - b)° + c$ for $-90 \leq x \leq 450$ is drawn on the grid below.



(b) Find the value of a, the value of b and the value of c.

$a =$ ………………

$b =$ ………………

$c =$ ………………

(3)

Examiner Comments: (mean score (ai) 0.47/1 (aii) 0.36/1 (b) 0.67/3)
(a) Transformations of functions is a new topic to this specification and set at a high grade.
Part (a) was done reasonably well.
In part (b) it was encouraging to see some students start the solution by drawing the graph of y = sin$x$. Clearly this part was a challenging question and difficult for many students.

Mark Scheme

Question		Working	Answer	Mark	Notes
18	a (i)		(3, −1)	1	B1
	(ii)		(−2, −0.5) oe	1	B1
	b		e.g. 2, 90, 1	3	B3 for all 3 correct values e.g. 2, 90, 1 **or** −2, 270, 1 If not B3 then B2 for any 2 correct values NB. 2 values from 2, 90, 1 **OR** 2 values from −2, 270, 1 NB: accept a value of $(90 + 360n)$ in place of 90 or $(270 + 360n)$ in place of 270 where n is an integer (could be negative) If not B2 then B1 for any 1 correct value **or** the graph of $y = \sin x°$ for $0 \leq x \leq 360$

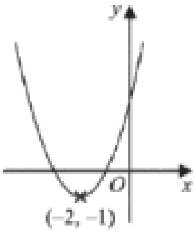
Examiner Comments:

(a)(i)(ii) this part was either correct or not with no method marks

(b) There was one mark for each correct value. Many students gave 3 rather than 2 as they looked at the maximum y value, rather than looking for the SF of the stretch.

Student Response A

18



The diagram shows the curve with equation $y = f(x)$
The coordinates of the minimum point of the curve are $(-2, -1)$

0 Q18a
0 Q18a

(a) Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(x - 5)$

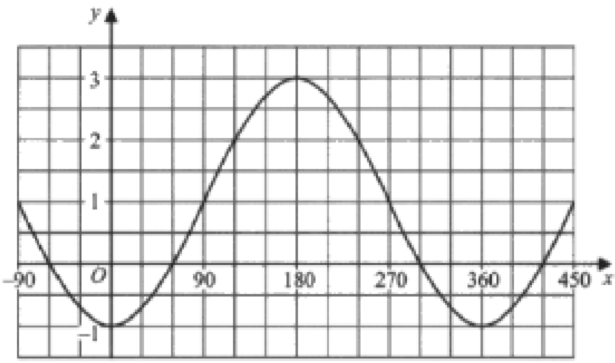
(–7 , 6)

(ii) $y = \frac{1}{2}f(x)$

(–1 , 0)
(2)

The graph of $y = a\sin(x - b)° + c$ for $-90 \leqslant x \leqslant 450$ is drawn on the grid below.

2 Q18b



(b) Find the value of a, the value of b and the value of c.

$a =$ 2

$b =$ 180

$c =$ 1
(3)

(Total for Question 18 is 5 marks) 2

Examiner Comments:

(a)(i)(ii) this student was unable to give the correct coordinates in either part, in (i) they have subtracted 5 from the *x* coordinate and added 7 to the *y* coordinate. (ii) the student has halved the *x* coordinate and added 1 to the *y* coordinate. B0B0 In part (b) the values of *a* and *c* are correct. B2

Student Response B

(i) $y = f(x - 5)$

(−2 , −b)

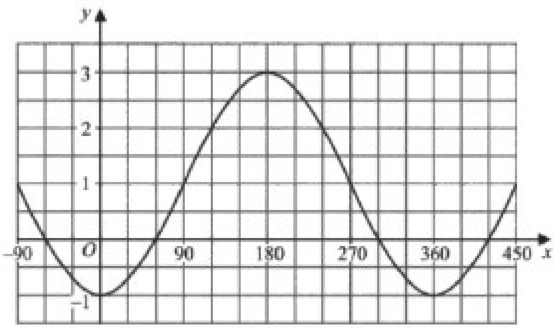
(ii) $y = \frac{1}{2}f(x)$

(−1 , −0.5)

(2)

The graph of $y = a\sin(x - b)° + c$ for $-90 \leqslant x \leqslant 450$ is drawn on the grid below.

0 Q18b



(b) Find the value of a, the value of b and the value of c.

a = ...

b = ...

c = ...

(3)

(Total for Question 18 is 5 marks) 0

Examiner Comments:

(a)(i)the student has in fact written the coordinates for $y = f(x) - 5$ B0
(a)(ii) the student has divided both the x and y coordinates by 2 rather than just the x coordinate. B0
(b) B0B0B0

Exemplar Question 8

Higher tier Paper 1H Question 19

Jack plays a game with two fair spinners, **A** and **B**.

Spinner **A** can land on the number 2 or 3 or 5 or 7.
Spinner **B** can land on the number 2 or 3 or 4 or 5 or 6.

Jack spins both spinners.
He wins the game if one spinner lands on an odd number **and** the other spinner lands on an even number.

Jack plays the game twice.
Work out the probability that Jack wins the game both times.

Examiner Comments: mean score: 1.83/4
A more challenging question on probability.
Listing winning outcomes was successful for some but the complexity of a tree diagram for two games defeated all but the most determined students. Some who found the probability of each combination of number, for example (2, 3) often failed to count the correct number of probabilities.

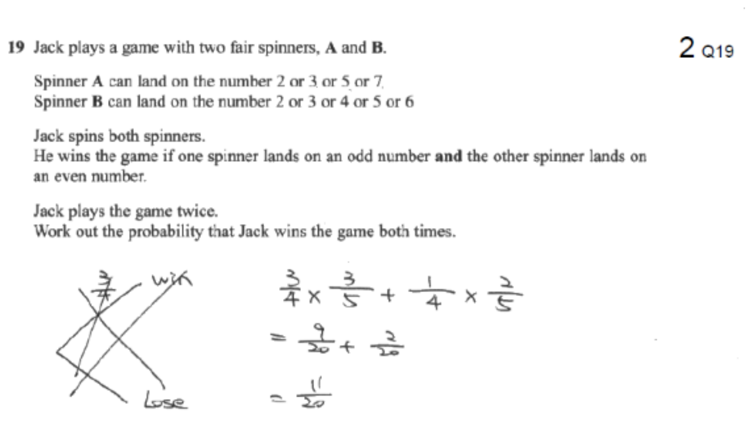
Mark Scheme

Question	Working	Answer	Mark	Notes
19	$\frac{1}{4}\times\frac{2}{5}\left(=\frac{2}{20}\right)$ or $\frac{3}{4}\times\frac{3}{5}\left(=\frac{9}{20}\right)$ or $\frac{1}{4}\times\frac{3}{5}\left(=\frac{3}{20}\right)$ or $\frac{3}{4}\times\frac{2}{5}\left(=\frac{6}{20}\right)$	$\frac{121}{400}$ oe	4	M1 for any one correct probability
	$\frac{1}{4}\times\frac{2}{5}+\frac{3}{4}\times\frac{3}{5}\left(=\frac{11}{20}\right)$ or $1-\left(\frac{1}{4}\times\frac{3}{5}+\frac{3}{4}\times\frac{2}{5}\right)\left(=\frac{11}{20}\right)$			M1 for a complete method
	"$\frac{11}{20}$"$\times$"$\frac{11}{20}$" or $\left(\text{"}\frac{2}{20}\text{"}+\text{"}\frac{9}{20}\text{"}\right)^2$			M1
				A1 for $\frac{121}{400}$ oe or 0.3025 or 30.25%

Examiner Comments:

The first method mark is for any correct probability which several students were able to achieve.
The second method mark is for a complete method for winning one game
The Third method mark for a complete method for winning 2 games
The Accuracy mark is for a fully correct answer written as a fraction, a decimal or a percentage.

Student Response A


19 Jack plays a game with two fair spinners, A and B. 2 Q19

Spinner A can land on the number 2 or 3 or 5 or 7.
Spinner B can land on the number 2 or 3 or 4 or 5 or 6

Jack spins both spinners.
He wins the game if one spinner lands on an odd number **and** the other spinner lands on an even number.

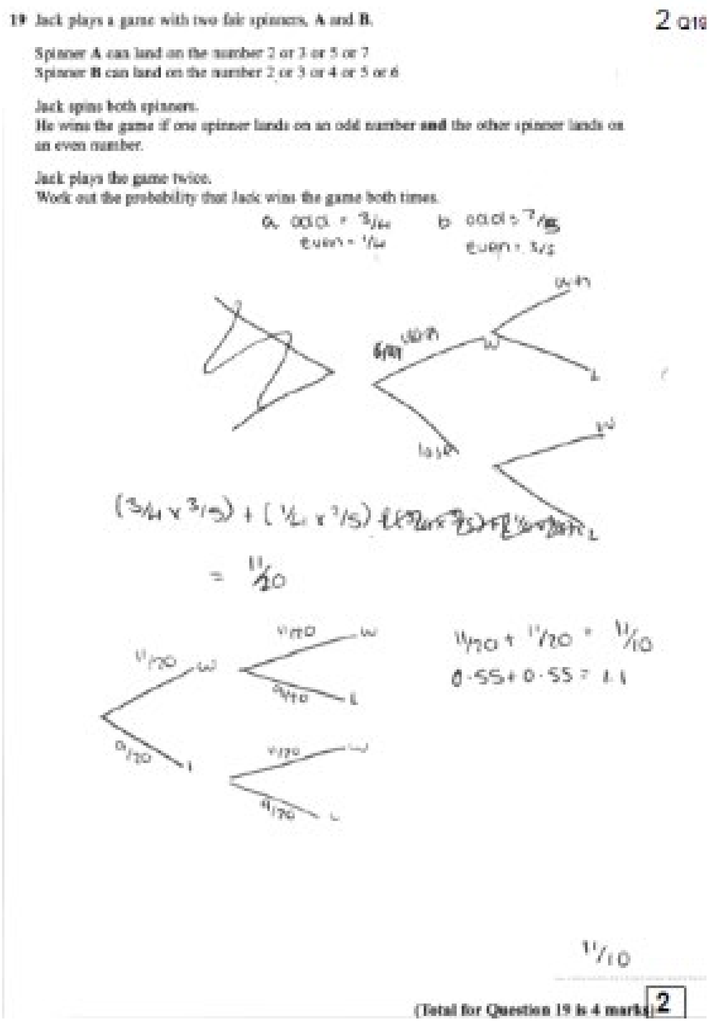
Jack plays the game twice.
Work out the probability that Jack wins the game both times.

Examiner Comments:

This student gains M1 for a correct probability and M1 for a complete method for winning one game.

No more marks as they don't consider winning the game twice.

Student Response B


19 Jack plays a game with two fair spinners, A and B. 2 Q19

Spinner A can land on the number 2 or 3 or 5 or 7
Spinner B can land on the number 2 or 3 or 4 or 5 or 6

Jack spins both spinners.
He wins the game if one spinner lands on an odd number **and** the other spinner lands on an even number.

Jack plays the game twice.
Work out the probability that Jack wins the game both times.

Examiner Comments:

A tree diagram approach is taken here.

M1 for a correct probability

M1 for a complete method for winning one game.

M0 as the student has added rather than multiplied the two correct probabilities.

A0 students should consider the accuracy of their work if a probability greater than 1 results!

Student Response C

19 Jack plays a game with two fair spinners, **A** and **B**. 4 Q19

Spinner **A** can land on the number 2 or 3 or 5 or 7
Spinner **B** can land on the number 2 or 3 or 4 or 5 or 6

Jack spins both spinners.
He wins the game if one spinner lands on an odd number **and** the other spinner lands on an even number.

Jack plays the game twice.
Work out the probability that Jack wins the game both times.

	2	3	5	7
2	✗	✓	✓	✓
3	✓	✗	✗	✗
4	✗	✓	✓	✓
5	✓	✗	✗	✗
6	✗	✓	✓	✓

prob. to win = $\frac{11}{20}$

prob to win 2times = $\frac{11}{20} \times \frac{11}{20}$

$= \frac{121}{400}$

$\frac{121}{400}$

(Total for Question 19 is 4 marks) 4

Examiner Comments:

This student has organised their work to enable them to see which probabilities to consider.

A fully correct method and answer result which gains

M1M1M1A1