Mark Scheme

Extra Assessment Material

for first teaching September 2017

International GCSE in Further Pure Mathematics

(4PM1/02) Paper 2

Further Pure International GCSE 4PM1/02					
Question	**Working**	**Answer**	**Mark**	**Spec Ref**	**Notes**
1		$y \geqslant 2x$ $y + 2x \ldots 4$ $x \geqslant 2$	B3	3E	
					Total 3 marks

Question	**Working**	**Answer**	**Mark**	**Spec Ref**	**Notes**
2	$\frac{\sqrt{3}}{2\sqrt{3}-2} = \frac{\sqrt{3}}{\left(2\sqrt{3}-2\right)} \times \frac{\left(2\sqrt{3}+2\right)}{\left(2\sqrt{3}+2\right)} = \frac{6+2\sqrt{3}}{12-4} = \frac{3+\sqrt{3}}{4}$ *		M1M1A1 (3)	1D	$\frac{3+\sqrt{3}}{4}$ with no working is M0M0A0
					Total 3 marks

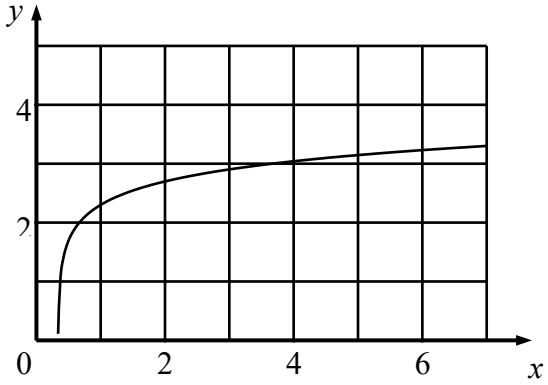
Question	Working	Answer	Mark	Spec Ref	Notes
3 (a)	$y^2 = 2x+3,\ \ x^2 = 2x+3 \Rightarrow x^2 - 2x - 3 = 0$ $\Rightarrow (x-3)(x+1) = 0 \Rightarrow x = 3,\ \ (-1)$	$x = 3$	M1A1 (2)	2A	
(b)	$\text{Volume} = \pi\int_0^3 (2x+3)\ \mathrm{d}x - \pi\int_0^3 x^2\ \mathrm{d}x$ $\text{Volume} = \pi\left[\frac{2x^2}{2}+3x\right]_0^3 - \pi\left[\frac{x^3}{3}\right]_0^3 = \pi\left[(9+9)-(9)\right] = 9\pi$ **ALT** $\text{Volume} = \pi\int_0^3 (2x+3)\ \mathrm{d}x - \frac{1}{3}\times\pi\times x^2\times h$ $\text{Volume} = \pi\left[\frac{2x^2}{2}+3x\right]_0^3 - \frac{1}{3}\times\pi\times 3^2\times 3 = \pi(9+9) - 9\pi = 9\pi$	9π 9π	M1 M1M1A1 (4) {M1 M1M1A1}	9C	
					Total 6 marks

Question	Working	Answer	Mark	Spec Ref	Notes
4 (a)	$\cos\angle BAC = \dfrac{8^2+9^2-13^2}{2\times 8\times 9} = -\dfrac{1}{6}$ *	$-\dfrac{1}{6}$ *	M1A1 (2)	10D	
(b)	(i) $\angle BAC = \cos^{-1}\left(\dfrac{8^2+13^2-9^2}{2\times 8\times 13}\right) = 43.049^\circ$ **ALT** $\angle BAC = \sin^{-1}\left(\dfrac{9\sin(99.594)}{13}\right) = 43.049^\circ$ (ii) $\angle BCA = 180 - 99.594 - 43.049 = 37.357 \approx 37.4^\circ$	43.0° 43.0º 37.4°	M1A1 M1A1 B1 (3)	10D	
(c)	$\angle BAX = 21.5245...^\circ \Rightarrow \angle BXA = 58.881^\circ$ $AX = \dfrac{8\sin(99.594)}{\sin(58.881)} = 9.214...\text{cm}$	9.21 cm	B1 M1A1 (3)	10D	
					Total 8 marks

Question	Working	Answer	Mark	Spec Ref	Notes
5 (a)	$\mathrm{f}(x)=5+3x-2x^2 \Rightarrow \mathrm{f}(x)=-2\left(x-\frac{3}{4}\right)^2+\frac{49}{8}$	$P=-2, Q=-\frac{3}{4}, R=\frac{49}{8}$	M1A1A1 (3)	2A	
(b)	$-2\left(x-\frac{3}{4}\right)^2+\frac{49}{8}-7=-2\left(x-\frac{3}{4}\right)^2-\frac{7}{8}$ $\Rightarrow$ **maximum** value of $-2\left(x-\frac{3}{4}\right)^2-\frac{7}{8}$ is $-\frac{7}{8}$ Hence $\mathrm{f}(x)-7<0$ for all values of x		M1A1 (2)		M1 – for the working A1 for the conclusion
					Total 5 marks

Question	Working	Answer	Mark	Spec Ref	Notes
6 (a)	$f(-1) = p(-1)^3 + q(-1)^2 - 1 + 6p = 0 \Rightarrow (5p + q = 1)$ $f(2) = p(2)^3 + q(2)^2 + 2 + 6p = -6 \Rightarrow (14p + 4q = -8)$ Solves simultaneous equations $5p + q = 1$ $14p + 4q = -8 \Rightarrow p = 2$ *, $q = -9$	$p = 2$ *, $q = -9$	M1 M1 M1A1 (4)	3B	
(b)	$x+1 \overline{)2x^3 - 9x^2 + x + 12}$ $2x^2 - 11x + 12$ $f(x) = (x+1)(2x^2 - 11x + 12) = (x+1)(2x-3)(x-4)$ **ALT** (comparing coefficients) $2x^3 - 9x^2 + x + 12 = (x+1)(Ax^2 + Bx + C) = Ax^3 + (A+B)x^2 + (B+C)x + C$ $\Rightarrow A = 2, \quad C = 12 \qquad -9 = A + B \Rightarrow B = -11$ $2x^2 - 11x + 12 = (2x-3)(x+4) \Rightarrow 2x^3 - 9x^2 + x + 12 = (2x-3)(x+4)(x+1)$	$(x+1)(2x-3)(x-4)$ $(x+1)(2x-3)(x-4)$	M1 M1A1 (3) {M1 M1 A1}	3A	
(c)	$(x+1)(2x-3)(x-4) = 0 \Rightarrow x = -1, \quad 4, \quad \frac{3}{2}$	$x = -1, \quad 4, \quad \frac{3}{2}$	M1A1 (2)	3C	
					Total 9 marks

Question	Working	Answer	Mark	Spec Ref	Notes
7 (a)	$(\alpha+\beta)^2=\frac{p^2}{4}$ and $\alpha\beta=\frac{q}{2}$		B1	2C	Award mark when both seen
	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=-\frac{57}{16}$		M1		Correct algebra on sum of roots
	$\frac{\frac{p^2-4q}{4}}{\frac{q}{2}}=\frac{p^2-4q}{2q}=-\frac{57}{16}\Rightarrow 16p^2-64q=-114q$		M1A1		Achieves equation in terms of p and q
	$\Rightarrow 8p^2=-25q$ $p+q=-3\Rightarrow q=-3-p$ $\Rightarrow 8p^2=-25(-3-p)$		M1		
	$\Rightarrow 8p^2-25p-75=0$		A1		Achieves 3TQ
	$\Rightarrow (8p+15)(p-5)=0\Rightarrow p=5\Rightarrow q=-8$	$p=5,\ q=-8$	M1A1 (8)		Correct values for p and q.
(b)	$(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta=(\alpha+\beta)^2-4\alpha\beta$		M1	2C	
	$\Rightarrow(\alpha-\beta)^2=\left(-\frac{5}{2}\right)^2-4\times-\frac{8}{2}=\frac{25}{4}+16=\frac{89}{4}$		M1		
	$\Rightarrow(\alpha-\beta)=+\sqrt{\frac{89}{4}}\ \left(=\frac{\sqrt{89}}{2}\right)$	$\sqrt{\frac{89}{4}}$	A1 (3)		
					Total 11 marks

Question	Working	Answer	Mark	Spec Ref	Notes
8(a)	Required values $x = 1.5 \quad y = 4.72$ $x = 3 \quad y = 6.52$ $x = 6 \quad y = 8.15$	 $x = 1.5 \quad y = 4.72$ $x = 3 \quad y = 6.52$ $x = 6 \quad y = 8.15$	 B2 (2)	4B	 B1 for one correct value B2 for all three correct values
(b)	Graph drawn. Their points plotted within half of one square Their points joined up in a smooth curve 		B2ft (2)	4B	B1ft for their point plotted within half on one square B1ft for a smooth curve passing through each of **their** points
(c)	$5\lg(3x-1) = x+1 \Rightarrow 5\lg(3x-1)+2 = x+3$ Draws line $y = x+3$	 $x = 1.3, \quad 4.5$	M1 M1 A1 (3)	1B 4B	 (Accept $x = 4.4$ or 4.5)

(d)	$3x = 10^{\left(2-\frac{x}{5}\right)} + 1 \Rightarrow 3x - 1 = 10^{\left(2-\frac{x}{5}\right)} \Rightarrow \lg(3x-1) = 2 - \frac{x}{5}$ $\lg(3x-1) = 2 - \frac{x}{5} \Rightarrow 5\lg(3x-1) + 2 = 12 - x$ Draws line $y = 12 - x$	$x = 4.5$	M1 M1 M1 A1 (4)		Rearranges and converts exponential into a logarithm So required line is $y = 12 - x$
					Total 11 marks

Question	Working	Answer	Mark	Spec Ref	Notes
9(a)	(i) $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{BD} = 2\mathbf{a} + \mathbf{b}$ (ii) $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = 3\mathbf{a} - \mathbf{b}$ (i) $\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB} = 3\mathbf{a} - \mathbf{b} - 2\mathbf{a} = \mathbf{a} - \mathbf{b}$	$2\mathbf{a} + \mathbf{b}$ $3\mathbf{a} - \mathbf{b}$ $\mathbf{a} - \mathbf{b}$	B1 B1 M1A1 (4)	7D	
(b)	$\overrightarrow{AX} = \frac{3}{5}\overrightarrow{AC} = \frac{3}{5}(3\mathbf{a} - \mathbf{b}) = \frac{9}{5}\mathbf{a} - \frac{3}{5}\mathbf{b}$ $\overrightarrow{BY} = \frac{2}{5}\overrightarrow{BD} = \frac{2}{5}(2\mathbf{a} + \mathbf{b}) = \frac{4}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ $\overrightarrow{AY} = \overrightarrow{AB} + \overrightarrow{BY} = \left(\frac{4}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}\right) + (\mathbf{a} - \mathbf{b}) = \frac{9}{5}\mathbf{a} - \frac{3}{5}\mathbf{b}$ Conclusion $\overrightarrow{AX} = \overrightarrow{AY}$ hence point X is the same as point Y		M1A1 M1 M1A1 A1 (6)	7A	Conclusion required for final mark
(c)	$\frac{\Delta BCX}{\Delta CDX} = \frac{2}{3}$ $\frac{\Delta AXD}{\Delta CDX} = \frac{3}{2}$ $\Rightarrow \frac{\frac{\Delta BCX}{\Delta CDX}}{\frac{\Delta AXD}{\Delta CDX}} = \frac{\Delta BCX}{\Delta AXD} = \frac{\frac{2}{3}}{\frac{3}{2}} = \frac{4}{9}$	$\frac{4}{9}$	M1 M1 M1A1 (4)	7F	
					Total 14 marks

Question	Working	Answer	Mark	Spec Ref	Notes
10 (a)	$160=(7-p)^2+(3p+1-(-2))^2$ $\Rightarrow 0=10p^2+4p-102=2(5p+17)(p-3)$ $\Rightarrow p=3$ $m=\frac{7-3}{10--2}=\frac{1}{3}$ $y-7=\frac{1}{3}(x-10)\Rightarrow 3y-x-11=0$ *	$3y-x-11=0$ (Shown)	M1 M1M1 A1 B1 M1A1 (7)	8A	
(b)	Gradient of perpendicular $=-\frac{1}{1/3}=-3$ $y-12=-3(x-5)\Rightarrow y+3x-27=0$	$y+3x-27=0$	B1 M1A1 (3)	8C 8D 8E	
(c)	$y+3x-27=0$ $3y-x-11=0$ $\Rightarrow x=7, y=6$ $AC=\sqrt{(7--2)^2+(6-3)^2}=3\sqrt{10}$ $BC=\sqrt{(7-6)^2+(10-7)^2}=\sqrt{10}$ $\Rightarrow AC:BC=3:1$ *	*	M1A1 M1 A1 (4)	8B	Solves simultaneous equations by any method
(d)	Length $CD=\sqrt{(0-7)^2+(27-6)^2}=7\sqrt{10}$ Area $=\frac{4\sqrt{10}\times 7\sqrt{10}}{2}=140$ (units^2)	140	M1 M1A1 (3)		

	ALT $\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & -2 & 10 & 0 \\ 27 & 3 & 7 & 27 \end{vmatrix} = 140$	140	{M1M1A1} (3)		
					Total 17 marks

Question	Working	Answer	Spec Ref	AO	Notes
11 (a)	$\cos 2A = \cos A\cos A - \sin A\sin A = \cos^2 A - \sin^2 A$ $\Rightarrow \cos 2A = \cos^2 A - \left(1-\cos^2 A\right) = 2\cos^2 A - 1$* $\sin 2A = 2\sin A\cos A$		M1A1 B1 (3)	10E 10F 10G	 cso
(b)	$\dfrac{\sin(2A)}{1+\cos(2A)} = \dfrac{2\sin A\cos A}{\cos^2 A+\sin^2 A+\cos^2 A-\sin^2 A} = \dfrac{2\sin A\cos A}{2\cos^2 A}$ $\dfrac{2\sin A\cos A}{2\cos^2 A} = \dfrac{\sin A}{\cos A} = \tan A$ *		M1 A1 (2)		 cso
(c)	$\tan\left(\theta+\dfrac{\pi}{3}\right)\left(1+\cos\left(2\theta+\dfrac{2\pi}{3}\right)\right) = 0.7 \Rightarrow \sin\left(2\theta+\dfrac{2\pi}{3}\right) = 0.7$ $2\theta+\dfrac{2\pi}{3} = \sin^{-1}(0.7) \Rightarrow 2\theta+\dfrac{2\pi}{3} = 0.77539,\ \ 2.36619,\ \ 7.05857,\ 8.64938$ $\theta = -0.659,\ \ 0.13589,\ \ 2.48208,\ \ (3.27749) \approx 0.136,\ \ 2.482,\ \ 3.277$	 0.136 2.482	M1 M1 M1A1 (4)	10H	Ignore angles given out of range. Deduct one mark for extra angles given within range up to a maximum of 2 marks.
(d) (i)	$\dfrac{2\sin\theta\cos^2\theta}{1+\cos 2\theta} = \dfrac{2\sin\theta\cos\theta\cos\theta}{1+\cos 2\theta} = \dfrac{\sin 2\theta\cos\theta}{1+\cos 2\theta} = \tan\theta\cos\theta = \sin\theta$ $\int \sin\theta\ \mathrm{d}\theta = -\cos\theta\ \ (+c)$	 $-\cos\theta\ \ (+c)$	M1M1 A1	9A	
(ii)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \mathrm{f}(\theta)\ \mathrm{d}\theta = \left[-\cos\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\cos\dfrac{\pi}{3}+\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}-1}{2}$	$\dfrac{\sqrt{3}-1}{2}$	B1 (4)		
					Total 13 marks