Mark Scheme

Extra Assessment Material

for first teaching September 2017

International GCSE in Further Pure Mathematics

(4PM1/01) Paper 1

Further Pure International GCSE 4PM1/01					
Question	**Working**	**Answer**	**Mark**	**Spec Ref**	**Notes**
1	$b^2 - 4ac > 0 \Rightarrow (2p+3)^2 - 4\times 2\times(-p) > 0$		M1	2B	
	$\Rightarrow 4p^2 + 20p + 9 > 0 \Rightarrow (2p+1)(2p+9) > 0$ critical values $p = -\frac{1}{2},\ -\frac{9}{2}$		M1 A1	2A	Substitutes into discriminant > 0 and forms 3TQ
		$x < -\frac{9}{2}$, $x > -\frac{1}{2}$	M1A1 (5)		Accept $x < -\frac{9}{2}$ OR $x > -\frac{9}{2}$ oe
					Total 5 marks

Question	**Working**	**Answer**	**Mark**	**Spec Ref**	**Notes**
2 (a)	$a = 3t^2 - 9t + 6$	$3t^2 - 9t + 6$	M1A1 (2)	9C	
(b)	$3t^2 - 9t + 6 = 0 \Rightarrow (3t-6)(t-1) = 0 \Rightarrow t = 1, 2$	$t = 1, 2$	M1A1 (2)	2A	
(c)	$s = \int_{(1)}^{(2)} t^3 - \frac{9}{2}t^2 + 6t + 3\ \mathrm{d}t = \left[\frac{t^4}{4} - \frac{9t^3}{6} + \frac{6t^2}{2} + 3t\right]_{(1)}^{(2)}$		M1A1	9C	Ignore any limits shown
	$s = \left(\frac{2^4}{4} - \frac{3\times 2^3}{2} + 3\times 2^2 + 3\times 2\right) - \left(\frac{1^4}{4} - \frac{3\times 1^3}{2} + 3\times 1^2 + 3\times 1\right)$ $s = 5.25$	$s = 5.25$ (oe)	M1 A1 (4)		
					Total 8 marks

Question	**Working**	**Answer**	**Mark**	**Spec Ref**	**Notes**
3	$l = r \times \theta \qquad A = \frac{1}{2} \times \theta \times r^2 \qquad \frac{\mathrm{d}l}{\mathrm{d}t} = 0.03$			10A	
	$r = \frac{l}{\theta} \qquad A = \frac{1}{2} \times \theta \times \left(\frac{l}{\theta}\right)^2 = \frac{l^2}{2\theta}$		B1	9A	Accept both formulae with either θ or $\frac{\pi}{3}$
	$\frac{\mathrm{d}A}{\mathrm{d}l} = \frac{l}{\theta}$		M1		
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}l}{\mathrm{d}t} \times \frac{\mathrm{d}A}{\mathrm{d}l} = 0.03 \times \frac{15}{\pi/3} = 0.42971\ldots \approx 0.43 \ (\mathrm{cm}^2/\mathrm{s})$	0.43 $(\mathrm{cm}^2/\mathrm{s})$	M1A1ftA1 (5)	9G	
ALT					
	$l = r \times \theta \qquad A = \frac{1}{2} \times \theta \times r^2 \qquad \frac{\mathrm{d}l}{\mathrm{d}t} = 0.03$			10A,9A	
	$\frac{\mathrm{d}l}{\mathrm{d}r} = \frac{\pi}{3} \quad \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}l}{\mathrm{d}t} \times \frac{\mathrm{d}r}{\mathrm{d}l} \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{3}{\pi} \times 0.03 = \frac{0.09}{\pi}$		B1	9G	Both formulae with either θ or $\frac{\pi}{3}$
	$A = \frac{1}{2} r^2 \frac{\pi}{3} = \frac{\pi}{6} r^2 \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}r} = \frac{\pi r}{3} \quad \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\pi r}{3} \times \frac{0.09}{\pi} = 0.03r$		M1M1		
	When $l = 15$, $r = \frac{45}{\pi}$ hence $\frac{\mathrm{d}A}{\mathrm{d}t} = 0.03 \times \frac{45}{\pi} = 0.4297\ldots$	0.43 $(\mathrm{cm}^2/\mathrm{s})$	A1ftA1		
					Total 5 marks

Question	Working	Answer	Mark	Spec Ref	Notes
4	$y=\mathrm{e}^{2x}\sin(3x)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{2x}\sin(3x)+3\mathrm{e}^{2x}\cos(3x)$		M1A1A1	9A 9B	
	$\Rightarrow\left[\frac{\mathrm{d}y}{\mathrm{d}x}=2y+3\mathrm{e}^{2x}\cos(3x)\Rightarrow 3\mathrm{e}^{2x}\cos(3x)=\frac{\mathrm{d}y}{\mathrm{d}x}-2y\Rightarrow \mathrm{e}^{2x}\cos(3x)=\frac{1}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}-2y\right)\right]$		M1		For finding an expression for $\mathrm{e}^{2x}\cos(3x)$ or method to eliminate $\mathrm{e}^{2x}\cos(3x)$ term.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\left[2\mathrm{e}^{2x}\sin(3x)+3\mathrm{e}^{2x}\cos(3x)\right]+3\left[2\mathrm{e}^{2x}\cos(3x)-3\mathrm{e}^{2x}\sin(3x)\right]$		M1		Second derivative
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}x}+3\left[2\mathrm{e}^{2x}\cos(3x)-3\mathrm{e}^{2x}\sin(3x)\right]=2\frac{\mathrm{d}y}{\mathrm{d}x}+6\mathrm{e}^{2x}\cos(3x)-9\mathrm{e}^{2x}\sin(3x)$		M1		Collecting terms only
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}x}+6\times\frac{1}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}-2y\right)-9y\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}x}+2\frac{\mathrm{d}y}{\mathrm{d}x}-4y-9y$		M1		Substitution of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\mathrm{e}^{2x}\cos(3x)=\frac{1}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}-2y\right)$
	$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-4\frac{\mathrm{d}y}{\mathrm{d}x}+13y=0$ cso		A1 (8)		
	ALT				
	$y=\mathrm{e}^{2x}\sin(3x)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{e}^{2x}\sin(3x)+3\mathrm{e}^{2x}\cos(3x)$ $\left\{\mathrm{e}^{2x}\cos(3x)=\frac{1}{3}\left(\frac{\mathrm{d}y}{\mathrm{d}x}-2y\right)\right\}$		M1A1A1 M1		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2y+3\mathrm{e}^{2x}\cos(3x)\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\frac{\mathrm{d}y}{\mathrm{d}x}+3\left[2\mathrm{e}^{2x}\cos(3x)-3\mathrm{e}^{2x}\sin(3x)\right]$		M1M1		

	$\Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 6\mathrm{e}^{2x}\cos(3x) - 9\mathrm{e}^{2x}\sin(3x)$ $\Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2\frac{\mathrm{d}y}{\mathrm{d}x} - 4y - 9y \Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \; - 4\frac{\mathrm{d}y}{\mathrm{d}x} + 13y = 0$ * cso		M1A1 (8)		
					Total 8 marks

Question	Working	Answer	Mark	Spec Ref	Notes
5 (a)		$r = -3$	B1 (1)	4A	Because $x + 3 = 0 \Rightarrow x = -3$
(b)	When $x = 0$, $\frac{5}{3} = \frac{q}{r} \Rightarrow q = \frac{5 \times -3}{3} = -5$ When $y = 0$, $\frac{5}{2}p - 5 = 0 \Rightarrow p = 2$	$p = 2, q = -5$	M1A1A1 (3)	4A	
(c)	Parallel to x-axis, $y = 2$	$y = 2$	B1 (1)	4A	
					Total 5 marks

Question	Working	Answer	Mark	Spec Ref	Notes
6 (a)	$n=30,\quad a=3$ $S_{30}=\frac{30}{2}\left(2\times3+\left(30-1\right)\times3\right)=1395$ **ALT** Last term $l=30\times3=90$ $S_{30}=\frac{1}{2}\times 30\left[3+90\right]=15\times 95=1395$	1395	M1A1 (2) [M1A1 (2)]	5A,5B	
(b)	$\frac{n}{2}\left(2\times3+\left(n-1\right)\times3\right)>4000\Rightarrow 3n^2+3n-8000>0$		M1A1	5B	
	$n=\frac{-3\pm\sqrt{3^2-4\times3\times\left(-8000\right)}}{2\times3}=51.142\ldots$ (discard negative root)	$n=52$	M1A1 (4)	2A	
					Total 6 marks

Question	Working	Answer	Mark	Spec Ref	Notes
7 (a)	$(2-x)^3 = 8+(3)(2^2)(-x)+(3)(2)(-x)^2 - x^3 = 8-12x+6x^2-x^3$	$8-12x+6x^2-x^3$	M1A1 (2)	6A	
(b)	(i) $\sqrt{(1+2x)} = (1+2x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(2x)+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(2x)^2}{2!}+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(2x)^3}{3!}$		M1A1	6A	
	$\sqrt{(1+2x)} = 1+x-\dfrac{x^2}{2}+\dfrac{x^3}{2}$	$1+x-\dfrac{x^2}{2}+\dfrac{x^3}{2}$	A1		
	(ii)	$\|x\| \leqslant \dfrac{1}{2}$	B1 (4)		Accept $\|x\| < \dfrac{1}{2}$
(c)	$\left(1+x-\dfrac{x^2}{2}+\dfrac{x^3}{2}\right)(8-12x+6x^2-x^3) = 8-4x-10x^2$	$A=-4,\ B=-10$	M1A1 (2)	6A	
(d)	$\int_{0.1}^{0.2}(8-4x-10x^2)\,\mathrm{d}x = \left[8x-2x^2-\dfrac{10}{3}x^3\right]_{0.1}^{0.2} = 0.71666... \approx 0.717$	0.717	M1M1A1 (3)	9C	
					Total 11 marks

Question	Working	Answer	Mark	Spec Ref	Notes
8	$8^{(2y+1)} = 4^{(2x-2)} \Rightarrow 2^{(6y+3)} = 2^{2(2x-2)} \Rightarrow 6y+3 = 4x-4$		M1A1	1B	
	$\log_2 y = \log_4 x + 1 \Rightarrow \log_2 y = \dfrac{\log_2 x}{\log_2 4} + 1 \Rightarrow \log_2 y = \dfrac{\log_2 x}{2} + 1$		M1	1B	M1 – for change of base
	$\Rightarrow 2\log_2 y = \log_2 x + 2 \Rightarrow \log_2 y^2 = \log_2 x + 2$		M1		M1 – for $2\log_2 y = \log_2 y^2$
	$\Rightarrow \log_2 y^2 = \log_2 x + \log_2 4 \Rightarrow \log_2 y^2 = \log_2 (4x) \Rightarrow y^2 = 4x$		B1M1A1		B1 – for $16 = \log_2 4$ M1 – for combining the logs A1 – for the final correct equation
	Solves the simultaneous equations $y^2 = 4x$ and $6y+3 = 4x-4$				
	$6y+3 = y^2 - 4 \Rightarrow y^2 - 6y - 7 = 0$		M1	3C	
	$y^2 - 6y - 7 = (y-7)(y+1) = 0 \Rightarrow y = 7, \quad -1$		M1 A1		
	(discard negative root) $x = \dfrac{y^2}{4} \Rightarrow x = \dfrac{(7)^2}{4} = \dfrac{49}{4}$	$y = 7$ $x = \dfrac{49}{4}$ oe	A1 (11)	2A	
					Total 11 marks

Question	Working	Answer	Mark	Spec Ref	Notes
9 (a)	$S_\infty = \frac{U_1}{1-r} \Rightarrow \frac{32}{3} = \frac{U_1}{1-r}$		B1	5B	
	$U_1 + U_1 r = \frac{14}{3}$		B1		
	$U_1 = \frac{32}{3}(1-r), \quad U_1(1+r) = \frac{14}{3} \Rightarrow \frac{32}{3}(1-r)(1+r) = \frac{14}{3}$		M1		
	$(1-r)(1+r) = \frac{14}{32} \Rightarrow 1-r^2 = \frac{14}{32} \Rightarrow r^2 = \frac{9}{16} \Rightarrow r = \frac{3}{4}$	$r = \frac{3}{4}$	M1A1		
	$\frac{32}{3} = \frac{U_1}{1-\frac{3}{4}} \Rightarrow U_1 = \frac{8}{3}$	$U_1 = \frac{8}{3}$ * cso	A1 (6)		
(b)	$S_{15} = \frac{\frac{8}{3}\left(1-\left(\frac{3}{4}\right)^{15}\right)}{1-\frac{3}{4}} = 10.5241208... \approx 10.5241$	10.5241	M1A1 (2)	5B	
(c)	$U_n < 0.04$ $\frac{8}{3} \times \left(\frac{3}{4}\right)^{n-1} < 0.04 \Rightarrow \left(\frac{3}{4}\right)^{n-1} < 0.015 \Rightarrow (n-1)\lg\left(\frac{3}{4}\right) < \lg(0.015)$		M1M1	5B	
	$n-1 > \frac{\lg 0.015}{\lg 0.75} \Rightarrow n > 15.598... \Rightarrow n = 16$	16	A1A1 (4)	1B	A1 for 15.598... seen A1 for n = 16
					Total 12 marks

Question	Working	Answer	Mark	Spec Ref	Notes
10 (a)	$GN = 4\tan 60^\circ = 4\sqrt{3}$ *		M1A1 (2)	10B 1C	
(b)	Vol = $\dfrac{4\sqrt{3}}{2}(8+12)\times 9 = 360\sqrt{3} = 623.538... \approx 624\ \left(\text{cm}^3\right)$	624 cm^3	M1A1 (2)	10C	
(c)	$FH = \sqrt{9^2+8^2} = \sqrt{145}$ $AH = \sqrt{145+\left(4\sqrt{3}\right)^2} = \sqrt{193} = 13.892... \approx 13.9$	13.9 cm	B1 M1A1 (3)	10C	
(d)	Required angle is $\angle GDN$ $\angle GDN = \sin^{-1}\left(\dfrac{4\sqrt{3}}{\sqrt{193}}\right) = 29.9142... \approx 29.9^\circ$	29.9º	M1 M1A1 (3)	10C	
(e)	Angle required is $\angle GBF$ $BG = \sqrt{\left(4\sqrt{3}\right)^2+4^2} = \sqrt{64} = 8$ $FB = \sqrt{\left(4\sqrt{3}\right)^2+12^2} = 8\sqrt{3}$		M1A1	10C	
	$\angle GBF = \cos^{-1}\left(\dfrac{8^2+\left(8\sqrt{3}\right)^2-8^2}{2\times 8\sqrt{3}\times 8}\right) = 30^\circ$	30º	M1A1 (4)		
	ALT ΔBGF **is isosceles** $\cos\angle FBG = \dfrac{\frac{1}{2}\sqrt{192}}{8} = \dfrac{\sqrt{3}}{2} \Rightarrow \angle FBG = 30^\circ$		M2A2 (4)		
					Total 14 marks

Question	Working	Answer	Mark	Spec Ref	Notes
11 (a)	$\frac{dy}{dx} = 3x^2 - 6x - \frac{9}{4}$		M1	9A 9B	
	$3x^2 - 6x - \frac{9}{4} = 0 \Rightarrow 4x^2 - 8x - 3 = 0$		A1		
	$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4\times4\times(-3)}}{2\times4} = \frac{2\pm\sqrt{7}}{2}$		M1		
	Hence stationary points $\Rightarrow x = \frac{2\pm\sqrt{7}}{2}$	$x = \frac{2\pm\sqrt{7}}{2}$	A1 (4)		
(b)	$f(x) = (x+a)(x-a)(x-2a) = x^3 - 2ax^2 - a^2x + 2a^3$		M1A1	3B	
	$A = \int_{-a}^{a} x^3 - 2ax^2 - a^2x + 2a^3 \ dx = \left[\frac{x^4}{4} - \frac{2ax^3}{3} - \frac{a^2x^2}{2} + 2a^3x\right]_{-a}^{a}$		M1M1A1	9A 9C	
	$A = \left(\frac{a^4}{4} - \frac{2a^4}{3} - \frac{a^4}{2} + 2a^4\right) - \left(\frac{a^4}{4} + \frac{2a^4}{3} - \frac{a^4}{2} - 2a^4\right) = \frac{8a^4}{3}$		M1A1		
	$\frac{27}{2} = \frac{8a^4}{3} \Rightarrow a = \pm\frac{3}{2}$	$a = \frac{3}{2}$	M1A1 (9)		
(c)	$f(x) = x^3 - 2ax^2 - a^2x + 2a^2 = x^3 - 3x^2 - \frac{9}{4}x + \frac{27}{4}$	$f(x) = x^3 - 3x^2 - \frac{9}{4}x + \frac{27}{4}$ + conclusion C and S are the same	M1A1 (2)	3B	
					Total 15 marks