Further Pure Mathematics
PAPER 2R
(Calculator)

Time: 2 hours

YOU MUST HAVE

Nil

YOU WILL BE GIVEN

Diagram Booklet
Formulae Pages
Answer Booklet

Q71642A

Pearson

Calculators may be used.

INSTRUCTIONS

Answer ALL questions.

Without sufficient working, correct answers may be awarded no marks.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on this Question Paper.

You must NOT write anything on the Formulae Pages. Anything you write on the Formulae Pages will gain NO credit.

INFORMATION

The total mark for this paper is **100**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams in case you need them.

You may be provided with models for Question 6 and Question 11.

ADVICE

Read each question carefully before you start to answer it.

Check your answers if you have time at the end.

Answer all ELEVEN questions.

Write your answers in the Answer Booklet.

You must write down all the stages in your working.

1. The position vector of the point **A** is $(3\underline{i} - 2\underline{j})$, referred to a fixed origin **O**

   The point **B** is such that $\overrightarrow{AB} = (6\underline{i} + 8\underline{j})$

   (a) Find the position vector of **B** as a simplified expression in terms of $\underline{i}$ and $\underline{j}$

   (2 marks)

   (b) Find the magnitude of vector $\overrightarrow{AB}$

   (1 mark)

   (c) Find a unit vector, in terms of $\underline{i}$ and $\underline{j}$, that is parallel to $\overrightarrow{AB}$

   (2 marks)

(Total for Question 1 is 5 marks)

2. When poured from a pipe, concrete is formed into the shape of a cuboid with a square base of side x and with a height of $3x$

   The volume of the cuboid increases at a constant rate of $8\,\text{m}^3/\text{s}$

   Find the rate of increase, in m/s, of x when $x = 2$ metres.

   (Total for Question 2 is 6 marks)

3. A geometric series has first term a and common ratio r, where $r > 0$

Given that the 3rd term of the series is 5 and that the 5th term of the series is $\dfrac{5}{2}$

(a) find

(i) the exact value of r

(ii) the value of a

(4 marks)

(b) Find the sum to infinity of this series.

Give your answer in the form $p + q\sqrt{2}$ where p and q are integers.

(2 marks)

(Total for Question 3 is 6 marks)

Turn over

4. $f(x) = x^3 + px^2 + qx + 7$

where p and q are integers.

$(x + 1)$ is a factor of $f(x)$

The remainder when $f(x)$ is divided by $(x + 2)$ is -5

(a) Find the value of p and the value of q

(5 marks)

(b) Hence, show that $f(x) = 0$ has only one real root.

(3 marks)

(Total for Question 4 is 8 marks)

Turn over

5. Look at the table for Question 5(a) in the Diagram Booklet.

(a) Complete the table of values for $y = e^{3x-2}$ giving your answers to 2 decimal places.

There are three spaces to fill.

(2 marks)

Look at the diagram for Question 5(b) in the Diagram Booklet.

It shows a grid.

(b) On the grid in the Diagram Booklet, draw the graph of $y = e^{3x-2}$ for $0 \leq x \leq 1$

(2 marks)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $3x = 2 + \ln(3 - x)$

(3 marks)

(Total for Question 5 is 7 marks)

6. Look at Diagram 1, Diagram 2 and Diagram 3 for Question 6 in the Diagram Booklet.

   You may be provided with a model.

   They are NOT accurate.

   Diagram 1 and the model show a right pyramid **VABCD** with vertex **V** and square base **ABCD**

   Diagram 2 shows the side **CVD** and Diagram 3 shows the base **ABCD**

   Each of the edges of the pyramid has the same length.

   Find the size, in degrees to one decimal place, of the angle between the plane **CVD** and the base **ABCD**

(Total for Question 6 is 6 marks)

7. (a) **Solve the equation**

$$\cos(3x - 15)^\circ = \frac{\sqrt{3}}{2}$$

for $0 \leq x < 180$

(4 marks)

(b) **Solve, giving your solutions to one decimal place where appropriate,**

$$3\tan y^\circ + 4\sin y^\circ = 0$$

for $-180 \leq y < 180$

(4 marks)

(continued on the next page)

Turn over

7.   **continued.**

(c)   **Solve, giving your solutions to one decimal place where appropriate,**

$$\cos \theta° = 3 \sin^2 \theta° - 1$$

for $-180 \leq \theta < 180$

(4 marks)

(Total for Question 7 is 12 marks)

8. **Look at the diagram for Question 8 in the Diagram Booklet.**

   The diagram shows part of the curve C with equation $y = e^{3x} - 1$ and part of the curve D with equation $y = 9 - 9e^{-3x}$

   The curves intersect at the origin O and the point A

   (a) (i) **Show that the x coordinate of the point A satisfies the equation**

   $$(e^{3x})^2 - 10e^{3x} + 9 = 0$$

   (ii) **Hence, show that the x coordinate of the point A is $\frac{1}{3}\ln 9$**

   (5 marks)

(continued on the next page)

8.   continued.

The finite region bounded by **C** and by **D** is shown shaded in the diagram.

(b)   Use calculus to find the exact area of this region.

(6 marks)

(Total for Question 8 is 11 marks)

9. (a) Write $\dfrac{3}{(3-x)^3}$ in the form $a(1-bx)^{-3}$

where a and b are fractions in their lowest terms.

(2 marks)

(b) Expand $\dfrac{3}{(3-x)^3}$ in ascending powers of x up to and including the term in x^3

Express each coefficient as a fraction in its lowest terms.

(3 marks)

(c) (i) Use a suitable value of x with your expansion in part (b), to obtain an approximation for $\dfrac{24}{125}$ to 5 decimal places.

(ii) Find the percentage error, to 2 decimal places, of your approximation from the actual value.

(4 marks)

(Total for Question 9 is 9 marks)

10. A curve **C** has equation

$$y = \frac{7x - 2}{2x - 3} \qquad x \neq \frac{3}{2}$$

    (a) Write down an equation of the asymptote to **C** that is

        (i) parallel to the **y**–axis,

        (ii) parallel to the **x**–axis.

(2 marks)

    (b) Find the coordinates of the points of intersection of **C** with the coordinate axes.

(2 marks)

    (c) Using calculus, show that at every point on the curve, the gradient of **C** is negative.

(4 marks)

(continued on the next page)

10. continued.

(d) Look at the diagram for Question 10(d) in the Diagram Booklet.

It shows a pair of axes.

Using the axes in the Diagram Booklet, sketch **C**

Show clearly and label with their equation any asymptotes and the coordinates of the points of intersection of **C** with the coordinate axes.

(3 marks)

The straight line **L** is the normal to **C** at the point **A**

The **x** coordinate of **A** is positive and the gradient of **L** is **17**

The line **L** also intersects **C** at the point **B**

(e) Find the exact coordinates of **B**

(7 marks)

(Total for Question 10 is 18 marks)

11. Look at Diagram 1, Diagram 2 and Diagram 3 for Question 11 in the Diagram Booklet.

You may be provided with a model.

They are NOT accurate.

Diagram 1 and the model show a solid metal right circular cylinder of radius r cm and height h cm

Diagram 2 shows the side view and Diagram 3 shows the top view of the cylinder.

The total surface area of the cylinder is 600 cm^2

The volume of the cylinder is V cm^3

(a) Show that

$$V = 300r - \pi r^3$$

(4 marks)

11. continued.

Given that **r** can vary,

(b) (i) use calculus to show that the exact value of **r** for which **V** is a maximum is

$$r = \sqrt{\dfrac{100}{\pi}}$$

(ii) justify that this value of **r** gives a maximum value of **V**

(5 marks)

The cylinder is melted down and reformed into a sphere of radius **p cm**

(c) Find, to one decimal place, the greatest possible value of **p**

(3 marks)

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER