# Further Pure Mathematics
## PAPER 1R
## (Calculator)

**Time: 2 hours**

## YOU MUST HAVE

Nil

## YOU WILL BE GIVEN

**Diagram Booklet**
**Formulae Pages**
**Answer Booklet**

Pearson

# Calculators may be used.

## INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions.

Without sufficient working, correct answers may be awarded no marks.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You must NOT write anything on the Formulae Pages. Anything you write on the Formulae Pages will gain NO credit.

## INFORMATION

The total mark for this paper is **100**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

**Turn over**

**ADVICE**

**Read each question carefully before you start to answer it.**

**Check your answers if you have time at the end.**

**Answer all ELEVEN questions.**

**Write your answers in the Answer Booklet.**

**You must write down all the stages in your working.**

1.   An arithmetic series has **5**th term **16** and **100**th term **301**

Find the sum of the first **50** terms of the series.

**(Total for Question 1 is 5 marks)**

2. A particle **P** is moving along a straight line, which passes through the fixed point **O**

   At time **t** seconds $(t \geq 0)$, the velocity, **v m/s**, of **P** is given by

   $$v = t^2 - 3t + 4$$

   At time **t** seconds the acceleration of **P** is **a m/s$^2$**

   (a) Find an expression for **a** in terms of **t**

   **(2 marks)**

   The displacement of **P** from **O** is **7** metres when $t = 2$

   (b) Find the exact displacement of **P** from **O** when $t = 4$

   **(5 marks)**

**(Total for Question 2 is 7 marks)**

3.  **Look at the diagram for Question 3 in the Diagram Booklet.**

    **It is NOT accurately drawn.**

    **The diagram shows sector ORS of a circle with centre O and radius 4 cm**

    **The size of angle ROS is $\theta$ radians.**

    **The area of sector ORS is $2\pi$ cm$^2$**

    (a)  **Find the exact value of $\theta$**

        **(2 marks)**

    (b)  **Find the perimeter, in cm to 3 significant figures, of the sector ORS**

        **(2 marks)**

**(continued on the next page)**

3.　continued.

The point **T** lies on **OR** such that **OT : TR = 1 : 3**

The region shown shaded in the diagram is bounded by the line **TR**, the line **TS** and the arc **RS** of the sector.

The area of this region is $A \, \text{cm}^2$

(c)　Find the exact value of **A**

(2 marks)

**(Total for Question 3 is 6 marks)**

4.   **Look at the diagram for Question 4 in the**

     **Diagram Booklet.**

     **It is NOT accurately drawn.**

     **The diagram shows a rectangle with width  x  metres**

     **and length (x + 4) metres.**

     **The perimeter of the rectangle is P metres and the**

     **area of the rectangle is A m$^2$**

     (a)  **Find, in terms of  x, an expression for**

          (i)   **P**

          (ii)  **A**

     **(2 marks)**

**(continued on the next page)**

4.　continued.

The perimeter of the rectangle has to be less than **30** metres.

The area of the rectangle has to be greater than **12 m$^2$**

(b)　Find the set of possible values for **x**

Give your answer in the form **a < x < b**

(5 marks)

**(Total for Question 4 is 7 marks)**

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**5.** **Differentiate with respect to x**

(a) $e^{4x}(6x + 2)^{\frac{3}{2}}$

**Give your answer in the form**

$e^{4x}\left(\sqrt{6x + 2}\right)(Ax + B)$ where **A** and **B** are

**integers.**

**(5 marks)**

(b) $\dfrac{\sin 3x}{(2x - 4)^3}$

**(3 marks)**

**(Total for Question 5 is 8 marks)**

6.  **Given that**

$$\frac{a + \sqrt{5}}{\sqrt{5} - 2} = 11 + 5\sqrt{5}$$

(a)  without using a calculator, find the value of **a**

Show your working clearly.

**(2 marks)**

**Triangle PQR is such that**

**PR = (x + 3)cm**

**QR = x cm**

angle **QPR = 30°**

angle **PQR = 45°**

(b)  Show that $x = 3 + 3\sqrt{2}$

**(3 marks)**

**(continued on the next page)**

**6.** **continued.**

**Given that**

$$\sin 105° = \frac{\sqrt{6} + \sqrt{2}}{4}$$ **and that the area of**

**triangle PQR is A cm$^2$**

(c) **find the exact value of A in the form**

$$\frac{9}{8}(p\sqrt{6} + q\sqrt{2} + r\sqrt{3} + s)$$

**where p, q, r and s are integers.**

**(3 marks)**

**(Total for Question 6 is 8 marks)**

7.   A curve $C$ has equation $y = \log_{10}(x + 2)$

(a)  Using the axes in the Answer Booklet on pages **34–39**, sketch the graph of $C$

Label the coordinates of the points of intersection of $C$ with the coordinate axes.

**(2 marks)**

(b)  Solve the equation $2(\log_a 4 + \log_a 16) = 1$

**(3 marks)**

(c)  Solve the equation $5\log_q 16 + 4\log_2 q = 24$

**(6 marks)**

**(Total for Question 7 is 11 marks)**

**Turn over**

8. (a) Using the binomial expansion, or otherwise, find the complete expansion of

$$(x + y)^3$$

(1 mark)

The quadratic equation

$$2x^2 + 3x + 4 = 0$$

has roots $\alpha$ and $\beta$

(b) Without solving the equation, find the value of

$$\alpha^3 + \beta^3$$

(4 marks)

(continued on the next page)

**Turn over**

8.   continued.

(c)   Hence, form a quadratic equation with integer coefficients that has roots

$$\frac{\alpha}{\beta^2} \text{ and } \frac{\beta}{\alpha^2}$$

**(5 marks)**

**(Total for Question 8 is 10 marks)**

9. **Look at the diagram for Question 9 in the Diagram Booklet.**

   **It is NOT accurately drawn.**

   **The diagram shows quadrilateral ABCD such that**

   $$\overrightarrow{AD} = 2\underline{a} + \underline{b}$$

   $$\overrightarrow{BC} = \frac{1}{3}\underline{b}$$

   $$\overrightarrow{BD} = -4\underline{a} - \underline{b}$$

   (a) **Prove that $\overrightarrow{AB}$ is parallel to $\overrightarrow{DC}$**

   **(4 marks)**

   **The diagonals, AC and BD, of the quadrilateral intersect at the point Y**

   (b) **Using a vector method, find $\overrightarrow{AY}$ as a simplified expression in terms of $\underline{a}$ and $\underline{b}$**

   **(6 marks)**

**(Total for Question 9 is 10 marks)**

10. Using suitable results for $\sin(A + B)$ and $\sin(A - B)$ from the Formulae Pages,

(a) show that $2\sin 4x \cos x = \sin 5x + \sin 3x$

(3 marks)

Look at the diagram for Question 10(b) in the Diagram Booklet.

It is NOT accurately drawn.

It shows part of a sketch of the curve

$y = 6\sin 4x \cos x$

(b) Using calculus, find the total area bounded by the curve and the $x$-axis between

$x = 0$ and

$x = \dfrac{\pi}{2}$

Give your answer to $3$ significant figures.

(8 marks)

(Total for Question 10 is 11 marks)

11.  An equation of the straight line $L$ is $y - 3x = 3$

The point $A$ on $L$ lies on the $y$-axis.

The point $B$ on $L$ has coordinates $(10, b)$, where $b$ is an integer.

The point $C$ divides $AB$ in the ratio $2 : 3$

The straight line $k$ passes through $C$ and is perpendicular to $L$

(a)  Show that an equation of $k$ is

$3y + x - 49 = 0$

(6 marks)

(continued on the next page)

**11.** continued.

The point **D** with coordinates $(p, q)$, where $q$ is positive, is such that **AD** is parallel to $k$ and the length of **AD** is $12\sqrt{10}$

**(b)** Find the coordinates of **D**

(6 marks)

The point **E** lies on $k$ such that **DE** is parallel to the $y$−axis.

The point **F** lies on **L** such that **DF** is parallel to the $y$−axis.

**(c)** Find the exact area of triangle **ECF**

(5 marks)

**(Total for Question 11 is 17 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

**END OF PAPER**