Pearson Edexcel

Mark Scheme (Results)

January 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=(x+p)(x+q), \text{ where } |pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=(mx+p)(nx+q) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0, \quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Marks
1(a)	$\left(1+\frac{x}{4}\right)^8 = 1+2x+\frac{7}{4}x^2+\frac{7}{8}x^3$ oe	B1M1A1 [3]
(b)	$1+\frac{x}{4}=1.035 \Rightarrow x=0.14$ oe $(1.035)^8 = 1+2(0.14)+1.75(0.14)^2+0.875(0.14)^3 = 1.3167$ [NB: the calculator value is 1.316809037]	B1 M1A1 [3]
		Total 6 marks

Question	Notes	Marks
1	$\left(1+\frac{x}{4}\right)^8$	
(a)	For an attempt at the binomial expansion of $\left(1+\frac{x}{4}\right)^8$ $\left(1+\frac{x}{4}\right)^8 = 1+2x+...$	B1
	For either correct third or fourth term. Need not be simplified. $...+C_2^8 \times \left(\frac{x}{4}\right)^2$ or $C_3^8 \times \left(\frac{x}{4}\right)^3$	M1
	For the correct expansion fully simplified. $\left(1+\frac{x}{4}\right)^8 = 1+2x+\frac{7}{4}x^2+\frac{7}{8}x^3$ OR $\left(1+\frac{x}{4}\right)^8 = 1+2x+1.75x^2+0.875x^3$ Do not ignore subsequent incorrect simplification.	A1 [3]
(b)	Finds the value of x to be substituted. $1+\frac{x}{4}=1.035 \Rightarrow x=0.14$ oe	B1
	Substitutes their value of x into their expansion provided it has at least 2 terms in x. *Their* $x \neq 1.035$ $(1.035)^8 = 1+2(0.14)+1.75(0.14)^2+0.875(0.14)^3$	M1
	For the correct value of $(1.035)^8 = 1.3167$ rounded correctly. [NB: the calculator value is 1.316809037]	A1 [3]
		Total 6 marks

Question	Scheme	Marks
2(a)	$3x-8<5x+3 \Rightarrow x>-\frac{11}{2}$	B1 [1]
(b)	$4x^2-7x+1>6-2x^2 \Rightarrow$ $6x^2-7x-5>0 \Rightarrow (3x-5)(2x+1)>0$ cv's are $x=\frac{5}{3},\ -\frac{1}{2} \Rightarrow x<-\frac{1}{2},\ x>\frac{5}{3}$	M1M1 M1A1 [4]
(c)	$-\frac{11}{2}<x<-\frac{1}{2},\ x>\frac{5}{3}$	B1 ft [1]
	Total	**6 marks**

Question	Notes	Marks
2(a)	$3x-8<5x+3$ $\Rightarrow x>-\frac{11}{2}$	B1 [1]
(b)	For rearranging to form a 3TQ $4x^2-7x+1>6-2x^2 \Rightarrow 6x^2-7x-5>0$	M1
	For method to solve their 3TQ. See general guidance for what constitutes an attempt to solve. $6x^2-7x-5>0 \Rightarrow (3x-5)(2x+1)>0$	M1
	For writing down their critical values from their factorisation and the correct inequality for their cv’s [outside region]. cv's are $x=\frac{5}{3},\ -\frac{1}{2}$ $x<-\frac{1}{2},\ x>\frac{5}{3}$	M1
	For the correct inequalities only $x<-\frac{1}{2},\ x>\frac{5}{3}$ A0 if rejects one region or if inequalities are incorrectly combined.	A1 [4]
(c)	For the correct inequality only $-\frac{11}{2}<x<-\frac{1}{2},\ x>\frac{5}{3}$ For follow through (a) must be a linear inequality and (b) must be of the form $x<a, x>b$ with $b>a$.	B1 ft [1]
	Total	**6 marks**

Question	Scheme	Marks
3	$\frac{dy}{dx}=3e^{3x}\sin 2x+2e^{3x}\cos 2x$	M1A1A1
	Method A $\frac{d^2y}{dx^2}=3\left(3e^{3x}\sin 2x+2e^{3x}\cos 2x\right)+6e^{3x}\cos 2x-4e^{3x}\sin 2x$ $\frac{d^2y}{dx^2}=3\frac{dy}{dx}+6e^{3x}\cos 2x-4y$ $2e^{3x}\cos 2x=\frac{dy}{dx}-3y\Rightarrow 6e^{3x}\cos 2x=3\frac{dy}{dx}-9y$ $\frac{d^2y}{dx^2}=3\frac{dy}{dx}+3\frac{dy}{dx}-9y-4y\Rightarrow 13y+\frac{d^2y}{dx^2}=6\frac{dy}{dx}$	M1 M1 M1 M1A1 [8]
	Method B $\frac{dy}{dx}=3y+2e^{3x}\cos 2x$ $\frac{d^2y}{dx^2}=3\frac{dy}{dx}+6e^{3x}\cos 2x-4e^{3x}\sin 2x,\ \frac{d^2y}{dx^2}=3\frac{dy}{dx}+6e^{3x}\cos 2x-4y$ $2e^{3x}\cos 2x=\frac{dy}{dx}-3y\Rightarrow 6e^{3x}\cos 2x=3\frac{dy}{dx}-9y$ $\frac{d^2y}{dx^2}=3\frac{dy}{dx}+3\frac{dy}{dx}-9y-4y\Rightarrow 13y+\frac{d^2y}{dx^2}=6\frac{dy}{dx}$	[M1,M1 M1 M1A1]
	Method C $\frac{d^2y}{dx^2}=3\left(3e^{3x}\sin 2x+2e^{3x}\cos 2x\right)+6e^{3x}\cos 2x-4e^{3x}\sin 2x$ $13y+\frac{d^2y}{dx^2}=13e^{3x}\sin 2x+(5e^{3x}\sin 2x+12e^{3x}\cos 2x)$ $6\frac{dy}{dx}=6(3e^{3x}\sin 2x+2e^{3x}\cos 2x)$ $(=18e^{3x}\sin 2x+12e^{3x}\cos 2x)$ $LHS=13e^{3x}\sin 2x+(5e^{3x}\sin 2x+12e^{3x}\cos 2x)$ $=18e^{3x}\sin 2x+12e^{3x}\cos 2x$ $RHS=6(3e^{3x}\sin 2x+2e^{3x}\cos 2x)$ $=18e^{3x}\sin 2x+12e^{3x}\cos 2x$ $LHS=RHS$	[M1 M1 M1 M1A1]
		Total 8 marks

Question	Notes	Marks
3	$y=e^{3x}\sin 2x$	
	For an attempt to differentiate the given expression.	

	• There needs to be an acceptable attempt to differentiate both terms. $e^{3x}\sin 2x \to ke^{3x}\sin 2x + le^{3x}\cos 2x$ with $k, l \neq 0$ • There need to be two terms added. $\frac{dy}{dx} = 3e^{3x}\sin 2x + 2e^{3x}\cos 2x$	M1
	One term completely correct. $\frac{dy}{dx} = 3e^{3x}\sin 2x + '2e^{3x}\cos 2x'$ or $\frac{dy}{dx} = '3e^{3x}\sin 2x' + 2e^{3x}\cos 2x$	A1
	Fully correct differentiated expression. $\frac{dy}{dx} = 3e^{3x}\sin 2x + 2e^{3x}\cos 2x$	A1
	Method A	
	For an attempt to find $\frac{d^2y}{dx^2}$ Minimally acceptable attempt is $ke^{3x}\sin 2x + le^{3x}\cos 2x \to k(me^{3x}\sin 2x + ne^{3x}\cos 2x) + l(pe^{3x}\cos 2x + qe^{3x}\sin 2x)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{d^2y}{dx^2} = 3\left(3e^{3x}\sin 2x + 2e^{3x}\cos 2x\right) + 6e^{3x}\cos 2x - 4e^{3x}\sin 2x$	M1
	For substituting y and $\frac{dy}{dx}$ into their $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 6e^{3x}\cos 2x - 4y$	M1
	For preparing to eliminate cos2x by rearranging their $\frac{dy}{dx}$ $2e^{3x}\cos 2x = \frac{dy}{dx} - 3y \Rightarrow 6e^{3x}\cos 2x = 3\frac{dy}{dx} - 9y$ Allow errors in arithmetic but not mathematically incorrect process.	M1
	For an unsimplified expression only in terms of y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 3\frac{dy}{dx} - 9y - 4y$	M1
	For the required simplified expression with fully correct working. $13y + \frac{d^2y}{dx^2} = 6\frac{dy}{dx}$	A1 [8]
	Method B	
	For an attempt to find $\frac{d^2y}{dx^2}$ using $\frac{dy}{dx} = 3y + 2e^{3x}\cos 2x$	

	Minimally acceptable attempt is $\frac{\mathrm{d}y}{\mathrm{d}x} = ky + le^{3x}\cos 2x \rightarrow$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = k\frac{\mathrm{d}y}{\mathrm{d}x} + l(pe^{3x}\cos 2x + qe^{3x}\sin 2x)$ k, l as in their first derivative, $p, q \neq 0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3\frac{\mathrm{d}y}{\mathrm{d}x} + 6\mathrm{e}^{3x}\cos 2x - 4\mathrm{e}^{3x}\sin 2x$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3\frac{\mathrm{d}y}{\mathrm{d}x} + 6\mathrm{e}^{3x}\cos 2x - 4y$	M1,M1
	For preparing to eliminate cos2x by rearranging their $\frac{\mathrm{d}y}{\mathrm{d}x}$ $2\mathrm{e}^{3x}\cos 2x = \frac{\mathrm{d}y}{\mathrm{d}x} - 3y \Rightarrow 6\mathrm{e}^{3x}\cos 2x = 3\frac{\mathrm{d}y}{\mathrm{d}x} - 9y$ Allow errors in arithmetic but not mathematically incorrect process.	M1
	For an unsimplified expression only in terms of y, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3\frac{\mathrm{d}y}{\mathrm{d}x} + 3\frac{\mathrm{d}y}{\mathrm{d}x} - 9y - 4y$	M1
	For the required simplified expression with fully correct working. $13y + \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6\frac{\mathrm{d}y}{\mathrm{d}x}$	A1
	Method C	
	For an attempt to find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ Minimally acceptable attempt is $ke^{3x}\sin 2x + le^{3x}\cos 2x \rightarrow$ $k(me^{3x}\sin 2x + ne^{3x}\cos 2x) + l(pe^{3x}\cos 2x + qe^{3x}\sin 2x)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3\left(3\mathrm{e}^{3x}\sin 2x + 2\mathrm{e}^{3x}\cos 2x\right) + 6\mathrm{e}^{3x}\cos 2x - 4\mathrm{e}^{3x}\sin 2x$	M1
	For substituting y and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into the LHS of the result.	M1
	For substituting $\frac{\mathrm{d}y}{\mathrm{d}x}$ into the RHS of the result. | For simplifying the LHS of the result and writing as $k(3e^{3x}\sin 2x + 2e^{3x}\cos 2x)$	M1
	For unsimplified expressions for the LHS and RHS of the result in terms of $e^{3x}\sin 2x$ and $e^{3x}\cos 2x$ only and method to simplify both sides	M1
	For correctly simplifying the LHS and RHS and showing equal.	A1
		Total 8 marks

Question	Scheme	Marks
4(a)(i)	$p = 2$	B1
(ii)	$r = 4$	B1 [2]
(b)	$\frac{3}{2} = \frac{2\times 0+q}{0+'4'} \Rightarrow q = 6$	M1A1 [2]
(c)	$0 = \frac{2s+'6'}{s+'4'} \Rightarrow s = -3$	M1A1 [2]
		Total 6 marks

Question	Notes	Marks
4(a)(i)	$p = 2$	B1 [1]
(ii)	$r = 4$	B1 [1]
(b)	For using the equation for C, substituting $x = 0$ and $y = \frac{3}{2}$ and attempt to rearrange to find the value of q $\frac{3}{2} = \frac{2\times 0+q}{0+4} \Rightarrow q = ...$	M1
	$q = 6$	A1 [2]
(c)	For using the equation of C to find s $0 = \frac{2s+6}{s+4} \Rightarrow s = ...$	M1
	$s = -3$	A1 [2]
		Total 6 marks

Question	Scheme	Marks
5(a)	$-\frac{1}{12}=\frac{10-0}{p-123}$ $\left[\text{or } -\frac{1}{12}=\frac{0-10}{123-p}\right]$ $\Rightarrow p-123=-12\times 10 \Rightarrow p=3*$	M1 A1 cso [2]
(b)	$y-10=-\frac{1}{12}(x-3)\Rightarrow 12y+x-123=0$	M1A1 [2]
(c)	$m_k=12$ $y-10=12(x-3)\Rightarrow y=12x-26$	B1 M1A1 [3]
(d)	At C: $y=12x-26$ when $y=0$, $x=\frac{26}{12}$ oe $\text{Area}_{ABC}=\frac{1}{2}\times 10\times\left(123-\frac{13}{6}\right)=\frac{3625}{6}$ ALT $\text{Area}=\frac{1}{2}\begin{vmatrix} 3 & 123 & {}^{13}/_{6} & 3 \\ 10 & 0 & 0 & 10 \end{vmatrix}=\frac{3625}{6}$	M1A1 M1A1 [M1A1] [4]
		Total 11 marks

Question	Notes	Marks
5(a)	States a correct expression for the gradient in terms of p either $-\frac{1}{12}=\frac{10-0}{p-123}$ or $-\frac{1}{12}=\frac{0-10}{123-p}$ and attempts to solve their equation in p $p-123=-12\times10\Rightarrow p=...$	M1
	Finds the value of p = 3* Must show an intermediate step e.g. $k = p \pm c$ or $c = \frac{p}{k}$ or $c = kp$	A1 cso [2]
(b)	Forms an equation using either the formula or $y = mx + c$ with the given values $y-10=-\frac{1}{12}(x-3)$ OR $y-0=-\frac{1}{12}(x-123)$	M1
	For the correct equation in the required form $12y+x-123=0$ Coefficients must be integers. Allow for $r = 1, s = 12, t = -123$	A1 [2]
(c)	For the gradient of the normal when x = 3 is 12	B1
	Forms an equation of the normal using either the formula or $y = mx + c$ with their values of the gradient of the normal. $y-12=12(x-3)$	M1
	For the correct equation in the required form. $y=12x-26$	A1 [3]
(d)	Attempts to find the x coordinate at point C $y=12x-26$ when $y=0,\ x=...$ Their linear equation from (c) with $y = 0$ and attempt to solve.	M1
	For the correct value of x $x=\frac{26}{12}$ oe	A1
	For attempting to find the area of triangle ABC Area = $\frac{1}{2}\times 10\times\left(123-\frac{13}{6}\right)=...$ or Area $=\frac{1}{2}\begin{vmatrix} 3 & 123 & {}^{13}/_{6} & 3 \\ 10 & 0 & 0 & 10 \end{vmatrix}$	M1
	For the correct area of the triangle $\frac{3625}{6}$ oe isw rounding	A1 [4]
		Total 11 marks

Question	Scheme	Marks
6	$A=\frac{r^2}{2}\times\frac{\pi}{6}=\left(\frac{\pi r^2}{12}\right)$	B1
	$\frac{\mathrm{d}A}{\mathrm{d}r}=\frac{2\times\pi r}{12}=\left(\frac{\pi r}{6}\right)$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$	M1
	Length of arc $AB=\frac{5\pi}{2}=r\frac{\pi}{6}\Rightarrow r=15$	M1
	$\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi\times 15}{6}\times 0.2=\frac{\pi}{2}\ \left(\mathrm{cm}^2/\mathrm{s}\right)$	M1A1 [6]
		Total 6 marks

Question	Notes	Marks
6	For using the correct formula for the area of the sector $A=\frac{r^2}{2}\times\frac{\pi}{6}=\left(\frac{\pi r^2}{12}\right)$	B1
	For an attempt to differentiate their expression for the area provided it is in the form $A=kr^2$ where $k\neq 1$ (see general guidance) $\frac{\mathrm{d}A}{\mathrm{d}r}=\frac{2\times\pi r}{12}=\left(\frac{\pi r}{6}\right)$	M1
	For a correct statement of chain rule to achieve $\frac{\mathrm{d}A}{\mathrm{d}t}$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ may be seen stated or implied by substitution.	M1
	For using the length of arc when the length of arc is $\frac{5\pi}{2}$ $\frac{5\pi}{2}=r\frac{\pi}{6}\Rightarrow(r=15)$	M1
	For attempting to find the rate of change of area using a correct chain rule, their $\frac{\mathrm{d}A}{\mathrm{d}r}$, the given $\frac{\mathrm{d}r}{\mathrm{d}t}$ and their value for r $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{'\pi'}{6}\times{'15'}\times 0.2=\ldots$	M1
	For the correct value of $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\pi}{2}\ \left(\mathrm{cm}^2/\mathrm{s}\right)$	A1 [6]
		Total 6 marks

Question	Scheme	Marks
7	$x=0,\ y=0^2+2=2$	B1
	$\left[y=x^2+2 \Rightarrow x^2=y-2\right]$	
	$V=\pi\int_2^a x^2\ \mathrm{d}y \Rightarrow 18\pi=\pi\int_2^a (y-2)\ \mathrm{d}y, =\pi\left[\dfrac{y^2}{2}-2y\right]_2^a$	M1,M1
	$18\pi=\pi\left[\left(\dfrac{a^2}{2}-2a\right)-\left(\dfrac{2^2}{2}-2\times 2\right)\right]$	M1
	$0=a^2-4a-32$	M1A1
	$a^2-4a-32=(a+4)(a-8)=0 \Rightarrow a=8$	M1A1 [8]
	ALT $x=0,\ y=0^2+2=2$	
	$\left[y=x^2+2 \Rightarrow x^2=y-2\right]$	[B1
	$18\pi=\pi\int_2^a (y-2)\ \mathrm{d}y,\ \ 18\pi=\pi\left[\dfrac{(y-2)^2}{2}\right]_2^a$	M1,M1
	$18\pi=\pi\left[\left(\dfrac{a-2}{2}\right)^2-\left(\dfrac{2-2}{2}\right)^2\right]$	M1
	$18=\left(\dfrac{a-2}{2}\right)^2 \Rightarrow 36=(a-2)^2$	M1A1
	$a=2\pm\sqrt{36} \Rightarrow a=8$	M1A1]
		Total 8 marks

Question	Notes	Marks
7	The intersection of S with the y-axis is at the point with coordinates (0, 2). May be given as when $x = 0, y = 2$	B1
	For a correct statement for the volume, condone missing or incorrect limits and missing π $\left[y = x^2 + 2 \Rightarrow x^2 = y - 2\right]$ $V = \pi\int_2^a x^2 \text{ d}y \Rightarrow 18\pi = \pi\int_2^a (y-2) \text{ d}y$	M1
	METHOD A For an attempt to integrate the expression which must be in the minimally acceptable form $y \pm 2$ $18\pi = \pi\left[\frac{y^2}{2} - 2y\right]_2^a$ [Ignore limits and π for this mark]	M1
	For substituting the limits into their integrated expression to form an equation in a. No simplification is required for this mark. $18\pi = \pi\left[\left(\frac{a^2}{2} - 2a\right) - \left(\frac{2^2}{2} - 2\times 2\right)\right]$	M1
	For forming a 3TQ in terms of a $0 = a^2 - 4a - 32$	M1
	For the correct 3TQ $0 = a^2 - 4a - 32$	A1
	For an attempt to solve their 3TQ $a^2 - 4a - 32 = (a+4)(a-8) = 0 \Rightarrow a = \ldots$	M1
	For the correct value of $a = 8$ If $a = -4$ is also stated then it must be rejected.	A1 [8]
	METHOD B For an attempt to integrate the expression which must be in the minimally acceptable form $y \pm 2$ $18\pi = \pi\left[\frac{(y-2)^2}{2}\right]_2^a$ [Ignore limits and π for this mark]	M1
	For substituting the limits into their integrated expression to form an equation in a. No simplification is required for this mark. $18\pi = \pi\left[\left(\frac{a-2}{2}\right)^2 - \left(\frac{2-2}{2}\right)^2\right]$	M1
	For forming an equation in a $18 = \left(\frac{a-2}{2}\right)^2 \Rightarrow 36 = (a-2)^2$	M1A1
	For an attempt to solve their equation. $a = 2 \pm \sqrt{36} \Rightarrow a = 8$	M1A1

Total 8 marks

Question	Scheme	Marks
8(a)	$\alpha+\beta=\dfrac{k}{3}$ and $\alpha\beta=-\dfrac{1}{3}$	B1
	$(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2\Rightarrow\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1
	$\alpha^2+\beta^2=\left(\dfrac{k}{3}\right)^3-2\left(-\dfrac{1}{3}\right)=\dfrac{k^2}{9}+\dfrac{2}{3}=\dfrac{k^2+6}{9}*$	A1 cso [3]
(b)	$\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2$	M1
	$\dfrac{466}{81}=\left(\dfrac{k^2+6}{9}\right)^2-2\left(-\dfrac{1}{3}\right)^2$	M1
	$\dfrac{466}{81}=\left(\dfrac{k^2+6}{9}\right)^2-2\left(-\dfrac{1}{3}\right)^2\Rightarrow k^4+12k^2-448=0$	M1
	$k^4+12k^2-448=\left(k^2-16\right)\left(k^2+28\right)=0\Rightarrow k^2=16\Rightarrow k=4$	M1A1 [5]
(c)	Sum: $\dfrac{\alpha^3+\beta}{\beta}+\dfrac{\beta^3+\alpha}{\alpha}=\dfrac{\alpha^4+\beta^4+2\alpha\beta}{\alpha\beta}$	M1
	$\dfrac{\alpha^3+\beta}{\beta}+\dfrac{\beta^3+\alpha}{\alpha}=\dfrac{\dfrac{466}{81}+2\left(-\dfrac{1}{3}\right)}{-\dfrac{1}{3}}=-\dfrac{412}{27}$	A1
	Product: $\left(\dfrac{\alpha^3+\beta}{\beta}\right)\left(\dfrac{\beta^3+\alpha}{\alpha}\right)=\dfrac{\alpha^4+\beta^4+(\alpha\beta)^3+\alpha\beta}{\alpha\beta}$	M1
	$\left(\dfrac{\alpha^3+\beta}{\beta}\right)\left(\dfrac{\beta^3+\alpha}{\alpha}\right)=\dfrac{\dfrac{466}{81}+\left(-\dfrac{1}{3}\right)^3+\left(-\dfrac{1}{3}\right)}{\left(-\dfrac{1}{3}\right)}=-\dfrac{436}{27}$	A1
	3TQ: $x^2+\dfrac{412}{27}x-\dfrac{436}{27}=0\Rightarrow 27x^2+412x-436=0$	M1A1 [6]
		Total 14 marks

Question	Scheme	Marks
8(a)	$\alpha+\beta=\dfrac{k}{3}$ and $\alpha\beta=-\dfrac{1}{3}$	B1
	For the correct algebra on $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1
	For substituting *their* sum and *their* product to give a simplified expression for $\alpha^2+\beta^2=\left(\dfrac{k}{3}\right)^3-2\left(-\dfrac{1}{3}\right)=\dfrac{k^2}{9}+\dfrac{2}{3}=\dfrac{k^2+6}{9}*$	A1 cso [3]

(b)	For the correct algebra on $\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2$ Or $\alpha^4+\beta^4=(\alpha+\beta)^4-4\alpha\beta(\alpha^2+\beta^2)-6\alpha^2\beta^2$	M1
	For substituting the values for $\alpha^2+\beta^2$ (in terms of *k*) and $\alpha\beta$ into their $\alpha^4+\beta^4$ and equating to $\frac{466}{81}$ $\frac{466}{81}=\left(\frac{k^2+6}{9}\right)^2-2\left(-\frac{1}{3}\right)^2$ Or for substituting the values for $\alpha^2+\beta^2$ (in terms of *k*), $\alpha+\beta$ and $\alpha\beta$ into their $\alpha^4+\beta^4$ and equating to $\frac{466}{81}$	M1
	For forming a simplified quadratic. $\frac{466}{81}=\left(\frac{k^2+6}{9}\right)^2-2\left(-\frac{1}{3}\right)^2\Rightarrow k^4+12k^2-448=0$ Or for $\left(\frac{k^2+6}{9}\right)^2=\frac{448}{81}$ Their quadratic must come from an attempt at $\alpha^4+\beta^4$	M1
	For solving their quadratic. $k^4+12k^2-448=\left(k^2-16\right)\left(k^2+28\right)=0\Rightarrow k^2=16\Rightarrow k=4$ Their quadratic must come from an attempt at $\alpha^4+\beta^4$	M1
	For the correct value of *k* = 4 If they give $k=\pm 4$ this is A0	A1 [5]
(c)	For the correct algebra on the sum. $\frac{\alpha^3+\beta}{\beta}+\frac{\beta^3+\alpha}{\alpha}=\frac{\alpha^4+\beta^4+2\alpha\beta}{\alpha\beta}$	M1
	For the correct value of the sum. $\frac{\alpha^3+\beta}{\beta}+\frac{\beta^3+\alpha}{\alpha}=\frac{\frac{466}{81}+2\left(-\frac{1}{3}\right)}{-\frac{1}{3}}=-\frac{412}{27}$	A1
	For the correct algebra on the product. $\left(\frac{\alpha^3+\beta}{\beta}\right)\left(\frac{\beta^3+\alpha}{\alpha}\right)=\frac{\alpha^4+\beta^4+(\alpha\beta)^3+\alpha\beta}{\alpha\beta}$	M1
	For the correct value of the product. $\left(\frac{\alpha^3+\beta}{\beta}\right)\left(\frac{\beta^3+\alpha}{\alpha}\right)=\frac{\frac{466}{81}+\left(-\frac{1}{3}\right)^3+\left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)}=-\frac{436}{27}$	A1
	For the 3TQ from their values of the sum and product (used correctly) [= 0 not required for this mark].	M1

	$x^2+\frac{412}{27}x-\frac{436}{27}=(0)$	
	For the correct 3TQ. Must have $= 0$. $27x^2+412x-436=0$ Must have integer coefficients. Accept a multiple of this.	A1 [6]
		Total 14 marks

Question	Scheme	Marks
9(a)	$ar^2=\frac{27}{2}$ $a+ar+\frac{27}{2}=\frac{57}{2}\left(\Rightarrow a+ar=15\Rightarrow a(1+r)=15\Rightarrow a=\frac{15}{1+r}\right)$ $\left(\frac{15}{1+r}\right)r^2=\frac{27}{2}\Rightarrow 30r^2-27r-27=0\Rightarrow\left(10r^2-9r-9=0\right)$	B1 M1 M1
	ALT $ar^2=\frac{27}{2}$ $\frac{57}{2}=\frac{a(r^3-1)}{r-1}$ $\frac{57}{2}=\frac{27}{2r^2}\times\frac{(r^3-1)}{r-1}\ \Rightarrow\ 30r^3-57r^2+27=0$ $\left(\Rightarrow 10r^3-19r^2+9=0\right)$	B1 M1 M1
	r: $10r^2-9r-9=0\Rightarrow(5r+3)(2r-3)=0\Rightarrow r=\frac{3}{2}$ a: $a=\frac{15}{1+\frac{3}{2}}=6$ $S_n=\sum_{r=1}^{n}\left('6'\div\frac{3}{2}\right)\left(\frac{3}{2}\right)^r\Rightarrow S_n=\sum_{r=1}^{n}4\left(\frac{3}{2}\right)^r$ *	M1A1 A1 M1A1 cso [8]
(b)	$\frac{6(1.5^k-1)}{1.5-1}>50\ 000$ $1.5^k>\frac{12\ 503}{3}\Rightarrow \lg 1.5^k>\lg\frac{12\ 503}{3}$ $k>\frac{\lg\frac{12\ 503}{3}}{\lg\frac{3}{2}}$ or $k>\frac{\lg\frac{12\ 503}{3}}{\lg 1.5}$ *	M1 M1 A1 [cso] [3]

(c)	For $k > 20.556...$ so $k = 21$	B1 [1]
		Total 12 marks

Question	Scheme	Marks
9(a)	For stating, $ar^2 = \frac{27}{2}$	B1
	For summing the first three terms; $a + ar + \frac{27}{2} = \frac{57}{2}\left(\Rightarrow a + ar = 15 \Rightarrow a(1+r) = 15 \Rightarrow a = \frac{15}{1+r}\right)$ OR $\frac{57}{2} = \frac{a(r^3-1)}{r-1}$	M1
	For attempting to form a 3TQ using their two expressions for U_3 and S_3 $\left(\frac{15}{1+r}\right)r^2 = \frac{27}{2} \Rightarrow 30r^2 - 27r - 27 = 0 \Rightarrow \left(10r^2 - 9r - 9 = 0\right)$ OR For attempting to form a cubic using their two expressions for U_3 and S_3 $\frac{57}{2} = \frac{27}{2r^2} \times \frac{(r^3-1)}{r-1} \Rightarrow 30r^3 - 57r^2 + 27 = 0$	M1
	For attempting to solve their 3TQ (see general guidance). $10r^2 - 9r - 9 = 0 \Rightarrow (5r+3)(2r-3) = 0 \Rightarrow r = ...$ OR If following the alt method then this mark is awarded for factorising the cubic into a linear expression and a 3TQ **and** attempting to solve their 3TQ. $(r-1)(10r^2 - 9r - 9) = 0 \Rightarrow (r-1)(5r+3)(2r-3) = 0$ $\Rightarrow r = \cdots$	M1
	For the correct value of $r = \frac{3}{2}$	A1
	For the correct value of $a = \frac{15}{1+\frac{3}{2}} = 6$	A1
	For attempting to use their *a* and *r* to find the correct expression for S_n	

	$S_n = \sum_{r=1}^{n} \left('6' \div \frac{3}{2}\right)\left(\frac{3}{2}\right)^r \Rightarrow S_n = ...$ OR For demonstrating that a particular value of r gives the correct term in the sequence.	M1
	For the correct expression $S_n = \sum_{r=1}^{n} 4\left(\frac{3}{2}\right)^r$ * OR For demonstrating that a particular value of r gives the correct term in the sequence and commenting on a correct common ratio.	A1 cso [8]
(b)	For using the summation formula > 50 000 $\frac{6\left(1.5^k - 1\right)}{1.5 - 1} > 50\ 000$	M1
	For rearranging the inequality to achieve, $1.5^k > \frac{12\ 503}{3}$ and takes logs base 10 of both sides $\lg 1.5^k > \lg \frac{12\ 503}{3}$	M1
	For using the laws of logs to make k the subject. $k > \frac{\lg \frac{12\ 503}{3}}{\lg \frac{3}{2}}$ or $k > \frac{\lg \frac{12\ 503}{3}}{\lg 1.5}$ *	A1 cso [3]
(c)	For $k > 20.556... \Rightarrow k = 21$	B1 [1]
		Total 12 marks

Question	Scheme	Marks
10(a)	$\frac{1}{243}=3^{-5}$	B1
	$9^{3y}=3^{6y}$	B1
	$\frac{9^{3y}}{243}=3^{-5}\times 3^{6y}\Rightarrow\frac{9^{3y}}{243}=3^{(6y-5)}$ *	M1A1 cso [4]
(b)	$27^{(x-2)}=3^{3(x-2)}=3^{(3x-6)}$	M1
	$6y-5=3x-6\Rightarrow(6y-3x-1=0)$	M1
	$\log_4 2=\frac{1}{2}$	B1
	$\log_{10}\sqrt{6xy}=\frac{1}{2}\log_{10}(6xy)\Rightarrow\log_{10}(6xy)=1$	M1
	$1=\log_{10}10\Rightarrow\log_{10}(6xy)=\log_{10}10\Rightarrow 6xy=10$	M1
	$3x-6y-1=0$ $6xy=10$	
	Method A	
	$6y=\frac{10}{x}\Rightarrow 3x-\frac{10}{x}-1=0\Rightarrow 3x^2-x-10=0$	M1
	$3x^2-x-10=(3x+5)(x-2)=0\Rightarrow x=2,\ -\frac{5}{3}$	M1
	$3\times 2-6y-1=0\Rightarrow 6y=5\Rightarrow y=\frac{5}{6}$	A1
	$3\times\left(-\frac{5}{3}\right)-6y-1=0\Rightarrow -6y=6\Rightarrow y=-1$	
	$x=2\quad y=\frac{5}{6}$ or $x=-\frac{5}{3}\quad y=-1$	A1 [9]
	Method B	
	$3x=\frac{5}{y}\Rightarrow\frac{5}{y}-6y-1=0\Rightarrow 6y^2+y-5=0$	M1
	$6y^2+y-5=(6y-5)(y+1)=0\Rightarrow y=\frac{5}{6},\ -1$	M1
	$3x-6\times\frac{5}{6}-1=0\Rightarrow 3x=6\Rightarrow x=2$	A1
	$3x-6\times(-1)-1=0\Rightarrow 3x=-5\Rightarrow x=-\frac{5}{3}$	
	$x=2\quad y=\frac{5}{6}$ or $x=-\frac{5}{3}\quad y=-1$	A1 [9]
		Total 13 marks

Question	Scheme	Marks
10	$$\frac{9^{3y}}{243}=27^{(x-2)}$$ $$\log_{10}\sqrt{6xy}=\log_4 2$$	
(a)	Writes down $\frac{1}{243}=3^{-5}$ May be seen as $\frac{1}{243}=\frac{1}{3^5}$ later correctly used as 3^{-5} when combining terms.	B1
	Writes down $9^{3y}=3^{6y}$	B1
	Combines the terms $\frac{9^{3y}}{243}=3^{-5}\times 3^{6y}=3^{(6y-5)}$	M1
	For the correct expression with no errors. $$\frac{9^{3y}}{243}=3^{(6y-5)}*$$	A1 cso [4]
(b)	For dealing with the power of 3 to give $27^{(x-2)}=3^{3(x-2)}=3^{(3x-6)}$	M1
	For equating the powers of 3 to give the equation $6y-5=3x-6\Rightarrow(6y-3x-1=0)$	M1
	For stating $\log_4 2=\frac{1}{2}$	B1
	For dealing with the square root $\log_{10}\sqrt{6xy}=\frac{1}{2}\log_{10}(6xy)=\left(\frac{1}{2}\right)\Rightarrow\log_{10}(6xy)=1$	M1
	For correctly removing all logarithms from the second equation $1=\log_{10}10\Rightarrow\log_{10}(6xy)=\log_{10}10\Rightarrow 6xy=10$	M1
	$$3x-6y-1=0$$ $$6xy=10$$	
	Method A	
	For substituting $6y=\frac{10}{x}$ into the linear equation to give $3x-\frac{10}{x}-1=0$ and attempting to form a 3TQ $3x^2-x-10=0$ OR for substituting $y=\frac{1}{6}(3x-1)$ into $6xy=10$ to give $x(3x-1)=10$ and attempting to form a 3TQ $3x^2-x-10=0$	M1
	For attempting to solve their 3TQ $3x^2-x-10=(3x+5)(x-2)=0\Rightarrow x=2,\ -\frac{5}{3}$	M1

	For finding the values of y $3\times 2-6y-1=0\Rightarrow 6y=5\Rightarrow y=\frac{5}{6}$ $3\times\left(-\frac{5}{3}\right)-6y-1=0\Rightarrow -6y=6\Rightarrow y=-1$ $x=2 \quad y=\frac{5}{6}$ or $x=-\frac{5}{3} \quad y=-1$ Answers must be given in pairs, pairing may be implied from working.	A1 A1 [9]
	Method B	
	For substituting $3x=\frac{10}{2y}\Rightarrow 3x=\frac{5}{y}$ into the linear equation to give $\frac{5}{y}-6y-1=0$ and attempting to form a 3TQ $6y^2+y-5=0$ OR for substituting $x=\frac{1}{3}(6y+1)$ into $6xy=10$ to give $2(6y+1)y=10$ and attempting to form a 3TQ $6y^2+y-5=0$	M1
	For attempting to solve their 3TQ $6y^2+y-5=(6y-5)(y+1)=0\Rightarrow y=\frac{5}{6},\ -1$	M1
	For finding the values of x $3x-6\times\frac{5}{6}-1=0\Rightarrow 3x=6\Rightarrow x=2$ $3x-6\times(-1)-1=0\Rightarrow 3x=-5\Rightarrow x=-\frac{5}{3}$ $x=2 \quad y=\frac{5}{6}$ or $x=-\frac{5}{3} \quad y=-1$	A1 A1 [9]
		Total 13 marks

Question	Scheme	Marks
11	$S=\pi r^2+2\pi rh$ $\pi r^2+2\pi rh=625\pi \Rightarrow 2rh=625-r^2 \Rightarrow h=\dfrac{625-r^2}{2r}$ *	B1 M1A1 [cso] [3]
(b)	$V=\pi r^2\left(\dfrac{625-r^2}{2r}\right)$ or $\left(V=\dfrac{625\pi}{2}r-\dfrac{\pi r^3}{2}\right)$ $\dfrac{\mathrm{d}V}{\mathrm{d}r}=\dfrac{625\pi}{2}-\dfrac{3\pi r^2}{2}$ $\dfrac{625\pi}{2}-\dfrac{3\pi r^2}{2}=0 \Rightarrow 625=3r^2 \Rightarrow r=14.4337... \Rightarrow r\approx 14.4$ For differentiating their $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}=-3\pi r$ $(=-136.03\ ...)$ For concluding that as $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}$ is negative then the volume is a maximum. (Explicit substitution of r is not required).	M1 M1 M1A1 M1 A1 [6]
(c)	$h=\dfrac{625-14.43375^2}{2\times 14.43375}=14.4337...\approx 14.4$	B1ft [1]
		Total 10 marks

Question	Scheme	Marks
11	For a correct expression for the surface area of the cup. $S=\pi r^2+2\pi rh$	B1
	For equating their expression for the surface area to 625π and attempting to make h the subject. $\pi r^2+2\pi rh=625\pi \Rightarrow 2rh=625-r^2 \Rightarrow h=\dfrac{625-r^2}{2r}$ Allow errors in arithmetic but not mathematically incorrect process.	M1
	For the correct expression for h as shown with no errors. $h=\dfrac{625-r^2}{2r}$ *	A1 cso [3]

(b)	For the correct volume of the container in terms of r only. $V=\pi r^2\left(\frac{625-r^2}{2r}\right)$ or $\left(V=\frac{625\pi}{2}r-\frac{\pi r^3}{2}\right)$	M1
	Minimally acceptable attempt at differentiation, see general guidance, no power to increase. $\frac{dV}{dr}=\frac{625\pi}{2}-\frac{3\pi r^2}{2}$	M1
	Places their derivative = 0 and attempts to rearrange to find r. Minimally acceptable derivative is of the form $a \pm b\pi r^2$ $\frac{625\pi}{2}-\frac{3\pi r^2}{2}=0 \Rightarrow 625=3r^2 \Rightarrow r=...$	M1
	For the correct value of r $r=14.4337... \Rightarrow r \approx 14.4$ (cm) Must reject negative value if found, award A0 if not rejected. May be implied from subsequent working.	A1
	For differentiating their $\frac{dV}{dr}$ For minimally acceptable attempt to differentiate their first derivative, see general guidance, no power to increase. Or testing gradients or a sketch. $\frac{d^2V}{dr^2}=-3\pi r$	M1
	For concluding that as $\frac{d^2V}{dr^2}$ is negative then the volume is a maximum. (Explicit substitution of r is not required).	A1 [6]
(c)	For the value of h using their value of r $h=\frac{625-14.43375^2}{2\times 14.43375}=14.4337... \approx 14.4$ Allow for 14.5 if substituting $r=14.4$	B1ft [1]
		Total 10 marks

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