Pearson Edexcel

Mark Scheme (Results)

January 2023

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In Further Pure Mathematics (4PM1)
Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0, \quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**
quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values
or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".
General policy is that if it could be done "in your head" detailed working would not be required.
(Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Marks
1	$\sqrt{48}=\sqrt{16}\sqrt{3}=4\sqrt{3}$	B1
	Method A	
	$\dfrac{a-\sqrt{48}}{\sqrt{3}+1}=b\sqrt{3}-9 \Rightarrow a-4\sqrt{3}=3b-9+\sqrt{3}(b-9)$	M1
	$\left[a=3b-9 \text{ and } -4=b-9\right]$	M1
	$b=5,\ a=6$	A1
		[4]
	Method B	
	$\dfrac{\left(a-4\sqrt{3}\right)}{\left(\sqrt{3}+1\right)}\times\dfrac{\left(\sqrt{3}-1\right)}{\left(\sqrt{3}-1\right)}=\dfrac{a\sqrt{3}-a-12+4\sqrt{3}}{2}=b\sqrt{3}-9$	[M1M1
	$b=5,\ a=6$	A1]
		Total 4 marks

Question	Notes	Marks
1	$\dfrac{a-\sqrt{48}}{\sqrt{3}+1}=b\sqrt{3}-9$	
	Simplifies $\sqrt{48}=\sqrt{16}\sqrt{3}=4\sqrt{3}$ - Seen anywhere.	B1
	Method A – Both sides multiplied by $\sqrt{3}+1$, collects terms and equates rational and irrational parts and obtains two equations at least one of which must be correct. $a-4\sqrt{3}=\left(b\sqrt{3}-9\right)\left(\sqrt{3}+1\right)=\left(3b-9\right)+\sqrt{3}\left(b-9\right)$ $\Rightarrow a=3b-9$ and $-4=b-9$	M1
	Solves their equations. The equation $-4=b-9$ must be solved correctly and the result substituted into the second equation to find a Allow one processing error here. **This is an A mark in Epen**	M1
	For a = 6 and b = 5	A1 [4]
	Simplifies $\sqrt{48}=\sqrt{16}\sqrt{3}=4\sqrt{3}$	B1
	Method B – rationalises the denominator, collects terms and equates rational and irrational parts and obtains two equations at least one of which must be correct. . $\dfrac{\left(a-4\sqrt{3}\right)}{\left(\sqrt{3}+1\right)}\times\dfrac{\left(\sqrt{3}-1\right)}{\left(\sqrt{3}-1\right)}=\dfrac{\sqrt{3}\left(a+4\right)-\left(a+12\right)}{2}=b\sqrt{3}-9$ $\Rightarrow\dfrac{-\left(a+12\right)}{2}=-9$ $\quad\dfrac{a+4}{2}=b$ oe	M1
	Solves their equations: The equation $\dfrac{-\left(a+12\right)}{2}=-9$ must be solved correctly and the result substituted into the second equation to find b Allow one processing error here. **This is an A mark in Epen**	M1
	For a = 6 and b = 5	A1 [4]
		Total 4 marks

Question	Scheme	Marks
2	$\frac{\sin \angle BCA}{10}=\frac{\sin 50}{9} \Rightarrow \angle BCA = 58.3381...^{o} \Rightarrow 58.3^{o},\ 121.7^{o}$	M1A1A1
		Total 3 marks

Question	Notes	Marks
2	Uses sine rule or any other appropriate trigonometry in triangle *ABC* $\frac{\sin \angle BCA}{10}=\frac{\sin 50}{9}$ Note: the perpendicular height of the triangle from *B* to *AC* is 7.66044 cm. Their method must be complete for the award of this mark.	M1
	$\angle BCA = 58.3381...^{o}$	A1
	One possible value is awrt 58.3^{o} and the other possible value is awrt 121.7°	A1 [3]
		Total 3 marks

Question	Scheme	Marks
3	$S_n < -450 \Rightarrow \frac{n}{2}\left(2\times16+[n-1](-5)\right) < -450$ $\Rightarrow 37n - 5n^2 < -900 \Rightarrow 5n^2 - 37n - 900 > 0$	M1A1
	$n = \frac{-(-37)\pm\sqrt{(-37)^2 - 4\times5\times(-900)}}{2\times5} \Rightarrow n = 17.617..$ so $n = 18$	M1A1 [4]
		Total 4 marks

Question	Notes	Marks
3	Uses the correct summation formula and sets <, > or = to -450 $S_n < -450 \Rightarrow \frac{n}{2}\left(2\times16+[n-1](-5)\right) < -450$	M1
	Forms a correct 3TQ with their expression $37n - 5n^2 < -900 \Rightarrow 5n^2 - 37n - 900 > 0$ Accept <, > or = 0 and accept terms in any order.	A1
	Attempts to solve their 3TQ using a valid method. [See General Guidance} $n = \frac{-(-37)\pm\sqrt{(-37)^2 - 4\times5\times(-900)}}{2\times5} \Rightarrow n = ...$	M1
	$n = 17.617...$ so $n = 18$ [Other root is – 10.217….]	A1
		Total 4 marks

Question	Scheme	Marks
4(a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} \Rightarrow \overrightarrow{AB} = (5\mathbf{i} + 9p\mathbf{j}) - (p\mathbf{i} + 2p\mathbf{j})$	M1A1
	$\mathbf{i}(5-p) + \mathbf{j}(7p) = Q(\mathbf{i} - 2\mathbf{j}) \Rightarrow 5 - p = Q$ and $7p = -2Q$	M1M1
	$7p = -2(5-p) \Rightarrow p = -2$	M1A1 [6]
(b)	$7('-2') = -2Q \Rightarrow Q = 7$, $\overrightarrow{AB} = 7(\mathbf{i} - 2\mathbf{j}) = 7\mathbf{i} - 14\mathbf{j}$	M1A1ft [2]
	OR	
	$\overrightarrow{AB} = (5\mathbf{i} + 9(-2)\mathbf{j}) - ((-2)\mathbf{i} + 2(-2)\mathbf{j}) = 7\mathbf{i} - 14\mathbf{j}$	[M1A1ft]
(c)	$\overrightarrow{OA} = -2\mathbf{i} - 4\mathbf{j} \Rightarrow \left\|\overrightarrow{OA}\right\| = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20}$	M1A1ft
	Unit vector is $\dfrac{1}{\sqrt{20}}(-2\mathbf{i} - 4\mathbf{j}) = \dfrac{\sqrt{5}}{5}(-\mathbf{i} - 2\mathbf{j})$	M1A1 [4]
		Total 12 marks

Question	Notes	Marks
4(a)	For the basic vector statement $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	M1
	For the correct vector (simplified or unsimplified) $\overrightarrow{AB} = (5\mathbf{i} + 9p\mathbf{j}) - (p\mathbf{i} + 2p\mathbf{j}) = [\mathbf{i}(5-p) + \mathbf{j}(7p)]$	A1
	For setting their $\overrightarrow{AB} = Q(\mathbf{i} - 2\mathbf{j})$ where $Q \neq 1,\ Q \neq 0$ $\mathbf{i}(5-p) + \mathbf{j}(7p) = Q(\mathbf{i} - 2\mathbf{j})$	M1
	For equating components of **i** and **j** **i** $5 - p = Q$ **j** $7p = -2Q$	M1
	Solving the simultaneous equations by any method to find the value of *p* $7p = -2(5-p) \Rightarrow p = \ldots$	M1
	For the value of $p = -2$	A1 [6]
(b)	For finding the value of *k* and using it to find the vector $\overrightarrow{AB}$ $7('-2') = -2Q \Rightarrow Q = 7$	M1
	For the correct vector $\overrightarrow{AB} = 7(\mathbf{i} - 2\mathbf{j}) = 7\mathbf{i} - 14\mathbf{j}$	A1ft [2]
	ALT	
	For substituting their value of *p* to find the vector $\overrightarrow{AB}$ $\overrightarrow{AB} = \mathbf{i}(5 - [-2]) + \mathbf{j}(7[-2]) = \ldots$	M1
	$\overrightarrow{AB} = \mathbf{i}(5-p) + \mathbf{j}(7p) \Rightarrow \overrightarrow{AB} = 7\mathbf{i} - 14\mathbf{j}$	A1ft [2]
(c)	$\overrightarrow{OA} = -2\mathbf{i} - 4\mathbf{j} \Rightarrow \left\|\overrightarrow{OA}\right\| = \sqrt{(-2)^2 + (-4)^2} = \ldots$	M1
	$\left\|\overrightarrow{OA}\right\| = \sqrt{20}$	A1ft
	Unit vector in the direction of $\overrightarrow{OA}$ is $\frac{1}{\sqrt{20}}(-2\mathbf{i} - 4\mathbf{j})$	M1
	Unit vector in the required form $\frac{\sqrt{5}}{5}(-\mathbf{i} - 2\mathbf{j})$ Allow $\frac{\sqrt{5}}{5}(\mathbf{i} + 2\mathbf{j})$ **provided** no processing errors seen.	A1 [4]
		Total 12 marks

Question	Scheme	Marks
5(a)	$f(-2) = -16a + 4 + 2b + 3a = 0$ and $f(1) = 2a + 1 - b + 3a = 0$ $-13a + 4 + 2b = 0$ and $5a + 1 - b = 0 \Rightarrow a = 2*$ $b = 11$	M1M1 M1A1cso A1 [5]
(b)	$f(x) = 4x^3 + x^2 - 11x + 6 = (4x-3)(x+2)(x-1)$	M1A1 [2]
(c)	$h(y) = 2^{(3y+2)} + 2^{2y} - 11(2^y) + 6 \Rightarrow h(y) = 4(2^y)^3 + (2^y)^2 - 11(2^y) + 6$ $x = 2^y \Rightarrow 2^y - 1 = 0,\ \ 2^y + 2 = 0,\ \ 4(2^y) - 3 = 0$ $2^y = 1 \Rightarrow y = 0$ $2^y = \frac{3}{4} \Rightarrow y = \log_2 \frac{3}{4} = -0.4150... \Rightarrow y = -0.415$ $\left[2^y = -2 \quad \text{no solution}\right]$	M1 M1 B1 M1A1 [5]
		Total 12 marks

Question	Notes	Marks
(a)	For substituting ± 2 into $f(x) = 0$ Allow sign errors	M1
	For substituting ± 1 into $f(x) = 0$ Allow sign errors	M1
	For solving the resulting pair of simultaneous equations. $-13a + 4 + 2b = 0$ $5a + 1 - b = 0$	M1
	For a = 2 *	A1 cso
	For b = 11	A1 [5]
	ALT uses polynomial division.	
	Multiplies out the product of the two factors to give: $(x+2)(x-1) = x^2 + x - 2$ This must be correct.	M1

	Uses polynomial division to give as a minimum: $2ax+(1-2M)$ where M is a constant. $x^2+x-2\overline{)2ax^3+x^2-bx+3a}$ with quotient $2ax+(1-2a)$	M1
	Equates coefficients: $(x^2+x-2)(2ax+(1-2a))=2ax^3+x^2-bx+3a$ $\Rightarrow 2ax^3+x^2+x(1-6a)+(-2+4a)=2ax^3+x^2-bx+3a$ $3a=4a-2$ $1-6a=-b$	M1
	For a = 2 *	A1 cso
	For b = 11	A1
(b)	Divides $f(x)=4x^3+x^2-11x+6$ by either [x^2+x-2] or x + 2 or $x-1$ Either, $\left[x^2+x-2\overline{)4x^3+x^2-11x+6}\right]$ with quotient $4x+k$ $x+2\overline{)4x^3+x^2-11x+6}$ with quotient $4x^2-7x\pm k$ $x-1\overline{)4x^3+x^2-11x+6}$ with quotient $4x^2+5x\pm k$	M1
	For the correct factorisation of $f(x)$ $4x^2-7x+3=(4x-3)(x-1)\Rightarrow f(x)=(4x-3)(x-1)(x+2)$	A1 [2]
(c)	For manipulating the indices to achieve $\Rightarrow h(y)=4(2^y)^3+(2^y)^2-11(2^y)+6$ OR Uses the factorised expression to obtain: $h(y)=(2^y+2)(2^y-1)(4(2^y)-3)$	M1
	Substitutes $x=2^y$ into the factorised f (x) to find either $2^y-1=0$ or $2^y+2=0$ or $4(2^y)-3=0$	M1
	For $2^y=1\Rightarrow y=0$	B1
	For $4(2^y)=3\Rightarrow 2^y=\frac{3}{4}\Rightarrow y=\log_2\frac{3}{4}$	M1
	For awrt $y=-0.4150...\Rightarrow y\approx-0.415$ [No solution for $2^y=-2$ - must reject]	A1 [5]
		Total 12 marks

Question	Scheme	Marks
6(a)	$\frac{dy}{dx}=\frac{(x^2+1)2xe^{(x^2+1)}-2xe^{(x^2+1)}}{(x^2+1)^2}$	M1A1A1
	$\frac{dy}{dx}=\frac{2xe^{(x^2+1)}(x^2+1-1)}{(x^2+1)^2}=\frac{2x^3e^{(x^2+1)}}{(x^2+1)^2}$	M1A1 cso [5]
(b)	When $x=-1$ $\quad \frac{dy}{dx}=\frac{-e^2}{2}, \quad y=\frac{e^2}{2}$	B1ft, B1
	$y-\frac{e^2}{2}=-\frac{e^2}{2}(x+1), \Rightarrow y=-\frac{e^2x}{2}$ oe	M1A1ft, A1 [5]
	Total 10 marks	

Question	Notes	Marks
6(a)	$y=\frac{e^{(x^2+1)}}{x^2+1}$	
	Using Quotient Rule $\frac{dy}{dx}=\frac{(x^2+1)2xe^{(x^2+1)}-2xe^{(x^2+1)}}{(x^2+1)^2}$ • For an attempt to differentiate both $e^{(x^2+1)}$ and x^2+1 Award for either $e^{x^2+1}\Rightarrow 2xe^{x^2+1}$ or $x^2+1\Rightarrow 2x$ but both must be **changed** expressions. • Numerator is to have two terms in either order **subtracted.** • Denominator must be $(x^2+1)^2$	M1
	At least one term fully correct in the numerator	A1
	Fully correct unsimplified.	A1
	For an attempt to take out a common factor of either $2x$ or $e^{(x^2+1)}$ $\frac{dy}{dx}=\frac{2xe^{(x^2+1)}(x^2+1-1)}{(x^2+1)^2}$ OR Multiplies out the first term in the numerator $\frac{2x^3e^{(x^2+1)}+2xe^{(x^2+1)}-2xe^{(x^2+1)}}{(x^2+1)^2}$	M1 [M1]

	Using Product Rule	
	$\frac{dy}{dx} = \left(x^2+1\right)^{-1} \times 2xe^{x^2+1} + (-2x)e^{x^2+1} \times \left(x^2+1\right)^{-2}$ • For an attempt to differentiate both $e^{\left(x^2+1\right)}$ and $\left(x^2+1\right)^{-1}$ Award for either $e^{x^2+1} \Rightarrow 2xe^{x^2+1}$ or $\left(x^2+1\right)^{-1} \Rightarrow (-2x)\left(x^2+1\right)^{-2}$ but both must be **changed** expressions. • Numerator is to have two terms added	M1
	At least one term correct	A1
	Fully correct unsimplified or simplified	A1
	For an attempt to take out a common factor of either $2x$ or $e^{\left(x^2+1\right)}$ and set a common denominator of $\left(x^2+1\right)^2$	M1
	For the fully correct expression for the derivative, $\frac{dy}{dx} = \frac{2x^3e^{\left(x^2+1\right)}}{\left(x^2+1\right)^2}$ NB: Any further simplification [e.g., cancelling $\left(x^2+1\right)$] following a correct answer seen, is A0.	A1 [5]
(b)	When $x = -1$, $\frac{dy}{dx} = \frac{-e^2}{2}$ ft their K in their $\frac{dy}{dx}$ Allow awrt $\frac{dy}{dx} = -3.7$ Even allow $\frac{dy}{dx} = \frac{-Ke^2}{4}$	B1ft
	When $x = -1$, $y = \frac{e^2}{2}$ Allow awrt $y = 3.7$	B1
	For a correctly used method for the equation of the tangent using their values for $\frac{dy}{dx}$ and y $y - '\frac{e^2}{2}' = '-\frac{e^2}{2}'(x+1)$ Also allow: $y - 3.7 = -3.7(x+1)$ If they use $y = mx + c$ they must obtain a value for c for the award of this mark.	M1
	For the correct equation in any form	A1ft
	For a correct simplified equation in any form but this must be in exact form. $y = -\frac{e^2x}{2}$ or $2y + e^2x = 0$	A1 [5]
		Total 10 marks

Question	Scheme	Marks
7(a)	$v = 1^2 - 10\times1 + 28 = 19$ [m/s]	B1 [1]
(b)	$s = \int\left(t^2 - 10t + 28\right)\,\mathrm{d}t = \frac{t^3}{3} - \frac{10t^2}{2} + 28t\left(+c\right)$ $24 = \frac{3^3}{3} - \frac{10\times3^2}{2} + 28\times3 + c \Rightarrow c = -24 \Rightarrow \left[s = \frac{t^3}{3} - \frac{10t^2}{2} + 28t - 24\right]$ $t = 5, \quad s = \frac{5^3}{3} - \frac{10\times5^2}{2} + 28\times5 - 24 = \frac{98}{3}$ [m]	M1 M1A1 M1A1 [5]
(c)	$\frac{\mathrm{d}v}{\mathrm{d}t} = 2t - 10$ when t = 9, acceleration = 8 [m/s ^{2}]	M1 A1 [2]
(d)	(i) $v = \left(t-5\right)^2 + 3$ Irrespective of the value of t $v \geqslant 3$ so the particle never comes to rest. **ALT** $b^2 - 4ac < 0 \Rightarrow \left(-10\right)^2 - 4\times1\times28 = -12$ No real solutions so the particle never comes to rest. (ii) Least value of v is 3 [m/s]	M1 A1 [M1 A1] B1 [3]
	Total 11 marks	

Question	Notes	Marks
7(a)	$v = t^2 - 10t + 28$	
	$v = 1^2 - 10\times1 + 28 = 19$ [m/s]	B1 [1]
(b)	For an attempt to integrate the given expression for v [See general guidance for the definition of an attempt] $s = \int\left(t^2 - 10t + 28\right)\,\mathrm{d}t = \frac{t^3}{3} - \frac{10t^2}{2} + 28t\left(+c\right)$	M1
	For finding the value of c. They cannot score this mark without $+c$ $24 = \frac{3^3}{3} - \frac{10\times3^2}{2} + 28\times3 + c \Rightarrow c = \ldots$	M1
	For the correct expression for s. This need not be explicitly stated. $s = \frac{t^3}{3} - \frac{10t^2}{2} + 28t - 24$	A1

	Award for the correct value of *c* seen -24 [m]	
	For using their integrated expression for *s* to find a value of the displacement when $t = 5$ $s = \frac{5^3}{3} - \frac{10\times5^2}{2} + 28\times5 - 24 = ...$	M1
	For the correct value of $s = \frac{98}{3}$ [m] Accept $s = 32.7$ [or better]	A1 [5]
(c)	For an attempt to differentiate the given *v* and substituting $t = 9$ into their differentiated expression. $\frac{dv}{dt} = 2t - 10 \Rightarrow \frac{dv}{dt} = 2\times9 - 10 = ...$	M1
	For acceleration = 8 [m/s ^{2}]	A1 [2]
(d)(i)	**Method A** Completes the square to give $v = (t-5)^2 + 3$	M1
	Concludes that at the minimum velocity [3 m/s] $t = 5$ so *P* never comes to rest	A1
	Method B Finds the value of the discriminant $b^2 - 4ac < 0 \Rightarrow (-10)^2 - 4\times1\times28 = -12$	M1
	Concludes that as there are no real solutions, **so *P* does not come to rest.**	A1
	Method C Solves the 3TQ to give the following two [non-real] values of *t*: $5+\sqrt{3}\text{i}$ and $5-\sqrt{3}\text{i}$ or $\left(t-\left[5+\sqrt{3}\text{i}\right]\right)\left(t-\left[5+\sqrt{3}\text{i}\right]\right)=0$	M1
	Concludes that as there are no real solutions, **so *P* does not come to rest.**	A1
	Method D Uses their result from (c) $\frac{dv}{dt} = 2t - 10 = 0 \Rightarrow t = 5$	M1
	Concludes that at the minimum velocity [3 m/s] $t = 5$ so *P* never comes to rest	A1
(ii)	For the correct value of $v = 3$ [m/s}	B1 [3]
		Total 11 marks

Question	Scheme	Marks
8(a)	$\int 17+2x-3x^2\ \mathrm{d}x = 17x+\frac{2x^2}{2}-\frac{3x^3}{3}+k$	M1A1
	$0=\left(17\times(-1)+\frac{2\times(-1)^2}{2}-\frac{3\times(-1)^3}{3}\right)+k \Rightarrow k=15$	M1
	$y=15+17x+x^2-x^3$	A1 [4]
(b)	$\frac{(15+17x+x^2-x^3)}{(x+1)}=-x^2+2x+15$	M1A1
	$-x^2+2x+15=(x+3)(5-x)$	M1A1
	$a=-3,\ [-1]$ and $b=5$	A1
	When $x=0$ $y=15$ so $c=15$	B1 [6]
(c)	$\int_0^5 (15+17x+x^2-x^3)\ \mathrm{d}x-\frac{1}{2}\times5\times15$	M1
	OR	
	$\int_0^5 (15+17x+x^2-x^3)\ \mathrm{d}x-\int_0^5 (15-3x)\ \mathrm{d}x$	
	$\int_0^5 (15+17x+x^2-x^3)\ \mathrm{d}x=\left[15x+\frac{17x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^5$	M1
	$\left(15\times5+\frac{17\times5^2}{2}+\frac{5^3}{3}-\frac{5^4}{4}\right)-(0)=\left(\frac{2075}{12}\right)$	M1
	Area of triangle Area $=\frac{1}{2}\times5\times15=37.5$	B1
	OR	
	Area = $\int_0^5 (15-3x)\ \mathrm{d}x=\left[15x-\frac{3x^2}{2}\right]_0^5=37.5$	[B1]
	For the correct area of $R=\frac{2075}{12}-37\frac{1}{2}=\frac{1625}{12}=135\frac{5}{12}$	A1 [5]
		Total 15 marks

Question	Notes	Marks
8(a)	$f'(x)=17+2x-3x^2$	
	For an attempt to integrate $f'(x)$ $y=17x+\frac{2x^2}{2}-\frac{3x^3}{3}+(k)$	M1
	For the correct integral **including** a constant of integration $y=17x+\frac{2x^2}{2}-\frac{3x^3}{3}+k$	A1
	For substituting (−1 , 0) into their integrated expression, which must include a constant of integration. $0=\left(17\times(-1)+\frac{2\times(-1)^2}{2}-\frac{3\times(-1)^3}{3}\right)+k\Rightarrow(k=15)$	M1
	For writing the equation in the required form $y=15+17x+x^2-x^3$ * **This is a given equation. Every step above must be seen for the award of full marks.**	A1 cso [4]
(b)	Divides $(15+17x+x^2-x^3)$ by $(x+1)$ $x+1\overline{)15+17x+x^2-x^3}$ with quotient $Q+2x+x^2$ **OR** Equates coefficients $(15+17x+x^2-x^3)=(x+1)(Ax^2+Bx+c)\Rightarrow(x+1)(-x^2+2x+Q)$ Where Q is a constant **Minimal working here is sight of the quadratic factor.**	M1
	For obtaining the correct 3TQ $-x^2+2x+15$	A1
	For factorising their 3TQ [or otherwise solving] $-x^2+2x+15=(x+3)(5-x)$	M1A1
	For $a=-3$, $[-1]$ and $b=5$ identified clearly.	A1
	SC No working seen [use of a root finder on a calculator]	
	For $(x+1)(x-5)(x+3)$ seen leading to $a=-3$ and $b=5$ with no working award: M0A0MA1A1	
	For $a=-3$ and $b=5$ seen with no other working seen award: M0A0M0A0A1	
	For the value of c = 15	B1
(c)	For writing a correct expression for the area of R with the correct limits $\int_0^5(15+17x+x^2-x^3)\,dx-\frac{1}{2}\times5\times15$ OR	M1

	$\int_0^5 \left(15+17x+x^2-x^3\right)\ \mathrm{d}x-\int_0^5 \left(15-3x\right)\ \mathrm{d}x$	
	For an attempt to integrate the expression for the curve. [Ignore limits for this mark] Area = $\int_0^5 \left(15+17x+x^2-x^3\right)\ \mathrm{d}x=\left[15x+\frac{17x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^5$	M1
	For evaluating their integral using their limits. Area = $\left(15\times5+\frac{17\times5^2}{2}+\frac{5^3}{3}-\frac{5^4}{4}\right)-(0)=\left(\frac{2075}{12}\right)$	M1
	For the area of the triangle Area $=\frac{1}{2}\times5\times15=37.5$ **ALT** Integrates the line Area = $\int_0^5 (15-3x)\ \mathrm{d}x=\left[15x-\frac{3x^2}{2}\right]_0^5=37.5$	B1 [B1]
	For the correct area of *R* $\frac{2075}{12}-37\frac{1}{2}=\frac{1625}{12}=135\frac{5}{12}$	A1 [5]
	ALT	
	For writing an expression for the area of *R* with the correct limits. $\int_0^5 \left(15+17x+x^2-x^3\right)\ \mathrm{d}x-\int_0^5 (15-3x)\ \mathrm{d}x=\left[\int_0^5 \left(20x+x^2-x^3\right)\ \mathrm{d}x\right]$ Award the B mark for a correct expression for the combined area.	M1 B1
	For an attempt to integrate the expression for the combined area or just the curve. [Ignore limits for this mark] Area = $\int_0^5 \left(20x+x^2-x^3\right)\ \mathrm{d}x=\left[\frac{20x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\right]_0^5$	M1
	For evaluating their integral using their limits, but the lower limit must be 0. Area = $\left(\frac{20\times5^2}{2}+\frac{5^3}{3}-\frac{5^4}{4}\right)=\left(\frac{1625}{12}\right)$	M1
	For the correct final area.	A1 [5]
		Total 15 marks

Question	Scheme	Marks
9(a)	$AC=\sqrt{12^2+12^2}=\sqrt{288}=12\sqrt{2}$	M1A1 [2]
(b)	$x=\dfrac{\frac{12\sqrt{2}}{2}}{\cos 30^{\circ}}=\sqrt{96}=\left(4\sqrt{6}\right)*$	M1A1 cso [2]
(c)	$\cos\angle AOB=\dfrac{96+96-12^2}{2\times\sqrt{96}\times\sqrt{96}}\Rightarrow\angle AOB=75.522...^{\circ}$ Area $\Delta OAB=\dfrac{1}{2}\times\sqrt{96}\times\sqrt{96}\times\sin'75.522^{\circ}'=\left(46.4758...\right)$ $4\times 46.4758+12^2=329.90....\Rightarrow$ Area = 330 [m^2]	M1A1 M1 M1A1 [5]
(d)	$\left[\angle OBC=\dfrac{180-75.522}{2}=52.239^{\circ}\right]$ Let Y on OB be the foot of the perpendicular from A to OB $AY=12\sin 52.239^{\circ}=\left(9.4868...\right)$ $\cos\angle AYC=\dfrac{'9.4868'^2+'9.4868'^2-288}{2\times'9.4868'\times'9.4868'}\Rightarrow\angle AYC=126.87...\approx 127^{[\circ]}$	M1 A1 M1A1 [4]
		Total 13 marks

Question	Notes	Marks
9(a)	For using Pythagoras theorem on triangle ABC or triangle ADC $AC=\sqrt{12^2+12^2}=...$	M1
	For the correct value of AC $AC=\sqrt{288}=12\sqrt{2}$	A1 [2]
(b)	Let the intersection of AC and BD be X Uses any appropriate trigonometry on triangle OAX For example; $x=\dfrac{\frac{12\sqrt{2}}{2}}{\cos 30^{\circ}}=...$ **OR** $\dfrac{\sin 120^{\circ}}{12\sqrt{2}}=\dfrac{\sin 30^{\circ}}{OA}\Rightarrow OA=...$ OR $\left(12\sqrt{2}\right)^2=x^2+x^2-2\times x\times x\cos 120\Rightarrow x=...$	M1
	For the correct value of x	A1 cso

	$x=\sqrt{96}=\left(4\sqrt{6}\right)$ *	[2]
(c)	For using trigonometry to find angle $\angle AOB$ $\cos\angle AOB=\dfrac{96+96-12^2}{2\times\sqrt{96}\times\sqrt{96}}=...$	M1
	$\angle AOB=75.522...^{o}$	A1
	Area of triangle *OAB* Area = $\dfrac{1}{2}\times\sqrt{96}\times\sqrt{96}\times\sin'75.522^{o}'=\left(46.4758...\right)$	M1
	Total area of the pyramid = $4\times46.4758+12^2=\left(329.90....\right)$	M1
	For the correct final area = awrt 330 (cm^2)	A1 [5]
	ALT	
	Height of the triangle of one of the triangular faces: $h=\sqrt{\left(4\sqrt{6}\right)^2-6^2}=...$	M1
	$h=2\sqrt{15}$	A1
	Area of triangle *OAB* = $\dfrac{1}{2}\times2\sqrt{15}\times12=12\sqrt{15}=\left(46.4758...\right)$	M1
	Total area of the pyramid = $4\times12\sqrt{15}+144=\left(329.903...\right)$	M1
	For the correct final area = awrt 330 (cm^2) Allow an exact answer of $48\sqrt{15}+144\quad\left(\text{cm}\right)^2$ oe	A1
(d)	$\cos\angle AOB=\dfrac{96+96-12^2}{2\times\sqrt{96}\times\sqrt{96}}=75.522^{o}$ or $\angle OBC=\dfrac{180^{o}-75.522^{o}}{2}=52.239^{o}$	
	Let *Y* on *OB* be the foot of the perpendicular from *A* to *OB* Length *AY* = $12\sin52.239^{o}=\left(9.4868...\right)$ OR Length *AY* = $4\sqrt{6}\sin75.522^{o}=\left(9.4868...\right)$	M1A1 [M1A1]
	For the appropriate trigonometry on triangle *AYC* to find angle *AYC* $\cos\angle AYC=\dfrac{'9.4868'^2+'9.4868'^2-288}{2\times'9.4868'\times'9.4868'}\Rightarrow\left(\angle AYC=126.87\right)$	M1
	Angle between plane *AOB* and plane *OBC* =awrt 127[°]	A1 [4]
		Total 13 marks

Question	Scheme	Marks
10(a)	$\cos(A-B)=\cos A\cos B+\sin A\sin B$ $\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos(A-B)-\cos(A+B)=\sin A\sin B-(-\sin A\sin B)=2\sin A\sin B$ *	M1A1cso [2]
(b)	$[A-B=5\theta,\ \ A+B=9\theta \Rightarrow A=7\theta,\ B=2\theta]$ $\cos 5\theta-\cos 9\theta=2\sin 7\theta\sin 2\theta$ *	B1 cso [1]
(c)	$\sqrt{3}\sin 7\theta=2\sin 7\theta\sin 2\theta \Rightarrow 0=2\sin 7\theta\sin 2\theta-\sqrt{3}\sin 7\theta$ $0=\sin 7\theta\left(2\sin 2\theta-\sqrt{3}\right)\Rightarrow \sin 7\theta=0,\ 2\sin 2\theta-\sqrt{3}=0$ $\sin 7\theta=0\Rightarrow 7\theta=0,\ \pi,\ 2\pi\Rightarrow\theta=\frac{\pi}{7},\ \frac{2\pi}{7}$ $2\sin 2\theta-\sqrt{3}=0\Rightarrow\sin 2\theta=\frac{\sqrt{3}}{2}\Rightarrow 2\theta=\frac{\pi}{3},\ \frac{2\pi}{3}\Rightarrow\theta=\frac{\pi}{6},\ \frac{\pi}{3}$	M1A1 M1 M1A1 M1A1 [7]
(d)	$\tan 2x=\frac{\sin 2x}{\cos 2x}\Rightarrow\tan 2x\cos 2x=\sin 2x$ $\int_0^{\frac{\pi}{7}}8\sin 7x\sin 2x\ \mathrm{d}x=\left[\int_0^{\frac{\pi}{7}}4\times(2\sin 7x\sin 2x)\ \mathrm{d}x\right]$ $\int_0^{\frac{\pi}{7}}4(\cos 5x-\cos 9x)\ \mathrm{d}x=4\left[\frac{\sin 5x}{5}-\frac{\sin 9x}{9}\right]_0^{\frac{\pi}{7}}$ $4\left[\frac{\sin 5x}{5}-\frac{\sin 9x}{9}\right]_0^{\frac{\pi}{7}}=4\left[\left(\frac{\sin 5\times\frac{\pi}{7}}{5}-\frac{\sin 9\times\frac{\pi}{7}}{9}\right)-\left(\frac{\sin 0}{5}-\frac{\sin 0}{9}\right)\right]$# $=0.9729...\approx 0.973$	M1 M1 M1M1 M1A1 [6]
		Total 16 marks

Question	Notes	Marks
10(a)	From the formula sheet $\cos(A-B)=\cos A\cos B+\sin A\sin B$ $\cos(A+B)=\cos A\cos B-\sin A\sin B$ Subtracts the two equations to give: $\cos(A-B)-\cos(A+B)=\sin A\sin B-(-\sin A\sin B)$	M1
	For the correct identity as shown with no errors, $\cos(A-B)-\cos(A+B)=2\sin A\sin B$ *	A1 cso [2]
(b)	For finding the value of A and the value of B $A-B=5\theta,\quad A+B=9\theta$ $\Rightarrow A=7\theta,\ B=2\theta$ Or as a minimum: $\cos(7\theta-2\theta)-\cos(7\theta+2\theta)=2\sin 7\theta\sin 2\theta$ *	B1cso [1]
(c)	Sets $\sqrt{3}\sin 7\theta=2\sin 7\theta\sin 2\theta$	M1
	Achieves the correct equation allow the terms in any order. $0=2\sin 7\theta\sin 2\theta-\sqrt{3}\sin 7\theta$	A1
	Factorises their equation $0=\sin 7\theta\left(2\sin 2\theta-\sqrt{3}\right)\Rightarrow \sin 7\theta=0,\ 2\sin 2\theta-\sqrt{3}=0$	M1
	For finding at least **one** correct value for θ using $\sin 7\theta=0$ $\sin 7\theta=0\Rightarrow 7\theta=0,\ \pi,\ 2\pi\Rightarrow\theta=\frac{\pi}{7},\ \frac{2\pi}{7}$	M1
	For both correct values $\theta=\frac{\pi}{7},\ \frac{2\pi}{7}$	A1
	For finding **one** correct value of θ using $2\sin 2\theta-\sqrt{3}=0$ $2\sin 2\theta-\sqrt{3}=0\Rightarrow\sin 2\theta=\frac{\sqrt{3}}{2}\Rightarrow 2\theta=\frac{\pi}{3},\ \frac{2\pi}{3}\Rightarrow\theta=\frac{\pi}{6},\ \frac{\pi}{3}$	M1
	For both correct values $\theta=\frac{\pi}{6},\ \frac{\pi}{3}$	A1 [7]
(d)	Uses the identity for $\tan 2x=\frac{\sin 2x}{\cos 2x}\Rightarrow\tan 2x\cos 2x=\sin 2x$	M1
	Substitutes the above into $8\sin 7x\cos 2x\tan 2x$ to give $\int_0^{\frac{\pi}{7}}8\sin 7x\sin 2x\ \mathrm{d}x=\left[\int_0^{\frac{\pi}{7}}4\times(2\sin 7x\sin 2x)\ \mathrm{d}x\right]$ Ignore integral sign and limits for this mark.	M1
	For substituting $\cos 5x-\cos 9x$ for $2\sin 7x\sin 2x$ to give $\int_0^{\frac{\pi}{7}}4(\cos 5x-\cos 9x)\ \mathrm{d}x$ Ignore integral sign and limits for this mark.	M1

	Integrates $\int_0^{\frac{\pi}{7}} 4(\cos 5x - \cos 9x)\ dx = 4\left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9}\right]_0^{\frac{\pi}{7}}$ Ignore limits for this mark. As a minimum they must obtain: $\left(\pm\frac{\sin 5x}{5} \pm \frac{\sin 9x}{9}\right)$ for the integration.	M1
	Substitutes the limits the correct way around $4\left[\frac{\sin 5x}{5} - \frac{\sin 9x}{9}\right]_0^{\frac{\pi}{7}} = 4\left[\left(\frac{\sin 5\times\frac{\pi}{7}}{5} - \frac{\sin 9\times\frac{\pi}{7}}{9}\right) - \left(\frac{\sin 0}{5} - \frac{\sin 0}{9}\right)\right]$ $= (0.9729...)$	M1
	For the correct value of $\int_0^{\frac{\pi}{7}} 8\sin 7x\cos 2x\tan 2x\ dx = 0.973$	A1 [6]
		Total 16 marks

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