Please check the examination details be	low before ente	ering your candidate information
Candidate surname		Other names
Centre Number Candidate N Pearson Edexcel Inter		al GCSE
Time 2 hours	Paper reference	4PM1/01
Further Pure Mat	hema	tics
Calculators may be used.		Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- You must NOT write anything on the formulae page.
 Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶







International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to *n* terms, $S_n = \frac{n}{2} [2a + (n-1)d]$

Geometric series

Sum to *n* terms,
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity, $S_{\infty} = \frac{a}{1-r} |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

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$$a_{10} + a_{11} + a_{12} = 129$$
 and $a_{19} + a_{20} + a_{21} = 237$

Find a_1		
		(4)

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(Total for Question 1 is 4 marks)



2	The point A has coordinates $(-5, 3)$, the point B has coordinates $(4, 0)$ and the point C has coordinates $(-1, 5)$.	
	The line l passes through C and is perpendicular to AB .	
	(a) Find an equation of l . Give your answer in the form $ax + by + c = 0$ where a , b and c are integers.	(4)
	The line l intersects AB at the point D .	
	(b) Show that the coordinates of D are $(-2, 2)$.	(0)
		(3)
	(c) Show that l is not the perpendicular bisector of AB .	(2)
	(d) Find the value of $\tan \angle ABC$.	
	Give your answer in its simplest form.	(4)





Question 2 continued





3 Curve C has equation $y = \frac{ax+3}{1-2x}$ where $x \neq \frac{1}{2}$ and a is a constant.

The asymptote to C that is parallel to the x-axis has equation y = 4

(a) Find the value of a

(2)

(b) Write down the equation of the asymptote to C that is parallel to the y-axis.

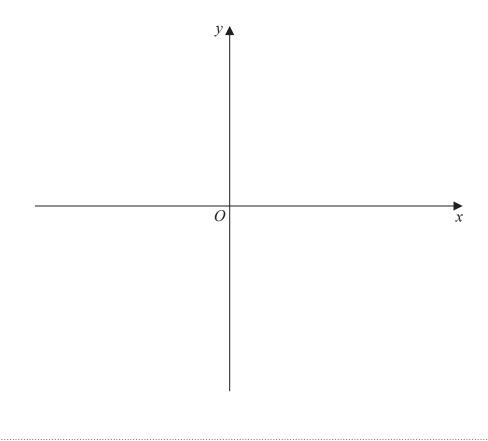
(1)

- (c) Find the coordinates of the point where C crosses
 - (i) the x-axis,
- (ii) the y-axis.

(2)

(d) Using the axes below, sketch C, showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(4)





4	$f(x) = x^3 + px^2 + qx + 6$ where p and q are constants.	
	Given that $(x - 1)$ is a factor of $f(x)$ and that when $f(x)$ is divided by $(x + 1)$ the remainder is 8	
	(a) (i) show that $p = -2$	
	(ii) find the value of q	(6)
	(b) Hence, solve the equation $f(x) = 0$	(3)



5 Given that k is a non-zero constant

curve C has equation
$$kx^2 - xy + (k+1)x = 1$$

straight line *l* has equation
$$y = \frac{k}{2}x + 1$$

The point A is the only point that lies on both C and l.

(a) Find the value of k

(6)

(b) Hence, find the coordinates of A.

(2)



- Given that $(8+3x)^{\overline{3}}$ can be expressed in the form $p(1+qx)^{\overline{3}}$ where p and q are constants.
 - (a) find the value of p and the value of q

(2)

(b) Hence, expand $(8 + 3x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2 , expressing each coefficient as an exact fraction in its lowest terms.

(3)

Using the expansion found in part (b) with a suitable value of x

(c) show that $\sqrt[3]{9} \approx \frac{599}{288}$

(2)



7 (a) Complete the table of values for

$$y = 0.5^{\left(\frac{x}{3}+1\right)} + 2$$

giving each value to 2 decimal places where appropriate.

x	-6	-5	-4	-3	-2	-1	0
y	4	3.59	3.26				2.5

(2)

(b) On the grid opposite, draw the graph of $y = 0.5^{\left(\frac{x}{3}+1\right)} + 2$ for $-6 \le x \le 0$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$\log_2(2x+2)^3 + x + 3 = 0$$
 in the interval $-6 \le x \le 0$

(6)

Question 7 continued 3 2 Turn over for a spare grid if you need to redraw your graph.



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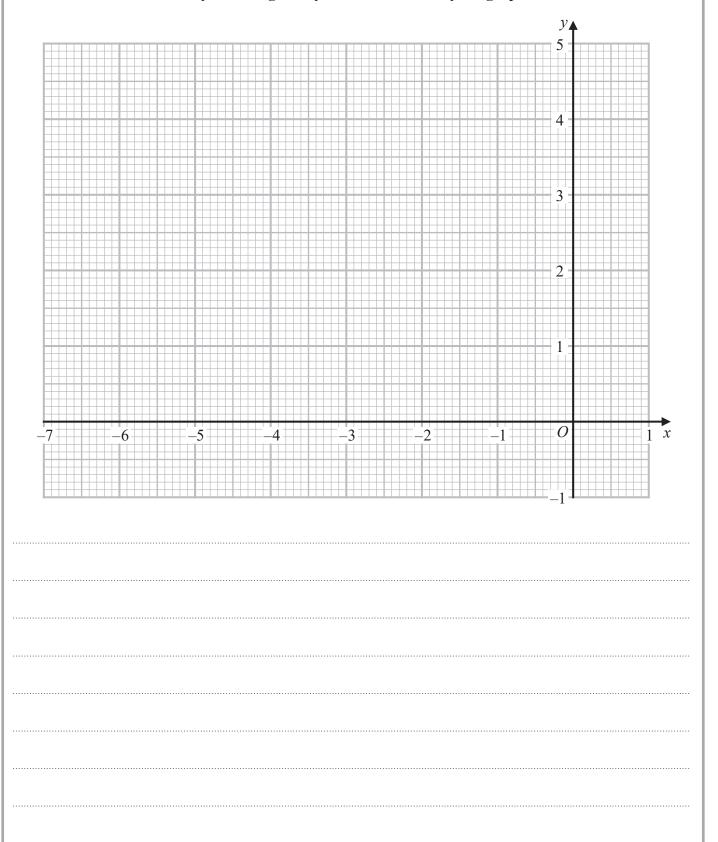
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Question 7 continued	



Question 7 continued

Only use this grid if you need to redraw your graph.



(Total for Question 7 is 10 marks)



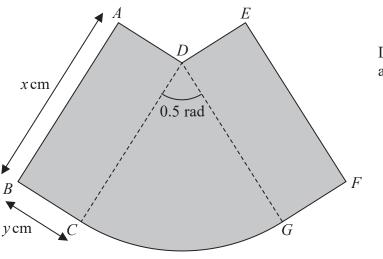


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a badge, shown shaded, made from two identical rectangles, ABCD and DEFG, and a sector DCG of a circle with centre D.

Each rectangle measures x cm by y cm.

The radius of the sector is x cm and the angle CDG is 0.5 radians.

The area of the badge is 50 cm²

The perimeter of the badge is Pcm.

(a) Show that

$$P = 2x + \frac{100}{x} \tag{5}$$

Given that x can vary,

(b) use calculus, to find the exact value of x for which P is a minimum. Justify that this value of x gives a minimum value for P

(6)

(c) Find the minimum value of P Give your answer in the form $k\sqrt{2}$, where k is an integer to be found.

(2)



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Question 8 continued	





9	Giving each value in your solution to 2 decimal places, solve the simultaneous equations						
	$e^{2y} - x + 2 = 0$						
	$\ln(x+3) - 2y - 1 = 0$						
		(8)					



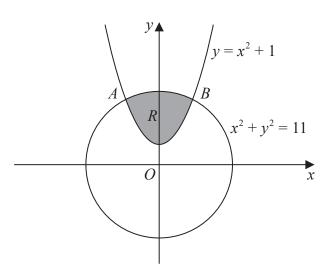


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Figure 2

The region R, shown shaded in Figure 2, is bounded by the curve with equation  $y = x^2 + 1$  and the curve with equation  $x^2 + y^2 = 11$ 

The two curves intersect at the point A and at the point B.

(a) Find the x coordinate of the point A and the x coordinate of the point B.

**(4)** 

The region R is rotated through 360° about the x-axis.

(b) Use algebraic integration to find the volume, to 2 decimal places, of the solid generated.

**(5)** 




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Question 10 continued



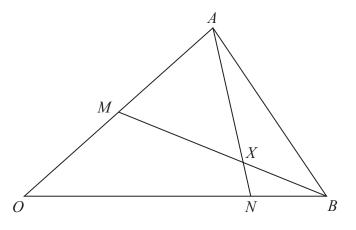


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows triangle  $\overrightarrow{OAB}$  with  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ 

M is the midpoint of OA.

N is the point on OB such that ON: NB = 3:1

The lines AN and BM intersect at the point X.

- (a) Find expressions, in terms of a and b, for
  - (i)  $\overrightarrow{AN}$
- (ii)  $\overrightarrow{BM}$

(3)

(b) Using a vector method, find AX:XN

(7)




Question 11 continued	
	(Total for Question 11 is 10 marks)
	TOTAL FOR PAPER IS 100 MARKS

