Pearson Edexcel

International GCSE in Further Pure Mathematics

(4PM1)

Two-year Scheme of Work

For first teaching from September 2017

Contents

Introduction

This scheme of work is based on a five-term model over two years. It is assumed that the sixth term will be dedicated to revision.

It can be used directly as a Scheme of Work for the International GCSE Further Pure Mathematics (4PM1).

The Scheme of Work is broken up into units, so that there is greater flexibility for moving topics around to meet planning needs.

Each unit contains:

- Contents, referenced back to the specification
- Prior knowledge
- Keywords

Each sub-unit contains:

- Recommended teaching time, although this is adaptable to individual teaching needs
- Objectives for students at the end of the sub-unit
- Possible success criteria for students at the end of the sub-unit
- Opportunities for reasoning/problem solving
- Common misconceptions
- Notes for general mathematical teaching points

Teachers should be aware that the estimated teaching hours are approximate and should be used a guideline only. This scheme of work is based on 45 minute teaching lessons.

Using this scheme of work

The units in this scheme are arranged by content area, and therefore do not provide in themselves an order for how the units could be delivered. Teachers will have their own preferences for how they order the content, and the scheme of work is provided as an editable Word document to enable easy reordering of the units.

No suggestion is made here for a possible order although some topics rely heavily on a solid knowledge of others so it is very useful to complete Units 1 – 6 and Unit 9 before others are tackled.

Unit	Number	Title	Estimated teaching hours
Logarithmic functions and indices	1	Indices	2
	2	Surds	2
	3	Use and properties of logarithms	7
	4	The functions a^x and $\log_b x$	2
The Quadratic equation	5	The manipulation of quadratic expressions and the solution of quadratic equations	4
	6	The roots of a quadratic equation	3
	7	Functions of the roots of a quadratic equation	6
Identities and inequalities	8	Factor and Remainder theorem	6
	9	Simultaneous equations	4
	10	Linear and quadratic inequalities	4
	11	Graphical representation of linear inequalities	2
Graphs and functions	12	Graphs of polynomials and rational functions	7
	13	The solution of equations by graphical methods	5
Series	14	Arithmetic series	6
	15	Geometric Series	6
Binomial series	16	Binomial Series	6
Vectors	17	Vectors	8
Rectangular Cartesian Coordinates	18	Rectangular Cartesian coordinates I	4
	19	The straight line and its equation	4
Calculus	20	Differentiation	7
	21	Applications of differentiation	4
	22	Integration	6
	23	Application of calculus to kinematics	4
	24	Application of calculus to rates of change	4
Trigonometry	25	Radian Measure	4
	26	The three basic trigonometrical ratios	4
	27	Applications of trigonometry in 2 and 3 dimensions	6
	28	Trigonometrical identities	8
		Total	135

1. Logarithmic functions and indices: Units 1 - 4

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**Logarithmic Functions and Indices**		**Est Teaching hours**
		Spec		
1	Indices	**1B**	Rules of indices e.g. simplify expressions by using rules of indices	2
2	Surds	**1C**	Simple manipulation of surds e.g. manipulate surds using the two rules $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$ and	2
		1D	Rationalising the denominator	
3	Use and properties of logarithms	**1B**	Use and properties of logarithms including change of base To include: $\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^k = k \log_a x$ $\log_a a = 1$ $\log_a 1 = 0$ The solution of equations of the form $a^x = b$ Students may use the change of base formulae:	5 – 7

			$\log_a x = \dfrac{\log_b x}{\log_b a}$ $\log_a b = \dfrac{1}{\log_b a}$	
4	The functions a^x and $\log_b x$	**1A**	A knowledge of the shape of the graphs of a^x and $\log_b x$ is expected	2

2. The Quadratic Function: Units 5 – 7

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**The Quadratic Function**		**Est Teaching hours**
		Spec		
5	The manipulation of quadratic equations and solution of quadratic equations	**2A**	Students should be able to factorise quadratic expressions and solve quadratic equations by factorisation.	3 – 4
			Students should be able to complete the square on a quadratic expression and solve a quadratic equation by completing the square.	
			Students should be able to solve a quadratic equation by using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
6	The roots of a quadratic equation	**2B**	Students should be able to use the discriminant to identify whether the roots are equal real, unequal real or not real.	2 – 3
7	Functions of the roots of a quadratic equation	**2C**	Simple examples involving the roots of a quadratic equation e.g. The roots of the equation $3x^2 + x - 6 = 0$ are α and β.	5 - 6

3. Identities and Inequalities: Units 8 – 11

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**Identities and Inequalities**		**Est Teaching hours**
		Spec		
8	Factor and Remainder Theorem	**3A**	Simple algebraic division	5 – 6
		3B	The factor and remainder theorems If $\mathrm{f}(x)=0$ when x = a, then $(x-a)$ is a factor of $\mathrm{f}(x)$. e.g. Factorise a cubic such as x^3+3x^2-4 when a factor has been provided. e.g. Be familiar with the terms 'quotient' and 'remainder', and be able to determine the remainder when the polynomial $\mathrm{f}(x)$ is divided by $(ax\pm b)$.	
9	Simultaneous Equations	**3C**	Solutions of simultaneous equations to include; e.g. two linear equations in two variables e.g. one linear and one quadratic equation in two variables	3 – 4
10	Linear and Quadratic inequalities	**3D**	Solving linear and quadratic inequalities.	3 – 4

			e.g. $ax+b>cx+d$ or $px^2+qx+r<sx^2+tx+u$	
11	Graphical representation of linear inequalities	**3E**	Graphical representation of linear inequalities in two variables. Simple problems on linear programming may be set	2

4. Graphs: Units 12 - 13

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**Graphs**		**Est Teaching hours**
		Spec		
12	Graphs of polynomials and rational functions	**4A**	Sketch graphs of the form e.g. $y=ax^2+bx+c$ and $y=dx^3+ex^2+fx+g$	6 – 7
			Sketch graphs of the form e.g. $y=\sin(ax)+b$ and $y=ae^{bx}+c$	
			Sketch graphs of rational functions with linear denominators e.g. $y=\dfrac{ax+b}{cx+d}$ and be able to identify equations of asymptotes	
13	The solution of equations by graphical methods	**4B**	The solution of equations and transcendental equations by graphical methods.	4 – 5

5. Series: Units 14 – 15

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		Series		Est Teaching hours
		Spec		
14	Use of the Σ notation	**5A**	The Σ notation may be employed wherever its use seems desirable	5 – 6
	Arithmetic Series	**5B**	Knowledge of the general term of an arithmetic series is required.	
			Knowledge of the sum to *n* terms of an arithmetic series is required	
15	Use of the Σ notation	**5A**	The Σ notation may be employed wherever its use seems desirable	5 – 6
	Geometric Series	**5B**	Knowledge of the general term of a geometric series is required.	
			Knowledge of the sum to *n* terms of a finite geometric series is required	
			Use of the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$ is required.	

6. Binomial Series: Unit 16

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**Binomial series**		**Est Teaching hours**
		Spec		
16	Use of the binomial series $(1+x)^n$	**6A**	Use of the series when:	5 – 6
			(i) *n* is a positive integer	
			(ii) *n* is rational and $\|x\| < 1$	
			The validity condition for (ii) is expected	

7. Scalar and Vector quantities: Unit 17

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**Vectors**		**Est Teaching hours**
		Spec		
17	Vectors	**7A**	The addition and subtraction of coplanar vectors and the multiplication of a vector by a scalar.	7 – 8
		7B	Components and resolved parts of a vector.	
		7C	Magnitude of a vector	
		7D	Position vector	
		7E	Unit vector	
		7F	Use of vectors to establish simple properties of geometrical figures	

8. Rectangular Cartesian co-ordinates: Units 18 and 19

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		Rectangular Cartesian co-ordinates		Est Teaching hours
		Spec		
18	Rectangular Cartesian coordinates I	**8A**	The distance *d* between two points (x_1, y_1) and (x_2, y_2), where $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	3 - 4
		8B	The coordinates of the point dividing the line joining (x_1, y_1) and (x_2, y_2) in the ration $m : n$ given by $\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$	

		8C	Gradient *m* of a line joining two points (x_1, y_1) and (x_2, y_2) $m = \frac{y_1 - y_2}{x_1 - x_2}$	
19	The straight line and its equation	**8D**	The straight line and its equation. The $y = mx + c$ and $y - y_1 = m(x - x_1)$ forms of the equation of a straight line are expected to be known. The interpretation of $ax + by + c = 0$ as a straight line is expected to be known.	3 – 4
		8E	The condition for two lines to be parallel or perpendicular is expected to be known. Where $m_1 \times m_2 = -1$	

9. Calculus: Units 20 - 24

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**Calculus**		**Est Teaching hours**
		Spec		
20	Differentiation	**9A**	Differentiation of sums of multiples of powers of $x, \sin ax, \cos ax$ and e^{ax}	6 – 7
		9B	Differentiation of a product, quotient and simple cases of a function of a function	
21	Applications of differentiation	**9D**	Stationary points and turning points	3 – 4
		9E	Maxima and minima problems which may be set in the context of a practical problem.	

			Justification of maxima and minima will be expected.	
		9F	The equations of tangents and normals to the curve $\mathrm{f}(x)$ which may be any function that students are expected to differentiate.	
22	Integration	**9A**	Integration of sums of multiple powers of x (excluding $\frac{1}{x}$), $\sin(ax), \cos(ax)$ and $\mathrm{e}^{(ax)}$.	5 – 6
		9C	Determination of areas and volumes	
23	Application of calculus to kinematics	**9C**	Applications to simple kinematics. Understand how displacement, velocity and acceleration are relates using calculus.	3 – 4
24	Application of calculus to connected rates of change	**9G**	Application of calculus to rates of change and connected rates of change. The emphasis will be on simple examples to test principles. A knowledge of $\mathrm{d}y \approx \frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x$ for small $\mathrm{d}x$ is expected	3 – 4

10. Trigonometry: Units 25 – 29

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		**Trigonometry**		**Est Teaching hours**
		Spec		
25	Radian Measure	**10A**	Radian measure, including use for arc length *l*, and area of the sector of a circle, *A*. The formulae $l = r\theta$ and $A = \frac{1}{2}r^2\theta$ are expected to be known.	4

26	The three basic trigonometrical ratios	**10B**	To include the exact values for sine, cosine and tangent of 30°, 45°, 60° (and the radian equivalents), and the use of these to find the trigonometric ratios of related values such as 120°, 300°	3 - 4
27	Applications of trigonometry in 2 and 3 dimensions and use of Cosine and sine formulae	**10C**	Applications to simple problems in two or three dimensions (including angles between a line and a plane and between two planes)	3 – 4
		10D	The cosine formula will be given but other formulae are expected to be known. The area of a triangle in the form: $\frac{1}{2}ab\sin C$ is expected to be known.	
28	Trigonometrical identities	**10E**	The identity $\cos^2\theta+\sin^2\theta=1$ is expected to be known.	8 - 10
		10F	Use of the identity $\tan\theta=\frac{\sin\theta}{\cos\theta}$ (which will be given on the formula sheet)	
		10G	The use of the basic formulae of addition of trigonometry Questions using the formulae $\sin(A\pm B)$, $\cos(A\pm B)$, $\tan(A\pm B)$ may be set; the formulae will be on the formula sheet.	
		10H	Solution of simple trigonometrical identities in a given interval.	

1. Indices Teaching time 2 hours

OBJECTIVES

1B	Rules of Indices e.g. simplify expressions by using rules of indices

POSSIBLE SUCCESS CRITERIA

Simplify $9x^2\times\left(3x^2\right)^{-3}$

Evaluate $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Links with other areas of mathematics, such as calculus.

Evaluate statements and justify which answer is correct by providing a counterargument by way of a correct solution.

COMMON MISCONCEPTIONS

Simplifying expressions such as $\left(2a^2\right)^3$ to give $2a^6$

$\frac{1}{2x^4}$ is often written as $2x^{-4}$

NOTES

Indices are used extensively in many areas of mathematics, and fluency and expertise in manipulating expressions with indices is essential.

EXAMPLE QUESTIONS FROM SAMs: (none)

2. Surds	**Teaching time** 2 hours

OBJECTIVES

1C	Simple manipulation of surds e.g. manipulate surds using the two rules $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
1D	Rationalising the denominator

POSSIBLE SUCCESS CRITERIA

Simplify: $\sqrt{12}+3\sqrt{48}+\sqrt{75}$

Rationalise: $\dfrac{5}{\sqrt{3}-\sqrt{2}}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Links with other areas of mathematics, where leaving answers in an exact form is required.

COMMON MISCONCEPTIONS

Some students add surds incorrectly. For example: $\sqrt{32}+\sqrt{18}=\sqrt{50}$

NOTES

Questions in this specification will ask for answers to a question to be left in exact form. Students should interpret this demand as leaving their answers in surd form.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 8 (d)

3. Use and properties of logarithms	**Teaching time** 5 – 7 hours

OBJECTIVES

1B	Use and properties of logarithms including change of base To include:

	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^k = k\log_a x$ $\log_a a = 1$ $\log_a 1 = 0$ The solution of equations of the form: $a^x = b$ Students may use the change of base formulae: $\log_a x = \frac{\log_b x}{\log_b a}$ $\log_a b = \frac{1}{\log_b a}$

POSSIBLE SUCCESS CRITERIA

For example;

- Find the value of $\log_5 525$
- Write as a single logarithm $3\log_3 7 - \log_3 2 + 2\log_3 8$
- Solve the equation $\log_3 x + 5\log_x 6 = 3$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Opportunities exist in Geometric series to find the number of terms

COMMON MISCONCEPTIONS

Some examples of erroneous work seen:

- $\log_x 60 - \log_x 6$ given as $\frac{\log_x 60}{\log_x 6}$
- $\log_x$ cancelled to simplify to an answer of 10
- Treating 'ln' as a variable. For example; $\ln(3x-4) = \ln 3x - \ln 4$

NOTES

Logarithms are used extensively in science so there are many cross-curricular opportunities.

Students need to be provided with plenty of practice to develop fluency in using logarithms.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 6

4. The functions a^x and $\log_b x$	**Teaching time** 2 hours

OBJECTIVES

1 A	A knowledge of the shape of the graphs of a^x and $\log_b x$ is expected.

POSSIBLE SUCCESS CRITERIA

On the same axis sketch the graphs of $y = 1^x$ and $y = \log_3 x$. Write down the coordinates of intersection of the point of intersection of these two graphs.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Logarithms are used extensively in science so there are many cross-curricular opportunities.

COMMON MISCONCEPTIONS

Students sometimesconfuse the shape of logarithmic and exponential functions.

5. The manipulation of quadratic expressions and solution of quadratic equations **Teaching time**
3 – 4 hours

OBJECTIVES

2A	Students should be able to factorise quadratic expressions and solve quadratic equations by factorisation.
	Students should be able to complete the square on a quadratic expression and solve a quadratic equation by completing the square.
	Students should be able to solve a quadratic equation by using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

POSSIBLE SUCCESS CRITERIA

Examples include;

Factorise $6x^2 - 11x - 10$

Solve the equation $x^2 - 5x + 18 = 2 + 3x$

Write $x^2 - 10x + 6$ in the form $(x-a)^2 + b$

Solve the equation $x^2 - 10x = 5$ leaving your answer in surd form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve quadratic equations.

COMMON MISCONCEPTIONS

Generally, students have no problems factorising quadratic expressions or completing the square on a quadratic, but all too frequently they go on to change an expression into an equation and attempt to solve it.

When solving quadratic equations, students sometimes forget to set the equation = 0 and proceed as follows;

$(x-2)(x+6) = 5$

$\Rightarrow x - 2 = 5 \text{ or } x + 6 = 5 \Rightarrow x = 7 \text{ or } -1$

NOTES

Forming quadratic expressions (and equations) comes up in many topics in this specification.

Students are expected to be able to interpret the maximum value of a function and write down the value of x for which this maximum occurs.

When a question requires the use of the quadratic formula, remind students that they should quote it before substituting in values.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Q9 (a) and (b)

6. The roots of a quadratic equation

Teaching time
2 – 3 hours

OBJECTIVES

2B	Students should be able to use the discriminant to identify whether the roots are equal real, unequal real or not real.

POSSIBLE SUCCESS CRITERIA

For example;

The equation $2x^2 + px - 5 = 0$ has unequal real roots. Find the range of values of *p*.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve quadratic equations and determine the nature of its roots.

COMMON MISCONCEPTIONS

Students sometimes confuse the conditions for real or not real roots.

NOTES

Remind students of the following conditions:

- Real equal roots $b^2 = 4ac$ (zero discriminant)
- Real unequal roots $b^2 > 4ac$ (positive discriminant)
- Not real roots $b^2 < 4ac$ (negative discriminant)

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 4

7. Functions of the roots of a quadratic equation	Teaching time 5 – 6 hours

OBJECTIVES

2C	Simple examples involving the roots of a quadratic equation.

POSSIBLE SUCCESS CRITERIA

A possible examination question could be;

The roots of the equation $3x^2 + x - 6 = 0$ are α and β.

Without solving the equation

(a) find the values of $\alpha + \beta$ and $\alpha\beta$.

(b) form a quadratic equation with roots $\frac{\alpha}{\alpha+\beta}$ and $\frac{\beta}{\alpha+\beta}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This topic is pure reasoning. Using properties of the roots of one equation and manipulating them algebraically into the required form to find another equation.

COMMON MISCONCEPTIONS

Students demonstrate a poor grasp on the expansion of brackets. For example, $(\alpha+\beta)^2 = \alpha^2 + \beta^2$ or even $(\alpha+\beta)^3 = \alpha^3 + \beta^3$ are too frequently seen.

The equation formed must be set = 0. Many students leave their answers in the form of an expression.

NOTES

This is a topic that relies on accurate and meticulous algebra, and it is important to show every step as well as working neatly and carefully.

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 9

8. Factor and Remainder Theorem	Teaching time 5 – 6 hours

OBJECTIVES

3A	Simple algebraic division
3B	The factor and remainder theorems If $\mathrm{f}(x)=0$ when x = a, then $(x-a)$ is a factor of $\mathrm{f}(x)$. e.g. Factorise a cubic such as x^3+3x^2-4 when a factor has been provided. e.g. Be familiar with the terms 'quotient' and 'remainder', and be able to determine the remainder when the polynomial $\mathrm{f}(x)$ is divided by $(ax \pm b)$.

POSSIBLE SUCCESS CRITERIA

For example;

$$\mathrm{f}(x)=x^3+x^2-4x-4$$

(a) Show that $(x-2)$ is a factor of $\mathrm{f}(x)$.

(b) Factorise $\mathrm{f}(x)$ completely.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Solve a cubic equation given one factor.

COMMON MISCONCEPTIONS

If $(x-2)$ is a factor of $\mathrm{f}(x)$, then it is necessary to test $\mathrm{f}(2)$. That is, substitute **2** (and not −2) into $\mathrm{f}(x)$, which must = 0 if it is a factor.

NOTES

Remind students that in algebraic division, they only divide successive terms by *x* or *ax*. For example, the first step in dividing $4x^3 + x^2 - 11x + 6$ by $4x - 3$ is as follows;

$$4x-3\overset{\displaystyle \qquad\qquad\quad x^2}{\overline{\left)4x^3 + x^2 - 11x + 6\right.}}$$ etc.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 4

9. Simultaneous Equations	**Teaching time** 3 – 4 hours

OBJECTIVES

3C	Solutions of simultaneous equations to include; e.g. two linear equations in two variables e.g. one linear and one quadratic equation in two variables

POSSIBLE SUCCESS CRITERIA

A typical example is;

Solve the simultaneous equations

$$x + 2y = 3$$
$$x^2 + 3xy = 10$$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up equations and to solve them.

COMMON MISCONCEPTIONS

Careless use of brackets. Sometimes students forget that brackets are not necessary or think they can manage without and forget them at the next step.

In the case of a quadratic equation and a linear equation, there will nearly always be two pairs of values of (x, y). Sometimes students do not attemptto find the values of y after they have found x or vice-versa.

NOTES

Remind students to use brackets with negative numbers, particularly when using a calculator.

Clear presentation of working out is essential.

Link with graphical representations.

EXAMPLE QUESTIONS FROM SAMs: (None)

10. Linear and quadratic inequalities — Teaching time 3 – 4 hours

OBJECTIVES

3D	Solving linear and quadratic inequalities. e.g. $ax+b>cx+d$ or $px^2+qx+r<sx^2+tx+u$

POSSIBLE SUCCESS CRITERIA

For example;

Find the set of values for which,

(a) $5x+9<x+20$

(b) $x(x+11)<3(1-x^2)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require a student to justify why certain values in a solution can be ignored.

Set up and solve problems involving linear and quadratic inequalities. For example, the set of values for which the roots of a quadratic are real.

COMMON MISCONCEPTIONS

Students sometimes state their final answer as a number quantity, and exclude the inequality or change it to = sign.

Students often make errors when multiplying or dividing through by a negative number.

Students frequently confuse a continuous set of values with a region that is not continuous, and apply incorrect notation.

We often see $-3 > x > 4$ given for what should be $x < -3$ or $x > 4$

Students also use the word **AND** incorrectly in a non-continuous region.

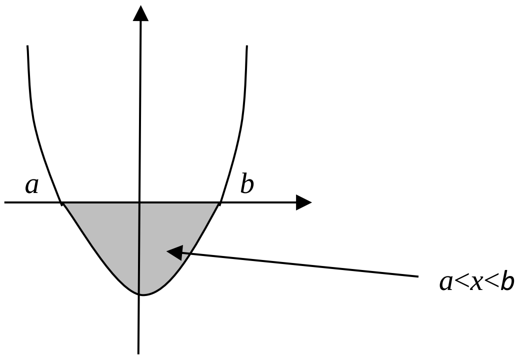
For example $x < -3$ **OR** $x > 4$ is given incorrectly as $x < -3$ **AND** $x > 4$

NOTES

Remind students to reverse the inequality if they multiply or divide by a negative number.

e.g. $-3x < 9 \Rightarrow -x < 3 \Rightarrow x > -3$

Reinforce the need to draw a sketch in a quadratic inequality. For example,



Remind students to leave their answers in the form of an inequality (and not changing to =).

Students can leave their answers in fractional or surd form, and note particularly where 'exact' values are required.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 2

11. Graphical representation of linear inequalities — Teaching time 2 hours

OBJECTIVES

3E	Graphical representation of linear inequalities in two variables Simple problems on linear programming can also be set.

POSSIBLE SUCCESS CRITERIA

Show, by shading, the region *R* defined by the inequalities

$$y \leq 3x - 4 \qquad 2x + 3y \leq 12 \qquad y \geq -2$$

For all points in *R* with coordinates (x, y) $P = 3x - 2y$

Find the greatest value of *P.*

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions in a real life context may be set.

COMMON MISCONCEPTIONS

Students can find visualisation of a question difficult.

Many students rely on the technique of finding the intersections on axes (by setting x= 0, then y = 0) The grid provided in the question does not always extend sufficiently and students at that point either extend the axes or give up.

NOTES

Remind students that substituting at least three values of *x* or *y* is also a solid technique to position a line correctly. Do not just rely on one method only.

Encourage students to set up a simple table of values for at least three points to draw a given equation of a line.

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 1

12. Graphs of polynomials and rational functions

Teaching time
6 – 7 hours

OBJECTIVES

4A	Sketch graphs of the form e.g. $y=ax^2+bx+c$ and $y=dx^3+ex^2+fx+g$
	Sketch graphs of the form e.g. $y=\sin(ax)+b$ and $y=ae^{bx}+c$
	Sketch graphs of rational functions with linear denominators e.g. $y=\frac{ax+b}{cx+d}$ and be able to identify equations of asymptotes

POSSIBLE SUCCESS CRITERIA

Examples could include;

- Sketch the graph of $y=(x-2)(2x+1)(x+5)$ showing the points where the curve crosses the coordinate axes.
- On the same axes sketch the curves given by $y=x^2(x-4)$ and $y=x(4-x)$ Find the coordinates of the points of intersection.
- Sketch the graph of $y=\cos(2x)-1$
- Sketch the graph of $y=3+\frac{2}{4-x}$ and state the equations of its asymptotes.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Solving multi-step problems in a logical manner.

COMMON MISCONCEPTIONS

Students can find the coordinates where the curve crosses the *x*-axis, but often do not find the coordinates where the curve crosses the *y*-axis.

Many students have difficulty in finding the equations of asymptotes of curves of the form $y = \frac{ax+b}{cx+d}$ or $y = a + \frac{b}{x+c}$

NOTES

Encourage students to sketch what information they are given in a question and emphasize that it is a sketch.

Careful annotation should be encouraged. It is good practice to label axes and write coordinates carefully at intersection points.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 7

13. The solution of equations by graphical methods	**Teaching time** 4 – 5 hours

OBJECTIVES

4B	The solution of equations and transcendental equations by graphical methods.

POSSIBLE SUCCESS CRITERIA

For example;

Complete the table of values of $y = 3 + 2e^{-\frac{1}{2}x}$, giving your values of *y* to 2 decimal places.

Use your graph to estimate, to 2 significant figures, the solution of the equation $e^{-\frac{1}{2}x} = \frac{1}{2}$

By drawing a suitable straight line on your graph estimate, to 2 significant figures, the solution of the equation $x = -2\ln\left(\frac{x-2}{2}\right)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Rearranging the equation to be solved into the form given in the question and finding the equation of a straight line.

COMMON MISCONCEPTIONS

Rounding is sometimescareless in these questions.

NOTES

Remind students that when the question states, 'use your graph to estimate', there must be visible evidence on the graph that it has been used.

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 7

14. Arithmetic Series

Teaching time
$5-6$ hours

OBJECTIVES

5A	Use of the Σ notation The Σ notation may be employed wherever its use seems desirable
5B	Knowledge of the general term of an arithmetic series is required. Knowledge of the sum to *n* terms of an arithmetic series is required

POSSIBLE SUCCESS CRITERIA

Possible questions could include;

- Given that $S_n = \sum_{r=1}^{r=n}(2r+1)$ show that $S_n = n(n+2)$
- Calculate $\sum_{r=1}^{r=20}(4r+1)$
- The fifth term of an arithmetic series is 24 and the sum of the first three terms of the series is 27.
  (a) Show that the first term of the series is 4 and calculate the common difference.
  (b) Given that the *n*th term of the series is greater than 300 find the least possible value of *n*.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This topic is a very rich in problem solving opportunities.

Problems will often require students to set up and solve linear simultaneous equations

There are opportunities for questions set in contextual situations. For example;

A polygon has 12 sides. The lengths of the sides starting with the smallest side, form an arithmetic series. The perimeter of the polygon is 222 cm and the length of the seventh side is ten times the length of the first side.

Find the shortest and longest sides of this polygon.

COMMON MISCONCEPTIONS

Students too frequently make errors in counting the number of terms.

Not being able to recall the correct formula for the *n*th term. The formula for the sum to *n* terms of an arithmetic series will be given to students on the formula page.

Errors in substitution into the formulae, and errors in using the formulae.

Students sometimes confuse substituting the values of *n* with the *n*th term.

NOTES

Reinforce the need to learn and practice using the only three formulae students will need to solve every arithmetic series question;

$$S_n = \frac{n}{2}\left(2a + \left(n-1\right)d\right)$$

$$S_n = \frac{n}{2}\left(a + l\right)$$

$$U_n = a + \left(n-1\right)d$$

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 8

15. Geometric Series	**Teaching time** 5 – 6 hours

OBJECTIVES

5A	Use of the Σ notation
5B	Knowledge of the general term of a geometric series is required. Knowledge of the sum to *n* terms of a finite geometric series is required
5B	Use of the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$ is required.

POSSIBLE SUCCESS CRITERIA

For example;

- Find $\sum_{r=1}^{10}\left(2\times 3^{r}\right)$
- Find the least value of *n* such that the sum of $3+9+27+81+.....$ to *n* terms would exceed 1 000 000
- Find the sum to infinity of $25+5+1+0.2+......$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This topic is a very rich in problem solving opportunities.

Problems will often require students to use logarithms to find the number of terms in a series.

There are opportunities for questions set in contextual situations. For example;

A ball is dropped from a height of 5 m. It bounces to a height of 4 m and continues to bounce. Subsequent heights to which it bounces follow a geometric sequence.

Find how high it will bounce after the fourth bounce.

COMMON MISCONCEPTIONS

A common error is not finding the value of *r* (the common ratio) correctly.

Students do not recall the formula for the *n*th term correctly.

Students sometimes do not use the correct formula when the number of terms is required to be found.

Poor use of logarithms when *n* is required to be found.

The condition for a series to sum to infinity.

NOTES

Reinforce the need to read questions correctly.

For example;

(a) Find the least value of *n* such that the sum of terms > 5000, requires the use of the summation formula and then logarithms.
(b) Find the least value of *n* such that the nth term exceeds 5000, requires the use of the *n*th term formula and then logarithms

The only three formulae students need, are as follows.

$U_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{(r-1)} = \frac{a(1-r^n)}{(1-r)}$ Either formula, but students must be consistent.

$S_\infty = \frac{a}{1-r}$

Reinforce the method of finding *r* from two terms e.g. $\frac{ar^n}{ar^{(n-1)}} = r$

A geometric series will sum to infinity if it is convergent. That is, $-1 < r < 1$ or $|r| < 1$

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 1

16. Binomial Series	Teaching time 5 – 6 hours

OBJECTIVES

6A	Use of the binomial series $(1+x)^n$ when;
	(i) *n* is a positive integer
	(ii) *n* is rational and $\lvert x \rvert < 1$
	The validity condition for (ii) is expected

POSSIBLE SUCCESS CRITERIA

For example:

- Expand $(1-2x^2)^{12}$ in ascending powers of x up to and including the term in x^4 and simplifying each term as far as possible.
  State the range of values of x for which this expansion is valid.

- Show that if x is small the expression $\sqrt{\frac{1+x}{1-x}}$ is approximated by $1+x+\frac{1}{2}x^2$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This topic brings together algebraic and numerical skills.

For example,

Expand $(1-x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^4.
Use your expansion with a suitable value for x to obtain, to 5 decimal places an approximate value for $\frac{1}{\sqrt{0.991}}$.

Opportunities for applying calculus to the binomial series expansion include, e.g. find an estimate for $\int_0^{0.5} \frac{(1+x)}{(1-x)}\,\mathrm{d}x$

COMMON MISCONCEPTIONS

Binomial expansion is particularly prone to;

- arithmetical errors
- algebraic errors – especially when using brackets
- errors in denominators – especially with using the factorial function

NOTES

Emphasize good practice by using brackets to separate terms.

For example,

$$(1-2x)^{-\frac{1}{3}} = 1 + -\frac{1}{3} \times -2x + \frac{-\frac{1}{3} \times -\frac{4}{3} \times -2x^2}{2!} + \ldots$$

This is correct but is highly error prone and may easily lead to an incorrect expansion

$$\left(1-2x\right)^{-\frac{1}{3}}=1+\left(-\frac{1}{3}\right)\left(-2x\right)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-2x\right)^{2}}{2!}+\ldots$$

This is also correct but because brackets have been used liberally will be much easier to evaluate and therefore much more likely to lead to a correctly simplified expansion.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 8

17. Scalar and Vector quantities	**Teaching time** 7 – 8 hours

OBJECTIVES

7A	The addition and subtraction of coplanar vectors and the multiplication of a vector by a scalar.
	Knowledge of the fact that if $\alpha_1\mathbf{a}+\beta_1\mathbf{b}=\alpha_2\mathbf{a}+\beta_2\mathbf{b}$ where **a** and **b** are non-parallel vectors, then $\alpha_1=\alpha_2$ and $\beta_1=\beta_2$ is expected.
7B	Use of the vectors **i** and **j** is expected.
7C	Magnitude of a vector
7D	Position vector $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\mathbf{b}-\mathbf{a}$
7E	Finding a unit vector
7F	Use of vectors to establish simple properties of geometrical figures

POSSIBLE SUCCESS CRITERIA

Add and subtract vectors algebraically and use column vectors.
Solve geometric problems and produce proofs
Find the magnitude of a vector

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

'Show that' type questions are an ideal opportunity for students to provide a clear logical chain of reasoning, providing links with other areas of mathematics, in particular algebra.

For example, find the area of a parallelogram defined by given vectors.

COMMON MISCONCEPTIONS

Students often confuse vectors with lengths when it is necessary to find the area of a polygon.

NOTES

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on their sketch. The geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors. Remind students to underline vectors and use an arrow above them, or they will be regarded as just lengths.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 3

18. Rectangular Cartesian co-ordinates I	**Teaching time** 3 - 4 hours

OBJECTIVES

8A	The distance *d* between two points (x_1, y_1) and (x_2, y_2), where $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
8B	The coordinates of the point dividing the line joining (x_1, y_1) and (x_2, y_2) in the ration $m:n$ given by $\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$
8C	Gradient *m* of a line joining two points (x_1, y_1) and (x_2, y_2)

	$m = \frac{y_1 - y_2}{x_1 - x_2}$

POSSIBLE SUCCESS CRITERIA

For example

- Find the distance *AB* where *A* is $(0,1)$ and *B* is $(6,9)$
- Find the coordinates of the point which divides*AB* in the ratio 3 : 1 where *A* is $(0,1)$ and *B* is $(6,9)$
- Find the gradient of the line segment *AB A* is $(0,1)$ and *B* is $(6,9)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use of Pythagoras theorem to find the length of a line segment.

Links to finding the equation of a straight line $y = mx + c$ including parallel lines and the perpendicular (normal) to a line.

Establishing properties of shapes using all three techniques.

COMMON MISCONCEPTIONS

Students must be consistent with the way they use coordinates.

For example; using $\frac{y_2 - y_1}{x_1 - x_2}$ or even $\frac{x_1 - x_2}{y_1 - y_2}$ to find the gradient of a line is sometimes seen in candidates work.

This also applies using Pythagoras to find the length of a line segment and to dividing a line in a given ratio which is particularly error prone.

NOTES

Students should be reminded to always show all steps in the working.

Use surds where appropriate and only give a decimal answer at the end of a question in order to maintain accuracy.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 10

19. The straight line and its equation

Teaching time
3 – 4 hours

OBJECTIVES

8D	The straight line and its equation. The $y = mx + c$ and $y - y_1 = m(x - x_1)$ forms of the equation of a straight line are expected to be known. The interpretation of $ax + by + c = 0$ as a straight line is expected to be known.

8E	The condition for two lines to be parallel or perpendicular is expected to be known. Where $m_1 \times m_2 = -1$

POSSIBLE SUCCESS CRITERIA

Examples may include;

- Work out the gradient of $10x - 2y + 5 = 0$
- Find the equation of the line that passes through the points $(0,1)$ and $(6,9)$
- Find an equation of the line which is parallel/perpendicular to $2y = 3x - 7$ and passes through the point (4, 5). Leave your answer in the form $ax + by + c = 0$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Set up equations of lines and find points of intersection using simultaneous equations.

Determine properties of shapes from equations of line.

COMMON MISCONCEPTIONS

The equation of a line must be in the form $y = mx + c$ to find the gradient of a line. So the gradient of $2y = 3x - 7$ is **NOT** 3.

Students often do not read questions carefully and confuse parallel and perpendicular lines.

NOTES

Encourage students to become fluent in using **both** $y = mx + c$ and $y - y_1 = m(x - x_1)$ to find an equation of a line.

EXAMPLE QUESTIONS FROM SAMs: Paper 2 Question 10

20. Differentiation — Teaching time 6 – 7 hours

OBJECTIVES

9A	Differentiation of sums of multiples of powers of $x, \sin ax, \cos ax$ and e^{ax}

9B	Differentiation of a product, quotient and simple cases of a function of a function

POSSIBLE SUCCESS CRITERIA

Examples may include;

- Differentiate; $3x^4 - \frac{1}{2x^2}$ $3\sin 2x$, $4e^{2x}$, $\cos\left(2x^2+3\right)$
- Differentiate; $3x^2\left(1-2x\right)^{-3}$, $\frac{2x^4}{2x+5}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Differentiation has multiple applications throughout this specification.

COMMON MISCONCEPTIONS

3 differentiates to 3 (rather than 0)

Students sometimes are not clear when to use chain, product or quotient rules.

Also whereas $\frac{\mathrm{d}\left(x^2+2x\right)}{\mathrm{d}x} = \frac{\mathrm{d}\left(x^2\right)}{\mathrm{d}x} + \frac{\mathrm{d}\left(2x\right)}{\mathrm{d}x}$, $\frac{\mathrm{d}\left(x^2.2x\right)}{\mathrm{d}x} \neq \frac{\mathrm{d}\left(x^2\right)}{\mathrm{d}x} . \frac{\mathrm{d}\left(2x\right)}{\mathrm{d}x}$

A frequent error is writing, for example, $\frac{1}{3x^2}$ as $3x^{-2}$

NOTES

Links with applications of differentiation

Remind students that in order to differentiate $\frac{1}{x^3}$ they must first change this to x^{-3}

because $y = \frac{1}{x^3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3x^2}$ is seen too often.

We sometimes see poor notation used. For example;

$$\mathrm{f}\left(x\right) = x^4 - 2x = 4x^3 - 2$$

Whilst the differentiation has been obviously performed correctly (and would be credited as such in an examination), $x^4 - 2x \neq 4x^3 - 2$.

Encourage students to use correct notation as follows;

$$f(x) = x^4 - 2x$$
$$f'(x) = 4x^3 - 2$$

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 6

21. Applications of differentiation	**Teaching time** 3 – 4 hours

OBJECTIVES

9D	Stationary points and turning points
9E	Maxima and minima problems which may be set in the context of a practical problem. Justification of maxima and minima will be expected
9F	The equations of tangents and normals to the curve $\mathrm{f}(x)$ which may be any function that students are expected to differentiate.

POSSIBLE SUCCESS CRITERIA

Examples include;

- Find the coordinates of the points where the gradient is zero on the curve $y = 4x^2 + 6x$ establishing whether this point is a maximum or minimum point.
- Find the gradient of the curve $y = x^2(3x-1)^3$ at the point
- $(1,8)$. Find the equation of the normal to the curve at this point.
- Find the maximum volume of a three dimensional shape, justifying that the value found is a maximum.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

There are many practical applications of this work. For example, finding the minimumtotalsurface area of an aluminium drink can given a specified volume.

COMMON MISCONCEPTIONS

Finding the gradient of the tangent to a curve when the normal is required, and vice versa is a very common error.

NOTES

Link with solving quadratic and linear equations.

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 5

Paper 2 Question 10

22. Integration	Teaching time 5 – 6 hours

OBJECTIVES

9A	Integration of sums of multiple powers of x (excluding $\frac{1}{x}$), $\sin ax, \cos ax$ and e^{ax}.
9C	Determination of areas and volumes

POSSIBLE SUCCESS CRITERIA

Typical questions may include;

- Find $\int\left(x^3+2x\right)\,dx$
- Evaluate $\int_3^4\left(\frac{2x^2+3}{x^2}\right)dx$
- Find the area bounded by the curve with equation $y=(4-x)(x+2)$ and the positive x- and *y*-axes.
- The region *R* is bounded by the curve with equation $y=\sin 2x,$ the *x* – axis and the lines $x=0$ and $x=\frac{\pi}{2}$. Find the volume of the solid formed when the region *R* is rotated through 2π radians about the *x* – axis.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Construct a clear chain of reasoning, showing every step in working.

Present an argument or proof.

COMMON MISCONCEPTIONS

Students forgetting to include the constant of integration when integrating an indefinite integral.

The correct formula for the volume of integration is $V=\pi\int_b^a y^2\,dx$. π is sometimes omitted.

NOTES

Encourage students to draw a sketch if one is not provided. (See Jan 2015 Paper 1 Q1)

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 11 part (b)
Paper 2 Question 9 part (d)

23. Application of calculus to kinematics

Teaching time
3 – 4 hours

OBJECTIVES

9C	Applications to simple kinematics Understand how displacement, velocity and acceleration are related using calculus

POSSIBLE SUCCESS CRITERIA

Solving a typical problem as follows;

The velocity, *v* m/s of a particle after *t* seconds is given by $v = t^2 + 10t + 5$
Find an expression for the acceleration, *a*.
The displacement $s = 0$ when $t = 0$
Find the displacement when $t = 2$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Recognising the mathematical processes required to solve a problem in kinematics using calculus.

COMMON MISCONCEPTIONS

Forgetting to include the constant of integration when for example integrating either acceleration or velocity.

Giving speed as a negative quantity.

NOTES

Remind students that;

Displacement is represented by *s* and is a measure of distance, where + or − indicates direction.

Speed is the absolute value of velocity.

It is absolutely crucial to include the constant of integration when integrating either velocity to achieve displacement, or acceleration to achieve velocity.

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 4

24. Application of calculus to connected rates of change — Teaching time 3 – 4 hours

OBJECTIVES

9G	Application of calculus to rates of change and connected rates of change The emphasis will be on simple examples to test principles. A knowledge of $\mathrm{d}y \approx \frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x$ for small $\mathrm{d}x$ is expected

POSSIBLE SUCCESS CRITERIA

The ability to use chain rule once, or several times, to connect rates of change in a question involving more than two variables.

For example:

A circle with radius r cm has area A cm^2. Find the rate of change of the area of the circle when the rate of change of the radius is 5 cm s^{-1}.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

These are problem set in real contexts and require students to be able to apply a logical set of steps using chain rule at least once.

COMMON MISCONCEPTIONS

Often, errors arise when students cannot remember standard formulae which they are required to differentiate.

Errors in differentiating the formulae for circles and cylinders. Remember that π is a constant and not a variable.

NOTES

Remind students that Chain Rule is not just a computational tool but it is a principle about rates of change. The key idea here is that average rates of change can be treated as ratios, so that for example $\frac{\Delta h}{\Delta x}.\frac{\Delta x}{\Delta t} = \frac{\Delta h}{\Delta t}$.

Dimensional analysis can be helpful to check calculations.

EXAMPLE QUESTIONS FROM SAMs: paper 2 Question 11

25. Radian measure	**Teaching time** 4 hours

OBJECTIVES

10A	Radian measure, including use for arc length, *l*, and area of the sector of a circle, *A*. The formulae $l = r\theta$ and $A = \frac{1}{2}r^2\theta$ are expected to be known.

POSSIBLE SUCCESS CRITERIA

- To be able to calculate using radians fluently.

Examples of possible questions.

- Calculate the area and perimeter of the sector of a circle given the angle of the sector in radians and the radius of the circle.
- Calculate the area given the arc length and angle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Links to other mensuration problems including using sine and cosine rules.

COMMON MISCONCEPTIONS

Students sometimes work with degrees and change back into radians. This should be avoided as even if it is carried out correctly, often results in loss of accuracy.

NOTES

Remind students that when evaluating integrals involving trigonometrical functions, radians must be used at all times.

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 2.

26. Trigonometrical ratios	Teaching time 3 – 4 hours

OBJECTIVES

10B	To include the exact values for sine, cosine and tangent of 30°, 45°, 60°(and the radian equivalents), and the use of these to find the trigonometric ratios of related values such as 120°, 300°

POSSIBLE SUCCESS CRITERIA

For example,

- Using the exact values of sine, cosine and tangent of 30°, 45°, 60° to find the exact values of, for example; cos (495°), tangent (— 135°) etc.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Links with identities and equations.

COMMON MISCONCEPTIONS

The concept of finding angles in a given range is not always understood.

Students sometimes work in degrees throughout and convert to radians at the end (usually introducing rounding errors), or even mix degrees with radians.

NOTES

Practise working in radians to achieve fluency, and know the equivalent angles in terms of π for 30°, 45° and 60°.

EXAMPLE QUESTIONS FROM SAMs: None

27. Applications of trigonometry in 2 or 3 dimensions

Teaching time
3 – 4 hours

OBJECTIVES

10C	Applications to simple problems in two or three dimensions (including angles between a line and a plane and between two planes)
10D	The cosine formula will be given but other formulae are expected to be known. The area of a triangle in the form: $\frac{1}{2}ab\sin C$ is expected to be known.

POSSIBLE SUCCESS CRITERIA

To include;

Calculate angles in two and three dimensional shapes using a combination of Pythagoras theorem and trigonometry.

Calculate the area of any triangle, and shapes constructed from triangles.

Calculate the volume of a prism.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangles that involve multiple uses of Pythagoras theorem and simple trigonometry and/or Sine and Cosine rules.

Link to 'real life' situations.

COMMON MISCONCEPTIONS

Students round answers prematurely leading to accuracy errors.

NOTES

Remind students to always draw a sketch if one is not given in the question, and in 3D shapes, sketch a carefully labelled triangle for each part of the shape.

Work in surds until the final answers to preserve accuracy or at least work to several decimal places and only round the final answer as directed.

Use Pythagoras theorem and trigonometry together.

The cosine rule is used when we have SAS and used to find the side opposite to the 'included' angle, or when we have SSS to find an angle.

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 2 and 10

28. Trigonometrical identities

Teaching time
8 – 10 hours

OBJECTIVES

10E	The identity $\cos^2\theta+\sin^2\theta=1$ is expected to be known.
10F	Use of the identity $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ (which will be given on the formula sheet)
10G	The use of the basic formulae of addition of trigonometry Questions using the formulae $\sin(A\pm B)$, $\cos(A\pm B)$, $\tan(A\pm B)$ may be set; the formulae will be on the formula sheet.
10H	Solution of simple trigonometrical equations for a given interval. Students should be able to solve equations such as; $\sin\left(x-\dfrac{\pi}{2}\right)=\dfrac{3}{4}\quad \text{for}\quad 0<x<\pi$ $6\cos^2(3x)^{\circ}+\sin(3x)^{\circ}-5=0\quad \text{for}\quad 0^{\circ}<x<90^{\circ}$

POSSIBLE SUCCESS CRITERIA

For example;

- Given that $2\sin(x+y)=3\cos(x+y)$ show that $\tan x=\dfrac{3-2\tan y}{2-3\tan y}$
- Solve, in degrees to 1 decimal place in the interval $0\le A\le 360^{\circ}$, $\cos^2 A-6\sin A=5$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step problems involving the trigonometrical identities requiring the application of quadratic equations.

Proofs such as;

Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Links to calculus, for example;

Find $\int_0^{\frac{\pi}{4}} \sin^3\theta \, d\theta$

COMMON MISCONCEPTIONS

$\sin(A+B) = \sin A + \sin B$ is seen occasionally.

Accuracy errors introduced by premature rounding.

Students sometimes work in degrees throughout and convert to radians at the end (usually introducing rounding errors), or even mix degrees with radians.

For example, it is not that uncommon to see the following;

$$\sin\left(2\theta - \frac{3\pi}{8}\right) = \frac{1}{2} \Rightarrow \left(2\theta - \frac{3\pi}{8}\right) = 30^{\text{o}} \Rightarrow \theta = \frac{30^{\text{o}} + \frac{3\pi}{8}}{2} = 15.589...$$

NOTES

<u>Proof</u>

- Remind students to show every step of working in a 'proof' to convince an examiner there is no 'fudging' of the answer.
- Start with the more complicated side.
- Only work on one side at a time.

Remind students that they must find **all** angles within a given range.

For example;

$$\sin\left(2\theta - \frac{3\pi}{8}\right) = \frac{1}{2}$$

$$\left(2\theta - \frac{3\pi}{8}\right) = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$2\theta = \frac{13\pi}{24}, \frac{29\pi}{24}, \frac{61\pi}{24}, \frac{77\pi}{24}$$

$$\theta = \frac{13\pi}{48}, \frac{29\pi}{48}, \frac{61\pi}{48}, \frac{77\pi}{48}$$

Note, there are four roots of this equation within the specified range

EXAMPLE QUESTIONS FROM SAMs: Paper 1 Question 3

Paper 2 Question 5