edexcel

# INTERNATIONAL GCSE

## Further Pure Mathematics (9-1)

### GETTING STARTED GUIDE

Pearson Edexcel International GCSE in Further Pure Mathematics (4PM1)

For first teaching September 2017
First examination June 2019

Pearson

Pearson Education Limited is one of the UK's largest awarding organisations, offering academic and vocational qualifications and testing to schools, colleges, employers and other places of learning, both in the UK and internationally. Qualifications offered include GCSE, AS and A Level, NVQ and our BTEC suite of vocational qualifications, ranging from Entry Level to BTEC Higher National Diplomas. Pearson Education Limited administers Edexcel GCE examinations.

Through initiatives such as onscreen marking and administration, Pearson is leading the way in using technology to modernise educational assessment, and to support teachers and learners.

This guide is Issue 1. We will inform centres of any changes to this issue. The latest issue can be found on the Edexcel website: www.edexcel.com/ial



ISBN 978-1-4469-4215-4

# Contents


## A Getting started for teachers 2

Introduction 2

Key Features 3

Qualification overview 4

What's new? 6

Understanding problem solving and mathematical reasoning 8

Delivery of the qualification – transferable skills 24

Course planner 26

## B Getting started for students 27

Student guide 27

# A Getting started for teachers

## Introduction

This getting started guide provides an overview of the new Pearson Edexcel International GCSE in Further Pure Mathematics qualification, to help you to get to grips with the changes to content and assessment, and to help you understand what this means for you and your students.

### Support for delivering the new specification

Our package of support to help you plan and implement the new specification includes:

**Planning** – In addition to the relevant section in this guide, we will provide a course planner and an editable scheme of work that you can adapt to suit your department.

**Teaching and learning** – To support you in delivering the new specification, we will provide suggested resource lists and suggested activities.

**Understanding the standard** – Sample assessment material will be provided.

**Tracking student progress** – Results Plus provides the most detailed analysis available of your students' exam performance. It can help identify topics and skills where students could benefit from further learning. We will also offer examWizard, which is a free exam preparation tool containing a bank of past Edexcel exam questions, mark schemes and examiner reports for a range of GCSE and GCE subjects.

**Support** – Our subject advisor service, and online community, will ensure you receive help and guidance from us as well as enabling you to share ideas with each other. You can sign up to receive e-newsletters to keep up to date with qualification updates, and product and service news. Email our subject advisor: TeachingMaths@pearson.com

## Key features of the qualification

- The content is very similar to the previous 4PM0 specification but has been updated to ensure progression to IAL and GCE A level Mathematics.
- This specification is intended for students who have a high ability in, or are motivated by, mathematics. The content and assessment approach has been designed to provide an additional Level 2 International GCSE qualification, which extends mathematical techniques beyond those covered in International GCSE/GCSE in mathematics.
- The assessment model has two, 2-hour papers with a single tier of entry at grades 9-4 with 3 allowed. It focuses on mathematical skills, techniques and concepts, and how to use them to solve problems. Both papers have some shorter questions that provide accessibility to all, and also longer in- depth questions that are useful preparation for the extended problems met at A Level. Both papers use calculators.
- Students develop problem-solving skills by translating problems in mathematical contexts and develop reasoning skills through exercises such as presenting arguments and proofs, and making deductions and drawing conclusions from mathematical information.

# A Getting started for teachers

## Qualification overview

This section provides an overview of the course to help you see what you will need to teach.

The overview gives a general summary of the examined papers.

The Pearson Edexcel International GCSE in Further Pure Mathematics comprises two externally- assessed papers.

| | |
|---|---|
| **Paper 1** | ***Component/paper code 4PM1/01** |
| **Paper 2** | ***Component/paper code 4PM1/02** |
| ■ Externally assessed<br>■ Availability in January and June<br>■ First assessment June 2019 | Each paper is 50% of the total International GCSE |
| Content summary<br>■ Number<br>■ Algebra and calculus<br>■ Geometry and trigonometry | |
| Assessment<br>■ Each paper is assessed through a two-hour examination set and marked by Pearson.<br>■ The total number of marks for each paper is 100.<br>■ Each paper will consist of around 11 questions with varying mark allocations per question, which will be stated on the paper.<br>■ Each paper will contain questions from any part of the specification content, and the solution of any questions may require knowledge of more than one section of the specification content.<br>■ The paper will have approximately 40% of the marks distributed evenly over grades 4 and 5, and approximately 60% of the marks distributed evenly over grades 6, 7, 8 and 9.<br>■ Grade 3 is allowed in this qualification.<br>■ A formula sheet will be included in the examination papers.<br>■ A calculator may be used in the examinations. | |

## Assessment Objectives and weightings

| | | % in International GCSE |
|---|---|---|
| **AO1** | Demonstrate a confident knowledge of the techniques of pure mathematics required in the specification | 30 – 40% |
| **AO2** | Apply a knowledge of mathematics to the solutions of problems for which an immediate method of solution is not available and which may involve knowledge of more than one topic in the specification | 20 – 30% |
| **AO3** | Write clear and accurate mathematical solutions | 35 – 50% |
| | **TOTAL** | **100%** |

## Relationship of Assessment Objectives to units

| **Unit number** | **Assessment Objectives** | | |
|---|---|---|---|
| | **A01** | **AO2** | **AO3** |
| Paper 1 | 15 – 20% | 10 – 15% | 17.5 – 25% |
| Paper 2 | 15 – 20% | 10 – 15% | 17.5 – 25% |
| **Total for International GCSE** | 30 – 40% | 20 – 30% | 35 – 50% |

## What's new ?

### *What's changed from 4PM0?*

The major change is that the grading will now run from grade 4 up to grade 9 (with 3 allowed) to ensure comparability with the regulated GCSE.

Teachers wanted few other changes and so we have retained the previous assessment model and content, but have enhanced one topic to enable differentiation at the top grades.

Teachers wanted us to provide a formulae sheet, so we have now included some of the formulae that students were previously required to memorise.

Changes in content are as follows:

**1. Logarithmic functions and indices**

**1D** Rationalising the denominator

Surds – we now expect candidates to be able to rationalise the denominator for expressions such as $\frac{1}{2-\sqrt{3}}$ ,as well as those with a denominator that is a pure surd.

(See SAMs Paper 2 Question 8)

---

**2. The quadratic function**

There are no changes in this section.

---

**3. Identities and inequalities**

There are no changes in this section.

---

**4. Graphs**

There are no changes in this section.

---

**5. Series**

**5B** Arithmetic and geometric series

The formula for sum to $n$ terms of an arithmetic series and of a geometric series will now be given in the formulae sheet. The formula for the sum to infinity of a converging geometric series will also be given.

## 6. The binomial series

**6A** Use of the binomial series

The formula for the binomial expansion of $(1 + x)^n$ will now be given in the formulae sheet.

## 7. Scalar and vector quantities

There are no changes in this section.

## 8. Rectangular Cartesian coordinates

There are no changes in this section.

## 9. Calculus

The formula for quotient rule will now be given in the formulae sheet. Candidates will be expected to know the formulae for product and chain rules.

## 10. Trigonometry

**10D** Use of the sine and cosine formulae

The cosine formula will be given, but other formulae are expected to be known. It is expected that candidates will know the area of the triangle in the form area = $\frac{1}{2}ab \sin C$.

**10F** Use of the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ will be now provided on the formulae sheet.

**10G** The use of the basic addition formulae of trigonometry

The formulae for sin $(A + B)$ and sin $(A - B)$, cos $(A + B)$ and cos $(A - B)$ and

tan $(A + B)$ and tan $(A - B)$ will now be given in the formulae sheet.

**10H** Solution of simple trigonometrical equations for a given interval

There is no new content here, but we have clarified the specification to explicitly include equations such as

$\sin\left(3x - 40°\right)\frac{1}{3} = \text{ for } -90° < x < 90°$

(See SAMs Paper 1 Question 3)

## Understanding problem solving and mathematical reasoning

Students need to be able to understand **problem-solving skills by translating problems** in mathematical or non-mathematical contexts into a process or a series of mathematical processes.

Students need to be able to demonstrate **reasoning skills** by:

- making deductions and drawing conclusions from mathematical information
- constructing chains of reasoning
- presenting arguments and proof
- interpreting and communicating information accurately.

Questions requiring the use of problem solving and mathematical reasoning are not new to the International GCSE specification. Papers from the previous specification (4PM0) along with papers from the other previous International GCSE specifications (4MB0 and 4MA0) and even the GCSE specifications (1MA0 linear) and 2MB0 (modular), will be a good source of both of these types of question. The new specifications (4MA1, 4MB1 and 1MA1) will also have useful questions requiring the use of mathematical reasoning and problem solving.

### Examples of questions requiring problem- solving skills from 4PM0

Many of the longer questions in Papers 1 and 2 give examples of problem solving. A few examples are given below.

### 4PM0/01 January 2016 Q2

This question requires candidates to translate a problem in a mathematical context into a series of mathematical processes.

| | |
|---|---|
| Find the set of values for which $(2x - 3)^2 > 7x - 3)$ | (5) |

| Mark | Working | Comments |
|---|---|---|
| M1 | $(2x - 3)^2 > 7x - 3$<br>$4x^2\ 12x + 9 > 7x - 3$<br>$4x^2\ 19x + 12 > 0$ | Candidates have to use their knowledge to expand the brackets and form a 3 term quadratic in readiness for factorising or solving using the formula. |
| M1 | $4x^2 - 19x + 12 > 0$<br>$(4x - 3)\ (x - 4) > 0$<br>$\Rightarrow x = \frac{3}{4},\ x = 4$ | Their equation needs factorising or solving using the quadratic formula in order to find the critical values.<br>For this mark to be awarded, candidates must reach a solution to their quadratic.<br>This mark would be awarded if they used a correct method to solve an incorrect 3 term quadratic inequality. |
| A1 | $x = \frac{3}{4},\ x = 4$ | This mark could be implied from a correctly stated final inequality. |
| M1 | $x <$ ‘$\frac{3}{4}$’, $x = 4$ or,<br>$x <$ ‘$\frac{3}{4}$’ OR $x =$ ‘4’ | This mark is for selecting the outside region for **their** critical values. The notation need not be necessarily correct for the award of this mark, but the region must be correct. |
| A1 | $x < \frac{3}{4},\ x = 4$ or,<br>$x < \frac{3}{4}$ OR $x = 4$ | The final mark is for the correct final answer and the notation must be correct.<br>An acceptable alternative using set language could be, for example, $\left(-\infty, \frac{3}{4}\right) \cup (4, \infty)$. |

**4PM0/01 June 2016 Q10 parts (a) and (b)**

This question requires candidates to translate a problem in a non-mathematical context into a series of mathematical processes. Questions involving reasoning are often flagged up by phrases such as "show that", as this example shows.

Diagram **NOT** accurately drawn

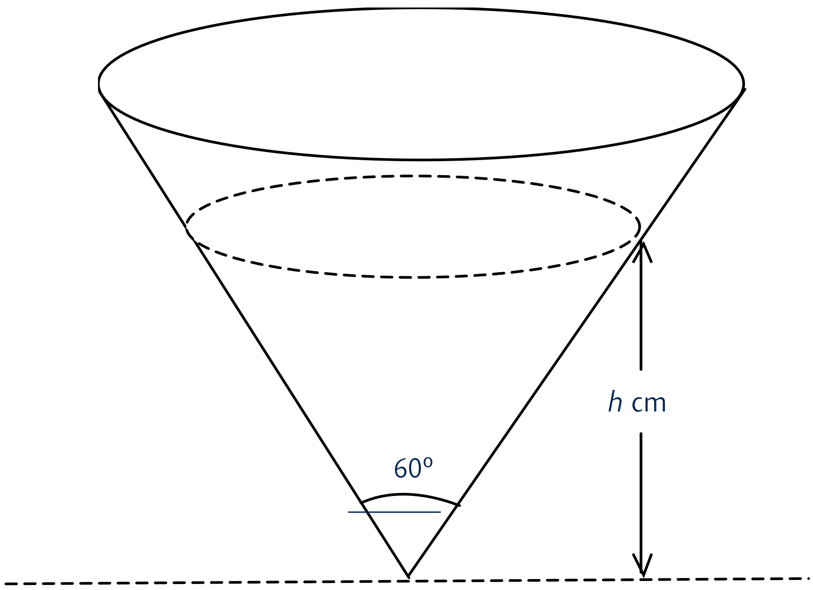



**Figure 1**

A conical container is fixed with its axis of symmetry vertical. Oil is dripping into the container at a constant rate of 0.4 $cm^3$ / s. At time $t$ seconds after the oil starts to drip into the container, the depth of the oil is $h$ cm. The vertical angle of the container is 60°, as shown in figure 1.

When $t = 0$ the container is empty.

(a) Show that $h^3 = \dfrac{18t}{5\pi}$ (4)

Given that the area of the top surface of the oil is $A$ $cm^2$

(b) show that $\dfrac{dA}{dt} = \dfrac{4}{5h}$ (5)

Find, in $cm^2$ / s to 3 significant figures, the rate of change of the area of the top surface of the oil when $t = 10$ (2)

Mark Scheme for parts (a) and (b) only

| | Mark | Working | Comments |
|---|---|---|---|
| (a) | B1 | $\frac{1}{3}\pi h^3 = 0.4t$<br>$V = \frac{1}{3}\pi h \times (h \tan 30)^3$<br>or $\left(V = \frac{1}{3}\pi h^3\right)$. | Candidates need to replace $r$ in the formula for the volume of a cone by using simple trigonometry. Candidates are expected to know and recall the formula for the volume of a cone $\left(V = \frac{1}{3}\pi r^2 h\right)$. |
| | B1 | $V = 0.4t$ | Candidates must use the information given in the question, (Oil is dripping into the container at a constant rate of 0.4 $cm^3$ / s), to write down that the volume of oil in the container is 0.4× time elapsed. |
| | M1 | $\frac{1}{3}\pi h^3 = 0.4t$ | This mark is for equating their two expressions for $V$, obtaining an equation in $t$ and $h$ only. |
| | A1 | Rearranges to $h^3 = \frac{18t}{5\pi}$ cso | cso (correct solution only) means that there must be no errors seen in any of the working given by candidates. |
| (b) | B1 | Area of top $= \frac{1}{3}\pi h^2$ | The formula for the area of a circle is $A = \pi r^2$<br>An expression in terms of $h$ is required, and is derived using simple trigonometry again where $h = r \tan 30$. |
| | M1 | $\frac{dA}{dh} = \frac{2}{3}\pi h$ | This mark is for an 'attempt' to differentiate their area of the top with respect to $h$. An attempt is defined as reducing the power of the variable by 1. |
| | M1 | Applies chain rule connecting $\frac{dA}{dt}$, $\frac{dA}{dh}$ and $\frac{dh}{dt}$ | Any equivalent correct form is acceptable for this mark.<br>For example; $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$<br>or alternatives such as even: $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dh} \times \frac{dh}{dr}$ |
| | A1 | $\frac{dh}{dt} = \frac{6}{5\pi h^2}$ | Or any equivalent expression such as:<br>$\frac{dt}{dh} = \frac{5\pi h^2}{6}$ |
| | A1 | Substitute for $\frac{dA}{dh}$ and $\frac{dh}{dt}$ in the chain rule to obtain the **given** expression for $\frac{dA}{dt}$ | No errors must be seen anywhere in the working for the award of this final mark. |

**4PM0/02 January 2016 Q 9 part (a) (i) and (ii)**

Sometimes, as in this example, candidates are asked to 'prove' and 'show that', requiring mathematical reasoning, together with a short written explanation.

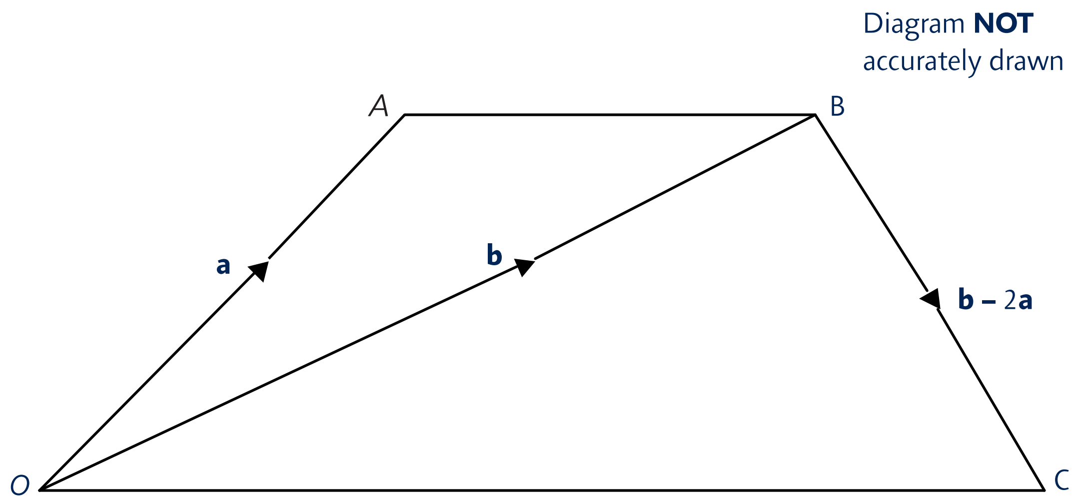



Figure 2 shows a quadrilateral *OABC*

$$\overrightarrow{OA} = \mathbf{a},\ \overrightarrow{OB} = \mathbf{b},\ \overrightarrow{BC} = \mathbf{a} - 2\mathbf{b}$$

(a) (i) Prove that $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$

(ii) Show that $AB : OC = 1 : 2$ (4)

The point $D$ lies on such that $OD : OB = 2 : 3$

(b) Find the ratio of the area of $\Delta ODC$: the area of $\Delta OAB$ (6)

Mark Scheme for part (a). Note that parts (i) and (ii) are marked together.

| | Mark | Working | Comments |
|---|---|---|---|
| (a) | B1 | $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ | Candidates begin the mathematical reasoning by finding the vector $\overrightarrow{AB}$ |
| | M1 | $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$<br>$\Rightarrow \overrightarrow{OC} = -2\mathbf{a} + 2\mathbf{b} = 2(\mathbf{a} + \mathbf{b})$<br>$\Rightarrow \overrightarrow{OC} = 2\overrightarrow{AB}$ | Candidates must show a complete method to find the vector $\overrightarrow{OC}$ and show that it is a multiple of $\overrightarrow{AB}$ . |
| | A1 | $\overrightarrow{OC} = 2\overrightarrow{AB}$ | Here a short written conclusion is required in addition to correct earlier work.<br>For example, 'hence $\overrightarrow{OC}$ is in the same direction as $\overrightarrow{AB}$ |
| | B1 | $\overrightarrow{OC} = 2\overrightarrow{AB}$ | Again, a short written explanation is required.<br>For example, '$\overrightarrow{OC}$ is twice the length of $\overrightarrow{AB}$ '. |

## Understanding problem solving

For all questions that require problem-solving skills candidates need to translate the problem into a series of mathematical processes. It will not be clear from the question what these processes are; it will be up to the candidate to interpret the question and determine the most appropriate method of solution. In many cases there will be a choice of different methods of solution.

As with any question, it is important that candidates show clear working to go with their final answer. Working is particularly important in these types of question, which are likely to attract more marks than those testing standard techniques. The majority of problem-solving questions will have a majority of method marks, with accuracy marks allocated to intermediate solutions and the final answer.

Another area that requires consideration in problem solving is the maintenance of accuracy throughout a solution. Some questions will require a series of processes in which case candidates should avoid rounding numbers prematurely (preferably using full calculator display), only rounding the final answer to the required number of significant figures or decimal places.

In order to develop good problem-solving skills, students need as much practise as possible in solving different types of problem. A good source of problems will be past examination questions on the previous specification 4PM0 and Core Mathematics A Level papers.

Students should be encouraged to work collaboratively and share their methods of solution, considering points such as whose method was the more efficient and why.

The following are all examples of questions where problem -solving skills, at varying levels, are required, taken from the SAMs papers.

### SAMs Paper 02 Q6

In this question, no structure has been given at all and candidates must determine all the necessary steps to solve the equation themselves.

In this question the bulk of the marks are method marks and are awarded for correct methods.

Solve the equation $\log_2 x + 6\log_x 2 = 7$

| Mark | Working | Comments |
|---|---|---|
| B1 | $\log_2 x = \dfrac{\log_x x}{\log_x 2} = \dfrac{1}{\log^x 2}$<br>OR<br>$\log_2 x = \dfrac{\log_x x}{\log_x 2} = \dfrac{1}{\log^x 2}$ | Candidates must realise that to start the question the base of either log must be changed. |
| M1 | $\log_2 x = \dfrac{6}{\log_2 x} = 7$ | They now substitute the changed log into the given equation. |
| M1 | $(\log_2 x)^2 - 7\log_2 x + 6 = 0$ | For this mark they must form a 3 term quadratic equation using their changed log. |
| A1 | $(\log_2 x)^2 - 7\log_2 x + 6 = 0$ | This is an intermediate accuracy mark awarded for the correct 3 term quadratic, equated to 0. |
| M1 | $(\log_2 x - 6)(\log_2 x - 1) = 0 \Rightarrow \log_2 x = \ldots$ | This method mark is awarded for an attempt to solve **their** 3 term quadratic by any method. There must be a complete method and two roots given as logs for the award of this mark.<br><br>The definition of an acceptable method is given on page 21. |
| M1 | $\log_2 x = 6$, $\log_2 x = 1 \Rightarrow x = \ldots, x = \ldots$ | For manipulating the logs to give values for $x$. |
| A1 | $x = 64$, $x = 2$ | The final accuracy mark is awarded for the correct values of $x$ only. |

**SAMs Paper 01 Q 4**

In this example, a problem in kinematics needs to be translated into a series of mathematical processes.

A particle $P$ is moving along the $x$-axis.

At time $t$ seconds ($t \geq 0$) the velocity, $v$ m/s, of $P$ is given by $v = 4t^2 - 19t + 12$

(a) Find the values of $t$ for which $P$ is instantaneously at rest. **(2)**

When $t = 0$, the displacement of $P$ from the origin is –4 m.

(b) Find the displacement of $P$ from the origin when $t = 6$ **(4)**

At time $t$ seconds the acceleration of $P$ is $a$ m/s$^2$.

(c) Find the value of $t$ when $a = 0$ **(3)**

| | Mark | Working | Comments |
|---|---|---|---|
| (a) | M1 | $v = 0$ so $4t^2 - 19t + 12 = 0$<br>$\Rightarrow (4t - 3)(t - 4) = 0 \Rightarrow t = \ldots, \ldots$ | For the award of the method mark, the method must be complete, and candidates must achieve a solution.<br>It is necessary to translate the phrase 'instantaneously at rest' into $v = 0$ and then attempt to solve the quadratic to achieve two values. There is a hint in the question that two values are required by specifically using the plural word 'values'. |
| | A1 | $t = \frac{3}{4}, 4$ | Correct values only. |
| (b) | M1 | $s = f4t^2 - 19t + 2dt$<br>$= \frac{4t^3}{3} - \frac{19t^2}{2} + 12t + c$ | Candidates now need to understand the relationship between displacement, velocity and acceleration in terms of calculus.<br>This mark is given purely for an attempt at integrating the expression for velocity.<br>The definition of an attempt is given on page 21. |
| | M1 | when $t = 0$, $s = -4 \Rightarrow c = -4$ | The mark is awarded for finding the value of the constant of integration. Note that if this mark has not been achieved, no further marks will be awarded in this part of the question. |
| | A1 | $s = \frac{4t^3}{3} - \frac{19t^2}{2} + 12t - 4$ | An intermediate accuracy mark for the correct expression for $s$. |
| | A1 | $s = \frac{4 \text{ x } 6^3}{3} - \frac{19 \text{ x } 6^2}{2} + 12 \times 6 - 4 = 14$ | The value of $t = 6$ is used in a **correct** expression for $s$, and a value of 14 (m) is achieved. |
| (c) | M1 | $a = \frac{dv}{dt} = 8t + 19$ | Knowledge that acceleration is the derivative of $v$ is required, and this method mark is awarded for an acceptable attempt to differentiate the **given** $v$. |
| | M1 | $8t - 19 = 0 \Rightarrow t = \ldots$ | For setting their $a$ (even if erroneous) = 0, and solving to find a value for $t$. |
| | A1 | $t = \frac{19}{8}$ | For the correct $t$ only. |

### SAMs Paper 01 Q 7 Part (c)

In this example, part (c) requires algebraic manipulation **and** use of the graph candidates have drawn in part (b).

(a) Complete the table of values for

$$y = 2^{\left(\frac{x}{2}+1\right)} + 1$$

giving your answers to 2 decimal places where appropriate. (2)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | 3 | | | | 9 | 12.31 |

(b) On the grid opposite, draw the graph of $y = 2^{\left(\frac{x}{2}+1\right)} + 1$ for $0 \leq x \leq 5$ (2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to 1 decimal place, of the root of the equation $\log_2 (4x - 6)^2 - x = 2$ in the interval $0 \leq x \leq 5$ (4)

Part (c) only

| Mark | Working | Comments |
|---|---|---|
| M1 | $\log^2 (4x - 6)^2 - x = 2 \Rightarrow 2\log_2$<br>$(4x - 6) = x + 2 \Rightarrow$<br>$\Rightarrow 4x - 6 = 2^{\left(\frac{x}{2}+1\right)}$ | This mark is awarded for a complete method to 'undo' the log, and apart from a possible misread of the question, all work must be correct.<br>So candidates must;<br>1. deal with the power of 2 in $\log_2 (4x - 6)^2$<br>2. leave only the log term on one side of the equation, and $(x + 2)$ on the other side<br>3. raise both sides of the equation to the power of 2. |
| M1 | $4x - 6 = 2^{\left(\frac{x}{2}+1\right)} \Rightarrow 4x - 5 = 2^{\left(\frac{x}{2}+1\right)} + 1$ | For correct algebraic manipulation of the previous log work and achieving the correct RHS of the given equation.<br>Note that aside from a possible misread of the question, all work must be correct here.<br>Note that this mark is dependent on the previous method mark. |
| M1 | $y = 4x - 5$ drawn on their graph | For drawing **their** $y = 4x - 5$ onto the curve drawn in part (b). |
| A1 | $x = 2.8$ | Correct answer only to the specified accuracy of 1 decimal place. |

## Method marks for solving a 3 term quadratic equation

### 1. Factorisation

$x^2 + bx + c = (x + p)(x + q)$ where $|pq| = |c|$

$ax^2 + bx + c = (mx + p)(nx + q)$ where $|mn| = |a|$ and $|pq| = |c|$

**NOTE:** In order for the method mark to be awarded for **attempting to solve** a quadratic, candidates must arrive at a final solution with two roots.

### 2. Formula

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for $a$, $b$ and $c$, leading to $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Errors in substitution are condoned providing the formula is quoted first. In the case of use of the formula, subject to the above criteria being met, the method mark is given automatically.

### 3. Completing the square

Solving $x^2 + bx + c = \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$ where $q \neq 0$

**NOTE:** In order for the method mark to be awarded for **attempting to solve** a quadratic, candidates must arrive at a final solution with two roots.

## Method marks for differentiation and integration

### 1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

### 2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n-1}$)

## Understanding mathematical reasoning

Questions testing candidates' mathematical reasoning skills can take on a number of different forms. We can ask candidates to reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

These types of question include those testing the ability to:

- construct a chain of reasoning, e.g. show all the steps when solving an equation
- present an argument or proof, e.g. give an algebraic proof using a series of logical steps.

When answering questions with instructions such as 'Show that' or 'Prove that', candidates must show every step in their working; failure to do so will result in the loss of marks as in these questions the answer is given. Examiners must be satisfied that sufficient working is present in a script to award marks at each stage, and will only award the final mark provided no errors have been seen in the candidate's work.

### SAMs Paper 02 Q 4

In this example, the question is structured in three parts and leads to the final demand, 'show that the equation $f(x) = 0$ has only one real root'.

$$f(x) = 2x^3 + px^2 + qx + 12 \quad p,q \in \mathbb{Z}$$

Given that $(x + 3)$ is a factor of $f(x)$ and that when $f'(x)$ is divided by $(x + 3)$ the remainder is 37

(a) show that $p = 1$ and find the value of $q$. (6)

(b) Hence factorise $f(x)$ completely. (2)

(c) Show that the equation $f(x) = 0$ has only one real root. (2)

| | Mark | Working | Comments |
|---|---|---|---|
| (a) | M1 | $f(-3) = 2 \times (-3)^2 + p \times (-3)^2 + q \times (-3) + 12 = 0$<br>$\Rightarrow 42 = 9p - 3q \Rightarrow 14 = 3p - q$<br>$f'(x) = 6x^2 + 2px + q$ | Candidates have to use their knowledge of factor theorem to substitute –3 into f($x$) equate to 0, and form an equation in $p$ and $q$. |
| | M1 | $f'(x) = 6x^2 + 2px + q$ | Candidates must understand that the notation f′($x$) requires them to differentiate f($x$) and this mark is awarded for an acceptable attempt at differentiation. The definition of an acceptable attempt in calculus is given below. |
| | M1 | $f'(-3) = 6 \times (-3)^2 + 2p \times (-3) + q = 37$<br>$\Rightarrow 6p + q = -17 \Rightarrow q = 17 - 6p$ | Candidates have to use their knowledge of factor theorem to substitute –3 into f′($x$), equate to 37 and form an equation in $p$ and $q$. |
| | A1 | $14 = 3p - q$ r equivalent equations<br>$-17 = -6p + q$ | Both equations in $p$ and $q$ correct in any form. |
| | M1 | $14 = 3p - 1(6p - 17) \Rightarrow p =$ etc.<br>OR<br>$-3 = -3p \Rightarrow p = \ldots$<br>$14 = 3 - q \Rightarrow q = \ldots$ | They then need to attempt to solve **their** two simultaneous equations by either method of their choosing (elimination or substitution). |
| | A1 | $p = 1, q = -11$ | The final A mark is for the correct values of $p$ and $q$. |
| (b) | M1 | $\frac{(2x^3 + x^2 - 11x + 12)}{(x + 3)} = 2x^2 - 5x + 4$ | Candidates must then use their values of $p$ and $q$ in f($x$) and attempt to divide f($x$) by the **given** $(x + 3)$ |
| | A1 | $(x + 3)(2x^2 - 5x + 4)$ | The final correct factorised f($x$) must be given exactly as shown for the award of this mark. |
| (c) | M1 | $b^2 - 4ac = (-5)^2 - 4 \times 2 \times 4 = -7$ | Candidates must now use the hint in the question that the only real root is –3, and test their quadratic root found in part (b) by finding the value of the discriminant. |
| | A1 | Discriminant = –7 so no real roots for the quadratic factor, hence $x = -3$ only real root. | This statement is required for the final mark. |

**SAMs Paper 01 Q 6**

In this question, no structure is given at all, although there is an obvious need to differentiate. The equation in the question is a product so requires the application of product rule. Whilst quotient rule is given in the formulae sheet, product rule is not and needs to have been memorised.

There are several ways of attempting this question though all must begin by applying product rule on $y = e^x (x^2 - 3x)$.

$$y = e^x (x^2 - 3x).$$

Show that $y - 2\frac{dy}{dx} + \frac{d^2y}{dx^2} = 2e^2$ (8)

| Mark | Working | Comments |
|---|---|---|
| M1 | $\frac{\mathrm{d}(\mathrm{e}^x)}{\mathrm{d}x} = \mathrm{e}^x$ or $\frac{\mathrm{d}(x^2 - 3x)}{\mathrm{d}x}\ 2x - 3$ | This mark is awarded for an acceptable attempt at differentiation on either $\mathrm{e}^x$ or $(x^2 - 3^x)$ |
| M1 | $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x\,(x^2 - 3x) + \mathrm{e}^x\,(2x - 3)$ | For a correct attempt at product rule.<br>This means, $\frac{\mathrm{d}(uv)}{\mathrm{d}x} = v\frac{\mathrm{d}u}{\mathrm{d}x} + u\frac{\mathrm{d}v}{\mathrm{d}x}$ using<br>**their** derivatives for $u$ and $v$. |
| A1 | $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x\,(x^2 - 3x) + \mathrm{e}^x\,(2x - 3)$ | For the correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ |
| M1 | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \ldots$ | For an attempt to reapply product rule. |
| A1 | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^x\,(x^2 - 3x) + \mathrm{e}^x\,(2x - 3) + \mathrm{e}^x\,(2x - 3) = 2\mathrm{e}^x$ | For the correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ |
| M1 | Substitution of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into the given expression. | Candidates can now use a variety of methods. Some are very efficient by noting that $\mathrm{e}^x\,(x^2 - 3x) = y$ and simplifying $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ before an attempt<br>is made to substitute into the given expression for which this mark is awarded, thereby also gaining the second M mark in one manipulation. |
| M1 | $y - a$ 'their' $\frac{\mathrm{d}y}{\mathrm{d}x}$ $+b$ 'their' $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\mathrm{e}^x$ | For an attempt at eliminating $\mathrm{e}^x$ and $(x^2 - 3x)$ from the LHS of the given expression, where $a$ and $b$ are constants. |
| A1 | $y - 2\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\mathrm{e}^x$ | Correct solution only.<br>Note there must be no errors of any kind in the solution for the award of this mark. |

## Delivery of the qualification – transferable skills

### Why transferable skills?

Ensuring students have opportunities to acquire transferable skills, as well as subject specific knowledge, understanding and skills to improve learners' progression outcomes is a central part of Pearson Edexcel's International GCSE qualifications.

In recent years, higher education institutions and employers have consistently flagged the need for students to develop a range of transferable skills to enable them to respond with confidence to the demands of undergraduate study and the world of work.

We have developed our teaching materials and support to:

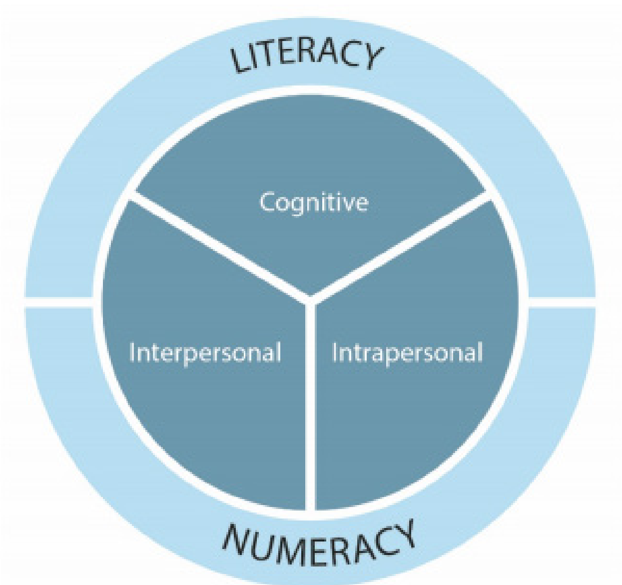
1) Increase awareness of transferable skills that are **already** being assessed (for both learners and teachers) and
2) Indicate where, for teachers, there are opportunities to teach additional skills that won't be formally assessed, but that would be of benefit to learners.

### What are transferable skills?

The Organisation for Economic Co-operation and Development (OECD) defines skills, or competencies, as 'the bundle of knowledge, attributes and capacities that can be learned and that enable individuals to successfully and consistently perform an activity or task and can be built upon and extended through learning.'[1]

To support the design of our qualifications, the Pearson Research Team selected and evaluated seven global 21st-century skills frameworks. Following on from this process, we identified the National Research Council's (NRC) framework [2] as the most evidence-based and robust skills framework, and have used this as a basis for our adapted skills framework.

The framework includes cognitive, intrapersonal skills and interpersonal skills.




[1] (OECD (2012), Better Skills, Better Jobs, Better Lives (2012):http://skills.oecd.org/documents/OECDSkillsStrategyFINALENG.pdf)

[2] Koenig, J. A. (2011) Assessing 21st Century Skills: Summary of a Workshop, National Research Council)

### What can I do if I want to see improved learner outcomes through the development of transferable skills?

For each of our International GCSE subjects we will provide a subject-specific interpretation of each of the identified skills and a comprehensive mapping as to how these elements can be developed and where they link to assessment.

The skills have been interpreted for this qualification to ensure they are appropriate for the subject. All the skills identified are evident or accessible in the teaching, learning and/or assessment of the qualification. Some skills are directly assessed.

Please refer to the 'Teaching and Learning Materials' section of the qualification webpage for more Pearson materials to support you in identifying and developing these skills in students.

## Course planner

You will find a course planner in the scheme of work document, which gives suggested teaching times for each unit.

### Suggested resources

We recognise that new resources will become available throughout the lifetime of a qualification. We will therefore supply a version of this resource list on our website, which will be updated on an ongoing basis.

| Name of resource | Link and information |
|---|---|
| Maths Emporium | This **free** website is intended for use by teachers of mathematics in secondary schools http://www.edexcelmaths.com/ |
| Sample assessment material and specimen papers | http://qualifications.pearson.com/en/qualifications/edexcel-international-gcses-and-edexcel-certificates/international-gcse-mathematics-b-2016.html |
| Dedicated Maths Subject Advisor | Teachingmaths@pearson.com |
| examWizard | examWizard is a free online resource for teachers containing a huge bank of past paper questions and support materials to help you create your own mock exams and tests. http://qualifications.pearson.com/en/support/Services/examwizard..html |
| ResultsPlus | ResultsPlus is a free online results analysis tool for teachers that gives you a detailed breakdown of your students' performance in Edexcel exams. http://qualifications.pearson.com/en/support/Services/ResultsPlus.html |
| Textbook | A new textbook matched to the new specification will be available in 2017. |

# B Getting started for students

## Student Guide

### Why study the Pearson Edexcel International GCSE in Further Pure Mathematics?

This course will enable you to:

- develop your problem-solving skills by translating problems in mathematical or non-mathematical contexts
- develop reasoning skills through exercises such as presenting arguments and proofs, and making deductions and drawing conclusions from mathematical information.

### What do I need to know, or able to do, before taking this course?

We recommend that students are able to read and write in English at Level B2 of the Common European Framework of Reference for Languages, otherwise there are no prior learning requirements for this qualification.

### Is this the right subject for me?

Have a look at our qualification overview to get an idea of what's included in this qualification. Then, why not get in touch with our student services, students@pearson.com to discuss any outstanding questions you might have?

You could also have a look http://qualifications.pearson.com/en/campaigns/pearson-qualifications-around-the -world.html#tab-Edexcel to find out what students and education experts around the world think about our qualifications.

We also offer International GCSE Mathematics A and Mathematics B and you may feel that the approach used in these specifications is more suitable for you.

### How will I be assessed?

This qualification is assessed through 100% written examination.

### What can I do after I've completed the course?

You can progress from this qualification to:

- the GCE Advanced Subsidiary (AS) and Advanced Level in Mathematics, Further Mathematics or Pure Mathematics
- the International Advanced Subsidiary (AS) and Advanced Level in Mathematics, Further Mathematics or Pure Mathematics
- other equivalent, Level 3 mathematics qualifications
- further study in other areas where mathematics is required
- further training or employment where mathematical skills and knowledge are required.

### What next?

Tall to your subject teacher at school or college for further guidance, or if you are a private candidate you should visit http://qualifications.pearson.com/en/support/support-for-you/students.html

For information about Edexcel, BTEC or LCCI qualifications
visit qualifications.pearson.com



Pearson Education Limited. Registered in England and Wales No. 872828
Registered Office: 80 Strand, London WC2R 0RL
VAT Reg No GB 278 537121

Getty Images: Alex Belmonlinsky

ISBN 978-1-4469-4215-4