edexcel

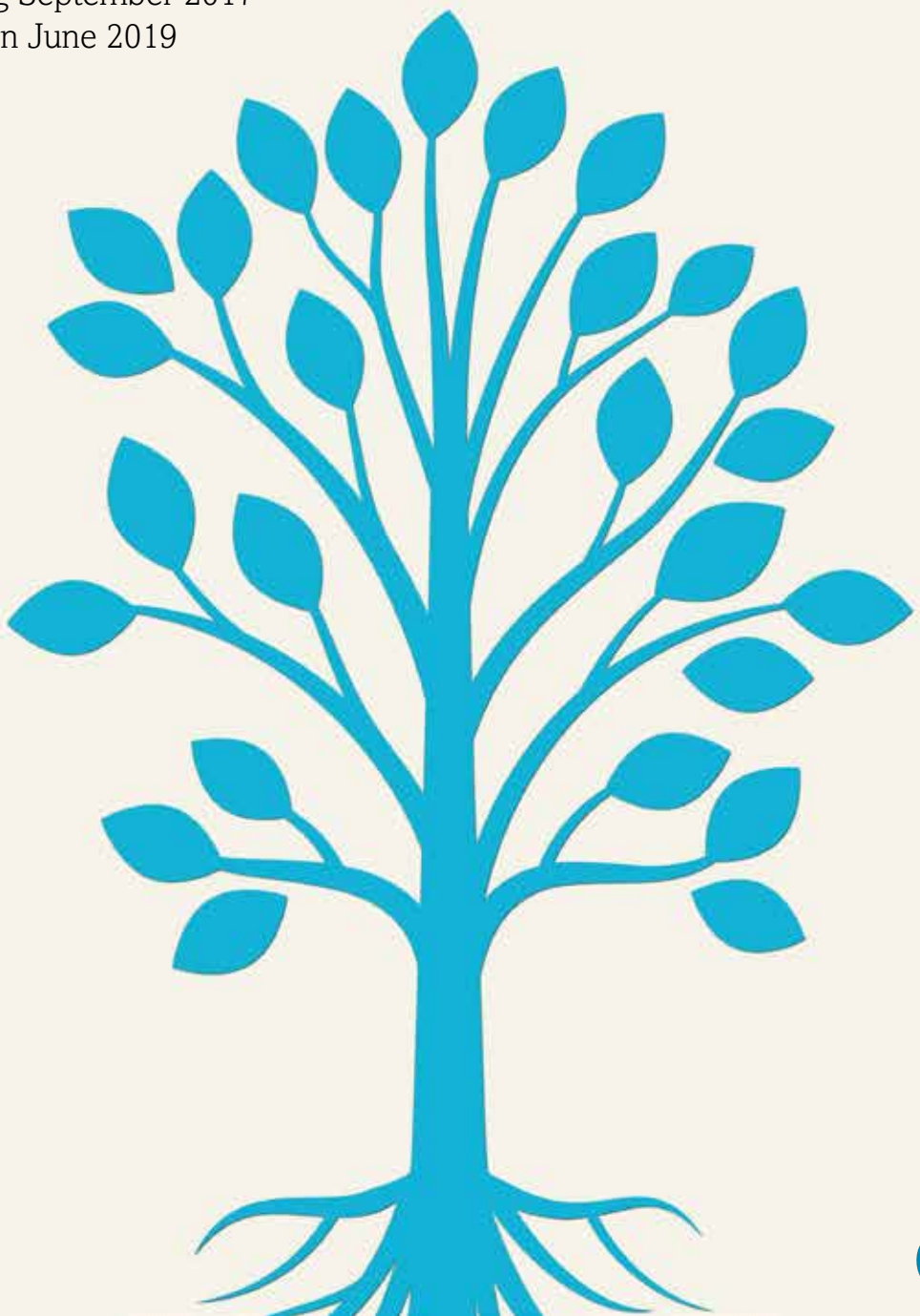
INTERNATIONAL GCSE

Further Pure Mathematics (9-1)

SAMPLE ASSESSMENT MATERIALS

Pearson Edexcel International GCSE in Further Pure Mathematics (4PM1)

For first teaching September 2017
First examination June 2019



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Acknowledgements

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Introduction

The Pearson Edexcel International GCSE in Further Pure Mathematics is designed for use in schools and colleges. It is part of a suite of International GCSE qualifications offered by Pearson.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

General marking guidance

These notes offer general guidance, but the specific notes for examiners appertaining to individual questions take precedence.

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Guidance on the use of abbreviations

M	method mark awarded for a correct method or partial method
B	unconditional accuracy mark (no method needed)
A	accuracy mark (awarded after a correct method or process; if no method or process is seen then full marks for the question are implied but see individual mark schemes for more details)
oe	or equivalent
cao	correct answer only
ft	follow through (when appropriate as per mark scheme)
sc	special case
dep	dependent (on a previous mark)
indep	independent
awrt	answer which rounds to
isw	ignore subsequent working
ee	each error
oo	or omission
cc	correct conclusion
ncc	not corrected correctly
dp	decimal place

Write your name here

Surname	Other names

Pearson Edexcel
International GCSE

Centre Number | Candidate Number

Further Pure Mathematics

Level 2
Paper 1

Sample assessment material for first teaching September 2017
Time: 2 hours

Paper Reference
4PM1/01

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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S 5 1 8 3 6 A 0 1 2 8

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (diferentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) On the axes below, sketch the lines with equations $2x + 3y = 8$ and $2y = 4x + 1$

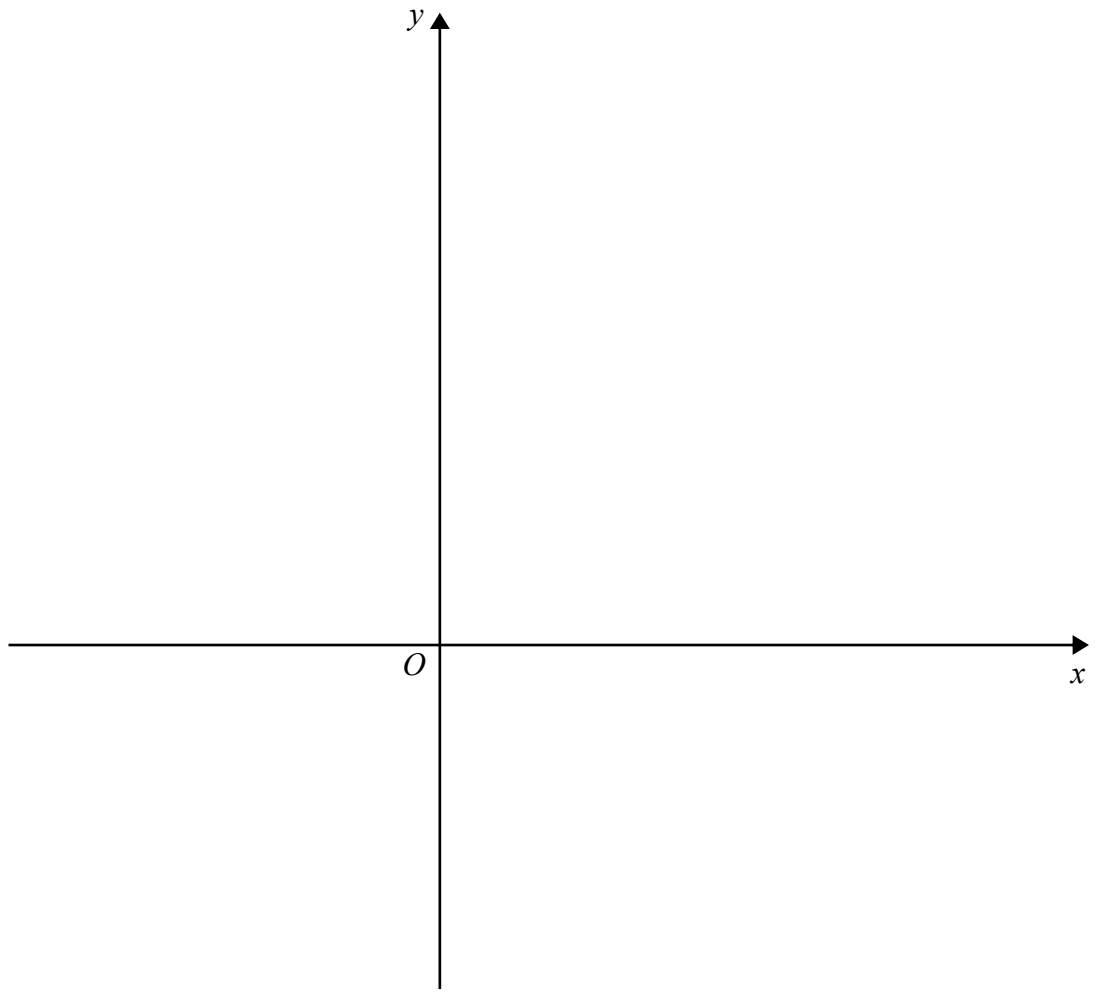
On your sketch, show the coordinates of the points where the lines cross the coordinate axes.

(2)

(b) Show, by shading on your sketch, the region R defined by the inequalities

$$2x + 3y \leqslant 8 \qquad 2y \leqslant 4x + 1 \qquad y \geqslant 0 \qquad x \leqslant 2$$

(2)



(Total for Question 1 is 4 marks)

2

Diagram **NOT** accurately drawn

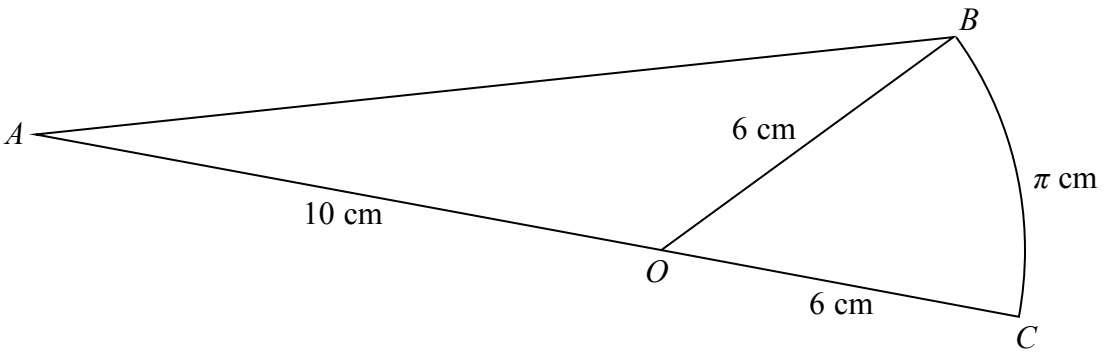


Figure 1

Figure 1 shows a shape ABC in which AOB is a triangle, AOC is a straight line and OBC is a sector of a circle with centre O.

$AO = 10$ cm, $OC = OB = 6$ cm and the length of arc $BC = \pi$ cm.

Find, to 3 significant figures,

(a) the length of AB,

(3)

(b) the area of the shape ABC.

(3)

Question 2 continued

(Total for Question 2 is 6 marks)

3 Solve, in degrees to 1 decimal place, for $0 \leqslant \theta < 180$

$$2\cos(2\theta + 30)^\circ + \tan(2\theta + 30)^\circ = 0$$

(6)

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Question 3 continued

(Total for Question 3 is 6 marks)

4 A particle P is moving along the x-axis.

At time t seconds ($t \geqslant 0$) the velocity, v m/s, of P is given by $v = 4t^2 - 19t + 12$

(a) Find the values of t for which P is instantaneously at rest.

(2)

When $t = 0$, the displacement of P from the origin is -4 m.

(b) Find the displacement of P from the origin when $t = 6$

(4)

At time t seconds the acceleration of P is a m/s^2.

(c) Find the value of t when $a = 0$

(3)

Question 4 continued

(Total for Question 4 is 9 marks)

5 Two numbers x and y are such that $2x + y = 13$

The sum of the squares of $2x$ and y is S.

(a) Show that $S = 8x^2 - 52x + 169$

(3)

Using calculus,

(b) find the value of x for which S is a minimum, justifying that this value of x gives a minimum value for S.

(4)

(c) find the minimum value of S.

(2)

Question 5 continued

(Total for Question 5 is 9 marks)

6 $y = e^x\left(x^2 - 3x\right)$

Show that $y - 2\frac{dy}{dx} + \frac{d^2y}{dx^2} = 2e^x$

(8)

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Question 6 continued

(Total for Question 6 is 8 marks)

7 (a) Complete the table of values for

$$y = 2^{\left(\frac{x}{2}+1\right)} + 1$$

giving your answers to 2 decimal places where appropriate.

(2)

x	0	1	2	3	4	5
y	3				9	12.31

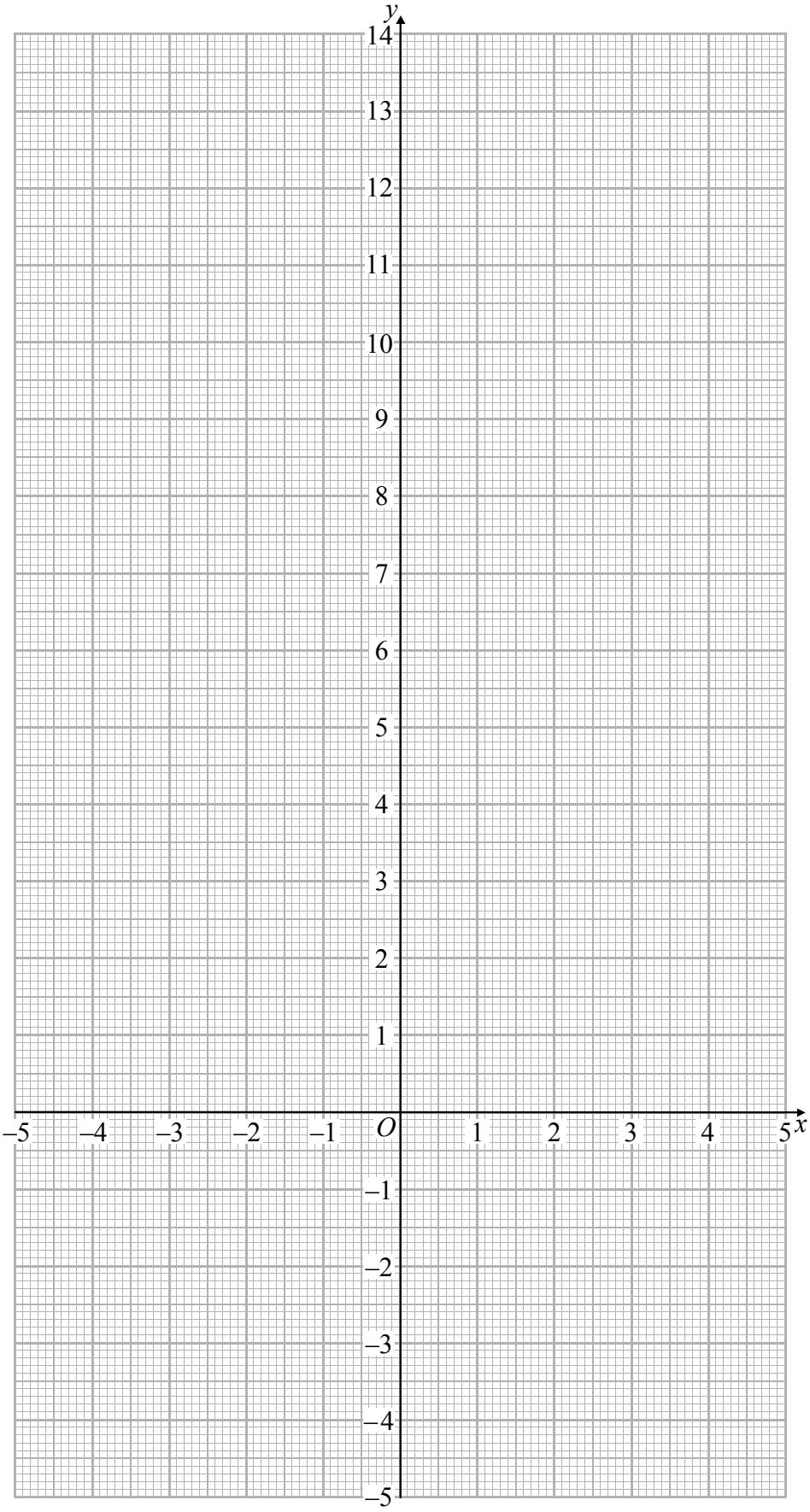
(b) On the grid opposite, draw the graph of $y = 2^{\left(\frac{x}{2}+1\right)} + 1$ for $0 \leqslant x \leqslant 5$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to 1 decimal place, of the root of the equation $\log_2(4x - 6)^2 - x = 2$ in the interval $0 \leqslant x \leqslant 5$

(4)

Question 7 continued



(Total for Question 7 is 8 marks)

8 The sum S_n of the first n terms of an arithmetic series is given by $S_n = 2n(n + 3)$

(a) Find the first term of the series.

(1)

(b) Find the common difference of the series.

(2)

The nth term of the series is T_n

Given that $6S_{(n-4)} = 7T_{(n+3)}$

(c) find the value of n.

(6)

Question 8 continued

(Total for Question 8 is 9 marks)

9 The roots of a quadratic equation are α and β where $\alpha + \beta = -\frac{7}{3}$ and $\alpha\beta = -2$

(a) Find a quadratic equation, with integer coefficients, which has roots α and β

(4)

Given that $\alpha > \beta$ and without solving the equation,

(b) show that $\alpha - \beta = \frac{11}{3}$

(2)

(c) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha + \beta}{\alpha} \text{ and } \frac{\alpha - \beta}{\beta}$$

(7)

Question 9 continued

(Total for Question 9 is 13 marks)

10

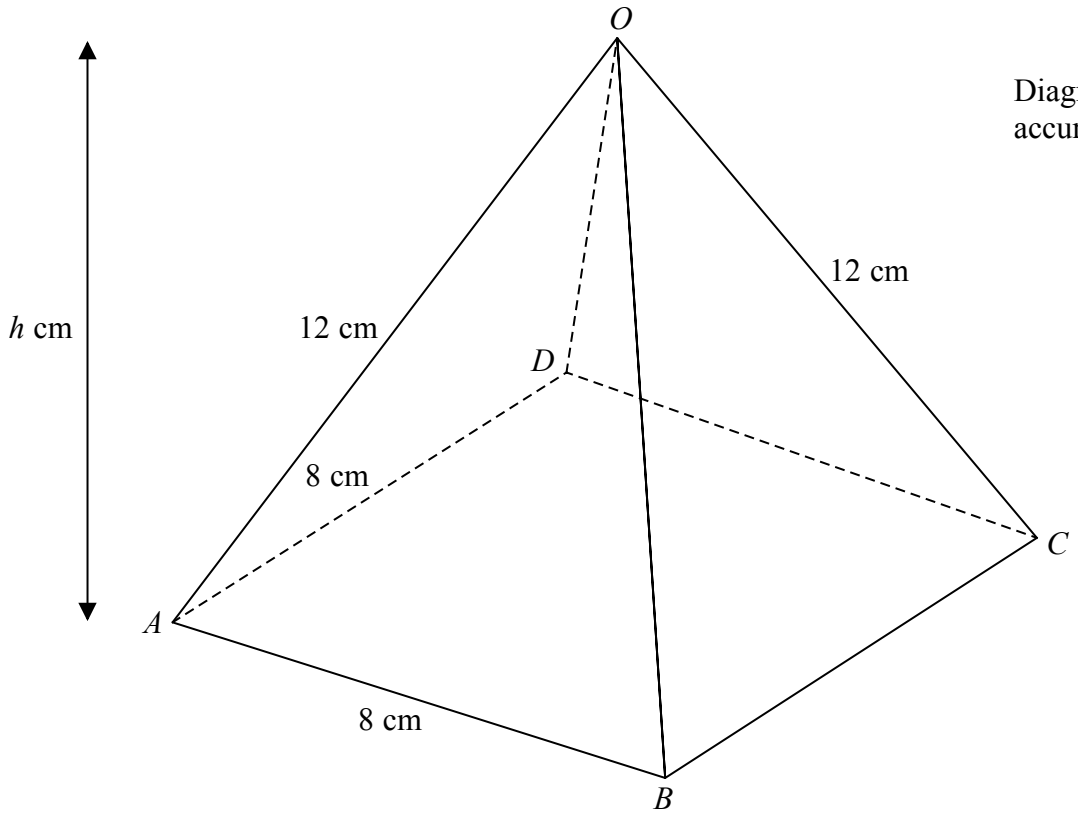


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Figure 2

Figure 2 shows a right pyramid $ABCDO$ with a horizontal square base of side 8 cm. The vertical height of the pyramid is h cm and $OA = OB = OC = OD = 12$ cm.

(a) Find the exact value of h.

(3)

(b) Find, to 1 decimal place, the size of the angle between OA and the plane $ABCD$.

(2)

(c) Find, to 1 decimal place, the size of the angle between the plane AOB and the plane $ABCD$.

(2)

The midpoint of OA is P and Q is the point on BC such that $BQ : QC = 3 : 1$

(d) Show that $PQ = 4\sqrt{5}$ cm.

(4)

(e) Find, to 1 decimal place, the size of angle PQA.

(4)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 = 15 marks)

11

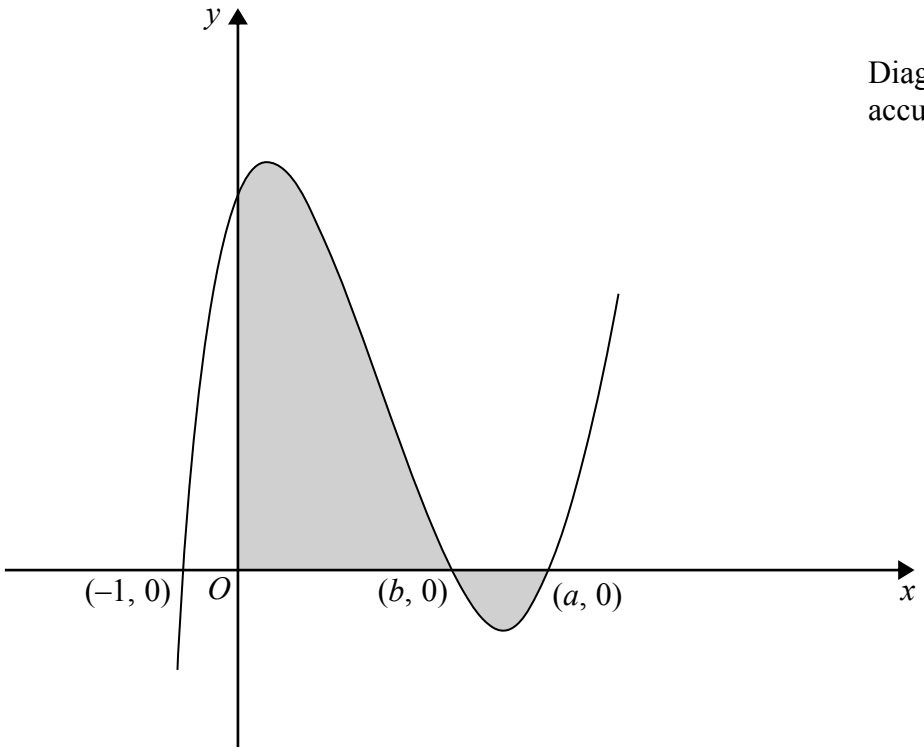

Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sketch of the curve with equation $y = f(x)$, which passes through the points with coordinates $(-1, 0)$, $(b, 0)$ and $(a, 0)$ where $0 < b < a$.

Given that $f'(x) = 6x^2 - 26x + 12$

(a) find,

(i) the value of a,

(ii) the value of b.

(8)

(b) Use algebraic integration to determine the exact value of the total area of the shaded regions shown in Figure 3.

(5)

Question 11 continued

Question 11 continued

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Question 11 continued

(Total for Question 11 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS

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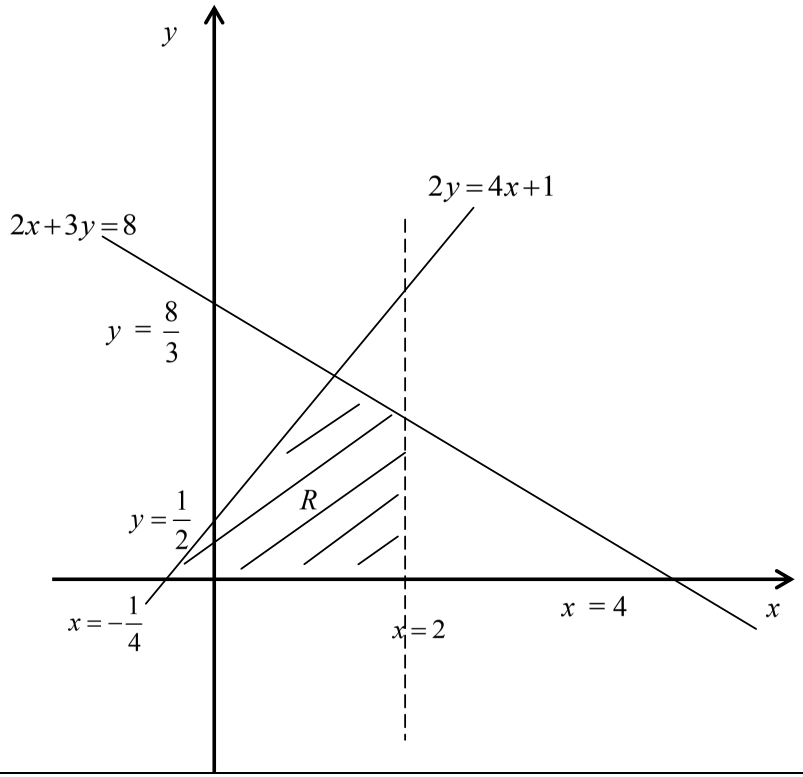
International GCSE Futher Pure Mathematics – Paper 1 mark scheme

Paper 1					
Question	Working	Answer	Mark	AO	Notes
1 **(a)**			B1 B1	1	One correct line Both correct lines
(b)			B1 B1	1	Correct line $x = 2$ drawn Correct region shaded in or out.
			(4)		

Question	Working	Answer	Mark	AO	Notes
2 **(a)**	$\pi = 6\theta \Rightarrow \theta = \frac{\pi}{6} \Rightarrow AOB = \frac{5\pi}{6} = (150^o)$		B1	1	
	$AB = \sqrt{10^2 + 6^2 - 2\times10\times6\times\cos\left(\frac{5\pi}{6}\right)} = 15.4894... = 15.5 \text{ cm}$	15.5 (cm)	M1A1	2	Accept working in degrees $AOB = 150^o$
(b)	$\text{Area} = \frac{1}{2}\times10\times6\times\sin\frac{5\pi}{6} + \frac{\pi}{6}\times\frac{6^2}{2} = 24.424...$	24.4 (cm^2)	M1M1A1	2	
	ALTERNATIVE $\text{Area} = \frac{1}{2}\times10\times6\times\sin\frac{5\pi}{6} + \frac{1}{2}\times\pi\times6 = 24.424...$		M1M1A1 **(6)**		
3	$2\cos(2\theta+30) + \frac{\sin(2\theta+30)}{\cos(2\theta+30)} = 0 \Rightarrow 2\cos^2(2\theta+30) + \sin(2\theta+30) = 0$		M1	2	
	$\Rightarrow 2 - 2\sin^2(2\theta+30) + \sin(2\theta+30) = 0$		M1A1		
	$\sin(2\theta+30) = \frac{1 \pm \sqrt{1-4\times2\times(-2)}}{2\times2} = 1.2807..., -0.7807....$		M1	3	Solves 3 TQ
	$2\theta + 30 = -51.33167..., 231.33167, 308.66833$		A1A1		Finds one angle from their 3TQ
	$\theta = 100.7,\ 139.3$	$\theta = 100.7,\ 139.3$	**(6)**		

Question	Working	Answer	Mark	AO	Notes
4					
(a)	$v=0$ so $4t^2-19t+12=0\Rightarrow(4t-3)(t-4)=0\Rightarrow t=\frac{3}{4},4$	$t=\frac{3}{4},4$	M1A1	1	
(b)	$s=\int 4t^2-19t+2\mathrm{d}t=\frac{4t^3}{3}-\frac{19t^2}{2}+12t+c$ when $t=0, s=-4 \Rightarrow c=-4$		M1M1A1	2	
	When $t=6$, $s=\frac{4\times6^3}{3}-\frac{19\times6^2}{2}+12\times6-4=14$	14	A1		
(c)	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=8t-19\Rightarrow 8t-19=0\Rightarrow t=\frac{19}{8}$	$t=\frac{19}{8}$	M1M1A1 **(9)**	3	
5	$2x+y=13\Rightarrow y=13-2x$		B1	1	
(a)	$S=4x^2+(13-2x)^2=4x^2+169-52x+4x^2=8x^2-52x+169$	$8x^2-52x+169$	M1A1		
(b)	$\frac{\mathrm{d}S}{\mathrm{d}x}=16x-52$, $\frac{\mathrm{d}S}{\mathrm{d}x}=0\Rightarrow16x-52=0\Rightarrow x=\frac{13}{4}$	$x=\frac{13}{4}$	M1M1A1	2,3	
	$\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=16$ $16>0$,	Hence minimum	B1		
(c)	$S=8\times\left(\frac{13}{4}\right)^2-52\times\frac{13}{4}+169=\frac{169}{2}=84.5$	$\frac{169}{2}=84.5$	M1A1 **(9)**	3	

Question	Working	Answer	Mark	AO	Notes
6	$\frac{dy}{dx} = e^x(x^2-3x)+e^x(2x-3)\left[\Rightarrow e^x(2x-3)=\frac{dy}{dx}-y\right]$		M1M1A1	4	
	$\frac{d^2y}{dx^2} = e^x(x^2-3x)+e^x(2x-3)+e^x(2x-3)+2e^x = y+2\left(\frac{dy}{dx}-y\right)+2e^x$		M1A1	4	
	$2e^x = \frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}-y\right)-y \Rightarrow 2e^x = y-2\frac{dy}{dx}+\frac{d^2y}{dx^2}$ *		M1M1A1 **(8)**		
7					
(a)	***x*** 0 1 2 3 4 5 ***y*** 3 **3.83** **5** **6.66** 9 12.31		B1B1	1	
(b)	All points plotted within an accuracy of half of a square. A smooth curve drawn through their points		B1B1	1	
(c)	$\log_2(4x-6)^2 - x = 2 \Rightarrow 2\log_2(4x-6) = x+2 \Rightarrow \log_2(4x-6) = \frac{x}{2}+1$		M1M1	2	
	$\Rightarrow 4x-6 = 2^{\left(\frac{x}{2}+1\right)} \Rightarrow 4x-5 = 2^{\left(\frac{x}{2}+1\right)}+1$				
	Line $y=4x-5$ drawn on graph $\Rightarrow$ so $x = 2.8(36)$	$x = 2.8$	M1A1 **(8)**		

Question	Working	Answer	Mark	AO	Notes
8					
(a)	$a = S_1 = 2\times1\times(1+3) = 8$	$a = 8$	B1	1	
(b)	$S_2 = 2\times2\times(2+3) = 20$				
	$S_2 = a + T_2 \Rightarrow T_2 = S_2 - a = 20 - 8 = 12$				
	$d = 12 - 8 = 4$	$d = 4$	M1A1	1	
(c)	Uses given formula for $S_n = 2n(n+3)$ and formula for *nth* term $T_n = a + (n-1)d$				
	$6\left[2(n-4)(n-4+3)\right] = 7\left[8+(n+3-1)4\right]$		M1M1	2, 3	
	$\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$		M1A1		
	$(n-8)(3n+2) = 0$				
	$\Rightarrow n = 8, \left(n = -\frac{2}{3}\right)$	$n = 8$	M1A1		
	ALT				
	Using formula $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$				
	$6\left[\frac{(n-4)}{2}\left(2\times8 + \left((n-4)-1\right)4\right)\right] = 7\left[8+(n+3-1)4\right]$		M1M1 M1A1	2, 3	
	$\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$				
	$(n-8)(3n+2) = 0$		M1A1		
	$\Rightarrow n = 8, \left(n = -\frac{2}{3}\right)$	$n = 8$	**(9)**		

Question	Working	Answer	Mark	AO	Notes
9					
(a)	$\alpha+\beta=-\frac{7}{3}\quad \alpha\beta=-2=-\frac{6}{3}$ so $a=3$, $b=7$ and $c=-6$		B1B1	1	
	Hence quadratic equation $\Rightarrow 3x^2+7x-6=0$ oe with integer coefficients		M1A1		
(b)	$(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta=(\alpha+\beta)^2-4\alpha\beta=\left(-\frac{7}{3}\right)^2-4\times-2=\frac{121}{9}$			3	
	$\alpha>\beta$ so $\alpha-\beta=\frac{11}{3}$ *		M1A1		
	Sum				
(c)	$\frac{\alpha+\beta}{\alpha}+\frac{\alpha-\beta}{\beta}=\frac{\beta(\alpha+\beta)+\alpha(\alpha-\beta)}{\alpha\beta}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{\left(-\frac{7}{3}\right)^2-2\times-2}{-2}=-\frac{85}{18}$		M1M1A1	3	
	Product $\frac{(\alpha+\beta)}{\alpha}\times\frac{(\alpha-\beta)}{\beta}=\frac{\left(-\frac{7}{3}\right)\times\left(\frac{11}{3}\right)}{-2}=\frac{77}{18}$		M1A1		
	Equation $18y^2+85y+77=0$ oe with integer coefficients		M1A1		
			(13)		

Question	Working	Answer	Mark	AO	Notes
10	Let M be the midpoint of diagonals AC or DB				
(a)	$AC = \sqrt{8^2+8^2} = 8\sqrt{2} \Rightarrow AM = 4\sqrt{2}$ $h = \sqrt{12^2 - \left(4\sqrt{2}\right)^2} = \sqrt{112} = \left(4\sqrt{7}\right)$	$4\sqrt{7}$	M1M1A1	1	
(b)	$\tan^{-1}\left(\frac{4\sqrt{7}}{4\sqrt{2}}\right) = 61.87449... \approx 61.9^o$	61.9°	M1A1	1	Or any equivalent trigonometry
(c)	Let N be the midpoint of AB Angle the plane AOB makes with horiz $= \tan^{-1}\left(\frac{4\sqrt{7}}{4}\right) = 69.295... \approx 69.3^o$	69.3°	M1A1	3	Or any equivalent trigonometry
(d)	**By using the symmetrical properties of the pyramid** Let S be the perpendicular from P to diagonal AC Let R be the perpendicular from S to side BC In triangle PSR $\rightarrow$ $PR = \sqrt{\left(2\sqrt{17}\right)^2 - 2^2} = 8$ In triangle PRQ $\rightarrow$ $PQ = \sqrt{8^2+4^2} = 4\sqrt{5}$	$4\sqrt{5}$	M1A1 M1A1	3	Or any equivalent system of right angle triangles
(e)	Length $AQ = \sqrt{8^2+6^2} = 10$ Angle of $PQA = \cos^{-1}\left(\frac{10^2 + \left(4\sqrt{5}\right)^2 - 6^2}{2\times 10\times 4\sqrt{5}}\right) = 36.39124... \approx 36.4^o$	36.4°	M1 M1A1A1 **(15)**	3	

Question	Working	Answer	Mark	AO	Notes
(d)	**ALTERNATIVE without using the symmetrical properties of the pyramid** $\cos OAB = \frac{12^2 + 8^2 - 12^2}{2 \times 8 \times 12} = \frac{1}{3}$ $\Rightarrow PB = \sqrt{6^2 + 8^2 - 2 \times 6 \times 8 \times \frac{1}{3}} = 2\sqrt{17}$ In triangle PBC $PC = \sqrt{6^2 + \left(8\sqrt{2}\right)^2 - 2 \times 6 \times \left(8\sqrt{2}\right) \times \cos\left(\tan^{-1}\left(\frac{\sqrt{7}}{\sqrt{2}}\right)\right)} = 10$ $\Rightarrow \text{Angle } PBC = \cos^{-1}\left(\frac{8^2 + 68 - 10^2}{2 \times 8 \times 2\sqrt{17}}\right) = 75.9637.....^{\circ}$ In triangle PBQ; $PQ = \sqrt{6^2 + 68 - 2 \times 6 \times 2\sqrt{17} \times \cos 75.96375...} = 4\sqrt{5}$	$4\sqrt{5}$	M1 M1 M1A1	3	

Question	Working	Answer	Mark	AO	Notes
11	Mark parts (i) and (ii) together			1, 3	
(a)	$y = \int 6x^2 - 26x + 12 \mathrm{d}x = \left[\frac{6x^3}{3} - \frac{26x^2}{2} + 12x + c \right]$		M1		
	At the point $(-1, 0)$ $0 = 2(-1)^3 - 13(-1)^2 + 12(-1) + C \Rightarrow C = 27$		M1A1		
	$\left(2x^3 - 13x^2 + 12x + 27\right) \div (x+1) = 2x^2 - 15x + 27 = (2x-9)(x-3)$		M1M1A1		
	$\Rightarrow a = \frac{9}{2}, b = 3$	$a = \frac{9}{2}, b = 3$	B1B1		
(b)	$\text{Area} = \int_0^3 2x^3 - 13x^2 + 12x + 27 \mathrm{d}x + \left\lvert \int_3^{\frac{9}{2}} 2x^3 - 13x^2 + 12x + 27 \mathrm{d}x \right\rvert =$		M1M1	3	
	$\left[\frac{2}{4}x^4 - \frac{13}{3}x^3 + \frac{12}{2}x^2 + 27x \right]_0^3 + \left\lvert \left[\frac{2}{4}x^4 - \frac{13}{3}x^3 + \frac{12}{2}x^2 + 27x \right]_3^{\frac{9}{2}} \right\rvert = \frac{2043}{32}$		A1M1A1		
	So Area $= \frac{2043}{32}$	Area $= \frac{2043}{32}$	**(13)**		
		Total	**100**		

Write your name here

Surname	Other names

Pearson Edexcel
International GCSE

Centre Number | Candidate Number

Further Pure Mathematics

Level 2
Paper 2

Sample assessment material for first teaching September 2017
Time: 2 hours

Paper Reference
4PM1/02

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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S 5 1 8 3 7 A 0 1 2 8

PEARSON

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (diferentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The nth term of a geometric series is $3e^{(1-2n)}$

Find the sum to infinity of this series.

Give your answer in the form $\frac{ae}{e^b - 1}$ where a and b are integers to be found.

(5)

(Total for Question 1 is 5 marks)

2 Find the set of values of x for which

(a) $3 + x < 2x - 1$

(1)

(b) $x(x - 1) > 6$

(3)

(c) **both** $3 + x < 2x - 1$ **and** $x(x - 1) > 6$

(1)

(Total for Question 2 is 5 marks)

3 O, A and B are fixed points such that

$$\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j} \qquad \overrightarrow{OB} = 8\mathbf{i} + p\mathbf{j} \qquad \text{and} \qquad \left|\overrightarrow{AB}\right| = 2\sqrt{13}$$

(a) Find the possible values of p.

(3)

Given that $p > 0$

(b) find a unit vector parallel to $\overrightarrow{AB}$

(2)

(Total for Question 3 is 5 marks)

4 $f(x) = 2x^3 + px^2 + qx + 12 \quad p, q \in \mathbb{Z}$

Given that $(x + 3)$ is a factor of $f(x)$ and that when $f'(x)$ is divided by $(x + 3)$ the remainder is 37

(a) show that $p = 1$ and find the value of q

(6)

(b) hence factorise $f(x)$ completely

(2)

(c) show that the equation $f(x) = 0$ has only one real root.

(2)

Question 4 continued

(Total for Question 4 is 10 marks)

5 (a) Show that $\cos(A - B) - \cos(A + B) = 2\sin A \sin B$

(2)

(b) Hence express $2\sin 5x \sin 3x$ in the form $\cos mx - \cos nx$ where m and n are integers, giving the value of m and the value of n,

(1)

(c) (i) Find $\int 4\sin 5\theta \sin 3\theta \, d\theta$

(ii) Hence evaluate $\int_0^{\frac{\pi}{6}} 4\sin 5\theta \sin 3\theta \, d\theta$, giving your answer in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers.

(4)

Question 5 continued

(Total for Question 5 is 7 marks)

6 Solve the equation $\log_2 x + 6\log_x 2 = 7$

Question 6 continued

(Total for Question 6 is 7 marks)

7 The curve C with equation

$$y = \frac{ax - 5}{x - b}$$

where a and b are integers, crosses the x-axis at the point (2.5, 0). The asymptote to C which is parallel to the y-axis has equation $x = 1$

(a) (i) Show that $a = 2$

(ii) Find the value of b.

(3)

(b) Find the coordinates of the point where C crosses the y-axis.

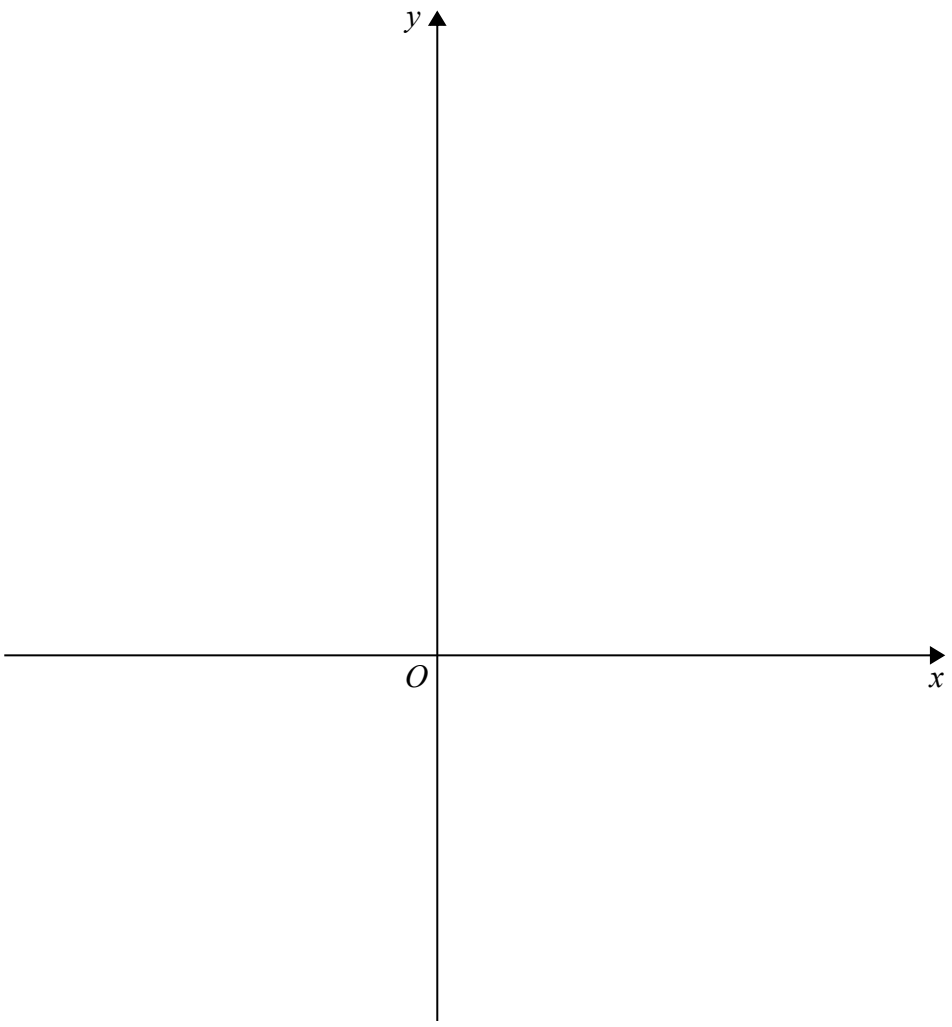
(1)

(c) Find the equation of the asymptote to C which is parallel to the x-axis.

(1)

(d) Using the axes below, sketch the curve C showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)



Question 7 continued

(Total for Question 7 is 8 marks)

8 (a) Expand $\frac{3}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^3 and simplifying each term as far as possible.

(4)

(b) Write down the range of values of x for which this expansion is valid.

(1)

(c) Show that $\frac{3}{\sqrt{0.9}} = \sqrt{10}$

(1)

(d) Express $\frac{1}{\sqrt{10}-3}$ in the form $a\sqrt{10} + b$, where a and b are integers.

(2)

(e) Hence, using your expansion with a suitable value for x, obtain an approximation to 5 decimal places of $\frac{1}{\sqrt{10}-3}$

(3)

Question 8 continued

(Total for Question 8 is 11 marks)

9 $f(x) = 7 + 4x - 2x^2$

Given that $f(x)$ can be written in the form $P(x + Q)^2 + R$ where P, Q and R are constants,

(a) find the value of P, the value of Q and the value of R.

(3)

(b) hence write down

(i) the maximum value of $f(x)$,

(ii) the value of x for which this maximum occurs.

(2)

The curve C has equation $y = 7 + 4x - 2x^2$

The line l with equation $y = 4 - x$ intersects C at two points.

(c) Find the x coordinates of these two points.

(3)

The finite region bounded by the curve C and the line l is rotated 360° about the x-axis.

(d) Use algebraic integration to find, to 3 significant figures, the volume of the solid generated.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

10

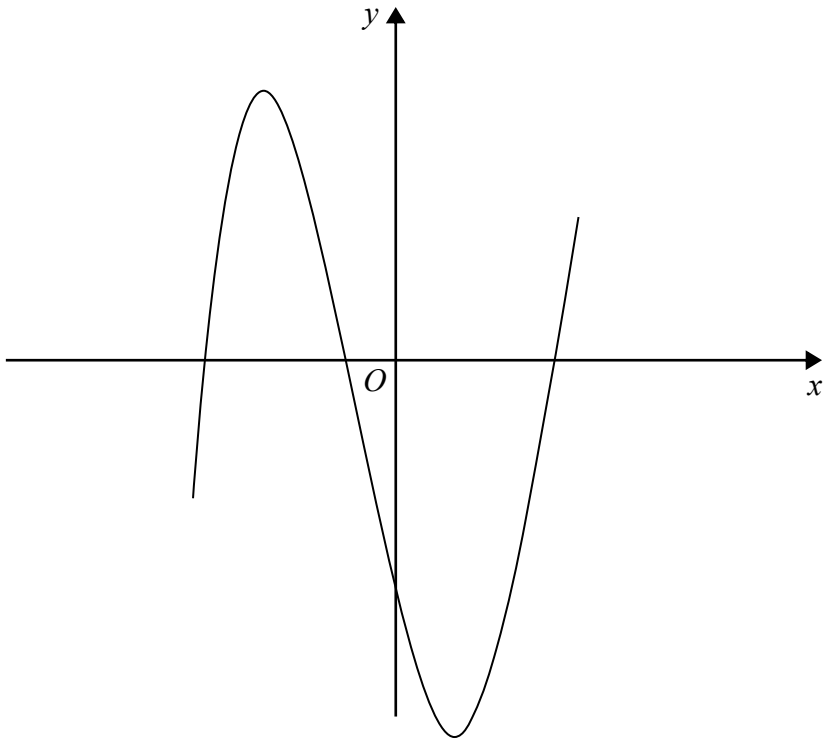


Figure 1

Figure 1 shows the curve M with equation $y = x^3 - 13x - 12$

The point P, with x coordinate -2, lies on M and line l_1 is the tangent to M at the point P.

(a) Find an equation for l_1

(5)

The point Q lies on M and the line l_2 is the tangent to M at the point Q.

Given that l_1 and l_2 are parallel,

(b) find an equation for l_2

(4)

The normal to M at P meets l_2 at the point R.

(c) Find the coordinates of R.

(4)

(d) Find the exact length of the line PR.

(2)

The tangent and normal at P and the tangent and normal at Q form a rectangle.

(e) Find the exact area of this rectangle.

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 18 marks)

11

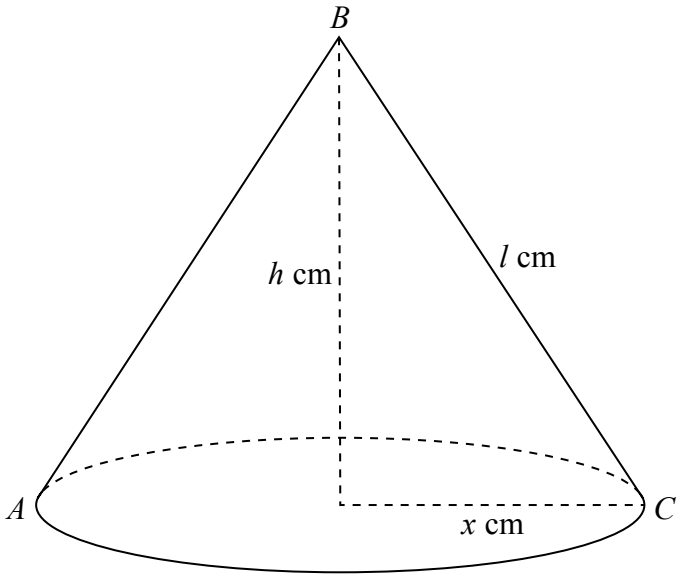


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a right circular cone with a base radius of x cm. The slant height of the cone is l cm and the height of the cone is h cm. The vertex of the cone is B and the points A and C, on the base of the cone, are such that AC is a diameter of the base.

The cone is increasing in size in such a way that the size of the angle ABC is constant at 60° and the **total** surface area of the cone is increasing at a constant rate of 10 cm^2/s.

Find the exact rate of increase of the volume of the cone when $x = 6$

(11)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

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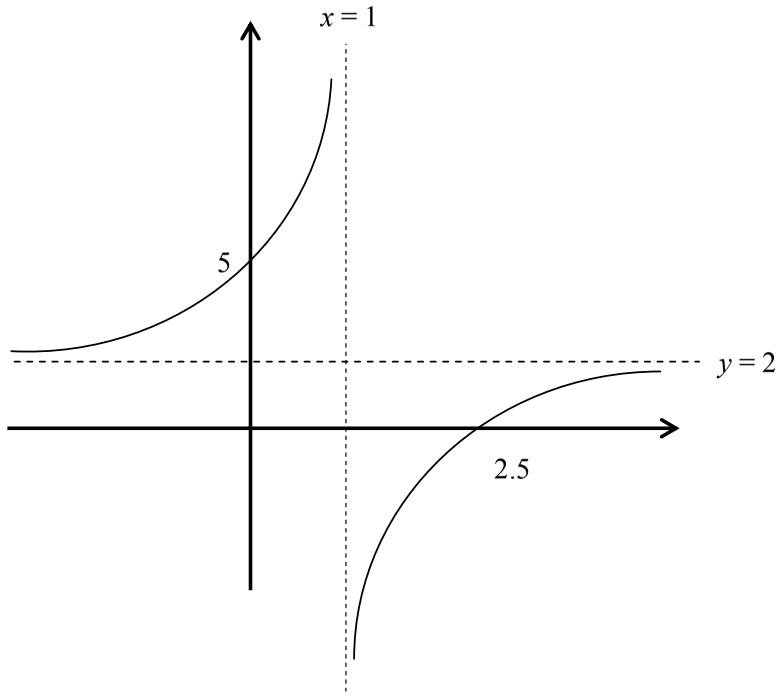
International GCSE Futher Pure Mathematics – Paper 2 mark scheme

Paper 2					
Question	**Working**	**Answer**	**Mark**	**AO**	**Notes**
1	First term = $3e^{-1}$, Common Ratio = e^{-2} $S=\frac{3e^{-1}}{1-e^{-2}}\times\left(\frac{e^2}{e^2}\right)=\frac{3e}{e^2-1}$ (So, $a = 3$ and $b = 2$)	$S_\infty=\frac{3e}{e^2-1}$	B1B1 M1M1 A1 **(5)**	1 2	
2 **(a)**	$3+x<2x-1\Rightarrow x>4$	$x > 4$	B1 M1	1	
(b)	$x(x-1)>6 \Rightarrow x^2-x-6>0\Rightarrow(x-3)(x+2)>0$ critical values are $x = 3, -2$ so $x > 3$ OR $x < -2$	$x > 3$ OR $x < -2$	M1A1	1	(Outside region)
(c)	$x > 3$ OR $x < -2$, $x > 4$ so $x > 4$	$x > 4$	B1 **(5)**	1	

Question	Working	Answer	Mark	AO	Notes
3 **(a)**	$\overrightarrow{AB} = -(4\mathbf{i}+3\mathbf{j})+(8\mathbf{i}+p\mathbf{j}) = 4\mathbf{i}+(p-3)\mathbf{j}$		B1	1	
	$\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{52} = \sqrt{4^2+(p-3)^2} \Rightarrow p^2-6p-27 = (p-9)(p+3) = 0$		M1A1		
	$\Rightarrow p = 9, p = -3$				
(b)	$(p>0 \quad \text{so } p = 9)$				
	$\overrightarrow{AB} = 4\mathbf{i}+(p-3)\mathbf{j} = 4\mathbf{i}+6\mathbf{j} \quad \Rightarrow \frac{1}{2\sqrt{13}}(4\mathbf{i}+6\mathbf{j}) = \frac{1}{\sqrt{13}}(2\mathbf{i}+3\mathbf{j})$ (oe)	$\frac{1}{2\sqrt{13}}(4\mathbf{i}+6\mathbf{j})$	B1B1	1	
			(5)		
4 **(a)**	$f(-3) = 2\times(-3)^2 + p\times(-3)^2 + q\times(-3)+12 = 0$				Mark parts (i) and (ii) together
	$\Rightarrow 42 = 9p-3q \Rightarrow 14 = 3p-q$		M1	3	
	$f'(x) = 6x^2+2px+q$		M1		
	$f'(-3) = 6\times(-3)^2+2p\times(-3)+q = 37$		M1A1		
	$\Rightarrow -6p+q = -17 \Rightarrow q = 17-6p$				
	$14 = 3p-1(6p-17) \Rightarrow p = 1, q = -11$	$q = -11$	M1A1		Solving simultaneous equations (by any method)
(b)	$\frac{(2x^3+x^2-11x+12)}{(x+3)} = 2x^2-5x+4$		M1A1	2	
	$\Rightarrow (x+3)(2x^2-5x+4)$				
(c)	$b^2-4ac = (-5)^2-4\times2\times4 = -7$ (7 hence no real roots for quadratic factor, so x = -3 only real root.)		M1A1	1	
			(10)		

Question	Working	Answer	Mark	AO	Notes
5 **(a)**	$\cos(A-B)-\cos(A+B)=\cos A\cos B+\sin A\sin B-\cos A\cos B+\sin A\sin B$ $\Rightarrow 2\sin A\cos B$		M1A1	1	
(b)	$2\sin 5x\sin 3x=\cos 2x-\cos 8x$	$\cos 2x-\cos 8x$	B1	2	
(c)(i)	$\int 4\sin 5\theta\sin 3\theta\,\mathrm{d}\theta=2\int\cos 2\theta-\cos 8\theta\,\mathrm{d}\theta=2\left[\frac{\sin 2\theta}{2}-\frac{\sin 8\theta}{8}\right](+c)$		M1A1	3	
(ii)	$\int_0^{\frac{\pi}{6}}4\sin 5\theta\sin 3\theta\,\mathrm{d}x=2\int_0^{\frac{\pi}{6}}(\cos 2\theta-\cos 8\theta)\,\mathrm{d}\theta=2\left[\frac{\sin 2\theta}{2}-\frac{\sin 8\theta}{8}\right]_0^{\frac{\pi}{6}}=\frac{5\sqrt{3}}{8}$	$\frac{5\sqrt{3}}{8}$	M1A1 **(7)**		
6	$\log_x 2=\frac{\log_2 2}{\log_2 x}=\frac{1}{\log_2 x}$		B1	2, 3	
	$\log_2 x+\frac{6}{\log_2 x}=7\Rightarrow(\log_2 x)^2-7\log_2 x+6=0$		M1M1 A1		
	$\Rightarrow(\log_2 x-6)(\log_2 x-1)=0\Rightarrow\log_2 x=6,\ \log_2 x=1$		M1M1		
	$\Rightarrow x=64,\ \ x=2$	64, 2	A1 **(7)**		

Question	Working	Answer	Mark	AO	Notes
7 **(a)**	(i) $0 = \frac{ax-5}{x-b} \Rightarrow ax-5=0 \Rightarrow a\frac{5}{2}-5=0 \Rightarrow a=2$		M1A1	2	
	(ii) $b=1$	$b=1$	B1		
(b)	When $x=0$, $y=\frac{-5}{-b} \Rightarrow y=\frac{5}{1} \Rightarrow y=5$	(0, 5)	B1	2	Accept $y=5$
(c)	$y=2$	$y=2$	B1	3	
(d)			B1	3	Curve in correct quadrants
			B1 B1		Correct asymptotes drawn Correct intersections with axes
			(8)		

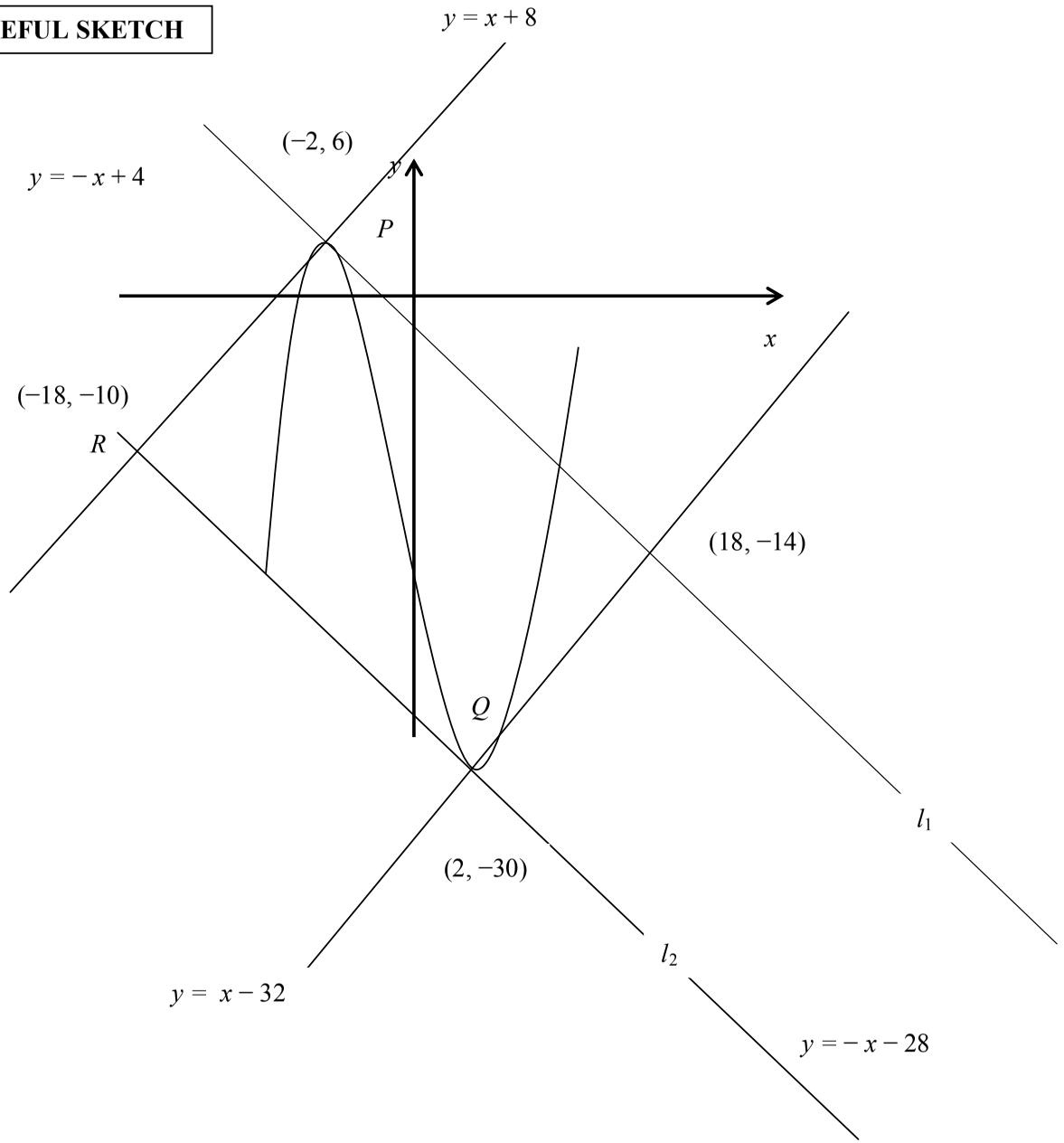
Question	Working	Answer	Mark	AO	Notes
8					
(a)	$\frac{3}{\sqrt{1-2x}} = 3(1-2x)^{-\frac{1}{2}} \Rightarrow$		B1	2	
	$= 3\left\{1 + \left(-\frac{1}{2}\right)(-2x) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{(-2x)^2}{2!} + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\frac{(-2x)^3}{3!}\ldots\right\}$		M1A1		
	$\frac{3}{\sqrt{1-2x}} = 3 + 3x + \frac{9}{2}x^2 + \frac{15}{2}x^3$		A1		
(b)	$-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x\rvert < \frac{1}{2}$	$-\frac{1}{2} < x < \frac{1}{2}$	B1	2	
(c)	$\frac{3}{\sqrt{0.9}} = \frac{3}{\sqrt{9/10}} = \frac{3}{3/\sqrt{10}} = \sqrt{10}$		B1	3	
(d)	$\frac{1}{\sqrt{10}-3} \times \frac{\left(\sqrt{10}+3\right)}{\left(\sqrt{10}+3\right)} = \frac{\left(\sqrt{10}+3\right)}{1} = \left(\sqrt{10}+3\right)$	$\left(\sqrt{10}+3\right)$	M1A1	3	
(e)	$1-2x = 0.9 \Rightarrow x = 0.05$		B1	3	
	$\sqrt{10}+3 = 3 + 3\times(0.05) + \frac{9}{2}\times(0.05)^2 + \frac{15}{2}\times(0.05)^3 + \ldots + 3 = 3.1621875\ldots + 3$	6.16219	M1A1		
	≈ 6.16219		**(11)**		

Question	Working	Answer	Mark	AO	Notes
9 **(a)**	$7+4x-2x^2=-2\left[(x-1)^2-1\right]+7=-2(x-1)^2+9$ $P=-2, Q=-1, R=9$		M1A1 A1	1	
(b)	(i) 9 (ii) 1	(i) 9 (ii) 1	B1B1	2	
(c)	$7+4x-2x^2=4-x \Rightarrow 2x^2-5x-3=0$ $\Rightarrow (2x+1)(x-3)=0 \Rightarrow x=-\frac{1}{2}, 3$	$x=-\frac{1}{2}, x=3$	M1M1 A1	2	
(d)	$V=\pi\int_{-0.5}^{3}(7+4x-2x^2)^2\,\mathrm{d}x=\pi\int_{-0.5}^{3}(4-x)^2\,\mathrm{d}x$ $\Rightarrow \pi\int_{-0.5}^{3} 33+64x-13x^2-16x^3+4x^4\,\mathrm{d}x$ $\Rightarrow \pi\left[33x+32x^2-\frac{13x^3}{3}-4x^4+\frac{4x^5}{5}\right]_{-0.5}^{3}=\pi\frac{4459}{30}=466.945\ldots\approx 467$	467	M1 M1A1 M1A1 **(13)**	3	

Question	Working	Answer	Mark	AO	Notes
10 (a)	$y = f(-2) = 6$		B1	1	
	$\frac{dy}{dx} = 3x^2 - 13 \Rightarrow f'(-2) = 13 - 13 = -1$		M1A1		
	Equation of tangent $y - 6 = -1(x - (-2)) \Rightarrow y + x - 4 = 0$	$y + x - 4 = 0$	M1A1		
(b)	$-1 = 3x^2 - 13 \Rightarrow 3x^2 - 12 = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow (x-2)(x+2) = 0$ at $Q, x = 2$		M1		
	$f(2) = -30$		B1	1, 2	
	$y - -30 = -(x-2) \Rightarrow \{y = -x - 28\}$	$y = -x - 28$	M1A1		
(c)	Gradient of normal = 1		B1	2	
	Equation of Normal $y - 6 = x - (-2) \Rightarrow \{y = x + 8\}$		M1		
	At R, $x + 8 = -x - 28 \Rightarrow x = -18$				
	When $x = -18$, $y = -10$	$(-18, -10)$	A1A1		

Question	Working	Answer	Mark	AO	Notes
10 cont'd	{Coordinates of R $(-18, -10)$}				
(d)	$PR = \sqrt{(6--10)^2 + (-2--18)^2} = 16\sqrt{2}$	$16\sqrt{2}$	M1A1	3	
(e)	Area of rectangle				
	Length of QR = $\sqrt{(-18-2)^2 + (-10--30)^2} = 20\sqrt{2}$		M1	3	
	Area = $16\sqrt{2} \times 20\sqrt{2} = 640$	640	M1A1		
(e)	**ALTERNATIVE** Equation of normal at Q $y--30 = x-2 \Rightarrow y = x-32$				
	Coordinates of 4th vertex of rectangle		M1	3	
	$x-32 = -x+4 \Rightarrow x = 18$ so $y = -14$				
	Area = $\frac{1}{2}\begin{bmatrix} -2 & -18 & 2 & 18 & -2 \\ 6 & -10 & -30 & -14 & 6 \end{bmatrix} = 640$	640	M1A1		
			(18)		

USEFUL SKETCH



Question	Working	Answer	Mark	AO	Notes
11	Total Surface Area of the cone $l = \dfrac{x}{\sin 30^\circ} = 2x$ $A = \pi rl + \pi r^2 \Rightarrow \pi x \times 2x + \pi x^2 = 3\pi x^2$ Volume of cone $h = \dfrac{x}{\tan 30^\circ} = \sqrt{3}x$ $\text{Vol} = \dfrac{1}{3}\pi r^2 h \Rightarrow V = \dfrac{\sqrt{3}}{3}\pi x^3$ $\dfrac{\mathrm{d}A}{\mathrm{d}t} = 10 \text{ (cm}^2/s) \quad \dfrac{\mathrm{d}V}{\mathrm{d}x} = \sqrt{3}\pi x^2, \dfrac{\mathrm{d}A}{\mathrm{d}x} = 6\pi x$ $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}x} \times \dfrac{\mathrm{d}A}{\mathrm{d}t} \times \dfrac{\mathrm{d}x}{\mathrm{d}A}$ $\Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}t} = \sqrt{3}\pi x^2 \times 10 \times \dfrac{1}{6\pi x} = \dfrac{5\sqrt{3}}{3}x \Rightarrow 10\sqrt{3} \text{ (cm}^3/s)$	$10\sqrt{3}$ (cm^3 / s)	B1 M1A1 B1 M1A1 B1M1 M1 M1A1 **(11)**	2, 3	M1 for differentiating **either** $\dfrac{\mathrm{d}V}{\mathrm{d}x}$ or $\dfrac{\mathrm{d}A}{\mathrm{d}x}$
		Total	**100**		

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