Mark Scheme (Results)

January 2017

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM0) Paper 1

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January 2017
Publications Code 4PM0_01_1701_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.
- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part ofthe question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$, where $|pq|=|c|$ leading to $x=....$

$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Jan 2017
4PMO Further Pure Mathematics Paper 1
Mark Scheme

Question number	Scheme	Marks
1	Mark both parts of this question together $18\pi = \theta r$ $126\pi = \frac{1}{2}\theta r^2 \quad \left(\Rightarrow 252\pi = \theta r^2\right)$ $\frac{252\pi}{18\pi} = \frac{\theta r^2}{\theta r} \Rightarrow 14 = r$ $18\pi = \theta \times 14 \Rightarrow \theta = \frac{9\pi}{7}$ oe **ALT** $A = \frac{1}{2}rl \Rightarrow 126\pi = \frac{1}{2} \times r \times 18\pi \Rightarrow r = 14$ $18\pi = 14\theta \Rightarrow \theta = \frac{9}{7}\pi$	B1 B1 M1A1 A1 **(5)** M1A1A1 B1B1 **(5)**

Notes	
B1	For the equation (or any equivalent) $18\pi = \theta r$
B1	For the equation (or any equivalent) $126\pi = \frac{1}{2}\theta r^2$
M1	For dividing their two equations, eliminating θ **and** finding a value for r
A1	For $r = 14$ (cm)
A1	For $\theta = \frac{9\pi}{7}$ oe
ALT	
M1	Attempts to use the (correct) $A = \frac{1}{2}rl$ formula to give $126\pi = \frac{1}{2} \times r \times 18\pi$
A1	Substitution of correct values of $A = 126\pi$ and $l = 18\pi$
A1	For r = 14 (cm)
B1	For the equation (or any equivalent) $18\pi = \theta \times$ their r
B1	For $\theta = \frac{9\pi}{7}$ oe
ALT using degrees	
B1	For the equation $\frac{\theta}{360} \times 2\pi r = 18\pi \Rightarrow \frac{\theta}{360} r = 9$
B1	For the equation $\frac{\theta}{360} \times \pi r^2 = 126\pi \Rightarrow \frac{\theta}{360} r^2 = 126$
M1	Divides their equations to eliminate θ to give $9r = 126$
A1	For $r = 14$
A1	For $\theta = \frac{9\pi}{7}$ oe

Question number	Scheme	Marks
2 **(a)**	$\text{f}(4) = 2\times 4^3 - 3p\times 4^2 + 4 + 4p = 0 \Rightarrow 128 + 4 = 48p + 4p \Rightarrow p = 3$ *	M1A1 (2)
(b)	$\text{f}(-2) = 2(-2)^3 - 9(-2)^2 + (-2) + 12 = -42$	M1A1 (2)
(c)	$\dfrac{2x^3 - 9x^2 + x + 12}{x-4} = 2x^2 - x - 3 = (x+1)(2x-3) \Rightarrow$	M1A1
	$2x^3 - 9x^2 + x + 12 = (x-4)(x+1)(2x-3)$	A1 (3)
(d)	$(x-4)(x+1)(2x-3) = 0 \Rightarrow x = 4, x = -1, x = \dfrac{3}{2}$	M1A1 (2) **(9)**

Notes

(a)	M1	For **either** f (–4) **or** f (4), equating f (±4) = 0 and finding a value for *p*. For the award of this mark the method must be complete.
	A1	$p = 3$
(b)	M1	For **either** f (–2) **or** f (2) and finding a value for f (±2) using the given *p*. For the award of this mark the method must be complete. **Division** Divides by $(x+2)$ and achieves at least $2x^2 - 13x + k$ (complete method)
	A1	$\text{f}(-2) = -42$ or remainder of -42 using division
(c)	M1	Divides f (*x*) – by $(x-4)$ or $(x+1)$ any method, achieves at least $2x^2 \pm ax \pm b$ where $a \neq 0,\ b \neq 0$, and attempts to factorise their 3TQ. (See general guidance for an acceptable attempt) Note: $(2x^3 - 9x^2 + x + 12) \div (x+1) = 2x^2 - 11x + 12$ **OR** by inspection; $(x-4)$ and $(x+1)$ are factors, hence third factor is $(2x \pm a)$
	A1	For achieving $2x^2 - x - 3 = (x+1)(2x-3)$ or $2x^2 - 11x + 12 = (2x-3)(x-4)$
	A1	For the correct factorisation of f (*x*) = $(x-4)(x+1)(2x-3)$
(d)	M1	For setting f (*x*) = 0 (can be implied by further work) **and** attempting to solve a factorised f (*x*) = 0. ie., $(x \pm 4)(x + '1')('2'x - '3') = 0 \Rightarrow x = \pm 4, '-1', '\dfrac{3}{2}'$
	A1	For $x = 4, x = -1, x = \dfrac{3}{2}$ Note: answers must be derived from correct algebra

Question number	Scheme	Marks
3	$3x^2-4x+1<6x-2 \Rightarrow 3x^2-10x+3<0$	M1
	$(x-3)(3x-1)<0 \Rightarrow$ c.v's $x=3, x=\frac{1}{3}$	M1A1
	Inside region for their values $\frac{1}{3}<x<3$	M1A1
		(5)

Notes

M1	For multiplying out the given inequality and achieving a 3TQ. Min acceptable 3TQ is $3x^2+bx+c$ Allow; $3x^2+bx+c$ = 0, $3x^2+bx+c$ < 0, $3x^2+bx+c$ > 0 or use of ≤ or ≥ or even just $3x^2+bx+c$
M1	For solving their 3TQ (see general guidance for the definition of an attempt) **and** finding two critical values
A1	For $x=3, x=\frac{1}{3}$
M1	For choosing the **INSIDE** region for **their** cvs.
A1	For a correctly defined region as shown $\frac{1}{3}<x<3$ Accept $\frac{1}{3}<x$ AND $x<3$ Do **not** accept $\frac{1}{3}<x$ OR $x<3$ (This is M1A0) Allow use of set language $\frac{1}{3}<x \cap x<3$ but **not** $\frac{1}{3}<x \cup x<3$ (This is M1A0)

NB:
Cancelling through by $(3x-1)$ and stating x < 3 is M0M0A0M0A0

The question states **'using algebra'.** There must be a minimal amount of working to award marks.

For just $\frac{1}{3}<x<3$ without evidence of algebra M0M0A0M0A0

Minimally acceptable attempt is as follows;

$(3x-1)(x+1)$ OR $(3x-1)(x-3) \Rightarrow x=\frac{1}{3}, -1$ or 3

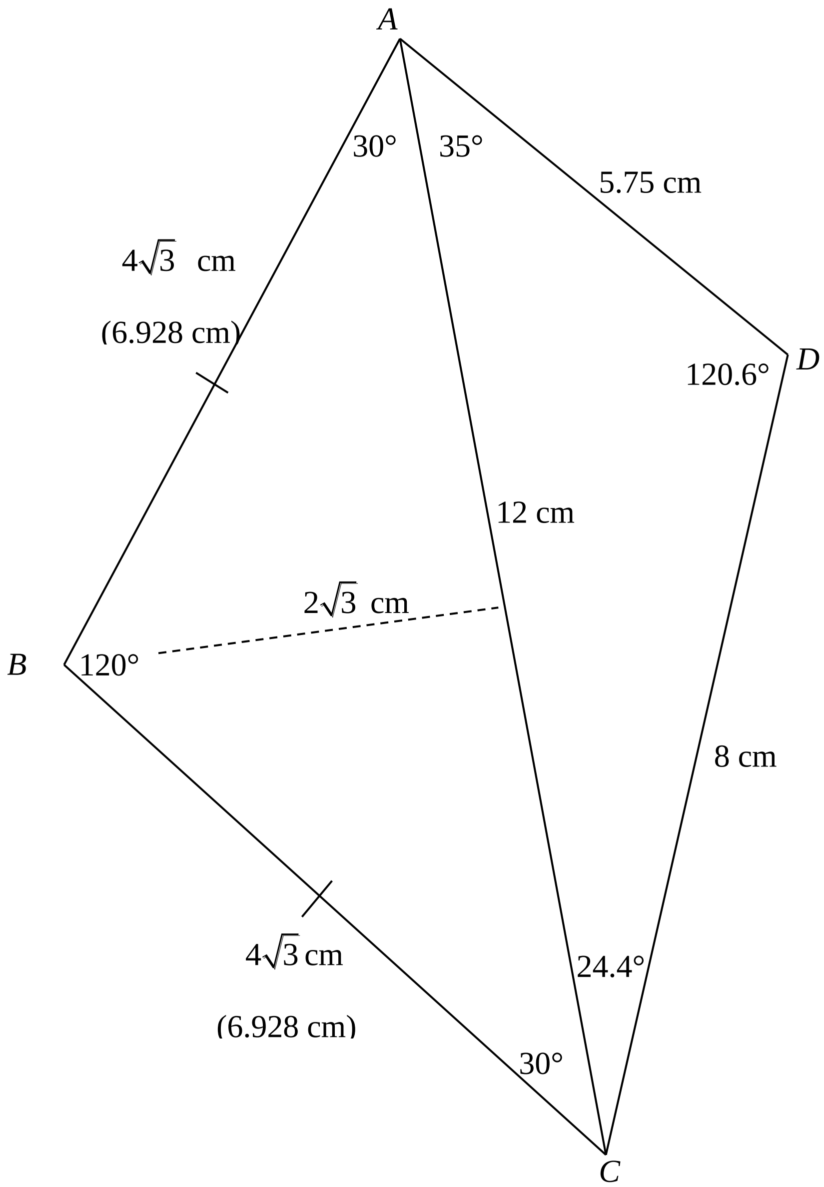
Question number	Scheme	Marks
4 (a)	$ar + ar^4 = \dfrac{28}{81}$, $ar - ar^4 = \dfrac{76}{405}$	M1
	$ar = \dfrac{4}{15}$, $ar^4 = \dfrac{32}{405}$	M1A1
(i)	$\dfrac{ar^4}{ar} = \dfrac{32}{405} \div \dfrac{4}{15} = \dfrac{8}{27} \Rightarrow r = \dfrac{2}{3}$ *	M1A1
(ii)	$a = \dfrac{2}{5}$	B1
		(6)
(b)	$S = \dfrac{\frac{2}{5}}{1 - \frac{2}{3}} = \dfrac{6}{5}$	M1A1
		(2)
		(8)

Notes		
(a)	M1	For setting up **both** equations for the sum and the difference. Accept any letter for the first term.
	M1	Adds or subtracts their equations to eliminate ar or ar^4
	A1	For both correct $ar=\dfrac{4}{15}$ and $ar^4=\dfrac{32}{405}$
(i)	M1	Divides ar^4 by ar to achieve an equation for r^3
	A1	For $r=\dfrac{2}{3}$ **Note:** This is a given result and every step must be shown to achieve this mark
(ii)	A1	For $a=\dfrac{2}{5}$ oe
ALT 1 for part (a)		
(a)	M1	Sets up **both** equations for the sum and the difference $ar+ar^4=ar\left(1+r^3\right)=\dfrac{28}{81}$ $\qquad ar-ar^4=ar\left(1-r^3\right)=\dfrac{76}{405}$
	M1	Factorises and divides equations above to eliminate ar to give $\left[ar\left(1+r^3\right)=\dfrac{28}{81}\right]\div\left[ar\left(1-r^3\right)=\dfrac{76}{405}\right]=\dfrac{\left(1+r^3\right)}{\left(1-r^3\right)}=\dfrac{28/81}{76/405}\ \left(=\dfrac{35}{19}\right)$
	A1	Achieves a correct equation in r^3 or r^4 $\dfrac{1+r^3}{1-r^3}=\dfrac{28\times405}{81\times76}$ or $\dfrac{r+r^4}{r-r^4}=\dfrac{28\times405}{81\times76}$
(i)	M1	Attempts to solve their equation in r^3 as far as r =
	A1	For $r=\dfrac{2}{3}$ **Note:** This is a given result so every step must be seen.
(ii)	B1	For $a=\dfrac{2}{5}$ oe
ALT 2 for part (a) using t_2 and t_5 or any other letters e.g x,y		
(a)	M1	Solves SE by elimination to give: $t_2+t_5=\dfrac{28}{81}$ and $t_2-t_5=\dfrac{76}{405}\Rightarrow t_2=\dfrac{4}{15}$ OR $t_5=\dfrac{32}{405}$
	M1	$t_2=ar=\dfrac{4}{15}$ OR $t_5=ar^4=\dfrac{32}{405}$ — Award these marks when they identify and use $t_2=ar=\dfrac{4}{15}$, $t_5=ar^4=\dfrac{32}{405}$
	A1	$t_2=ar=\dfrac{4}{15}$ AND $t_5=ar^4=\dfrac{32}{405}$
Then follow ms for (i) and (ii).		
(b)	M1	Uses **correct** formula for the sum to infinity of a geometric series $S=\dfrac{a}{1-r}=\dfrac{\text{their } a}{1-\dfrac{2}{3}}='\dfrac{6}{5}'$ They must reach a value for S_∞ for this mark
	A1	For $S=\dfrac{6}{5}$

Question number	Scheme	Marks
5.		
(a)	$12^2 = 2BA^2 - 2\times BA\times BC\times\cos 120 \Rightarrow 144 = 3AB^2 \Rightarrow AB = \sqrt{48} = (4\sqrt{3})$	M1A1
	ALT	
	$AB = \dfrac{12\sin 30}{\sin 120} = 4\sqrt{3}$ (6.9282...)	(M1A1)
		(2)
(b)	$\dfrac{\sin D}{12} = \dfrac{\sin(35)}{8} \Rightarrow D = \sin^{-1}\left(\dfrac{12\sin(35)}{8}\right) = 59.357...$	M1A1
	$D = 180 - 59.3755 = 120.64245.. \approx 120.6$	A1ft
		(3)
(c)	ACD= 24.3541°	B1
	Area of ABC = $\dfrac{1}{2}\times\left(\sqrt{48}\right)^2\times\sin 120 = 12\sqrt{3}\,(=20.78...)$	M1A1
	Area of ADC = $\dfrac{1}{2}\times 12\times 8\times\sin(24.3576...) = 19.7966...$	M1A1
	Area of $ABCD$= 40.5812… = 40.6 cm^2 (3sf)	A1
		(6)
	ALT	
	$\left[AD = \dfrac{8\sin('24.3576..')}{\sin(35)} = 5.7524..\right]$	(B1)
	Area of ABC = $\dfrac{1}{2}\times\left(\sqrt{48}\right)^2\times\sin 120 = 12\sqrt{3}$	(M1A1)
	Area ADC = $\dfrac{1}{2}\times 5.752...\times 8\times\sin(120.6424...) = 19.7966...$	(M1A1)
	Area of $ABCD$ = 40.5812… = 40.6 cm^2 (3sf)	(A1) (6)
		(11)

Notes		
(a)	M1	Uses a correct cosine rule to find length *AB*
	A1	For $AB = 4\sqrt{3}$
ALT 1		
(a)	M1	For using a correct sine rule to find length *AB*
	A1	For $AB = 4\sqrt{3}$
ALT 2		
(a)	M1	Divides triangle *ABC* into two congruent right angle triangles. $AB = \frac{6}{\sin 60^{\circ}}$
	A1	For $AB = 4\sqrt{3}$
(b)	M1	For using a correct sine rule to find $\angle ADC$
	A1	For the acute angle resulting from their sine rule = 59.357...$^{\circ}$ (accept minimum accuracy of 59.4°)
	A1	For the correct obtuse angle $\angle ADC = 120.6^{\circ}$
The general principle of marking part (c) is; First M1A1 for triangle *ABC*, second M1A1 for triangle *ADC*		
(c)	B1	$\angle ACD = 24.3576^{\circ}$ (accept minimum accuracy of 24.4°)
	M1	Area of ΔABC using correct formula for area of a triangle using 120° and their length *AB* or *BC* (but their *AB* = *BC*)
	A1	Area ΔABC = $12\sqrt{3}$ (oe., accept minimum accuracy of 20.8)
	M1	Area of ΔADC using correct formula and their $\angle ADC$ and the given lengths 12 cm and 8 cm.
	A1	Area ΔADC = 19.79662... (accept minimum 19.8)
	A1	Area of quadrilateral *ABCD* = 40.6 (cm^2)
ALT 1		
(c)	B1	For finding length *AD* = 5.7524.. (accept minimum accuracy of 5.7)
	M1	Area of ΔABC using correct formula for area of a triangle using 120° and their length *AB* or *BC* (but their *AB* = *BC*)
	A1	For substitution of correct values. [Area ΔABC = $12\sqrt{3}$ (oe., accept minimum accuracy of 20.8)]
	M1	Area of using correct formula and their *AD* and the given length 12 cm and angle 35°.
	A1	For substitution of correct values. [Area ΔADC = 19.79662... (accept minimum 19.8)]
	A1	Area of quadrilateral *ABCD* = 40.6 (cm^2)
ALT 2		
(c)	B1	Divides triangle *ABC* into two congruent right angle triangles. (midpoint of *AB* is *M*) $BM = \frac{6}{\tan 60^{\circ}} = 2\sqrt{3}$ accept 3.46...
	M1	Area of ΔABC using 2 × correct formula for area of a triangle $2 \times \frac{1}{2} \times 6 \times '2\sqrt{3}' = '12\sqrt{3}'$
	A1	Area ΔABC = $12\sqrt{3}$ (oe., accept minimum accuracy of 20.8)
Areas of ΔADC and quadrilateral *ABCD* as above.		

Useful Sketch



Area $ABC = 12\sqrt{3}$ or $20.78...$ cm^2

Area of $ADC = 19.79...$ cm^2

Total area = 40.6 cm^2

Penalise rounding only once. If they their answer to (b) as awrt120.6 (e.g.120.64) deduct the A mark. If they then give their answer to (c) as 40.61 do not penalise.

Question number	Scheme	Marks
6. **(a)**	$a = 2$, $b = 3$	B1B1 (2)
(b)	At intersection of the curve with the y-axis , $x = 0$ $y = \dfrac{3\times 0 + c}{0 + '2'} = \dfrac{c}{'2'}\left(= \dfrac{7}{2}\right) \Rightarrow c = 7$	M1A1 (2)
(c)	At intersection of the curve with the x-axis, $y = 0$ $0 = \dfrac{'3'x + '7'}{x + '2'} \Rightarrow '3'x + '7' = 0 \Rightarrow x = -\dfrac{7}{3} \Rightarrow s = -\dfrac{7}{3}$	M1A1ft (2) **(6)**

Notes

(a)	B1	For $a = 2$ **or** $b = 3$
	B1	For $a = 2$ **and** $b = 3$
(b)	M1	For using the given equation and setting $x = 0$ and $y = 3.5$ (oe) . They must achieve a value for c for the award of this mark Follow through their values for a and b. If their b is incorrect or they even use the letter b allow $b\times 0 = 0$.
	A1	$c = 7$
(c)	M1	Uses their values for a, b and c and sets $y = 0$. They must achieve a value for x for the award of this mark
	A1ft	For $s = -\dfrac{7}{3}$

Question number	Scheme	Marks
7. (a)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>y</td><td>2</td><td>**3.79**</td><td>4.40</td><td>4.77</td><td>5.04</td><td>**5.26**</td><td>5.43</td><td>**5.58**</td></tr></table>	B1B1 (2)
(b)	Correct points plotted	B1B1 (2)
(c)	$\ln(5x+1)=x \Rightarrow \ln(5x+1)+2=x+2$ Line $y=x+2$ drawn $\Rightarrow x=2.6$ or 2.7	M1M1A1 (3)
(d)	$e^{(3x-1)}=5x+1 \Rightarrow 3x-1=\ln(5x+1) \Rightarrow 3x+1=\ln(5x+1)+2$ Line $y=3x+1$ drawn on graph $\Rightarrow x=0.9$	M1M1 M1A1 (4) **(11)**

Notes

(a)	B1	For any two of three correct values, correctly rounded
	B1	For all three correct values, correctly rounded
NB: Accept for B0B1 three values which all round to the correctly rounded values.		
(b)	B1ft	Their points plotted correctly to within half of one square
	B1ft	Their points joined up in a smooth curve from x = 1 onwards. Allow a straight line between x = 0 and 1.
Note: these follow through marks are from their table only.		
(c)	M1	For forming the linear equation $\ln(5x+1)+2=x+2$ or for identifying that the line with equation $y=x+2$ is required. This can be implied from a correct line drawn.
	M1	For drawing their ‘ $y=x+2$ ’ Coordinates of the correct line $y=x+2$ are (0,2) , (1,3) , (2,4) , (3,5) etc
	A1	For x = 2.6 or 2.7 (Note: must be 1 dp)
(d)	M1	For taking natural logarithms of both sides of the given equation to give $3x-1=\ln(5x+1)$
	M1	For forming the linear equation $\ln(5x+1)+2=3x+1$ or for identifying that the line with equation $y=3x+1$ is required. This can be implied from a correct line drawn.
	M1	For drawing their ‘ $y=3x+1$ ’. Coordinates of the correct line $y=3x+1$ are (0,1), (1,4)
	A1	For x = 0.9 Do not penalise rounding in (d) if penalised in (c). The value in (d) must round to 0.9.

Question number	Scheme	Marks
8. **(a) (i)**	$\left(1+\frac{x}{2}\right)^{-3}=\left[1+(-3)\left(\frac{x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{x}{2}\right)^3\text{.........}\right]$	M1
	$=1-\frac{3x}{2}+\frac{3x^2}{2}-\frac{5x^3}{4}$	A1A1
(ii)	$-2<x<2$	B1 (4)
(b)	$(2+x)^{-3}=2^{-3}.\left(1+\frac{x}{2}\right)^{-3}=\frac{1}{8}.\left(1+\frac{x}{2}\right)^{-3}$ so, $A=\frac{1}{8}$, $B=\frac{1}{2}$	B1B1 (2)
(c)	$\frac{(1+4x)}{(2+x)^3}=(1+4x)\left(\frac{1}{8}-\frac{3x}{16}+\frac{3x^2}{16}-\frac{5x^3}{32}...\right)=\frac{1}{8}+\frac{5x}{16}-\frac{9x^2}{16}...$	M1A1 (2)
(d)	$\int_0^{0.2}\frac{(1+4x)}{(2+x)}\mathrm{d}x=\int_0^{0.2}\frac{1}{8}+\frac{5x}{16}-\frac{9x^2}{16}\mathrm{d}x=\left[\frac{x}{8}+\frac{5x^2}{32}-\frac{3x^3}{16}\right]_0^{0.2}=0.0298$	M1dM1A1 (3) **(11)**

Notes

(a) (i)	M1	For an attempt at a binomial expansion. There must be as a minimum; the expansion must start with 1; there must be a minimum of 4 terms (accept a list); the power of *x* must be correct; the factorial denominator must be correct. $\frac{x}{2}$ must be seen at least once.
	A1	Two terms in *x* simplified and correct
	A1	Fully correct as shown ie., $1-\frac{3x}{2}+\frac{3x^2}{2}-\frac{5x^3}{4}$
(ii)	B1	For $-2<x<2$ or $\lvert x\rvert<2$
(b)	B1	For $A=\frac{1}{8}$ **OR** $B=\frac{1}{2}$ or embedded as $\frac{1}{8}\left(1+\frac{1}{2}x\right)^{-3}$ **OR** $\frac{1}{8}\left(1+\frac{x}{2}\right)^{-3}$
	B1	For $A=\frac{1}{8}$ **AND** $B=\frac{1}{2}$ or embedded as $\frac{1}{8}\left(1+\frac{1}{2}x\right)^{-3}$ **AND** $\frac{1}{8}\left(1+\frac{x}{2}\right)^{-3}$
(c)	M1	For expanding $(1+4x)(\text{their A})(\text{their expansion from (a) at least as far as } x^2)$
	A1	Fully correct as shown $\frac{1}{8}+\frac{5x}{16}-\frac{9x^2}{16}$ ignore further terms
(d)	M1	For attempting to integrate their answer to part (c) (minimum of two terms) For an attempt to integrate, see general guidance
	dM1	For substituting 0.2 (0 not required) into their **integrated** expression.
	A1	For a value of 0.0298 only

Note: If there is no evidence of integration in (d) M0M0A0

Question number	Scheme	Marks
9. **(a) (i)**	$\alpha+\beta=\left(\frac{4}{3}\right)$	B1
(ii)	$\alpha\beta=\frac{6}{3}=2$	B1 (2)
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\Rightarrow\left(\frac{4}{3}\right)^3-3\times2\times\left(\frac{4}{3}\right)=-\frac{152}{27}$ *	M1M1A1 (3)
(c)	$\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}=\frac{-\frac{152}{27}}{4}=-\frac{38}{27}$	M1A1
	$\frac{\alpha}{\beta^2}\times\frac{\beta}{\alpha^2}=\frac{1}{\alpha\beta}=\frac{1}{2}$	B1
	$x^2+\frac{38}{27}x+\frac{1}{2}=0\Rightarrow54x^2+76x+27=0$ oe (integer multiples)	M1A1 (5) **(10)**

Notes		
(a) (i)	B1	For the sum $\alpha+\beta=\left(\frac{4}{3}\right)$
(ii)	B1	For the product $\alpha\beta=\frac{6}{3}$ oe
(b)	M1	For the **correct** algebra to find $\alpha^3+\beta^3$ e.g., $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ $\alpha^3+\beta^3=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ Their final expansion must be given in a form such that they can substitute their sum and product directly.
	M1	For substituting their values for the sum and product into their $\alpha^3+\beta^3$ Note $\alpha^2+\beta^2=-\frac{20}{9}$
	A1	For $-\frac{152}{27}$ **Note:** This is a 'show' question. Every step must be correct for the award of this mark.

(c)	M1	For the correct algebra on the sum $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$ **and** substitution of their $\alpha+\beta$ and $\alpha\beta$.
	A1	For the correct sum of $-\frac{38}{27}$ allow $\frac{-\frac{152}{27}}{4}$
	B1	For the correct product of $\frac{1}{2}$
	M1	For using their sum and their product correctly to form an equation. $\left(x^2+(-\text{sum})\times x+\text{product}\right)=0$ (condone missing = 0)
	A1	For the correct equation as shown. Accept any integer multiples. e.g $108x^2+152x+54=0$ etc
ALT (c)	M1	Attempts to form the equation as follows. Must be $-$ve sum, + ve product $\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2-\left(-x\left(\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}\right)\right)+\frac{\alpha\beta}{(\alpha\beta)^2}$ $(=0)$
	M1	$\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2-\left(-x\left(\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}\right)\right)+\frac{\alpha\beta}{(\alpha\beta)^2}$ Correct algebra only
	First A1	$\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2+x\left(\frac{\mathbf{\frac{152}{27}}}{\mathbf{4}}\right)+\frac{\alpha\beta}{(\alpha\beta)^2}=x^2+x\left(\frac{\mathbf{38}}{\mathbf{27}}\right)+\frac{\alpha\beta}{(\alpha\beta)^2}$
	B1	$\left(x-\frac{\alpha}{\beta^2}\right)\left(x-\frac{\beta}{\alpha^2}\right)=x^2+x\left(\frac{38}{27}\right)+\frac{\mathbf{2}}{\mathbf{4}}$
	Final A1	$x^2+\frac{38}{27}x+\frac{1}{2}=0\Rightarrow 54x^2+76x+27=0$ oe with integer multiples

Question number	Scheme	Marks
10. **(a)**	$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = 3t^2 - 8t + 5$	M1A1 (2)
(b)	$3t^2 - 8t + 5 = 0 \Rightarrow (3t-5)(t-1) = 0 \Rightarrow t = \dfrac{5}{3}, 1$	M1A1 (2)
(c)	$s = \int t^3 - 4t^2 + 5t + 1 = \dfrac{t^4}{4} - \dfrac{4t^3}{3} + \dfrac{5t^2}{2} + t + c$	M1A1
	When $t = 0, s = 3 \Rightarrow c = 3$	B1
	$s = \dfrac{t^4}{4} - \dfrac{4t^3}{3} + \dfrac{5t^2}{2} + t + 3$	dM1
	So $s = 8\dfrac{1}{3}$ m	A1 (5)
	ALT	
	$s = 3 + \int_0^2 t^3 - 4t^2 + 5t + 1 \, \mathrm{d}x = 3 + \left[\dfrac{t^4}{4} - \dfrac{4t^3}{3} + \dfrac{5t^2}{2} + t\right]_0^2 = 8\dfrac{1}{3}$ m	{M1A1B1
	For correct substitution and evaluation	dM1A1} {(5)} **(9)**

Notes		
(a)	M1	For an attempt to differentiate the given v. See general guidance for the definition of an attempt
	A1	For the correct $a = 3t^2 - 8t + 5$
(b)	M1	Sets their a = 0 **and** attempts to solve their 3TQ. They must achieve 2 values only for t for the award of this mark.
	A1	For $t = \frac{5}{3}, 1$
Please check the whole method in part (c) before you begin to award marks.		
(c)	M1	Attempts to integrate the given v. See general guidance for the definition of an attempt. Award this mark if the constant of integration is not seen.
	A1	For the correct integrated expression for s, which **must** include $+c$.
	B1	For c = 3 (Or any other letter given for the constant of integration)
	dM1	For substituting the value of t = 2 into an **integrated** expression
	A1	For $s = 8\frac{1}{3}$
ALT 1		
(c)	M1	Attempts to integrate the given v. See general guidance for the definition of an attempt. The limits of integration not required for this mark
	A1	For the correct integrated expression
	B1	For +3
	dM1	For substituting their limits of integration.
	A1	For $s = 8\frac{1}{3}$ Note: if their limits were the wrong way around they will achieve $s = -8\frac{1}{3}$. Even if they give the final answer as $s = 8\frac{1}{3}$ this is A0.
ALT 2 Only apply this scheme when see they have added the additional displacement of 3m at t = 0		
	M1	Attempts to integrate the given v. See general guidance for the definition of an attempt. The limits of integration not required for this mark
	A1	For the correct integrated expression $+c$ not required
	dM1	For substituting the value of t = 2 into an **integrated** expression
	A1	For achieving $s = \frac{16}{3}$
	B1	For adding 3 to their s to achieve $s = \frac{25}{3}$ oe

Question number	Scheme	Marks
11. **(a)**	Mark parts (i) and (ii) together $f'(x) = p + 2qx = 0 \Rightarrow p + 2q(3) = 0 \Rightarrow p + 6q = 0$	M1
	$9 = p(3) + q(3)^2 \Rightarrow 9 = 3p + 9q \Rightarrow (3 = p + 3q)$	M1A1
	Solves simultaneous equations by substitution or elimination	
	(i) $[6p + q = 0] - [3 = p + 3q] = 3q = -3 \Rightarrow q = -1 \Rightarrow p = 6$	M1A1
	$q = -1$	B1
	(ii) $f''(x) = -2 \Rightarrow$ negative constant so point is a maximum	B1 (7)
(b)	$-x + 10 = 6x - x^2 \Rightarrow 0 = x^2 - 7x + 10 \Rightarrow (x-2)(x-5) = 0 \Rightarrow x = 2, 5$	M1M1A1 (3)
(c)	Volume $= \pi\int_2^5 (-x^2 + 6x)^2 \mathrm{d}x - \pi\int_2^5 (-x + 10)^2 \mathrm{d}x$	M1
	Volume $= \pi\int_2^5 \{(x^4 - 12x^3 + 36x^2) - (x^2 - 20x + 100)\}\mathrm{d}x$	
	$= \pi\left[\frac{x^5}{5} - 3x^4 + \frac{35}{3}x^3 + 10x^2 - 100x\right]_2^5$ (or integrate without simplification)	M1A1
	$= \pi\left[625 - 3\times 625 + \frac{35\times 125}{3} + 250 - 500\right] - \left[\frac{32}{5} - 48 + \frac{35\times 8}{3} + 40 - 200\right]$	M1
	$V = \frac{333\pi}{5}$	A1 (5) **(15)**

Notes		
(a)	M1	Attempts to differentiate the given equation for curve *C*, equates to 0, **and** substitutes in $x = 3$ to form an equation in *p* and *q*.
	M1	Substitutes (3,9) into the given equation to form an equation in *p* and *q*.
	A1	For both correct equations; $p+6q=0$ and $3=p+3q$ or any equivalent to either equation.
	M1	Attempts to solve the simultaneous equations by any method.
	A1	For $p = 6$. This is a show so check that the method is correct.
	B1	For $q=-1$
	B1	Finds the second derivate, substitutes the value of *q* and finds $f''(x)=-2$ **with a conclusion** hence maximum. E.g. Minimally acceptable -2 hence maximum
		OR Completes the square to show that the maximum value of *y* is 9 when $x = 3$ $y=-x^2+6=-(x^2-6)=-\left[(x-3)^2-9\right]=-(x-3)^2+9$ **with a conclusion** that the maximum value of $y = 9$ occurs when $x = 3$
(b)	M1	Sets the equation of *l* = equation of *c* with their values of *p* and *q* and forms a 3TQ.
	M1	Attempts to solve their 3TQ by any method, but must achieve **two** values of *x*.
	A1	For $x=2,5$
Marks in part (c) are dependent on their method being dimensionally correct and complete		
(c)	**Method 1 (Combined integration)**	
	M1	For a statement using the correct formula for the volume of rotation $V=\pi\int y^2\ \mathrm{d}x$, using the equation for *C* with their value of *q*, minus the equation for line *l* rearranged to make *y* the subject. Ignore missing d*x* and ignore limits for this mark. π must be present and the equations must be squared.
	M1	For integrating their statement for *V*. Their limits of integration found in (b) must be shown, the correct way around for the award of this mark. The highest power of *x* must be a term in x^4. Ignore missing π for this mark.
	A1	For the correct integrated expression for *V*, complete with limits. It need not be simplified for this mark and ignore missing π for this mark.
	ddM1	For substituting in both of their values from (b) and subtracting them.
	A1	For the correct volume in terms of π only of $V=\dfrac{333\pi}{5}$ or 66.6π oe. isw erroneous attempts to simplify after 66.6π oe seen

	Method 2 (Integration of curve and volume of truncated cone)	
	M1	For a statement using the correct formula for the volume of rotation $V=\pi\int y^2\ \mathrm{d}x$, using the equation for C with their value of q. π must be present and the equations must be squared. Evidence of an attempt to find the volume of a truncated cone must be seen for this mark.
	M1	For integrating their statement for V. Their limits of integration found in (b) must be shown, the correct way around for the award of this mark, and substituted into their integrated expression. The highest power of x must be a term in x^4. Ignore missing π for this mark.
	A1	For the correct volume for C $\left(V=195.6\ (\pi)\right)$
	ddM1	For a correct method to find the volume of a truncated cone using their values of x from (b) to find y and substitute into the volume of a truncated cone. When $x=5, y=5$ and $x=2, y=8$ $V=\frac{1}{3}\times\pi\times'8'^2\times'8'-\frac{1}{3}\times\pi\times'5'^2\times'5'$ $(=129\pi)$
	A1	For the correct volume in terms of π only of $V=\frac{333\pi}{5}$ or 66.6π oe. isw erroneous attempts to simplify after 66.6π oe seen
	Method 3 (Integration of curve and line separately)	
	M1	For a statement using the correct formula for the volume of rotation $V=\pi\int y^2\ \mathrm{d}x$, using the equation for C with their value of q. Ignore missing dx and ignore limits for this mark. π must be present and the equation must be squared. **AND** For a statement using the correct formula for the volume of rotation $V=\pi\int y^2\ \mathrm{d}x$, using the equation for l with their values of p and q. Ignore missing dx and ignore limits for this mark. π must be present and the equation must be squared.
	M1	For integrating their statements for V. Their limits of integration found in (b) must be shown, the correct way around for the award of this mark. The highest power of x in C must be a term in x^4, and x^2 in l. Ignore missing π for this mark.
	A1	For the correct integrated expressions for C and l, complete with limits. They need not be simplified for this mark and ignore missing π for this mark.
	ddM1	For substituting in their values from (b) and subtracting them. **AND** subtracting the volume of the truncated cone fromm the volume of the curve.
	A1	For the correct volume in terms of π only of $V=\frac{333\pi}{5}$ or 66.6π oe. isw erroneous attempts to simplify after 66.6π oe seen
NOTE: Volume of revolution of C = 195.6π Volume of truncated come = 129π		

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