

Write your name here

Surname

Other names

**Edexcel**

**International GCSE**

Centre Number

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Candidate Number

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# Further Pure Mathematics

## Paper 1

Thursday 19 January 2012 – Morning

**Time: 2 hours**

Paper Reference

**4PM0/01**

**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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6/6/6/6/4/2/2



**PEARSON**

**Answer all ELEVEN questions**

**Write your answers in the spaces provided**

**You must write down all stages in your working**

**1** Show that the two lines with equations

$$6x + 4y = -15$$

$$10x - 15y = 9$$

are perpendicular.

(4)

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**(Total for Question 1 is 4 marks)**



2 Solve the equation

$$\frac{x}{x+1} - \frac{1}{x+2} = 2$$

Give your answers correct to 3 significant figures.

(4)

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(Total for Question 2 is 4 marks)



3 Solve the inequality  $6x^2 - 19x - 7 < 0$

(4)

Handwriting practice area consisting of 20 horizontal dotted lines.

(Total for Question 3 is 4 marks)



4 Find the coefficient of  $x^7$  in the expansion of  $\left(1 + \frac{x}{\sqrt{3}}\right)^{10}$ , giving your answer in the form  $a\sqrt{3}$ , where  $a$  is a rational number.

(4)

(Total for Question 4 is 4 marks)



5 Differentiate with respect to  $x$

(a)  $y = x^2 e^x$

(2)

(b)  $y = (x^3 + 2x^2 + 3)^5$

(3)

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6 For  $x$  radians,

$$y = 3 \cos \frac{x}{2}$$

(a) Complete the table, giving the three missing values correct to 2 decimal places.

(2)

$x$	0	0.5	1	1.5	2	2.5	3	3.5
$y$	3		2.63	2.20		0.95	0.21	

On the axes opposite,

(b) draw the graph of  $y = 3 \cos \frac{x}{2}$  for  $0 \leq x \leq 3.5$

(2)

(c) Using your graph, find an estimate, to 1 decimal place, for the root of the equation

$$2x = 1 + 2 \cos \frac{x}{2}$$

(4)

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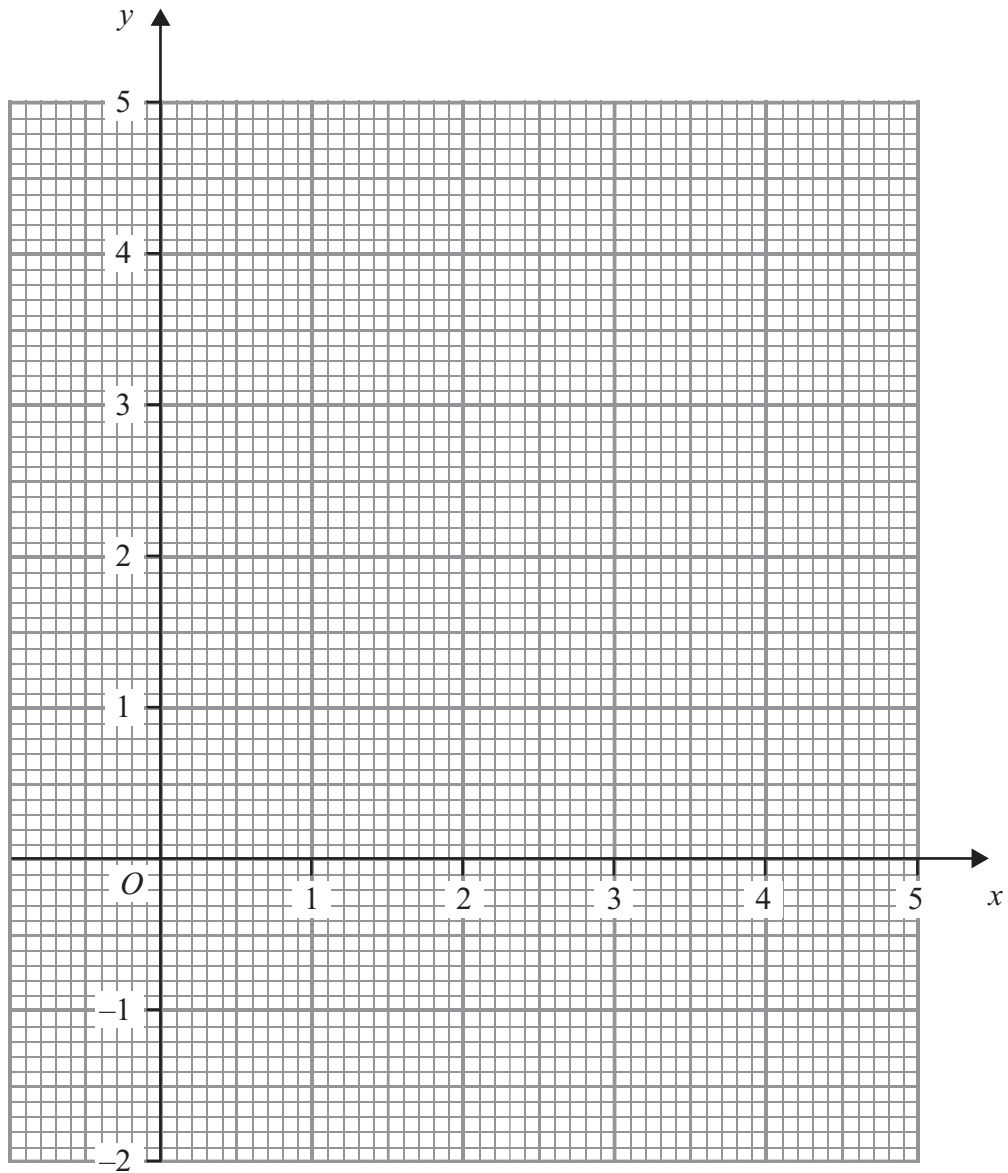
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Question 6 continued



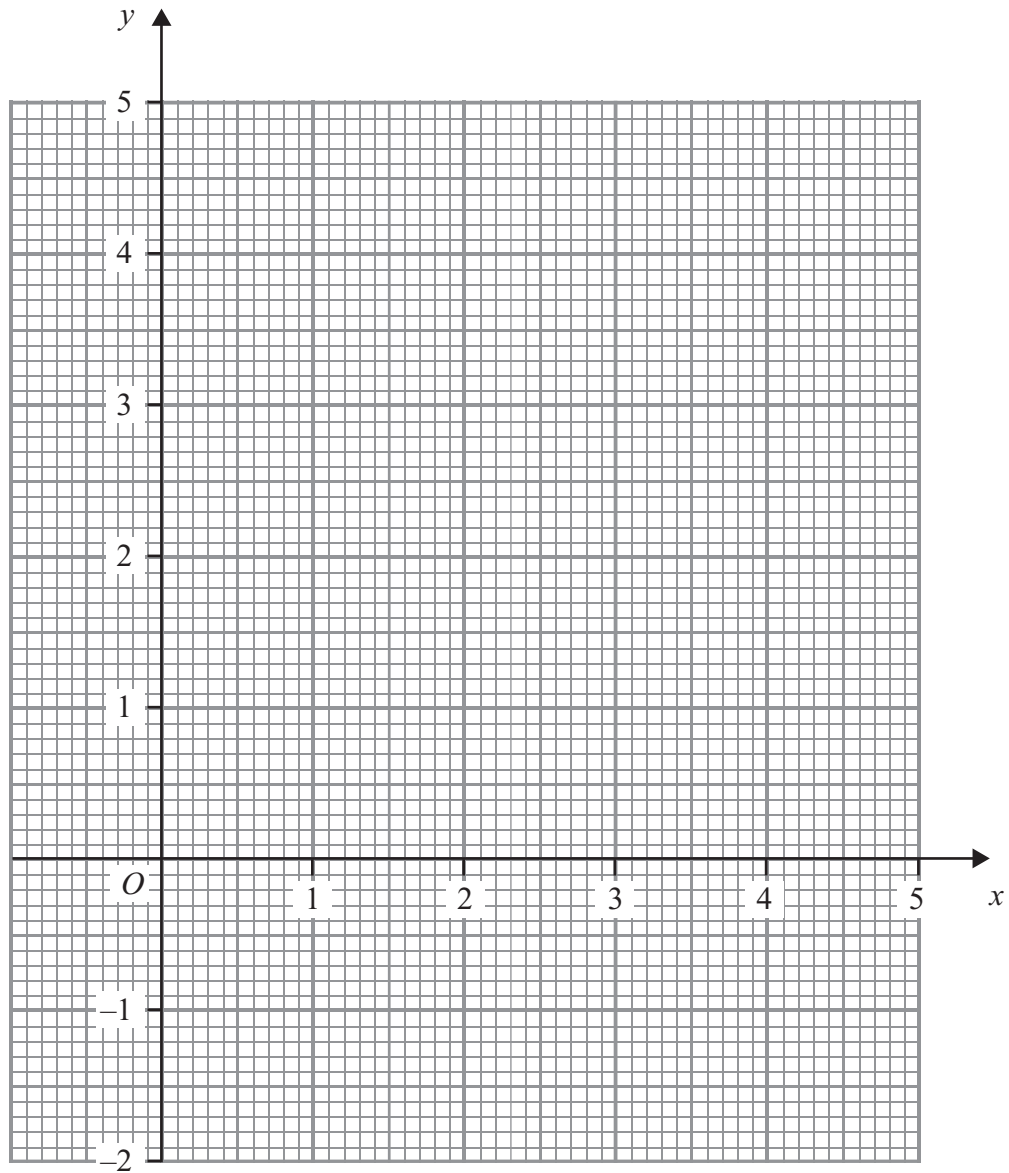
**Question 6 continued**

Handwriting practice area consisting of 25 horizontal dotted lines.



Question 6 continued

Use this page only if you need to redraw your graph.



(Total for Question 6 is 8 marks)



7 The curve  $C$  with equation  $y = \frac{2x-3}{x-3}$ ,  $x \neq 3$ , crosses the  $x$ -axis at the point  $A$  and the  $y$ -axis at the point  $B$ .

(a) Find the coordinates of  $A$  and the coordinates of  $B$ . (2)

(b) Write down an equation of the asymptote to  $C$  which is

(i) parallel to the  $y$ -axis,

(ii) parallel to the  $x$ -axis. (2)

(c) Sketch  $C$  showing clearly the asymptotes and the coordinates of the points  $A$  and  $B$ . (3)

(d) Find an equation of the normal to  $C$  at the point  $B$ . (5)

The normal to  $C$  at the point  $B$  crosses the curve again at the point  $D$ .

(e) Find the  $x$ -coordinate of  $D$ . (4)

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**Question 7 continued**

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**Question 7 continued**

A series of horizontal dotted lines for writing.





8 The equation  $x^2 + mx + 15 = 0$  has roots  $\alpha$  and  $\beta$  and the equation  $x^2 + hx + k = 0$  has roots  $\frac{\alpha}{\beta}$  and  $\frac{\beta}{\alpha}$

(a) Write down the value of  $k$  (1)

(b) Find an expression for  $h$  in terms of  $m$  (6)

Given that  $\beta = 2\alpha + 1$

(c) find the two possible values of  $\alpha$  (3)

(d) Hence find the two possible values of  $m$  (3)

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**Question 8 continued**

A series of horizontal dotted lines for writing.

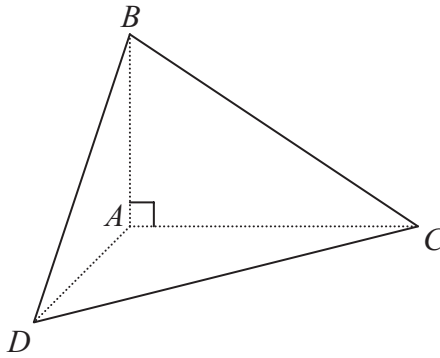


**Question 8 continued**

Dotted lines for writing.







**Figure 1**

Figure 1 shows a triangular pyramid  $ABCD$ .

$$\angle BAC = \angle DAC = \angle BAD = 90^\circ$$

$AD = 5$  cm,  $AC = 8$  cm and  $AB = 6$  cm.

(a) Find, in degrees to the nearest  $0.1^\circ$ , the size of  $\angle BDC$ . (6)

(b) Find, to 3 significant figures, the area of triangle  $BDC$ . (3)

(c) Find the area of triangle  $DAC$ . (1)

The point  $E$  lies on  $CD$  so that  $AE$  is perpendicular to  $CD$ .

(d) Find the exact length of  $AE$ . (2)

(e) Hence, or otherwise, find in degrees to the nearest  $0.1^\circ$ , the size of the angle between the planes  $DAC$  and  $BDC$ . (4)

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**Question 9 continued**

Dotted lines for writing.



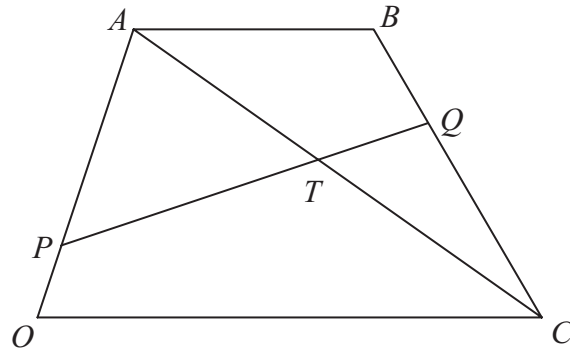


**Question 9 continued**

Ruled area for student response.

**(Total for Question 9 is 16 marks)**





**Figure 2**

Figure 2 shows a trapezium  $OABC$  in which  $AB$  is parallel to  $OC$  and  $AB = \frac{1}{2} OC$ . The point  $P$  divides  $OA$  in the ratio  $1:3$  and the point  $Q$  divides  $BC$  in the ratio  $1:2$

The line  $AC$  intersects the line  $PQ$  at the point  $T$ .

$\vec{OA} = \mathbf{a}$  and  $\vec{OC} = \mathbf{c}$

(a) Find, as simplified expressions in terms of  $\mathbf{a}$  and  $\mathbf{c}$

(i)  $\vec{BC}$

(ii)  $\vec{PQ}$

(5)

(b) (i) Given that  $\vec{PT} = \lambda \vec{PQ}$ , find an expression for  $\vec{AT}$  in terms of  $\lambda$ ,  $\mathbf{a}$  and  $\mathbf{c}$

(ii) Given also that  $\vec{AT} = \mu \vec{AC}$ , find an expression for  $\vec{AT}$  in terms of  $\mu$ ,  $\mathbf{a}$  and  $\mathbf{c}$

(2)

(c) Use your answers from part (b) to find the value of  $\lambda$  and hence write down the ratio  $PT : TQ$

(6)

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**Question 10 continued**

Ruled lines for writing the answer to Question 10.





**Question 10 continued**

A series of horizontal dotted lines for writing answers.

**(Total for Question 10 is 13 marks)**



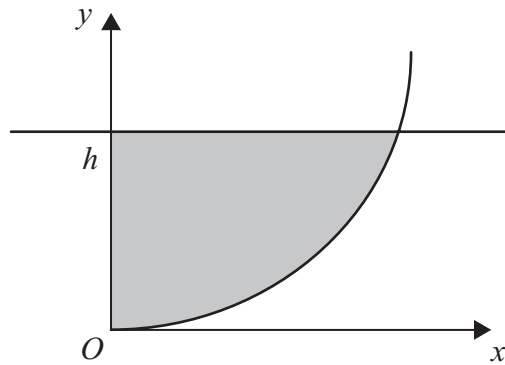


Figure 3

The centre of the circle  $C$ , with equation  $x^2 + y^2 - 10y = 0$ , has coordinates  $(0, 5)$ . The circle passes through the origin  $O$ . The region bounded by the circle, the positive  $y$ -axis and the line  $y = h$ , where  $h < 5$ , is shown shaded in Figure 3. The shaded region is rotated through  $2\pi$  radians about the  $y$ -axis.

(a) Show that the volume of the solid formed is  $\frac{1}{3}\pi h^2(15 - h)$ .

(5)

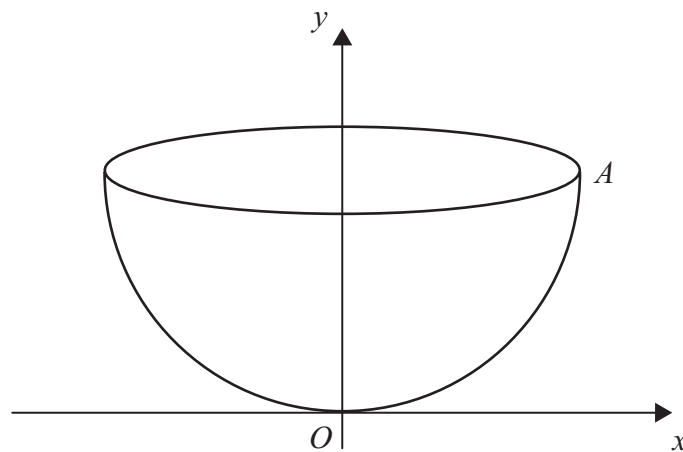


Figure 4

The point  $A$  with coordinates  $(5, 5)$  lies on  $C$ . A bowl is formed by rotating the arc  $OA$  through  $2\pi$  radians about the  $y$ -axis, as shown in Figure 4. Water is poured into the bowl at a constant rate of  $6 \text{ cm}^3/\text{s}$ . The volume of water in the bowl is  $V \text{ cm}^3$  when the depth of water above  $O$  is  $h \text{ cm}$ .

(b) Use the formula given in part (a) to find an expression for  $\frac{dV}{dh}$  in terms of  $h$ .

(1)

(c) Find, to 3 significant figures, the rate at which  $h$  is changing when the water above  $O$  is  $1.5 \text{ cm}$  deep.

(4)

The area of the surface of the water is  $W \text{ cm}^2$  when the depth of water above  $O$  is  $h \text{ cm}$ .

(d) Show that, for  $0 < h < 5$ , the rate of change of the depth of water above  $O$  is  $\frac{k}{W}$ , stating the value of  $k$ .

(3)



**Question 11 continued**

Lined writing area with 25 horizontal dotted lines.



P 4 0 6 1 6 A 0 2 9 3 2

**Question 11 continued**

A series of horizontal dotted lines for writing.



**Question 11 continued**

A series of horizontal dotted lines for writing.



**Question 11 continued**

Dotted lines for writing.

**(Total for Question 11 is 13 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

