INTERNATIONAL ADVANCED LEVEL

PHYSICS

TEACHER MATHEMATICS SUPPORT

Pearson Edexcel International Advanced Subsidiary in Physics (XPH11)
Pearson Edexcel International Advanced Level in Physics (YPH11)
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Contents

Introduction

Many A level physics students find mathematical concepts difficult, not only those who are not also taking maths at IAL/GCE.

We have also all had our patience tested by capable students saying physics 'is just maths' or 'all about equations'. These students become adept at putting numbers into selected formulae correctly for International GCSE/GCSE without necessarily having developed the type of mathematical reasoning needed for more advanced physics.

Both groups of students find the transition to A level difficult. The new specifications raise the bar further by demanding 40% of marks be for Level 2 maths, moving away from questions where students put the numbers into a formula and towards 'problem solving' where students have more responsibility for selecting both the correct data and approach to finding the solution.

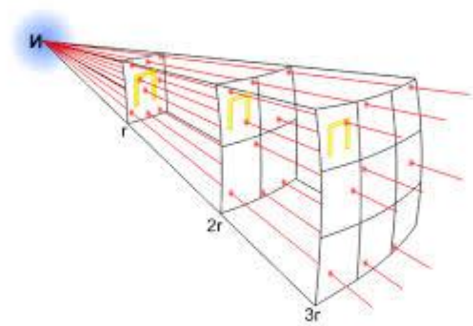
Why is this transition so difficult and how can we support students?

The relationship between maths and physics

Part of the reason for students' difficulty is that physicists and mathematicians have different relationships with maths and so students' learning in maths doesn't transfer well to our classrooms.

For a physicist the maths describes a picture, a representation (usually approximate) of a relationship (observed experimentally) describing a physical reality (putting aside for the moment the unresolved issues with quantum theory). For students, maths is an absolute that gives you 'the answer' when you put the numbers in.

For example, when you are asked to consider the intensity at two distances from a point source you probably visualise a diagram like this one representing the inverse square law, whereas many students will instead begin to look for intensity and distance on the formula sheet. In doing so they also miss all the hidden assumptions you make that aren't part of the formula: isotropy, large numbers of photons, constancy in time etc. You also have a good idea of the order of magnitude of the answer from the context.



There are countless other examples: the different ways we use negative numbers e.g. direction, choice of zero; limits of validity e.g. critical angle, threshold frequency; the hidden geometrical content of formulae e.g. $F = BIl$.

Teaching maths for physicists

Unfortunately because there are countless examples we cannot identify areas in the new specifications that are 'hotspots', areas that lend themselves to the new Level 2 maths content: any area of the spec is fertile ground! How can we prepare students given such a wide scope?

The following sections aim to describe some of the maths skills students should develop to become good physicists. These can be embedded into the teaching of the subject content in the scheme of work. This is arguably the best way to deliver it because it enables ideas to be revisited many times in the course of delivering the content, gradually building and reinforcing students' skills. Indications of where there are opportunities for teaching maths skills are given in the following text. The challenge with this approach is to be systematic in ensuring maths skills actually do form part of many lessons.

Alternatively students can be taught maths skills separately by devoting lesson time (or extra sessions) to this, ideally timed to precede their use within the subject content. The sub-sections below could be used as an outline for a scheme of work for such an approach. The challenge here of course is finding such dedicated time.

In either case every opportunity should be taken to relate mathematical ideas to real physical situations, if possible in a practical way, as well as creating explicit links to where the same skills are used in the other sciences.

Maths Skills for Physicists

Standard Form, Units, Prefixes and Significant Figures

Mathematical skills: C0.1, C0.2, C1.1

Students should be familiar with these ideas from International GCSE/GCSE maths and science but it is worth explicitly teaching these skills again as some additional complications arise at A level. For example:

- conversion of complex units such as those for density, for example from g cm^{-3} to kg m^{-3}
- using more unit prefixes e.g. nano-, giga- and micro-
- learning more about measurements and significant figures (sf).

Tip: two voltmeter readings of, say, 0.935 V and 1.025 V are correctly recorded despite different sf as the meter resolution is ±0.005 V.

Worked example

An ultrasound pulse travels through a metal bar and reflects off a defect inside the bar. The time for the pulse echo to be detected is 12 μs. The speed of ultrasound in the bar is 1.1 km s^{-1}. Calculate the distance between the defect and the bar's surface in mm.

Answer

$d = vt$

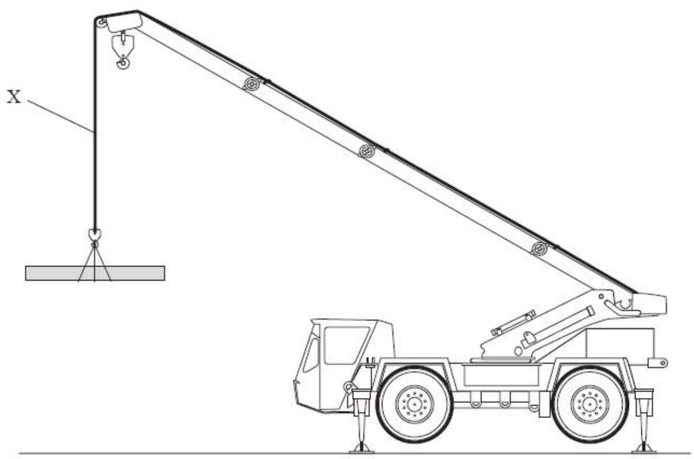
$= 1.1 \times 10^{3} \times (½ \times 12 \times 10^{-6})$

$= 6.6 \times 10^{-3}$ m

$= 6.6$ mm

Practice question 1

The diagram shows a crane lifting a concrete beam.



Weight of beam = 13 kN diameter of steel cable = 1.1 cm

Show that the stress in the cable at point X is about 0.1 GPa.

Answer

$$\text{Stress} = \frac{F}{A}$$

$$= \frac{13000}{\pi(\frac{1}{2}(1.1 \times 10^{-2}))^2}$$

$$= 1 \times 10^7$$

$$\approx 0.1 \text{ GPa}$$

Students will benefit from a refresh of simple calculator skills as they will be required to enter more complex calculations at A level. For example, they should learn to do the following (key sequences in the text refer to the Casio FX-82ES).

- Reset it back to 'normal' mode (SHIFT 9 3 = AC).
- Make sure × and ÷ happen in the right order, perhaps using brackets to make sure. (What is the value of $3.2 \times 10^7 \div 2.5 \times 10^4 + 3.0 \times 10^3$?)
- Enter terms appreciating the order of evaluation
- Enter standard form accurately (some have EXP, others $x10^x$) including negative values (some allow the subtraction key others have a (-) or a +/- key).
- Convert answers given by the calculator in different forms (S<->D, ENG).
- Change final answer to given sf (SHIFT MODE 7 (SCI) 3 for 3 sf).
- Change to and from radian mode (SHIFT MODE 4 or 3).
- Use a reciprocal button ($\frac{1}{x}$ or x^{-1}).

Tip: encourage students to enter unit prefixes as they occur i.e. enter 650 nm as 6 5 0 EXP – 9 rather than converting the exponent mentally.

There are several good, free calculator emulators on the internet which can be used to explicitly teach calculator skills, especially if your establishment adopts a standard model.

Considering the units of quantities will help both in certain exam questions such as the example below but also in problem solving where the units of the required answer can signpost a direction to proceed (see later).

Worked example

The current I in a length of aluminium of cross-sectional area A is given by the formula

$$I = nevA$$

where e is the charge on an electron.

Show that the units on the left-hand side of the equation are consistent with those on the right-hand side.

Answer

Write out the units for each quantity. In this example we're thinking of the ones as being electrons (but not writing it this way).

$$nevA = \frac{\cancel{1}}{\cancel{m^3}} \quad \frac{C}{\cancel{1}} \quad \frac{\cancel{m}}{s} \quad \cancel{m^2}$$

$$= C\,s^{-1}$$

$$= A$$

Tip: give students pre-starter evaluation or estimation activities which they can tackle on arrival to class e.g. a starter to a lesson on specific charge might ask students to estimate (non-calculator):

$$\frac{1.60 \times 10^{-19}}{9.11 \times 10^{-31}} \approx \frac{2}{10} \times 10^{(-19\,-\,-31)} = 0.2 \times 10^{12} = 2 \times 10^{11}$$

Students should be able to estimate a numerical answer mentally before using the calculator. They do find this challenging initially but students of all abilities can learn to do approximate sums mentally with a little practice.

Practice question 2

A submarine has a volume of 7100 m^3. Calculate the upthrust force.

Density of sea water = 1.03 g cm^{-3}

Answer

$F = m_w g$

$= V\rho_w g$

$= 7100\ \cancel{m^3} \times 1.03\ \cancel{g}\ \cancel{cm^{-3}} \times 10^6\ \cancel{cm^3}\ \cancel{m^{-3}} \times 9.81\ \cancel{N}\ kg^{-1} \div \cancel{10^3}\ \cancel{g}\ kg^{-1}$

$= 71700 \times 10^3$ N

$= 7.17 \times 10^7$ N

Further examples are given in the section on estimation below.

Transformation of Formulae

Mathematical skills: C2.2–4

Many students will transition from International GCSE/GCSE familiar with this process but perhaps using the 'magic triangle' approach rather than the more formal mathematical method. This of course causes problems for linear equations with two or more terms i.e. those found in 'suvat', the photoelectric effect, resistors in parallel and the lens equation. Again, lots of practice gradually reduces students' errors.

Tip: when a new formula is introduced, make it a rule that students practise transposing it to make each variable the subject.

Transforming equations where students end up with an inversion or 'fractions within fractions' causes some difficulty. Although the situation can be avoided by pre-calculating parts of the final sum, this approach can cause rounding errors so it is better to teach students how to handle the algebra.

Practice question

Calculate the angular velocity of a satellite that makes 8 complete orbits in 3 days.

Answer

A student may substitute a version of $\frac{3}{8}$ for the period into $\omega = \frac{2\pi}{T}$.

This gives $\dfrac{2 \times \pi}{(\frac{3 \times 24 \times 60 \times 60}{8})}$

This can easily be entered into a calculator incorrectly.

Instead, the 'bottom of the bottom' moves to the top to give $\dfrac{2 \times \pi \times 8}{3 \times 24 \times 60 \times 60}$.

This is easier to do on the calculator.

Estimation

Mathematical skills: C0.4, 1.4

An area which students are unlikely to be familiar with is estimating the value or order of magnitude of a quantity from prior knowledge. Questions in exams on the new specifications can expect students to make a guess at the value of a familiar quantity, so it is important to prepare students for this. As with mentally estimating the result of a calculation above, this requires some practice. Fortunately an estimation problem makes an interesting starter activity which if used regularly develops students' confidence and understanding.

Tip: give students pre-starter evaluation or estimation activities which they can tackle on arrival to class

For example, to estimate the weight of the air in an average house the students might use

$$\text{mass} = \text{density} \times \text{volume}$$

Having looked up or been given the density of air they should be encouraged to remember such common data values. In estimating the dimensions they should be encouraged to simplify (to a cube, for example) and not to dwell on the exact values (as neither 1 m nor 10 m is appropriate, 5 m is good enough, leading to a volume of 125 m^3).

So the answer to the question would be about 1300 N. Many students will stop at the mass in kg and you can emphasise the importance of carefully reading the question for the quantity required.

Practice question

Estimate:

- the number of photocopies made in your school in a year
- the sea level rise caused if the entire world's population went swimming
- the rate at which rubber wears off the tyres of a family car (in $mm^3\ s^{-1}$).

Answer

Any reasonable answers should be accepted.

Be sure to have students emphasise the assumptions they make e.g. each teacher copies one worksheet per day for a class of 30, as this begins to develop their understanding that our equations have limits to their application.

Proportion, Percentage Change and Uncertainty

Mathematical skills: C0.3, 1.5, 2.1

Proportional reasoning is a skill we use as physicists all the time: when this thing doubles, this other thing doubles (or halves, or quadruples…). Most students have not developed this skill by International GCSE/GCSE and when faced with problems that require it, resort to multiple applications of a formula which takes a long time and is prone to error. This is particularly serious in exactly the type of question where we see this skill being tested – multiple choice items.

Worked example

The current through a bulb connected to a 6 V cell is 2 A. The bulb is replaced with one of half the original resistance. Calculate the new current.

The long way (by calculation)

Resistance of original bulb, using Ohm's law, $R = \frac{V}{I} = \frac{6}{2} = 3\ \Omega$.

Therefore new bulb has resistance 1.5 Ω.

New current, using Ohm's law, $I = \frac{V}{R} = \frac{6}{1.5} = 4$ A.

Using proportion

Ohm's law tells us $I \propto \frac{1}{R}$ (if V constant) so if R halves then I doubles.

So the answer is 4 A.

Practice question 1

A football has a diameter of 22.5 cm. It contains air at a temperature of 20°C and a pressure of 1.65×10^5 Pa. When the football is left in direct sunlight, the temperature of the air in the football increases to 40°C.

Show that the new pressure exerted by the air in the football is about 2×10^5 Pa. Assume that the volume of the football remains constant.

Answer

T increases by a factor $\frac{40+273}{20+273} = 1.068…$

$\frac{p}{T}$ = constant so p increases by the same factor

p becomes $1.65 \times 10^5 \times 1.068 = 1.76 \times 10^5 \approx 2\text{x}10^5$ Pa

Questions can also be set with letters (algebraic symbols) rather than values: in this case you can only use proportion.

Practice question 2

Two protons, separated by a distance x, experience a repulsive force F.

If the separation is reduced to $\frac{x}{3}$ the force between the protons will be:

A $\frac{F}{9}$ **B** $\frac{F}{3}$ **C** $3F$ **D** $9F$

Answer

Separation multiplied by a factor $\frac{1}{3}$.

$F \propto \frac{1}{(\text{separation})^2}$ so F divided by this factor squared.

F becomes $9F$ – answer **D**.

Testing whether a relationship is direct or inverse proportion (sometimes inverse square or exponential) is also a common task. Students should be familiar with the reasoning behind and application of such a test to a given data set.

Practice question 3

A wire carries a constant current. A Hall probe is used to investigate how the magnetic flux density produced by the wire varies with distance from the wire.

r/cm	V/V
1.0	0.725
1.5	0.483
2.0	0.363
2.5	0.29
3.0	0.242
3.5	0.21

The potential difference V was recorded for a range of distances r.

It is suggested that V and r are related by the equation

$$V = \frac{k}{r}$$

where k is a constant.

Determine by calculation whether this suggestion is valid.

Answer

Test whether Vr = constant to within experimental uncertainty.

Calculate value of Vr for each row to give: 0.725, 0.724, 0.726 …

Vr is constant to within <1% therefore suggestion is valid.

Combining experimental uncertainties is usually taught as a 'new' topic. However, it uses the same ideas, albeit in approximation. Showing students the reasoning below is useful both in mental calculations and in then introducing combination of uncertainties. (It might be handy to know that 'Big O' notation will be taught to any of your students studying maths or computer science.)

$$1.01a \times 1.02b = (1 + 0.01)(1 + 0.02)ab = (1 + 0.01 + 0.02 + \ldots)ab \approx 1.03ab$$

If a increases by 1% and b increases by 2% then the product increases by about 3%.

Likewise, having students check that $\frac{1}{0.98} = 1.0204\ldots \approx (1+1.02)$ leads to

$$\frac{1.01a}{0.98b} \approx (1 + 0.01)(1 + 0.02)ab = (1 + 0.01 + 0.02 + O(10 - 4))ab \approx 1.03ab$$

If a increases by 1% and b decreases by 2% then the ratio increases by about 3%. Adding the ideas that powers are multiple multiplications and investigating how far the approximation works will make excellent training for physicists.

If $z = \frac{a}{x^2}$ and x increases by 7% then z decreases by about 14%.

This works to within ½% for changes of this size.

Worked example

A 1 m length of wire of cross-section 1.00×10^{-2} mm^2 has resistance 2.00 Ω.

a Calculate the resistivity of the material.

b What is the effect on the resistance of a 2% increase in diameter?

Answer

a $\rho = \frac{RA}{l} = \frac{2 \times 1 \times 10^{-2} \times 10^{-6}}{1} = 2 \times 10^{-8}$ Ω m

b

Expert teacher solution

$R \propto \frac{1}{d^2}$ so increasing d by 2% makes R decrease by 4%.

So the answer is 96% of 2 Ω which is 1.92 Ω to 3 sf.

Transitioning student's solution

$R = \frac{\rho l}{A} = \frac{2 \times 10^{-8} \times 1}{A}$

$A = \pi(\frac{d}{2})^2$ so $d = 2\sqrt{\frac{A}{\pi}} = 0.113$ mm

So new $d = 0.115$ mm, $A = 1.04 \times 10^{-2}$ mm^2

So $R = \frac{2 \times 10^{-8} \times 1}{1.04 \times 10^{-2}} = 1.92$ Ω (3 sf)

Vectors

Mathematical skills: C0.6, C4.1–5

Geometrical aspects of physics are incorporated using the concept of vectors. Students will have met vectors formally in International GCSE/GCSE maths if not physics. However, be aware of the focus in your maths department: the treatment of vectors is likely to be via column vectors and any addition problems may well focus on abstract tasks such as finding midpoints rather than be done in a context. In short, students might not see much similarity between vectors in maths and physics!

Tip: capitalise on what students know from maths by starting with column vectors then focusing on vector addition by sliding vectors along one another so they are 'head to tail' and using Pythagoras to find resultants.

Although vectors live firmly in mechanics, we can refer back to those ideas whenever we use negative numbers to represent a reverse direction (e.g. current flow, gas laws). Vectors are also hidden in the geometry of radiation and field lines (inverse square laws), polarisation, superposition, and all aspects of magnetism. Maths students will learn about dot and (in further maths) cross products so links could be made here for those students.

Practice question 1

Which set of quantities is all scalar?

A acceleration, displacement, velocity

B energy, mass, power

C extension, force, gravitational potential energy

D weight, kinetic energy, work

Answer

B energy, mass, power

Practice question 2

A model boat is crossing a stream. The stream is travelling east at a speed of 1.5 m s^{-1}.

The boat is heading north at a speed of 0.5 m s^{-1}.

The magnitude of the resultant velocity is:

A $(1.5 + 0.5)$ m s^{-1}

B $(1.5^2 + 0.5^2)$ m s^{-1}

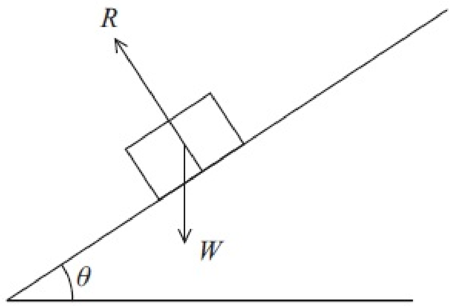
C $\sqrt{1.5 + 0.5}$ m s^{-1}

D $\sqrt{(1.5^2 + 0.5)^2}$ m s^{-1}

Answer

D $\sqrt{(1.5^2 + 0.5)^2}$ m s^{-1}

Practice question 3



The diagram shows an object on an inclined surface.

The component of the weight *W* parallel to the surface is

A 0

B 1

C *W* cos θ

D *W* sin θ

Answer

C *W* cos θ

Practice question 4

Films made to be watched in three dimensions (3D) are produced by projecting two slightly different images on to the screen, one to be seen by each eye.

In one technique the images are polarised. The viewers wear special glasses where the lenses are replaced by two separate plane polarising filters.

The light from the screen reaching each eye passes through a different filter so each eye sees a different image. The filter for one eye has a plane of polarisation of 45° and the filter for the other eye has a plane of polarisation of 135°.

Explain this choice of angles.

Answer

They are 90 degrees apart.

The first filter selects one direction of vibration; at the second filter the component of this vibration along the axis will be zero (because cos 90 = 0).

Rate

Mathematical skills: C2.1, C3.5–7

The idea of rate is expressed in maths via calculus which of course students are not required to have studied; however they do need to be familiar with the delta notation and finding gradients from graphs. In the former case, a useful technique is to consider what happens 'in each second' effectively setting $\Delta t = 1$ (see examples below in problem solving). The latter, using graphical methods, helps to visualise something that is otherwise abstract. (See the section on simple harmonic motion below.)

Rate occurs in several places in physics, both in time (acceleration, frequency, capacitor discharge) and in space (absorption of beta particles with thickness, density with height in the atmosphere).

Students may well be familiar with rates in the other sciences and other subjects.

Tip: wherever possible link the use of rate in the scheme of work to other subjects *e.g.* reaction rate in chemistry, photosynthesis rate in biology, GDP growth in economics, attrition rate in a psychological study

In many cases rate is linked to the idea of probability e.g. in radioactive decay and absorption, the probability of emission in given time or absorption in given thickness and in the gases or black body radiation, the probability distribution of particle or photon energies. (See example in following section.)

Problem Solving

I understand the lessons: I just can't do the questions.

Problem solving is one of those areas that has been the subject of a fair amount of academic research. How do expert physicists approach 'problems' and how can we teach students to do the same?

If you have read any of this work you will know some of the arguments: the ability to categorise the problem ('this is a question about terminal velocity'); recall the solution to a similar problem; visualise the problem; working backwards; creativity etc.

In the new specifications, problem solving involves students deciding the direction to proceed in a question. For A level then, we just need students to be able to complete an intermediate step in a calculation (sometimes using a formula from the sheet, sometimes a 'common sense' relationship) or to recognise the need to use a principle of physics that isn't explicitly stated, such as a conservation law.

The first step must be to draw a diagram and to add all the information from the question, including the target quantity (the unknown).

Tip: students should be made to draw a diagram for all but the simplest of examples of substitution into a single formula.

The next step is to consider what principles are involved: this only improves with experience. In the meantime, working backwards from the target quantity using equations from the formula sheet is an algorithm students can usually apply.

When a relationship is not available it can be revealed by thinking about units: we might not immediately see how to get a current but realising that amps can be expressed as electrons per second might help.

Good starting points for principles are the conservation of energy and momentum and any of Newton's laws. Often, adding an energy transfer ('Sankey') diagram alongside the main diagram is very helpful.

Worked example

A bulb of resistance 12 Ω is connected to a 6 V battery. Calculate the charge passing through the bulb in 1 minute.

Answer

The current would tell us how much charge passes in given time ($Q = It$).

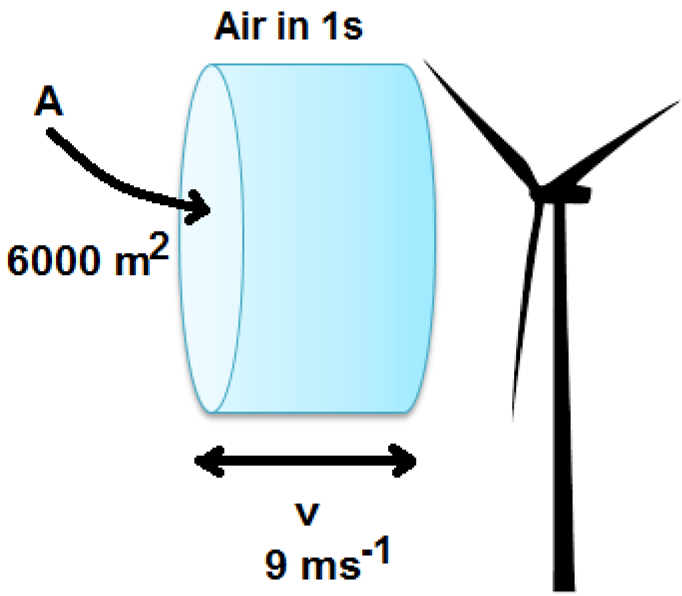
We can find current from the information we have using Ohm's law ($V = IR$).

$$I = \frac{6}{12} = 0.5\ \text{A}$$

$$Q = 0.5 \times 60 = 30\ \text{C}$$

Worked example

Kinetic energy of wind is transferred to electrical energy by a wind turbine as the blades rotate.



The area swept out by one blade, as it turns through 360°, is 6000 m^2. Wind at a speed of 9 m s^{-1} passes the turbine.

Calculate the maximum power available to the wind turbine.

density of air = 1.2 kg m^{-3}

Answer

We need to find power. $P = \frac{E}{t}$

The energy transfer is from the wind's KE to the turbine's.

So we need the KE available from the wind in 1 s. $KE = ½mv^2$

We don't know *m* but we know how much wind passes through the turbine in 1 s from our diagram: it is $A\rho x$. x is the distance the wind goes in 1 s so that's v.

So the power is the wind's KE available in 1 s which is $½(A\rho v)v^2$.

So the answer is 2.62 MW.

Practice question 1

The heating element of an electric shower has a power of 6.0 kW.

Water enters the shower at a temperature of 7.5°C.

Calculate the water flow rate required to give an output temperature of 37.5°C.

specific heat capacity of water = 4200 $J\,kg^{-1}\,K^{-1}$

Answer

We need flow rate – thinking about units, that must be $m^3\,s^{-1}$

The energy transfer is from electrical heating to the water.

We will think about what happens in 1 s.

Electrical energy transfer is 6 kW × 1 s = 6000 J.

Energy in the water is $E = mc\Delta T$; We know c and T and m is the mass being heated in 1 s which is the flow rate.

So using conservation of energy 6000 J = flow rate × 4200 × (37.5-7.5) .

So the answer is $4.76 \times 10^{-2}\,m^3\,s^{-1}$.

Practice question 2

A CCD sensor of area 1 mm^2 is illuminated with light of wavelength 700 nm and intensity 2.84 $mW\,m^{-2}$. The quantum efficiency of the CCD is 70%. Find the rate at which electrons are released in the sensor.

Answer

Photon energy = hc/λ= 2.84×10^{-19} J

Power incident on sensor = $\dfrac{2.84 \times 10^{-2}\ W\,m^{-2}}{(10^3)^2\ mm^2\,m^{-2}}$ = 2.84×10^{-9} W

Photons incident per second = $\dfrac{2.84 \times 10^{-9}\ J\,s^{-1}}{2.84 \times 10^{-19}\ J}$ =$10^{10}\,s^{-1}$

70% release an electron so the answer is $7 \times 10^9\,s^{-1}$

Common Mathematical Models in Physics

This section covers four specific models which recur in different parts of the course.

If integrating the maths content across the course this provides opportunities for recap and consolidation.

Simple Harmonic Motion

Mathematical skills: C0.6, C4.5–7

Although this is a single topic in the specification the ideas appear throughout physics e.g. in circular motion and waves.

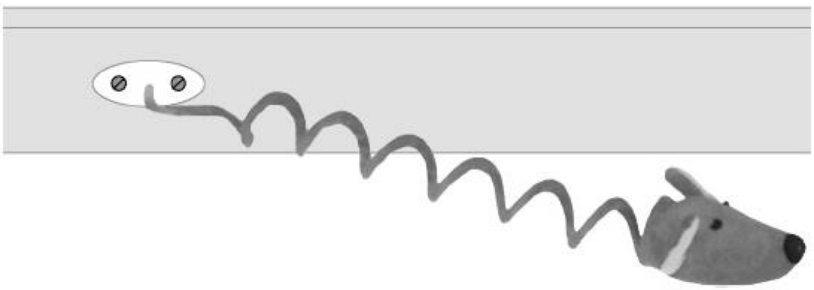
Simple harmonic motion (SHM) is defined as the motion that occurs when the acceleration of an object, a, is proportional and opposite to its displacement from equilibrium ($a \propto -x$).

Students can be introduced to this graphically: for example, datalogging a mass–spring system then carefully using their International GCSE/GCSE knowledge that v is the gradient of this d–t graph (and a is the gradient of the resulting v–t graph) to see the relationship. Reversing the logic they can then be told that it turns out the **only** way to get a to be an upside-down (scaled) x is with this (sine) shape of curve.

After introducing radian measure, the resulting concepts and relationships are assumed knowledge in many areas: frequency, angular velocity, phase etc.

Worked example

A toy for cats consists of a plastic mouse of mass ***m*** attached to a spring. When the mouse is on a low-friction horizontal surface, with the spring attached to a rigid support as shown, it performs simple harmonic motion when given a small displacement ***x*** from its equilibrium position and released.



The mouse has a mass m = 0.15 kg and the spring extends by 20 cm when the mouse is supported vertically by the spring.

Calculate the frequency of oscillation of the mouse if it is set into oscillation on a low friction horizontal surface.

Answer

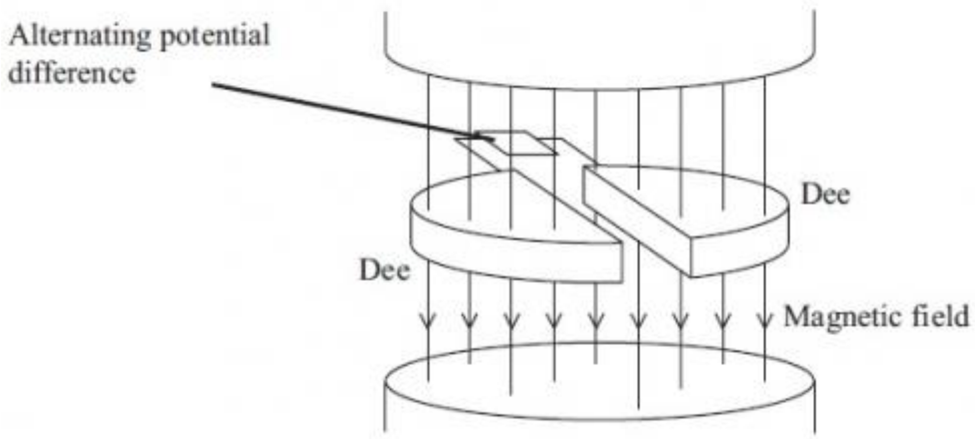
$$k = \frac{0.15\,\text{kg} \times 9.81\,\text{N kg}^{-1}}{0.2\,\text{m}} = 7.4\,\text{N m}^{-1}$$

$$\omega = \sqrt{\frac{7.4\,\text{N m}^{-1}}{0.15\,\text{kg}}} = 7.0(\text{rad s}^{-1})$$

$$f = \frac{\omega}{2\pi} = \frac{7\,\text{s}^{-1}}{2\pi} = 1.1\,\text{Hz}$$

Practice question 1

A cyclotron can be used to accelerate charged particles.



A beam of low-speed protons is introduced into a cyclotron.

Show that the number of revolutions per second, *f*, completed by the protons is given by:

$$f = \frac{eB}{2\pi m}$$

where *e* is the electronic charge
B is the uniform magnetic flux density within the cyclotron
m is the mass of the proton.

Answer

Magnetic force provides centripetal force, therefore:

$$Bev = \frac{mv^2}{r}$$

$$Ber = mv$$

$$Ber = mr\omega$$

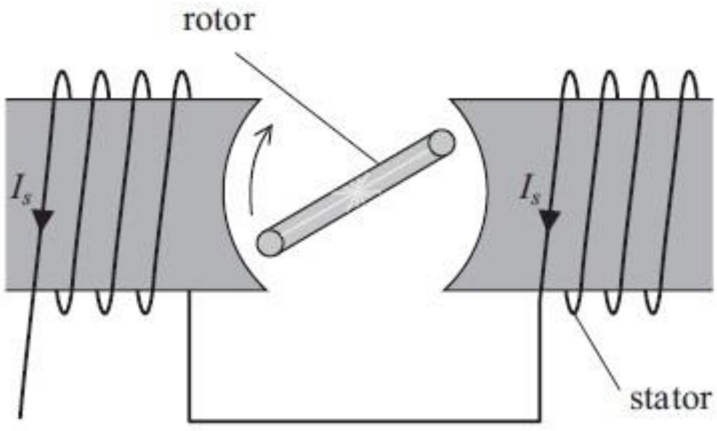
$$Be = m(2\pi f)$$

$$f = \frac{eB}{2\pi m}$$

Practice question 2

The diagram represents a simple induction motor. An alternating current I_s is supplied to a stationary coil (stator). This coil is wrapped around an iron core.

A rotating coil (rotor) is shown end on in the diagram.



An induction motor is used to rotate the turntable in a record deck. Long-play records require the turntable to rotate at 33 revolutions per minute.

Calculate the angular velocity of the turntable.

Answer

$T = \frac{60}{33\,(1.82\,\text{s})}$ **Or** $f = \frac{33}{60\,(0.55\,\text{s}^{-1})}$

Use of $\omega = \frac{2\pi}{T}$ **Or** $w = \frac{2\pi}{f}$

$\omega = 3.5\ \text{rad s}^{-1}$

Exponential Decay

Mathematical skills: C0.5, C2.5, C3.10

Students will have met this idea already in International GCSE/GCSE radioactivity. At A level it appears in the absorption of beta particles with thickness, in capacitor discharge and decay of damped SHM. This model is based on the idea that the amount something changes by depends on how much of it you've got in the first place. Students find this idea easy to grasp with an everyday analogy such as the emptying of water from a bath (faster when it's full) or for growth, the idea of 'trending' or 'viral' social media.

Worked example

In September 1987, two youngsters in Brazil removed a stainless steel cylinder from a machine in an abandoned clinic. Five days later they sold the cylinder to a scrap dealer who prised open a platinum capsule inside to reveal a glowing blue powder. The powder was found to contain caesium-137 and had an activity of 5.2×10^{13} Bq.

Caesium-137 is a β^--emitter with a half-life of 30 years.

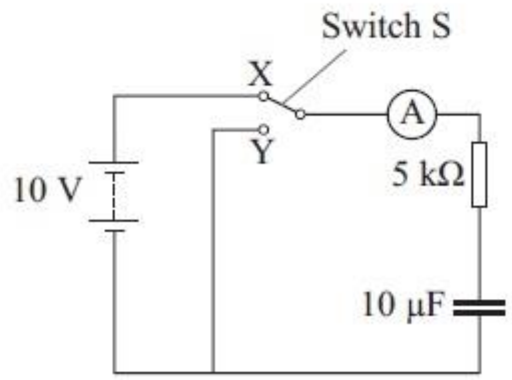
Show that the decay constant for the caesium-137 is about $7 \times 10^{-10}\ \text{s}^{-1}$.

Answer

$$\lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{30 \times 365 \times 24 \times 3600\text{s}} = 7.32 \times 10^{-10}\ \text{s}^{-1}$$

Practice question

A student sets up the circuit shown in the diagram.



The student wants to use this circuit to produce a short time delay, equal to the time it takes for the potential difference across the capacitor to fall to 0.07 of its maximum value.

Calculate this time delay.

Answer

Time constant, $\tau = CR$

$= 5 \times 10^3 \times 10 \times 10^{-6}$

$= 0.05\text{ s}$

Time to fall from 10 V to 0.7 V …

$$ln\left(\frac{10\text{ V}}{0.7\text{ V}}\right) = \frac{t}{0.05\text{ s}}$$

$t = 0.13\text{ s}$

This is another area in which to make links to other sciences: the growth of bacteria and other populations, first order kinetics in chemistry, Moore's law in computing etc. You might like to tell the story of the rice and chessboard (search online for 'shahnameh rice chess').

Inverse Square Laws

Mathematical skills: C3.1-2, C3.12, C4.3, C4.6, C4.7

Inverse square laws occur wherever there is a flux of some quantity through a surface e.g. any kind of radiation or field lines from a point source.

It is worth revisiting at each point because it is so familiar to us that it is easy to miss the wide range of mathematical concepts required: surface area of a sphere, proportion, radian measure and for applications especially in astronomy, small angle approximations etc.

Practice question 1

X and Y are identical stars. When viewed from Earth the flux from star X is 4 times the flux from star Y. Which of the following explanations is possible?

A X is twice as far away as Y.

B X is four times as far away as Y.

C Y is twice as far away as X.

D Y is four times as far away as X.

Answer

C Y is twice as far away as X.

Practice question 2

A small satellite has a weight of 1200 N at the Earth's surface. It is launched into a circular orbit with radius equal to twice the radius of the Earth. The weight of the satellite in this orbit is

A 0 N

B 300 N

C 600 N

D 1200 N

Answer

B 300 N

Power Laws, Logarithms and Graphs

Mathematical skills: C0.5, C2.5, C3.1-4, C3.10-12

The assumption that two variables have a power law relationship i.e. $y \propto x^n$ is the final recurring theme covered here. his is fertile ground for Level 2 maths questions in the exam because it requires the use of logarithms to cast into linear form i.e.

$$\log y = \log k + n \log x$$

In earlier work on graphs students should be taught why the linear form is so useful to us (confirmation of linearity, or proportionality, identification of systematic errors and possible physical interpretations of gradient, intercept and area underneath). They will meet this when studying 'suvat', internal resistance, photoelectric effect and will perhaps plot functions of variables to establish relationships in SHM (T^2), in optics (sin θ) and for Boyle's law ($\frac{1}{V}$).

Having established the importance of linearity it makes sense to try to cast $y \propto x^n$ into this form, the new problem being that students' previous experience won't work because n is unknown.

Practice question

A physicist investigates how light intensity varies with distance from a light bulb. She sets up the apparatus as shown.

The relationship between R and d is given by $R = k\,d^p$ where k and p are constants. Explain why a graph of $\ln R$ against $\ln d$ should give a straight line.

Answer

Shows expansion $\ln R = p \ln(d) + \ln(k)$.

Compares with $y = mx + c$ and states that the gradient is p which is constant.

