# Edexcel International AS/A Level

# Mathematics

## Understanding assessment and improving delivery Module 1 YMA01_20IO7

First teaching in 2018, first assessment 2019

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# Aims and Objectives

- - Delegates will

- be introduced to the idea of assessment objectives: what are they and why they are used when writing examination papers,

- analyse recent question papers and learn which types of question match the different assessment objectives,

- understand how the demand of a question can be altered while keeping the content essentially the same.

# Session Agenda

10:00 Introduction and overview

10:10 What are Assessment Objectives ? (AOs)

10:40 Assigning AOs to Edexcel mathematics papers

11:00 BREAK

11:30 Looking at AO1 and AO2 in detail

12 :00 End

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Polls to get to know the delegates.

# Introduction to Assessment Objectives

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# Examples in this presentation

- Edexcel exam questions undergo a rigorous process before any student sees the examination paper.
  In several slides in this presentation the language and style are not fully that of the exams – indeed there are some problems that would not do at all as exam questions but do have a use as a teaching application.

- The questions themselves are indicative also of the range that students should see in class. They are not intended in any way as a 'pointer' to examination questions.
  The Edexcel team have produced material which teachers will be able to use to support their teaching – especially of the new topics.

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# Activities in this presentation

There are several activities in the modules.

Some are as material for delegates to do some mathematics.

In all of the activities delegates are encouraged to consider such issues as:

Alternative methods of solution
Teaching implications
Demand
How activities/tasks/questions could be adapted.
How well they fit the content
Assigning the Assessment Objective(s)

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# General Structure of an Assessment

- **Content**
- Facts
- Techniques
- Relationships
- Models

**Assessment Objectives**

Demonstrate knowledge of facts, techniques and relationships

Demonstrate application of facts, techniques and relationships to solve problems

Demonstrate processes to model real situations and to interpret results of calculations involving models

# General Structure of an Assessment

- Content coverage
  - sufficient for each separate assessment (samples from (nearly) all sections of the content list)
  - complete coverage over a cycle of assessments
- Assessment Objectives
  - **fixed** from assessment to assessment
  - **same weightings** from assessment to assessment (some leeway allowed)

# Structure of the Edexcel P1 Assessment

- Content - as given in the specification
- e.g.. Laws of indices for all rational exponents.
- e.g.. Interpret linear and quadratic inequalities graphically.
- e.g.. Solve simultaneous equations; analytical solution by substitution.

Assessment Objectives (AOs)

1. Recall, select and use their knowledge of mathematical facts, concepts and techniques…..
2. Construct rigorous mathematical arguments and proofs…
3. Recall, select and use their knowledge of standard mathematical models to represent situations in the real world….
4. Comprehend translations of common realistic contexts into mathematics….. results
5. Use contemporary calculator technology and other permitted resources……..

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# Structure of the Edexcel pure units assessment

- All figures in the following table are expressed as marks out of 75.

| | AO1 | AO2 | AO3 | AO4 | AO5 |
|---|---|---|---|---|---|
| **P1** | 30–35 | 25–30 | 5–15 | 5–10 | 1–5 |
| **P2** | 30–35 | 25–30 | 5–15 | 5–10 | 1–5 |
| **P3** | 30–35 | 25–30 | 5–15 | 5–10 | 1–5 |
| **P4** | 30–35 | 25–30 | 5–15 | 5–10 | 1–5 |

# Structure of the Edexcel applications units assessment

- All figures in the following table are expressed as marks out of 75.

| | AO1 | AO2 | AO3 | AO4 | AO5 |
|---|---|---|---|---|---|
| **M1** | 20 – 25 | 20 – 25 | 15 – 20 | 6 – 11 | 4 – 9 |
| **M2** | 20 – 25 | 20 – 25 | 10 – 15 | 7 – 10 | 5 –10 |
| **S1** | 20 – 25 | 20 – 25 | 15 – 20 | 5 – 10 | 5 –10 |
| **S2** | 20 – 25 | 20 – 25 | 10 – 15 | 5 – 10 | 5 –10 |
| **D1** | 20 – 25 | 20 – 25 | 15 – 20 | 5 – 10 | 5 –10 |

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# Structure of the Edexcel P1 assessment

- In practice most questions on our examination papers have more than one AO assigned to them.

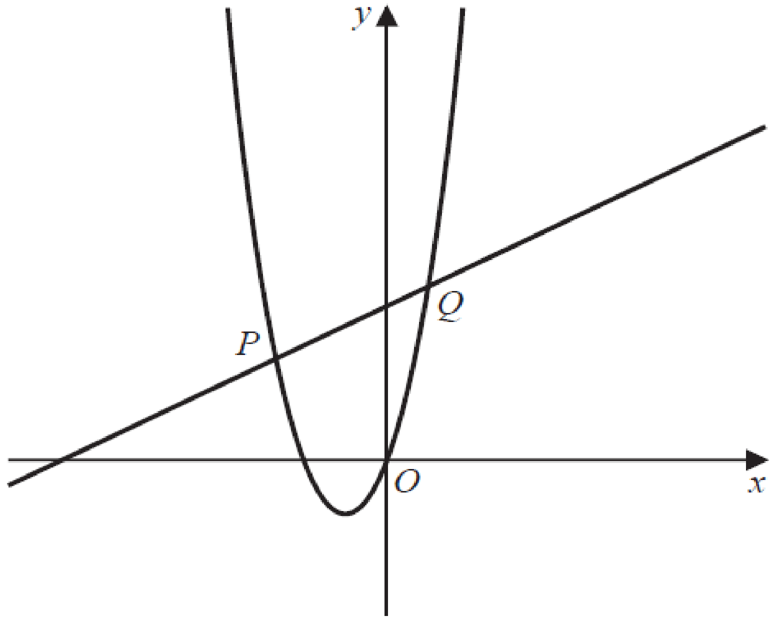


**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = 2x^2 + 3x$ and the straight line with equation $y = \frac{1}{2}x + 3$

AO1 - 3 marks
AO2 - 2 marks
AO4 – 2 marks

The line meets the curve at the points $P$ and $Q$, as shown in Figure 1.

(a) Using algebra, find the coordinates of $P$ and the coordinates of $Q$.

**(5)**

(b) Hence write down the range of values of $x$ for which $2x^2 + 3x \geqslant \frac{1}{2}x + 3$

**(2)**

# Activity 1

There are two questions in Activity 1 of your pack

The first Question has the associated mark scheme. Use it to assign the marks to AOs.

Then do the second Question. There is an opportunity on the second question to take part in a poll for the AOs awarded to each part

# Structure of the Edexcel P1 assessment

(a)

- $2x^2 + 3x = \frac{1}{2}x + 3$
- $4x^2 + 5x - 6 = 0$ AO1 (1 mark)
- $(4x - 3)(x + 2) = 0$ AO2 (2 marks)

- $x = \frac{3}{4}$ or $x = -2$ AO1 (1 mark)

- $y = \frac{1}{2} \times \frac{3}{4} + 3 = 29/8$ or $y = \frac{1}{2} \times (-2) + 3 = 2$ AO1 (1 mark)

(b)

- $x \leq \frac{3}{4}$ or $x \geq -2$ AO4 (2 marks)

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# Structure of the Edexcel P1 assessment

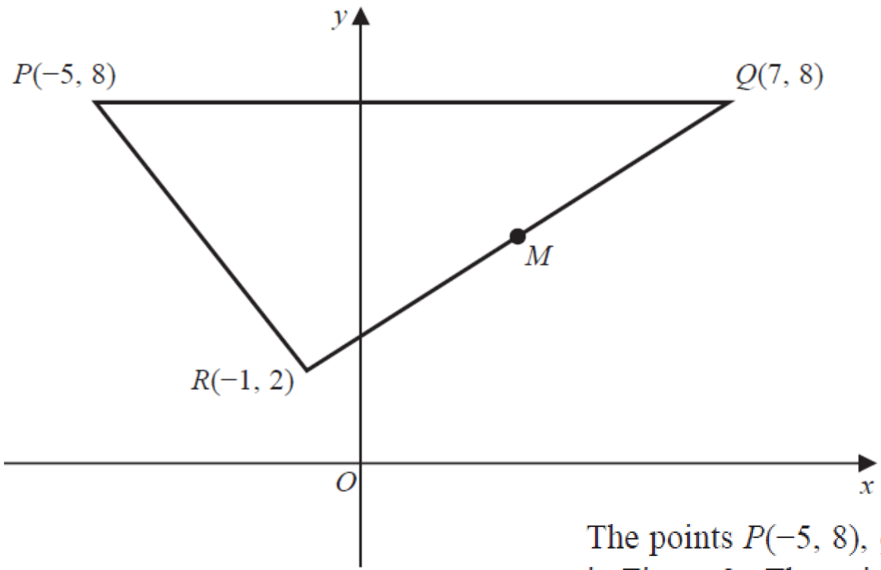


**Figure 3**

- Decide how to allocate the marks to the AOs

The points $P(-5, 8)$, $Q(7, 8)$ and $R(-1, 2)$ form the vertices of a triangle $PQR$, as shown in Figure 3. The point $M$ is the midpoint of $QR$.

The line $l$ passes through $M$ and is parallel to $PR$.

(a) Find an equation for $l$, writing your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(4)**

The line $l$ cuts $PQ$ at the point $N$.

(b) Find

(i) the coordinates of $N$,

(ii) the area of triangle $MNQ$.

**(3)**

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Polls for Activity 1 question 2

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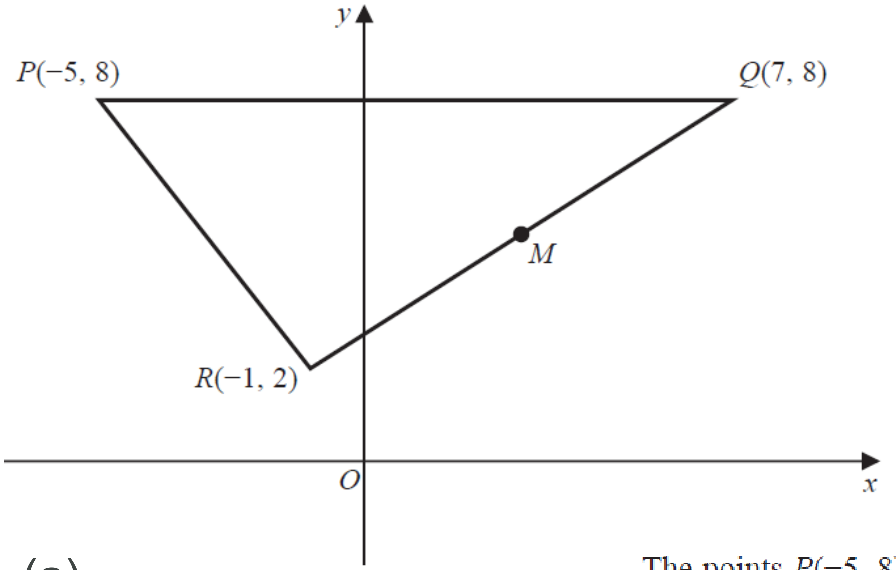
# Structure of the Edexcel P1 assessment




- Decide how to allocate the marks to the AOs

(a)
- AO1, AO2, AO2, AO2

(b)
- (i) AO1
- (ii) AO2, AO2

The points $P(-5, 8)$, $Q(7, 8)$ and $R(-1, 2)$ form the vertices of a triangle $PQR$, as shown in Figure 3. The point $M$ is the midpoint of $QR$.

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(a) Find an equation for $l$, writing your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(4)**

The line $l$ cuts $PQ$ at the point $N$.

(b) Find

(i) the coordinates of $N$,

(ii) the area of triangle $MNQ$.

**(3)**

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# AO1

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# Looking at AO1 on P1, P2, P3 and P4

Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts

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# Looking at AO1 on P1, P2, P3 and P4

**Knowledge / Recall**

Simplify  $a \times (\sqrt{a})^{-1}$

Write down an equation of the straight line with gradient 4 passing through P (2, 1).

Differentiate $3x^4$

**Knowledge / Recall**

Find the area of a sector radius 3 cm and angle at the centre 0.4 radians

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# Looking at AO1 on P1, P2, P3 and P4

- **Concepts**

If $y = x^3$ then $y' = 3x^2$ so $\int x^2\, dx = \frac{x^3}{3} + C$

- **Concepts**

If $ab = 0$ then $a = 0$ or $b = 0$

If $\log_a b = n$ then $a^n = b$

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# Looking at AO1 on P1, P2, P3 and P4

- **Techniques**

- Solve quadratic equations using the formula

Given $y = 48x - 10x^2$ find the maximum value of $y$

**Techniques**

Write $\dfrac{7}{(x-3)(2x+1)}$ as the sum of partial fractions

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# Activity 2

Use the sheet for activity 2 to enter some ideas of:

- Knowledge/recall
- Concepts
- Techniques

Share your ideas through Chat

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# Looking at AO1 on P1, P2, P3 and P4

Questions which <u>only</u> assess AO1 are rare:

- e.g. Specimen Pure 1 Q1 (Differentiation, integration)
- e.g. Practice Pure 1 Q1 (Transformations)
- e.g. Specimen Pure 2 Q1a (Binomial expansion)
- e.g. June 19 Pure 1 Q1 (Integration)
- e.g. June 19 (C34) Q3 (Parametric to Cartesian)

# Looking at AO1 on P1, P2, P3 and P4

- e.g.. June 19 (C34) Q3 (Parametric to Cartesian)

**3.** A curve $C$ has parametric equations

$$x = \sqrt{3}\tan\theta, \qquad y = \sec^2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{3}$$

The cartesian equation of $C$ is

$$y = \mathrm{f}(x), \qquad 0 \leqslant x \leqslant k, \qquad \text{where } k \text{ is a constant}$$

(a) State the value of $k$.

**(1)**

(b) Find $\mathrm{f}(x)$ in its simplest form.

**(2)**

(c) Hence, or otherwise, find the gradient of the curve at the point where $\theta = \dfrac{\pi}{6}$

**(3)**

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# Looking at AO1 on P1, P2, P3 and P4

Activity3

- Look briefly through Activity 3

Use Chat to write any comments

- The 3 exam questions were given AO1 only
- Decide on the Knowledge/Concepts/Techniques being assessed
- Do you agree with the assignment of only AO1?

Use the poll to [put your opinion

# Looking at AO1 on P1, P2, P3 and P4

However AO1 appears in most questions:

- - as an explicit part (a)
- - as underlying knowledge/skills/ concepts.

AO1

**2.** (a) Find $\displaystyle\int \frac{4x+3}{x}\,\mathrm{d}x, \qquad x > 0$

**(2)**

(b) Given that $y = 25$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(4x+3)y^{\frac{1}{2}}}{x} \qquad x > 0,\ y > 0$$

giving your answer in the form $y = [\mathrm{g}(x)]^2$.

**(5)**

AO2

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# Looking at AO1 on P1, P2, P3 and P4

As AO1 appears in most questions:

- Students need to be able to have a good understanding of the basics

- as underlying knowledge/concepts/techniques

- in doing so they become more **fluent** in their work

Use Chat to give an opinion

In the next Activity 4 (Yellow) you are asked to think about Pure 3

Activity 4 is a test intended to assess a student’s AO1

Are there any significant omissions ?

Are there any questions which do not solely address AO1?

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# AO2

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Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.

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# Arguments and Proofs

- ‘Construct rigorous arguments and proofs’

An argument is a series of statements which support a belief/hypothesis.

Arguments can be **deductive** – using the laws of logical reasoning

Or **inductive** – using evidence/observation to support the hypothesis.

(Mathematical) proofs are discussed in subsequent slides .

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# Types of Proof

A proof must show all **assumptions**, have a clear **sequential list of steps** that logically follow and must cover **all possible cases**.
You should usually make a **concluding statement** by restating the original conjecture that you have proven.

## a. Proof by Deduction

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

**Prove that the product of two square numbers is a square number.**

Let the two square numbers be $x$ and $y$

Then $x = n^2$ and $y = m^2$

$xy = n^2 m^2 = (nm)^2$ which is a square number

What is missing from this proof?

# Types of Proof

## a. Proof by Deduction

**Exam Tip**: This is quite a common last part.

**Prove that $x^2 + 4x + 5$ is positive for all values of $x$.**

?

**Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.**

?

# Types of Proof

## a. Proof by Deduction

**Prove that $x^2 + 4x + 5$ is positive for all values of $x$.**

$$x^2 + 4x + 5 = (x+2)^2 + 1$$

$(x+2)^2 \geq 0$ for all $x$

$$\therefore (x+2)^2 + 1 \geq 1 > 0$$

Anything squared is at least 0. This is formally known as the '*trivial inequality*'.

## Test Your Understanding

**Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.**

Let $2n-1$ and $2n+1$ be any two consecutive odd numbers, where $n$ is an integer.

$$(2n-1)^2 + (2n+1)^2 = 4n^2 - 4n + 1 + 4n^2 + 4n + 1$$

$= 8n^2 + 2$ which is 2 more than a multiple of 8.

# Looking at AO2 on P1, P2, P3 and P4

(a) Show that $(x - 2)$ is a factor of $x^3 - x^2 - x - 2$

(b) Show that the equation $x^3 = x^2 + x + 2$ has exactly one real root

'Show' could be replaced by 'Prove'.

# Looking at AO1 on P1, P2, P3 and P4

(a) Let $P(x) = x^3 - x^2 - x - 2$

$P(2) = 8 - 4 - 2 - 2 = 0$

So, by the factor theorem $(x - 2)$ is a factor of $P(x)$

$$x^3 - x^2 - x - 2 = (x - 2)(x^2 + x + 1)$$

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# Looking at AO2 on P1, P2, P3 and P4

(b) $x^3 = x^2 + x + 2$

$x^3 - x^2 - x - 2 = 0$

$(x - 2)(x^2 + x + 1) = 0$

So $x = 2$ or $x^2 + x + 1 = 0$

The quadratic has no real roots as the discriminant is – 3

Hence the equation $x^3 = x^2 + x + 2$ has only one real root.

There should be a conclusion to complete.

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# Types of Proof

## b. Proof by Exhaustion

This means breaking down the statement into **all possible cases**, where we prove each individual case.
(This technique is sometimes known as 'case analysis')

---

**Prove that $n^2 + n$ is even for all integers $n$.**

?

# Types of Proof

## b. Proof by Exhaustion

This means breaking down the statement into **all possible cases**, where we prove each individual case.
(This technique is sometimes known as ‘case analysis’)

---

**Prove that $n^2 + n$ is even for all integers $n$.**

$n$ is either even or odd.

If $n$ is even:

$$\begin{aligned} n^2 + n &= even \times even + even \\ &= even + even \\ &= even \end{aligned}$$

If $n$ is odd:

$$\begin{aligned} n^2 + n &= odd \times odd + odd \\ &= odd + odd \\ &= even \end{aligned}$$

$\therefore n^2 + n$ is even for all integers $n$.

# Looking at AO2 on P1, P2, P3 and P4

Prove that any square number when divided by 5 leaves a remainder of 0 or 1 or 4

?

?

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# Looking at AO2 on P1, P2, P3 and P4

## Prove that any square number when divided by 5 leaves a remainder of 0 or 1 or 4

| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $n^2$ | 0 | 1 | 4 | 9 | (1)6 | (2)5 | (3)6 | (4)9 | (6)4 | (8)1 |

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $N^2$ | 0 | 1 | 4 | 9 | 6 | 5 | 6 | 9 | 4 | 1 |
| Rem | 0 | 1 | 4 | 4 | 1 | 0 | 1 | 4 | 4 | 1 |

To find the remainder when any number is divided by 5, only the units digit of the number needs to be considered.
Looking at the table above, the possible remainders are 0, 1, 4, 4, 2, 0, 1, 4, 4, 1

So any square number will leave a remainder of 0 or 1 or 4 when divided by 5

# Looking at AO2 on P1, P2, P3 and P4

- Problems which involve multiplication or division properties by a particular number can often be tackled by using an algebraic exhaustive set.
- For example:
- Show that the square of any integer is a multiple of 3 or one more than a multiple of 3

An exhaustive set is ?

Squaring gives ?

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# Looking at AO2 on P1, P2, P3 and P4

- Problems which involve multiplication or division properties by a particular number can often be tackled by using an algebraic exhaustive set.
- For example:
- Show that the square of any integer is a multiple of 3 or one more than a multiple of 3

An exhaustive set is $3n$, $3n + 1$ and $3n + 2$

Squaring gives $9n^2$, $3(3n^2 + 2n) + 1$ and $3(3n^2 + 4n + 1) + 1$

So any square number will leave a remainder of 0 or 1 when divided by 3

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# Looking at AO2 on P1, P2, P3 and P4

**SAMs (IAL – P2)**

Prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.

**(4 marks)**

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# SAMs Solution

| $n$ | $n^2$ | $n^2+2$ | |
|---|---|---|---|
| **1** | **1** | **3** | **Odd** |
| **2** | **4** | **6** | **Even** |
| **3** | **9** | **11** | **Odd** |
| **4** | **16** | **18** | **Even** |
| **5** | **25** | **27** | **Odd** |
| **6** | **36** | **38** | **Even** |

**SAMs (IAL – P2)**
Prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.

| | |
|---|---|
| When $n$ is odd, $n^2$ is odd (odd $\times$ odd $=$ odd) so $n^2+2$ is also odd | M1 |
| So for all odd numbers $n$, $n^2+2$ is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even) | A1 |
| When $n$ is even, $n^2$ is even **and a multiple** of 4, so $n^2+2$ cannot be a multiple of 4 | M1 |
| Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all $n$, $n^2+2$ cannot be divisible by 4" | A1* |
| | **(4)** |

# Types of Proof

## c. Disproof by Counter-Example

While to prove a statement is true, we need to prove every possible case (potentially infinitely many!), **we only need one example to disprove** a statement.
This is known as a **counterexample**.

**Disprove the statement:**
**"$n^2 - n + 41$ is prime for all integers $n$."**

?

# Types of Proof

## c. Disproof by Counter-Example

While to prove a statement is true, we need to prove every possible case (potentially infinitely many!), **we only need one example to disprove** a statement.
This is known as a **counterexample**.

---

**Disprove the statement:**
**"$n^2 - n + 41$ is prime for all integers $n$."**

If $n = 41$, then we have $41^2 - 41 + 41$
$= 41^2$
Which is not prime as it has a factor of 41.
Thus the statement is not true.

# Types of Proof

Not all mathematical statements are amenable to these types of proof

Let $f(n) = n^2 - n + 41$

Then

$f(n)$ is prime for an infinite number of values of $n$

So counter-example works best when there are words like 'always' or 'never' in the statement

# Looking at AO2 on P1, P2, P3 and P4

Disproof by counter-example.

Some further examples

‘Any odd number greater than 1 has an even number of distinct divisors.’

?

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# Looking at AO2 on P1, P2, P3 and P4

- A common use of deductive proof is in that of trigonometric identities.

(a) Prove that

$$\frac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x, \qquad x \neq (2n + 1)90°, \, n \in \mathbb{Z}$$

**(3)**

?

This solution shows every step so is fully acceptable as a proof

# Looking at AO2 on P1, P2, P3 and P4

- A common use of deductive proof is in that of trigonometric identities.

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°, n \in \mathbb{Z}$$

(3)

| | Examples: | |
|---|---|---|
| (a) | $\frac{1-\cos 2x}{1+\cos 2x} = \frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)} = \frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x$ | M1dM1A1 |

This solution shows every step so is fully acceptable as a proof

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# Looking at AO2 on P1, P2, P3 and P4

Poll

Please award a mark for each of the attempts on the next slide at::

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°,\ n \in \mathbb{Z}$$

**(3)**

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# Looking at AO2 on P1, P2, P3 and P4

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°,\ n \in \mathbb{Z}$$

(3)

| | | |
|---|---|---|
| **A** | $\dfrac{1-\cos 2x}{1+\cos 2x} = \dfrac{\sin^2 x}{\cos^2 x} = \tan^2 x$ | ? |
| **B** | $\dfrac{1-\cos 2x}{1+\cos 2x} = \tan^2 x \Rightarrow 1-\cos 2x = \tan^2 x\left(1+\cos 2x\right)$<br>$1-\left(1-2\sin^2 x\right) = \tan^2 x\left(1+2\cos^2 x-1\right)$<br>$2\sin^2 x = \dfrac{\sin^2 x}{\cos^2 x}\left(2\cos^2 x\right)$<br>$2\sin^2 x = 2\sin^2 x$ | ? |

How many marks should these approaches get?

# Looking at AO2 on P1, P2, P3 and P4

(a) Prove that

$$\frac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x, \qquad x \neq (2n + 1)90°, n \in \mathbb{Z}$$

(3)

| | |
|---|---|
| $\frac{1-\cos 2x}{1+\cos 2x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$ | M0dM0A0 |

How many marks should these approaches get?

| | |
|---|---|
| $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x \Rightarrow 1-\cos 2x = \tan^2 x(1+\cos 2x)$<br>$1-(1-2\sin^2 x) = \tan^2 x(1+2\cos^2 x - 1)$<br>$2\sin^2 x = \frac{\sin^2 x}{\cos^2 x}(2\cos^2 x)$<br>$2\sin^2 x = 2\sin^2 x$ | M1dM1A1 |

# Looking at AO2 on P1, P2, P3 and P4

## Proof (by deduction) Trig identities

- Ideally:
- Start with the more complex side of the identity.
- Simplify the more complex to the less complex side.

Less than ideal (but usually still valid)
Work on both sides to reach a trig statement which is obviously true
(e.g. $\sin^2 x = \sin^2 x$)

There should be a conclusion to complete in this case

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# Looking at AO2 on P1, P2, P3 and P4

## Proof (by deduction) Trig identities

Proof by deduction requires you to start from **known facts**
What are these?

Knowledge of

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$,

and $\sin^2\theta + \cos^2\theta = 1$.

In the Specification for P2

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

As required knowledge for P3

…..and formulae in the formulae book.

# Activity 5

Activity 5 has examples of attempted proofs of two fairly standard results – one from P2 and one from P3

Look through the proofs and decide whether they are valid or not

Please use the poll to record your decisions for 1A, !B and 2A

# Looking at AO2 on P1, P2, P3 and P4

## Proof (by deduction) Trig and logs

| | |
|---|---|
| **B** | $\log_4 r = \log_{2^2} r$ |
| | $\log_4 r = \log_2 \sqrt{r}$ |
| | $\log_4 r = \frac{1}{2} \log_2 r$ |

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# Looking at AO2 on P1, P2, P3 and P4

## Proof (by deduction) Trig and logs

**A** $\dfrac{1+\sin 2x}{\cos 2x} = \dfrac{1+\tan x}{1-\tan x}$

$$(1+\sin 2x)(1-\tan x) = \cos 2x(1+\tan x)$$

$$1+\sin 2x - \tan x - \sin 2x\tan x = \cos 2x\tan x$$

$$\cos x + \cos x\sin 2x - \sin x - \sin 2x\sin x = \cos 2x\cos x + \cos 2x\sin x$$

$$\cos x + \cos x\sin 2x - \sin x - \cos 2x\sin x = \cos 2x\cos x + \sin 2x\sin x$$

$$\cos x - \sin x = \cos x - \sin x$$

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# Types of Proof

## d. Proof by contradiction

P4 only

✎ To prove a statement is true by contradiction:

- **Assume** that the statement is in fact **false**.
- Prove that this **leads to a contradiction**.
- Therefore we were wrong in assuming the statement was false, and therefore it must be true.

**How to structure/word proof:**

1. "Assume that *[negation of statement]*."
2. [*Reasoning followed by…*] "*This contradicts the assumption that*…" or "*This is a contradiction*".
3. "Therefore [*restate original statement*]."

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# Looking at AO2 on P1, P2, P3 and P4

The first part of a proof by contradiction requires you to negate the original statement. What is the negation of each of these statements?

“There are infinitely many prime numbers.”

“There are infinitely many non-prime (i.e. composite) numbers.”

“There are finitely many prime numbers.”

“There are finitely many non-composite numbers.”

Proved by contradiction over 2000 years ago

# Looking at AO2 on P1, P2, P3 and P4

The first part of a proof by contradiction requires you to negate the original statement. What is the negation of each of these statements?

"All teachers are clever"

"There exists a teacher who is not clever."

"No teachers are clever."

"The trainer is clever."

**Comments:** The negation of "all are" is not "none are". So the negation of "everyone likes green" wouldn't be "no one likes green", but: "not everyone likes green". Do not confuse a 'negation' with the 'opposite'.

# Looking at AO2 on P1, P2, P3 and P4

The first part of a proof by contradiction requires you to negate the original statement. What is the negation of each of these statements?

"If it is raining, then my garden is wet."

"If it is not raining, my garden is dry."

"If it is not raining, my garden is wet."

"If it is raining, my garden is not wet."

**Comments:** If you have a conditional statement like "*If A then B*", then the negation is "*If A then not B*", i.e. the same condition applies, but the implication is negated.

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# Negating the original statement

An important negation which students should be aware of is that of

| | |
|---|---|
| Original statement | $a < b$ |
| Negation | |

Less common is

| | |
|---|---|
| Original statement | $a = b$ |
| Negation | |

# Negating the original statement

An important negation which students should be aware of is that of

| | |
|---|---|
| Original statement | $a < b$ |
| Negation | $a \geq b$ (or $b \leq a$) |

Less common is

| | |
|---|---|
| Original statement | $a = b$ |
| Negation | $a \neq b$ |

# An Example

Prove by contradiction that if $n^2$ is even, then $n$ must be even.

Assume there exists a number $n$ such that $n^2$ is even, but $n$ is not even.

Assumption must be stated

$n$ is odd therefore $n = 2k + 1$ for some integer $k$.

$$\begin{aligned} n^2 &= (2k+1)^2 = 4k^2 + 4k + 1 \\ &= 2(2k^2 + 1) + 1 \end{aligned}$$

which is odd.

This contradicts the assumption that $n^2$ is even.

'Contradiction' must be stated

Therefore if $n^2$ is even then $n$ must be even.

Conclusion must be stated

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# Looking at AO2 on P1, P2, P3 and P4

Proof on P4

Students should be familiar with the proofs of the infinity of prime numbers and with the irrationality of the square root of 2 In the Spec

It's then easy to prove, for example that $(\sqrt{2} + 1)$ is irrational – by contradiction

Students don't find it easy to adapt the proof of the irrationality of $\sqrt{2}$ to, for example, $\sqrt{3}$ – but should be encouraged to do so.

Students should also be encouraged to decide why the 'proof' breaks down for $\sqrt{4}$ .

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# Looking at AO2 on P1, P2, P3 and P4

As an example, here is a slightly different proof to Euclid for the infinity of primes

1. (a) Assume the number of primes is finite.
   (b) So there must be a largest prime, $p$, say

2. Let $N = p! + 1$
   Clearly $N > p$

3. Then $N$ leaves a remainder of 1 when divided by any prime $\leq p$

4. So either $N$ is prime or has a prime factor $> p$.

5. So the assumption is false and the original statement proved..

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# Looking at AO2 on P1, P2, P3 and P4

‘Proof ‘ in the units – a summary

- P1 – no formal proofs, but ‘show that’s’’ may be set

- P2 – 1.1 (deductive proof),1.2 (proof by exhaustion) and 1.3 (Disproof by counter example)

- P3 – 2.2 and 2.3 (proofs of trigonometric identities)

- P4 – 1.1 (proof by contradiction)

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# Looking at AO2 on P1, P2, P3 and P4

‘Activity 6 (Light Blue) – Proof

Activity 6 contains 10 proofs

Select any that interest you and attempt a proof.

What knowledge/ concepts and techniques are required?

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# Looking at AO2 on P1, P2, P3 and P4

‘Extended arguments involving the manipulation of mathematical expressions.’

Stage 1

↓ Translation and understanding

Stage 2

↓ Calling upon relevant knowledge ↓ ↑ Planning a strategy

Stage 3

↓ Execution and interpretation

Stage 4

# Looking at AO2 on P1, P2, P3 and P4

‘Extended arguments involving the manipulation of mathematical expressions.’

Solving extended problems is hard – especially in a limited time!

The difficulty of a problem depends upon several factors!

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# Looking at AO2 on P1, P2, P3 and P4

‘Extended arguments involving the manipulation of mathematical expressions.’

…..whether the answer is given or not:

‘Show that…’ instead of

‘Find’

For an argument supporting reasons do not usually have to be given

We could claim that ‘Show that’ is easier than ‘Find’ because it gives the student a definite end point

We could claim that ‘Show that’ is harder than ‘Find’ because it could force the student to use a specific method.

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# Scaffolding

**Scaffolding** is the term used to add structure to a question which will usually require extended mathematics .

$$f(x) = \frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \qquad x > 2$$

Prove that f($x$) is a decreasing function.

Just think for a moment about what strategies would students plan to use to answer this question……

The question was eventually scaffolded as………

# Scaffolding

**Scaffolding** is the term used to add structure to a question which will usually require extended mathematics to answer it.

Gives a start. Makes it clear what is required

$$\frac{6-5x-4x^2}{(2-x)(1+2x)} \equiv A + \frac{B}{(2-x)} + \frac{C}{(1+2x)}$$

(a) Find the values of the constants $A$, $B$ and $C$.

$$f(x) = \frac{6-5x-4x^2}{(2-x)(1+2x)} \qquad x > 2$$

(b)

(c)

# Scaffolding

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

find the exact coordinates of the point on the curve where $\dfrac{dy}{dx} = 2\sqrt{3}$

No scaffolding.
This Q could be classed as basically all AO2

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# Looking at AO2 on P1, P2, P3 and P4

:As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

Show that

(*a*) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k \cot x$, where $k$ is a constant to be found.

Hence find the exact coordinates of the point on the curve where

(*b*) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\sqrt{3}$

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# Looking at AO2 on P1, P2, P3 and P4

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

(ii) (*a*) Use the substitution $x = \sec\theta$ to show that

$$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x = \int_0^{\frac{\pi}{3}} \tan^2\theta\,\mathrm{d}\theta$$

(*b*) Hence find the exact value of

$$\int_1^2 \sqrt{1-\frac{1}{x^2}}\,\mathrm{d}x$$

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# Looking at AO2 on P1, P2, P3 and P4

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

find the exact value of

# Looking at AO2 on P1, P2, P3 and P4

Questions assessing AO2 are almost always assigned AO1 marks also. This is because the processes and knowledge required to solve a complex problem are based on AO1

We can see this by looking at a particular topic from the Specification.

To see how this works we'll look at **parameters**.

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# Looking at AO2 on P1, P2, P3 and P4

| | | |
|---|---|---|
| 3.1 | Parametric equations of curves and conversion between cartesian and parametric forms. | |
| 5.1 | Differentiation of simple functions defined implicitly or parametrically. | The finding of equations of tangents and normals to curves given parametrically or implicitly is required. |
| 6.1 | Evaluation of volume of revolution. | $\pi\int y^2 \, dx$ is required, but *not* $\pi\int x^2 \, dy$. Students should be able to find a volume of revolution, given parametric equations. |

The scheme of work – on the website – gives some further details

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# P4 Parameters – how AO1 and AO2 work together (also with AO4)

| | |
|---|---|
| 3.1 | Parametric equations of curves and conversion between cartesian and parametric forms. |

From a Principal Examiner Report
………only a minority of students were able to use one of the trigonometric forms of Pythagoras to eliminate *t* and manipulate the resulting equation to obtain an answer in the required form.

$x = 6\cos 2t$, $y = 2\sin t$

$x = \tan t$, $y = 2\sin^2 t$

$x = 1 + \sqrt{3}\tan t$, $y = 5\sec t$

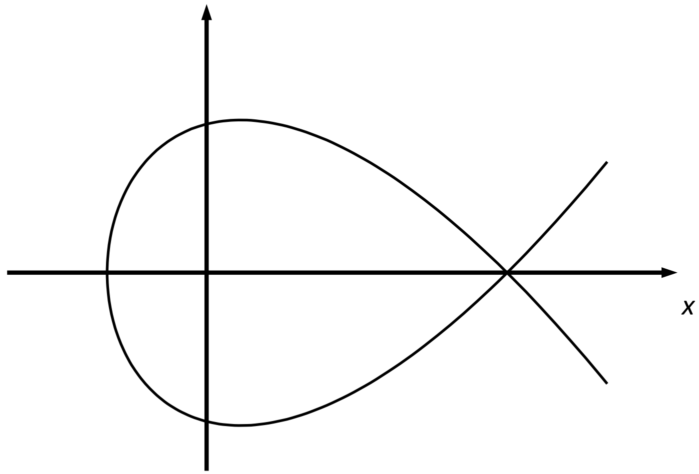
$x = 3\cos t$, $y = 9\sin 2t$

$x = \tan t$, $y = 2\sin^2 t$

$x = 8\cos^3 t$, $y = 6\sin^2 t$

These were taken from recent C34 papers. Most involve use of $\sin^2 + \cos^2 = 1$ or its equivalent.

# P4 Parameters – how AO1 and AO2 work together (also with AO4)




$x = 2t^2 - 2, \quad y = t(t^2 - 4)$

Find the coordinates of points where
- the curve crosses the axes
- crosses the line $x = 2$

# P4 Parameters – how AO1 and AO2 work together (also with AO4)

| | | |
|---|---|---|
| 5.1 | Differentiation of simple functions defined implicitly or parametrically. | The finding of equations of tangents and normals to curves given parametrically or implicitly is required. |

$x = 6\cos 2t$, $y = 2\sin t$

*Show that $y' = \lambda \operatorname{cosec} t$, giving the exact value of the constant $\lambda$.*

$x = 1 + \sqrt{3}\tan t$, $y = 5\sec t$

*Show that $y' = \lambda \sin t$, giving the exact value of the constant $\lambda$.*

$x = 8\cos^3 t$, $y = 6\sin^2 t$

It's almost always easier to use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$

This also can involve use of trig identities to achieve a given answer

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# P4 Parameters – how AO1 and AO2 work together (also with AO4)

## e.g.. Parameters

$x = 8\cos^3 t$, $y = 6\sin^2 t$

Find the equation of the normal to the curve at $t = \pi/3$
Give the equation in the form $ax + by = c$

# P4 Parameters – how AO1 and AO2 work together (also with AO4)

| | | |
|---|---|---|
| 6.1 | Evaluation of volume of revolution. | $\pi \int y^2 \, dx$ is required, but *not* $\pi \int x^2 \, dy$. Students should be able to find a volume of revolution, given parametric equations. |

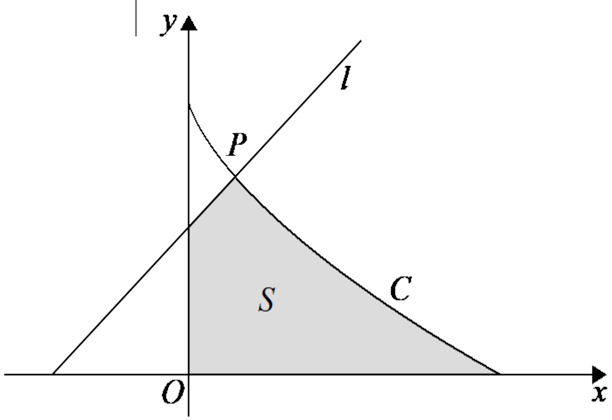


**Figure 6**

$x = 8\cos^3 \theta$, $y = 6\sin^2 \theta$,

The line l is the normal to C at P with $t = \pi/3$

(*c*) Show that the area of $S$ is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,d\theta$$

(*d*) Hence, by integration, find the exact area of $S$.

So students have to be able to transform an integral in *x* to one in *θ*

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# Any questions?

# Thank you for attending this event.

***How did we do?***
***Please fill in the evaluation form that you'll receive via e-mail in a few minutes.***

ALWAYS LEARNING