Pearson Edexcel

INTERNATIONAL ADVANCED LEVEL

MATHEMATICS/ FURTHER MATHEMATICS/ PURE MATHEMATICS

SCHEME OF WORK

PURE MATHEMATICS 1

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA01)
Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM01)
Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM01)
Pearson Edexcel International Advanced Level in Mathematics (YMA01)
Pearson Edexcel International Advanced Level in Further Mathematics (YFM01)
Pearson Edexcel International Advanced Level in Pure Mathematics (YPM01)
First teaching September 2018
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First certification from August 2019 (International Advanced Subsidiary) and August 2020 (International Advanced Level)

Contents

Introduction

This scheme of work can be used directly as a scheme of work for Pure Mathematics 1 of International Advanced Subsidiary/ Advanced Level in Mathematics/ Further Mathematics/ Pure Mathematics specification (XMA01/XFM01/XPM01/YMA01/YFM01/YPM01).

The scheme of work is broken up into units and sub-units, so that there is greater flexibility for moving topics around to meet planning needs.

Each unit contains:

- Specification references
- Prior knowledge
- Keywords
- Notes.

Each sub-unit contains:

- Recommended teaching time, though of course this is adaptable according to individual teaching needs
- Objectives for students at the end of the sub-unit
- Teaching points
- Opportunities for problem-solving and modelling
- Common misconceptions and examiner report quotes (from legacy Specifications)
- Notes

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

Our free support for the International AS and A level Mathematics/Further Mathematics/Pure Mathematics specifications can be found on the Pearson Edexcel Mathematics website and on the Emporium.

Pure Mathematics 1

	Title	Estimated hours
1	**Algebra and functions**	
a	Algebraic expressions: basic algebraic manipulation, indices and surds	**4**
b	Quadratic functions: factorising, solving, graphs and discriminants	**4**
c	Equations: quadratic/linear simultaneous	**4**
d	Inequalities: linear and quadratic (including graphical solutions)	**5**
e	Graphs: cubic and reciprocal	**5**
f	Transformations: transforming graphs; f(x) notation	**5**
2	**Trigonometry**	
a	Trigonometric ratios and graphs, and area of a triangle in the form $\frac{1}{2}ab\sin C$	**6**
b	Radians (exact values), arcs and sectors	**4**
3	**Coordinate geometry in the (x, y) plane:** Straight-line graphs, parallel/perpendicular, length and area problems	**6**
4	**Differentiation**	
a	Definition, differentiating polynomials, second derivatives	**6**
b	Gradients, tangents and normals	**5**
5	**Integration:** Definition as opposite of differentiation, indefinite integrals of x^n	**6**
		60 hours

1: Algebra and functions

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SPECIFICATION REFERENCES

1.1 Laws of indices for all rational exponents
1.2 Use and manipulation of surds
1.3 Quadratic functions and their graphs
1.4 The discriminant of a quadratic function
1.5 Completing the square. Solution of quadratic equations
1.6 Solve simultaneous equations; analytical solution by substitution
1.7 Interpret linear and quadratic inequalities graphically
1.8 Represent linear and quadratic inequalities graphically
1.9 Solutions of linear and quadratic inequalities
1.10 Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation
1.11 Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations
1.12 Knowledge of the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$

PRIOR KNOWLEDGE

<u>INTERNATIONAL GCSE/GCSE (9-1) in Mathematics at Higher Tier</u>

- Collecting like terms and factorising
- Manipulating surds and rationalising the denominator
- Solving linear simultaneous equations
- Solving quadratic equations (by factorising and completing the square)
- Working with inequalities
- Solving quadratic inequalities
- Functional notation and shapes of standard graphs (e.g. parabola, cubic, reciprocal)
- Rules of indices

KEYWORDS

Expression, function, constant, variable, term, unknown, coefficient, index, linear, identity, simultaneous, elimination, substitution, factorise, completing the square, intersection, change the subject, cross-multiply, power, exponent, base, rational, irrational, reciprocal, root, standard form, surd, rationalise, exact, manipulate, sketch, plot, quadratic, maximum, minimum, turning point, transformation, translation, polynomial, discriminant, real roots, repeated roots, quotient, intercepts, inequality, asymptote.

1a. Algebraic expressions: basic algebraic manipulation, indices and surds (1.1) (1.2) (1.10)	Teaching time 4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to perform essential algebraic manipulations, such as expanding brackets, collecting like terms, factorising etc;
- understand and be able to use the laws of indices for all rational exponents;
- be able to use and manipulate surds, including rationalising the denominator.

TEACHING POINTS

Recap the skills taught at INTERNATIONAL GCSE/GCSE Higher Tier (9-1).

Emphasise that in many cases, only a fraction or surd can express the exact answer, so it is important to be able to calculate with surds.

Ensure students understand that $\sqrt{a} + \sqrt{b}$ is *not* equal to $\sqrt{a+b}$ and that they know that $a^{\frac{m}{n}}$ is equivalent to $\sqrt[n]{a^m}$ and that a^{-m} is equivalent to $\frac{1}{a^m}$.

Most students understand the skills needed to complete these calculations but make basic errors with arithmetic leading to incorrect solutions.

Questions involving squares, for example $(2\sqrt{3})^2$, will need practice.

Students should be exposed to lots of simplifying questions involving fractions as this is where most marks are lost in exams.

Recap the difference of two squares $(x + y)(x - y) = x^2 - y^2$. Link this to $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = x - y$, explaining the choice of term when rationalising the denominator of a fraction involving surds.

Provide students with plenty of practice and ensure that they check their answers.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Include examples which involve calculating areas of shapes with side lengths expressed as surds. Exact solutions for Pythagoras questions is another place where surds occur naturally.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Common errors include: misinterpreting $\left(a\sqrt{b}\right)^2$ as $(a+\sqrt{b})^2$; evaluating $(\sqrt{2})^2$ as 4 instead of 2; slips when multiplying out brackets; basic arithmetic errors; and leaving surds in the denominator rather than fully simplifying fractions. Two examples of errors with indices are, writing $\frac{1}{3x}$ as $3x^{-1}$and writing $\frac{4}{\sqrt{x}}$ as $4x^{\frac{1}{2}}$; these have significant implications later in the course (e.g. differentiation).

Many of these errors can be avoided if students carefully check their work and have plenty of practice.

NOTES

Make use of matching activities (e.g. Tarsia puzzles)

1b. Quadratic functions: factorising, solving, graphs and discriminants (1.3) (1.4) (1.5)	Teaching time 4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve a quadratic equation by factorisation and use of the formula;
- be able to work with quadratic functions and their graphs;
- know and be able to use the discriminant of a quadratic function, including the conditions for real and repeated roots;
- be able to complete the square. e.g. $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$;
- be able to solve quadratic equations, including in a function of the unknown

TEACHING POINTS

Lots of practice is needed as these algebraic skills are fundamental to all subsequent work. Students must become fluent, and continue to develop thinking skills such as choosing an appropriate method, and interpreting the language in a question. Emphasise correct setting out and notation.

Students will need lots of practice with negative coefficients for *x* squared and be reminded to always use brackets if using a calculator. e.g. $(-2)^2$. They should also be aware of how their calculator can be used to solve quadratic equations.

Include manipulation of surds when using the formula for solving quadratic equations. [Link with previous sub-unit.]

Where examples are in a real-life contexts, students should check that solutions are appropriate and be aware that a negative solution may not be appropriate in some situations.

Students must be made aware that this sub-unit is about finding the links between completing the square and factorised forms of a quadratic and the effect this has on the graph. Use graph drawing packages to see the effect of changing the value of the '+ *c*' and link this with the roots and hence the discriminant.

Start by drawing $y = x^2$ and add different *x* terms followed by different constants in a systematic way. Then move on to expressions where the coefficient of x^2 is not 1.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

The path of an object thrown can be modelled using quadratic graphs. Various questions can be posed about the path:

- When is the object at a certain height?
- What is the maximum height?
- Will it clear a wall of a certain height, a certain distance away?

Areas of shapes where the side lengths are given as algebraic expressions.
Proof of the quadratic formula.

Working backwards, e.g. find a quadratic equation whose roots are $\frac{-5\pm\sqrt{17}}{4}$

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When completing the square, odd coefficients of *x* can cause difficulties. Students do not always relate finding the minimum point and line of symmetry to completing the square. Students should be provided with plenty of practice on completing the square with a wide range of quadratic forms.

Notation and layout can also be a problem; students must remember to show all the necessary working out at every stage of a calculation, particularly on 'show that' questions.

Examiners often refer to poor use of the quadratic formula. In some cases the formula is used without quoting it first and there are errors in substitution. In particular, the use of $-b \pm \frac{\sqrt{b^2-4ac}}{2a}$ so that the division does not extend under the '$-b$', is relatively common.

Another common mistake is to think that the denominator is always 2. Also, students sometimes include *x*'s in their expressions for the discriminant. Such methods are likely to lose a significant number of marks.

NOTES

Encourage the use of graphing packages or graphing Apps (e.g. Desmos or Autograph), so students can graph as they go along and 'picture' their solutions. You can link the discriminant with complex numbers if appropriate for students also studying Further Maths.

1c. Equations: quadratic/linear simultaneous (1.6)	Teaching time 4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve linear simultaneous equations using elimination and substitution;
- be able to use substitution to solve simultaneous equations where one equation is linear and the other quadratic

TEACHING POINTS

Simultaneous equations are important both in future pure topics but also for applied maths. Students will need to be confident solving simultaneous equations including those with non-integer coefficients of either or both variables.

The quadratic may involve powers of 2 in one unknown or in both unknowns, e.g. Solve $y = 2x + 3$, $y = x^2 - 4x + 8$ or $2x - 3y = 6$, $x^2 - y^2 + 3x = 50$.

Emphasise that simultaneous equations lead to a pair or pairs of solutions, and that both variables need to be found.

Make sure students practise examples of worded problems where the equations need to be set up.

Students should be encouraged to check their answers using substitution.
Sketches can be used to check the number of solutions and whether they will be positive or negative. This will be reviewed and expanded upon as part of the curve sketching topic.

Use graphing packages or graphing Apps (e.g. Desmos or Autograph), so students can visualise their solutions e.g. straight lines crossing an ellipse or a circle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Simultaneous equations in contexts, such as costs of items given total cost, can be used. Students must be aware of the context and ensure that the solutions they give are appropriate to that context.

Simultaneous equations will be drawn on heavily in curve sketching and coordinate geometry.

Investigate when simultaneous equations cannot be solved or only give rise to one solution rather than two.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Mistakes are often due to signs errors or algebraic slips which result in incorrect coordinates. Students should be encouraged to check their working and final answers, and if the answer seems unlikely, to go back and look for errors in their working.

Examiners often notice that it is the more successful candidates who check their solutions.

Students should remember to find the values of both variables as stopping after finding one is a common cause of lost marks in exam situations. Students who do remember to find the values of the second variable must take care that they substitute into a correct equation or a correctly rearranged equation.

1d. Inequalities: linear and quadratic (including graphical solutions) (1.7) (1.8) (1.9)	Teaching time 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to solve linear and quadratic inequalities;
- know how to express solutions through correct use of 'and' and 'or' or through set notation;
- be able to interpret linear and quadratic inequalities graphically;
- be able to represent linear and quadratic inequalities graphically

TEACHING POINTS

Provide students with plenty of practice at expressing solutions in different forms using the correct notation. Students must be able to express solutions using 'and' and 'or' appropriately, or by using set notation.

So, for example:

$x < a$ **or** $x > b$ is equivalent to $\{x: x < a\} \cup \{x: x > b\}$

and $\{x: c < x\} \cap \{x: x < d\}$ is equivalent to $x > c$ **and** $x < d$.

Inequalities may contain brackets and fractions, but these will be reducible to linear or quadratic inequalities. For example, $\frac{a}{x} < b$ becomes $ax < bx^2$.

Students' attention should be drawn to the effect of multiplying or dividing by a negative value, this must also be taken into consideration when multiplying or dividing by an unknown constant.

Sketches are the most commonly used method for identifying the correct regions for quadratic inequalities, though other methods may be used. Whatever their method, students should be encouraged to make clear how they obtained their answer.

Students will need to be confident interpreting and sketching both linear and quadratic graphs in order to use them in the context of inequalities.

Make sure that students are also able to interpret combined inequalities.
For example, solving

$ax + b > cx + d$
$px^2 + qx + r \geq 0$
$px^2 + qx + r < ax + b$

and interpreting the third inequality as the range of x for which the curve $y = px^2 + qx + r$ is below the line with equation $y = ax + b$.

When representing inequalities graphically, shading and correctly using the conventions of dotted and solid lines is required. Students using graphical calculators or computer graphing software will need to ensure they understand any differences between the conventions required and those used by their graphical calculator.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Financial or material constraints within business contexts can provide situations for using inequalities in modelling. For those doing Decision maths this will link to linear programming.

Inequalities can be linked to length, area and volume where side lengths are given as algebraic expressions and a maximum or minimum is given.

Following on from using a quadratic graph to model the path of an object being thrown, inequalities could be used to find the time for which the object is above a certain height.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students may make mistakes when multiplying or dividing inequalities by negative numbers.

In exam questions, some students stop when they have worked out the critical values rather than going on to identify the appropriate regions. Sketches are often helpful at this stage for working out the required region.

It is quite common, when asked to solve an inequality such as $2x^2 - 17x + 36 < 0$ to see an incorrect solution such as
$2x^2 - 17x + 36 < 0 \Rightarrow (2x - 9)(x - 4) < 0 \Rightarrow x < \frac{9}{2}, x < 4$.

1e. Graphs: cubic and reciprocal (1.11)	Teaching time 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and use graphs of functions;
- be able to sketch curves defined by simple equations including polynomials;
- be able to use intersection points of graphs to solve equations.

TEACHING POINTS

Students should be familiar with the general shape of cubic curves from INTERNATIONAL GCSE/GCSE (9-1) Mathematics, so a good starting point is asking them to identify key features and draw sketches of the general shape of a positive or negative cubic. Equations can then be given from which to sketch curves.

Cubic equations given at this point should either already be factorised or be easily simplified
(e.g. $y = x^3 + 4x^2 + 3x$) as students will not yet have encountered algebraic division.

The coordinates of all intersections with the axes will need to be found. Where equations are already factorised, students will need to find where they intercept the axes. Repeated roots will need to be explicitly covered as this can cause confusion.

Students should also be able to find an equation when given a sketch on which all intersections with the axes are given. To do this they will need to be confident multiplying out multiple brackets.

Reciprocal graphs in the form $y = \frac{a}{x}$ are covered at INTERNATIONAL GCSE/GCSE but those in the form $y = \frac{a}{x^2}$ will be new. When sketching reciprocal graphs such as $y = \frac{a}{x}$ and $= \frac{a}{x^2}$, the asymptotes will be parallel to the axes.

Intersecting points of graphs can be used to solve equations, a curve and a line and two curves should be covered. When finding points of intersection students should be encouraged to check that their answers are sensible in relation to the sketch.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to justify the number of solutions to simultaneous equations using the intersections of two curves.
Students can be given sketches of curves or photographs of curved objects (e.g. roller coasters, bridges, etc.) and asked to suggest possible equations that could have been used to generate each sketch.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

When sketching cubic graphs, most students are able to gain marks by knowing the basic shape and sketching it passing through the origin. Recognising whether the cubic is positive or negative sometimes causes more difficulty. Students sometimes fail to recognise the significance of a square factor in the factorised form of a polynomial.

When sketching graphs, marks can easily be lost by not labelling all the key points or labelling them incorrectly e.g. (0, 6) instead of (6, 0).

1f. Transformations: transforming graphs, f(x) notation (1.12)	Teaching time 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$;
- be able to sketch the result of a simple transformation given the graph of any function $y = \mathrm{f}(x)$

TEACHING POINTS

Transformations to be covered are: $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ and $y = \mathrm{f}(ax)$.

Students should be able to apply one of these transformations to any of the functions listed and sketch the resulting graph:
quadratics, cubics, reciprocals, $y = \frac{a}{x^2}$, $\sin x$, $\cos x$, and $\tan x$.

Students will need to be able to transform points and asymptotes both when sketching a curve and to give either the new point or the equation of the line.

Given a curve or an equation that has been transformed, students should be able to state the transformation that has been used.

Links can be made with sketching specific curves. Students should be able to sketch curves like $y = (x - 3)^2 + 2$ and $y = \frac{2}{x-3} + 2$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Examples can be used in which the graph is transformed by an unknown constant and students encouraged to think about the effects this will have.
The use of graphing packages or graphing Apps (e.g. Desmos or Autograph) can be invaluable here.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

One of the most common errors is translating the curve in the wrong direction for f(x + a) or f(x) + a. Students sometimes also apply the wrong scale factor when sketching f(ax).

Other errors involve algebraic mistakes and incomplete sketches, or sketches without key values marked.

Students should be encouraged to check any answers they have calculated against their sketches to check they make sense.

NOTES

Dynamic geometry packages can be used to help students investigate and visualise the effect of transformations.

2: Trigonometry

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SPECIFICATION REFERENCES

3.1 The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab\sin C$
3.2 Radian measure, including use for arc length and area of sector
3.3 Sine, cosine and tangent functions. Their graphs, symmetries and periodicity

PRIOR KNOWLEDGE

Algebra covered so far:
Basic algebraic manipulation
Quadratics
Graph transformations

<u>INTERNATIONAL GCSE/GCSE (9-1) in Mathematics at Higher Tier</u>

- Pythagoras' theorem
- Trigonometry in right-angled triangles
- The sine rule
- The cosine rule
- The area of a triangle: $\frac{1}{2}ab\sin C$
- Bearings
- Exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$ for $\theta = 30°, 45°$ and $60°$ (GCSE(9-1) only)

KEYWORDS

Sine, cosine, tangent, interval, period, amplitude, function, inverse, angle of elevation, angle of depression, bearing, degree, identity, special angles, unit circle, symmetry, hypotenuse, opposite, adjacent, intercept.

2a. Trigonometric ratios and graphs, and area of a triangle in the form $\frac{1}{2}ab\sin C$ (3.1) (3.3)	Teaching time 6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the sine and cosine rules;
- understand and be able to use the area of a triangle in the form $\frac{1}{2}ab\sin C$;
- understand and be able to use the sine, cosine and tangent functions; their graphs, symmetries and periodicity

TEACHING POINTS

Students should be shown how the *x* and *y* coordinates of points on the unit circle can be used to give cosine and sine respectively.

Use of trigonometric ratios will have been covered at INTERNATIONAL GCSE/GCSE (9-1) Mathematics; questions should now be focused more on multi-step problems and questions set in context.

When using the sine rule the ambiguous case should be covered. Students could be encouraged to prove the area of a triangle.

Students should be encouraged to write down any formulae they will be using before substituting in the numbers.

Students should be able to solve questions in various contexts; these could include coordinate geometry or real-life situations. Questions may involve bearings, which may not be well remembered from INTERNATIONAL GCSE/GCSE so should be reviewed. Students should be encouraged to check that their answers are realistic as this check can show up errors.

When completing multi-step questions emphasise to students that they should show all working out and use the answer function on their calculators to avoid rounding errors. It can be a useful teaching point to divide the class asking one side to round all answers and the other to keep values stored in their calculator to show how this affects the final answer.

The unit circle can again be used to show how the trigonometric graphs are formed. Characteristics such as the period and amplitude should be discussed. Knowledge of graphs of curves with equations such as $y = \sin x$, $y = \cos(x + 30)$, $y = \tan 2x$ is expected so this is a good opportunity to recap transformations.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use of the graphs can be linked to modelling situations such as yearly temperatures, wave lengths and tidal patterns. Proof of the sine and cosine rules.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students occasionally assume that triangles given in exam questions are right-angled and so use right-angled trigonometric ratios rather than the sine and cosine rules.

A frequently seen error in these questions is students using the cosine rule to calculate an incorrect angle, sometimes despite having drawn a correctly labelled diagram. This indicates a lack of understanding of how the labelling of edges and angles on a diagram relates to the application of the cosine rule formula.

2b. Radians (exact values), arcs and sectors (3.2)	Teaching time 4 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand the definition of a radian and be able to convert between radians and degrees;
- be able to derive and use the formulae for arc length and area of sector.

TEACHING POINTS

Ensure all students know how to change between radian and degree mode on their own calculators and emphasise the need to check which mode it is in.

Radian measure will be new to students and it is important that they understand what 1 radian actually is.

Make sure students know that 'exact value' implies an answer must be given in surd form or as a multiple of π.
They need to know the exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ (and their multiples) and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ (and their multiples).

Emphasise the need to always put a scale on both axes when drawing trigonometric graphs; students must be able to do this in radians.

Make links between writing the trig ratio of any angle (obtuse/reflex/negative) to the trig ratio of an acute angle and to the trig graphs. (Do not rely on the CAST method as this tends to show a lack of understanding.)

Derive the formulae for arc length and area of a sector by replacing the $\frac{\theta}{360°}$ in the INTERNATIONAL GCSE/GCSE formulae with $\frac{\theta}{2\pi}$. The πs cancel giving length of arc = $r\theta$ and area of sector = $\frac{1}{2}r^2\theta$.

Cover examples which will involve finding the area of a segment by subtracting a triangle from a sector.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

One radian can be defined as 'the angle at the centre of a circle which measures out exactly one radius around the circumference.' Therefore, using $C = 2\pi r$, we can conclude that the full circumference, C is made up of 2π radians. This means 360 is equivalent to 2π radians.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

A common exam mistake is for students to have their calculators set in the wrong mode resulting in the loss of accuracy marks.

3: Coordinate geometry in the (*x*, *y*) plane:	**Teaching time** 6 hours

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SPECIFICATION REFERENCES

2.1 Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$

2.2 Conditions for two straight lines to be parallel or perpendicular to each other

PRIOR KNOWLEDGE

Algebraic manipulation covered so far
Simultaneous equations

<u>INTERNATIONAL GCSE/GCSE (9-1) in Mathematics at Higher Tier</u>
Equation of a line
Parallel and perpendicular lines
Pythagoras
Calculating the proportionality constant k
Circle theorems

KEYWORDS

Equation, bisect, centre, chord, circle, circumcircle, coefficient, constant, diameter, gradient, hypotenuse, intercept, isosceles, linear, midpoint, parallel, perpendicular, proportion, Pythagoras, radius, right angle, segment, semicircle, simultaneous, tangent.

OBJECTIVES

By the end of the unit, students should:

- understand and use the equation of a straight line;
- know and be able to apply the gradient conditions for two straight lines to be parallel or perpendicular;
- be able to find lengths and areas using equations of straight lines;
- be able to use straight-line graphs in modelling

TEACHING POINTS

Students should be encouraged to draw sketches when answering questions or, if a diagram is given, annotate the diagram.

Equations can be given or asked for in the forms $y = mx + c$ and $ax + by + c = 0$ where a, b and c are integers. Students will need to be familiar with both forms, so questions should be asked where different forms are given or required in the answer. Given either form, students should be able to find the intercepts with the axes and the gradient. The x-intercept often causes students more difficulty, so will need more practice, but is useful for sketches and questions involving area or perimeter.

Students should be able to find the equation of a line given the gradient and a point, either the formula $y - y_1 = m(x - x_1)$ can be used or the values substituted into $y = mx + c$. To find the equation of a line from two points the gradient can be found and then one of the previous two methods used or the formula $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$ can be used.

If this formula is used, care needs to be taken to ensure that the y-values are substituted into the correct places and that negative signs are taken into account. It should be emphasised that in the majority of cases, the form $y - y_1 = m(x - x_1)$ is far more efficient and less prone to errors than other methods.

The gradient conditions for parallel and perpendicular lines may be remembered from INTERNATIONAL GCSE/GCSE (9-1), but are still worth revising. They need to be well understood as they are used further when dealing with circles and in differentiation.

Students should be able to identify whether lines are parallel, perpendicular or neither and find the equation of a parallel or perpendicular line when given a point on the line.

The length of a line segment is found by using Pythagoras' theorem, which can be written as the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Students could be encouraged to show how to go from Pythagoras to the formula. Answers to length and distance questions are likely to be given in surd form, giving further practice in simplifying surds. Students should be encouraged to give answers in exact form unless specified otherwise.

Make shapes using lines and the axes; students can then be asked to find the area or perimeter of composite shapes. Answers should be given in exact form to practise combining and simplifying surds.

Real-life situations such as conversions can be modelled using straight-line graphs, this is likely to be familiar from INTERNATIONAL GCSE/GCSE (9-1) Mathematics.

Students should also be familiar with finding the relationship between two variables and expressing this using the proportion symbol $\propto$ or using an equation involving a constant (k). This can be extended to straight-line graphs through the origin with a gradient of k. Students should be able to calculate and interpret the gradient.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

To help students see how much information is given in the equation of a line, a good activity is to give an equation and ask students to find everything they know about that line, e.g. the intercepts, a point on the line, the gradient, a sketch, a parallel line, etc.

Students can be given sketches and asked to suggest equations that would/would not work.

Modelling with straight-line graphs gives the opportunity to collect data that can then be plotted and a line of best fit used to find an equation. It might also be possible to compare the data to a theoretical model. Students should be encouraged to consider strengths and limitations of modelling.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

In exams, students should be encouraged to quote formulae before using them. This allows method marks to be awarded even if arithmetical slips are made or incorrect values substituted.

Questions may specify a particular form for an answer (for example integer coefficients). Emphasise to students the importance of following these instructions carefully so as not to lose marks.

Students should be encouraged to draw diagrams while working on solutions as this often results in fewer mistakes and can act as a sense check for answers. At the same time, where diagrams are given in questions, students should be aware that these are not to be relied upon and ‘spotting’ answers by looking at a diagram without providing evidence to support this will not gain full marks. However, candidates should be encouraged to use any diagrams provided to help them answer the question.

The usual sorts of algebraic and numerical slips cause marks to be lost and students should be encouraged to carefully check their working. A common error is to incorrectly calculate the gradient of a straight line when it is given in the form $ax + by + c = 0$, so students should be encouraged to practise this technique.

NOTES

Dynamic geometry programs can be used to make changes and observe the effect, helping students to discover and visualise the effect of changing equations.

4: Differentiation

Return to overview

SPECIFICATION REFERENCES

4.1 The derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a point; the gradient of the tangent as a limit; interpretation as a rate of change, second order derivatives
4.2 Differentiation of x^n and related sums, differences and constant multiples
4.3 Application of differentiation to gradients, tangents and normals

PRIOR KNOWLEDGE

Covered so far
- Solving quadratics
- Coordinate geometry
- Function notation
- Indices

<u>INTERNATIONAL GCSE/GCSE (9-1) in Mathematics at Higher Tier</u>
- Fractions
- Area of 2D shapes
- Volume and surface area of 3D shapes
- Rearranging equations
- Differentiate integer powers of x, stationary points (INTERNATIONAL GCSE (9-1) only)

KEYWORDS

Differentiation, derivative, first principles, rate of change, rational, constant, tangent, normal, integer, calculus, function, parallel, perpendicular.

4a. Definition, differentiating polynomials, second derivatives (4.1) (4.2)	Teaching time 6 hours

OBJECTIVES

By the end of the sub-unit, students should:

- understand and be able to use the derivative of f(x) as the gradient of the tangent to the graph of
  y = f(x) at a general point (x, y);
- understand the gradient of the tangent as a limit and its interpretation as a rate of change;
- understand differentiation from first principles for small positive integer powers of x;
- be able to find second derivatives;
- be able to differentiate x^n, for rational values of n, and related constant multiples, sums and differences

TEACHING POINTS

Students should know that $\frac{\mathrm{d}y}{\mathrm{d}x}$ is the rate of change of y with respect to x.
Knowledge of the chain rule is not required.

The notation f ′(x) may be used for the first derivative and f ″(x) may be used for the second order derivative.

Students should know how to differentiate from first principles. Students should be able to use, for n = 2 and n = 3, the gradient expression $\lim_{h\to 0}\left(\frac{(x+h)^n - x^n}{h}\right)$.
The alternative notations $h \to 0$ rather than $\delta x \to 0$ are acceptable.

Students will need to be confident in algebraic manipulation of functions to ensure that they are in a suitable format for differentiation. For example, students will be expected to be able to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2+3x-5}{4x^{\frac{1}{2}}}$, for $x > 0$. Mistakes are easily made with negative and/or fractional indices so there should be plenty of practice with these.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Consider the case where the gradient is zero and interpret the meaning in various contexts, such as practical problems of minimising and maximising.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Algebraic manipulation, particularly where surds are involved, can cause problems for students. For example, when multiplying out brackets and faced with $-4\sqrt{x} \times -4\sqrt{x}$ common incorrect answers are $-4\sqrt{x}$, $\pm 16\sqrt{x}$ and $\pm 16x^{\frac{1}{4}}$. Similarly, when dividing by $\sqrt{x}$, some students think that $\frac{x}{\sqrt{x}} = 1$.

4b. Gradients, tangents and normals (4.3)	Teaching time 5 hours

OBJECTIVES

By the end of the sub-unit, students should:

- be able to apply differentiation to find gradients, tangents and normals.

TEACHING POINTS

Students should be able to use differentiation to find equations of tangents and normals at specific points on a curve. This reviews and extends the earlier work on coordinate geometry.

Students will need plenty of practice at setting up equations from a given context, in some cases this may include showing that it can be written in a particular form.

Where students are given the answer to work towards they must be aware that they need to work forwards showing all steps clearly rather than starting with the answer and working backwards.

Use graph plotting software that allows the derivative to be plotted so that students can see the relationship between a function and its derivative graphically.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Differentiation can be linked to many real-world applications, there can be discussion with students about contexts and the validity of solutions.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students may have difficulty differentiating fractional terms such as $\frac{8}{x}$ if they are unable to rewrite this as $8x^{-1}$ before differentiating.

When working out the equations of tangents and normal, some students mix the gradients and equations up and end up substituting in the wrong place.

5: Integration	**Teaching time** 6 hours

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SPECIFICATION REFERENCES

5.1 Indefinite integration as the reverse of differentiation
5.2 Integration of x^n, and related sums, differences and constant multiples

PRIOR KNOWLEDGE

Covered so far
- Algebraic manipulation
- Differentiation

KEYWORDS

Calculus, differentiate, integrate, reverse, indefinite, definite, constant, evaluate, intersection.

OBJECTIVES

By the end of the unit, students should:

- understand indefinite integration as the reverse of differentiation;
- be able to integrate x^n (excluding $n = -1$), and related sums, differences and constant multiples.

TEACHING POINTS

Integration can be introduced as the reverse process of differentiation. Students need to know that for indefinite integrals a constant of integration is required.

Similarly to differentiation, students should be confident with algebraic manipulation. For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected. Introduce students to the integral sign; this can be useful in setting work out clearly on these sorts of questions and will be used later in definite integration.

Given f ′(x) and a point on the curve, students should be able to find an equation of the curve in the form y = f(x).

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to explain the need for the + c in indefinite integration.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students sometimes have difficulty when integrating expressions involving negative indices. Forgetting to add + c when working out indefinite integrals is also a very common mistake.