INTERNATIONAL ADVANCED LEVEL

MATHEMATICS/
FURTHER MATHEMATICS/
PURE MATHEMATICS

SAMPLE ASSESSMENT
MATERIALS

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA01)
Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM01)
Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM01)
Pearson Edexcel International Advanced Level in Mathematics (YMA01)
Pearson Edexcel International Advanced Level in Further Mathematics (YFM01)
Pearson Edexcel International Advanced Level in Pure Mathematics (YPM01)

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Issue 3
Edexcel, BTEC and LCCI qualifications

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Acknowledgements

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### Summary of changes made between previous issue and this current issue

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<td>An alternative solution for Statistics 3, Question 5 has been inserted in the mark scheme.</td>
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If you need further information on these changes or what they mean, contact us via our website at: qualifications.pearson.com/en/support/contact-us.html.
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Introduction


These sample assessment materials have been developed to support these qualifications and will be used as the benchmark to develop the assessment students will take.

For units P1, P2, P3, P4 and D1, the sample assessment materials have been formed using questions from different past papers from legacy qualifications, together with some new questions. For units FP1-FP3, M1-M3 and S1-S3, the sample assessment materials have been formed using whole past question papers from legacy qualifications.

The booklet ‘Mathematical Formulae and Statistical Tables’ will be provided for use with these assessments and can be downloaded from our website, qualifications.pearson.com.
General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked unless the candidate has replaced it with an alternative response.

Specific guidance for mathematics

1. These mark schemes use the following types of marks:
   - M marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
2. Abbreviations
   - bod benefit of doubt
   - ft follow through
   - √ this symbol is used for correct ft
   - cao correct answer only
   - cso correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw ignore subsequent working
   - awrt answers which round to
   - SC: special case
   - o.e. or equivalent (and appropriate)
   - d... dependent or dep
   - indep independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
3. All M marks are follow through. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don’t logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – **there may be more space than you need**.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – **use this as a guide as to how much time to spend on each question**.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Answer ALL questions. Write your answers in the spaces provided.

1. Given that \( y = 4x^3 - \frac{5}{x^2}, \ x \neq 0 \), find in their simplest form

(a) \( \frac{dy}{dx} \)  

(b) \( \int y \, dx \)
Question 1 continued

(Total for Question 1 is 6 marks)
2. (a) Given that \(3^{-1.5} = a\sqrt{3}\) find the exact value of \(a\) \(\quad (2)\)

(b) Simplify fully \(\frac{1}{(2x^2)^3} \quad \frac{4x^2}{4x^2} \quad (3)\)
2. (a) Given that \( 3 - 1.5 = \) 

(b) Simplify fully 

\( \frac{4}{2} \left( \frac{4}{x} \right)^3 \times \) 

Leave blank

(Total for Question 2 is 5 marks)
3. Solve the simultaneous equations

\[ y + 4x + 1 = 0 \]
\[ y^2 + 5x^2 + 2x = 0 \]

(Total for Question 3 is 6 marks)
Question 3 continued

(Solve the simultaneous equations)

\[ 3x + 4y^2 + 5x^2 + 2x = 0 \]

(Total for Question 3 is 6 marks)
4. The straight line with equation \( y = 4x + c \), where \( c \) is a constant, is a tangent to the curve with equation \( y = 2x^2 + 8x + 3 \)

Calculate the value of \( c \) \( \text{ (5 marks) } \)
Question 4 continued

(Total for Question 4 is 5 marks)
5. (a) On the same axes, sketch the graphs of \( y = x + 2 \) and \( y = x^2 - x - 6 \) showing the coordinates of all points at which each graph crosses the coordinate axes. 

(b) On your sketch, show, by shading, the region \( R \) defined by the inequalities 
\[
y < x + 2 \quad \text{and} \quad y > x^2 - x - 6
\]

(c) Hence, or otherwise, find the set of values of \( x \) for which \( x^2 - 2x - 8 < 0 \)
Question 5 continued

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(Total for Question 5 is 8 marks)
6.

Figure 1 shows a sketch of the curve $C$ with equation $y = f(x)$

The curve $C$ passes through the origin and through $(6, 0)$

The curve $C$ has a minimum at the point $(3, -1)$

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$

(b) $y = f(x + p)$, where $p$ is a constant and $0 < p < 3$

On each diagram show the coordinates of any points where the curve intersects the $x$-axis and of any minimum or maximum points.
Question 6 continued

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.

The curve C passes through the origin and through (6, 0).

The curve C has a minimum at the point (3, –1).

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$

(b) $y = f(x + p)$, where $p$ is a constant and $0 < p < 3$.

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

(Total for Question 6 is 7 marks)
7. A curve with equation \( y = f(x) \) passes through the point \((4, 25)\)

Given that

\[
f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \quad x > 0
\]

find \( f(x) \), simplifying each term. \((5)\)
Given that $f(x) = \frac{x^8}{3} - 10x^2 - x + 1$, find $f'(x)$, simplifying each term.

(Total for Question 7 is 5 marks)
8.

Figure 2

The line \( l_1 \), shown in Figure 2 has equation \( 2x + 3y = 26 \)

The line \( l_2 \) passes through the origin \( O \) and is perpendicular to \( l_1 \)

(a) Find an equation for the line \( l_2 \)

\( \text{The line } l_2 \text{ intersects the line } l_1 \text{ at the point } C. \) Line \( l_1 \) crosses the \( y \)-axis at the point \( B \) as shown in Figure 2.

(b) Find the area of triangle \( OBC \). Give your answer in the form \( \frac{a}{b} \), where \( a \) and \( b \) are integers to be found.
Question 8 continued

The line \( l_1 \), shown in Figure 2 has equation \( 2x + 3y = 26 \).

The line \( l_2 \) passes through the origin \( O \) and is perpendicular to \( l_1 \). 

(a) Find an equation for the line \( l_2 \).

(b) Find the area of triangle \( OBC \). Give your answer in the form \( \frac{a}{b} \), where \( a \) and \( b \) are integers to be found.

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Turn over
9.

A sketch of part of the curve $C$ with equation

$$y = 20 - 4x - \frac{18}{x}, \quad x > 0$$

is shown in Figure 3.

Point $A$ lies on $C$ and has $x$ coordinate equal to 2

(a) Show that the equation of the normal to $C$ at $A$ is $y = -2x + 7$.  \hspace{1cm} (6)

The normal to $C$ at $A$ meets $C$ again at the point $B$, as shown in Figure 3.

(b) Use algebra to find the coordinates of $B$.  \hspace{1cm} (5)
Question 9 continued

Figure 3: A sketch of part of the curve $C$ with equation $y = 20 - 4x - 18x$, $x > 0$ is shown in Figure 3.

Point $A$ lies on $C$ and has $x$ coordinate equal to 2.

(a) Show that the equation of the normal to $C$ at $A$ is $y = -2x + 7$.

(b) Use algebra to find the coordinates of $B$.

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The triangle \( \triangle XYZ \) in Figure 4 has \( XY = 6 \text{ cm} \), \( YZ = 9 \text{ cm} \), \( ZX = 4 \text{ cm} \) and angle \( \angle ZXY = \alpha \).

The point \( W \) lies on the line \( XY \).

The circular arc \( ZW \), in Figure 4, is a major arc of the circle with centre \( X \) and radius 4 cm.

(a) Show that, to 3 significant figures, \( \alpha = 2.22 \) radians.

(b) Find the area, in \( \text{cm}^2 \), of the major sector \( \triangle XZWX \).

(c) The region, shown shaded in Figure 4, is to be used as a design for a logo. Calculate

\( \text{(c) the area of the logo} \)

\( \text{(d) the perimeter of the logo.} \)

(Total for Question 9 is 11 marks)
10.

**Figure 4**

The triangle $XYZ$ in Figure 4 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$.

The point $W$ lies on the line $XY$.

The circular arc $ZW$, in Figure 4, is a major arc of the circle with centre $X$ and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.  

(b) Find the area, in cm$^2$, of the major sector $XZW$.  

The region, shown shaded in Figure 4, is to be used as a design for a logo.

Calculate

(c) the area of the logo  

(d) the perimeter of the logo.
Question 10 continued
Question 10 continued

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS
### Pure Mathematics P1 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>$y = 4x^3 - \frac{5}{x^2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x^n \rightarrow x^{n-1}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>e.g. sight of $x^2$ or $x^{-3}$ or $\frac{1}{x}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3 \times 4x^2$ or $-5 \times -2x^{-3}$ (o.e.) (Ignore $+ c$ for this mark)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$12x^2 + \frac{10}{x}$ or $12x^2 + 10x^{-3}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>all on one line and no $+ c$</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$x^n \rightarrow x^{n+1}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>e.g. sight of $x^4$ or $x^{-1}$ or $\frac{1}{x}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Do not award for integrating their answer to part (a)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4 \frac{x^4}{4}$ or $-5 \times \frac{x^{-1}}{-1}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>For fully correct and simplified answer with $+ c$ all on one line. Allow</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow$ Allow $x^4 + 5 \times \frac{1}{x} + c$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow$ Allow $1x^4$ for $x^4$</td>
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(6 marks)
### Question 2(a)

\[ 3^{-1.5} = \frac{1}{3^{\frac{1}{2}}} \left( \frac{\sqrt{3}}{\sqrt{3}} \right) \]

\[ = \frac{\sqrt{3}}{9} \text{ so } a = \frac{1}{9} \]

**Alternative**

\[ 3^{-1.5} = a\sqrt{3} \Rightarrow a = \frac{3^{-1.5}}{3^{0.5}} = 3^{-1.5 - 0.5} \]

\[ \Rightarrow a = 3^{-2} = \frac{1}{9} \]

**Notes:**

(a) **M1:** Scored for a full attempt to write \( 3^{-1.5} \) in the form \( a\sqrt{3} \) or, as an alternative, makes \( a \) the subject and attempts to combine the powers of 3

**A1:** For \( a = \frac{1}{9} \) Note: A correct answer with no working scores full marks

(b) **M1:** For an attempt to expand \[ \left( 2x^{\frac{1}{2}} \right)^3 \] Scored for one correct power either \( 2^3 \) or \( x^\frac{3}{2} \).

\[ \left( 2x^{\frac{1}{2}} \right)^3 \times \left( 2x^{\frac{1}{2}} \right) \times \left( 2x^{\frac{1}{2}} \right) \text{ on its own is not sufficient for this mark.} \]

**dM1:** For dividing their coefficients of \( x \) and subtracting their powers of \( x \). Dependent upon the previous M1

**A1:** Correct answer \( 2x^{-\frac{3}{2}} \) or \[ \frac{2}{\sqrt{x}} \]
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| **3**    | $y = -4x - 1$  
$\Rightarrow (-4x - 1)^2 + 5x^2 + 2x = 0$ | Attempts to make $y$ the subject of the linear equation and substitutes into the other equation. M1 |
|          | $21x^2 + 10x + 1 = 0$ | Correct 3 term quadratic A1 |
|          | $(7x+1)(3x+1) = 0 \Rightarrow (x) = -\frac{1}{7}, -\frac{1}{3}$ | dM1: Solves a 3 term quadratic by the usual rules dM1A1 |
|          | $y = -\frac{3}{7}, \frac{1}{3}$ | M1: Substitutes to find at least one $y$ value A1: $y = -\frac{3}{7}, \frac{1}{3}$ M1 A1 |

**Alternative**

\[
x = -\frac{1}{4}y - \frac{1}{4}
\Rightarrow y^2 + 5\left(\frac{-1}{4}y - \frac{1}{4}\right)^2 + 2\left(\frac{-1}{4}y - \frac{1}{4}\right) = 0
\]
Attempts to make $x$ the subject of the linear equation and substitutes into the other equation. M1

|          | $\frac{21}{16}y^2 + \frac{1}{8}y - \frac{3}{16} = 0$ | Correct 3 term quadratic A1 |
|          | $(21y^2 + 2y - 3 = 0)$ | |
|          | $(7y + 3)(3y - 1) = 0 \Rightarrow (y) = -\frac{3}{7}, \frac{1}{3}$ | Solves a 3 term quadratic dM1 |
|          | $(y) = -\frac{3}{7}, \frac{1}{3}$ | A1 |
|          | $x = -\frac{1}{7}, -\frac{1}{3}$ | Substitutes to find at least one $x$ value. M1 |
|          | $x = -\frac{1}{7}, -\frac{1}{3}$ | A1 |

(6 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4</strong></td>
<td><strong>Sets</strong> $2x^2 + 8x + 3 = 4x + c$ and collects $x$ terms together</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td><strong>Obtains</strong> $2x^2 + 4x + 3 - c = 0$ o.e.</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>States that</strong> $b^2 - 4ac = 0$</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td>$4^2 - 4 \times 2 \times (3 - c) = 0$ and so $c =$</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td>$c = 1 \text{ cs o}$</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Alternative 1A</strong></td>
<td><strong>Sets derivative</strong> &quot;$4x + 8&quot; = 4 \Rightarrow x =$</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td>$x = -1$</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Substitute</strong> $x = -1$ in $y = 2x^2 + 8x + 3$ ( \Rightarrow y = -3)</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Substitute</strong> $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y + 3) = 4(x + 1)$ and expand</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td>$c = 1$ or writing $y = 4x + 1 \text{ cs o}$</td>
<td><strong>A1</strong></td>
<td></td>
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<tr>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
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<tr>
<td><strong>Alternative 1B</strong></td>
<td><strong>Sets derivative</strong> &quot;$4x + 8&quot; = 4 \Rightarrow x =$,</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td>$x = -1$</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Substitute</strong> $x = -1$ in $2x^2 + 8x + 3 = 4x + c$</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Attempts to find value of</strong> $c$</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td>$c = 1$ or writing $y = 4x + 1 \text{ cs o}$</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Alternative 2</strong></td>
<td><strong>Sets</strong> $2x^2 + 8x + 3 = 4x + c$ and collects $x$ terms together</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td><strong>Obtains</strong> $2x^2 + 4x + 3 - c = 0$ or equivalent</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>States that</strong> $b^2 - 4ac = 0$</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td>$4^2 - 4 \times 2 \times (3 - c) = 0$ and so $c =$</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td>$c = 1 \text{ cs o}$</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Alternative 3</strong></td>
<td><strong>Sets</strong> $2x^2 + 8x + 3 = 4x + c$ and collects $x$ terms together</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td><strong>Obtains</strong> $2x^2 + 4x + 3 - c = 0$ or equivalent</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Uses</strong> $(x+1)^2 - 2 + 3 - c = 0$ or equivalent</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Writes</strong> $-2 + 3 - c = 0$</td>
<td><strong>dM1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>So</strong> $c = 1 \text{ cs o}$</td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(5 marks)
**Question 4 continued**

**Notes:**

**Method 1A**

**M1:** Attempts to solve their \( \frac{dy}{dx} = 4 \). They must reach \( x = \ldots \) (Just differentiating is M0 A0).

**A1:** \( x = -1 \) (If this follows \( \frac{dy}{dx} = 4x + 8 \), then give M1 A1 by implication).

**dM1:** (Depends on previous M mark) Substitutes their \( x = -1 \) into \( f(x) \) or into “their \( f(x) \)” from (b)” to find \( y \).

**dM1:** (Depends on both previous M marks) Substitutes their \( x = -1 \) and their \( y = -3 \) values into \( y = 4x + c \) to find \( c \) or uses equation of line is \( (y + “3”) = 4(x + “1”) \) and rearranges to \( y = mx+c \)

**A1:** \( c = 1 \) or allow for \( y = 4x + 1 \) cso.

**Method 1B**

**M1A1:** Exactly as in Method 1A above.

**dM1:** (Depends on previous M mark) Substitutes their \( x = -1 \) into \( 2x^2 + 8x + 3 = 4x + c \)

**dM1:** Attempts to find value of \( c \) then A1 as before.

**Method 2**

**M1:** Sets \( 2x^2 + 8x + 3 = 4x + c \) and tries to collect x terms together.

**A1:** Collects terms e.g. \( 2x^2 + 4x + 3 - c = 0 \) or \( -2x^2 - 4x - 3 + c = 0 \) or \( 2x^2 + 4x + 3 = c \) or even \( 2x^2 + 4x = c - 3 \). Allow “=0” to be missing on RHS.

**dM1:** Then use completion of square \( 2(x+1)^2 - 2 + 3 - c = 0 \) (Allow \( 2(x+1)^2 - k + 3 - c = 0 \)) where \( k \) is non zero. It is enough to give the correct or almost correct (with \( k \)) completion of the square.

**dM1:** \( -2 + 3 - c = 0 \) AND leading to a solution for \( c \) (Allow -1 + 3 - c = 0) \( (x = -1 \) has been used)

**A1:** \( c = 1 \) cso

**Method 3**

**M1:** Sets \( 2x^2 + 8x + 3 = 4x + c \) and tries to collect x terms together. May be implied by \( 2x^2 + 8x + 3 - 4x \pm c \) on one side.

**A1:** Collects terms e.g. \( 2x^2 + 4x + 3 - c = 0 \) or \( -2x^2 - 4x - 3 + c = 0 \) or \( 2x^2 + 4x + 3 = c \) even \( 2x^2 + 4x = c - 3 \). Allow “=0” to be missing on RHS.

**dM1:** Then use completion of square \( 2(x+1)^2 - k + 3 - c = 0 \) (Allow \( 2(x+1)^2 - k + 3 - c = 0 \)) where \( k \) is non zero. It is enough to give the correct or almost correct (with \( k \)) completion of the square.

**dM1:** \( -2 + 3 - c = 0 \) AND leading to a solution for \( c \) (Allow -1 + 3 - c = 0) \( (x = -1 \) has been used)

**A1:** \( c = 1 \) cso
<table>
<thead>
<tr>
<th>Question</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5(a)</strong></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>(b)</td>
<td>Finite region between line and curve shaded</td>
</tr>
<tr>
<td>(c)</td>
<td>((x^2 - x - 6 &lt; x + 2 ) \Rightarrow x^2 - 2x - 8 &lt; 0) ((x - 4)(x + 2) &lt; 0 \Rightarrow \text{Line and curve intersect at } x = 4 \text{ and } x = -2)</td>
</tr>
<tr>
<td></td>
<td>(-2 &lt; x &lt; 4)</td>
</tr>
</tbody>
</table>

**Notes:**

(a) As scheme.

(b) As scheme.

(c)  
**M1:** For a valid attempt to solve the equation \(x^2 - 2x - 8 = 0\)  
**A1:** For \(x = 4\) and \(x = -2\)  
**A1:** \(-2 < x < 4\)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6(a)</strong></td>
<td><img src="image" alt="Graph" /></td>
<td>Shape ( \cup ) through (0, 0) B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3, 0) B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.5, -1) B1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td><img src="image" alt="Graph" /></td>
<td>Shape ( \cup ), not through (0, 0) B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Minimum in 4th quadrant B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>((-p, 0)) and ( (6-p, 0) ) B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>((3-p, -1)) B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4) (7 marks)</td>
</tr>
</tbody>
</table>

**Notes:**

(a)
- **B1:** U shaped parabola through origin.
- **B1:** (3,0) stated or 3 labelled on x - axis (even (0,3) on x - axis).
- **B1:** (1.5, -1) or equivalent e.g. (3/2, -1) labelled or stated and matching minimum point on the graph.

(b)
- **B1:** Is for any translated curve to left or right or up or down not through origin
- **B1:** Is for minimum in 4th quadrant and x intercepts to left and right of y axis (i.e. correct position).
- **B1:** Coordinates stated or shown on x axis (Allow \( 0-p, 0 \) instead of \((-p, 0)\))
- **B1:** Coordinates stated.

Note: If values are taken for \( p \), then it is possible to give M1A1B0B0 even if there are several attempts. (In this case none of the curves should go through the origin for M1 and all minima should be in fourth quadrant and all x intercepts need to be to left and right of y axis for A1.)
### Question 7

\[
f(x) = \int \left( \frac{3}{8} x^2 - 10x^{-\frac{1}{2}} + 1 \right) \, dx
\]

\[
x^\alpha \rightarrow x^{\alpha+1} \Rightarrow f(x) = \frac{3}{8} \times \frac{x^3}{3} - 10 \times \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + x (c)
\]

Substitute \( x = 4, y = 25 \) \( \Rightarrow 25 = 8 - 40 + 4 + c \)

\[
\Rightarrow c =
\]

\[
f(x) = \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53
\]

(5 marks)

#### Notes:

**M1:** Attempt to integrate \( x^\alpha \rightarrow x^{\alpha+1} \)

**A1:** Term in \( x^3 \) or term in \( x^{\frac{1}{2}} \) correct, coefficient need not be simplified, no need for +x nor +c

**A1:** ALL three terms correct, coefficients need not be simplified, no need for +c

**M1:** For using \( x = 4, y = 25 \) in their \( f(x) \) to form a linear equation in \( c \) and attempt to find \( c \)

**A1:** \( \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53 \) cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be \( f(x) \) or \( y \)). Need full expression with 53. These marks need to be scored in part (a).
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8(a)</strong></td>
<td>$2x + 3y = 26 \Rightarrow 3y = 26 - 2x$ and attempt to find $m$ from $y = mx + c$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( $\Rightarrow y = \frac{26}{3} - \frac{2}{3}x$ ) so gradient $= -\frac{2}{3}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Gradient of perpendicular $= \frac{-1}{\text{their gradient}} = \left(\frac{3}{2}\right)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Line goes through $(0, 0)$ so $y = \frac{3}{2}x$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>($4$)</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Solves their $y = \frac{3}{2}x$ with their $2x + 3y = 26$ to form equation in $x$ or in $y$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Solves their equation in $x$ or in $y$ to obtain $x = \text{or } y =$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$x = 4$ or any equivalent e.g. $\frac{156}{39}$ or $y = 6$ o.a.e</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$B = (0, \frac{26}{3})$ used or stated in (b)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area $= \frac{1}{2} \times 4 \times \frac{26}{3}$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{52}{3}$ (o.e. with integer numerator and denominator)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>($6$)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)

**M1:** Complete method for finding gradient. (This may be implied by later correct answers.) e.g.
Rearranges $2x + 3y = 26 \Rightarrow y = mx + c$ so $m =$
Or finds coordinates of two points on line and finds gradient e.g.
$(13,0)$ and $(1,8)$ so $m = \frac{8 - 0}{1 - 13}$

**A1:** States or implies that gradient $= -\frac{2}{3}$ condone $= -\frac{2}{3}x$ if they continue correctly. Ignore errors in constant term in straight line equation.

**M1:** Uses $m_1 \times m_2 = -1$ to find the gradient of $l_2$. This can be implied by the use of $\frac{-1}{\text{their gradient}}$

**A1:** $y = \frac{3}{2}x$ or $2y - 3x = 0$ Allow $y = \frac{3}{2}x + 0$ Also accept $2y = 3x, y = \frac{39}{26}x$ or even
$y - 0 = \frac{3}{2}(x - 0)$ and isw.
Question 8 notes continued

(b)

M1: Eliminates variable between their \( y = \frac{3}{2}x \) and their (possibly rearranged) \( 2x + 3y = 26 \) to form an equation in \( x \) or \( y \). (They may have made errors in their rearrangement).

dM1: (Depends on previous M mark) Attempts to solve their equation to find the value of \( x \) or \( y \)

A1: \( x = 4 \) or equivalent or \( y = 6 \) or equivalent

B1: \( y \) coordinate of \( B \) is \( \frac{26}{3} \) (stated or implied) - isw if written as \( (\frac{26}{3}, 0) \).

Must be used or stated in (b)

dM1: (Depends on previous M mark) Complete method to find area of triangle \( OBC \) (using their values of \( x \) and/or \( y \) at point \( C \) and their \( \frac{26}{3} \))

A1: CaO \( \frac{52}{3} \) or \( \frac{104}{6} \) or \( \frac{1352}{78} \) o.e

Alternative 1
Uses the area of a triangle formula \( \frac{1}{2} \times OB \times (x \) coordinate of \( C) \)

Alternative methods: Several Methods are shown below. The only mark which differs from Alternative 1 is the last M mark and its use in each case is described below:

Alternative 2
In 8(b) using \( \frac{1}{2} \times BC \times OC \)

A1: Uses the area of a triangle formula \( \frac{1}{2} \times BC \times OC \) Also finds \( OC = \sqrt{52} \) and \( BC = \left( \frac{4}{3} \sqrt{13} \right) \)

Alternative 3
In 8(b) using \( \frac{1}{2} \begin{vmatrix} 0 & 4 & 0 \\ 0 & 6 & \frac{26}{3} \\ 0 & 0 & 0 \end{vmatrix} \)

A1: States the area of a triangle formula \( \frac{1}{2} \begin{vmatrix} 0 & 4 & 0 \\ 0 & 6 & \frac{26}{3} \\ 0 & 0 & 0 \end{vmatrix} \) or equivalent with their values

Alternative 4
In 8(b) using area of triangle \( OBX \) – area of triangle \( OCX \) where \( X \) is point \( (13, 0) \)

A1: Uses the correct subtraction \( \frac{1}{2} \times 13 \times \frac{26}{3} - \frac{1}{2} \times 13 \times 6 \)

Alternative 5
In 8(b) using area \( = \frac{1}{2} \left( 6 \times 4 \right) + \frac{1}{2} \left( 4 \times \frac{8}{3} \right) \) drawing a line from \( C \) parallel to the \( x \) axis and dividing triangle into two right angled triangles

A1: For correct method area \( = \frac{1}{2} \left( \left. 6 \times \left( 4 \right) \right) + \frac{1}{2} \left( \left. 4 \times \left[ \left( \frac{26}{3} \right) - \left( 6 \right) \right] \right) \right) \)

Method 6 Uses calculus

\[
\int_{0}^{4} \left( \frac{26}{3} - \frac{2x}{3} - \frac{3x^2}{2} \right) \, dx = \left[ \frac{26}{3} - \frac{x^2}{3} - \frac{3x^4}{4} \right]_{0}^{4}
\]
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
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<tbody>
<tr>
<td><strong>9(a)</strong></td>
<td>Substitutes $x = 2$ into $y = 20 - 4x - \frac{18}{x}$ and gets 3</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\frac{dy}{dx} = -4 + \frac{18}{x^2}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal ($-2$)</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>States or uses $y = -3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3) to deduce that $y = -2x + 7$</td>
<td>ddM1</td>
</tr>
</tbody>
</table>

(b)  
Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$  
Or put $y = 20 - 4\left(\frac{7 - y}{2}\right) - \frac{18}{\left(\frac{7 - y}{2}\right)}$ to give $y^2 - y - 6 = 0$  
$(2x - 9)(x - 2) = 0$ so $x = \text{or } (y - 3) (y + 2) = 0$ so $y =$  
$\left(\frac{9}{2}, -2\right)$  
A1 A1

(11 marks)

Notes:

(a)  
**B1:** Substitutes $x = 2$ into expression for $y$ and gets 3 cao (must be in part (a) and must use curve equation – not line equation). This must be seen to be substituted.  
**M1:** For an attempt to differentiate the negative power with $x^{-1}$ to $x^{-2}$.  
**A1:** Correct expression for $\frac{dy}{dx} = -4 + \frac{18}{x^2}$  
**dM1:** Dependent on first M1 substitutes $x = 2$ into their derivative to obtain a numerical gradient and find negative reciprocal or states that $-2 \times \frac{1}{2} = -1$

Alternative 1  
**dM1:** Dependent on first M1. Finds equation of line using changed gradient (not their $\frac{1}{2}$ but $-\frac{1}{2}$ 2 or $-2$) e.g. $y = "3" = "2"(x - 2)$ or $y = "-2" x + c$ and use of (2, "3") to find $c =$  
**A1:** cao. This is a given answer $y = -2x + 7$ obtained with no errors seen and equation should be stated.

Alternative 2 – checking given answer  
**dM1:** Uses given equation of line and checks that (2, 3) lies on the line.  
**A1:** cao. This is a given answer $y = -2x + 7$ so statement that normal and line have the same gradient and pass through the same point must be stated.
(b)  
**M1:** Equate the two given expressions, collect terms and simplify to a 3TQ. There may be sign errors when collecting terms but putting for example $20x - 4x^2 - 18 = -2x + 7$ is M0 here.  
**A1:** Correct 3TQ = 0 (need = 0 for A mark) $2x^2 - 13x + 18 = 0$  
**dM1:** Attempt to solve an appropriate quadratic by factorisation, use of formula, or completion of the square (see general instructions).  
**A1:** $x = \frac{9}{2}$ o.e or $y = -2$ (allow second answers for this mark so ignore $x = 2$ or $y = 3$)  
**A1:** Correct solutions only so both $x = \frac{9}{2}, y = -2$ or $\left(\frac{9}{2}, -2\right)$  
If $x = 2, y = 3$ is included as an answer and point B is not identified then last mark is A0.  
Answer only – with no working – send to review. The question stated ‘use algebra’.
Question 10(a)

10(a) 9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha = \ldots.

\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} = \frac{-29}{48} = -0.604..

\alpha = 2.22 * \text{cso}

Correct use of cosine rule leading to a value for \cos \alpha M1

A1: Correct 3TQ = 0 (need = 0 for A mark)

2x^2 - 13x + 18 = 0

Attempt to solve an appropriate quadratic by factorisation, use of formula, or completion of the square (see general instructions).

A1: 9x \Rightarrow o.e or y = -2 (allow second answers for this mark so ignore x = 2 or y = 3)

A1: Correct solutions only so both 9x \Rightarrow, y = -2 or \frac{9}{2},22

If x = 2, y = 3 is included as an answer and point B is not identified then last mark is A0.

Answer only – with no working – send to review. The question stated 'use algebra'.

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<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(a)</td>
<td>Scheme</td>
</tr>
<tr>
<td>10(a)</td>
<td>9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha = \ldots.</td>
</tr>
<tr>
<td>10(a)</td>
<td>\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} = \frac{-29}{48} = -0.604..</td>
</tr>
<tr>
<td>10(a)</td>
<td>\alpha = 2.22 * \text{cso}</td>
</tr>
<tr>
<td>10(a)</td>
<td></td>
</tr>
<tr>
<td>10(a)</td>
<td>Alternative</td>
</tr>
<tr>
<td>10(a)</td>
<td>(XY^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos 2.22 \Rightarrow XY^2 = .. )</td>
</tr>
<tr>
<td>10(a)</td>
<td>XY = 9.00....</td>
</tr>
<tr>
<td>10(a)</td>
<td>(2)</td>
</tr>
<tr>
<td>10(a)</td>
<td>2\pi - 2.22(= 4.06366......)</td>
</tr>
<tr>
<td>10(a)</td>
<td>\frac{1}{2} \times 4^2 \times 4.06&quot;</td>
</tr>
<tr>
<td>10(a)</td>
<td>32.5</td>
</tr>
<tr>
<td>10(a)</td>
<td>(3)</td>
</tr>
<tr>
<td>10(a)</td>
<td>Alternative – Circle Minor – sector</td>
</tr>
<tr>
<td>10(a)</td>
<td>\pi \times 4^2</td>
</tr>
<tr>
<td>10(a)</td>
<td>\pi \times 4^2 - \frac{1}{2} \times 4^2 \times 2.22 = 32.5</td>
</tr>
<tr>
<td>10(a)</td>
<td>= 32.5</td>
</tr>
<tr>
<td>10(a)</td>
<td>(3)</td>
</tr>
<tr>
<td>10(a)</td>
<td>(c) Area of triangle = \frac{1}{2} \times 4 \times 6 \times \sin 2.22(= 9.56)</td>
</tr>
<tr>
<td>10(a)</td>
<td>\text{So area required} = &quot;9.56&quot; + &quot;32.5&quot;</td>
</tr>
<tr>
<td>10(a)</td>
<td>Area of logo = 42.1 \text{ cm}^2 or 42.0 \text{ cm}^2</td>
</tr>
<tr>
<td>10(a)</td>
<td>(3)</td>
</tr>
<tr>
<td>10(a)</td>
<td>(d) Arc length = 4 \times 4.06 (= 16.24) or 8\pi - 4 \times 2.22</td>
</tr>
<tr>
<td>10(a)</td>
<td>Perimeter = ZY + WY + Arc Length</td>
</tr>
<tr>
<td>10(a)</td>
<td>Perimeter of logo = 27.2 or 27.3</td>
</tr>
<tr>
<td>10(a)</td>
<td>(4)</td>
</tr>
<tr>
<td>10(a)</td>
<td>(12 marks)</td>
</tr>
</tbody>
</table>
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

• Use black ink or ball-point pen.
• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions and ensure that your answers to parts of questions are clearly labelled.
• Answer the questions in the spaces provided – there may be more space than you need.
• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
• Inexact answers should be given to three significant figures unless otherwise stated.

Information

• A booklet 'Mathematical Formulae and Statistical Tables' is provided.
• There are 9 questions in this question paper. The total mark for this paper is 75.
• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Try to answer every question.
• Check your answers if you have time at the end.
• If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

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- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Answer ALL questions. Write your answers in the spaces provided.

1. \( f(x) = x^4 + x^3 + 2x^2 + ax + b, \)

where \( a \) and \( b \) are constants.

When \( f(x) \) is divided by \( (x - 1) \), the remainder is 7

(a) Show that \( a + b = 3 \) \hspace{1cm} (2)

When \( f(x) \) is divided by \( (x + 2) \), the remainder is \(-8\)

(b) Find the value of \( a \) and the value of \( b \) \hspace{1cm} (5)
Question 1 continued

(Total for Question 1 is 7 marks)
2. The first term of a geometric series is 20 and the common ratio is \( \frac{7}{8} \). The sum to infinity of the series is \( S_\infty \)

(a) Find the value of \( S_\infty \) \( \quad \) (2)

The sum to \( N \) terms of the series is \( S_N \)

(b) Find, to 1 decimal place, the value of \( S_{12} \) \( \quad \) (2)

(c) Find the smallest value of \( N \), for which \( S_\infty - S_N < 0.5 \) \( \quad \) (4)
Question 2 continued
Question 2 continued

(Total for Question 2 is 8 marks)
3. \( y = \sqrt{3^2 + x} \)

(a) Complete the table below, giving the values of \( y \) to 3 decimal places.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>1.251</td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

(b) Use the trapezium rule with all the values of \( y \) from your table to find an approximation for the value of

\[ \int_0^1 \sqrt{3^2 + x} \, dx \]

You must show clearly how you obtained your answer.

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate for the value of

\[ \int_0^1 \sqrt{3^2 + x} \, dx \]
(c) Explain how the trapezium rule could be used to obtain a more accurate estimate for

You must show clearly how you obtained your answer.

(a) Complete the table below, giving the values of

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Use the trapezium rule with all the values of

the value of

to 3 decimal places.
Given $n$, prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.

(Total for Question 3 is 7 marks)
4. Given \( n \in \mathbb{N} \), prove, by exhaustion, that \( n^2 + 2 \) is not divisible by 4.

(4)
Question 4 continued

(b) Find the value of $n$.

(c) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week $n$.

A company, which is making 200 mobile phones each week, plans to increase its production.

An arithmetic series has first term $a$ and common difference $d$. The company then plans to continue to make 600 mobile phones each week.

(Total for Question 4 is 4 marks)
5. An arithmetic series has first term $a$ and common difference $d$.

(a) Prove that the sum of the first $n$ terms of the series is

$$\frac{1}{2}n[2a + (n - 1)d]$$

(4)

A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week $N$.

(b) Find the value of $N$

(2)

The company then plans to continue to make 600 mobile phones each week.

(c) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

(5)
Question 5 continued
(Total for Question 5 is 11 marks)
6. (i) Find the exact value of \( x \) for which
\[
\log_2(2x) = \log_2(5x + 4) - 3
\]
(ii) Given that
\[
\log_2 y + 3\log_2 2 = 5
\]
express \( y \) in terms of \( a \). Give your answer in its simplest form.
The circle with equation $x^2 + y^2 - 20x - 16y + 139 = 0$ had centre $C$ and radius $r$.

(a) Find the coordinates of $C$.

(b) Show that $r = 5$.

The line with equation $x = 13$ crosses the circle at the points $P$ and $Q$ as shown in Figure 1.

(c) Find the $y$ coordinate of $P$ and the $y$ coordinate of $Q$.

A tangent to the circle from $O$ touches the circle at point $X$.

(d) Find, in surd form, the length $OX$. 

(Total for Question 6 is 7 marks)
7. 

The circle with equation

\[ x^2 + y^2 - 20x - 16y + 139 = 0 \]

had centre \( C \) and radius \( r \).

(a) Find the coordinates of \( C \). (2)

(b) Show that \( r = 5 \) (2)

The line with equation \( x = 13 \) crosses the circle at the points \( P \) and \( Q \) as shown in Figure 1.

(c) Find the \( y \) coordinate of \( P \) and the \( y \) coordinate of \( Q \). (3)

A tangent to the circle from \( O \) touches the circle at point \( X \).

(d) Find, in surd form, the length \( OX \). (3)
Question 7 continued
Question 7 continued
Question 7 continued

(Question 7 is 10 marks)
Figure 2 shows a sketch of part of the curves $C_1$ and $C_2$ with equations

$$C_1: y = 10x - x^2 - 8 \quad x > 0$$
$$C_2: y = x^3 \quad x > 0$$

The curves $C_1$ and $C_2$ intersect at the points $A$ and $B$.

(a) Verify that the point $A$ has coordinates $(1, 1)$ (1)

(b) Use algebra to find the coordinates of the point $B$ (6)

The finite region $R$ is bounded by $C_1$ and $C_2$

(c) Use calculus to find the exact area of $R$ (5)
Question 8 continued
Question 8 continued
Question 8 continued

(ii) Given that

\[ g \]

[Equation]

[giving your answers in terms of]

\[ x \]

[Expression]

[Total for Question 8 is 12 marks]
9. (i) Solve, for $0 \leq \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3} \cos 3\theta = 0$$

giving your answers in terms of $\pi$

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \quad 0 \leq k \leq 3$$

(a) find $\cos x$ in terms of $k$

(b) When $k = 3$, find the values of $x$ in the range $0 \leq x < 360^\circ$
Question 9 continued
Question 9 continued

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS
### Pure Mathematics P2 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>$f(x) = x^4 + x^3 + 2x^2 + ax + b$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Attempting $f(1)$ or $f(-1)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$f(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$</td>
<td>A1* eso</td>
</tr>
<tr>
<td></td>
<td>(as required) AG</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Attempting $f(-2)$ or $f(2)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$f(-2) = 16 - 8 + 8 - 2a + b = -8 {\Rightarrow -2a + b = -24}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Solving both equations simultaneously to get as far as $a = \ldots$ or $b = \ldots$ dM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Any one of $a = 9$ or $b = -6$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Both $a = 9$ and $b = -6$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7 marks)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)  
**M1:** For attempting either $f(1)$ or $f(-1)$.  
**A1:** For applying $f(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a).  
**Alternative**  
**M1:** For long division by $(x - 1)$ to give a remainder in $a$ and $b$ which is independent of $x$.  
**A1:** Or $\{\text{Remainder} = \} b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given).

(b)  
**M1:** Attempting either $f(-2)$ or $f(2)$.  
**A1:** correct underlined equation in $a$ and $b$; e.g. $16 - 8 + 8 - 2a + b = -8$ or equivalent, e.g. $-2a + b = -24$.  
**dM1:** An attempt to eliminate one variable from 2 linear simultaneous equations in $a$ and $b$. Note that this mark is dependent upon the award of the first method mark.  
**A1:** Any one of $a = 9$ or $b = -6$.  
**A1:** Both $a = 9$ and $b = -6$ and a correct solution only.  
**Alternative**  
**M1:** For long division by $(x + 2)$ to give a remainder in $a$ and $b$ which is independent of $x$.  
**A1:** For $\{\text{Remainder} = \} b - 2(a - 8) = -8 \{\Rightarrow -2a + b = -24\}$.  
Then dM1A1A1 are applied in the same way as before.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2(a)</strong></td>
<td>$S_\infty = \frac{20}{1-\frac{7}{8}} = 160$</td>
<td>Use of a correct $S_\infty$ formula M1 160 A1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$S_{12} = \frac{20\left(1-(\frac{7}{8})^2\right)}{1-\frac{7}{8}} = 127.77324... = 127.8 \text{ (1 dp)}$</td>
<td>M1: Use of a correct $S_n$ formula with $n = 12$ (condone missing brackets around $\frac{7}{8}$) M1 A1</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>$160 - \frac{20\left(1-(\frac{7}{8})^N\right)}{1-\frac{7}{8}} &lt; 0.5$</td>
<td>Applies $S_N$ (GP only) with $a = 20$, $r = \frac{7}{8}$ and “uses” 0.5 and their $S_\infty$ at any point in their working. M1</td>
</tr>
<tr>
<td></td>
<td>$160\left(\frac{7}{8}\right)^N &lt; (0.5)$ or $\left(\frac{7}{8}\right)^N &lt; \left(\frac{0.5}{160}\right)$</td>
<td>Attempt to isolate $160\left(\frac{7}{8}\right)^N$ or $\left(\frac{7}{8}\right)^N$ dM1</td>
</tr>
<tr>
<td></td>
<td>$N\log\left(\frac{7}{8}\right) &lt; \log\left(\frac{0.5}{160}\right)$</td>
<td>Uses the law of logarithms to obtain an equation or an inequality of the form $N\log\left(\frac{7}{8}\right) &lt; \log\left(\frac{0.5}{\text{their } S_\infty}\right)$ or $N &gt; \log_{0.875}\left(\frac{0.5}{\text{their } S_\infty}\right)$ M1</td>
</tr>
<tr>
<td></td>
<td>$N &gt; \frac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \text{ cso}$</td>
<td>$N = 44$ (Allow $N \geq 44$ but no $N &gt; 44$) A1 cso</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow N = 44$</td>
<td>An incorrect inequality statement at any stage in a candidate’s working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. <strong>BUT</strong> it is possible to gain full marks for using $=,$ as long as no incorrect working seen.</td>
</tr>
<tr>
<td><strong>Alternative: Trial &amp; Improvement Method in (c):</strong></td>
<td>Attempts $160 - S_N$ or $S_N$ with at least one value for $N &gt; 40$ M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Attempts $160 - S_N$ or $S_N$ with $N = 43$ or $N = 44$ dM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>For evidence of examining $160 - S_N$ or $S_N$ for <strong>both</strong> $N = 43$ and $N = 44$ with <strong>both</strong> values correct to 2 DP M1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Eg: $160 - S_{43} = \text{awrt } 0.51$ and $160 - S_{44} = \text{awrt } 0.45$ or $S_{43} = \text{awrt } 159.49$ and $S_{44} = \text{awrt } 159.55$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$N = 44$ A1 cso</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Answer of $N = 44$ only with no working scores no marks</strong></td>
<td>(4)</td>
</tr>
</tbody>
</table>

(8 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(a)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 B1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1 M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1ft</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7 marks)</td>
</tr>
</tbody>
</table>

Notes:

(a)  
B1: For 1.494  
B1: For 1.741 (1.740 is B0). Wrong accuracy e.g. 1.49, 1.74 is B1B0

(b)  
B1: Need ½ of 0.25 or 0.125 o.e.  
M1: Requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however) x values: M0 if values used in brackets are x values instead of y values  
A1ft: Follows their answers to part (a) and is for {correct expression}  
A1: Accept 1.4965, 1.497, or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table).

Separate trapezia may be used: B1 for 0.125, M1 for \( \frac{1}{2}h(a+b) \) used 3 or 4 times (and A1ft if it is all correct) e.g. 0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2) is M1 A0 equivalent to missing one term in { } in main scheme.
A solution based around a table of results

<table>
<thead>
<tr>
<th>$n$</th>
<th>$n^2$</th>
<th>$n^2 + 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>38</td>
</tr>
</tbody>
</table>

When $n$ is odd, $n^2$ is odd (odd $\times$ odd = odd) so $n^2 + 2$ is also odd

M1

So for all odd numbers $n$, $n^2 + 2$ is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even)

A1

When $n$ is even, $n^2$ is even and a multiple of 4, so $n^2 + 2$ cannot be a multiple of 4

M1

Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all $n$, $n^2 + 2$ cannot be divisible by 4"

A1*

Alternative - (algebraic) proof

If $n$ is even, $n = 2k$, so $\frac{n^2 + 2}{4} = \frac{(2k)^2 + 2}{4} = \frac{4k^2 + 2}{4} = k^2 + \frac{1}{2}$

M1

If $n$ is odd, $n = 2k + 1$, so $\frac{n^2 + 2}{4} = \frac{(2k+1)^2 + 2}{4} = \frac{4k^2 + 4k + 3}{4} = k^2 + k + \frac{3}{4}$

M1

For a partial explanation stating that

- either of $k^2 + \frac{1}{2}$ or $k^2 + k + \frac{3}{4}$ are not a whole numbers.
- with some valid reason stating why this means that $n^2 + 2$ is not a multiple of 4.

A1

Full proof with no errors or omissions. This must include

- The conjecture
- Correct notation and algebra for both even and odd numbers
- A full explanation stating why, for all $n$, $n^2 + 2$ is not divisible by 4

A1*

(4 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S = a + (a + d) + \ldots + [a + (n - 1)d])</td>
<td>B1: requires at least 3 terms, must include first and last terms, an adjacent term and dots!</td>
<td>B1</td>
</tr>
<tr>
<td>((S =) [a + (n - 1)d] + \ldots + a)</td>
<td>M1: for reversing series (dots needed)</td>
<td>M1</td>
</tr>
<tr>
<td>(2S = [2a + (n - 1)d] + \ldots + [2a + (n - 1)d])</td>
<td>dM1: for adding, must have 2S and be a genuine attempt. Either line is sufficient. Dependent on 1st M1.</td>
<td>dM1</td>
</tr>
<tr>
<td>(2S = n[2a + (n - 1)d])</td>
<td>(NB – Allow first 3 marks for use of (l) for last term but as given for final mark)</td>
<td>A1</td>
</tr>
<tr>
<td>(S = \frac{n}{2} [2a + (n - 1)d]) cso</td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td>(b) 600 = 200 + ((N - 1)20) (\Rightarrow) (N = \ldots) Use of 600 with a correct formula in an attempt to find (N).</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>(N = 21) cso</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(c) Look for an AP first:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S = \frac{21}{2} (2 \times 200 + 20 \times 20)) or (\frac{21}{2} (200 + 600))</td>
<td>M1: Use of correct sum formula with their integer (n = N) or (N - 1) from part (b) where (3 &lt; N &lt; 52) and (a = 200) and (d = 20).</td>
<td>M1A1</td>
</tr>
<tr>
<td>(S = \frac{20}{2} (2 \times 200 + 19 \times 20)) or (\frac{20}{2} (200 + 580))</td>
<td>M1: Use of correct sum formula with their integer (n = N) or (N - 1) from part (b) where (3 &lt; N &lt; 52) and (a = 200) and (d = 20).</td>
<td></td>
</tr>
<tr>
<td>((= 8400 \text{ or } 7800))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Then for the constant terms:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(600 \times (52 - &quot;N&quot;) (\Rightarrow) 18600)</td>
<td>M1: (600 \times k) where (k) is an integer and (3 &lt; k &lt; 52)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>A1: A correct un-simplified follow through expression with their (k) consistent with (n) so that (n + k = 52)</td>
<td>A1ft</td>
</tr>
<tr>
<td>So total is 27000 cso</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>There are no marks in (c) for just finding (S_{52})</td>
<td></td>
<td>(5)</td>
</tr>
<tr>
<td>(11 marks)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>6(i)</td>
<td>$\log_2 \left( \frac{2x}{5x + 4} \right) = -3$ or $\log_2 \left( \frac{5x + 4}{2x} \right) = 3$ or $\log_2 \left( \frac{5x + 4}{x} \right) = 4$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\left( \frac{2x}{5x + 4} \right)^2 = 2^{-3}$ or $\left( \frac{5x + 4}{2x} \right)^3 = 2^3$ or $\left( \frac{5x + 4}{x} \right)^4 = 2^4$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$16x = 5x + 4 \Rightarrow x = (\text{depends on Ms and must be this equation or equiv})$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{4}{11}$ or exact recurring decimal 0.36 after correct work</td>
<td>A1 cso</td>
</tr>
<tr>
<td>Alternative</td>
<td>$\log_2 (2x) + 3 = \log_2 (5x + 4)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>So $\log_2 (2x) + \log_2 (8) = \log_2 (5x + 4)$ earns 2nd M1 (3 replaced by $\log_2 8$)</td>
<td>2nd M1</td>
</tr>
<tr>
<td></td>
<td>Then $\log_2 (16x) = \log_2 (5x + 4)$ earns 1st M1 (addition law of logs)</td>
<td>1st M1</td>
</tr>
<tr>
<td></td>
<td>Then final M1 A1 as before</td>
<td>dM1 A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\log_a y + \log_a 2^3 = 5$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\log_a 8y = 5$</td>
<td>Applies product law of logarithms</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{1}{8} a^5$</td>
<td>cso</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y = \frac{1}{8} a^5$</td>
</tr>
<tr>
<td></td>
<td>(3 marks)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(i) M1: Applying the subtraction or addition law of logarithms correctly to make **two log terms** into one log term.

M1: For RHS of either $2^{-3}$, $2^1$, $2^4$ or $\log_2 \left( \frac{1}{8} \right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an error. **Use of $3^2$ is M0**

dM1: Obtains **correct** linear equation in $x$. usually the one in the scheme and attempts $x =$

A1: cso. Answer of 4/11 with no suspect log work preceding this.

(ii) M1: Applies power law of logarithms to replace $3 \log_a 2$ by $\log_a 2^3$ or $\log_a 8$

dM1: (Should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7(a)</strong></td>
<td>Obtain ((x \pm 10)^2) and ((y \pm 8)^2)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>((10, 8))</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>See ((x \pm 10)^2 + (y \pm 8)^2 = 25 = r^2) or ((r^2) = &quot;100&quot; + &quot;64&quot; - 139)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(r = 5^*)</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Substitute (x = 13) into the equation of circle and solve quadratic to give (y = )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>e.g. (x = 13 \Rightarrow (13 - 10)^2 + (y - 8)^2 = 25 \Rightarrow (y - 8)^2 = 16) so (y = 4) or 12</td>
<td>A1 A1</td>
</tr>
<tr>
<td></td>
<td>N.B. This can be attempted via a 3, 4, 5 triangle so spotting this and achieving one value for (y) is M1 A1. Both values scores M1 A1 A1</td>
<td></td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>(OC = \sqrt{10^2 + 8^2} = \sqrt{164})</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Length of tangent = (\sqrt{164 - 5^2} = \sqrt{139})</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td><strong>(10 marks)</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a) M1: Obtains \((x \pm 10)^2\) and \((y \pm 8)^2\) May be implied by one correct coordinate

A1: \((10, 8)\) Answer only scores both marks.

Alternative: Method 2: From \(x^2 + y^2 + 2gx + 2fy + c = 0\) centre is \((\pm g, \pm f)\)

M1: Obtains \((\pm 10, \pm 8)\)

A1: Centre is \((-g, -f)\), and so centre is \((10, 8)\).

(b) M1: For a correct method leading to \(r = \ldots\), or \(r^2 =\)

Allow "100"+"64"−139 or an attempt at using \((x \pm 10)^2 + (y \pm 8)^2 = r^2\) form to identify \(r = \)

A1*: \(r = 5\) This is a printed answer, so a correct method must be seen.

Alternative:

(b)

M1: Attempts to use \(\sqrt{g^2 + f^2 - c}\) or \((r^2) = "100" + "64" - 139\)

A1*: \(r = 5\) following a correct method.

(c) M1: Substitutes \(x = 13\) into either form of the circle equation, forms and solves the quadratic equation in \(y\)

A1: Either \(y = 4\) or 12

A1: Both \(y = 4\) and 12
(d) M1: Uses Pythagoras' Theorem to find length OC using their \((10,8)\)

M1: Uses Pythagoras' Theorem to find \(OX\). Look for \(\sqrt{OC^2 - r^2}\)

A1: \(\sqrt{139}\) only
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8(a)</strong></td>
<td>Substitutes $x = 1$ in $C_1: y = 10x - x^2 - 8 = 10 - 1 - 8 = 1$ and in $C_2: y = x^3 = 1 = 1 \Rightarrow (1, 1)$ lies on both curves.</td>
<td>B1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$10x - x^2 - 8 = x^3$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$x^3 + x^2 - 10x + 8 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(x - 1)(x^2 + 2x - 8) = 0$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$(x - 1)(x + 4)(x - 2) = 0$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$x = 2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(2, 8)$</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>$\int \left{ (10x - x^2 - 8) - x^3 \right} dx$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= 5x^2 - \frac{x^3}{3} - 8x - \frac{x^4}{4}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Using limits 2 and 1: $\left[ \frac{20}{3} - \frac{8}{3} - 16 - 4 \right] - \left[ \frac{5}{3} - \frac{1}{3} - 8 - \frac{1}{4} \right]$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{11}{12}$</td>
<td>A1</td>
</tr>
</tbody>
</table>

Total: (12 marks)

**Notes:**

(a) **B1:** Substitutes $x = 1$ to both $y = 10x - x^2 - 8$ and $y = x^3$ AND achieves $y = 1$ in both.

(b) **B1:** Sets equations equal to each other and proceeds to $x^3 + x^2 - 10x + 8 = 0$

**M1:** Divides by $(x - 1)$ to form a quadratic factor. Allow any suitable algebraic method including division or inspection.

**A1:** Correct quadratic factor $(x^2 + 2x - 8)$

**M1:** For factorising of their quadratic factor.

**A1:** Achieves $x = 2$

**A1:** Coordinates of $B = (2, 8)$

(c) **M1:** For knowing that the area of $R = \int \left\{ (10x - x^2 - 8) - x^3 \right\} dx$

This may also be scored for finding separate areas and subtracting.

**M1:** For raising the power of $x$ seen in at least three terms.

**A1:** Correct integration. It may be left un-simplified. That is allow $\frac{10x^2}{2}$ for $5x^2$
### Question 8 notes continued

<p>| M1: | For using the limits &quot;2&quot; and 1 in their integrated expression. If separate areas have been attempted, &quot;2&quot; and 1 must be used in both integrated expressions. |
| A1: | For $\frac{11}{12}$ or exact equivalent. |</p>
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9(i)</strong></td>
<td><strong>Way 1</strong></td>
<td>M1</td>
</tr>
<tr>
<td>Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $\Rightarrow (3\theta) = \frac{\pi}{3}$</td>
<td><strong>Way 2</strong></td>
<td></td>
</tr>
<tr>
<td>Or Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm \frac{1}{2}$ or $\sin 3\theta = \pm \frac{\sqrt{3}}{2}$ so $\Rightarrow (3\theta) = \frac{\pi}{3}$</td>
<td>Adds $\pi$ or $2\pi$ to previous value of angle to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$</td>
<td></td>
</tr>
<tr>
<td><strong>(3)</strong></td>
<td><strong>(3)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>(ii)(a)</strong></td>
<td>$4(1 - \cos^2 x) + \cos x = 4 - k$</td>
<td>M1</td>
</tr>
<tr>
<td>Applies $\sin^2 x = 1 - \cos^2 x$</td>
<td>Attempts to solve $4 \cos^2 x - \cos x - k = 0$, to give $\cos x =$</td>
<td>dM1</td>
</tr>
<tr>
<td>$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \frac{1}{\sqrt{64 + 4k}}$</td>
<td>or other correct equivalent</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(3)</strong></td>
<td><strong>(3)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)</td>
<td>M1</td>
</tr>
<tr>
<td>Obtains two solutions from 0, 139, 221</td>
<td>dM1</td>
<td></td>
</tr>
<tr>
<td>$0$ or 2.42 or 3.86 in radians</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = 0$ and 139 and 221 (allow awrt 139 and 221) must be in degrees</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td><strong>(3)</strong></td>
<td><strong>(3)</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(i) **M1:** Obtains $\frac{\pi}{3}$. Allow $\frac{\pi}{3}$ or even $\theta = \frac{\pi}{3}$. Need not see working here. May be implied by $\theta = \frac{\pi}{9}$ in final answer (allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark). Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm \frac{1}{\sqrt{3}}$.

**M1:** Adding $\pi$ or $2\pi$ to a previous value however obtained. It is not dependent on the previous mark. (May be implied by final answer of $\theta = \frac{4\pi}{9}$ or $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420).
**Question 9 notes continued**

**A1:** Need all three correct answers in terms of $\pi$ and **no extras in range.**

**NB:** $\theta = 20^\circ, 80^\circ, 140^\circ$ earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0

<table>
<thead>
<tr>
<th>(ii)(a)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1:</strong> Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$). This must be awarded in (ii) (a) for an expression with $k$ not after $k = 3$ is substituted.</td>
<td></td>
</tr>
<tr>
<td><strong>dM1:</strong> Uses formula or completion of square to obtain $\cos x = \text{expression in } k$ (Factorisation attempt is M0)</td>
<td></td>
</tr>
<tr>
<td><strong>A1:</strong> cao - award for their final simplified expression</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(ii)(b)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1:</strong> <strong>Either</strong> attempts to substitute $k = 3$ into their answer to obtain two values for $\cos x$ <strong>Or</strong> restarts with $k = 3$ to find two values for $\cos x$ (They cannot earn marks in ii(a) for this). <strong>In both cases</strong> they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) and correct method for solving their quadratic (usual rules – see notes) The <strong>values for $\cos x$ may be &gt;1 or &lt; -1.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>dM1:</strong> Obtains <strong>two correct</strong> values for $x$</td>
<td></td>
</tr>
<tr>
<td><strong>A1:</strong> Obtains <strong>all three correct values</strong> in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.</td>
<td></td>
</tr>
</tbody>
</table>
Question 9 notes
continued

A:

1.

Need all three correct answers in terms of $\pi$ and no extras in range.

NB:

20, 80, 140 $\theta = \degree$ earns M1A0 and 0.349, 1.40 and 2.44 earns M1A0.

(ii)

(a) M1:

Applies

$22\sin x - \cos x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos 2x$).

This must be awarded in (ii) (a) for an expression with $k$ not after $k = 3$ is substituted.

dM1:

Uses formula or completion of square to obtain $\cos x$ (Factorisation attempt is M0).

A1:

cao for their final simplified expression.

(ii)(b) M1:

Either attempts to substitute $k = 3$ into their answer to obtain two values for $\cos x$ or

restarts with $k = 3$ to find two values for $\cos x$ (They cannot earn marks in ii(a) for this).

In both cases they need to have applied $22\sin x - \cos x$ (brackets may be missing) and correct method for solving their quadratic (usual rules – see notes) The values for $\cos x$ may be $>1$ or $< -1$.

dM1:

Obtains two correct values for $x$.

A1:

Obtains all three correct values in degrees (allow awrt 139 and 221) including 0.

Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
1. Express \[
\frac{6x + 4}{9x^2 - 4} - \frac{2}{3x + 1}
\] as a single fraction in its simplest form. 

(Total for Question 1 is 4 marks)
Question 1 continued

(Total for Question 1 is 4 marks)
2. \( f(x) = x^3 + 3x^2 + 4x - 12 \)

(a) Show that the equation \( f(x) = 0 \) can be written as

\[
x = \sqrt[3]{\frac{4(3 - x)}{(3 + x)}} \quad x \neq -3
\]

The equation \( x^3 + 3x^2 + 4x - 12 = 0 \) has a single root which is between 1 and 2

(b) Use the iteration formula

\[
x_{n+1} = \sqrt[3]{\frac{4(3 - x_n)}{(3 + x_n)}} \quad n \geq 0
\]

with \( x_0 = 1 \) to find, to 2 decimal places, the value of \( x_1, x_2 \) and \( x_3 \)

(c) By choosing a suitable interval, prove that \( \alpha = 1.272 \) to 3 decimal places.
Question 2 continued
Question 2 continued

(Total for Question 2 is 8 marks)
3.

Figure 1 shows a sketch of part of the graph $y = f(x)$ where

$$f(x) = 2|3 - x| + 5 \quad x \geq 0$$

(a) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where $k$ is a constant, has two distinct roots,

(b) state the set of possible values for $k$. 

(2)
Figure 1 shows a sketch of part of the graph $y = f(x)$ where $f(x) = 2^{\frac{1}{3} - \frac{1}{2}x} + 5x^0$.

(a) Solve the equation $f(x) = \frac{1}{2}x + 30$.

(b) State the set of possible values for $k$ where the equation $f(x) = k$ has two distinct roots.
4. (i) Find

\[ \int_{5}^{13} \frac{1}{2x - 1} \, dx \]

writing your answer in its simplest form. \hfill (4)

(ii) Use integration to find the exact value of

\[ \int_{0}^{\frac{\pi}{2}} \sin 2x + \sec \frac{1}{3} x \tan \frac{1}{3} x \, dx \]

\hfill (3)
Question 4 continued
Question 4 continued
Question 4 continued

(Total for Question 4 is 7 marks)
5. Given that

\[ y = \frac{5x^2 - 10x + 9}{(x - 1)^2} \quad x \neq 1 \]

show that \( \frac{dy}{dx} = \frac{k}{(x - 1)^3} \), where \( k \) is a constant to be found. \( \text{(6)} \)
Given that 

\[ y = (x - k)^2 + 2x - 2 \] 

is a constant to be found. 

Leave blank 

(Total for Question 5 is 6 marks)
6. The functions \( f \) and \( g \) are defined by

\[
\begin{align*}
  f &: x \mapsto e^x + 2 \quad x \in \mathbb{R} \\
  g &: x \mapsto \ln x \quad x > 0
\end{align*}
\]

(a) State the range of \( f \).

(b) Find \( fg(x) \), giving your answer in its simplest form.

(c) Find the exact value of \( x \) for which \( f(2x + 3) = 6 \)

(d) Find \( f^{-1} \) stating its domain.

(e) On the same axes sketch the curves with equation \( y = f(x) \) and \( y = f^{-1}(x) \), giving the coordinates of all the points where the curves cross the axes.
Question 6 continued
Question 6 continued
7. The point $P$ lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that $P$ has $(x, y)$ coordinates $\left(p, \frac{\pi}{2}\right)$, where $p$ is a constant,

(a) find the exact value of $p$  \hspace{1cm} (1)

The tangent to the curve at $P$ cuts the $y$-axis at the point $A$.

(b) Use calculus to find the coordinates of $A$.  \hspace{1cm} (6)
Question 7 continued
Question 7 continued

(Total for Question 7 is 7 marks)
8. In a controlled experiment, the number of microbes, $N$, present in a culture $T$ days after the start of the experiment were counted.

$N$ and $T$ are expected to satisfy a relationship of the form

$$N = aT^b$$

where $a$ and $b$ are constants

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving $m$ and $c$ in terms of the constants $a$ and/or $b$.

Figure 2

Figure 2 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(c) With reference to the model, interpret the value of the constant $a$. 
In a controlled experiment, the number of microbes, $N$, present in a culture $T$ days after the start of the experiment were counted. $N$ and $T$ are expected to satisfy a relationship of the form $N = aT^b$ where $a$ and $b$ are constants.

(a) Show that this relationship can be expressed in the form $\log_{10} N = m \log_{10} T + c$ giving $m$ and $c$ in terms of the constants $a$ and/or $b$.

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(c) With reference to the model, interpret the value of the constant $a$. 
Question 8 continued
9. (a) Prove that
\[ \sec 2A + \tan 2A = \frac{\cos A + \sin A}{\cos A - \sin A} \quad A \neq \frac{(2n + 1)\pi}{4}, \quad n \in \mathbb{Z} \] (5)

(b) Hence solve, for \(0 \leq \theta < 2\pi\)
\[ \sec 2\theta + \tan 2\theta = \frac{1}{2} \]

Give your answers to 3 decimal places. (4)
Question 9 continued
Question 9 continued

(Total for Question 9 is 9 marks)
10. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

\[ x = De^{-0.2t} \]

where \( x \) is the amount of the antibiotic in the bloodstream in milligrams, \( D \) is the dose given in milligrams and \( t \) is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(b) show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

No more doses of the antibiotic are given. At time \( T \) hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that \( T = a \ln\left( b + \frac{b}{c} \right) \), where \( a \) and \( b \) are integers to be determined.
Question 10 continued

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the first dose.

(b) Show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is 7.5 mg.

(c) Show that the time in hours after the second dose is given when the total amount of the antibiotic in the bloodstream is 7.5 mg. Your answer in mg to 3 decimal places.

1. [Blank]
2. [Blank]
3. [Blank]
4. [Blank]
5. [Blank]
6. [Blank]
7. [Blank]
8. [Blank]
9. [Blank]
10. [Blank]
Question 10 continued

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9(x^2 - 4 = (3x - 2)(3x + 2)) at any stage</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Eliminating the common factor of ((3x + 2)) at any stage</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Use of a common denominator</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(\frac{2}{32(3x-2)(3x+1)} = \frac{2}{(9x-4)(3x+1)} ) or (\frac{2}{32(3x-1)(3x+2)} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Amongst possible (incorrect) options scoring method marks are</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(\frac{2}{23\cdot2} \cdot \frac{2}{(3x-2)(3x+1)} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Invisible brackets, single fraction.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This is not a given answer so you can allow recovery from ‘invisible’ brackets.</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
- B1: For factorising \(29 - 4 = (3x - 2)(3x + 2)\) using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark.
- B1: For eliminating/cancelling out a factor of \((3x + 2)\) at any stage of the answer.
- M1: For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme.
- A1: \(6 \cdot \frac{2}{(3x-2)(3x+1)} \) is now 1,1,1,1
### Pure Mathematics P3 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$9x^2 - 4 = (3x - 2)(3x + 2)$ at any stage</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Eliminating the common factor of $(3x + 2)$ at any stage</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{2(3x+2)}{(3x-2)(3x+2)} = \frac{2}{3x-2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use of a common denominator</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)} = \frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\frac{2(3x+1)}{(3x-2)(3x+1)} - \frac{2(3x-2)}{(3x+1)(3x-2)}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{6}{(3x-2)(3x+1)}$ or $\frac{6}{9x^2-3x-2}$</td>
<td>A1</td>
</tr>
</tbody>
</table>

(4 marks)

**Notes:**

**B1:** For factorising $9x^2 - 4 = (3x - 2)(3x + 2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark.

**B1:** For eliminating/cancelling out a factor of $(3x+2)$ at any stage of the answer.

**M1:** For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are $\frac{2(3x+2)}{(9x^2-4)(3x+1)} - \frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$. Only one numerator adapted, separate fractions $\frac{2 \times 3x + 1 - 2 \times 3x - 2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction.

**A1:** $\frac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from ‘invisible’ brackets.

**Alternative**

$\frac{2(3x+2)}{(9x^2-4)} - \frac{2}{(3x+1)} = \frac{2(3x+2)(3x+1) - 2(9x^2-4)}{(9x^2-4)(3x+1)} = \frac{18x+12}{(9x^2-4)(3x+1)}$

The alternative has scored 0,0,1,0 so far

$\frac{6(3x+2)}{(3x+2)(3x-2)(3x+1)}$ is now 1,1,1,0

$\frac{6}{(3x-2)(3x+1)}$ and now 1,1,1,1
Question | Scheme | Marks
--- | --- | ---
2(a) | \( x^3 + 3x^2 + 4x - 12 = 0 \) \( \Rightarrow x^3 + 3x^2 = 12 - 4x \) | M1
| \( \Rightarrow x^2(x + 3) = 12 - 4x \) | dM1
| \( \Rightarrow x^2 = \frac{12 - 4x}{x + 3} \) | A1*
| \( \Rightarrow x = \sqrt{\frac{4(3-x)}{(x+3)}} \) | (3)

(b) | \( x_1 = \sqrt{\frac{4(3-1)}{3+1}} = 1.41 \) | M1 A1
| awrt \( x_2 = 1.20 \) \( x_3 = 1.31 \) | A1
| (3)

(c) | Attempts \( f(1.2725) = (+)0.00827... \) \( f(1.2715) = -0.00821.... \) | M1
| Values correct with reason (change of sign with \( f(x) \) continuous) and conclusion \( (\Rightarrow \alpha = 1.272) \) | A1
| (2)

Notes:
(a)  
**M1**: Moves from \( f(x) = 0 \), which may be implied by subsequent working, to \( x^2(x \pm 3) = \pm 12 \pm 4x \) by separating terms and factorising in either order. No need to factorise rhs for this mark.

**dM1**: Divides by \((x+3)\) term to make \( x^2 \) the subject, then takes square root. No need for rhs to be factorised at this stage.

**A1**: CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The \( 12-4x \) needs to have been factorised.

(b)  
**M1**: An attempt to substitute \( x_o = 1 \) into the iterative formula to calculate \( x_1 \). This can be awarded for the sight of \( \sqrt[4]{\frac{8}{4}}, \sqrt[4]{2}, \sqrt[4]{2} \) and even 1.4

**A1**: \( x_1 = 1.41 \). The subscript is not important. Mark as the first value found, \( \sqrt[4]{2} \) is A0

**A1**: \( x_2 = \text{awrt} \ 1.20 \) \( x_3 = \text{awrt} \ 1.31 \). Mark as the second and third values found. Condone 1.2 for \( x_2 \)

(c)  
**M1**: Calculates \( f(1.2715) \) and \( f(1.2725) \), or the tighter interval with at least 1 correct to 1 sig fig rounded or truncated. Accept \( f(1.2715) = -0.008 \) 1sf rounded or truncated. Also accept \( f(1.2715) = -0.01 \) 2dp. Accept \( f(1.2725) = (+) \ 0.008 \) 1sf rounded or truncated. Also accept \( f(1.2725) = (+)0.01 \) 2dp

**A1**: Both values correct (see above),
A valid reason; Accept change of sign, or \( >0 <0 \), or \( f(1.2715) \times f(1.2725)<0 \)
And a (minimal) conclusion; Accept hence root or \( \alpha=1.272 \) or QED or \( \square \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(a)</td>
<td>Uses (-2(3 - x) + 5 = \frac{1}{2}x + 30)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Attempts to solve by multiplying out bracket, collect terms etc.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(x = \frac{62}{3}) only</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Makes the connection that there must be two intersections. Implied by either end point (k &gt; 5) or (k \leq 11)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(5 &lt; k \leq 11)</td>
<td>A1</td>
</tr>
</tbody>
</table>

(3)

(5 marks)

**Notes:**

(a)

**M1:** Deduces that the solution to \(f(x) = \frac{1}{2}x + 30\) can be found by solving \(-2(3 - x) + 5 = \frac{1}{2}x + 30\)

**M1:** Correct method used to solve their equation. Multiplies out bracket/collects like terms.

**A1:** \(x = \frac{62}{3}\) only. Do not allow 20.6

(b)

**M1:** Deduces that two distinct roots occurs when \(y = k\) intersects \(y = f(x)\) in two places. This may be implied by the sight of either end point. Score for sight of either \(k > 5\) or \(k \leq 11\)

**A1:** Correct solution only \(\{k : k \in \mathbb{R}, 5 < k \leq 11\}\)
### Question 4

#### (i)

\[
\int \frac{1}{(2x-1)} \, dx = \frac{1}{2} \ln(2x-1)
\]

\[
\int_5^{13} \frac{1}{(2x-1)} \, dx = \frac{1}{2} \ln 25 - \frac{1}{2} \ln 9 = \frac{1}{2} \ln \left( \frac{25}{9} \right)
\]

\[
= \ln \left( \frac{5}{3} \right)
\]

**Marks:** M1 A1 dM1 A1

#### (ii)

Integrates to give \( \alpha \cos 2x + \beta \sec \frac{1}{3}x \{+c\} \) where \( \alpha \neq 0, \beta \neq 0 \)

\[
\left[ -\frac{1}{2} \cos 2x + 3 \sec \frac{1}{3}x \{+c\} \right]
\]

\[
\left( -\frac{1}{2} \cos \left( \frac{2 \pi}{2} \right) + 3 \sec \left( \frac{1}{3} \times \frac{2}{2} \right) \right) - \left( -\frac{1}{2} \cos(0) + 3 \sec(0) \right)
\]

Substitutes limits of 0 and \( \pi \) and subtracts the correct way around

\[
= 2\sqrt{3} - 2
\]

**Marks:** M1 dM1 A1 (7 marks)

### Notes:

#### (i)

**M1:** For \( \int \frac{1}{(2x-1)} \, dx = k \ln(2x-1) \) where \( k \) is a constant.

**A1:** Correct integration \( \int \frac{1}{(2x-1)} \, dx = \frac{1}{2} \ln(2x-1) \)

**dM1:** Scored for substituting in the limits, subtracting and using correctly at least one log law.

You may see the subtraction law \( k \ln 25 - k \ln 9 = k \ln \left( \frac{25}{9} \right) \) or the index law

\[
\frac{1}{2} \ln 25 - \frac{1}{2} \ln 9 = \ln 5 - \ln 3
\]

**A1:** \( \text{cao} \ \ln \left( \frac{5}{3} \right) \)

#### (ii)

**M1:** Integrates to a form \( \alpha \cos 2x + \beta \sec \frac{1}{3}x \{+c\} \) where \( \alpha \neq 0, \beta \neq 0 \)

**dM1:** Dependent upon the previous M1. It is scored for substituting limits of 0 and \( \pi \) and subtracting the correct way around.

**A1:** \( \text{cao} \ 2\sqrt{3} - 2 \)
# Question 5

\[ y = \frac{5x^2 - 10x + 9}{(x-1)^2} \]

Differentiates numerator to \(10x - 10\) and denominator to \(2(x - 1)\) o.e. \(\text{B1}\)

Uses the quotient rule
\[
\frac{dy}{dx} = \frac{(x-1)^2 (10x-10) - (5x^2 - 10x + 9)2(x-1)}{(x-1)^4}
\]

Takes out a common factor from the numerator and cancels
\[
\frac{dy}{dx} = \frac{(x-1)\left((x-1)(10x-10) - (5x^2 - 10x + 9)2\right)}{(x-1)^4}
\]

Simplifies the numerator by multiplying and collecting terms
\[
\frac{dy}{dx} = \frac{10x^2 - 20x + 10 - 10x^2 + 20x - 18}{(x-1)^3}
\]

\[
\frac{dy}{dx} = \frac{-8}{(x-1)^3}
\]

(6 marks)

### Notes:

**B1:** See scheme.

**M1:** Uses the quotient rule to reach a form
\[
\frac{dy}{dx} = \frac{(x-1)^2 (Ax + B) - (5x^2 - 10x + 9)(Cx + D)}{(x-1)^4}
\]

Alternatively uses the product rule to reach a for
\[
\frac{dy}{dx} = (x-1)^{-2} (Ax + B) + (5x^2 - 10x + 9)C(x-1)^{-3}
\]

**A1:** Fully correct \(\frac{dy}{dx}\) if the product rule is used
\[
\frac{dy}{dx} = (x-1)^{-2} (10x - 10) - (5x^2 - 10x + 9)2(x-1)^{-3}
\]

**M1:** This is for using a correct method to reach a form \(\frac{dy}{dx} = \frac{g(x)}{(x-1)^3}\). See scheme when using the quotient rule. If the product rule is used it is for combining the terms using a common denominator.

**M1:** Scored for simplifying the numerator (By multiplying out and collecting terms).

**A1:**
\[
\frac{dy}{dx} = \frac{-8}{(x-1)^3}
\]
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td>( f(x) &gt; 2 )</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>( f(x) = e^{\ln x} + 2, = x + 2 )</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(c)</td>
<td>( e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4 )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( 2x + 3 = \ln 4 )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{\ln 4 - 3}{2} ) or ( \ln 2 - \frac{3}{2} )</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(d)</td>
<td>Let ( y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y - 2) = x )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( f^{-1}(x) = \ln(x - 2), x &gt; 2 )</td>
<td>A1 B1 ft</td>
</tr>
<tr>
<td>(e)</td>
<td>Shape for ( f(x) )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>((0, 3))</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Shape for ( f^{-1}(x) )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>((3, 0))</td>
<td>B1</td>
</tr>
</tbody>
</table>

(14 marks)

Notes:

(a)  
**B1:** Range of \( f(x) > 2 \). Accept \( y > 2, (2, \infty), f > 2 \), as well as ‘range is the set of numbers bigger than 2’ but **don’t accept** \( x > 2 \)

(b)  
**M1:** For applying the correct order of operations. Look for \( e^{\ln x} + 2 \). Note that \( \ln e^x + 2 \) is M0  
**A1:** Simplifies \( e^{\ln x} + 2 \) to \( x + 2 \). Just the answer is acceptable for both marks.

(c)  
**M1:** Starts with \( e^{2x+3} + 2 = 6 \) and proceeds to \( e^{2x+3} = \ldots \)  
**A1:** \( e^{2x+3} = 4 \)  
**M1:** Takes \( \ln \)’s both sides, \( 2x + 3 = \ln \ldots \) and proceeds to \( x = \ldots \).
**Question 6 notes continued**

| (d) | M1: Starts with \( y = e^x + 2 \) or \( x = e^y + 2 \) and attempts to change the subject. All ln work must be correct. The 2 must be dealt with first. Eg. \( y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2 \) is M0.  
A1: \( f^{-1}(x) = \ln(x - 2) \) or \( y = \ln(x - 2) \) or \( y = \ln|x - 2| \) There must be some form of bracket.  
B1ft: Either \( x > 2 \), or follow through on their answer to part (a), provided that it wasn’t \( y \in \mathbb{R} \). Do not accept \( y > 2 \) or \( f^{-1}(x) > 2 \). |
|----|----|
| (e) | B1: Shape for \( y = e^x \). The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the x axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.  
B1: (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve.  
B1: Shape for \( y = \ln x \). The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the y axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects \( y = e^x \).  
B1: (3, 0) lies on the curve. Accept 3 written on the x axis as long as the point lies on the curve. |
### Question 7

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a)</strong></td>
<td>( p = 4\pi^2 ) or ((2\pi)^2)</td>
<td>B1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>( x = (4y - \sin 2y)^2 \Rightarrow \frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y) )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Sub ( y = \frac{\pi}{2} \Rightarrow \frac{dx}{dy} = 24\pi ) ((= 75.4)) OR (\Rightarrow \frac{dy}{dx} = \frac{1}{24\pi} (= 0.013))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Equation of tangent ( y - \frac{\pi}{2} = \frac{1}{24\pi} x - 4\pi^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Using ( y - \frac{\pi}{2} = \frac{1}{24\pi} x - 4\pi^2 ) with ( x = 0 \Rightarrow y = \frac{\pi}{3} ) cos</td>
<td>M1 A1</td>
</tr>
<tr>
<td><strong>Alternative I for first two marks</strong></td>
<td>(x = (4y - \sin 2y)^2 \Rightarrow x^{0.5} = 4y - \sin 2y)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(\Rightarrow 0.5x^{-0.5} \frac{dx}{dy} = 4 - 2\cos 2y)</td>
<td></td>
</tr>
<tr>
<td><strong>Alternative II for first two marks</strong></td>
<td>(x = (16y^2 - 8y\sin 2y + \sin^2 2y))</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(\Rightarrow 1 = 32y \frac{dy}{dx} - 8\sin 2y \frac{dy}{dx} - 16y\cos 2y \frac{dy}{dx} + 4\sin 2y\cos 2y \frac{dy}{dx})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Or (1dx = 32y dy - 8\sin 2y dy - 16y\cos 2y dy + 4\sin 2y\cos 2y dy)</td>
<td></td>
</tr>
</tbody>
</table>

#### (7 marks)

### Notes:

(a) 
**B1:** \( p = 4\pi^2 \) or exact equivalent \(2\pi^2\). Also allow \( x = 4\pi^2 \)

(b) 
**M1:** Uses the chain rule of differentiation to get a form 
\[ A(4y - \sin 2y)(B \pm C\cos 2y), \quad A, B, C \neq 0 \] on the right hand side.

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form 
\[ x = (16y^2 - 8y\sin 2y + \sin^2 2y) \Rightarrow \frac{dx}{dy} = P y \pm Q \sin 2y \pm R \cos 2y \pm S \sin 2y \cos 2y \quad P, Q, R, S \neq 0 \]

A second method is to take the square root first. To score the method look for a differentiated expression of the form \( P_{x}^{-0.5} = 4 - Q \cos 2y \)

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.
Question 7 notes continued

A1: \[
\frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y) \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{2(4y - \sin 2y)(4 - 2\cos 2y)}
\] with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs. In the alternative \[
\frac{dx}{dy} = 32y - 8\sin 2y - 16y\cos 2y + 4\sin 2y\cos 2y
\]

M1: Sub \( y = \frac{\pi}{2} \) into \( \frac{dx}{dy} \) or inverted \( \frac{dy}{dx} \). Evidence could be minimal, eg \( y = \frac{\pi}{2} \Rightarrow \frac{dx}{dy} = ... \)

It is not dependent upon the previous M1 but it must be a changed \( x = (4y - \sin 2y)^2 \)

M1: Score for a correct method for finding the equation of the tangent at \( \left( 4\pi^2, \frac{\pi}{2} \right) \).

Allow for \( y - \frac{\pi}{2} = \frac{1}{\text{their numerical } \frac{dx}{dy}} x - \text{their } 4\pi^2 \)

Allow for \( \left( y - \frac{\pi}{2} \right) \times \text{their numerical } \frac{dx}{dy} = x - \text{their } 4\pi^2 \)

Even allow for \( y - \frac{\pi}{2} = \frac{1}{\text{their numerical } \frac{dx}{dy}} x - p \)

It is possible to score this by stating the equation \( y = \frac{1}{24\pi} x + c \) as long as \( \left( 4\pi^2, \frac{\pi}{2} \right) \) is used in a subsequent line.

M1: Score for writing their equation in the form \( y = mx + c \) and stating the value of \( c' \)

or setting \( x = 0 \) in their \( y - \frac{\pi}{2} = \frac{1}{24\pi} x - 4\pi^2 \) and solving for \( y \).

Alternatively using the gradient of the line segment \( AP = \text{gradient of tangent} \).

Look for \( \frac{\pi}{2} - y = \frac{1}{24\pi} \Rightarrow y = .. \) Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1: \( \cos y = \frac{\pi}{3} \). You do not have to see \( \left( 0, \frac{\pi}{3} \right) \)
<table>
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<tbody>
<tr>
<td><strong>8(a)</strong></td>
<td>( N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow \log_{10} N = \log_{10} a + b \log_{10} T ) ( m = b \text{ and } c = \log_{10} a )</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(2)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Uses the graph to find either ( a ) or ( b ) ( a = 10^{\text{intercept}} \text{ or } b = \text{gradient} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Uses the graph to find both ( a ) and ( b ) ( a = 10^{\text{intercept}} \text{ and } b = \text{gradient} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Uses ( T = 3 ) in ( N = aT^b \text{ with their } a \text{ and } b )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Number of microbes ( \approx 800 )</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(4)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>States that 'a' is the number of microbes 1 day after the start of the experiment.</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>(1)</strong></td>
</tr>
<tr>
<td></td>
<td>(7 marks)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)  
**M1:** Takes \( \log_{10}'s \) of both sides and attempts to use the addition law. Condone \( \log = \log_{10} \) for this mark.  
**A1:** Proceeds correctly to \( \log_{10} N = \log_{10} a + b \log_{10} T \) and states \( m = b \) and \( c = \log_{10} a \)

(b)  
**Way One: Main scheme**  
**M1:** For attempting to use the graph to find either \( a \) or \( b \) using \( a = 10^{\text{intercept}} \text{ or } b = \text{gradient} \) This may be implied by \( a = 10^{1.75 \text{ to } 1.85} \text{ or } b = 2.27 \text{ to } 2.33 \)  
**M1:** For attempting to use the graph to find BOTH \( a \) and \( b \) (See previous M1)  
**M1:** Uses \( T = 3 \) in \( N = aT^b \text{ with their } a \text{ and } b \)  
**A1:** Number of microbes \( \approx 800 \)

**Way Two: Alternative using line of best fit techniques.**  
**M1:** For \( \log_{10} 3 \approx 0.48 \) and using the graph to find \( \log_{10} N \)  
**M1:** For using the graph to find \( \log_{10} N \) (FYI \( \log_{10} N \approx 2.9 \))  
**M1:** For \( \log_{10} N = k \Rightarrow N = 10^k \)  
**A1:** Number of microbes \( \approx 800 \)

(c)  
**B1:** See scheme.
\[
\sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A} \\
= \frac{1 + \sin 2A}{\cos 2A} \\
= \frac{1 + 2 \sin A \cos A}{\cos^2 A - \sin^2 A} \\
= \frac{\cos^2 A + \sin^2 A + 2 \sin A \cos A}{\cos^2 A - \sin^2 A} \\
= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)} \\
= \frac{\cos A + \sin A}{\cos A - \sin A}
\]

\[
\sec 2\theta + \tan 2\theta = \frac{1}{2} \Rightarrow \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{1}{2} \\
\Rightarrow 2 \cos \theta + 2 \sin \theta = \cos \theta - \sin \theta \\
\Rightarrow \tan \theta = -\frac{1}{3} \\
\Rightarrow \theta = \text{awrt} 2.820, 5.961
\]

**Notes:**

(a) **B1:** A correct identity for \(\sec 2A = \frac{1}{\cos 2A}\) or \(\tan 2A = \frac{\sin 2A}{\cos 2A}\).

It need not be in the proof and it could be implied by the sight of \(\sec 2A = \frac{1}{\cos^2 A - \sin^2 A}\).

**M1:** For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for \(\frac{1 + \cos 2A \tan 2A}{\cos 2A}\) or \(\frac{1 + \sin 2A}{\cos 2A}\).

**M1:** For getting an expression in just \(\sin A\) and \(\cos A\) by using the double angle identities \(\sin 2A = 2 \sin A \cos A\) and \(\cos 2A = \cos^2 A - \sin^2 A\), \(2 \cos^2 A - 1\) or \(1 - 2 \sin^2 A\).

Alternatively for getting an expression in just \(\sin A\) and \(\cos A\) by using the double angle identities \(\sin 2A = 2 \sin A \cos A\) and \(\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}\) with \(\tan A = \frac{\sin A}{\cos A}\).

For example \(\frac{1}{\cos^2 A - \sin^2 A} + \frac{2 \sin A}{\cos A} \frac{\cos A}{1 - \sin^2 A} \frac{\cos A}{\cos^2 A}\) is B1M0M1 so far.
**Question 9 notes continued**

| M1: | In the main scheme it is for replacing 1 by \( \cos^2 \theta + \sin^2 \theta \) and factorising both numerator and denominator. |
| A1*: | Cancelling to produce given answer with no errors. Allow a consistent use of another variable such as \( \theta \), but mixing up variables will lose the A1*. |

### (b)

<p>| M1: | For using part (a), cross multiplying, dividing by ( \cos \theta ) to reach ( \tan \theta = k ) |
| A1: | ( \tan \theta = -\frac{1}{3} ) |
| dM1: | Scored for ( \tan \theta = k ) leading to at least one value (with 1 dp accuracy) for ( \theta ) between 0 and 2( \pi ). You may have to use a calculator to check. Allow answers in degrees for this mark. |
| A1: | ( \theta ) awrt 2.820, 5.961 with no extra solutions within the range. Condone 2.82 for 2.820. You may condone different/ mixed variables in part (b) |</p>
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10(a)</strong></td>
<td>Subs $D = 15$ and $t = 4$ [x = 15e^{-0.2\times 4} = 6.740 \text{ (mg)}]</td>
<td>M1 A1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$15e^{-0.2\times 7} + 15e^{-0.2\times 2} = 13.754 \text{ (mg)}$</td>
<td>M1 A1*</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>$15e^{-0.2\times T} + 15e^{-0.2\times (T+5)} = 7.5$[15e^{-0.2\times T} + 15e^{-0.2\times T e^{-1}} = 7.5]$[15e^{-0.2\times T} (1+e^{-1}) = 7.5 \Rightarrow e^{-0.2\times T} = \frac{7.5}{15(1+e^{-1})}]</td>
<td>M1 dM1</td>
</tr>
<tr>
<td></td>
<td>[T = -5\ln \left( \frac{7.5}{15(1+e^{-1})} \right) = 5\ln \left( \frac{2+\frac{2}{e}}{1} \right)]</td>
<td>A1 A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td><strong>Notes:</strong></td>
<td></td>
<td>(8 marks)</td>
</tr>
<tr>
<td><strong>(a)</strong></td>
<td>Attempts to substitute both $D = 15$ and $t = 4$ in $x = D e^{-0.2t}$. It can be implied by sight of $15e^{-0.8}$, $15e^{-0.2\times 4}$ or awrt 6.7. Condone slips on the power. Eg you may see -0.02</td>
<td>M1: Cao. 6.740 (mg) Note that 6.74 (mg) is A0</td>
</tr>
<tr>
<td></td>
<td>Alternatively finds the amount after 5 hours, $15e^{0.2\times 5} = \text{awrt } 5.52$ adds the second dose = 15 to get a total of awrt 20.52 then multiplies this by $e^{-0.4}$ to get awrt 13.75. Sight of 5.52+15=20.52 $\rightarrow$ 13.75 is fine.</td>
<td>A1*: Cso so both the expression $15e^{-0.2\times 7} + 15e^{-0.2\times 2}$ and $13.754 \text{ (mg)}$ are required</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Attempt to find the sum of two expressions with $D = 15$ in both terms with $t$ values of 2 and 7. Evidence would be $15e^{-0.2\times 7} + 15e^{-0.2\times 2}$ or similar expressions such as $(15e^{-1} + 15)e^{-0.2\times 2}$. Award for the sight of the two numbers awrt 3.70 and awrt 10.05, followed by their total awrt 13.75. Alternatively finds the amount after 5 hours, $15e^{-1} = \text{awrt } 5.52$ adds the second dose = 15 to get a total of awrt 20.52 then multiplies this by $e^{-0.4}$ to get awrt 13.75. Sight of 5.52+15=20.52 $\rightarrow$ 13.75 is fine.</td>
<td>M1: Attempts to write down a correct equation involving $T$ or $t$. Accept with or without correct bracketing Eg. accept $15e^{-0.2\times T} + 15e^{-0.2\times (T+5)} = 7.5$ or similar equations $(15e^{-1} + 15)e^{-0.2\times T} = 7.5$</td>
</tr>
<tr>
<td></td>
<td>Alternatively both the expression $(15e^{-0.2\times 5} + 15)\times e^{-0.2\times 2}$ and 13.754 (mg) are required. Sight of just the numbers is not enough for the A1*</td>
<td>dM1: Attempts to solve their equation, dependent upon the previous mark, by proceeding to $e^{-0.2\times T} = \ldots$ An attempt should involve an attempt at the index law $x^{m+n} = x^m \times x^n$ and taking out a factor of $e^{-0.2\times T}$ Also score for candidates who make $e^{+0.2\times T}$ the subject using the same criteria.</td>
</tr>
</tbody>
</table>
Question 10 notes continued

A1: Any correct form of the answer, for example, $-5\ln\left(\frac{7.5}{15(1+e^{-1})}\right)$

A1: Cso. $T = 5\ln\left(2 + \frac{2}{e}\right)$ Condone $t$ appearing for $T$ throughout this question.

(e) Alternative 1
1st Mark (Method): $15e^{-0.2\times T} + \text{awrt} 5.52e^{-0.2\times T} = 7.5 \Rightarrow e^{-0.2\times T} = \text{awrt} 0.37$

2nd Mark (Accuracy): $T = -5\ln(\text{awrt} 0.37)$ or awrt 5.03 or $T = -5\ln\left(\frac{7.5}{\text{awrt} 20.52}\right)$

Alternative 2
1st Mark (Method): $13.754e^{-0.2\times T} = 7.5 \Rightarrow T = -5\ln\left(\frac{7.5}{13.754}\right)$ or equivalent such as 3.03

2nd Mark (Accuracy): $3.03 + 2 = 5.03$ Allow $-5\ln\left(\frac{7.5}{13.754}\right) + 2$

Alternative 3 (by trial and improvement)
1st Mark (Method): $15e^{-0.2\times5} + 15e^{-0.2\times10} = 7.55$ or $15e^{-0.2\times5.1} + 15e^{-0.2\times10.1} = 7.40$ or any value between.

2nd Mark (Accuracy): Answer $T = 5.03$. 
A1: Any correct form of the answer, for example,

\( \left( \frac{7.55}{\ln 15} \right) - 15 \)

Condone \( t \) appearing for \( T \) throughout this question.

(c) Alternative 1

1st Mark (Method):

\( 0.2 \times 5.52e^{T} - 7.5 \times 0.215e^{T} \)

\( \Rightarrow 0.2e^{T} - 0.37 \)

2nd Mark (Accuracy):

\( T = -5\ln 0.37 \) or awrt 5.03 or awrt 7.55ln 213.754

Alternative 2

1st Mark (Method):

\( 0.2 \times 13.754 - 7.5 \times 5\ln 5.1 \)

\( \Rightarrow 3.03 \)

2nd Mark (Accuracy):

\( 3.03 + 2 = 5.03 \)

Allow 7.55ln 213.754

Alternative 3 (by trial and improvement)

1st Mark (Method):

\( 0.2 \times 15 - 7.5 \times 0.2 \times 15 \)

\( \Rightarrow 0.2 \times 10.115 - 15 \times 0.2 \times 15 \)

\( \Rightarrow 10.115 - 22.75 \)

\( \Rightarrow 0.2 \times 10.115 - 15 \times 0.2 \times 15 \)

\( \Rightarrow 10.115 - 22.75 \)

\( \Rightarrow 20.52 \)

or any value between.

2nd Mark (Accuracy):

Answer \( T = 5.03 \).

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Answer ALL questions. Write your answers in the spaces provided.

1. Use the binomial series to find the expansion of

\[ \frac{1}{(2 + 5x)^3} \quad |x| < \frac{2}{5} \]

in ascending powers of \(x\), up to and including the term in \(x^3\)

Give each coefficient as a fraction in its simplest form. (6)
Question 1 continued

(Total for Question 1 is 6 marks)
2. A curve $C$ has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of $x$ and $y$.

(b) Find an equation of the tangent to $C$ at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.
Question 2 continued

(Total for Question 2 is 7 marks)
3. \[ f(x) = \frac{1}{x(3x - 1)^2} = \frac{A}{x} + \frac{B}{(3x - 1)} + \frac{C}{(3x - 1)^2} \]

(a) Find the values of the constants \( A, B \) and \( C \)

(b) (i) Hence find \( \int f(x) \, dx \)

(ii) Find \( \int_{1}^{2} f(x) \, dx \), giving your answer in the form \( a + \ln b \), where \( a \) and \( b \) are constants.
Question 3 continued
4.

Figure 1 shows a sketch of the curve \( C \) with parametric equations

\[
\begin{align*}
x &= \sqrt{3} \sin 2t \\
y &= 4 \cos^2 t
\end{align*}
\]

0 \( \leq t \leq \pi \)

(a) Show that \( \frac{dy}{dx} = k\sqrt{3} \tan 2t \), where \( k \) is a constant to be found.  

(b) Find an equation of the tangent to \( C \) at the point where \( t = \frac{\pi}{3} \)

Give your answer in the form \( y = ax + b \), where \( a \) and \( b \) are constants.
Figure 1 shows a sketch of the curve $C$ with parametric equations

\[ x = 3 \sin 2t \]
\[ y = 4 \cos 2t \]

(a) Show that \( \frac{dy}{dx} = k \tan 2t \), where \( k \) is a constant to be found.

(b) Find an equation of the tangent to $C$ at the point where \( t = \frac{3}{2} \pi \). Give your answer in the form $y = ax + b$, where $a$ and $b$ are constants.

Question 4 continued
Question 4 continued

(Total for Question 4 is 9 marks)
5.

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$

The curve meets the $x$-axis at the origin $O$ and cuts the $x$-axis at the point $A$.

(a) Find, in terms of $\ln 2$, the $x$ coordinate of the point $A$.  

(b) Find $\int xe^{\frac{1}{2}x} \, dx$  

The finite region $R$, shown shaded in Figure 2, is bounded by the $x$-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$

(c) Find, by integration, the exact value for the area of $R$.

Give your answer in terms of $\ln 2$
Question 5 continued
Question 5 continued
Question 5 continued
Question 5 continued

(Total for Question 5 is 8 marks)
6. Prove by contradiction that, if \( a, b \) are positive real numbers, then \( a + b \geq 2\sqrt{ab} \)
Question 6 continued

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(Total for Question 6 is 4 marks)
Figure 3 shows a sketch of the curve \( C \) with parametric equations

\[
x = 4 \cos \left( t + \frac{\pi}{6} \right) \quad y = 2 \sin t \quad 0 \leq t \leq 2\pi
\]

(a) Show that

\[
x + y = 2\sqrt{3} \cos t
\]

(b) Show that a cartesian equation of \( C \) is

\[
(x + y)^2 + ay^2 = b
\]

where \( a \) and \( b \) are integers to be found.
Question 7 continued

Given that \( \theta = 20 \) when \( t = 0 \) where \( \lambda \) is a positive constant.

The rate of increase of the temperature of the water at time \( t \) seconds, the temperature of the water is \( \theta ^\circ \text{C} \).

Water is being heated in a kettle. At time \( t = 0.01 \), find the time, to the nearest second, when the kettle switches off.

The temperature of the water is modelled by the differential equation

\[
\frac{d\theta}{dt} = 120 - \lambda t
\]

(Total for Question 7 is 5 marks)
8. Water is being heated in a kettle. At time \( t \) seconds, the temperature of the water is \( \theta \) °C.

The rate of increase of the temperature of the water at time \( t \) is modelled by the differential equation

\[
\frac{d\theta}{dt} = \lambda(120 - \theta) \quad \theta \leq 100
\]

where \( \lambda \) is a positive constant.

Given that \( \theta = 20 \) when \( t = 0 \)

(a) solve this differential equation to show that

\[
\theta = 120 - 100e^{-\lambda t}
\]

(b) Given that \( \lambda = 0.01 \), find the time, to the nearest second, when the kettle switches off.

When the temperature of the water reaches 100 °C, the kettle switches off.
Question 8 continued
Question 8 continued

(c) Find the value of the distance $AP$.

Given that $P \equiv (1, 2, 3)$ and $E \equiv (5, 4, 2)$ is a scalar parameter. $\mu$ lies on the line $l_2$ and is parallel to the line $l_1$ is given by the equation

$$l_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \mu$$

The acute angle between $l_1$ and $l_2$ is $\theta$.

Where $l_2$ passes through the point $A$, where

$$l_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \mu$$

The point $E$ lies on $l_2$ and $\mu = 1$ where

$$\theta = \cos^{-1}\left( \frac{\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}}{\| \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \| \| \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \|} \right)$$

Find the coordinates of $A$. The point $E$ lies on $l_2$ and $\mu = 1$ and is parallel to the line $l_1$. The point $E$ lies on $l_2$ and $\mu = 1$.

$f$ find the coordinates of the two possible positions of $A$.

$g$ Investigate the distance $AE$.

$h$ Given that $PE$ is a vector equation for the line $l_2$ passes through the point $A$.

(i) Find the value of the distance $APE$.

(ii) Find the area of triangle $APE$.

(iii) Find the equation of the plane $l$.

(iv) Find the value of the distance $PAE$.

(v) Find the coordinates of $B$.

(vi) Find the equation of the line $l$.

(vii) Find the value of the distance $PAE$.

(viii) Find the equation of the plane $l$.

(ix) Find the coordinates of $C$.

(x) Find the equation of the line $l$.

(xi) Find the value of the distance $PAE$.

(xii) Find the equation of the plane $l$.

(xiii) Find the coordinates of $D$.

(xiv) Find the equation of the line $l$.

(xv) Find the value of the distance $PAE$.

(xvi) Find the equation of the plane $l$.

(xvii) Find the coordinates of $E$.

(xviii) Find the equation of the line $l$.

(xix) Find the value of the distance $PAE$.

(xx) Find the equation of the plane $l$.

Question 8 continued

(Total for Question 8 is 11 marks)
9. With respect to a fixed origin $O$, the line $l_1$ is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where $\mu$ is a scalar parameter.

The point $A$ lies on $l_1$ where $\mu = 1$.

(a) Find the coordinates of $A$. (1)

The point $P$ has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$.

The line $l_2$ passes through the point $P$ and is parallel to the line $l_1$.

(b) Write down a vector equation for the line $l_2$. (2)

(c) Find the exact value of the distance $AP$.

Give your answer in the form $k\sqrt{2}$, where $k$ is a constant to be found. (2)

The acute angle between $AP$ and $l_2$ is $\theta$.

(d) Find the value of $\cos \theta$. (3)

A point $E$ lies on the line $l_2$.

Given that $AP = PE$,

(e) find the area of triangle $APE$, (2)

(f) find the coordinates of the two possible positions of $E$. (5)
Question 9 continued

(Total for Question 9 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS
**Pure Mathematics P4 Mark scheme**

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ \left( \frac{1}{2 + 5x} \right)^3 = (2 + 5x)^{-3} ]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[ = (2)^{-3} \left( \frac{1 + 5x}{2} \right)^3 = \frac{1}{8} \left( \frac{1 + 5x}{2} \right)^{-3} ]</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>[ = \left[ \frac{1}{8} \left( 1 + 3(5kx) + \frac{(-3)(-4)}{2!} (5kx)^2 + \frac{(-3)(-4)(-5)}{3!} (5kx)^3 + ... \right) \right] ]</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>[ = \left[ \frac{1}{8} \left( 1 + 3 \left( \frac{5x}{2} \right) \right) + \frac{(-3)(-4)}{2!} \left( \frac{5x}{2} \right)^2 + \frac{(-3)(-4)(-5)}{3!} \left( \frac{5x}{2} \right)^3 + ... \right] ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ = \frac{1}{8} \left[ 1 - \frac{15}{2} x + \frac{75}{2} x^2 - \frac{625}{4} x^3 + ... \right] ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ = \frac{1}{8} \left[ 1 - 7.5x + 37.5x^2 -156.25 x^3 + ... \right] ]</td>
<td>A1 A1</td>
</tr>
<tr>
<td></td>
<td>[ \text{or} \quad \frac{1}{8} \left[ \frac{15}{16} x; + \frac{75}{16} x^2 - \frac{625}{32} x^3 + ... \right] ]</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

**M1:** Mark can be implied by a constant term of \((2)^{-3}\) or \(\frac{1}{8}\).

**B1:** \(2^{-3}\) or \(\frac{1}{8}\) outside brackets or \(\frac{1}{8}\) as candidate’s constant term in their binomial expansion.

**M1:** Expands \((...+kx)^{-3}\), \(k\) a value \(\neq 1\) to give any 2 terms out of 4 terms simplified or unsimplified, Eg: \(1 + (-3)(kx) + \frac{(-3)(-4)}{2!} (kx)^2 + \frac{(-3)(-4)(-5)}{3!} (kx)^3 \) or \(1 + \ldots + \frac{(-3)(-4)}{2!} (kx)^2 + \frac{(-3)(-4)(-5)}{3!} (kx)^3 \) are fine for M1.

**A1:** A correct simplified or unsimplified \(1 + (-3)(kx) + \frac{(-3)(-4)}{2!} (kx)^2 + \frac{(-3)(-4)(-5)}{3!} (kx)^3 \) expansion with consistent \((kx)\). Note that \((kx)\) must be consistent and \(k\) a value \(\neq 1\). (on the RHS, not necessarily the LHS) in a candidate’s expansion.

**A1:** For \(\frac{1}{8} - \frac{15}{16} x \text{ (simplified)} \) or also allow \(0.125 - 0.9375x\).

**A1:** Accept only \(\frac{75}{16} x^2 - \frac{625}{32} x^3 \) or \(\frac{11}{16} x^2 - \frac{17}{32} x^3 \) or \(4.6875x^2 - 19.53125x^3 \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2(a)</td>
<td>[ x^3 + 2xy - x - y^3 - 20 = 0 ]</td>
<td>M1 A1 B1</td>
</tr>
<tr>
<td></td>
<td>[ \left{ \frac{\partial}{\partial x} \times \frac{\partial}{\partial y} \right} \left( 3x^2 + \left( 2y + 2x \frac{dy}{dx} \right) - 1 - 3y^2 \frac{dy}{dx} = 0 \right) ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ 3x^2 + 2y - 1 + \left( 2x - 3y^2 \right) \frac{dy}{dx} = 0 ]</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>[ \frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x} \text{ or } \frac{1 - 3x^2 - 2y}{2x - 3y^2} \text{ eso} ]</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>At P(3, -2), ( m(T) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)} = \frac{22}{6} = \frac{11}{3} ) ( ) and either ( T: y - 2 = \left( \frac{11}{3} \right) (x - 3) ) or ( (-2) = \left( \frac{11}{3} \right)(3) + c \Rightarrow c = ... )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>T: ( 11x - 3y - 39 = 0 ) or ( K(11x - 3y - 39) = 0 ) eso</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(7 marks)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)

**M1:** Differentiates implicitly to include either \( 2y \frac{dx}{dy} \) or \( x^3 \rightarrow \pm kx^5 \frac{dx}{dy} \) or \( -x \rightarrow - \frac{dx}{dy} \)

(Ignore \( \frac{dx}{dy} = \) ).

**A1:** \( x^3 \rightarrow 3x^2 \frac{dx}{dy} \) and \( -x - y^3 - 20 = 0 \rightarrow - \frac{dx}{dy} - 3y^2 = 0 \)

**B1:** \( 2xy \rightarrow 2y \frac{dx}{dy} + 2x \)

**dM1:** Dependent on the first method mark being awarded. An attempt to factorise out all the terms in \( \frac{dx}{dy} \) as long as there are at least two terms in \( \frac{dx}{dy} \).

**A1:** For \( \frac{1 - 2y - 3x^2}{2x - 3y^2} \) or equivalent. Eg: \( \frac{3x^2 + 2y - 1}{3y^2 - 2x} \)

(b)

**M1:** Some attempt to substitute both \( x = 3 \) and \( y = -2 \) into their \( \frac{dy}{dx} \) which contains both \( x \) and \( y \) to find \( m_r \) and

- either applies \( y - 2 = (\text{their } m_r)(x - 3) \), where \( m_r \) is a numerical value.
- or finds \( c \) by solving \( (-2) = (\text{their } m_r)(3) + c \), where \( m_r \) is a numerical value.

**A1:** Accept any integer multiple of \( 11x - 3y - 39 = 0 \) or \( 11x - 39 - 3y = 0 \) or \( -11x + 3y + 39 = 0 \), where their tangent equation is equal to 0.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(a)</strong></td>
<td>$1 = A(3x - 1)^2 + Bx(3x - 1) + Cx$</td>
<td><strong>B1</strong></td>
</tr>
<tr>
<td>$x \to 0$</td>
<td>$(1 = A)$</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td>$x \to \frac{1}{3}$</td>
<td>$1 = \frac{1}{3}C \Rightarrow C = 3$</td>
<td>any two constants correct coefficients of $x^2$</td>
</tr>
<tr>
<td>$0 = 9A + 3B \Rightarrow B = -3$</td>
<td>all three constants correct</td>
<td><strong>A1</strong></td>
</tr>
</tbody>
</table>

| (b)(i) | $\int \left( \frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) \, dx$ | **M1** |
| & | $= \ln x - \frac{3}{3} \ln (3x-1) + \frac{3}{(-1)^3} \ln (3x-1)^{-1} \quad (+C)$ | **A1**ft |
| & | $(= \ln x - \ln (3x-1) - \frac{1}{3x-1} \quad (+C))$ | **A1**ft |

| (b)(ii) | $\int_{1}^{2} f(x) \, dx = \left[ \ln x - \ln (3x-1) - \frac{1}{3x-1} \right]_1^2$ | **M1** |
| & | $= \left( \ln 2 - \ln 5 - \frac{1}{5} \right) - \left( \ln 1 - \ln 2 - \frac{1}{2} \right)$ | **M1** |
| & | $= \ln \frac{2 \times 2}{5} + \ldots$ | **A1** |
| & | $= \frac{3}{10} + \ln \left( \frac{4}{5} \right)$ | **A1** |

(10 marks)

**Notes:**

(a)

- **B1:** Obtaining $1 = A(3x - 1)^2 + Bx(3x - 1) + Cx$ at any stage. This will usually be at the beginning of the solution but, if the cover-up rule is used, it could appear later.

- **M1:** A complete method of finding any one of the three constants. If either $A = 1$ or $C = 3$ is given without working or, at least, without incorrect working, allow this M1 – use of the cover-up rule is acceptable. In principle, an alternative method is equating coefficients (or substituting three values other than 0 and $\frac{1}{3}$), obtaining a sufficient set of equations and solving for any one of the three constants.

- **A1:** Any two of $A, B$ and $C$ correct. These will usually, but not always, be $A$ and $C$.

- **A1:** All three of $A, B$ and $C$ correct. If all three constants are correct and the answers do not clearly conflict with any working, allow all 4 marks (including the B1 bod). There are a number of possible ways of finding $B$ but, as long as the M has been gained, you need not consider the method used.
### Question 3 notes continued

(b)(ii)

**M1:** Dependent upon the M mark in (b). Substituting in the correct limits and subtracting, not necessarily the right way round. There must be evidence that both 1 and 2 have been used but errors in substitution do not lose the mark.

**M1:** Dependent upon both previous Ms. Applies the addition and/or subtraction rules of logs to obtain a single logarithm. Either the addition or the subtraction rule of logs must be used correctly at least once to gain this mark and this must be seen in the attempt at (b)(ii).

**A1:** The correct answer in the form specified. Accept equivalent fractions including exact decimals for \( a \) and or \( b \).

Accept \( \ln \frac{4}{5} + \frac{3}{10} \).

\( \frac{3}{10} - \ln \frac{5}{4} \) is not acceptable.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(a)</td>
<td>( \frac{dx}{dt} = 2\sqrt{3} \cos 2t )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \frac{dy}{dt} = -8 \cos t \sin t )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( \frac{dy}{dx} = \frac{-8 \cos t \sin t}{2\sqrt{3} \cos 2t} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = \frac{4 \sin 2t}{2\sqrt{3} \cos 2t} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{dy}{dx} = -\frac{2}{3} \sqrt{3} \tan 2t ) ( k = -\frac{2}{3} )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( k = -\frac{2}{3} )</td>
<td>(5)</td>
</tr>
<tr>
<td>(b)</td>
<td>When ( t = \frac{\pi}{3} ), ( x = \frac{3}{2} ), ( y = 1 ) can be implied</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( m = -\frac{2}{3} \sqrt{3} \tan \left( \frac{2\pi}{3} \right) ) ( = 2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( y - 1 = 2 \left( x - \frac{3}{2} \right) )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>( y = 2x - 2 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>( 2 )</td>
<td>(4)</td>
</tr>
<tr>
<td>(9 marks)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:

(a)

**B1:** The correct \( \frac{dx}{dt} \)

**M1:** \( \frac{dy}{dt} = \pm k \cos t \sin t \) or \( \pm k \sin 2t \), where \( k \) is a non-zero constant. Allow \( k = 1 \)

**A1:** \( \frac{dy}{dt} = -8 \cos t \sin t \) or \( -4 \sin 2t \) or equivalent. In this question, it is possible to get a correct answer after incorrect working, e.g. \( 2 \cos 2t - 2 \rightarrow -4 \sin 2t \). This should lose this mark and the next A but ignore in part (b).

**M1:** Their \( \frac{dy}{dt} \) divided by their \( \frac{dx}{dt} \), or their \( \frac{dy}{dt} \) multiplied by their \( \frac{dx}{dt} \). The answer must be a function of \( t \) only.
A1: The correct answer in the form specified. They don’t have to explicitly state \( k = -\frac{2}{3} \) but there must be evidence that the constant is \( -\frac{2}{3} \). Accept equivalent fractions.

(b)
B1: That when \( t = \frac{\pi}{3} \), \( x = \frac{3}{2} \) and \( y = 1 \). Exact numerical values are required but the values can be implied, for example by a correct final answer, and can occur anywhere in the question.

M1: Substituting \( t = \frac{\pi}{3} \) into their \( \frac{dy}{dx} \). Trigonometric terms, e.g. \( \tan \frac{2\pi}{3} \) need not be evaluated.

dM1: Dependent on the previous M. Finding an equation of a tangent with their point and their numerical value of the gradient of the tangent, not the normal. Expressions like \( \tan \frac{2\pi}{3} \) must be evaluated. The equation must be linear. Using \( y - y' = m(x - x') \). They should get \( x' \) and \( y' \) the right way round. Alternatively writing \( y = (\text{their } m)x + c \) and using their point, the right way round, to find \( c \).

A1: cao. The correct answer in the form specified.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(a)</td>
<td>$y = 4x - x e^{\frac{1}{2}x}, x \geq 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\left{ y = 0 \Rightarrow 4x - x e^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow \right}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$e^{\frac{1}{2}x} = 4 \Rightarrow x_d = 4 \ln 2$</td>
<td>Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = \ldots$ in terms of $\pm \lambda \ln \mu$ where $\mu &gt; 0$ M1</td>
</tr>
<tr>
<td></td>
<td>$4 \ln 2$ cao (Ignore $x = 0$) A1</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>$\left{ \int x e^{\frac{1}{2}x} , dx \right} = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x} {dx}$</td>
<td>$\alpha xe^{\frac{1}{2}x} - \beta \int e^{\frac{1}{2}x} {dx}, \alpha &gt; 0, \beta &gt; 0$ M1</td>
</tr>
<tr>
<td></td>
<td>$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x} {dx}, \text{with or without } dx$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} {+ c}$</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>$\left{ \int 4x dx \right} = 2x^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\left{ \int_0^{4 \ln 2} (4x - x e^{\frac{1}{2}x}) , dx \right} = \left[ 2x^2 - \left( 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} \right) \right]_0^{4 \ln 2 \text{ or } \ln 16 \text{ or their limits}}$</td>
<td>$4 \ln 2$ or $2 \ln 2$ or $\ln 16$ or their limits</td>
</tr>
<tr>
<td></td>
<td>$= \left( 2(4 \ln 2)^2 - 2(4 \ln 2)e^{\frac{1}{2}(4 \ln 2)} + 4e^{\frac{1}{2}(4 \ln 2)} \right) - \left( 2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)} \right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= (32 \ln 2)^2 - 32(\ln 2) + 16 - (4)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 32(\ln 2)^2 - 32(\ln 2) + 12$</td>
<td></td>
</tr>
</tbody>
</table>

(8 marks)

Notes:

(a) M1: Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = \ldots$ in terms of $\pm \lambda \ln \mu$ where $\mu > 0$
A1: $4 \ln 2$ cao stated in part (a) only (Ignore $x = 0$)

(b) M1: Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x} - \beta \int e^{\frac{1}{2}x} \{dx\}, \text{where } \alpha > 0, \beta > 0$.
   (must be in this form) with or without $dx$
Question 5 notes continued

A1: \[2x e^{x^2} - \int 2e^{x^2} \{dx\} \] or equivalent, with or without \(dx\). Can be un-simplified.

A1: \[2x e^{x^2} - 4e^{x^2} \] or equivalent with or without + \(c\). Can be un-simplified.

(c)

B1: \[4x \rightarrow 2x^2 \text{ or } \frac{4x^2}{2} \text{ oe} \]

M1: Complete method of applying limits of their \(x_0\) and 0 to all terms of an expression of the form \(\pm Ax^2 \pm Bx e^{x^2} \pm Ce^{x^2}\). (Where \(A \nless 0, B \nless 0\) and \(C \nless 0\)) and subtracting the correct way round.

A1: A correct three term exact quadratic expression in \(\ln 2\). For example allow for A1

- \[32(\ln 2)^2 - 32(\ln 2) + 12\]
- \[8(2\ln 2)^2 - 8(4\ln 2) + 12\]
- \[2(4\ln 2)^2 - 32(\ln 2) + 12\]
- \[2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 12\]

Note that the constant term of 12 needs to be combined from \(4e^{2(4\ln 2)} - 4e^{2(0)}\) oe.

Also allow \[32\ln 2(\ln 2 - 1) + 12\] or \[32\ln 2\left(\ln 2 - 1 + \frac{12}{32\ln 2}\right)\] for A1.

Allow \[32(\ln^2 2) - 32(\ln 2) + 12\] for the final A1.
6

Assumption: there exists positive real numbers $a$, $b$ such that

$$a + b < 2\sqrt{ab}$$

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a + b - 2\sqrt{ab} &lt; 0$</td>
<td>$(a + b)^2 = (2\sqrt{ab})^2$</td>
</tr>
<tr>
<td>$(\sqrt{a} - \sqrt{b})^2 &lt; 0$</td>
<td>$a^2 + 2ab + b^2 &lt; 4ab$</td>
</tr>
<tr>
<td>$a^2 - 2ab + b^2 &lt; 0$</td>
<td>$(a - b)^2 &lt; 0$</td>
</tr>
</tbody>
</table>

This is a contradiction, therefore

If $a$, $b$ are positive real numbers, then $a + b \geq 2\sqrt{ab}$

(4 marks)

Notes:

B1: As this is proof by contradiction, the candidate is required to start their proof by assuming that the contrary. That is "if $a$, $b$ are positive real numbers, then $a + b \geq 2\sqrt{ab}$" is true.

Accept, as a minimum, there exists $a$ and $b$ such that $a + b < 2\sqrt{ab}$

M1: For starting with $a + b < 2\sqrt{ab}$ and proceeding to either $(\sqrt{a} - \sqrt{b})^2 < 0$ or $(a - b)^2 < 0$

A1: All algebra is required to be correct. Do not accept, for instance, $(a + b)^2 = 2\sqrt{ab}^2$ even when followed by correct lines.

A1: A fully correct proof by contradiction. It must include a statement that $(a - b)^2 < 0$ is a contradiction so if $a$, $b$ are positive real numbers, then $a + b \geq 2\sqrt{ab}$
7(a) \[ x = 4 \cos \left(t + \frac{\pi}{6}\right), \quad y = 2 \sin t \]

\[ x = 4 \left( \cos t \cos \left(\frac{\pi}{6}\right) - \sin t \sin \left(\frac{\pi}{6}\right) \right) \]

So, \( \{x + y\} = 4 \left( \cos t \cos \left(\frac{\pi}{6}\right) - \sin t \sin \left(\frac{\pi}{6}\right) \right) + 2 \sin t \) Adds their expanded \( x \) (which is in terms of \( t \)) to \( 2 \sin t \)

\[ = 4 \left( \frac{\sqrt{3}}{2} \cos t - \frac{1}{2} \sin t \right) + 2 \sin t \]

\[ = 2 \sqrt{3} \cos t \quad \text{cso} \]

M1

\( \{x + y\} = 4 \cos \left(\frac{\pi}{6}\right) - 4 \sin t \sin \left(\frac{\pi}{6}\right) + 2 \sin t \)

\[ = 4 \left( \frac{\sqrt{3}}{2} \cos t - \frac{1}{2} \sin t \right) + 2 \sin t \]

\( \Rightarrow \ (x+y)^2 + 3y^2 = 12 \)  

M1

(3)

(b) \( \left( \frac{x + y}{2\sqrt{3}} \right)^2 + \left( \frac{y}{2} \right)^2 = 1 \) Applies \( \cos^2 t + \sin^2 t = 1 \) to achieve an equation containing only \( x \)'s and \( y \)'s.

\[ \Rightarrow \ (x + y)^2 + 3y^2 = 12 \]

M1

(2)

Alternative

\( (x + y)^2 = 12 \cos^2 t = 12(1 - \sin^2 t) = 12 - 12 \sin^2 t \)

\( (x + y)^2 = 12 - 3y^2 \) Applies \( \cos^2 t + \sin^2 t = 1 \) to achieve an equation containing only \( x \)'s and \( y \)'s.

\[ \Rightarrow \ (x + y)^2 + 3y^2 = 12 \]

M1

(2)

Notes:

(a)

M1: \( \cos \left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos \left(\frac{\pi}{6}\right) \pm \sin t \sin \left(\frac{\pi}{6}\right) \) or \( \cos \left(t + \frac{\pi}{6}\right) \rightarrow \left( \frac{\sqrt{3}}{2} \right) \cos t \pm \left( \frac{1}{2} \right) \sin t \)

dM1: Adds their expanded \( x \) (which is in terms of \( t \)) to \( 2 \sin t \).

A1*: Evidence of \( \cos \left(\frac{\pi}{6}\right) \) and \( \sin \left(\frac{\pi}{6}\right) \) evaluated and the proof is correct with no errors.

(b)

M1: Applies \( \cos^2 t + \sin^2 t = 1 \) to achieve an equation containing only \( x \)'s and \( y \)'s.

A1: leading \( (x + y)^2 + 3y^2 = 12 \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>8(a)</td>
<td>( \frac{d\theta}{dt} = \lambda(120 - \theta), \ 0 \leq \theta \leq 100 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \int \frac{1}{120 - \theta} \ d\theta = \int \lambda \ dt )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( -\ln(120 - \theta); = \lambda t + c )</td>
<td>For integrating lhs M1 A1 For integrating rhs M1 A1</td>
</tr>
<tr>
<td></td>
<td>{t = 0, \ \theta = 20 \Rightarrow } -\ln(100) = \lambda(0) + c )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>\Rightarrow -\ln(120 - \theta) = \lambda t - \ln 100 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>\Rightarrow -\lambda t = \ln \left( \frac{120 - \theta}{100} \right) )</td>
<td>dddM1</td>
</tr>
<tr>
<td></td>
<td>( e^{-\lambda t} = \frac{120 - \theta}{100} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 100 e^{-\lambda t} = 120 - \theta ) leading to ( \theta = 120 - 100e^{-\lambda t} )</td>
<td>A1*</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{ \lambda = 0.01, \ \theta = 100 \Rightarrow } 100 = 120 - 100 e^{-0.01t} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>\Rightarrow 100 e^{-0.01t} = 120 - 100 \Rightarrow )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( -0.01t = \ln \left( \frac{120 - 100}{100} \right) ) Uses correct order of operations by moving from 100 = 120 - 100e^{-0.01t} to give t = ... and ( t = A \ln B ), where ( B &gt; 0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t = \frac{1}{-0.01} \ln \left( \frac{120 - 100}{100} \right) ) dM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \left{ t = \frac{1}{-0.01} \ln \left( \frac{1}{5} \right) = 100 \ln 5 \right} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( t = 160.94379… \ 161 ) (s) (nearest second) awrt 161</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>(11 marks)</td>
</tr>
</tbody>
</table>

Notes:

(a) **B1M1A1M1A1**: Mark as in the scheme.

**M1**: Substitutes \( t = 0 \) AND \( \theta = 20 \) in an integrated equation leading to \( \pm \lambda t = \ln(f(\theta)) \)

**dddM1**: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.

**A1***: Correct answer with no errors. This is a given answer

(b) **M1**: Substitutes \( \lambda = 0.01, \ \theta = 100 \) into given equation

**M1**: See scheme

**A1**: Awrt 161 seconds.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9 (a)</strong></td>
<td>$A(3, 5, 0)$</td>
<td>B1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>[ l_2 : \mathbf{r} = \begin{pmatrix} 1 \ 5 \ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \ 4 \ 3 \end{pmatrix} ] [ \mathbf{a} + \lambda \mathbf{d} \text{ or } \mathbf{a} + \mu \mathbf{d}, \mathbf{a} \neq 0, \mathbf{d} \neq 0 ] with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$</td>
<td>M1</td>
</tr>
</tbody>
</table>

Correct vector equation using $\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$ or $l = \mathbf{a} + \lambda \mathbf{d}$ | A1 |
| **(c)** | $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ | M1 |

$\overrightarrow{AP} = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$ Full method for finding $\overrightarrow{AP}$ | A1 |

$2\sqrt{2}$ | (2) |
| **(d)** | So $\overrightarrow{AP} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$ Realisation that the dot product is required between $\overrightarrow{AP}$ or $\overrightarrow{PA}$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | M1 |

$\cos \theta = \frac{\mathbf{A}\mathbf{P} \cdot \mathbf{d}_2}{|\mathbf{A}\mathbf{P}| |\mathbf{d}_2|} = \pm \left[ \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} \right] \left[ \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix} \right] \quad \frac{\sqrt{(-2)^2 + (0)^2 + (2)^2 \cdot \sqrt{(-5)^2 + (4)^2 + (3)^2}}}{dM1}$

$\cos \theta = \pm \frac{(10 + 0 + 6)}{\sqrt{8 \cdot \sqrt{50}}} = \frac{4}{5}$ | A1 cso |

$\cos \theta = \frac{\pm (10 + 0 + 6)}{\sqrt{8 \cdot \sqrt{50}}} = \frac{4}{5}$ | (3) |
| **(e)** | $\{\text{Area } APE\} = \frac{1}{2} (\text{their } 2\sqrt{2})^2 \sin \theta$ | M1 |

$= 2.4$ | A1 |

$= 2.4$ | (2) |
### Question 9(f)

PE = (-5\lambda)i + (4\lambda)j + (3\lambda)k and PE = their 2\sqrt{2} from part (c)

\[\{PE^2\} = (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (their \ 2\ \sqrt{2})^2\]

This mark can be implied. **M1**

\[\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \frac{4}{25} \Rightarrow \lambda = \pm \frac{2}{5}\]

Either \(\lambda = \frac{2}{5}\) or \(\lambda = -\frac{2}{5}\) **A1**

\[l_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \pm \frac{2}{5} \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}\]

dependent on the previous M mark
Substitutes at least one of their values of \(\lambda\) into \(l_2\). **dM1**

\[\{OE\} = \begin{pmatrix} 3 \\ 17 \\ 5 \\ 4 \\ 5 \end{pmatrix} \text{ or } \begin{pmatrix} 3 \\ 3.4 \\ 0.8 \end{pmatrix}, \{OE\} = \begin{pmatrix} -1 \\ 33 \\ 5 \\ 16 \\ 5 \end{pmatrix} \text{ or } \begin{pmatrix} -1 \\ 6.6 \\ 3.2 \end{pmatrix}\]

At least one set of coordinates are correct. **A1**

Both sets of coordinates are correct. **A1**

(5)

### Notes:

(a)

**B1:** Allow \(A(3, 5, 0)\) or \(3i + 5j\) or \(3i + 5j + 0k\) or \(\begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}\) or benefit of the doubt \(\begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}\)

(b)

**A1:** Correct vector equation using \(\mathbf{r} = \) or \(l = \) or \(l_2 = \) or Line 2 =

i.e. Writing \(\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}\) or \(\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \mathbf{d}\), where \(\mathbf{d}\) is a multiple of \(\begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}\).

**Note:** Allow the use of parameters \(\mu\) or \(t\) instead of \(\lambda\).

(c)

**M1:** Finds the difference between \(\overrightarrow{OP}\) and their \(\overrightarrow{OA}\) and applies Pythagoras to the result to find \(AP\)

Note: Allow M1A1 for \(\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}\) leading to \(AP = \sqrt{(2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}\).
Question 9 notes continued

(d)  
**M1:** Realisation that the dot product is required between $\overrightarrow{AP}$ or $\overrightarrow{PA}$

**dM1:** Full method to find $\cos \theta$ (dependent upon the previous M),

**A1:** $\cos \theta = \frac{4}{5}$ or exact equivalent

(e)  
**M1 A1:** For $\frac{1}{2}(2\sqrt{2})^2 \sin(36.869\ldots)$ or $\frac{1}{2}(2\sqrt{2})^2 \sin(180^\circ - 36.869\ldots)\ldots = \text{awrt 2.40}

Candidates must use their $\theta$ from part (d) or apply a correct method of finding their $\sin \theta = \frac{3}{5}$ from their $\cos \theta = \frac{4}{5}$

(f)  
**M1:** Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working.

For $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$

1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent.

1st M1 for $\lambda = \frac{\text{their } \overrightarrow{AP} = \frac{2\sqrt{2}}{5} \overrightarrow{\lambda}}{\overrightarrow{\lambda}}$ and 1st A1 for $\lambda = \frac{2\sqrt{2}}{5\sqrt{2}}$

So $\mathbf{d_1} = \left[ \frac{1}{5\sqrt{2}} \begin{array}{c} -5 \\ 4 \\ 3 \end{array} \right] \Rightarrow \text{"vector"} = \frac{2\sqrt{2}}{5\sqrt{2}} \left[ \begin{array}{c} -5 \\ 4 \\ 3 \end{array} \right]$ is M1A1

**dM1:** In part (f) can be implied for at least 2 (out of 6) correct $x$, $y$, $z$ ordinates from their values of $\lambda$. 

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*(continued on the next page)*
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided — there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets — use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Answer ALL questions. Write your answers in the spaces provided.

1. Use the standard results for \( \sum_{r=1}^{n} r \) and for \( \sum_{r=1}^{n} r^3 \) to show that, for all positive integers \( n \),

\[
\sum_{r=1}^{n} r(r^2 - 3) = \frac{n}{4}(n + a)(n + b)(n + c)
\]

where \( a, b \) and \( c \) are integers to be found.

(Total for Question 1 is 4 marks)
Question 1 continued

Leave blank

(Total for Question 1 is 4 marks)
2. A parabola $P$ has cartesian equation $y^2 = 28x$. The point $S$ is the focus of the parabola $P$.

(a) Write down the coordinates of the point $S$. 

(1)

Points $A$ and $B$ lie on the parabola $P$. The line $AB$ is parallel to the directrix of $P$ and cuts the $x$-axis at the midpoint of $OS$, where $O$ is the origin.

(b) Find the exact area of triangle $ABS$. 

(4)
Question 2 continued

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(Total for Question 2 is 5 marks)
3.

\[ f(x) = x^2 + \frac{3}{x} - 1, \quad x < 0 \]

The only real root, \( \alpha \), of the equation \( f(x) = 0 \) lies in the interval \([-2, -1]\).

(a) Taking \(-1.5\) as a first approximation to \( \alpha \), apply the Newton-Raphson procedure once to \( f(x) \) to find a second approximation to \( \alpha \), giving your answer to 2 decimal places. \((5)\)

(b) Show that your answer to part (a) gives \( \alpha \) correct to 2 decimal places. \((2)\)
Question 3 continued

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(5)
(2)

α

2 + 3

α

f(α) =

x

0 <

DO NOT WRITE IN THIS AREA

(Total for Question 3 is 7 marks)
4. Given that

\[
A = \begin{pmatrix} k & 3 \\ -1 & k + 2 \end{pmatrix}, \text{ where } k \text{ is a constant}
\]

(a) show that \(\det(A) > 0\) for all real values of \(k\),

(b) find \(A^{-1}\) in terms of \(k\).
Question 4 continued

(Total for Question 4 is 5 marks)
5.

\[2z + z^* = \frac{3 + 4i}{7 + i}\]

Find \(z\), giving your answer in the form \(a + bi\), where \(a\) and \(b\) are real constants. You must show all your working.

(5)
Question 5 continued

(Total for Question 5 is 5 marks)
6. The rectangular hyperbola $H$ has equation $xy = 25$

(a) Verify that, for $t \neq 0$, the point $P\left(5t, \frac{5}{t}\right)$ is a general point on $H$.\hfill (1)

The point $A$ on $H$ has parameter $t = \frac{1}{2}$

(b) Show that the normal to $H$ at the point $A$ has equation

$$8y - 2x - 75 = 0$$\hfill (5)

This normal at $A$ meets $H$ again at the point $B$.

(c) Find the coordinates of $B$.\hfill (4)
Question 6 continued

(Total for Question 6 is 10 marks)
7.

\[
\mathbf{P} = \begin{pmatrix}
\frac{5}{13} & -\frac{12}{13} \\
\frac{12}{13} & \frac{5}{13}
\end{pmatrix}
\]

(a) Describe fully the single geometrical transformation \( U \) represented by the matrix \( \mathbf{P} \). \( \text{(3 marks)} \)

The transformation \( V \), represented by the \( 2 \times 2 \) matrix \( \mathbf{Q} \), is a reflection in the line with equation \( y = x \).

(b) Write down the matrix \( \mathbf{Q} \). \( \text{(1 mark)} \)

Given that the transformation \( V \) followed by the transformation \( U \) is the transformation \( T \), which is represented by the matrix \( \mathbf{R} \),

(c) find the matrix \( \mathbf{R} \). \( \text{(2 marks)} \)

(d) Show that there is a value of \( k \) for which the transformation \( T \) maps each point on the straight line \( y = kx \) onto itself, and state the value of \( k \). \( \text{(4 marks)} \)
Question 7 continued
Question 7 continued
Question 7 continued

(Total for Question 7 is 10 marks)
8.

\[ f(z) = z^4 + 6z^3 + 76z^2 + az + b \]

where \( a \) and \( b \) are real constants.

Given that \(-3 + 8i\) is a complex root of the equation \( f(z) = 0 \)

(a) write down another complex root of this equation. \((1)\)

(b) Hence, or otherwise, find the other roots of the equation \( f(z) = 0 \) \((6)\)

(c) Show on a single Argand diagram all four roots of the equation \( f(z) = 0 \) \((2)\)
Question 8 continued

(a) Write down another complex root of this equation.

(b) Hence, or otherwise, find the other roots of the equation \( f(z) = 0 \)

given that \(-3 + 8i\) is a complex root of the equation \( f(z) = 0 \) and \( a \) and \( b \) are real constants.
Question 8 continued
Question 8 continued

(Total for Question 8 is 9 marks)
9. The quadratic equation

\[2x^2 + 4x - 3 = 0\]

has roots \(\alpha\) and \(\beta\).

Without solving the quadratic equation,

(a) find the exact value of

(i) \(\alpha^2 + \beta^2\)

(ii) \(\alpha^3 + \beta^3\)

(b) Find a quadratic equation which has roots \((\alpha^2 + \beta)\) and \((\beta^2 + \alpha)\), giving your answer in the form \(ax^2 + bx + c = 0\), where \(a\), \(b\) and \(c\) are integers.
Question 9 continued

(ii)

(b) Find a quadratic equation which has roots $(\alpha, \beta)$, giving your answer in the form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are integers.

The quadratic equation $\alpha^2 \beta + \alpha^2 \beta^2 + \beta^2 = 0$, where $\alpha$ and $\beta$ are integers.

Turn over

Leave blank
Question 9 continued
10. (i) A sequence of positive numbers is defined by

\[ u_1 = 5 \]
\[ u_{n+1} = 3u_n + 2, \quad n \geq 1 \]

Prove by induction that, for \( n \in \mathbb{Z}^+ \),

\[ u_n = 2 \times (3)^n - 1 \]

(ii) Prove by induction that, for \( n \in \mathbb{Z}^+ \),

\[ \sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3 + 2n)}{3^n} \]
Question 10 continued
Question 10 continued

(Total for Question 10 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS
### Further Pure Mathematics FP1 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1        | \[\sum_{r=1}^{n} r(r^2 - 3) = \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r\]  
\[= \frac{1}{4} n^2(n+1)^2 - 3\left(\frac{1}{2} n(n+1)\right)\]  
Attempts to expand \(r(r^2 - 3)\) and attempts to substitute at least one correct standard formula into their resulting expression.  
M1  
Correct expression (or equivalent)  
A1  
dependent on the previous M mark  
Attempt to factorise at least \(n(n+1)\) having attempted to substitute both the standard formulae  
dM1  
\[= \frac{1}{4} n(n+1)[n(n+1) - 6]\]  
{this step does not have to be written}  
A1 cso  
\[= \frac{1}{4} n(n+1)[n^2 + n - 6]\]  
Correct completion with no errors  
(4)  
\[= \frac{1}{4} n(n+1)(n+3)(n-2)\]  
(4 marks) |

**Notes:**

Applying eg. \(n=1, n=2, n=3\) to the printed equation without applying the standard formulae to give \(a=1, b=3, c=-2\) or another combination of these numbers is M0A0M0A0.

**Alternative Method:**

Obtains  
\[\sum_{r=1}^{n} r(r^2 - 3) = \frac{1}{4} n(n+1)[n(n+1) - 6] = \frac{1}{4} n(n+a)(n+b)(n+c)\]

So \(a=1, n=1 \Rightarrow -2 = \frac{1}{4}(1)(2)(1+b)(1+c)\) and \(n=2 \Rightarrow 0 = \frac{1}{4}(2)(3)(2+b)(2+c)\)

leading to either \(b=-2, c=3\) or \(b=3, c=-2\)

**dM1: dependent on the previous M mark.**

- Substitutes in values of \(n\) and solves to find \(b=...\) and \(c=...\)

**A1:**

- Finds \(a=1, b=3, c=-2\) or another combination of these numbers.

Using only a method of “proof by induction” scores 0 marks unless there is use of the standard formulae when the first M1 may be scored.

Allow final dM1 A1 for \(\frac{1}{4} n^4 + \frac{1}{2} n^3 - \frac{5}{4} n^2 - \frac{3}{2} n\) or \(\frac{1}{4} n^3 + 2n^2 - 5n - 6\)

or \(\frac{1}{4} (n^4 + 2n^3 - 5n^2 - 6n) \rightarrow \frac{1}{4} n(n+1)(n+3)(n-2)\), from no incorrect working.

Give final A0 for eg. \(\frac{1}{4} n(n+1)[n^2 + n - 6] \rightarrow \frac{1}{4} n(n+1)(x+3)(x-2)\) unless recovered.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(a)</td>
<td>( P : y^2 = 28x ) or ( P(7t^2, 14t) )</td>
<td>B1</td>
</tr>
<tr>
<td>( y^2 = 4ax \Rightarrow a = 7 ) ( \Rightarrow S(7,0) )</td>
<td>Accept ((7,0)) or ( x = 7, y = 0 ) or ( 7 ) marked on the x-axis in a sketch</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>( {A \text{ and } B \text{ have } x \text{ coordinate}} \frac{7}{2} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Divides their ( x ) coordinate from (a) by 2 and substitutes this into the parabola equation and takes the square root to find ( y = \ldots ) or applies</td>
<td></td>
</tr>
<tr>
<td>( y = \sqrt{(2(7) - 3.5)^2 - (3.5)^2} = \sqrt{(10.5)^2 - (3.5)^2} ) or ( 7t^2 = 3.5 \Rightarrow t = \sqrt{0.5} \Rightarrow y = 2(7)\sqrt{0.5} )</td>
<td>( y = (\pm)7\sqrt{2} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>At least one correct exact value of ( y ). Can be unsimplified or simplified.</td>
<td>A1</td>
</tr>
<tr>
<td>( A, B ) have coordinates ( \left(\frac{7}{2}, 7\sqrt{2}\right) ) and ( \left(\frac{7}{2} - 7\sqrt{2}\right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area triangle ( ABS = )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} \left( 2(7\sqrt{2}) \right) \left( \frac{7}{2} \right) ) or ( \frac{1}{2} \left( \begin{array}{c} 7 \ 3.5 \ 7 \ 0 \ 7\sqrt{2} \ -7\sqrt{2} \ 0 \end{array} \right) )</td>
<td>dependent on the previous M mark</td>
<td></td>
</tr>
<tr>
<td>( \frac{49}{2}\sqrt{2} )</td>
<td>Correct exact answer.</td>
<td>A1</td>
</tr>
<tr>
<td>(4)</td>
<td>(5 marks)</td>
<td></td>
</tr>
</tbody>
</table>
Question 2 continued

Notes:

(a)
You can give B1 for part (a) for correct relevant work seen in either part (a) or part (b).

(b)

1st M1: Allow a slip when candidates find the x coordinate of their midpoint as long as
0 < their midpoint < their a

Give 1st M0 if a candidate finds and uses \( y = 98 \)

1st A1: Allow any exact value of either \( 7\sqrt{2}, -7\sqrt{2}, \sqrt{98}, -\sqrt{98}, 14\sqrt{0.5}, \) awrt 9.9 or awrt \(-9.9\)

2nd dM1: Either \( \frac{1}{2} \left( 2 \times \text{their } 7\sqrt{2} \right) \left( \text{their } x_{\text{midpoint}} \right) \) or \( \frac{1}{2} \left( 2 \times \text{their } 7\sqrt{2} \right) \left( \text{their } 7 - x_{\text{midpoint}} \right) \)

Condone area triangle \( \triangle ABS = \left( 7\sqrt{2} \right) \left( \frac{7}{2} \right), \) i.e. \( \left( \text{their } 7\sqrt{2} \right) \left( \frac{\text{their } 7}{2} \right) \)

2nd A1: Allow exact answers such as \( \frac{49}{2} \sqrt{2}, \frac{49}{2}, 24.5\sqrt{2}, \frac{\sqrt{4802}}{2}, \frac{\sqrt{4802}}{4}, 3.5\sqrt{2}, 49 \frac{1}{2}, \) or \( \frac{7}{2} \sqrt{98} \) but do not allow \( \frac{1}{2} \left( 3.5 \right) \left( 2\sqrt{98} \right) \) seen by itself.

Give final A0 for finding 34.64823228… without reference to a correct exact value.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(a)</strong></td>
<td>(f(x) = x^2 + \frac{3}{x} - 1, \quad x &lt; 0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(f'(x) = 2x - 3x^{-2})</td>
<td>At one of either (x^2 \to \pm A x) or (\frac{3}{x} \to \pm B x^{-2}) where (A) and (B) are non-zero constants. M1</td>
</tr>
<tr>
<td></td>
<td>(f(-1.5) = -0.75), (f'(-1.5) = -\frac{13}{3})</td>
<td>Either (f(-1.5) = -0.75) or (f'(-1.5) = -\frac{13}{3}) or awrt (-4.33) or a <strong>correct numerical expression</strong> for either (f(-1.5)) or (f'(-1.5)) <strong>Can be implied by later working</strong> B1</td>
</tr>
<tr>
<td></td>
<td>(\alpha = -1.5 - \frac{f(-1.5)}{f'(-1.5)}) (\Rightarrow \alpha = -1.5 - \frac{-0.75}{-4.333333...})</td>
<td>dependent on the previous M mark Valid attempt at Newton-Raphson using their values of (f(-1.5)) and (f'(-1.5)) dM1</td>
</tr>
<tr>
<td></td>
<td>(\alpha = -1.67307692...) or (-\frac{87}{52}) (\Rightarrow \alpha = -1.67)</td>
<td>dependent on all 4 previous marks (-1.67) on their first iteration (Ignore any subsequent iterations) A1 cso cao</td>
</tr>
</tbody>
</table>

**Correct differentiation followed by a correct answer scores full marks in (a)**

**Correct answer with no working scores no marks in (a)**

(b) **Way 1**

| \(f(-1.675) = 0.01458022...\) | Chooses a suitable interval for \(x\), which is within \(\pm 0.005\) of their answer to (a) and at least one attempt to evaluate \(f(x)\). M1 |
| \(f(-1.665) = -0.0295768...\) | Both values correct awrt (or truncated) 1 sf, sign change and conclusion. A1 cso |

| \(\alpha = -1.67\) (2 dp) | |

(2)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(b) continued</strong></td>
<td><strong>Way 2</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Alt 1: Applying Newton-Raphson again</strong></td>
<td>Eg. Using</td>
<td></td>
</tr>
<tr>
<td>$\alpha = -1.67, -1.673$ or $\frac{87}{52}$</td>
<td>Evidence of applying Newton-Raphson for a second time on their answer to part (a)</td>
<td>M1</td>
</tr>
<tr>
<td>$\alpha = -1.67 - \frac{-0.007507185629\ldots}{-4.415692926\ldots} {= -1.671700115\ldots}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = -1.673 - \frac{0.005743106396\ldots}{-4.41783855\ldots} {= -1.671700019\ldots}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = -\frac{87}{52} - \frac{0.006082942257\ldots}{-4.417893838\ldots} {= -1.67170036\ldots}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>So $\alpha = -1.67$ (2 dp)</td>
<td>$\alpha = -1.67$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
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</tr>
</tbody>
</table>

**Notes:**

(a) Incorrect differentiation followed by their estimate of $\alpha$ with no evidence of applying the NR formula is final dM0A0.

**B1:** B1 can be given for a correct numerical expression for either $f'(-1.5)$ or $f(-1.5)$

Eg. either $(-1.5)^2 + \frac{3}{(-1.5)} - 1$ or $2(-1.5) - \frac{3}{(-1.5)^2}$ are fine for B1.

Final -This mark can be implied by applying at least one correct value of either $f(-1.5)$ or $f'(-1.5)$

**dM1:** in $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$. So just $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$ with an incorrect answer and no other evidence scores final dM0A0.

Give final dM0 for applying $1.5 - \frac{f(-1.5)}{f'(-1.5)}$ without first quoting the correct N-R formula.

(b) **A1:** **Way 1:** correct solution only

Candidate needs to state both of their values for $f(x)$ to awrt (or truncated) 1 sf along with a reason and conclusion. Reference to change of sign or eg. $f(-1.675) \times f(-1.665) < 0$

or a diagram or $< 0$ and $> 0$ or one positive, one negative are sufficient reasons. There must be a (minimal, not incorrect) conclusion, eg. $\alpha = -1.67$, root (or $\alpha$ or part (a)) is correct, QED and a square are all acceptable. Ignore the presence or absence of any reference to continuity.

A minimal acceptable reason and conclusion is “change of sign, hence root”.

No explicit reference to 2 decimal places is required.

Stating “root is in between – 1.675 and – 1.665” without some reference to is not sufficient for A1

Accept 0.015 as a correct evaluation of $f(-1.675)$
(b) **Way 2:** correct solution only

Their conclusion in Way 2 needs to convey that they understand that \( \alpha = -1.67 \) to 2 decimal places. Eg. “therefore my answer to part (a) [which must be \(-1.67\)] is correct” is fine for A1.

\[-1.67 - \frac{f(-1.67)}{f'(1.67)} = -1.67 \text{ (2 dp)} \]

is sufficient for M1A1 in part (b).

The root of \( f(x) = 0 \) is \(-1.67169988...\), so candidates can also choose \( x_1 \) which is less than \(-1.67169988... \) and choose \( x_2 \) which is greater than \(-1.67169988... \) with both \( x_1 \) and \( x_2 \) lying in the interval \([-1.675, -1.665]\) and evaluate \( f(x_1) \) and \( f(x_2) \).

**Helpful Table**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.675</td>
<td>0.014580224</td>
</tr>
<tr>
<td>-1.674</td>
<td>0.010161305</td>
</tr>
<tr>
<td>-1.673</td>
<td>0.005743106</td>
</tr>
<tr>
<td>-1.672</td>
<td>0.001325627</td>
</tr>
<tr>
<td>-1.671</td>
<td>-0.003091136</td>
</tr>
<tr>
<td>-1.670</td>
<td>-0.007507186</td>
</tr>
<tr>
<td>-1.669</td>
<td>-0.011922523</td>
</tr>
<tr>
<td>-1.668</td>
<td>-0.016337151</td>
</tr>
<tr>
<td>-1.667</td>
<td>-0.020751072</td>
</tr>
<tr>
<td>-1.666</td>
<td>-0.025164288</td>
</tr>
<tr>
<td>-1.665</td>
<td>-0.029576802</td>
</tr>
</tbody>
</table>
**4(a)**

A = \[
\begin{pmatrix}
k & 3 \\
-1 & k + 2
\end{pmatrix}
\]
where \( k \) is a constant and let \( g(k) = k^2 + 2k + 3 \)

\{det(A)\} = k(k + 2) + 3 or \( k^2 + 2k + 3 \)  \( \text{Correct det}(A), \text{un-simplified or simplified} \)  \( \text{B1} \)

**Way 1**

\[(k + 1)^2 - 1 + 3 \quad \text{Attempts to complete the square} \]

\[(k + 1)^2 + 2 > 0 \quad (k + 1)^2 + 2 \text{ and } > 0 \quad \text{A1 cso} \]

(b)

\[A^{-1} = \frac{1}{k^2 + 2k + 3} \begin{pmatrix} k + 2 & -3 \\ 1 & k \end{pmatrix} \]

\[1 \quad \text{their det}(A) \begin{pmatrix} k + 2 & -3 \\ 1 & k \end{pmatrix} \]

\( \text{Correct answer in terms of } k \)  \( \text{A1} \)

**Way 2**

\{det(A)\} = k(k + 2) + 3 or \( k^2 + 2k + 3 \)  \( \text{Correct det}(A), \text{un-simplified or simplified} \)  \( \text{B1} \)

\{b^2 - 4ac\} = 2^2 - 4(1)(3) \quad \text{Applies “} b^2 - 4ac \text{” to their det}(A) \quad \text{M1}

**All of**

- \( b^2 - 4ac = -8 < 0 \)
- some reference to \( k^2 + 2k + 3 \) being above the \( x \)-axis
- so \( \text{det}(A) > 0 \)

\( \text{Complete solution} \quad \text{A1 cso} \)

**Way 3**

\{g(k) = det(A)\} = k(k + 2) + 3 or \( k^2 + 2k + 3 \)  \( \text{Correct det}(A), \text{un-simplified or simplified} \)  \( \text{B1} \)

\( g'(k) = 2k + 2 = 0 \Rightarrow k = -1 \)

\( g_{\text{min}} = (-1)^2 + 2(-1) + 3 \)

\( g_{\text{min}} = 2, \text{ so det}(A) > 0 \)

\( g_{\text{min}} = 2 \text{ and states det}(A) > 0 \quad \text{A1 cso} \)

\( \text{(3 marks)} \)
Question 4 continued

Notes:

(a) B1: Also allow $k(k+2)=−3$

**Way 2:** Proving $b^2−4ac=−8<0$ by itself could mean that $\det(A)>0$ or $\det(A)<0$.

To gain the final A1 mark for Way 2, candidates need to show $b^2−4ac=−8<0$ and make some reference to $k^2+2k+3$ being above the $x$-axis (eg. states that coefficient of $k^2$ is positive or evaluates $\det(A)$ for any value of $k$ to give a positive result or sketches a quadratic curve that is above the $x$-axis) before then stating that $\det(A)>0$.

Attempting to solve $\det(A)=0$ by applying the quadratic formula or finding $−1±\sqrt{2}i$ is enough to score the M1 mark for Way 2. To gain A1 these candidates need to make some reference to $k^2+2k+3$ being above the $x$-axis (eg. states that coefficient of $k^2$ is positive or evaluates $\det(A)$ for any value of $k$ to give a positive result or sketches a quadratic curve that is above the $x$-axis) before then stating that $\det(A)>0$.

(b) A1: Allow either

$$\frac{1}{(k+1)^2 + 2} \begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix} \text{ or } \begin{pmatrix} k+2 & -3 \\ k^2 + 2k + 3 & 1 \\ k & k^2 + 2k + 3 \end{pmatrix} \text{ or equivalent.}$$
### Question 5

\[ 2z + z^* = \frac{3 + 4i}{7 + i} \]

#### Way 1

\[ \{2z + z^*\} = 2(a + ib) + (a - ib) \]

Left hand side = \[2(a + ib) + (a - ib)\]
Can be implied by eg. \[3a + ib\]

**Note:** This can be seen anywhere in their solution.

\[ \ldots = \frac{(3 + 4i)(7 - i)}{(7 + i)(7 - i)} \]

Multiplies numerator and denominator of the right hand side by \(7 - i\) or \(-7 + i\)

\[ \ldots = \frac{25 + 25i}{50} \]

Applies \(i^2 = -1\) to and collects like terms to give right hand side = \(\frac{25 + 25i}{50}\) or equivalent

So, \(3a + ib = \frac{1}{2} + \frac{1}{2}i\)

\[ a = \frac{1}{6}, \ b = \frac{1}{2} \text{ or } z = \frac{1}{6} + \frac{1}{2}i \]

**dependent on the previous B and M marks**

Equates either real parts or imaginary parts to give at least one of \(a = \ldots\) or \(b = \ldots\)

Either \(a = \frac{1}{6}\) and \(b = \frac{1}{2}\) or \(z = \frac{1}{6} + \frac{1}{2}i\)

**ddM1**

**ddM1**

\[ \text{(5 marks)} \]

#### Way 2

\[ \{2z + z^*\} = 2(a + ib) + (a - ib) \]

Left hand side = \[2(a + ib) + (a - ib)\]
Can be implied by eg. \[3a + ib\]

\[ (3a + ib)(7 + i) = \ldots \]

Multiplies their \((3a + ib)\) by \((7 + i)\)

\[ 21a + 3ai + 7bi - b = \ldots \]

Applies \(i^2 = -1\) to give left hand side = \(21a + 3ai + 7bi - b\)

So, \((21a - b) + (3a + 7b) = 3 + 4i\)

\[ 21a - b = 3, \ 3a + 7b = 4 \]

\[ a = \frac{1}{6}, \ b = \frac{1}{2} \text{ or } z = \frac{1}{6} + \frac{1}{2}i \]

**dependent on the previous B and M marks**

Equates both real parts and imaginary parts to give at least one of \(a = \ldots\) or \(b = \ldots\)

Either \(a = \frac{1}{6}\) and \(b = \frac{1}{2}\) or \(z = \frac{1}{6} + \frac{1}{2}i\)

**ddM1**

**ddM1**

\[ \text{(5 marks)} \]
### Question 5 continued

**Notes:**

Some candidates may let \( z = x + iy \) and \( z^* = x - iy \).  
So apply the mark scheme with \( x = a \) and \( y = b \).  
For the final A1 mark, you can accept exact equivalents for \( a, b \).
Question 6(a)

$H: xy = 25, \ P\left(5t, \frac{5}{t}\right)$ is a general point on $H$

Either $5\left(\frac{5}{t}\right) = 25$ or $y = \frac{25}{x} = \frac{25}{5t} = \frac{5}{t}$ or $x = \frac{25}{y} = \frac{25}{\frac{5}{t}} = 5t$ or states $c = 5$

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
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<tbody>
<tr>
<td></td>
<td>B1</td>
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</tbody>
</table>

(b)

$y = \frac{25}{x} = 25x^{-1} \Rightarrow \frac{dy}{dx} = -25x^{-2} = -\frac{25}{x^2}$

$\frac{dy}{dx} = \pm kx^{-2}$

where $k$ is a numerical value

$xy = 25 \Rightarrow x\frac{dy}{dx} + y = 0$

Correct use of product rule. The sum of two terms, one of which is correct.

$\frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = -\frac{5}{t^2} \left(\frac{1}{5}\right)$

Correct numerical gradient at $A$, which is found using calculus.

Can be implied by later working

$\left\{ \text{At } A, \ t = \frac{1}{2}, \ x = \frac{5}{2}, \ y = 10 \right\} \Rightarrow \frac{dy}{dx} = -4$

So, $m_N = \frac{1}{4}$

Applies $m_N = -\frac{1}{m_T}$, to find a numerical $m_N$, where $m_T$ is found from using calculus.

Can be implied by later working

$y - 10 = \frac{1}{4}(x - \frac{5}{2})$

Correct line method for a normal where a numerical $m_N (\neq m_T)$ is found from using calculus.

Can be implied by later working

$10 = \frac{1}{4}(\frac{5}{2}) + c \Rightarrow c = \frac{75}{8} \Rightarrow y = \frac{1}{4}x + \frac{75}{8}$

leading to $8y - 2x - 75 = 0$ (*)

Correct solution only A1

<table>
<thead>
<tr>
<th>Marks</th>
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<tbody>
<tr>
<td></td>
<td>(5)</td>
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<tr>
<td>Question</td>
<td>Scheme</td>
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</tr>
<tr>
<td><strong>6(c)</strong></td>
<td>[ y = \frac{25}{x} \Rightarrow 8\left(\frac{25}{x}\right) - 2x - 75 = 0 \text{ or } x = \frac{25}{y} \Rightarrow 8y - 2\left(\frac{25}{y}\right) - 75 = 0 ] or [ x = 5t, \ y = \frac{5}{t} \Rightarrow 8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 ] Substitutes [ y = \frac{25}{x} \text{ or } x = \frac{25}{y} \text{ or } x = 5t \text{ and } y = \frac{5}{t} ] into the printed equation or their normal equation to obtain an equation in either (x) only, (y) only or (t) only.</td>
</tr>
<tr>
<td></td>
<td>[ 2x^2 + 75x - 200 = 0 \text{ or } 8y^2 - 75y - 50 = 0 \text{ or } 2t^2 + 15t - 8 = 0 \text{ or } ] [ 10t^2 + 75t - 40 = 0 ] [ (2x - 5)(x + 40) = 0 \Rightarrow x = \ldots \text{ or } (y - 10)(8y + 5) = 0 \Rightarrow y = \ldots \text{ or } ] [ (2t - 1)(t + 8) = 0 \Rightarrow t = \ldots ] <strong>dependent on the previous M mark</strong> Correct attempt of solving a 3TQ to find either (x = \ldots, y = \ldots) or (t = \ldots)</td>
</tr>
<tr>
<td></td>
<td>Finds at least one of either (x = -40) or (y = -\frac{5}{8})</td>
</tr>
<tr>
<td></td>
<td>[ B\left(-40, \ -\frac{5}{8}\right) \text{ Both correct coordinates (If coordinates are not stated they can be paired together as } x = \ldots, y = \ldots \text{ )} ]</td>
</tr>
<tr>
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</tbody>
</table>

**Notes:**

(a) A conclusion is not required on this occasion in part (a).

**B1:** Condone reference to \(c = 5\) (as \(xy = c^2\) and \(\left(\frac{ct}{t}, \frac{c}{t}\right)\) are referred in the Formula book.)

(b) \[ \frac{dy}{dx} = \frac{dy}{dr} \cdot \frac{dr}{dx} = -\frac{5}{t^2}\left(\frac{1}{5}\right) = -\frac{1}{t^2} \Rightarrow m = t^2 \Rightarrow y - 10 = t^2\left(x - \frac{5}{2}\right) \] scores only the first M1.

When \(t = \frac{1}{2}\) is substituted giving \(y - 10 = \frac{1}{4}\left(x - \frac{5}{2}\right)\) the response then automatically gets A1(implied) M1(implied) M1
Question 6 notes continued

(c) You can imply the final three marks (dM1A1A1) for either

- \(8 \left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow \left(-40, \frac{-5}{8}\right)\)
- \(8y - 2 \left(\frac{25}{y}\right) - 75 = 0 \rightarrow \left(-40, \frac{-5}{8}\right)\)
- \(8(5t) - 2 \left(\frac{5}{t}\right) - 75 = 0 \rightarrow \left(-40, \frac{-5}{8}\right)\)

with no intermediate working.

You can also imply the middle dM1A1 marks for either

- \(8 \left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow x = -40\)
- \(8y - 2 \left(\frac{25}{y}\right) - 75 = 0 \rightarrow y = -\frac{5}{8}\)
- \(8(5t) - 2 \left(\frac{5}{t}\right) - 75 = 0 \rightarrow x = -40\) or \(y = -\frac{5}{8}\)

with no intermediate working.

**Writing** \(x = -40, y = -\frac{5}{8}\) followed by \(B\left(40, \frac{5}{8}\right)\) or \(B\left(-\frac{5}{8}, -40\right)\) is final A0.

Ignore stating \(B\left(\frac{5}{2}, 10\right)\) in addition to \(B\left(-40, -\frac{5}{8}\right)\).
Rotation Rotation B1

67 degrees (anticlockwise) Either arctan(12/13), tan⁻¹(12/13), sin⁻¹(12/13), cos⁻¹(5/13), awrt 67 degrees, awrt 1.2, truncated 1.1 (anticlockwise), awrt 293 degrees clockwise or awrt 5.1 clockwise B1 o.e.

about (0, 0) The mark is dependent on at least one of the previous B marks being awarded.

Note: Give 2nd B0 for 67 degrees clockwise o.e.

(b) \( \{Q = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \} \) Correct matrix B1

(c) \( \{R = PQ\} \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & -\frac{12}{13} \end{pmatrix} \) Multiplies P by their Q in the correct order and finds at least one element M1

Correct matrix A1

(d) Way 1

Forming the equation "their matrix R" \( \begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix} \) M1

Allow x being replaced by any non-zero number eg. 1. Can be implied by at least one correct ft equations below.

\[-\frac{12}{13}x + \frac{5}{13}kx = x \text{ or } \frac{5}{13}x + \frac{12}{13}kx = kx \Rightarrow k = ... \]

Uses their matrix equation to form an equation in k and progresses to give k = numerical value M1

So \( k = 5 \) dependent on only the previous M mark k = 5 A1 cao

Dependent on all previous marks being scored in this part. Either

- Solves both \( -\frac{12}{13}x + \frac{5}{13}kx = x \) and \( \frac{5}{13}x + \frac{12}{13}kx = kx \) to give \( k = 5 \) A1 cso

- Finds \( k = 5 \) and checks that it is true for the other component

- Confirms that \( \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & -\frac{12}{13} \end{pmatrix} \begin{pmatrix} x \\ 5x \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix} \)
### Question 7(d) continued

#### Way 2

Either

\[
\cos 2\theta = -\frac{12}{13}, \quad \sin 2\theta = \frac{5}{13} \quad \text{or} \quad \tan 2\theta = -\frac{5}{12}
\]

Correct follow through equation in \(2\theta\) based on their matrix \(R\)

\[
\{k\} = \tan \left(\frac{1}{2} \arccos \left(\frac{12}{13}\right)\right)
\]

Full method of finding \(2\theta\), then \(\theta\) and applying \(\tan\theta\)

\[
\tan \left(\frac{1}{2} \arccos \left(\frac{12}{13}\right)\right) \quad \text{or} \quad \tan(\text{awrt } 78.7^\circ) \quad \text{or} \quad \tan(\text{awrt } 1.37) \quad \text{Can be implied.}
\]

So \(k = 5\)

\[k = 5 \quad \text{by a correct solution only}\]

(4)

#### Marks

- M1
- M1
- A1
- A1

### Notes:

#### (a)
Condone “Turn” for the 1\(^{st}\) B1 mark.
Penalise the first B1 mark for candidates giving a combination of transformations.

#### (c)
Allow 1\(^{st}\) M1 for eg. “their matrix \(R^n\) = \( \begin{pmatrix} 1 \\ k \end{pmatrix} \)

or "their matrix \(R^2\) = \( \begin{pmatrix} k \\ k^2 \end{pmatrix} \)

or "their matrix \(R^\circ\) = \( \begin{pmatrix} \frac{1}{k} \\ \frac{1}{k} \end{pmatrix} \)

or equivalent

\[
y = (\tan \theta)x:
\begin{pmatrix}
\cos 2\theta & \sin 2\theta \\
\sin 2\theta & -\cos 2\theta
\end{pmatrix}
\begin{pmatrix}
-\frac{12}{13} \\
\frac{5}{13}
\end{pmatrix}
\begin{pmatrix}
\frac{5}{13} \\
\frac{12}{13}
\end{pmatrix}
\]


**Question 8**

(a) \( f(z) = z^4 + 6z^3 + 76z^2 + az + b, \) \( a, b \) are real constants. \( z_i = -3 + 8i \) is given.

\[ -3 - 8i \]  \[ -3 - 8i \]  \[ B1 \]

(b) \[ z^2 + 6z + 73 \]

Attempt to expand \((z - (-3 + 8i))(z - (-3 - 8i))\) or any valid method to establish a quadratic factor

eg. \[ z = -3 \pm 8i \Rightarrow z + 3 = \pm 8i \Rightarrow z^2 + 6z + 9 = -64 \]

or sum of roots \(-6, \) product of roots \(73 \) to give \( z^2 \pm (\text{sum})z + \text{product} \)

\[ z^2 + 6z + 73 \]  \[ A1 \]

\[ f(z) = (z^2 + 6z + 73)(z^2 + 3) \]

Attempts to find the other quadratic factor.

eg. using long division to get as far as \( z^2 + \ldots \)

or eg. \( f(z) = (z^2 + 6z + 73)(z^2 + \ldots) \)

\[ z^2 + 3 \]  \[ A1 \]

\( \left\{ \begin{array}{l} z^2 + 3 = 0 \Rightarrow z = \pm \sqrt{3}i \end{array} \right\} \)

Correct method of solving the 2nd quadratic factor

\( \sqrt{3}i \) and \( -\sqrt{3}i \)

\[ dM1 \]

(c) \( \left\{ \begin{array}{l} z^2 + 3 = 0 \Rightarrow z = \pm \sqrt{3}i \end{array} \right\} \)

Criteria

- \( -3 \pm 8i \) plotted correctly in quadrants 2 and 3 with some evidence of symmetry
- Their other two complex roots (which are found from solving their 2nd quadratic in (b)) are plotted correctly with some evidence of symmetry about the \( x \)-axis

Satisfies at least one of the two criteria \[ B1 \] ft

Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other.

\[ B1 \] ft

\( (2) \)  \( (9 \text{ marks}) \)
Question 8 continued

Notes:

(b)  
Give $3^\text{rd}$ M1 for $z^2 + k = 0, \ k > 0 \Rightarrow \text{at least one of either } z = \sqrt{k} i \ \text{or} \ z = -\sqrt{k} i$

Give $3^\text{rd}$ M0 for $z^2 + k = 0, \ k > 0 \Rightarrow z = \pm ki$

Give $3^\text{rd}$ M0 for $z^2 + k = 0, \ k > 0 \Rightarrow z = \pm k \ \text{or} \ z = \pm \sqrt{k}$

Candidates do not need to find $a = 18, \ b = 219$
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>9(a)</strong></td>
<td>$2x^2 + 4x - 3 = 0$ has roots $\alpha$, $\beta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha + \beta = -\frac{4}{2}$ or $-2$, $\alpha\beta = -\frac{3}{2}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Both $\alpha + \beta = -\frac{4}{2}$ and $\alpha\beta = -\frac{3}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This may be seen or implied anywhere in this question.</td>
<td></td>
</tr>
<tr>
<td><strong>(i)</strong></td>
<td>$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta =$ ------</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Use of a correct identity for $\alpha^2 + \beta^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(May be implied by their work)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= (-2)^2 - 2\left(-\frac{1}{2}\right) = 7$</td>
<td>A1 cso</td>
</tr>
<tr>
<td></td>
<td>7 from correct working</td>
<td></td>
</tr>
<tr>
<td><strong>(ii)</strong></td>
<td>$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) =$ ------</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Use of an appropriate and correct identity for $\alpha^3 + \beta^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(May be implied by their work)</td>
<td></td>
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<tr>
<td></td>
<td>$= (-2)^3 - 3\left(-\frac{1}{2}\right)(-2) = -17$</td>
<td>A1 cso</td>
</tr>
<tr>
<td></td>
<td>$= (-2)\left(7 - \frac{1}{2}\right) = -17$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-17$ from correct working</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Sum $= \alpha^2 + \beta + \beta^2 + \alpha$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \alpha^2 + \beta^2 + \alpha + \beta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 7 + (-2) = 5$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Uses at least one of their $\alpha^2 + \beta^2$ or $\alpha + \beta$ in an attempt to find a numerical value for the sum of $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$</td>
<td></td>
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<tr>
<td></td>
<td>Product $= (\alpha^2 + \beta)(\beta^2 + \alpha)$</td>
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<tr>
<td></td>
<td>$= (\alpha\beta)^2 + \alpha^3 + \beta^3 + \alpha\beta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \left(-\frac{1}{2}\right)^2 - 17 - \frac{1}{2} = -\frac{65}{4}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Expands $(\alpha^2 + \beta)(\beta^2 + \alpha)$ and uses at least one of their $\alpha\beta$ or $\alpha^3 + \beta^3$ in an attempt to find a numerical value for the product of $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x^2 - 5x - \frac{65}{4} = 0$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Applies $x^2 - \text{(sum)}x + \text{product}$ (Can be implied)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(“$= 0$” not required)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4x^2 - 20x - 65 = 0$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Any integer multiple of $4x^2 - 20x - 65 = 0$, including the “$= 0$”</td>
<td>(4)</td>
</tr>
</tbody>
</table>
### Question 9(b) continued

**Alternative:** Finding $\alpha^2 + \beta$ and $\beta^2 + \alpha$ explicitly

Eg. Let $\alpha = \frac{-4 + \sqrt{40}}{4}$, $\beta = \frac{-4 + \sqrt{40}}{4}$ and so  

\[ \alpha^2 + \beta = \frac{5 - 3\sqrt{10}}{2}, \quad \beta^2 + \alpha = \frac{5 + 3\sqrt{10}}{2} \]

\[ \left(x - \left(\frac{5 - 3\sqrt{10}}{2}\right)\right) \left(x - \left(\frac{5 + 3\sqrt{10}}{2}\right)\right) \]

Uses \( \left(x - (\alpha^2 + \beta)\right) \left(x - (\beta^2 + \alpha)\right) \) with exact numerical values. (May expand first)  

\[ = x^2 - \left(\frac{5 + 3\sqrt{10}}{2}\right)x - \left(\frac{5 - 3\sqrt{10}}{2}\right)x + \left(\frac{5 - 3\sqrt{10}}{2}\right) \left(\frac{5 + 3\sqrt{10}}{2}\right) \]

Attempts to expand using exact numerical values for $\alpha^2 + \beta$ and $\beta^2 + \alpha$  

\[ \Rightarrow x^2 - 5x - \frac{65}{4} = 0 \]

Collect terms to give a 3TQ. ("= 0" not required)  

\[ 4x^2 - 20x - 65 = 0 \]

Any integer multiple of $4x^2 - 20x - 65 = 0$, including the "= 0"  

(4)

### Notes:

**(a)**

**1st A1:** $\alpha + \beta = 2, \quad \alpha\beta = -\frac{3}{2} \Rightarrow \alpha^2 + \beta^2 = 4 - 2\left(-\frac{3}{2}\right) = 7$ is M1A0 cso

Finding $\alpha + \beta = -2, \quad \alpha\beta = -\frac{3}{2}$ by writing down or applying $\frac{-4 + \sqrt{40}}{4}, \quad \frac{-4 + \sqrt{40}}{4}$ but then writing $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 + 3 = 7$ and $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -8 - 9 = -17$ scores B0M1A0M1A0 in part (a).

Applying $\frac{-4 + \sqrt{40}}{4}, \quad \frac{-4 + \sqrt{40}}{4}$ explicitly in part (a) will score B0M0A0M0A0

Eg: Give no credit for $\left(\frac{-4 + \sqrt{40}}{4}\right)^2 + \left(\frac{-4 + \sqrt{40}}{4}\right)^2 = 7$

or for $\left(\frac{-4 + \sqrt{40}}{4}\right)^3 + \left(\frac{-4 + \sqrt{40}}{4}\right)^3 = -17$

**(b)**

Candidates are allowed to apply $\frac{-4 + \sqrt{40}}{4}, \quad \frac{-4 + \sqrt{40}}{4}$ explicitly in part (b).

A correct method leading to a candidate stating $a = 4, \quad b = -20, \quad c = -65$ without writing a final answer of $4x^2 - 20x - 65 = 0$ is final M1A0
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$u_1 = 5, u_{n+1} = 3u_n + 2, \ n \geq 1$. Required to prove the result, $u_n = 2 \times (3)^n - 1, \ n \in \mathbb{Z}^+$</td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>$n = 1$: $u_1 = 2(3) - 1 = 5$ $u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(Assume the result is true for $n = k$) $u_{k+1} = 3(2(3)^k - 1) + 2$ Substitutes $u_k = 2(3)^k - 1$ into $u_{k+1} = 3u_k + 2$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= 2(3)^{k+1} - 1$ dependent on the previous M mark Expresses $u_{k+1}$ in term of $3^{k+1}$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$u_{k+1} = 2(3)^{k+1} - 1$ by correct solution only</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>If the result is true for $n = k$, then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all $n$</td>
<td>A1 cso</td>
</tr>
<tr>
<td></td>
<td><strong>Required to prove the result</strong> $\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{3 + 2n}{3^n}, \ n \in \mathbb{Z}^+$</td>
<td>(5)</td>
</tr>
<tr>
<td>(ii)</td>
<td>$n = 1$: LHS $= \frac{4}{3}, \ RHS = 3 - \frac{5}{3} = \frac{4}{3}$ Shows or states both LHS $= \frac{4}{3}$ and RHS $= \frac{4}{3}$ or states LHS $= \frac{4}{3}$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(Assume the result is true for $n = k$) $\sum_{r=1}^{k+1} \frac{4r}{3^r} = 3 - \frac{3 + 2k}{3^k} + \frac{4(k + 1)}{3^{k+1}}$ Adds the $(k + 1)^{th}$ term to the sum of $k$ terms</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= 3 - \frac{3(3 + 2k)}{3^{k+1}} + \frac{4(k + 1)}{3^{k+1}}$ dependent on the previous M mark Makes $3^{k+1}$ or $(3)3^k$ a common denominator for their fractions.</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$= 3 - \frac{(3 + 2k) - 4(k + 1)}{3^{k+1}}$ Correct expression with common denominator $3^{k+1}$ or $(3)3^k$ for their fractions.</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= 3 - \frac{5 + 2k}{3^{k+1}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 3 - \frac{(3 + 2(k + 1))}{3^{k+1}}$ $3 - \frac{(3 + 2(k + 1))}{3^{k+1}}$ by correct solution only</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>If the result is true for $n = k$, then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all $n$</td>
<td>A1 cso</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td><strong>(11 marks)</strong></td>
<td></td>
</tr>
</tbody>
</table>
Question 10 continued

Notes:

(i) & (ii) Final A1 for parts (i) and (ii) is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of all four underlined points either at the end of their solution or as a narrative in their solution.

(i) 
\[ u_1 = 5 \] by itself is not sufficient for the 1st B1 mark in part (i).
\[ u_1 = 3 + 2 \] without stating \[ u_1 = 2(3) - 1 = 5 \] or \[ u_1 = 6 - 1 = 5 \] is B0

(ii) 
LHS = RHS by itself is not sufficient for the 1st B1 mark in part (ii).
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
• Use black ink or ball-point pen.
• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions and ensure that your answers to parts of questions are clearly labelled.
• Answer the questions in the spaces provided – there may be more space than you need.
• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
• Inexact answers should be given to three significant figures unless otherwise stated.

Information
• A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
• There are 8 questions in this question paper. The total mark for this paper is 75.
• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
• Read each question carefully before you start to answer it.
• Try to answer every question.
• Check your answers if you have time at the end.
• If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over
Answer ALL questions. Write your answers in the spaces provided.

1. Using algebra, find the set of values of $x$ for which

$$\frac{x}{x + 2} < \frac{2}{x + 5}$$

(7)
Question 1 continued
2. (a) Express \( \frac{1}{(r + 6)(r + 8)} \) in partial fractions.

(b) Hence show that

\[
\sum_{r=1}^{n} \frac{2}{(r + 6)(r + 8)} = \frac{n(an + b)}{56(n + 7)(n + 8)}
\]

where \( a \) and \( b \) are integers to be found.
Hence show that

(a) Express $a$ and $b$ are integers to be found.

$$
\sum r^n = \frac{1}{r-1} \left( r^n - 1 \right) + \frac{a}{r-1} + b \left( r^n + 1 \right)
$$

(1)

(4)

Leave blank

Question 2 continued
Question 2 continued

(Total for Question 2 is 5 marks)
3. (a) Show that the substitution \( z = y^{-2} \) transforms the differential equation

\[
\frac{dy}{dx} + 2xy = xe^{-x^2}y^3 \quad (I)
\]

into the differential equation

\[
\frac{dz}{dx} - 4xz = -2xe^{-x^2} \quad (II)
\]  

(b) Solve differential equation (II) to find \( z \) as a function of \( x \).

(c) Hence find the general solution of differential equation (I), giving your answer in the form \( y^2 = f(x) \).
Question 3 continued
Question 3 continued

(Total for Question 3 is 10 marks)
4. A transformation $T$ from the $z$-plane to the $w$-plane is given by
\[ w = \frac{z - 1}{z + 1}, \quad z \neq -1 \]

The line in the $z$-plane with equation $y = 2x$ is mapped by $T$ onto the curve $C$ in the $w$-plane.

(a) Show that $C$ is a circle and find its centre and radius. \hfill (7)

The region $y < 2x$ in the $z$-plane is mapped by $T$ onto the region $R$ in the $w$-plane.

(b) Sketch circle $C$ on an Argand diagram and shade and label region $R$. \hfill (2)
Question 4 continued
Question 4 continued
5. Given that \( y = \cot x \),

(a) show that

\[
\frac{d^2y}{dx^2} = 2 \cot x + 2 \cot^3 x
\]  

(3)

(b) Hence show that

\[
\frac{d^3y}{dx^3} = p \cot^4 x + q \cot^2 x + r
\]

where \( p, q \) and \( r \) are integers to be found.  

(3)

(c) Find the Taylor series expansion of \( \cot x \) in ascending powers of \( \left( x - \frac{\pi}{3} \right) \) up to and including the term in \( \left( x - \frac{\pi}{3} \right)^3 \).  

(3)
Question 5 continued
Question 5 continued

(Total for Question 5 is 9 marks)
6. (a) Find the general solution of the differential equation
\[
\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - 3y = 2\sin x \quad (I)
\]

Given that \( y = 0 \) and \( \frac{dy}{dx} = 1 \) when \( x = 0 \)

(b) find the particular solution of differential equation (I).
Question 6 continued
Figure 1 shows the two curves given by the polar equations

\[ r = 3 \sin \theta, \quad 0 \leq \theta \leq \pi \]
\[ r = 1 + \cos \theta, \quad 0 \leq \theta \leq \pi \]

(a) Verify that the curves intersect at the point \( P \) with polar coordinates \( \left( \frac{3}{2}, \frac{\pi}{3} \right) \). \( \hspace{1cm} (2) \)

The region \( R \), bounded by the two curves, is shown shaded in Figure 1.

(b) Use calculus to find the exact area of \( R \), giving your answer in the form \( a \left( \pi - \sqrt{3} \right) \), where \( a \) is a constant to be found. \( \hspace{1cm} (6) \)
Question 7 continued
Question 7 continued
8. (a) Show that
\[(z + \frac{1}{z})^3 \left( z - \frac{1}{z} \right)^3 = z^6 - \frac{1}{z^6} - k \left( z^3 - \frac{1}{z^3} \right) \]
where \(k\) is a constant to be found. \(\quad\) (3)

Given that \(z = \cos \theta + i \sin \theta\), where \(\theta\) is real,

(b) show that
\[
\begin{align*}
(i) & \quad z^n + \frac{1}{z^n} = 2 \cos n\theta \\
(ii) & \quad z^n - \frac{1}{z^n} = 2i \sin n\theta 
\end{align*}
\] \(\quad\) (3)

(c) Hence show that
\[
\cos^3 \theta \sin^3 \theta = \frac{1}{32} (3 \sin 2\theta - \sin 6\theta) \] \(\quad\) (4)

(d) Find the exact value of
\[
\int_{0}^{\pi} \cos^3 \theta \sin^3 \theta d\theta \] \(\quad\) (4)
Question 8 continued
Question 8 continued
Question 8 continued
## Further Pure Mathematics FP2 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{x}{x+2} &lt; \frac{2}{x+5}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Critical Values $-2$ and $-5$</td>
<td>Seen anywhere in solution (dM1): Attempt an interval inequality using one of $-2$ or $-5$ with another cv (A1, A1) Correct intervals Can be in set notation One correct scores $A1A0$ Award on basis of the inequalities seen - ignore any and/or between them Set notation answers do not need the union sign.</td>
</tr>
</tbody>
</table>

**Alternative**

<p>| Critical Values $-2$ and $-5$ | Seen anywhere in solution | (B1, B1) |
| $\frac{x}{x+2} &lt; \frac{2}{x+5}$ | (\Rightarrow x(x+5)^2(x+2) &lt; 2(x+2)^2(x+5)) (\Rightarrow (x+5)(x+2)[x(x+5)-2(x+2)] &lt; 0) | (M1) |
| Critical values $-4$ and $1$ | Correct critical values | (A1) |
| $-5 &lt; x &lt; -4$, $-2 &lt; x &lt; 1$ ((-5,-4) \cup (-2,1)) | (\Rightarrow (x+5)(x+2)[(x-1)(x+4)] &lt; 0) (Multiply by ((x+5)^2(x+2)^2) and attempt to factorise a quartic or use quad formula) | (A1, A1) |</p>
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2(a)</strong></td>
<td>[ \frac{1}{(r+6)(r+8)} ]</td>
<td><strong>(1)</strong></td>
</tr>
<tr>
<td></td>
<td>[ \frac{1}{2(r+6)} - \frac{1}{2(r+8)} \text{ oe} ]</td>
<td><strong>B1</strong></td>
</tr>
<tr>
<td></td>
<td>Correct partial fractions, any equivalent form</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>[ \left( 2 \times \frac{1}{2} \right) \left( \frac{1}{7} - \frac{1}{9} + \frac{1}{8} - \frac{1}{10} + \frac{1}{9} - \frac{1}{11} \ldots + \frac{1}{n+5} - \frac{1}{n+7} + \frac{1}{n+6} - \frac{1}{n+8} \right) ]</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td></td>
<td>Expands at least 3 terms at start and 2 at end (may be implied)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The partial fractions obtained in (a) can be used without multiplying by 2.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fractions may be [ \frac{1}{2} \times \frac{1}{7} - \frac{1}{2} \times \frac{1}{9} \text{ etc} ] These comments apply to both M1 and A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \frac{1}{7} + \frac{1}{8} - \frac{1}{n+7} - \frac{1}{n+8} ]</td>
<td><strong>A1</strong></td>
</tr>
<tr>
<td></td>
<td>Identifies the terms that do not cancel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \frac{15(n+7)(n+8) - 56(2n+15)}{56(n+7)(n+8)} ]</td>
<td><strong>M1</strong></td>
</tr>
<tr>
<td></td>
<td>Attempt common denominator</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Must have multiplied the fractions from (a) by 2 now</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \frac{n(15n+113)}{56(n+7)(n+8)} ]</td>
<td><strong>A1 cso</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>(4)</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>(5 marks)</strong></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
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<td>-------</td>
</tr>
<tr>
<td><strong>3(a)</strong></td>
<td>[\frac{dy}{dx} + 2xy = xe^{-x^2}y^3]</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>$z = y^{-2} \Rightarrow y = z^{-\frac{1}{2}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[\frac{dy}{dx} = \frac{\frac{1}{2}z^{-\frac{3}{2}}}{dz} \frac{dz}{dx}]</td>
<td>M1: $\frac{dz}{dx} = kz^{-\frac{3}{2}} \frac{dz}{dx}$ M1 A1</td>
</tr>
<tr>
<td></td>
<td>$-\frac{1}{2}z^{-\frac{3}{2}} \frac{dz}{dx} + \frac{2x}{z^{\frac{1}{2}}} = xe^{-x^2}z^{\frac{1}{2}}$</td>
<td>A1: Correct differentiation</td>
</tr>
<tr>
<td></td>
<td>[\frac{dz}{dx} - 4xz = -2xe^{-x^2} ] *</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Correct completion to printed answer with no errors seen</td>
<td>A1 cso</td>
</tr>
</tbody>
</table>

**Alternative 1**

\[\frac{dz}{dy} = -2y^{-3} \text{ oe}\] M1: $\frac{dz}{dy} = ky^{-3}$ M1 A1

|          | $-\frac{1}{2}y^{-3} \frac{dz}{dx} + 2xy = xe^{-x^2}y^3$ | Substitutes for $dy/dx$ M1 |
|          | \[\frac{dz}{dx} - 4xz = -2xe^{-x^2} \] * | Correct completion to printed answer with no errors seen A1 |

**Alternative 2**

\[\frac{dz}{dx} = -2y^{-3} \frac{dy}{dx}\] M1: $\frac{dz}{dx} = ky^{-3} \frac{dy}{dx}$ inc chain rule M1 A1

|          | $-\frac{1}{2}y^{-3} \frac{dz}{dx} + 2xy = xe^{-x^2}y^3$ | Substitutes for $dy/dx$ M1 |
|          | \[\frac{dz}{dx} - 4xz = -2xe^{-x^2} \] * | Correct completion to printed answer with no errors seen A1 |

**Alternative 2**

\[I = e^{\int_{-4}^{x} dy} = e^{-2x^2}\] M1: $I = e^{\int_{-4}^{x} dy}$ A1: $e^{-2x^2}$

|          | $ze^{-2x^2} = \int -2xe^{-3x^2} \ dx$ | dM1 |
|          | $z\times I = \int -2xe^{-x^2} I \ dx$ | dM1 |
|          | \[\frac{1}{3}e^{-3x^2} (+c)\] | M1 |
|          | $\int xze^{3x^2} \ dx = pe^{3x^2} (+c)$ | M1 |
|          | $z = ce^{2x^2} + \frac{1}{3}e^{-x^2}$ | Or equivalent A1 |

(5)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(c)</strong></td>
<td>( \frac{1}{y^2} = ce^{2x^2} + \frac{1}{3}e^{-x^2} \Rightarrow y^2 = \frac{1}{ce^{2x^2}} + \frac{1}{3}e^{-x^2} )</td>
<td>( y^2 = \frac{1}{(b)} \left( \frac{3e^{x^2}}{1+ke^{3x^2}} \right) )</td>
</tr>
</tbody>
</table>

(10 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4(a)</strong></td>
<td>$w = \frac{z - 1}{z + 1}$</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$w = \frac{z - 1}{z + 1} \Rightarrow wz + w = z - 1 \Rightarrow z = \ldots$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$z = \frac{w + 1}{1 - w}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{u + iv + 1}{1 - u - iv} \times \frac{1 - u + iv}{1 - u + iv}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{-u^2 - v^2 + 1}{\ldots}, \quad y = 2v$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$y = 2x \Rightarrow 2v = -2u^2 - 2v^2 + 2$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$u^2 + (v + \frac{1}{2})^2 - \frac{1}{4} = 1$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Centre $(0, -\frac{1}{2})$, radius $\frac{\sqrt{5}}{2}$</td>
<td>A1: Correct centre (allow $-\frac{1}{2}i$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1: Correct radius</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>(7)</td>
</tr>
<tr>
<td><strong>Special Case:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$w = \frac{x + iy - 1}{x + iy + 1} = \frac{(x - 1) + 2xi}{(x + 1) + 2xi} \times \frac{(x + 1) - 2xi}{(x + 1) - 2xi}$</td>
<td>M1: rationalise the denominator, may have $2x$ or $y$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{(x^2 - 1) + 4x^2 + 2xi(x + 1 - (x - 1))}{(x + 1)^2 + 4x^2}$</td>
<td>A1: Correct result in terms of $x$ only. Must have rational denominator shown, but no other simplification needed</td>
</tr>
<tr>
<td></td>
<td>B1ft: Their circle correctly positioned provided their equation does give a circle</td>
<td>B1ft B1</td>
</tr>
<tr>
<td></td>
<td>B1: Completely correct sketch and shading</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9 marks)</td>
<td></td>
</tr>
</tbody>
</table>
**Question 5(a)**

\[ y = \cot x \]

\[
\frac{dy}{dx} = -\csc^2 x
\]

\[
\frac{d^2y}{dx^2} = (-2\csc x)(-\csc x \cot x) = 2\csc^2 x \cot x = 2 \cot x + 2 \cot^3 x
\]

M1: Differentiates using the chain rule or product/quotient rule

A1: Correct derivative

A1: Correct completion to printed answer

A1cso*

**Alternative**

\[ y = \frac{\cos x}{\sin x} \]

\[
\frac{dy}{dx} = \frac{\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}
\]

\[
\frac{d^2y}{dx^2} = -\left(-2\sin^{-3} x \cos x\right) = ...
\]

M1A1

Correct completion to printed answer see above A1

(3)

**Question 5(b)**

\[
\frac{d^3y}{dx^3} = -2\csc^2 x - 6 \cot^2 x \csc^2 x
\]

Correct third derivative B1

\[
= -2(1 + \cot^2 x) - 6 \cot^2 x(1 + \cot^2 x)
\]

Uses \( 1 + \cot^2 x = \csc^2 x \) M1

\[
= -6 \cot^4 x - 8 \cot^2 x - 2
\]

cso

A1

(3)

**Question 5(c)**

\[
f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}
\]

M1: Attempts all 4 values at \( \frac{\pi}{3} \) No working need be shown

\[
(y =) \frac{1}{\sqrt{3}} - \frac{4}{3}\left(x - \frac{\pi}{3}\right) + \frac{4}{3\sqrt{3}} \left(x - \frac{\pi}{3}\right)^2 - \frac{8}{9}\left(x - \frac{\pi}{3}\right)^3
\]

M1: Correct application of Taylor using their values. Must be up to and including \( \left(x - \frac{\pi}{3}\right)^3 \)

M1A1

A1: Correct expression Must start \( y = ... \) or \( \cot x \)

f(x) allowed provided defined here or above as \( f(x) = \cot x \) or \( y \)

Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698.., so accept 0.77) 0.889

(3)

(9 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| **6(a)** | \[
\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - 3y = 2 \sin x
\] | **AE:**  \[m^2 - 2m - 3 = 0\] Forms Auxiliary Equation and attempts to solve (usual rules) M1 \[
m^2 - 2m - 3 = 0 \implies m = ...(-1,3)
\] \[(y = ) Ae^{3x} + Be^{-x}\] Cao A1 \[
(y' = ) p \cos x - q \sin x
\] \[
(y'' = ) - p \sin x - q \cos x
\] Differentiates twice and substitutes 2q - 4p = 2, 4q + 2p = 0 Correct equations A1 \[
p = -\frac{2}{5}, \quad q = \frac{1}{5}\] A1 A1 \[
y = \frac{1}{5} \cos x - \frac{2}{5} \sin x
\] y = \[Ae^{3x} + Be^{-x} + \frac{1}{5} \cos x - \frac{2}{5} \sin x\] Follow through their p and q and their CF B1ft \[
(8)
\] **(b)** \[
y' = 3Ae^{3x} - Be^{-x} - \frac{1}{5} \sin x - \frac{2}{5} \cos x
\] Differentiates their GS M1 \[
0 = A + B + \frac{1}{5}, \quad 1 = 3A - B - \frac{2}{5}\] M1: Uses the given conditions to give two equations in A and B A1: Correct equations \[
A = \frac{3}{10}, \quad B = -\frac{1}{2}\] Solves for A and B Both correct A1 \[
y = \frac{3}{10} e^{3x} - \frac{1}{2} e^{-x} + \frac{1}{5} \cos x - \frac{2}{5} \sin x
\] Sub their values of A and B in their GS A1ft \[
(5)
\] **(13 marks)**
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>7(a)</td>
<td>Attempt to verify coordinates in at least one of the polar equations.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Coordinates verified in both curves (Coordinate brackets not needed).</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Alternative**

Equate $\sqrt{3} \sin \theta = 1 + \cos \theta$ and verify (by substitution) that $\theta = \frac{\pi}{3}$ is a solution or solve by using $t = \tan \frac{\theta}{2}$ or writing

$$\frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta = \frac{1}{2} \sin \left( \theta - \frac{\pi}{6} \right) = \frac{1}{2} \quad \theta = \frac{\pi}{3}$$

Squaring the original equation allowed as $\theta$ is known to be between 0 and $\pi$.

Use $\theta = \frac{\pi}{3}$ in either equation to obtain $r = \frac{3}{2}$.

**b)**

$$\frac{1}{2} \int ( \sqrt{3} \sin \theta )^2 \, d\theta, \quad \frac{1}{2} \int (1 + \cos \theta)^2 \, d\theta$$

Correct formula used on at least one curve (1/2 may appear later). Integrals may be separate or added or subtracted. M1

$$\frac{1}{2} \int 3 \sin^2 \theta \, d\theta, \quad \frac{1}{2} \int (1 + 2 \cos \theta + \cos^2 \theta) \, d\theta$$

Attempt to use $\sin^2 \theta$ or $\cos^2 \theta = \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ on either integral. M1

**Not** dependent 1/2 may be missing

$$= \frac{3}{4} \left[ \theta - \frac{1}{2} \sin 2\theta \right]^{\left(\frac{\pi}{3}\right)}_0, \quad \frac{3}{2} \left[ \frac{1}{2} \theta + 2 \sin \theta + \frac{1}{4} \sin 2\theta \right]^{\left(\frac{\pi}{3}\right)}_0$$

Correct integration (ignore limits) A1A1 or A1A0

$$R = \frac{3}{4} \left[ \frac{\pi - \sqrt{3}}{4} \right] + \frac{1}{2} \left[ \frac{3 \pi}{2} - \left( \frac{\pi + \sqrt{3} + \sqrt{3}}{8} \right) \right]$$

Correct use of limits for both integrals. Integrals must be added. Dep on both previous M marks ddM1

$$= \frac{3}{4} \left( \pi - \sqrt{3} \right)$$

Cao

No equivalents allowed A1

(8 marks)
Question | Scheme | Marks
--- | --- | ---
8(a) | \[(z + \frac{1}{z})^3 (z - \frac{1}{z})^3 = (z^2 - \frac{1}{z^2})^3\] | \[= z^6 - 3z^2 + \frac{3}{z^2} - z^{-6}\] M1: Attempt to expand A1: Correct expansion \[= z^6 - \frac{1}{z^6} - 3\left(z^2 - \frac{1}{z^2}\right)\] Correct answer with no errors seen (3) Alternative \[(z + \frac{1}{z})^3 = z^3 + 3z + \frac{3}{z} + \frac{1}{z^3}, \quad (z - \frac{1}{z})^3 = z^3 - 3z + \frac{3}{z} - \frac{1}{z^3}\] M1A1 M1: Attempt to expand both cubic brackets A1: Correct expansions \[= z^6 - \frac{1}{z^6} - 3\left(z^2 - \frac{1}{z^2}\right)\] Correct answer with no errors A1 (3) (b)(i)(ii) \[z^n = \cos n\theta + i\sin n\theta\] Correct application of de Moivre B1 \[z^{-n} = \cos (-n\theta) + i\sin (-n\theta) = \pm \cos n\theta \pm i\sin n\theta\] Attempt \(z^{-n}\) M1 \[z^n + \frac{1}{z^n} = 2\cos n\theta^*, \quad z^n - \frac{1}{z^n} = 2i\sin n\theta^*\] \[z^{-n} = \cos n\theta - i\sin n\theta\] must be seen A1* (3) (c) \[(z + \frac{1}{z})^3 (z - \frac{1}{z})^3 = (2\cos \theta)^3 (2i\sin \theta)^3\] B1 \[z^6 - \frac{1}{z^6} - 3\left(z^2 - \frac{1}{z^2}\right) = 2i\sin 6\theta - 6i\sin 2\theta\] Follow through their \(k\) in place of 3 B1ft \[\text{Equating right hand sides and simplifying } 2^3 \times (2i)^3 \text{ (B mark needed for each side to gain M mark) }\] M1 \[\cos^3 \theta \sin^3 \theta = \frac{1}{32} (3\sin 2\theta - \sin 6\theta) \] A1cso (4)
<table>
<thead>
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</thead>
</table>
| 8(d) | \[
\int_{0}^{\frac{\pi}{2}} \cos^3 \theta \sin^3 \theta \, d\theta = \int_{0}^{\frac{\pi}{2}} \frac{1}{32} (3 \sin 2\theta - \sin 6\theta) \, d\theta
\]

\[
= \frac{1}{32} \left[ -\frac{3}{2} \cos 2\theta + \frac{1}{6} \cos 6\theta \right]_{0}^{\frac{\pi}{2}}
\]

M1: \[ p \cos 2\theta + q \cos 6\theta \]

A1: Correct integration
Differentiation scores M0A0

M1 A1

| | dM1: Correct use of limits – lower limit to have non-zero result. Dep on previous M mark |
| | A1: Cao (oe) but must be exact |
| | (4) |

(14 marks)
\begin{align*}
&\int_0^\pi \cos \theta \, d\theta = 0 \\
&\int_0^\pi \sin \theta \, d\theta = 0 \\
&\int_0^\pi \cos 2\theta \, d\theta = 0 \\
&\int_0^\pi \sin 3\theta \, d\theta = 0 \\
&\int_0^\pi \cos 6\theta \, d\theta = 0 \\
&\int_0^\pi \sin 3\theta \, d\theta = 0 \\
&\int_0^\pi \sin 2\theta \, d\theta = 0 \\
&\int_0^\pi \cos 6\theta \, d\theta = 0 \\
\end{align*}
Answer ALL questions. Write your answers in the spaces provided.

1. The curve \( C \) has equation

\[ y = 9 \cosh x + 3 \sinh x + 7x \]

Use differentiation to find the exact \( x \) coordinate of the stationary point of \( C \), giving your answer as a natural logarithm.

(6)
Question 1 continued

(Total for Question 1 is 6 marks)
2. An ellipse has equation
\[ \frac{x^2}{25} + \frac{y^2}{4} = 1 \]

The point \( P \) lies on the ellipse and has coordinates \((5 \cos \theta, 2 \sin \theta)\), \(0 < \theta < \frac{\pi}{2}\)

The line \( L \) is a normal to the ellipse at the point \( P \).

(a) Show that an equation for \( L \) is

\[ 5x \sin \theta - 2y \cos \theta = 21 \sin \theta \cos \theta \]  

(5)

Given that the line \( L \) crosses the \( y \)-axis at the point \( Q \) and that \( M \) is the midpoint of \( PQ \),

(b) find the exact area of triangle \( OPM \), where \( O \) is the origin, giving your answer as a multiple of \( \sin 2\theta \)  

(6)
Question 2 continued

(Total for Question 2 is 11 marks)
3. Without using a calculator, find

\[(a) \int_{-2}^{1} \frac{1}{x^2 + 4x + 13} \, dx, \text{ giving your answer as a multiple of } \pi,\]  

\[(b) \int_{-1}^{4} \frac{1}{\sqrt{4x^2 - 12x + 34}} \, dx, \text{ giving your answer in the form } p \ln(q + r\sqrt{2}),\]  

where \(p, q\) and \(r\) are rational numbers to be found.
Without using a calculator, find

\[ \int \frac{1}{x(x^2 + 1)} \, dx \]

where \( a, b, c, d, e \) and \( f \) are rational numbers to be found.

Leave blank giving your answer as a multiple of \( \pi \).

DO NOT WRITE IN THIS AREA
Question 3 continued

(Total for Question 3 is 12 marks)
4.

\[ M = \begin{pmatrix} 1 & k & 0 \\ -1 & 1 & 1 \\ 1 & k & 3 \end{pmatrix}, \text{ where } k \text{ is a constant} \]

(a) Find \( M^{-1} \) in terms of \( k \).

Hence, given that \( k = 0 \)

(b) find the matrix \( N \) such that

\[ MN = \begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix} \]
Question 4 continued
Question 4 continued

(Total for Question 4 is 9 marks)
5. Given that \( y = \text{artanh}(\cos x) \)

(a) show that

\[
\frac{dy}{dx} = -\csc x
\]

(b) Hence find the exact value of

\[
\int_{0}^{\frac{\pi}{6}} \cos x \, \text{artanh}(\cos x) \, dx
\]

giving your answer in the form \( a \ln(b + c\sqrt{3}) + d\pi \), where \( a \), \( b \), \( c \) and \( d \) are rational numbers to be found.

(5)
Question 5 continued
Question 5 continued

(Total for Question 5 is 7 marks)
6. The coordinates of the points $A$, $B$ and $C$ relative to a fixed origin $O$ are $(1, 2, 3)$, $(-1, 3, 4)$ and $(2, 1, 6)$ respectively. The plane $II$ contains the points $A$, $B$ and $C$.

(a) Find a cartesian equation of the plane $II$. \hspace{1cm} (5)

The point $D$ has coordinates $(k, 4, 14)$ where $k$ is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of $k$. \hspace{1cm} (4)
Question 6 continued
Question 6 continued

(Total for Question 6 is 9 marks)
7. The curve $C$ has parametric equations

\[ x = 3t^4, \quad y = 4t^3, \quad 0 \leq t \leq 1 \]

The curve $C$ is rotated through $2\pi$ radians about the $x$-axis. The area of the curved surface generated is $S$.

(a) Show that

\[ S = k\pi \int_0^1 t^5 \left( t^2 + 1 \right)^{\frac{1}{2}} dt \]

where $k$ is a constant to be found. \hspace{1cm} (4)

(b) Use the substitution $u^2 = t^2 + 1$ to find the value of $S$, giving your answer in the form

\[ p\pi \left( 11\sqrt{2} - 4 \right) \]

where $p$ is a rational number to be found. \hspace{1cm} (7)
Question 7 continued
Question 7 continued
Question 7 continued

(Total for Question 7 is 11 marks)
8. \( I_n = \int_0^{\ln 2} \tanh^2 x \, dx, \quad n \geq 0 \)

(a) Show that, for \( n \geq 1 \)

\[
I_n = I_{n-1} - \frac{1}{2n-1} \left( \frac{3}{5} \right)^{2n-1} \tag{5}
\]

(b) Hence show that

\[
\int_0^{\ln 2} \tanh^4 x \, dx = p + \ln 2
\]

where \( p \) is a rational number to be found. \( \tag{5} \)
Question 8 continued
Further Pure Mathematics FP3 Mark scheme

Question Scheme Marks
1 9cosh 3sinh 7 y x xx =++

Correct derivative B1

\( e^e + e^{3x} + e^{7x} \)

Replaces sinh \( x \) and cosh \( x \) by the correct exponential forms M1

Note that the first 2 marks can score the other way round:

M1: \( e^{9x} + e^{3x} + e^{7x} \)

B1: \( e^{9x} + e^{3x} + e^{7x} \)

212e 14e 6 0 xx x x + − =

Oe

M1: Obtains a quadratic in \( e^x \)

M1 A1

A1: Correct quadratic \( 3e^x - 1 + 2e^x - 3 = 0 \) ...

Solves their quadratic as far as \( e^x = ... \)

M1

1ln 3 x   = (Allow – ln3)

3e 2 x =−

need not be seen. Extra answers, award A0

Alternative
d 9sinh 3cosh 7d y xxx =++

Correct derivative B1

229sinh 3cosh 7 81sinh 9cosh 42cosh 49 xx xxx =− −⇒ =++

272cosh 42cosh 130 0 xx − − =

Squares and attempts quadratic in cosh \( x \) M1

\( 53cosh 5 + 12cosh 3 = 0 \)

M1: Solves quadratic M1

M1 A1

A1: Correct value

2

55ln 133 x   = ±−

Use of ln form of arcosh M1

1ln 3 x   = (Allow – ln3) A1

NB:

Ignore any attempts to find the \( y \) coordinate (6 marks)
### Further Pure Mathematics FP3 Mark scheme

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<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y = 9 \cosh x + 3 \sinh x + 7x )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{dy}{dx} = 9 \sinh x + 3 \cosh x + 7 )</td>
<td>Correct derivative</td>
</tr>
<tr>
<td></td>
<td>( 9 \left( \frac{e^x - e^{-x}}{2} \right) + 3 \left( \frac{e^x + e^{-x}}{2} \right) + 7 = 0 )</td>
<td>Replaces ( \sinh x ) and ( \cosh x ) by the correct exponential forms</td>
</tr>
<tr>
<td></td>
<td>Note that the first 2 marks can score the other way round:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1: ( y = 9 \left( \frac{e^x + e^{-x}}{2} \right) + 3 \left( \frac{e^x - e^{-x}}{2} \right) + 7x )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B1: ( \frac{dy}{dx} = 9 \left( \frac{e^x - e^{-x}}{2} \right) + 3 \left( \frac{e^x + e^{-x}}{2} \right) + 7 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 12e^{2x} + 14e^x - 6 = 0 ) oe</td>
<td>M1: Obtains a quadratic in ( e^x )</td>
</tr>
<tr>
<td></td>
<td>A1: Correct quadratic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( (3e^x - 1)(2e^x + 3) = 0 \Rightarrow e^x = ... )</td>
<td>Solves their quadratic as far as ( e^x = ... )</td>
</tr>
<tr>
<td></td>
<td>( x = \ln \left( \frac{1}{3} \right) )</td>
<td>cso (Allow (-\ln 3)) ( e^x = -\frac{3}{2} ) need not be seen. Extra answers, award A0</td>
</tr>
<tr>
<td><strong>Alternative</strong></td>
<td>( \frac{dy}{dx} = 9 \sinh x + 3 \cosh x + 7 )</td>
<td>Correct derivative</td>
</tr>
<tr>
<td></td>
<td>( 9 \sinh x = -3 \cosh x - 7 \Rightarrow 81 \sinh^2 x = 9 \cosh^2 x + 42 \cosh x + 49 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 72 \cosh^2 x - 42 \cosh x - 130 = 0 )</td>
<td>Squares and attempts quadratic in ( \cosh x )</td>
</tr>
<tr>
<td></td>
<td>( (3 \cosh x - 5)(12 \cosh x + 13) = 0 \Rightarrow \cosh x = \frac{5}{3} ) M1: Solves quadratic</td>
<td>A1: Correct value</td>
</tr>
<tr>
<td></td>
<td>( x = \ln \left( \frac{5}{3} + \sqrt{\left( \frac{5}{3} \right)^2 - 1} \right) )</td>
<td>Use of ( \ln ) form of ( \text{arcosh} )</td>
</tr>
<tr>
<td></td>
<td>( x = \ln \left( \frac{1}{3} \right) )</td>
<td>cso (Allow (-\ln 3))</td>
</tr>
<tr>
<td><strong>NB:</strong></td>
<td>Ignore any attempts to find the ( y ) coordinate</td>
<td></td>
</tr>
</tbody>
</table>

(6 marks)
## Question 2

### (a)

\[
\frac{x^2}{25} + \frac{y^2}{4} = 1, \quad P(5 \cos \theta, 2 \sin \theta)
\]

\[
\frac{dx}{d\theta} = -5 \sin \theta, \quad \frac{dy}{d\theta} = 2 \cos \theta
\]

or

\[
\frac{2x}{25} + \frac{2y}{4} \frac{dy}{dx} = 0
\]

Correct derivatives or correct implicit differentiation \( \text{B1} \)

\[
\frac{dy}{dx} = \frac{2 \cos \theta}{-5 \sin \theta}
\]

Divides their derivatives correctly or substitutes and rearranges \( \text{M1} \)

\[
M_N = \frac{5 \sin \theta}{2 \cos \theta}
\]

Correct perpendicular gradient rule \( \text{M1} \)

\[
y - 2 \sin \theta = \frac{5 \sin \theta}{2 \cos \theta} \left( x - 5 \cos \theta \right)
\]

Correct straight line method (any complete method) \textbf{Must} use their gradient of the normal. \( \text{M1} \)

\[
5x \sin \theta - 2y \cos \theta = 21 \sin \theta \cos \theta^* \quad \text{cso}
\]

### (b)

At \( Q, x = 0 \) \( \Rightarrow y = -\frac{21}{2} \sin \theta \) \( \text{B1} \)

\[
M \text{ is } \left( \frac{0 + 5 \cos \theta}{2}, \frac{2 \sin \theta - \frac{21}{4} \sin \theta}{2} \right)
\]

Correct mid-point method for at least one coordinate

\[
= \left( \frac{5}{2} \cos \theta, -\frac{17}{4} \sin \theta \right)
\]

Can be implied by a correct \( x \) coordinate \( \text{M1} \)

\[
L \text{ cuts } x\text{-axis at } \frac{21}{5} \cos \theta
\]

\[
\text{Area } OPM = OLP + OLM
\]

\[
= \frac{1}{2} \frac{21}{5} \cos \theta \cdot 2 \sin \theta + \frac{1}{2} \frac{21}{5} \cos \theta \cdot \frac{17}{4} \sin \theta
\]

\[
= \frac{105}{16} \sin 2\theta
\]

Or \ 6.5625 \sin 2\theta \ must be positive \( \text{A1} \)

(5)

(6)
<table>
<thead>
<tr>
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<th>Marks</th>
</tr>
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<tbody>
<tr>
<td><strong>2(b)</strong></td>
<td><strong>Alternative 1: Using Area OPM</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>See above for B1M1</td>
<td>B1 M1</td>
</tr>
<tr>
<td></td>
<td>Area $\Delta OPM = \frac{1}{2} \begin{vmatrix} 0 &amp; 5\cos\theta &amp; \frac{5}{2}\cos\theta &amp; 0 \ 0 &amp; 2\sin\theta &amp; -\frac{12}{7}\sin\theta &amp; 0 \end{vmatrix}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2} \left( 0 + 5\sin\theta\cos\theta + 0 - 0 + \frac{85}{4}\sin\theta\cos\theta - 0 \right)$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{105}{4} \sin\theta\cos\theta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \frac{105}{16} \sin 2\theta$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td><em>(6)</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Alternative 2: Using Area OPQ</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>At Q, $x = 0 \Rightarrow y = -\frac{21}{2}\sin\theta$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Area $\Delta OPQ = \frac{1}{2} \begin{vmatrix} 5\cos\theta &amp; 0 \ 2\sin\theta &amp; -\frac{21}{2}\sin\theta \end{vmatrix}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2} \times \frac{105}{2} \sin\theta\cos\theta$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{105}{8} \sin 2\theta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area $OPM = \frac{1}{2} \times $ Area $OPQ$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{105}{16} \sin 2\theta$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td><em>(6)</em></td>
<td></td>
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</tbody>
</table>
### Question 2(b) continued

**Alternative 3**

At $Q$, $x = 0 \implies y = -\frac{21}{2} \sin \theta$

$M$ is \( \left( \frac{0 + 5 \cos \theta}{2}, \frac{2 \sin \theta - \frac{21}{4} \sin \theta}{2} \right) \) \quad \left( = \left( \frac{5}{2} \cos \theta, -\frac{17}{4} \sin \theta \right) \right) \quad \text{B1}

\[ OP = \sqrt{4 \sin^2 \theta + 25 \cos^2 \theta} \left( = \sqrt{4 + 21 \cos^2 \theta} \right) \quad \text{B1} \]

\[ d = \frac{5 \cos \theta \times \frac{2 \sin \theta}{5 \cos \theta} + \frac{17}{4} \sin \theta}{\sqrt{\frac{4 \sin^2 \theta}{25 \cos^2 \theta} + 1}} = \frac{21}{4} \sin \theta \quad \text{M1} \]

\[ \text{Area} = \frac{1}{2} \times \frac{21 \sin \theta}{\sqrt{\frac{4 + 21 \cos^2 \theta}{25 \cos^2 \theta}}} \times \sqrt{4 + 21 \cos^2 \theta} \quad \text{M1 A1} \]

\[ = \frac{105}{16} \sin 2\theta \quad \text{A1} \]

(11 marks)
### Question 3(a)

\[ x^2 + 4x + 13 \equiv (x + 2)^2 + 9 \]

\[
\int \frac{1}{(x + 2)^2 + 9} \, dx = \frac{1}{3} \arctan \left( \frac{x + 2}{3} \right) \\
M1: \arctan f(x) \\
A1: \text{Correct expression}
\]

\[
\left[ \frac{1}{3} \arctan \left( \frac{x + 2}{3} \right) \right]_{-2}^{1} = \frac{1}{3} (\arctan 1 - \arctan 0) \\
\text{Correct use of limits} \\
\text{arctan0 need not be shown}
\]

\[ \frac{\pi}{12} \]

cao

A1

(5)

### Alternative

**Sub** \( x + 2 = 3 \tan t \)

\[ x^2 + 4x + 13 \equiv (x + 2)^2 + 9 \]

\[
\frac{dx}{dt} = 3 \sec^2 t \\
\int \frac{3 \sec^2 t}{9 \tan^2 t + 9} \, dt = \frac{1}{3} \int \frac{1}{3} \, dt = \frac{1}{3} t \\
M1 \text{ sub and integrate inc use of } \tan^2 + 1 = \sec^2 \\
A1 \text{ Correct expression} \text{ Ignore limits}
\]

\[ \left[ \frac{\pi}{4} \right]_{0}^{\frac{3}{2}} \]

Either change limits and substitute

Or reverse substitution and substitute original limits

\[ \frac{\pi}{12} \]

cao

A1

(5)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(b)</strong></td>
<td>4x^2 - 12x + 34 = 4 \left( x - \frac{3}{2} \right)^2 + 25</td>
<td>M1: 4(x \pm p)^2 \pm q, (p, q \neq 0)</td>
</tr>
<tr>
<td></td>
<td>or ( (2x - 3)^2 + 25 )</td>
<td>A1: 4 \left( x - \frac{3}{2} \right)^2 + 25</td>
</tr>
<tr>
<td></td>
<td>( \int \frac{1}{\sqrt{4 \left( x - \frac{3}{2} \right)^2 + 25}} , dx = \frac{1}{2} \int \frac{1}{\sqrt{\left( x - \frac{3}{2} \right)^2 + \frac{25}{4}}} , dx = \frac{1}{2} \text{arsinh} \left( \frac{x - \frac{3}{2}}{\frac{5}{2}} \right) )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>M1: \text{arsinh} f(x). A1: Correct expression</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[ \left[ \frac{1}{2} \text{arsinh} \left( \frac{x - \frac{3}{2}}{\frac{5}{2}} \right) \right]_{-1}^{4} = \frac{1}{2} \left( \text{arsinh} (1) - \text{arsinh} (-1) \right) ]</td>
<td>\text{Correct use of limits}</td>
</tr>
<tr>
<td></td>
<td>= \frac{1}{2} \left( \ln (1 + \sqrt{2}) - \ln (-1 + \sqrt{2}) \right)</td>
<td>\text{Uses the logarithmic form of \text{arsinh}}</td>
</tr>
<tr>
<td></td>
<td>= \frac{1}{2} \ln (3 + 2\sqrt{2}) or \ln (1 + \sqrt{2})</td>
<td>cao</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7)</td>
</tr>
<tr>
<td><strong>Alternative: Second M1 A1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>\text{Sub } 2x - 3 = u \text{ or } 2x - 3 = 5 \sinh u</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \int \text{arsinh} \left( \frac{1}{\sqrt{25 \sinh^2 u + 25}} \right) , 5 \cosh u , du = \left[ \frac{1}{2} \text{arsinh} \left( \frac{u}{5} \right) \right]_{-5}^{5} )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( \int_{-5}^{5} \frac{1}{\sqrt{u^2 + 25}} , du = \left[ \frac{1}{2} \text{arsinh} \left( \frac{u}{5} \right) \right]_{-5}^{5} )</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
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</tr>
<tr>
<td><strong>4(a)</strong></td>
<td>$M = \begin{pmatrix} 1 &amp; k &amp; 0 \ -1 &amp; 1 &amp; 1 \ 1 &amp; k &amp; 3 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>M</td>
<td>= 3 - k - k(-3 - 1)(= 3k + 3)$</td>
</tr>
<tr>
<td>$M^T = \begin{pmatrix} 1 &amp; -1 &amp; 1 \ k &amp; 1 &amp; k \ 0 &amp; 1 &amp; 3 \end{pmatrix}$ or minors $\begin{pmatrix} 3 - k &amp; -4 &amp; -k - 1 \ 3k &amp; 3 &amp; 0 \ k &amp; 1 &amp; 1 + k \end{pmatrix}$</td>
<td>M1: Identifiable full attempt at inverse including reciprocal of determinant. Could be indicated by at least 6 correct elements. A1ft: Two rows or two columns correct (follow through their determinant but not incorrect entries in the matrices used) A1ft: Fully correct inverse (follow through as before)</td>
<td>M1 A1ft A1ft</td>
</tr>
<tr>
<td>$M^{-1} = \frac{1}{3 + 3k} \begin{pmatrix} 3 - k &amp; -3k &amp; k \ 4 &amp; 3 &amp; -1 \ -k - 1 &amp; 0 &amp; 1 + k \end{pmatrix}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$MN = \begin{pmatrix} 3 &amp; 5 &amp; 6 \ 4 &amp; -1 &amp; 1 \ 3 &amp; 2 &amp; -3 \end{pmatrix}$ $\Rightarrow N = M^{-1} \begin{pmatrix} 3 &amp; 5 &amp; 6 \ 4 &amp; -1 &amp; 1 \ 3 &amp; 2 &amp; -3 \end{pmatrix}$</td>
<td>Correct statement</td>
</tr>
<tr>
<td>$N = \frac{1}{3} \begin{pmatrix} 3 &amp; 0 &amp; 0 \ 4 &amp; 3 &amp; -1 \ -1 &amp; 0 &amp; 1 \end{pmatrix} \begin{pmatrix} 3 &amp; 5 &amp; 6 \ 4 &amp; -1 &amp; 1 \ 3 &amp; 2 &amp; -3 \end{pmatrix} = \begin{pmatrix} 3 &amp; 5 &amp; 6 \ 7 &amp; 5 &amp; 10 \ 0 &amp; -1 &amp; -3 \end{pmatrix}$</td>
<td>M1: Multiplies the given matrix by their $M^{-1}$ in the correct order Must include the &quot;$\frac{1}{3}$&quot; A2: Correct matrix $(-1$ each error). If left with $\frac{1}{3}$ outside the matrix award A0</td>
<td>M1 A(2, 1, 0)</td>
</tr>
</tbody>
</table>

**NB:** If every element is the negative of the correct element, allow M1A1A0

|  |  | (5) |
| (b) |  | | | (9 marks) |
**Question** 5(a)  
\( y = \text{artanh}(\cos x) \)

\[
\frac{dy}{dx} = \frac{1}{1 - \cos^2 x} \times -\sin x
\]
Correct use of the chain rule  
M1

\[
= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\csc x
\]
A1: Correct completion with no errors  
A1

**Alternative 1**

\[
\tanh y = \cos x \Rightarrow \text{sech}^2 y \frac{dy}{dx} = -\sin x
\]

\[
\frac{dy}{dx} = -\sin x \quad \text{sech}^2 y = -\sin x \quad 1 - \cos^2 x
\]
Correct differentiation to obtain a function of \( x \)  
M1

\[
= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\csc x
\]
A1: Correct completion with no errors  
A1

**Alternative 2**

\[
\text{artanh}(\cos x) = \frac{1}{2} \ln \left( \frac{1 + \cos x}{1 - \cos x} \right)
\]

\[
\frac{dy}{dx} = \frac{1}{2} \times \frac{1 - \cos x}{1 + \cos x} \times -\sin x \left( 1 - \cos x \right) - \sin x \left( 1 + \cos x \right) \quad \text{Correct differentiation to obtain a function of } x \quad \text{M1}
\]

\[
= \frac{-2\sin x}{2 \left( 1 - \cos^2 x \right)} = -\csc x
\]
A1: Correct completion with no errors  
A1

**Question** (b)  
\[ \int \cos x \, \text{artanh} (\cos x) \, dx = \sin x \, \text{artanh} (\cos x) - \int \sin x \times -\csc x \, dx \]
M1: Parts in the correct direction  
A1

\[
\left[ \sin x \, \text{artanh} (\cos x) + x \right]_0^\pi = \frac{1}{2} \, \text{artanh} \left( \frac{\sqrt{3}}{2} \right) + \frac{\pi}{6} \left( -0 \right)
\]
M1: Correct use of limits on either part (provided both parts are integrated). Lower limit need not be shown

\[
= \frac{1}{4} \ln \left( 1 + \frac{\sqrt{3}}{2} \right) + \frac{\pi}{6}
\]
Use of the logarithmic form of \( \text{artanh} \)  
M1

\[
= \frac{1}{4} \ln \left( 7 + 4\sqrt{3} \right) + \frac{\pi}{6} \quad \text{or} \quad \frac{1}{2} \ln \left( 2 + \sqrt{3} \right) + \frac{\pi}{6}
\]
Cao (oe)  
A1

The last 2 M marks may be gained in reverse order.  
(5)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td>$\overrightarrow{AB} = \begin{pmatrix} -2 \ 1 \ 1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 1 \ -1 \ 3 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 3 \ -2 \ 2 \end{pmatrix}$</td>
<td>Two correct vectors in $\Pi$ Can be negatives of those shown</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\begin{vmatrix} i &amp; j &amp; k \ -2 &amp; 1 &amp; 1 \ 1 &amp; -1 &amp; 3 \end{vmatrix} = \begin{pmatrix} 4 \ 7 \ 1 \end{pmatrix}$</td>
<td>M1: Attempt cross product of two vectors lying in $\Pi$ (At least one no. to be correct.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$\begin{pmatrix} 4 \ 7 \ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \ 2 \ 3 \end{pmatrix} = 4 + 14 + 3$</td>
<td>Attempt scalar product with their normal and a point in the plane</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$4x + 7y + z = 21$</td>
<td>Cao (oe)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>Alternative 1</td>
<td>$a + 2b + 3c = d$</td>
<td>Correct equations</td>
</tr>
<tr>
<td></td>
<td>$-a + 3b + 4c = d$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$2a + b + 6c = d$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a = \frac{4}{21}d$, $b = \frac{1}{3}d$, $c = \frac{1}{21}d$</td>
<td>M1: Solve for $a$, $b$ and $c$ in terms of $d$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$d = 21 \Rightarrow a = \ldots$, $b = \ldots$, $c = \ldots$</td>
<td>Obtains values for $a$, $b$, $c$ and $d$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$4x + 7y + z = 21$</td>
<td>Cao (oe)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>Alternative 2: Using $\mathbf{r} = a + b\mathbf{s} + c\mathbf{t}$ where $\mathbf{b}$ and $\mathbf{c}$ are vectors in $\Pi$</td>
<td>Two correct vectors in the plane</td>
<td>See main scheme</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{r} = \begin{pmatrix} x \ y \ z \end{pmatrix} = \begin{pmatrix} 1 \ 2 \ 3 \end{pmatrix} + s \begin{pmatrix} -2 \ 1 \ 1 \end{pmatrix} + t \begin{pmatrix} 1 \ -1 \ 3 \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$x = 1 - 2s + t$</td>
<td>Deduce 3 correct equations</td>
</tr>
<tr>
<td></td>
<td>$y = 2 + s - t$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$z = 3 + s + 3t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4x + 7y + z = 21$</td>
<td>M1: Eliminate $s$, $t$</td>
</tr>
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<td></td>
<td></td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$4x + 7y + z = 21$</td>
<td>A1: Cao</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(5)</td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
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</tr>
<tr>
<td>6(b)</td>
<td>$\mathbf{AD} \cdot \mathbf{AB} \times \mathbf{AC}$</td>
<td>Attempt suitable triple product</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$M_1$</td>
</tr>
<tr>
<td></td>
<td>$= \begin{pmatrix} 4 \ 7 \end{pmatrix} \begin{pmatrix} k - 1 \ 2 \ 11 \end{pmatrix}$</td>
<td>$M_1$</td>
</tr>
<tr>
<td></td>
<td>$= 4k - 4 + 14 + 11$</td>
<td>$dM_1$</td>
</tr>
<tr>
<td>$\therefore \frac{1}{6} (4k + 21) = 6$</td>
<td>$M_1$: Set $\frac{1}{6}$ (their triple product) = 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_1$: Correct equation</td>
<td></td>
</tr>
<tr>
<td>$k = \frac{15}{4}$</td>
<td>Cao (oe)</td>
<td></td>
</tr>
<tr>
<td><strong>Alternative</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area $ABC$</td>
<td>Attempt area $ABC$ and distance between $D$ and $II$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2} \left</td>
<td>\mathbf{AB} \right</td>
</tr>
<tr>
<td></td>
<td>$D$ to $II$ is $\frac{4k + 28 + 14 - 21}{\sqrt{16 + 49 + 1}}$</td>
<td>$dM_1$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{6} \sqrt{6} \sqrt{11} \frac{4k + 28 + 14 - 21}{\sqrt{16 + 49 + 1}} = 6$</td>
<td>$A_1$: Correct equation</td>
</tr>
<tr>
<td></td>
<td>$k = \frac{15}{4}$</td>
<td>Cao (oe)</td>
</tr>
<tr>
<td><strong>(4)</strong></td>
<td></td>
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<tr>
<td><strong>(9 marks)</strong></td>
<td></td>
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</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
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</tr>
<tr>
<td><strong>7(a)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = 3t^4, \quad y = 4t^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{dx}{dt} = 12t^3, \quad \frac{dy}{dt} = 12t^2$</td>
<td>Correct derivatives</td>
<td>B1</td>
</tr>
<tr>
<td>$S = (2\pi) \int y \left( \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right)^{\frac{1}{2}} dt = (2\pi) \int 4t^3 \left( (12t^3)^2 + (12t^2)^2 \right)^{\frac{1}{2}} dt$</td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td>$\left{ (2\pi) \int 4t^3 \left( 144t^6 + 144t^4 \right)^{\frac{1}{2}} dt \right}$</td>
<td>M1: Substitutes their derivatives into a correct formula (2\pi not required)</td>
<td></td>
</tr>
<tr>
<td>$S = (2\pi) \int 4t^3 \left( 144t^6 \right)^{\frac{1}{2}} dt$</td>
<td>Attempt to factor out at least $t^4$ - numerical factor may be left</td>
<td>M1</td>
</tr>
<tr>
<td>$S = 96\pi \int_0^1 t^5 \left( t^2 + 1 \right)^{\frac{1}{2}} dt$</td>
<td>Correct completion</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u^2 = t^2 + 1 \Rightarrow 2u \frac{du}{dt} = 2t$ or $2u = 2t \frac{dr}{du}$</td>
<td>Correct differentiation</td>
<td>B1</td>
</tr>
<tr>
<td>$t = 0 \Rightarrow u = 1, \quad t = 1 \Rightarrow u = \sqrt{2}$</td>
<td>Correct limits</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Alternative:</td>
<td>Reverse the substitution later. (Treat as M1 in this case and award later when work seen)</td>
</tr>
<tr>
<td>$S = (96\pi) \int t^5 \times u \times \frac{u}{t} du$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S = (96\pi) \int (u^2 - 1)^2 \times u^2 du$</td>
<td>M1: Complete substitution</td>
<td></td>
</tr>
<tr>
<td>$S = (96\pi) \int (u^6 - 2u^4 + u^2) du = (96\pi) \left[ \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} \right]$</td>
<td>A1: Correct integral in terms of $u$. Ignore limits, need not be simplified</td>
<td>M1 A1</td>
</tr>
<tr>
<td>M1: Expands and attempts to integrate</td>
<td>dM1</td>
<td></td>
</tr>
<tr>
<td>$S = 96\pi \left[ \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} \right] \bigg</td>
<td>_{u} = 96\pi \left{ \left[ \frac{\sqrt{2}}{7} - \frac{2\sqrt{2}}{5} + \frac{\sqrt{2}}{3} \right] - \left( \frac{1}{7} - \frac{2}{5} + \frac{1}{3} \right) \right}$</td>
<td>ddM1</td>
</tr>
<tr>
<td>M1: Correct use of their changed limits (both to be changed)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternative: If sub reversed, substitute the original limits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S = \frac{192\pi}{105} (11\sqrt{2} - 4)$</td>
<td>Cao eg $\frac{64\pi}{35}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
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<td>(7)</td>
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<tr>
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<td></td>
<td>(11 marks)</td>
</tr>
</tbody>
</table>
### Question 8(a)

The integral is given by:

\[ I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geq 0 \]

1. **Scheme:**
   - **Marks:** B1

   \[ \tanh^{2n} x = \tanh^{2(n-1)} x \tanh^2 x \]

2. **Scheme:**
   - **Marks:** M1

   \[ \tanh^{2n} x = \pm \tanh^{2(n-1)} x \left(1 - \text{sech}^2 x\right) \]

3. **Scheme:**
   - **Marks:** B1

   \[ I_n = \int_0^{\ln 2} \tanh^{2(n-1)} x \, dx - \int_0^{\ln 2} \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx \]

4. **Scheme:**
   - **Marks:** M1 A1

   \[ I_n = I_{n-1} - \left[ \frac{1}{2n-1} \tanh^{2n-1} x \right]_0^{\ln 2} \]

   \[ = I_{n-1} - \frac{1}{2n-1} \left( \frac{3}{5} \right)^{2n-1} \]

   **M1:** Correctly substitutes for \( I_{n-1} \) and obtains

   \[ \int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k \tanh^{2n-1} x \]

   **A1:** Correct expression

5. **Scheme:**
   - **Marks:** A1*

   Correct completion with no errors

---

### Alternative

The integral is given by:

\[ I_n - I_{n-1} = \int_0^{\ln 2} \left( \tanh^{2n} x - \tanh^{2(n-1)} x \right) \, dx \]

6. **Scheme:**
   - **Marks:** B1

   \[ = \int_0^{\ln 2} \tanh^{2(n-1)} x \left( \tanh^2 x - 1 \right) \, dx \]

7. **Scheme:**
   - **Marks:** M1

   \[ = \int_0^{\ln 2} \tanh^{2(n-1)} x \left( - \text{sech}^2 x \right) \, dx \]

   **M1:**

   \[ \int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k \tanh^{2n-1} x \]

   **A1:** Correct expression

8. **Scheme:**
   - **Marks:** A1*

   Correct completion with no errors

---

(5 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8(b)</strong></td>
<td>$I_0 = \ln 2$</td>
<td>The integration must be seen. B1</td>
</tr>
<tr>
<td></td>
<td>$I_2 = I_1 - \frac{1}{3} \left( \frac{3}{5} \right)^3$</td>
<td>Applies the reduction formula once M1</td>
</tr>
<tr>
<td></td>
<td>$I_2 = I_0 - \frac{1}{1} \left( \frac{3}{5} \right)^3 - \frac{1}{3} \left( \frac{3}{5} \right)^3$</td>
<td>M1: Second application of the reduction formula M1A1</td>
</tr>
<tr>
<td></td>
<td>$I_2 = \ln 2 - \frac{84}{125}$</td>
<td>cao A1</td>
</tr>
</tbody>
</table>

**Special Case:** If $I_4$ is found award B1 for $I_0$ or $I_1$ and M1M0A0A0

**(5) Alternative**

$I_1 = \int_0^{\ln 2} \tanh^2 x \, dx = \int_0^{\ln 2} \left( 1 - \text{sech}^2 x \right) \, dx$

$I_1 = [x - \tanh x]_0^{\ln 2}$ Correct integration B1

$I_2 = I_1 - \frac{1}{3} \left( \frac{3}{5} \right)^3$ Applies the reduction formula once M1

$I_1 = \ln 2 - \tanh (\ln 2) = \ln 2 - \frac{3}{5}$ M1: Uses limits A1: Correct expression M1A1

$I_2 = \ln 2 - \frac{3}{5} - \frac{1}{3} \left( \frac{3}{5} \right)^3$

$= \ln 2 - \frac{84}{125}$ A1

**(5) (10 marks)**
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

• Use black ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

• Fill in the boxes at the top of this page with your name, centre number and candidate number.

• Answer all questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions in the spaces provided – there may be more space than you need.

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Inexact answers should be given to three significant figures unless otherwise stated.

Information

• A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

• There are 7 questions in this question paper. The total mark for this paper is 75.

• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

• If you change your mind about an answer, cross it out and put your new answer and any working underneath.
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Turn over
1. A car is moving along a straight horizontal road with constant acceleration $a \text{ m s}^{-2} (a > 0)$. At time $t = 0$ the car passes the point $P$ moving with speed $u \text{ m s}^{-1}$. In the next 4 s, the car travels 76 m and then in the following 6 s it travels a further 219 m.

Find

(i) the value of $u$,

(ii) the value of $a$. (7)
Question 1 continued

(Total for Question 1 is 7 marks)
2. Two particles $P$ and $Q$ are moving in opposite directions along the same horizontal straight line. Particle $P$ has mass $m$ and particle $Q$ has mass $km$. The particles collide directly. Immediately before the collision, the speed of $P$ is $u$ and the speed of $Q$ is $2u$. As a result of the collision, the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of $k$. 

(b) Find, in terms of $m$ and $u$ only, the magnitude of the impulse exerted on $Q$ by $P$ in the collision.
Question 2 continued

(Total for Question 2 is 6 marks)
3. A block $A$ of mass 9 kg is released from rest from a point $P$ which is a height $h$ metres above horizontal soft ground. The block falls and strikes another block $B$ of mass 1.5 kg which is on the ground vertically below $P$. The speed of $A$ immediately before it strikes $B$ is 7 m s$^{-1}$. The blocks are modelled as particles.

(a) Find the value of $h$.  

Immediately after the impact the blocks move downwards together with the same speed and both come to rest after sinking a vertical distance of 12 cm into the ground. Assuming that the resistance offered by the ground has constant magnitude $R$ newtons,

(b) find the value of $R$. 


Question 3 continued
Question 3 continued
A diving board $AB$ consists of a wooden plank of length 4 m and mass 30 kg. The plank is held at rest in a horizontal position by two supports at the points $A$ and $C$, where $AC = 0.6$ m, as shown in Figure 1. The force on the plank at $A$ acts vertically downwards and the force on the plank at $C$ acts vertically upwards.

A diver of mass 50 kg is standing on the board at the end $B$. The diver is modelled as a particle and the plank is modelled as a uniform rod. The plank is in equilibrium.

(a) Find

(i) the magnitude of the force acting on the plank at $A$, 

(ii) the magnitude of the force acting on the plank at $C$. 

(b) Find, in kg, the greatest integer mass of a diver who can stand on the board at $B$ without breaking the support at $A$.

(c) Explain how you have used the fact that the diver is modelled as a particle.
Question 4 continued
Question 4 continued
Question 4 continued

(Total for Question 4 is 10 marks)
5. Two forces, \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \), act on a particle \( A \).

\[
\mathbf{F}_1 = (2\mathbf{i} - 3\mathbf{j}) \text{ N and } \mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j}) \text{ N, where } p \text{ and } q \text{ are constants.}
\]

Given that the resultant of \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \) is parallel to \( (i + 2j) \),

(a) show that \( 2p - q + 7 = 0 \) \( (5) \)

Given that \( q = 11 \) and that the mass of \( A \) is 2 kg, and that \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \) are the only forces acting on \( A \),

(b) find the magnitude of the acceleration of \( A \). \( (5) \)
Question 5 continued

(Total for Question 5 is 10 marks)
6.

\[ P \]

\[ Q \]

\[ \text{Figure 2} \]

Two cars, \( A \) and \( B \), move on parallel straight horizontal tracks. Initially \( A \) and \( B \) are both at rest with \( A \) at the point \( P \) and \( B \) at the point \( Q \), as shown in Figure 2. At time \( t = 0 \) seconds, \( A \) starts to move with constant acceleration \( a \) m s\(^{-2}\) for 3.5 s, reaching a speed of 14 m s\(^{-1}\). Car \( A \) then moves with constant speed 14 m s\(^{-1}\).

(a) Find the value of \( a \).  

(2)

Car \( B \) also starts to move at time \( t = 0 \) seconds, in the same direction as car \( A \). Car \( B \) moves with a constant acceleration of 3 m s\(^{-2}\). At time \( t = T \) seconds, \( B \) overtakes \( A \). At this instant \( A \) is moving with constant speed.

(b) On a diagram, sketch, on the same axes, a speed-time graph for the motion of \( A \) for the interval \( 0 \leq t \leq T \) and a speed-time graph for the motion of \( B \) for the interval \( 0 \leq t \leq T \).  

(3)

(c) Find the value of \( T \).  

(8)

(d) Find the distance of car \( B \) from the point \( Q \) when \( B \) overtakes \( A \).  

(1)

(e) On a new diagram, sketch, on the same axes, an acceleration-time graph for the motion of \( A \) for the interval \( 0 \leq t \leq T \) and an acceleration-time graph for the motion of \( B \) for the interval \( 0 \leq t \leq T \).  

(3)
Question 6 continued
Question 6 continued

(Total for Question 6 is 17 marks)
7.

A particle $P$ of mass 4 kg is attached to one end of a light inextensible string. A particle $Q$ of mass $m$ kg is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at a point on the intersection of two fixed inclined planes. The string lies in a vertical plane that contains a line of greatest slope of each of the two inclined planes. The first plane is inclined to the horizontal at an angle $\alpha$, where $\tan \alpha = \frac{3}{4}$ and the second plane is inclined to the horizontal at an angle $\beta$, where $\tan \beta = \frac{4}{3}$. Particle $P$ is on the first plane and particle $Q$ is on the second plane with the string taut, as shown in Figure 3.

The first plane is rough and the coefficient of friction between $P$ and the plane is $\frac{1}{4}$. The second plane is smooth. The system is in limiting equilibrium.

Given that $P$ is on the point of slipping down the first plane,

(a) find the value of $m$, \hspace{1cm} (10)

(b) find the magnitude of the force exerted on the pulley by the string, \hspace{1cm} (4)

(c) find the direction of the force exerted on the pulley by the string. \hspace{1cm} (1)
Question 7 continued
Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS
## Mechanics M1 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[76 = 4u + \frac{1}{2}a \cdot 4^2 \quad \text{or} \quad 76 = \frac{1}{2}(u + u + 4a) \times 4] Use of (s = ut + \frac{1}{2}at^2) for (t = 4, s = 76) and (u \neq 0) (use of (u = 0) is M0)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>((38 = 2u + 4a)) Correctly substituted equation</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[295 = 10u + \frac{1}{2}a \cdot 10^2] (\text{or} \quad 295 = \frac{1}{2}(u + u + 10a) \times 10] (\text{or} \quad 295 = (u + 10a) \times 10 - \frac{1}{2}a \times 100] Use of (s = ut + \frac{1}{2}at^2) for (t = 10, s = 295) (\text{or} \quad s = u't + \frac{1}{2}at^2) for (t = 6, s = 219, u' \neq u)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>((59 = 2u + 10a)) Correctly substituted equation</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(\text{or} \quad 219 = (19 + 2a) \times 6 + \frac{1}{2}a \times 6^2] (\text{or} \quad 219 = (38 - u) \times 6 + \frac{1}{2}a \times 6^2] (\text{or} \quad 219 = (u + 4a) \times 6 + \frac{1}{2}a \times 6^2] (\text{or} \quad 219 = \frac{1}{2}(u + 4a + u + 10) \times 6] (\text{or} \quad 219 = (u + 10a) \times 6 - \frac{1}{2}a \times 36) Solve simultaneous for (u) or for (a). This marks is not available if they have assumed a value for (u) or (a) in the preceding work - it is dependent on the first 2 M marks.</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>(u = 12)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(a = 3.5)</td>
<td>A1</td>
</tr>
<tr>
<td><strong>Alternative</strong></td>
<td></td>
<td>(7)</td>
</tr>
<tr>
<td></td>
<td>(t = 2, \ v_2 = \frac{76}{4} = 19) (t = 7, \ v_7 = \frac{219}{6} = 36.5) Find the speed at (t = 2, t = 7) Both values correct Averages with no links to times is M0</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(36.5 = 19 + 5a \Rightarrow a = 3.5) Use of (v = u + 5a) with their (u, v) Correct (a)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(19 = u + 2a) Complete method for finding (u) Correct equation in (u)</td>
<td>DM1 A1</td>
</tr>
<tr>
<td></td>
<td>(u = 19 - 7 = 12)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7)</td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
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<tr>
<td>----------</td>
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<tr>
<td>2(a)</td>
<td>$mu - 2kmu = \frac{1}{2}mu + kmu$ or $m\left(\frac{1}{2}u + u\right) = km\left(-u - 2u\right)$</td>
<td>Use of CLM or Equal and opposite impulses. Need all 4 terms dimensionally correct. Masses and speeds must be paired correctly. Condone sign errors. Condone factor of g throughout.</td>
</tr>
<tr>
<td></td>
<td>Un simplified equation with at most one error</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Correct unsimplified equation</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$k = \frac{1}{2}$ From correct working only</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td>(b)</td>
<td>Impulse on $P$ or impulse on $Q$. Mass must be used with the correct speeds. E.g. $km \times \frac{1}{2}u$ is M0</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{3mu}{2}$ Only From correct working only</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
</tbody>
</table>

(6 marks)
<table>
<thead>
<tr>
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<th>Scheme</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>3(a)</strong></td>
<td>$7^2 = 2 \times 9.8h$</td>
<td>Use of $v^2 = u^2 + 2as$ with $u = 0, v = 7$ or alternative complete method to find $h$</td>
</tr>
<tr>
<td></td>
<td>$h = 2.5$</td>
<td>Condone $h = -2.5$ in the working but the final answer must be positive.</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$9 \times 7 = 10.5u$</td>
<td>Use CLM to find the speed of the blocks after the impact. Condone additional factor of $g$ throughout.</td>
</tr>
<tr>
<td></td>
<td>$u = 6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0^2 = 6^2 - 2a \times 0.12$</td>
<td>Use of $v^2 = u^2 + 2as$ with $u = 6, v = 0$ Allow for their $u$ and $v = 0$ Allow for $u = 7, v = 0$ Accept alternative $suvat$ method to form an equation in $a$. Condone use of 12 for 0.12</td>
</tr>
<tr>
<td></td>
<td>Correctly substituted equation in $a$ with $u = 6, s = 0.12$ (implied by $a = 150$)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$(\downarrow) 10.5g - R = 10.5 \times (-a)$</td>
<td>Use of $F = ma$ with their $a \neq \pm g$. Must have all 3 terms and 10.5 Condone sign error(s)</td>
</tr>
<tr>
<td></td>
<td>$(\downarrow) 10.5g - R = 10.5 \times (-150)$</td>
<td>Unsimplified equation with $a$ substituted and at most one error (their $a$ with the wrong sign is 1 error)</td>
</tr>
<tr>
<td></td>
<td>Correct unsimplified equation with $a$ substituted</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$R = 1680$ or 1700</td>
<td>A1</td>
</tr>
<tr>
<td><strong>Alternative for the last 6 marks:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2} \times 10.5 \times 6^2 + 10.5 \times 9.8 \times 0.12 = R \times 0.12$</td>
<td>Energy equation (needs all three terms)</td>
</tr>
<tr>
<td></td>
<td>$-1$ each error A1A1A0 for 1 error, A1A0A0 for 2 errors</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R = 1680$ or 1700</td>
<td>A1</td>
</tr>
<tr>
<td>Question</td>
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<tr>
<td>----------</td>
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</tr>
<tr>
<td><strong>4(a)</strong></td>
<td><img src="" alt="Diagram" /></td>
<td></td>
</tr>
</tbody>
</table>

- **M(A)** \((30g \times 2) + (50g \times 4) = 0.6 \times S\)
  
  Moments equation. Requires all terms and dimensionally correct. Condone sign errors. Allow M1 if g missing. M1

- **M(C)** \((0.6 \times R) = (1.4 \times 30g) + (3.4 \times 50g)\)
  
  Correct unsimplified equation. A1

- **M(G)** \((2 \times R) = (1.4 \times S) + (2 \times 50g)\)
  
  **M(B)** \((4 \times R) + (2 \times 30g) = (3.4 \times S)\)

- \((R + 30g + 50g = S)\)

  Resolve vertically. Requires all 4 terms. Condone sign errors. M1

- Correct equation (with R or their R)
  
  NB: The second M1A1 can also be earned for a second moments equation. A1

- **R** = 3460 or 3500 or \(\frac{1060g}{3}\) (N)

  One force correct. A1

- Not 353.3g

- **S** = 4250 or 4200 or \(\frac{1300g}{3}\) (N)

  Both forces correct. If both forces are given as decimal multiples of g mark this as an accuracy penalty A0A1. A1

- Not 433.3g

  \((6)\)

<table>
<thead>
<tr>
<th><strong>(b)</strong></th>
<th></th>
<th></th>
</tr>
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- **M(C)** \((30g \times 1.4) + (Mg \times 3.4) = 0.6 \times 5000\)

  Use \(R = 5000\) and complete method to form an equation in \(M\) or weight. Needs all terms present and dimensionally correct. Condone sign errors. Accept inequality. Use of \(R\) and \(S\) from (a) is M0. M1

- **M = 77 kg**

  77.7 is A0 even if the penalty for over-specified answers has already been applied. A1

  \((3)\)
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<tbody>
<tr>
<td>4(c)</td>
<td>The weight of the diver acts at a point. Accept “the mass of the diver is at a point”.</td>
<td>B1(1)</td>
</tr>
</tbody>
</table>

(10 marks)
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<tbody>
<tr>
<td><strong>5(a)</strong></td>
<td>Resultant force = $F_1 + F_2$ in the form $ai + bj$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Use parallel vector to form a scalar equation in $p$ and $q$.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Correct equation (accept any equivalent form)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Depend on no errors seen in comparing the vectors.</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>Rearrange to obtain given answer.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Given Answer</td>
<td>A1</td>
</tr>
<tr>
<td><strong>5(b)</strong></td>
<td>$q = 11 \implies p = 2$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$R = 4i + 8j$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$(2 + p)i + 8j$ for their $p$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4i + 8j = 2a \ (a = 2i + 4j)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Use of $F = ma$</td>
<td></td>
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<tr>
<td></td>
<td>$</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{20} = 4.5$ or 4.47 or better (m s$^{-2}$)</td>
<td>A1</td>
</tr>
<tr>
<td><strong>Alternative</strong> for the last two M marks:</td>
<td></td>
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<tr>
<td></td>
<td>$</td>
<td>F</td>
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<tr>
<td></td>
<td>$\sqrt{80} = 2 \times</td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>Use of $</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>Dependent on the preceding M1</td>
<td></td>
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<tr>
<td></td>
<td>(10 marks)</td>
<td></td>
</tr>
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<td>Marks</td>
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<td>----------</td>
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</tr>
<tr>
<td><strong>6(a)</strong></td>
<td>( v = u + at \Rightarrow 14 = 3.5a )</td>
<td>Use of ( suvat ) to form an equation in ( a ) M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( a = 4 ) A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2)</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td></td>
<td>Graph for ( A ) or ( B ) B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Second graph correct and both graphs extending beyond the point of intersection B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Values 3.5, 14, ( T ) shown on axes, with ( T ) not at the point of intersection. Accept labels with delineators. B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>NB: 2 separate diagrams scores max B1B0B1 (3)</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>( \frac{1}{2}T.3T, \quad \frac{(T + T - 3.5)}{2}.14 )</td>
<td>Find distance for ( A ) or ( B ) in terms of ( T ) only. Correct area formulae: must see ( \frac{1}{2} ) in area formula and be adding in trapezium M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One distance correct A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Both distances correct A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \frac{1}{2}T.3T = \frac{(T + T - 3.5)}{2}.14 ) Equate distances and simplify to a 3 term quadratic in ( T ) in the form ( aT^2 + bT + c = 0 ) M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 3T^2 - 28T + 49 = 0 ) Correct quadratic A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( (3T - 7)(T - 7) = 0 ) Solve 3 term quadratic for ( T ) M1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( T = \frac{7}{3} ) or ( 7 ) Correct solution(s) - can be implied if only ever see ( T = 7 ) from correct work. A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>but ( T &gt; 3.5, \quad T = 7 ) A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8)</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>73.5 m</td>
<td>From correct work only. B0 if extra answers. B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>6(e)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Graph" /></td>
<td>(A) Condone missing 4</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(B) Condone graph going beyond $T = 7$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Must go beyond 3.5. Condone no 3.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(A) Condone graph going beyond $T = 7$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Must go beyond 3.5. B0 if see a solid vertical line.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sometimes very difficult to see. If you think it is there, give the mark.</td>
<td></td>
</tr>
</tbody>
</table>

Condone separate diagrams.

**Alternative for (c) for candidates with a sketch like this:**

![Graph](image)

\[
\frac{1}{2} \times 3 \times (T + 3.5)^2 = \frac{1}{2} \times 4 \times 3.5^2 + 14T
\]

Use diagram to find area M1

One distance correct A1

Both distances correct A1

\[12T^2 - 28T - 49 = 0\]

Simplify to a 3 term quadratic in $T$ M1

Correct quadratic A1

\[(2T - 7)(6T + 7) = 0\]

Complete method to solve for the $T$ in the question M1

Correct solution(s) - can be implied if only ever see Total = 7 A1

Total time = 7 A1

(17 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>7(a)</td>
<td>( F = 0.25R )</td>
</tr>
<tr>
<td></td>
<td>( \sin \alpha = \frac{3}{5} ) or ( \cos \alpha = \frac{4}{5} ) ( \sin \beta = \frac{2}{5} ) or ( \cos \beta = \frac{3}{5} )</td>
</tr>
<tr>
<td></td>
<td>( R = 4g \cos \alpha ) ( (31.36) )</td>
</tr>
<tr>
<td></td>
<td>Correct equation</td>
</tr>
<tr>
<td></td>
<td>( T + F = 4g \sin \alpha )</td>
</tr>
<tr>
<td></td>
<td>( T = mg \sin \beta )</td>
</tr>
<tr>
<td></td>
<td>( (T + 7.84 = 23.52) ) ( (T = 15.68) )</td>
</tr>
<tr>
<td></td>
<td>( T = 7.84m )</td>
</tr>
<tr>
<td></td>
<td>Solve for ( m )</td>
</tr>
<tr>
<td></td>
<td>( m = 2 )</td>
</tr>
<tr>
<td></td>
<td>NB Condone a whole system equation ( 4g \sin \alpha - F = mg \sin \beta ) followed by ( m = 2 ) for 6/6</td>
</tr>
<tr>
<td></td>
<td>M2 for an equation with all 3 terms. Condone trig confusion. Condone an acceleration ( \neq 0 )</td>
</tr>
<tr>
<td></td>
<td>A2 (-1 each error) for a correct equation:</td>
</tr>
<tr>
<td>7(b)</td>
<td>( F = \frac{\sqrt{T^2 + T^2}}{2} ) or ( 2T \cos 45^\circ ) or ( \frac{T}{\cos 45} )</td>
</tr>
<tr>
<td></td>
<td>Correct expression in ( T )</td>
</tr>
<tr>
<td></td>
<td>Substitute their ( T ) into a correct expression. Dependent on the previous M mark</td>
</tr>
<tr>
<td></td>
<td>( F = \sqrt{\frac{8g}{5}} = 22 ) or 22.2 (N)</td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>7(c)</td>
<td>Along the angle bisector at the pulley Or equivalent - accept angle + arrow shown on diagram. (8.1° to downward vertical) Do not accept a bearing</td>
</tr>
</tbody>
</table>
Along the angle bisector at the pulley or equivalent – accept angle + arrow shown on diagram.

Do not accept a bearing.
Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A car of mass 900 kg is travelling up a straight road inclined at an angle $\theta$ to the horizontal, where $\sin \theta = \frac{1}{25}$. The car is travelling at a constant speed of $14 \text{ m s}^{-1}$ and the resistance to motion from non-gravitational forces has a constant magnitude of 800 N. The car takes 10 seconds to travel from $A$ to $B$, where $A$ and $B$ are two points on the road.

   (a) Find the work done by the engine of the car as the car travels from $A$ to $B$. (4)

   When the car is at $B$ and travelling at a speed of $14 \text{ m s}^{-1}$ the rate of working of the engine of the car is suddenly increased to $P \text{ kW}$, resulting in an initial acceleration of the car of $0.7 \text{ m s}^{-2}$. The resistance to motion from non-gravitational forces still has a constant magnitude of 800 N.

   (b) Find the value of $P$. (4)
Question 1 continued
2. A particle $P$ of mass 0.7 kg is moving in a straight line on a smooth horizontal surface. The particle $P$ collides with a particle $Q$ of mass 1.2 kg which is at rest on the surface. Immediately before the collision the speed of $P$ is $6\text{ms}^{-1}$. Immediately after the collision both particles are moving in the same direction. The coefficient of restitution between the particles is $e$.

(a) Show that $e < \frac{7}{12}$

Given that $e = \frac{1}{4}$

(b) find the magnitude of the impulse exerted on $Q$ in the collision.
Question 2 continued
Question 2 continued
Question 2 continued

(Total for Question 2 is 10 marks)
3. At time $t$ seconds ($t \geq 0$) a particle $P$ has velocity $\mathbf{v}$ m s$^{-1}$, where

$$\mathbf{v} = (6t^2 + 6t)i + (3t^2 + 24)j$$

When $t = 0$ the particle $P$ is at the origin $O$. At time $T$ seconds, $P$ is at the point $A$ and $\mathbf{v} = \lambda \mathbf{i} + \mathbf{j}$, where $\lambda$ is a constant.

Find

(a) the value of $T$, 

(b) the acceleration of $P$ as it passes through the point $A$, 

(c) the distance $OA$. 

(Total for Question 3 is 11 marks)
Question 3 continued

(Total for Question 3 is 11 marks)
4.

Two particles $P$ and $Q$, of mass 2 kg and 4 kg respectively, are connected by a light inextensible string. Initially $P$ is held at rest at the point $A$ on a rough fixed plane inclined at $\alpha$ to the horizontal ground, where $\sin \alpha = \frac{3}{5}$. The string passes over a small smooth pulley fixed at the top of the plane. The particle $Q$ hangs freely below the pulley and 2.5 m above the ground, as shown in Figure 1. The part of the string from $P$ to the pulley lies along a line of greatest slope of the plane. The system is released from rest with the string taut. At the instant when $Q$ hits the ground, $P$ is at the point $B$ on the plane. The coefficient of friction between $P$ and the plane is $\frac{1}{4}$.

(a) Find the work done against friction as $P$ moves from $A$ to $B$. (4)

(b) Find the total potential energy lost by the system as $P$ moves from $A$ to $B$. (3)

(c) Find, using the work-energy principle, the speed of $P$ as it passes through $B$. (4)
Question 4 continued
Question 4 continued
Question 4 continued

(Total for Question 4 is 11 marks)
The uniform lamina $ABCDEF$, shown in Figure 2, consists of two identical rectangles with sides of length $a$ and $3a$. The mass of the lamina is $M$. A particle of mass $kM$ is attached to the lamina at $E$. The lamina, with the attached particle, is freely suspended from $A$ and hangs in equilibrium with $AF$ at an angle $\theta$ to the downward vertical.

Given that $\tan \theta = \frac{4}{7}$, find the value of $k$. (10)
Question 5 continued
Question 5 continued
Question 5 continued

(Total for Question 5 is 10 marks)
A uniform rod $AB$, of mass $3m$ and length $2a$, is freely hinged at $A$ to a fixed point on horizontal ground. A particle of mass $m$ is attached to the rod at the end $B$. The system is held in equilibrium by a force $F$ acting at the point $C$, where $AC = b$. The rod makes an acute angle $\theta$ with the ground, as shown in Figure 3. The line of action of $F$ is perpendicular to the rod and in the same vertical plane as the rod.

(a) Show that the magnitude of $F$ is $\frac{5mg\cos \theta}{b}$.

The force exerted on the rod by the hinge at $A$ is $R$, which acts upwards at an angle $\phi$ above the horizontal, where $\phi > \theta$.

(b) Find

(i) the component of $R$ parallel to the rod, in terms of $m$, $g$ and $\theta$,

(ii) the component of $R$ perpendicular to the rod, in terms of $a$, $b$, $m$, $g$ and $\theta$.

(c) Hence, or otherwise, find the range of possible values of $b$, giving your answer in terms of $a$. 

---

**Figure 3**

The diagram shows a uniform rod $AB$ with a particle attached at $B$. The rod makes an angle $\theta$ with the ground and is held in equilibrium by a force $F$ acting at point $C$. The rod is hinged at $A$. The line of action of $F$ is perpendicular to the rod. A particle of mass $m$ is attached to the end $B$.
Question 6 continued

(Total for Question 6 is 11 marks)
7. (5 marks)

Figure 4

At time $t = 0$, a particle $P$ of mass 0.7 kg is projected with speed $u$ m s$^{-1}$ from a fixed point $O$ at an angle $\theta^\circ$ to the horizontal. The particle moves freely under gravity. At time $t = 2$ seconds, $P$ passes through the point $A$ with speed 6 m s$^{-1}$ and is moving downwards at $45^\circ$ to the horizontal, as shown in Figure 4.

Find

(a) the value of $\theta$,  \hspace{1cm} (6)

(b) the kinetic energy of $P$ as it reaches the highest point of its path. \hspace{1cm} (3)

For an interval of $T$ seconds, the speed, $v$ m s$^{-1}$, of $P$ is such that $v \leq 6$

(c) Find the value of $T$. \hspace{1cm} (5)
Question 7 continued
Question 7 continued

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS
### Mechanics M2 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(a)</strong></td>
<td>Resolving parallel to the plane</td>
<td>Condone trig confusion</td>
</tr>
<tr>
<td>$D = 900g \sin \theta + 800$</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>$\frac{900}{25} \times g + 800(=1152.8) \text{ (N)}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Work done : Their $D \times $distance $=1152.8 \times 14 \times 10$</td>
<td>Independent. For use of $14 \times 10 \times$ their $D$</td>
<td>M1</td>
</tr>
<tr>
<td>$= 161392 = 161 \text{ kJ (160)}$</td>
<td>Accept $161000 \text{ (J), 160000 (J). Ignore incorrect units.}$</td>
<td>A1</td>
</tr>
</tbody>
</table>

**Alternative using energy**

Work done $= 900gd \sin \theta + 800d$ | Allow with incorrect $d$ | M1A1 |

Use of $d = 14 \times 10$ | Independent – allow in an incorrect expression | M1 |

$= 161392 = 161 \text{ kJ (160)}$ | | A1 |

(4 marks)

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(b)</strong></td>
<td>Equation of motion</td>
<td>All terms required. Condone trig confusion and sign errors. Allow with $900a$</td>
</tr>
<tr>
<td>$D - 900g \sin \theta - 800 = 900 \times 0.7$</td>
<td>Correct unsimplified with $a = 0.7$ used</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Accept with their $1152.8 \text{ arising from a 2 term expression in (a)}$</td>
<td></td>
</tr>
</tbody>
</table>

$D = 1782.8 \text{ (N)}$

Use of $P = Fv$ | $P = 14 \times \frac{\text{their } D}{1000}$ | Independent Treat missing $1000$ as misread, so allow for $14 \times$ their $D$ |

Allow for $\frac{1000P}{14}$ (or $\frac{P}{14}$) in their equation of motion | M1 |

$P = 25.0 \text{ (25)}$ | cao | A1 |

(4 marks)

(8 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2(a)</strong></td>
<td><img src="image" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>CLM: (0.7 \times 6 = 0.7 \times v + 1.2w)</td>
<td>Requires all terms &amp; dimensionally correct</td>
<td>M1</td>
</tr>
<tr>
<td>((42 = 7v + 12w))</td>
<td>Correct unsimplified</td>
<td>A1</td>
</tr>
<tr>
<td>Impact:</td>
<td>Used the right way round Condone sign errors</td>
<td>M1</td>
</tr>
<tr>
<td>(w - v = 6e)</td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>Equation in (e) and (v) only: (42 - 72e = 19v)</td>
<td>Dependent on the two previous M marks</td>
<td>DM1</td>
</tr>
<tr>
<td>Use direction to form an inequality:</td>
<td>Independent. Applied correctly for their (v)</td>
<td>M1</td>
</tr>
<tr>
<td>(42 - 72e &gt; 0 \Rightarrow e &lt; \frac{7}{12})</td>
<td><em>Given answer</em></td>
<td>A1</td>
</tr>
<tr>
<td><strong>(7)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2(b)</strong></td>
<td>Impulse on (Q): (I = w \times 1.2)</td>
<td>M1</td>
</tr>
<tr>
<td>Solve for (w): (w = v + 6e = \frac{42 - 72}{19} \times \frac{1}{4} + 6 \times \frac{1}{4})</td>
<td>Accept unsimplified with (e) substituted. Have to be using (w) in part (b) (w = \frac{105}{38} = 2.763\ldots) seen or implied</td>
<td>B1</td>
</tr>
<tr>
<td>(I = 1.2 \times \frac{42}{19} \times \frac{5}{4} = \frac{63}{19}(=3.32)) (N s)</td>
<td>3.3 or better</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(3)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Alternative</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Impulse on (Q = -) impulse on (P)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(= -0.7(v - 6))</td>
<td>Accept negative here</td>
<td>M1</td>
</tr>
<tr>
<td>(= -0.7 \left(\frac{42}{19} - \frac{72}{19} - 6\right))</td>
<td>Substitute for (e) in their (v) (v = \frac{24}{19} = 1.263\ldots) seen or implied Accept negative here.</td>
<td>B1</td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>2(b) (\text{continued}) [\frac{63}{19}]</td>
<td>Final answer must be positive. 3.3 or better</td>
<td>A1</td>
</tr>
</tbody>
</table>

(3)

(10 marks)
### Question 3(a)

Use \( \mathbf{v} = \lambda (\mathbf{i} + \mathbf{j}) \):

\[
6T^2 + 6T = 3T^2 + 24
\]

Form an equation in \( t, T \) or \( \lambda \)

\[
\lambda^2 - 108\lambda + 2592 = 0
\]

Simplify to quadratic in \( t, T \) or \( \lambda \) and solve.

\[
(T + 4)(T - 2) = 0, \quad T = 2
\]

\( T = 2 \) only

If they score M1 and then state \( T = 2 \) allow 3/3

If they guess \( T = 2 \) and show that it works then allow 3/3.

If all we see is \( T = 2 \) with no equation then 0/3 for (a) but full marks are available for (b) and (c).

(3)

### Question 3(b)

Differentiate: \( \mathbf{a} = (12t + 6)i + 6j \)

Majority of powers going down

Need to be considering both components

Correct in \( t \) or \( T \)

\( = 30i + 12j \) (m s\(^{-2}\))

Cao

A1

(3)

### Question 3(c)

Integrate:

\[
\mathbf{r} = \left(2r^3 + 3r^2 (+A)\right)i + \left(t^3 + 24t(+B)\right)j
\]

Clear evidence of integration.

Need to be considering both components.

Do not need to see the constant(s).

-1 each error

If the integration is seen in part (a) it scores no marks at that stage, but if the result is used in part (c) then the M1A2 is available in part (c)

\[
\mathbf{OA} = 28i + 56j \quad \text{Use their } T
\]

Distance = \( 28\sqrt{5} = 62.6 \) (m)

Use of Pythagoras on their \( \mathbf{OA} \)

DM1

63 or better , \( \sqrt{3920} \)

A1

NB: Incorrect \( T \) can score 2/3 in (b) and 4/5 in (c)

(5)

(11 marks)
<table>
<thead>
<tr>
<th>Question</th>
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<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4(a)</strong></td>
<td>Resolve perpendicular to the plane: ( R = 2g \cos \alpha )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Use ( F = \mu R : F = \frac{1}{4} \times 2g \times \frac{4}{5} = \frac{2g}{5} ) with ( \frac{1}{4} ) and their ( R ) (3.92)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Work done: ( WD = 2.5 \times F ) For their ( F ) dM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 2.5 \times \frac{2g}{5} = 9.8 , \text{(J)} ) Accept ( g ) A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>If a candidate has found the total work done but you can see the correct terms/processes for finding the work done against friction, give B1M1DM1A0 (3/4)</td>
<td></td>
</tr>
<tr>
<td><strong>4(b)</strong></td>
<td>Change in PE : ( \pm (4g \times 2.5 - 2g \times 2.5 \sin \alpha) ) Requires one gaining and one losing Condone trig confusion</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = \pm (4g \times 2.5 - 2g \times 1.5) ) ( \pm ) (correct unsimplified) A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PE lost = ( 7g = 68.6 , \text{(J)} ) or 69 (J) Accept 7g A1</td>
<td></td>
</tr>
<tr>
<td><strong>4(c)</strong></td>
<td>KE gained + WD = loss in GPE The question requires the use of work-energy. Alternative methods score 0/4. Requires all terms but condone sign errors (must be considering both particles)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{2} \times 4v^2 + \frac{1}{2} \times 2v^2 + \text{(their (a))} = \text{(their (b))} ) Correct unsimplified. -1 each error A2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 3v^2 = 6g )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v = \sqrt{2g} = 4.43 , \text{(m s}^{-1}) ) or 4.4. Accept ( \sqrt{2g} ) A1</td>
<td></td>
</tr>
<tr>
<td><strong>Alternative</strong></td>
<td>Equations of motion for each particle leading to ( T = \frac{12g}{5} = 23.52 ) followed by a W-E equation for ( P ): ( 2.5T = \frac{1}{2} \times 2v^2 + 2g \times 2.5 \sin \alpha + (a) ) M1A2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equations of motion for each particle leading to ( T = \frac{12g}{5} = 23.52 ) followed by a W-E equation for ( Q ): ( \frac{1}{2} \times 4v^2 + 2.5T = 4g \times 2.5 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( v = \sqrt{2g} = 4.43 , \text{(m s}^{-1}) ) A1</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>4(c) continued</strong></td>
<td>Use of $\alpha = 36.9$ gives correct answers to 3 sf</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use of $\alpha = 37$ gives correct answers to 2 sf and more than this is not justified, so A0 if they give 3 sf in this case.</td>
<td></td>
</tr>
</tbody>
</table>

(11 marks)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5</strong></td>
<td><strong>Moments about vertical axis (AF):</strong>&lt;br&gt;Requires all terms and dimensionally correct but condone $g$ missing</td>
<td>M1</td>
</tr>
<tr>
<td> </td>
<td>$\frac{Mg}{2} \times 1 + \frac{Mg}{2} \times 1.5a + 3akMg = Mg(1 + k)\bar{x}$&lt;br&gt;-1 each error&lt;br&gt;Accept with $M$ and/or $g$ not seen.</td>
<td>A2</td>
</tr>
<tr>
<td> </td>
<td>$\left( \bar{x} = \frac{1 + 3k}{1 + k}a \right)$</td>
<td></td>
</tr>
<tr>
<td> </td>
<td><strong>Moments about horizontal axis (AB or FE):</strong>&lt;br&gt;Requires all terms and dimensionally correct but condone $g$ missing</td>
<td>M1</td>
</tr>
<tr>
<td> </td>
<td>$\frac{Mg}{2} \times 1.5a + \frac{Mg}{2} \times 3.5a + 4akMg = Mg(1 + k)\bar{y}$&lt;br&gt;-1 each error.&lt;br&gt;Accept with $M$ and/or $g$ not seen.&lt;br&gt;Do not penalise repeated errors.</td>
<td>A2</td>
</tr>
<tr>
<td> </td>
<td>$\left( \bar{y} = \frac{2.5 + 4k}{1 + k}a \right)$</td>
<td></td>
</tr>
<tr>
<td> </td>
<td>Working with axes through F gives&lt;br&gt;$\bar{x} = \frac{1 + 3k}{1 + k}a$ and&lt;br&gt;$\bar{y} = \frac{1.5}{1 + k}a$</td>
<td></td>
</tr>
<tr>
<td> </td>
<td>SR: A candidate working with a mixture of mass and mass ratio can score 4/6&lt;br&gt;M1A0A0M1A2</td>
<td></td>
</tr>
<tr>
<td> </td>
<td><strong>Use of $\tan\theta$ with their distances from AF &amp; AB</strong>&lt;br&gt;Must be considering the whole system. Allow for inverted ratio.</td>
<td>M1</td>
</tr>
<tr>
<td> </td>
<td>$\tan\theta = \frac{M + 3kM}{2.5M + 4kM} = \frac{4}{7}$&lt;br&gt;or exact equivalent</td>
<td>A1</td>
</tr>
<tr>
<td> </td>
<td>Equate their $\tan\theta$ to $\frac{4}{7}$ and solve for $k$:&lt;br&gt;$7M + 21kM = 10M + 16kM$&lt;br&gt;$k = \frac{3}{5}$&lt;br&gt;cs</td>
<td>M1</td>
</tr>
<tr>
<td> </td>
<td><strong>Alternative</strong> for the people who start by considering only the L shape.</td>
<td>(10)</td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>5 continued</strong></td>
<td><strong>M1</strong> (for either) requires all terms and dimensionally correct but condone ( \frac{g}{M} ) missing. A1 for each correct.</td>
<td><strong>M1A2</strong></td>
</tr>
<tr>
<td><strong>Combine with the particle</strong></td>
<td><strong>M1</strong> (for both) requires all terms and dimensionally correct but condone ( \frac{g}{M} ) missing. A1 for each correct.</td>
<td><strong>M1A2</strong></td>
</tr>
<tr>
<td><strong>See over for a more geometrical approach</strong></td>
<td><strong>Candidate starts by finding centre of mass at ( \left( a, \frac{3}{2}a \right) ) relative to ( F ) (or equivalent), M1A2 scored</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Use of ( \tan \theta ) with their distances for finding ( d_1 ) or ( d_2 ).</strong></td>
<td><strong>M1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Obtain length of a side in a triangle containing ( d_i ):</strong></td>
<td><strong>A1</strong></td>
<td></td>
</tr>
<tr>
<td>[ \left( \frac{5}{2}a \right) \tan \theta - a \left( = \frac{3}{7}a \right) ] Correct for their centre of mass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
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<td>-------</td>
</tr>
<tr>
<td><strong>5 continued</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>      $d_1 = \left(\frac{3}{7}a\right)\cos\theta$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>      Correct for their centre of mass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>      $d_2 = \frac{5}{7}a\cos\theta$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>      Use of $\tan\theta$ to find second distance</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>      $3a - 4a\tan\theta = \frac{5}{7}a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>      $\frac{3}{7}a\cos\theta = k \times \frac{5}{7}a\cos\theta \implies k = \frac{3}{5}$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>      Moments about $A$: $Md_1 = kMd_2$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td><strong>(10 marks)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>6(a)</strong></td>
<td>Taking moments about $A$: Requires all terms - condone trig confusion and sign errors</td>
<td>M1</td>
</tr>
<tr>
<td>$bF = 3mga \cos \theta + mg \times 2a \cos \theta$</td>
<td>-1 each error</td>
<td>A2</td>
</tr>
<tr>
<td>$bF = 5mga \cos \theta$</td>
<td><em>Given answer</em></td>
<td>A1</td>
</tr>
<tr>
<td>$F = \frac{5mga}{b} \cos \theta$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| **6(b)** | Component of $R$ parallel to $AB$: Requires all terms - condone trig confusion | M1 |
| $R \cos (\phi - \theta)$ | $= 3mg \sin \theta + mg \sin \theta = 4mg \sin \theta$ | Correct unsimplified | A1 |
| Component of $R$ perpendicular to $AB$: Requires all terms - condone consistent trig confusion and sign errors | M1 |
| $(R \sin(\phi - \theta)) + F = 4mg \cos \theta$ | Correct unsimplified | A1 |
| **Alternatives** for: $M(B)$ | $2aR \sin(\phi - \theta) + 3mga \cos \theta = F(2a - b)$ | M1A1 |
| $M(C)$ | | |
| $(R \sin(\phi - \theta)) = 4mg \cos \theta - \frac{5mga}{b} \cos \theta$ | Correct with $F$ substituted. | A1 |
| ISW for incorrect work after correct components seen | | |
| **Alternative** | | |
| $X = F \sin \theta = \frac{5mga}{b} \cos \theta \sin \theta$ | Allow with $F$. Requires all terms - condone trig confusion | M1 |
| $F$ substituted | | A1 |
| $Y = 4mg - F \cos \theta = 4mg - \frac{5mga}{b} \cos^2 \theta$ | Allow with $F$. Requires all terms - condone trig confusion and sign errors. | M1 |
| Correct unsimplified | | A1 |
| Correct substituted | | A1 |

| **6(c)** | Use of $R \sin(\phi - \theta) > 0$ | M1 |
| Solve for $b$ in terms of $a$: $4 > \frac{5a}{b}, \ (2a \geq b > \frac{5a}{4}$ | $2a$ not required CSO | A1 |

**Special case:**

Misread of directions in (b) | **NB:** This MR can score full marks | (2) |
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6(c)</strong> continued</td>
<td>Alternative</td>
<td></td>
</tr>
<tr>
<td></td>
<td>For $\varphi &gt; \theta$, $\tan \varphi &gt; \tan \theta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tan \varphi = \frac{Y}{X} = \frac{4 - \frac{5a}{b} \cos^2 \theta}{\frac{5a}{b} \cos \theta \sin \theta} &gt; \tan \theta$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$4 - \frac{5a}{b} \cos^2 \theta &gt; \frac{5a}{b} \sin^2 \theta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$4 &gt; \frac{5a}{b} \left( \cos^2 \theta + \sin^2 \theta \right) \Rightarrow b &gt; \frac{5a}{4}$</td>
<td>cso A1</td>
</tr>
<tr>
<td></td>
<td><em>Correct unsimplified</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11 marks)</td>
<td></td>
</tr>
<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
</tr>
<tr>
<td>----------</td>
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</tr>
<tr>
<td><strong>7(a)</strong></td>
<td>Equate horizontal components of speeds: $u \cos \theta = 6 \cos 45^\circ \left(= 3\sqrt{2} \right)$ (4.24...)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Correct unsimplified A1</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Use suvat for vertical speeds: $u \sin \theta - 2g = -6 \sin 45^\circ$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Correct unsimplified A1</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Divide to find $\tan \theta$: $\tan \theta = \frac{2g - 3\sqrt{2}}{3\sqrt{2}} = 3.61..$</td>
<td>DM1</td>
</tr>
<tr>
<td></td>
<td>$\theta = 74.6$ (75)</td>
<td></td>
</tr>
</tbody>
</table>

| **7(b)** | At max height, speed $= u \cos \theta (= 3\sqrt{2}$ (m s$^{-1}$)) | B1 |
|          | KE $= \frac{1}{2} \times 0.7 \times (3\sqrt{2})^2$ (J) | M1 |
|          | Correct for their $v$ at the top, $v \neq 0$ | |
|          | $= 6.3$ (J) | A1 |
|          | accept awrt 6.30. CSO | | (3) |

| **7(c)** | When P is moving upwards at 6 m s$^{-1}$ | M1 |
|          | Use suvat to find first time $v = 6$ | A1 |
|          | $u \sin \theta - gt = 3\sqrt{2}$ | |
|          | $2g - 3\sqrt{2} - gt = 3\sqrt{2}$ | M1 |
|          | Solve for $t$ | |
|          | $t = \frac{2g - 6\sqrt{2}}{g} = 1.13..$ | A1 |
|          | $T = 2 - 1.13 = 0.87$ | A1 |
|          | CAO accept awrt 0.87 | | (5) |

**Alternative**

|          | $6 \sin 45 = 0 + gt$ | M1A1 |
|          | find time from top to $A$: | |
|          | $T = 2t = \frac{12\sqrt{2}}{g} = 0.87$ | M1 |
|          | Correct strategy | A1 |
|          | Correct unsimplified | A1 | (5) |
## Question 7(c) continued

### Alternative

: \( u \sin \theta = gt \) (their \( u, \theta \))

\( t = 1.567... \)

\( T = 2(2 - 1.567...) \)

\( = 0.87 \)

### Alternative

Vertical speed at \( A = - (\text{vertical speed at } B) = \sqrt{36 - (3\sqrt{2})^2} = 3\sqrt{2} \)

Or use the 45° angle

Use \( v = u + at \) for \( A \rightarrow B \)

\(-3\sqrt{2} = 3\sqrt{2} - gT \)

\( T = 0.87 \)

See below for alt 7d

### Alternative 7d

\[
\begin{align*}
\nu^2 &= (3\sqrt{2})^2 + (u \sin \theta - gt)^2 \\
&\leq 36
\end{align*}
\]

Form expression for \( \nu^2 \). Inequality not needed at this stage

Correct inequality for \( \nu^2 \).

\[
\begin{align*}
-\sqrt{18} &\leq u \sin \theta - gt \leq \sqrt{18} \\
\frac{u \sin \theta - \sqrt{18}}{g} &\leq t \leq \frac{u \sin \theta + \sqrt{18}}{g}
\end{align*}
\]

\( T = \frac{u \sin \theta + \sqrt{18}}{g} - \frac{u \sin \theta - \sqrt{18}}{g} = 2\frac{\sqrt{18}}{g} = 0.866 \)

(14 marks)
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

• Use black ink or ball-point pen.
• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
• Fill in the boxes at the top of this page with your name, centre number and candidate number.
• Answer all questions and ensure that your answers to parts of questions are clearly labelled.
• Answer the questions in the spaces provided – there may be more space than you need.
• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
• Inexact answers should be given to three significant figures unless otherwise stated.

Information
• A booklet 'Mathematical Formulae and Statistical Tables' is provided.
• There are 6 questions in this question paper. The total mark for this paper is 75.
• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
• Read each question carefully before you start to answer it.
• Try to answer every question.
• Check your answers if you have time at the end.
• If you change your mind about an answer, cross it out and put your new answer and any working underneath.
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• If you change your mind about an answer, cross it out and put your new answer and any working underneath.
1. A hemispherical bowl, of internal radius \( r \), is fixed with its circular rim upwards and horizontal. A particle \( P \) of mass \( m \) moves on the smooth inner surface of the bowl. The particle moves with constant angular speed in a horizontal circle. The centre of the circle is at a distance \( \frac{1}{2} r \) vertically below the centre of the bowl, as shown in Figure 1.

The time taken by \( P \) to complete one revolution of its circular path is \( T \).

Show that \( T = \pi \sqrt{\frac{2r}{g}} \).  

(8)
Question 1 continued

(Continued from previous page)

(Continued to next page)
2. A spacecraft $S$ of mass $m$ moves in a straight line towards the centre of the Earth. The Earth is modelled as a sphere of radius $R$ and $S$ is modelled as a particle. When $S$ is at a distance $x$, $x \geq R$, from the centre of the Earth, the force exerted by the Earth on $S$ is directed towards the centre of the Earth. The force has magnitude $\frac{K}{x^2}$, where $K$ is a constant.

(a) Show that $K = mgR^2$ \hspace{1cm} (2)

When $S$ is at a distance $3R$ from the centre of the Earth, the speed of $S$ is $V$. Assuming that air resistance can be ignored,

(b) find, in terms of $g$, $R$ and $V$, the speed of $S$ as it hits the surface of the Earth. \hspace{1cm} (7)
Question 2 continued
Question 2 continued

(Total for Question 2 is 9 marks)
3. At time \( t = 0 \), a particle \( P \) is at the origin \( O \), moving with speed 8 m s\(^{-1}\) in the positive \( x \) direction. At time \( t \) seconds, \( t \geq 0 \), the acceleration of \( P \) has magnitude \( 2(t + 4)^{\frac{1}{2}} \) m s\(^{-2}\) and is directed towards \( O \).

(a) Show that, at time \( t \) seconds, the velocity of \( P \) is \( 16 - 4(t + 4)^{\frac{1}{2}} \) m s\(^{-1}\). (5)

(b) Find the distance of \( P \) from \( O \) when \( P \) comes to instantaneous rest. (7)
Question 3 continued
Question 3 continued
A particle of mass $3m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle is held at the point $A$, where $OA$ is horizontal and $OA = a$. The particle is projected vertically downwards from $A$ with speed $u$, as shown in Figure 2. The particle moves in complete vertical circles.

(a) Show that $u^2 \geq 3ag$.  

(7)

Given that the greatest tension in the string is three times the least tension in the string,

(b) show that $u^2 = 6ag$.  

(5)
Question 4 continued
Question 4 continued
Question 4 continued

(Total for Question 4 is 12 marks)
Two fixed points $A$ and $B$ are 5 m apart on a smooth horizontal floor. A particle $P$ of mass 0.5 kg is attached to one end of a light elastic string, of natural length 2 m and modulus of elasticity 20 N. The other end of the string is attached to $A$. A second light elastic string, of natural length 1.2 m and modulus of elasticity 15 N, has one end attached to $P$ and the other end attached to $B$.

Initially $P$ rests in equilibrium at the point $O$, as shown in Figure 3.

(a) Show that $AO = 3$ m.  

The particle is now pulled towards $A$ and released from rest at the point $C$, where $ACB$ is a straight line and $OC = 1$ m.

(b) Show that, while both strings are taut, $P$ moves with simple harmonic motion.  

(c) Find the speed of $P$ at the instant when the string $PB$ becomes slack.  

The particle first comes to instantaneous rest at the point $D$.

(d) Find the distance $DB$.  

Figure 3

---

**Diagram:**

- A straight line of 5 m is shown with points $A$, $O$, $P$, and $B$. Point $O$ is the equilibrium point. Point $P$ is attached to a mass 0.5 kg, and the string is stretched between $P$ and $B$. Another string is attached between $P$ and $A$, and the string is elongated to form a triangle $AOC$.
Question 5 continued
6.

The shaded region $R$ is bounded by part of the curve with equation $y = x^2 + 3$, the $x$-axis, the $y$-axis and the line with equation $x = 2$, as shown in Figure 4. The unit of length on each axis is one centimetre. The region $R$ is rotated through $2\pi$ radians about the $x$-axis to form a uniform solid $S$.

Using algebraic integration,

(a) show that the volume of $S$ is $\frac{202}{5}\pi \text{ cm}^3$, \hspace{1cm} (4)

(b) show that, to 2 decimal places, the centre of mass of $S$ is 1.30 cm from $O$. \hspace{1cm} (5)

A uniform right circular solid cone, of base radius 7 cm and height 6 cm, is joined to $S$ to form a solid $T$. The base of the cone coincides with the larger plane face of $S$, as shown in Figure 5. The vertex of the cone is $V$.

The mass per unit volume of $S$ is twice the mass per unit volume of the cone.

(c) Find the distance from $V$ to the centre of mass of $T$. \hspace{1cm} (5)

The point $A$ lies on the circumference of the base of the cone. The solid $T$ is suspended from $A$ and hangs freely in equilibrium.

(d) Find the size of the angle between $VA$ and the vertical. \hspace{1cm} (3)
Question 6 continued
Question 6 continued
Question 6 continued

(Total for Question 6 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS
# Mechanics M3 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(30° or θ for the first 3 lines)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R \sin 30° = mg$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$R \cos 30° = m(r \cos 30°)\omega^2$</td>
<td>M1 A1 A1</td>
</tr>
<tr>
<td></td>
<td>$\omega^2 = \frac{R}{mr} = \frac{g}{r \sin 30°}$</td>
<td>DM1 A1</td>
</tr>
<tr>
<td></td>
<td>$\omega = \sqrt{\frac{2g}{r}}$</td>
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<tr>
<td></td>
<td>Time $= \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r}{2g}} = \pi \sqrt{\frac{2r}{g}}$ *</td>
<td>A1 cso</td>
</tr>
<tr>
<td><strong>Alternative:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Resolve perpendicular to the reaction:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$mg \cos 30° = m \times \text{rad} \times \omega^2 \cos 60°$</td>
<td>M2 A1 (LHS) A1 (RHS)</td>
</tr>
<tr>
<td></td>
<td>$= mr \cos 30° \omega^2 \cos 60°$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Obtain $\omega$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Correct time</td>
<td>A1</td>
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<td></td>
<td></td>
<td>(8)</td>
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<tr>
<td><strong>Notes:</strong></td>
<td></td>
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</tr>
<tr>
<td><strong>M1:</strong></td>
<td>Resolving vertically 30° or θ</td>
<td></td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>Correct equation 30° or θ</td>
<td></td>
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<tr>
<td><strong>M1:</strong></td>
<td>Attempting an equation of motion along the radius, acceleration in either form 30° or θ</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Allow with $r$ for radius.</td>
<td></td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>LHS correct 30° or θ</td>
<td></td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>RHS correct, 30° or θ but not $r$ for radius.</td>
<td></td>
</tr>
<tr>
<td><strong>DM1:</strong></td>
<td>Obtaining an expression for $\omega^2$ or for $v^2$ and the length of the path 30° or θ Dependent on both previous M marks.</td>
<td></td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>Correct expression for $\omega$ Must have the numerical value for the trig function now.</td>
<td></td>
</tr>
<tr>
<td><strong>A1 cso:</strong></td>
<td>Deducing the GIVEN answer.</td>
<td></td>
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<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
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<tr>
<td>----------</td>
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</tr>
<tr>
<td>2(a)</td>
<td>( F = \frac{K}{x^2} )</td>
<td></td>
</tr>
<tr>
<td>( x = R \Rightarrow F = mg \quad \therefore mg = \frac{K}{R^2} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( K = mgR^2 ) *</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(b) ( \frac{mgR^2}{x^2} = -mv \frac{dv}{dx} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( g \int \frac{R^2}{x^2} , dx = - \int v , dv )</td>
<td>dM1</td>
<td>A1ft</td>
</tr>
<tr>
<td>( -g \frac{R^2}{x} = -\frac{1}{2}v^2 \quad (+c) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = 3R, \ v = V \Rightarrow -g \frac{R^2}{3R} = -\frac{1}{2}V^2 + c )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( c = -\frac{Rg}{3} + \frac{1}{2}V^2 )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( x = R \Rightarrow \frac{1}{2}v^2 = -\frac{Rg}{3} + \frac{1}{2}V^2 + g \frac{R^2}{R} )</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>( v^2 = V^2 + \frac{4Rg}{3} )</td>
<td>A1</td>
<td>cso</td>
</tr>
<tr>
<td>( v = \sqrt{V^2 + \frac{4Rg}{3}} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)
- **M1:** Setting \( F = mg \) **and** \( x = R \)
- **A1:** Deducing the GIVEN answer

(b)
- **M1:** Attempting an equation of motion with acceleration in the form \( v \frac{dv}{dx} \). The minus sign may be missing.
- **dM1:** Attempting the integration.
- **A1ft:** Correct integration, follow through on a missing minus sign from line 1, constant of integration may be missing.
- **M1:** Substituting \( x = 3R, v = V \) to obtain an equation for \( c \)
- **A1:** Correct expression for \( c \).
- **M1:** Substituting \( x = R \) and their expression for \( c \).
- **A1:** Correct expression for \( v \), any equivalent form.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(a)</strong></td>
<td>$\frac{dv}{dt} = -2(t + 4)^{\frac{1}{2}}$</td>
<td>M1</td>
</tr>
<tr>
<td> </td>
<td>$v = -\int 2(t + 4)^{\frac{1}{2}} , dt$</td>
<td></td>
</tr>
<tr>
<td> </td>
<td>$v = -4(t + 4)^{\frac{1}{2}} ( + c)$</td>
<td>dM1 A1</td>
</tr>
<tr>
<td> </td>
<td>$t = 0, v = 8 \Rightarrow c = 16$</td>
<td>M1</td>
</tr>
<tr>
<td> </td>
<td>$v = 16 - 4(t + 4)^{\frac{1}{2}} \text{ (m s}^{-1}) \ast$</td>
<td>A1 cso</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$v = 0 \quad 16 = 4(t + 4)^{\frac{1}{2}}$</td>
<td>M1</td>
</tr>
<tr>
<td> </td>
<td>$16 = t + 4 \quad t = 12$</td>
<td>A1</td>
</tr>
<tr>
<td> </td>
<td>$x = 4\int \left( 4 - (t + 4)^{\frac{1}{2}} \right) , dt$</td>
<td></td>
</tr>
<tr>
<td> </td>
<td>$x = 4\left( 4t - \frac{2}{3}(t + 4)^{\frac{3}{2}} \right) \ast ( + d)$</td>
<td>M1 A1</td>
</tr>
<tr>
<td> </td>
<td>$t = 0, \ x = 0 \quad d = 4\times\frac{2}{3}\times4^{\frac{3}{2}} = \frac{64}{3} \text{ oe}$</td>
<td>A1</td>
</tr>
<tr>
<td> </td>
<td>$t = 12 \quad x = 4\left( 4\times12 - \frac{2}{3}\times16^{\frac{3}{2}} \right) + \frac{64}{3} = 42\frac{2}{3} \text{ (m) oe eg 43 or better}$</td>
<td>dM1 A1</td>
</tr>
</tbody>
</table>

**Notes:**

**3(a)**

**M1:** Attempting an expression for the acceleration in the form $\frac{dv}{dt}$; minus may be omitted.

**DM1:** Attempting the integration

**A1:** Correct integration, constant of integration may be omitted (no ft)

**M1:** Using the initial conditions to obtain a value for the constant of integration

**A1:** cso. Substitute the value of $c$ and obtain the final GIVEN answer

**3(b)**

**M1:** Setting the given expression for $v$ equal to 0

**A1:** Solving to get $t = 12$

**M1:** Setting $v = \frac{dx}{dt}$ and attempting the integration wrt $t$. At least one term must clearly be integrated.

**A1:** Correct integration, constant may be omitted.
**Question 3 notes continued**

M1: Substituting $t = 0, \ x = 0$ and obtaining the correct value of $d$. Any equivalent number, inc decimals.

dM1: Substituting their value for $t$ and obtaining a value for the required distance. Dependent on the second M mark.

A1: Correct final answer, any equivalent form.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4(a)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy to top: $\frac{1}{2} \times 3m \times u^2 - \frac{1}{2} \times 3m v^2 = 3m a$</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td>NL2 at top: $T + 3mg = 3m \frac{v^2}{a}$</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td>$T = 3m \frac{u^2}{a} - 6mg - 3mg$</td>
<td>dM1</td>
<td></td>
</tr>
<tr>
<td>$T \geq 0 \Rightarrow \frac{u^2}{a} \geq 3g$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>$u^2 \geq 3ag$</td>
<td>A1 cso</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension at bottom:</td>
<td>(7)</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2} \times 3m \times v^2 - \frac{1}{2} \times 3m u^2 = 3m a$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>$T_{max} - 3mg = 3m \frac{v^2}{a}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>$T_{max} = 3mg + 6mg + 3m \frac{u^2}{a}$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>$T_{min} = 3m \frac{u^2}{a} - 9mg$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$9mg + 3m \frac{u^2}{a} = 3 \left( 3m \frac{u^2}{a} - 9mg \right)$</td>
<td>dM1</td>
<td></td>
</tr>
<tr>
<td>$u^2 = 6ag \ast$</td>
<td>A1 cso</td>
<td></td>
</tr>
<tr>
<td><strong>(12 marks)</strong></td>
<td></td>
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</tbody>
</table>

**Notes:**

**4(a)**

- **M1**: Attempting an energy equation, can be to a general point for this mark. Mass can be missing but use of $v^2 = u^2 + 2as$ scores M0
- **A1**: Correct equation from $A$ to the top.
- **M1**: Attempting an equation of motion along the radius at the top, acceleration in either form.
- **A1**: Correct equation, acceleration in form $\frac{v^2}{r}$
- **dM1**: Eliminate $v^2$ to obtain an expression for $T$ dependent on both previous M marks.
- **M1**: Use $T \geq 0$ at top to obtain an inequality connecting $a$, $g$ and $u$
- **A1**: Re-arrange to obtain the GIVEN answer.
Question 4 notes continued

(b)
M1: Attempting an energy equation to the bottom, maybe from $A$ or from the top.
M1: Attempting an equation of motion along the radius at the bottom.
A1: Correct expression for the max tension.
dM1: Forming an equation connecting *their* tension at the top with *their* tension at the bottom. If the 3 is multiplying the wrong tension this mark can still be gained. Dependent on both previous M marks.
A1: cso. Obtaining the GIVEN answer.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5(a)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ T = \frac{20e}{2} = \frac{15(1.8-e)}{1.2} ]</td>
<td>M1A1</td>
<td></td>
</tr>
<tr>
<td>[ 10e \times 1.2 = 15(1.8 - e) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ e = 1 ]</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>[ AO = 3\text{m} ] *</td>
<td>A1cso</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 0.5\ddot{x} = \frac{20(1-x)}{2} - \frac{15(0.8+x)}{1.2} ]</td>
<td>M1 A1 A1</td>
<td></td>
</tr>
<tr>
<td>[ \ddot{x} = -45x \therefore \text{SHM} ]</td>
<td>A1 cso</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>String becomes slack when ( x = (-0.8) ) (allow wo sign due to symmetry)</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>[ v^2 = \omega^2 (a^2 - x^2) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ v^2 = 45(1 - 0.8^2) ] ((=16.2))</td>
<td>M1 A1 ft</td>
<td></td>
</tr>
<tr>
<td>[ v = 4.024\ldots \text{m/s} ] (4.0 or better)</td>
<td>A1 ft</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \frac{1}{2} \times 20y^2 - \frac{1}{2} \times 20 \times 1.8^2 = \frac{1}{2} \times 0.5 \times 16.2 \text{ ft on } v ]</td>
<td>M1 A1 A1 ft</td>
<td></td>
</tr>
<tr>
<td>[ 20y^2 - 64.8 = 16.2 ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ y^2 = 4.05 \quad v = 2.012\ldots ]</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>Distance (DB =</td>
<td>5 - 4.012...</td>
<td>= 0.988\ldots \text{m} ) (accept 0.99 or better)</td>
</tr>
<tr>
<td><strong>Alternative</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 0.5a = -10(1.8 + x) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ v \frac{dv}{dx} = -36 - 10x ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \int v , dv = -\int (36 + 10x) , dx ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \frac{v^2}{2} = -36x + 5x^2 + c ]</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td>[ x = 0, \quad v = \frac{9\sqrt{5}}{5} \therefore c = 8.1 ]</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>Then (v = 0) etc</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td></td>
</tr>
</tbody>
</table>

(17 marks)
Question 5 continued

Notes:

(a)
M1: Attempting to obtain and equate the tensions in the two parts of the string.
A1: Correct equation, extension in AP or BP can be used or use OA as the unknown.
A1: Obtaining the correct extension in either string (ext in BP = 0.8 m) or another useful distance.
A1: cso. Obtaining the correct GIVEN answer.

(b)
M1: Forming an equation of motion at a general point. There must be a difference of tensions, both with the variable. May have $m$ instead of 0.5 Accel can be $a$.
A1 A1: Deduct 1 for each error, $m$ or 0.5 allowed, acceleration to be $\ddot{x}$ now.
A1: cso Correct equation in the required form, with a concluding statement; $m$ or 0.5 allowed.

Question 5 notes continued

(c)
B1: For $x = \pm 0.8$ Need not be shown explicitly.
M1: Using $v^2 = \omega^2 \left( a^2 - x^2 \right)$ with their (numerical) $\omega$ and their $x$
A1ft: Equation with correct numbers ft their $\omega$
A1ft: Correct value for $v$ 2sf or better or exact.

(d)
M1: Attempting an energy equation with 2 EPE terms and a KE term.
A1: 2 correct terms may have $(1.8 + x)$ instead of $y$.
A1ft: Completely correct equation, follow through their $v$ from (c)
A1: Correct value for distance travelled after $PB$ became slack. $x = 0.21$
A1ft: Complete to the distance $DB$. Follow through their distance travelled after $PB$ became slack.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6(a)</strong></td>
<td>Vol = ( \pi \int_0^2 (x^2 + 3)^2 , dx )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= ( \pi \int_0^2 (x^4 + 6x^2 + 9) , dx )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= ( \pi \left[ \frac{1}{5} x^3 + 2x^2 + 9x \right]_0 )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>= ( \frac{202}{5} \pi ) cm(^3) *</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(4)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>( \pi \int_0^2 (x^2 + 3)^2 , dx = \pi \int_0^2 (x^5 + 6x^3 + 9x) , dx )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= ( \pi \left[ \frac{1}{6} x^6 + \frac{3}{2} x^4 + \frac{9}{2} x^2 \right]_0 )</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>= ( \frac{158}{3} \pi )</td>
<td>A1</td>
</tr>
<tr>
<td>(Or by chain rule or substitution)</td>
<td>C of m = ( \frac{158}{3} \times \frac{5}{202} ), ( = 1.3036... ) ( = 1.30 ) cm</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = \frac{158}{3} \pi )</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>Mass ratio = ( 2 \times \frac{202}{5} \pi ) ( \frac{1}{3} \pi \times 7^2 \times 6 ) ( \left( \frac{404}{5} + 98 \right) \pi )</td>
<td>B1</td>
</tr>
<tr>
<td>Dist from ( V )</td>
<td>6.7</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>( \frac{404}{5} \times 6.7 + 98 \times 4.5 = \left( \frac{404}{5} + 98 \right) \bar{x} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \bar{x} = \frac{\left( \frac{404}{5} + 98 \right)}{\frac{404}{5} \times 6.7 + 98 \times 4.5} = 5.494... = 5.5 ) cm</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Accept 5.49 or better</td>
<td></td>
</tr>
<tr>
<td><strong>(5)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>( \tan \theta = \frac{6 - \bar{x}}{7} = 0.5058... )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \alpha = \tan^{-1} \left( \frac{6}{7} \right) - \tan^{-1} \left( \frac{0.5058...}{7} \right) = 36.468...^\circ ) ( = 36^\circ ) or better</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = \tan^{-1} \left( \frac{6}{7} \right) - \tan^{-1} \left( \frac{0.5058...}{7} \right) = 36.468...^\circ ) ( = 36^\circ ) or better</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(3)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

(a)  
**M1:** Using \( \pi \int y^2 \, dx \) with the equation of the curve, no limits needed
<table>
<thead>
<tr>
<th>Question 6 notes continued</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>dM1:</strong> Integrating their expression for the volume.</td>
</tr>
<tr>
<td><strong>A1:</strong> Correct integration inc limits now.</td>
</tr>
<tr>
<td><strong>A1:</strong> Substituting the limits to obtain the GIVEN answer.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>(b)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1:</strong> Using ( (\pi)\int xy^2 ,dx ) with the equation of the curve, no limits needed, ( \pi ) can be omitted.</td>
</tr>
<tr>
<td><strong>A1:</strong> Correct integration, including limits; no substitution needed for this mark.</td>
</tr>
<tr>
<td><strong>A1:</strong> Correct substitution of limits.</td>
</tr>
<tr>
<td><strong>M1:</strong> Use of ( \frac{\pi\int xy^2 ,dx}{\pi\int y^2 ,dx} ) with their ( \pi\int xy^2 ,dx ). ( \pi ) must be seen in both numerator and denominator or in neither.</td>
</tr>
<tr>
<td><strong>A1:</strong> cso. Correct answer. Must be 1.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>(c)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B1:</strong> Correct mass ratio.</td>
</tr>
<tr>
<td><strong>B1:</strong> Correct distances, from ( V ) or any other point, provided consistent.</td>
</tr>
<tr>
<td><strong>M1:</strong> Attempting a moments equation.</td>
</tr>
<tr>
<td><strong>A1ft:</strong> Correct equation, follow through their distances and mass ratio.</td>
</tr>
<tr>
<td><strong>A1:</strong> Correct distance from ( V )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>(d)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1:</strong> Attempting the tan of an appropriate angle, numbers either way up.</td>
</tr>
<tr>
<td><strong>M1:</strong> Attempting to obtain the required angle.</td>
</tr>
<tr>
<td><strong>A1:</strong> Correct final answer 2sf or more.</td>
</tr>
</tbody>
</table>
dM1: Integrating their expression for the volume.

A1: Correct integration inc limits now.

A1: Substituting the limits to obtain the GIVEN answer.

(b)

M1: Using \( \pi \int_2^x y \, dx \) with the equation of the curve, no limits needed, \( \pi \) can be omitted.

A1: Correct integration, including limits; no substitution needed for this mark.

A1: Correct substitution of limits.

M1: Use of \( \pi \int_2^x y \, dx \) with their \( \pi \int_2^x y \, dx \).

A1: cso.

Correct answer. Must be 1.30

(c)

B1: Correct mass ratio.

B1: Correct distances, from \( V \) or any other point, provided consistent.

M1: Attempting a moments equation.

A1ft: Correct equation, follow through their distances and mass ratio.

A1: Correct distance from \( V \)

(d)

M1: Attempting the tan of an appropriate angle, numbers either way up.

M1: Attempting to obtain the required angle.

A1: Correct final answer 2sf or more.
Answer ALL questions. Write your answers in the spaces provided.

1. The percentage oil content, \( p \), and the weight, \( w \) milligrams, of each of 10 randomly selected sunflower seeds were recorded. These data are summarised below.

\[
\sum w^2 = 41252 \quad \sum wp = 27557.8 \quad \sum w = 640 \quad \sum p = 431 \quad S_{pp} = 2.72
\]

(a) Find the value of \( S_{ww} \) and the value of \( S_{wp} \)  

(3)

(b) Calculate the product moment correlation coefficient between \( p \) and \( w \)  

(2)

(c) Give an interpretation of your product moment correlation coefficient.  

(1)

The equation of the regression line of \( p \) on \( w \) is given in the form \( p = a + bw \)

(d) Find the equation of the regression line of \( p \) on \( w \)  

(4)

(e) Hence estimate the percentage oil content of a sunflower seed which weighs 60 milligrams.  

(2)
Question 1 continued

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(Total for Question 1 is 12 marks)
2. The time taken to complete a puzzle, in minutes, is recorded for each person in a club. The times are summarised in a grouped frequency distribution and represented by a histogram.

One of the class intervals has a frequency of 20 and is shown by a bar of width 1.5 cm and height 12 cm on the histogram. The total area under the histogram is 94.5 cm².

Find the number of people in the club. (3)
Question 2 continued

(Total for Question 2 is 3 marks)
3. The discrete random variable $X$ has probability distribution

$$P(X = x) = \frac{1}{5} \quad x = 1, 2, 3, 4, 5$$

(a) Write down the name given to this distribution. (1)

Find

(b) $P(X = 4)$ (1)

(c) $F(3)$ (1)

(d) $P(3X - 3 > X + 4)$ (2)

(e) Write down $E(X)$ (1)

(f) Find $E(X^2)$ (2)

(g) Hence find $\text{Var}(X)$ (2)

Given that $E(aX - 3) = 11.4$

(h) find $\text{Var}(aX - 3)$ (4)
Question 3 continued

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Question 3 continued

(Total for Question 3 is 14 marks)
4. A researcher recorded the time, $t$ minutes, spent using a mobile phone during a particular afternoon, for each child in a club.

The researcher coded the data using $v = \frac{t - 5}{10}$ and the results are summarised in the table below.

<table>
<thead>
<tr>
<th>Coded Time ($v$)</th>
<th>Frequency ($f$)</th>
<th>Coded Time Midpoint ($m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq v &lt; 5$</td>
<td>20</td>
<td>2.5</td>
</tr>
<tr>
<td>$5 \leq v &lt; 10$</td>
<td>24</td>
<td>$a$</td>
</tr>
<tr>
<td>$10 \leq v &lt; 15$</td>
<td>16</td>
<td>12.5</td>
</tr>
<tr>
<td>$15 \leq v &lt; 20$</td>
<td>14</td>
<td>17.5</td>
</tr>
<tr>
<td>$20 \leq v &lt; 30$</td>
<td>6</td>
<td>$b$</td>
</tr>
</tbody>
</table>

(You may use $\sum fm = 825$ and $\sum fm^2 = 12012.5$)

(a) Write down the value of $a$ and the value of $b$.  

(b) Calculate an estimate of the mean of $v$.  

(c) Calculate an estimate of the standard deviation of $v$.  

(d) Use linear interpolation to estimate the median of $v$.  

(e) Hence describe the skewness of the distribution. Give a reason for your answer.  

(f) Calculate estimates of the mean and the standard deviation of the time spent using a mobile phone during the afternoon by the children in this club.
Question 4 continued
5. A biased tetrahedral die has faces numbered 0, 1, 2 and 3. The die is rolled and the number face down on the die, \( X \), is recorded. The probability distribution of \( X \) is

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

If \( X = 3 \) then the final score is 3
If \( X \neq 3 \) then the die is rolled again and the final score is the sum of the two numbers.

The random variable \( T \) is the final score.

(a) Find \( P(T = 2) \)

(b) Find \( P(T = 3) \)

(c) Given that the die is rolled twice, find the probability that the final score is 3
The random variable $X$ is recorded. The probability distribution of $X$ is shown below.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$P(X=x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) Find $P(T = 3)$.

c) Given that the die is rolled twice, find the probability that the final score is 3 if the face down on the die is 0, 1, 2 or 3.

Question 5 continued
6. Three events $A$, $B$ and $C$ are such that

$$P(A) = \frac{2}{5}, \quad P(C) = \frac{1}{2}, \quad P(A \cup B) = \frac{5}{8}$$

Given that $A$ and $C$ are mutually exclusive find

(a) $P(A \cup C)$ \hspace{2cm} (1)

Given that $A$ and $B$ are independent

(b) show that $P(B) = \frac{3}{8}$ \hspace{2cm} (4)

(c) Find $P(A \mid B)$ \hspace{2cm} (1)

Given that $P(C' \cap B') = 0.3$

(d) draw a Venn diagram to represent the events $A$, $B$ and $C$ \hspace{2cm} (5)
Question 6 continued
7. A machine fills bottles with water. The volume of water delivered by the machine to a bottle is $X$ ml where $X \sim N(\mu, \sigma^2)$

One of these bottles of water is selected at random.

Given that $\mu = 503$ and $\sigma = 1.6$

(a) find

(i) $P(X > 505)$

(ii) $P(501 < X < 505)$

(b) Find $w$ such that $P(1006 - w < X < w) = 0.9426$

Following adjustments to the machine, the volume of water delivered by the machine to a bottle is such that $\mu = 503$ and $\sigma = q$

Given that $P(X < r) = 0.01$ and $P(X > r + 6) = 0.05$

(c) find the value of $r$ and the value of $q$
Question 7 continued
Question 7 continued
Question 7 continued

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS
### Statistics S1 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>( S_{ww} = 41252 - \frac{640^2}{10} = 292 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( S_{wp} = 27557.8 - \frac{640 \times 431}{10} = -26.2 )</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>( r = \frac{-26.2}{\sqrt{26.2 \times 2.72}} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= -0.9297 awrt -0.930</td>
<td>A1</td>
</tr>
<tr>
<td>(c)</td>
<td>As weight increases the percentage of oil content decreases o.e.</td>
<td>B1</td>
</tr>
<tr>
<td>(d)</td>
<td>( b = \frac{-26.2}{292} = -0.0897... ) awrt -0.09</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>( a = \frac{431}{10} - \left( -\frac{26.2}{292} \right) \times \left( \frac{640}{10} \right) = 48.842... )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( p = 48.8 - 0.0897w )</td>
<td>A1</td>
</tr>
<tr>
<td>(e)</td>
<td>( p = 48.8 - 0.0897 \times 60 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= 43.4/43.5 awrt 43.4/43.5</td>
<td>A1</td>
</tr>
</tbody>
</table>

(12 marks)

**Notes:**

(a) **M1:** for a correct expression for \( S_{ww} \) or \( S_{wp} \) (may be implied by one correct answer)
- **1st A1:** for either \( S_{ww} = 292 \) or \( S_{wp} = -26.2 \)
- **2nd A1:** for both \( S_{ww} = 292 \) and \( S_{wp} = -26.2 \)

(b) **M1:** for a correct expression (Allow ft of their \( S_{ww} \) or \( S_{wp} \) provided \( S_{ww} \neq 41252 \) and \( S_{wp} \neq 27557.8 \). Condone missing “-“
- **A1:** for awrt -0.930 (Condone -0.93 for M1A1 if correct expression is seen)
  (Answer only awrt -0.930 scores 2/2 but answer only -0.93 is M1A0)

(c) **B1:** For a correct contextual description of negative correlation which must include weight and oil (but \( w \) increases as \( p \) decreases is not sufficient)

(d) **1st M1:** for a correct expression for \( b \) (Allow ft)
- **1st A1:** for awrt -0.09
- **2nd M1:** for a correct method for \( a \) ft their value of \( b \) (Allow \( a = 43.1 + b \times 64 \))
- **2nd A1:** for a correct equation for \( p \) and \( w \) with \( a = \) awrt 48.8 and \( b = \) awrt -0.0897 No fractions. Equation in \( x \) and \( y \) is A0

(e) **M1:** substituting \( w = 60 \) into their equation
- **A1:** awrt 43.4 or 43.5 (Answer only scores 2/2)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 2 | 1.5×12 = 18  
20 people represented by 18 (cm²) or 1 person is represented by 0.9 (cm²) | M1 |
| | $x = \frac{20\times94.5}{18}$ oe  
$= 105$ (people) | M1 A1 cao |

(3 marks)

Notes:
M1: For an attempt to relate area to frequency (e.g. $\frac{20}{18}$ or $\frac{18}{20}$ seen)
M1: For a correct expression/equation for total frequency e.g. $\frac{18}{20} = \frac{94.5}{x}$
A1: For 105 cao
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3(a)</strong></td>
<td>(Discrete) <strong>Uniform</strong></td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>[P(X = 4) = \frac{1}{5} \text{ oe}]</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(c)</td>
<td>[F(3) = \frac{3}{5} \text{ oe}]</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(d)</td>
<td>[P(3X - 3 &gt; X + 4) = P(X &gt; 3.5)] [= \frac{2}{5} \text{ oe}]</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(e)</td>
<td>[E(X) = 3]</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(f)</td>
<td>[E(X^2) = \frac{1}{5}(1^2 + 2^2 + 3^2 + 4^2 + 5^2)]</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(g)</td>
<td>[\text{Var} (X) = 11 - 3^2 \quad \text{or} \quad \frac{(5 + 1)(5 - 1)}{12}]</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(h)</td>
<td>11.4 = aE(X) - 3 \quad \text{or} \quad 11.4 = 3a - 3]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[a = 4.8]</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>[\text{Var} (4.8X - 3) = 4.8^2 \times 2]</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>[= 46.08 \quad \text{awrt 46.1}]</td>
<td>A1 (4)</td>
</tr>
</tbody>
</table>

(14 marks)
**Question 3 continued**

**Notes:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>B1: For uniform.</td>
</tr>
<tr>
<td>(d)</td>
<td>M1: For identifying the correct probabilities i.e. ( P(X &gt; 3.5) ) or ( P(X = 4) + P(X = 5) )</td>
</tr>
<tr>
<td>(f)</td>
<td>M1: For a correct expression.</td>
</tr>
<tr>
<td>(g)</td>
<td>M1: For either ‘their (f)’ – ‘their (e)’(^2) or for a correct expression ( \frac{(5+1)(5-1)}{12} )</td>
</tr>
</tbody>
</table>
| (h) | 1\(^{st}\) M1: For setting up a correct linear equation using \( aE(X) - 3 = 11.4 \)  
    1\(^{st}\) A1: May be implied by a correct answer.  
    2\(^{nd}\) M1: For "their \( a^2 \) "x"their Var(X)" (must see values substituted) (may be implied by a correct answer or correct ft answer)  
    NB: ‘their Var(X)’ < 0 is M0 here. |
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(a)</td>
<td>7.5 and 25</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>Mean = 10.3125</td>
<td>awrt 10.3</td>
</tr>
<tr>
<td>(c)</td>
<td>$\sigma = \sqrt{\frac{120125 - 10.3125^2}{80}}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= 6.6188.. (s = 6.6605…)</td>
<td>awrt 6.62</td>
</tr>
<tr>
<td>(d)</td>
<td>Median = ${5} + \frac{20}{24} \times 5$ or ${10} - \frac{4}{24} \times 5$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= 9.16666</td>
<td>awrt 9.17</td>
</tr>
<tr>
<td>(e)</td>
<td>Mean &gt; median (\therefore) positive skew</td>
<td>M1A1</td>
</tr>
<tr>
<td>(f)</td>
<td>$t = 10v + 5$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Mean = 10\times10.3125 + 5</td>
<td>awrt 108</td>
</tr>
<tr>
<td></td>
<td>= 108.125</td>
<td>awrt 108</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 10 \times 6.6188$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= 66.188.. (66.605 from s)</td>
<td>awrt 66.2</td>
</tr>
</tbody>
</table>

Notes:

(a) B1: Both values correct (may be seen in table)

(b) B1: For awrt 10.3 (Do not allow improper fractions).

(c) M1: For a correct expression including the square root (allow ft from their mean)

A1: For awrt 6.62 (Allow $s = $ awrt 6.66)

(d) M1: For a correct fraction: $\frac{20}{24} \times 5$ or if using $n + 1$ for $\frac{20.5}{24} \times 5$ may be scored from working down $\frac{4}{24} \times 5$

A1: For awrt 9.17 or (if using $n + 1$) for awrt 9.27
(e) M1: For a correct comparison of ‘their b’ and ‘their d’ (must have an answer to both (b) and (d))
Comparison may be part of bigger expression e.g. 3(mean – median)/s.d.
Allow use of \( Q_3 - Q_2 > Q_2 - Q_1 \) only if \( Q_1 = 5 \) and \( Q_3 = 15 \) are both seen
A1: For positive skew (which must follow from their values)

(f) M1: (1st M1) For 10×"their mean"+5
M1: (2nd M1) or 10×"their sd"
Use of decoded data to find mean must be fully correct,
i.e. 8650/80 = awrt 108 (M1A1)
Use of decoded data to find s.d. must be fully correct,
i.e. \( \sqrt{\frac{1285750}{80} - \left(\frac{8650}{80}\right)^2} \) = awrt 66.2 (M1A1)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5(a)</strong></td>
<td>$P(T = 2) = 3 \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{12}$ oe</td>
<td>M1 A1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$P(T = 3) = [P(0, 3) + P(1, 2) + P(2, 1)] + P(3)$</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>$= \left( \frac{1 \times \frac{1}{2}}{6} \right) + \left( \frac{1 \times \frac{1}{2}}{6} \right) + \left( \frac{1 \times \frac{1}{2}}{2} \right) + \frac{1}{2}$</td>
<td>M1 M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{23}{36}$ oe</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>$P(T = 3 \mid \text{rolled twice}) = \frac{P((T = 3) \cap \text{die rolled twice})}{P(\text{die rolled twice})}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{5}{36}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{5}{18}$ oe</td>
<td>(3)</td>
</tr>
<tr>
<td><strong>(8 marks)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

Correct answer only in (a), (b) or (c) scores full marks for that part. Methods leading to answers > 1 score 0 marks

(a) M1: For a correct expression.
A1: Allow exact equivalent ($\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ is M0A0).

(b) M1: For $\frac{1}{2} + \text{at least one correct product.}$
M1: For fully correct expression.
A1: Allow exact equivalent.

(c) M1: For correct conditional probability ratio (this mark may be implied by 2nd M1) but going on to assume independence [using numerator $P(T = 3) \times P(\text{rolled twice})$] is M0M0A0.
M1: For a correct numerical ratio of probabilities (allow ft of (their (b) – $\frac{1}{2}$) as numerator).
A1: Allow exact equivalent.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| **6(a)** | \[
\frac{9}{10} \text{ oe}
\] | B1 |
| **(b)** | \[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\] | M1 A1 |
| | \[
\frac{5}{8} = \frac{2}{5} + P(B) - \frac{2}{5} P(B)
\] | M1 A1 |
| | \[
P(B) = \frac{3}{8} \text{ *}
\] | A1 cso |
| **(c)** | \[
P(A | B) = \frac{2}{5} \text{ oe}
\] | B1 |
| **(d)** | \[
\begin{align*}
&0.25 & &0.15 & &0.05 \\
&0.15 & &0 & &0.05 \\
&0 & &0.175 & &0.325 \\
&0.325 & &0.05 & &0
\end{align*}
\] | A1 |

**Diagram**:  
0.15 and 0.25  
0.05 and 0.05  
0.175 and 0.325

**(11 marks)**

**Notes:**

**(b)**  
**M1:** For use of \(P(A \cup B) = P(A) + P(B) - P(A \cap B)\)

**M1:** For use of \(P(A \cap B) = P(A) \times P(B)\) (But just seeing \(\frac{2}{5} \times \frac{3}{8} = \frac{3}{20}\) on its own is M0M0)

**A1:** A correct equation

**A1:** (No wrong working seen dependent on all previous marks)  
(allow a full verification method, however, substitution of \(P(B) = \frac{3}{8}\) into only one \(P(B)\) to find the other \(P(B)\) (e.g. using \(3/20\) to find \(3/8\)) can score M1M0A0A0)
**Question 6 notes continued**

(d)

**B1:** 3 circles intersecting, see diagram above, (at least 2 labelled) with the two zeros showing $A$ does not intersect $C$ (Do not allow blank spaces for the two zeros)  
**or** 3 circles, see diagram below, (at least 2 labelled) where $B$ intersects $A$ and $C$ but $A$ and $C$ do not intersect.

**M1:** 0.15 placed in $(A \cap B \cap C')$ and 0.25 placed in $(A \cap B' \cap C')$

**M1:** 0.3 – ‘their 0.25’ **and** 1 – (‘their 0.15’ + ‘their 0.25’ + ‘their 0.05’ + $\frac{1}{2}$)

**M1:** $\frac{3}{8}$ – (“their 0.15” + “their 0.05”), i.e. $P(B) = \frac{3}{8} \text{ and } \frac{1}{2} - \text{“their 0.175”}$, i.e. $P(C) = \frac{1}{2}$

For the 3rd M mark, blank regions inside $P(B)$ and $P(C)$ are not treated as 0s and score M0

**A1:** fully correct with box

---

**Diagram**

![Diagram of intersecting circles with numbers labeled](image)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7(a)(i)</td>
<td>( P(X &gt; 505) = P\left( Z &gt; \frac{505 - 503}{1.6} \right) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = 1 - P(Z &lt; 1.25) = 1 - 0.8944 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = 0.1056 ) awrt 0.106 A1</td>
<td>(3)</td>
</tr>
<tr>
<td>(ii)</td>
<td>( P(501 &lt; X &lt; 505) = 1 - 2 \times 0.1056 ) or ( 0.8944 - 0.1056 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = 0.7888 ) awrt 0.789 A1</td>
<td>(2)</td>
</tr>
<tr>
<td>(b)</td>
<td>( P(X &lt; w) = 0.9713 ) or ( P(X &gt; w) = 0.0287 ) (may be implied by ( z = \pm 1.9 ))</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( \frac{w - 503}{1.6} = 1.9 ) or ( \frac{1006 - w - 503}{1.6} = -1.9 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( w = 506.04... ) awrt 506 A1</td>
<td>(3)</td>
</tr>
<tr>
<td>(c)</td>
<td>( \frac{r - 503}{q} = -2.3263 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( \frac{r + 6 - 503}{q} = 1.6449 )</td>
<td>M1A1</td>
</tr>
<tr>
<td></td>
<td>( 1.6449q - 6 = -2.3263q ) ddM1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( q = 1.51... ) awrt 1.51 A1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( r = 499.48..... ) awrt 499 A1</td>
<td>(7)</td>
</tr>
</tbody>
</table>

Notes:

(a)
(i)  
M1: Standardising with 505, 503 and 1.6. May be implied by use of 1.25 (Allow ±)
M1: For 1 - P(Z < 1.25) i.e. a correct method for finding P(Z > 1.25),
  e.g. 1 - p where 0.5 < p < 0.99
(ii) 
M1: 1 - 2 \times \text{their(i)}

(b) 
M1: For using symmetry to find the area of one tail (may be seen in a diagram)
M1: A single standardisation with 503, 1.6 and w (or 1006 - w)
  and set = ± z value (1.8 < |z| < 2)
A1: For awrt 506 which must come from correct working. \textbf{(Answer only)}: 506 scores 0/3, but
  506.0...with no working send to review
### Question 7 notes continued

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(c)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>( \frac{r - 503}{q} = z \text{ value where }</td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>( \frac{r - 503}{q} = \text{awrt } -2.3263 ) (signs must be compatible)</td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>( \frac{r + 6 - 503}{q} = z \text{ value where }</td>
</tr>
<tr>
<td><strong>A1:</strong></td>
<td>( \frac{r + 6 - 503}{q} = \text{awrt 1.6449} ) (signs must be compatible)</td>
</tr>
</tbody>
</table>

**Special Case:**
Less than 4dp \( z \)-values: use of \( \text{awrt 2.32/2.33/2.34 and awrt 1.64/1.65} \) could score M1 A0 M1 and then A1 provided both equations have compatible signs.

**3rd M1:** (dep on both Ms) attempt to solve simultaneous equations leading to a value for \( q \) or \( r \)

**3rd A1:** Or awrt 1.51

**4th A1:** For awrt 499 (allow 499.5)
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – **there may be more space than you need**.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – **use this as a guide as to how much time to spend on each question**.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Answer ALL questions. Write your answers in the spaces provided.

1. The number of cars caught speeding per day, by a particular camera, has a Poisson distribution with mean 0.8

   (a) Find the probability that in a given 4 day period exactly 3 cars will be caught speeding by this camera. (3)

   A car has been caught speeding by this camera.

   (b) Find the probability that the period of time that elapses before the next car is caught speeding by this camera is less than 48 hours. (3)

   Given that 4 cars were caught speeding by this camera in a two day period,

   (c) find the probability that 1 was caught on the first day and 3 were caught on the second day. (5)

   Each car that is caught speeding by this camera is fined £60

   (d) Using a suitable approximation, find the probability that, in 90 days, the total amount of fines issued will be more than £5000 (5)
Question 1 continued
Question 1 continued
2. A continuous random variable $X$ has cumulative distribution function

$$F(x) = \begin{cases} 
0 & x < 1 \\
\frac{1}{5} (x - 1) & 1 \leq x \leq 6 \\
1 & x > 6 
\end{cases}$$

(a) Find $P(X > 4)$ \hspace{1cm} (2)

(b) Write down the value of $P(X \neq 4)$ \hspace{1cm} (1)

(c) Find the probability density function of $X$, specifying it for all values of $x$ \hspace{1cm} (2)

(d) Write down the value of $E(X)$ \hspace{1cm} (1)

(e) Find $\text{Var}(X)$ \hspace{1cm} (2)

(f) Hence or otherwise find $E(3X^2 + 1)$ \hspace{1cm} (3)
Question 2 continued

(a) Find $P(X \neq 4)$

(b) Write down the value of $P(X > 4)$

(c) Find the probability density function of $X$, specifying it for all values of $X$.

(d) Write down the value of $E(X)$

(e) Find $\text{Var}(X)$

(f) Hence or otherwise find $E(3X^2 + 1)$
Question 2 continued
Question 2 continued

(Total for Question 2 is 11 marks)
3. Explain what you understand by

(a) a statistic, (1)

(b) a sampling distribution. (1)

A factory stores screws in packets. A small packet contains 100 screws and a large packet contains 200 screws. The factory keeps small and large packets in the ratio 4:3 respectively.

(c) Find the mean and the variance of the number of screws in the packets stored at the factory. (3)

A random sample of 3 packets is taken from the factory and \( Y_1, Y_2 \) and \( Y_3 \) denote the number of screws in each of these packets.

(d) List all the possible samples. (2)

(e) Find the sampling distribution of \( \bar{Y} \) (4)
Question 3 continued
Question 3 continued

(Total for Question 3 is 11 marks)
4. Accidents occur randomly at a crossroads at a rate of 0.5 per month. A researcher records the number of accidents, $X$, which occur at the crossroads in a year.

(a) Find $P(5 \leq X < 7)$

A new system is introduced at the crossroads. In the first 18 months, 4 accidents occur at the crossroads.

(b) Test, at the 5% level of significance, whether or not there is reason to believe that the new system has led to a reduction in the mean number of accidents per month. State your hypotheses clearly.
Question 4 continued

(Total for Question 4 is 7 marks)
5. The continuous random variable $X$ has probability density function $f(x)$ given by

\[
f(x) = \begin{cases} 
  k(x^2 + a) & -1 < x \leq 2 \\
  3k & 2 < x \leq 3 \\
  0 & \text{otherwise}
\end{cases}
\]

where $k$ and $a$ are constants.

Given that $E(X) = \frac{17}{12}$

(a) find the value of $k$ and the value of $a$  

(b) Write down the mode of $X$  

Leave blank
Question 5 continued

(Total for Question 5 is 9 marks)
6. The Headteacher of a school claims that 30% of parents do not support a new curriculum. In a survey of 20 randomly selected parents, the number, \( X \), who do not support the new curriculum is recorded.

Assuming that the Headteacher’s claim is correct, find

(a) the probability that \( X = 5 \)  

(b) the mean and the standard deviation of \( X \) 

The Director of Studies believes that the proportion of parents who do not support the new curriculum is greater than 30%. Given that in the survey of 20 parents 8 do not support the new curriculum,

(c) test, at the 5% level of significance, the Director of Studies’ belief. State your hypotheses clearly.

The teachers believe that the sample in the original survey was biased and claim that only 25% of the parents are in support of the new curriculum. A second random sample, of size \( 2n \), is taken and exactly half of this sample supports the new curriculum.

A test is carried out at a 10% level of significance of the teachers’ belief using this sample of size \( 2n \)

Using the hypotheses \( H_0: p = 0.25 \) and \( H_1: p > 0.25 \)

(d) find the minimum value of \( n \) for which the outcome of the test is that the teachers’ belief is rejected.
Question 6 continued
Question 6 continued
Question 6 continued

(Total for Question 6 is 13 marks)
7. A multiple choice examination paper has \( n \) questions where \( n > 30 \)
Each question has 5 answers of which only 1 is correct. A pass on the paper is obtained 
by answering 30 or more questions correctly.

The probability of obtaining a pass by randomly guessing the answer to each question 
should not exceed 0.0228

Use a normal approximation to work out the greatest number of questions that could be 
used. 

(8)
Question 7 continued
Question 7 continued

(Total for Question 7 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS
## Statistics S2 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1(a)</strong></td>
<td>$X \sim \text{Po}(3.2)$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$P(X = 3) = \frac{e^{-3.2}3^{3.2^3}}{3!}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= 0.2226 awrt 0.223</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$Y \sim \text{Po}(1.6)$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$P(Y \geq 1) = 1 - P(Y = 0)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= $1 - e^{-1.6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 0.7981 awrt 0.798</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td></td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>$X \sim \text{Po}(0.8)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P(X = 1) \times P(X = 3) \div P(Y = 4) = \frac{(e^{-0.8} \times 0.8) \times \left(\frac{e^{-0.8} \times 0.8^3}{3!}\right)}{e^{-1.6} \times \frac{1.6^4}{4!}}$</td>
<td>M1 M1</td>
</tr>
<tr>
<td></td>
<td>= $\frac{0.3594 \times 0.0383}{0.05513}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>= 0.25</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td></td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>$A \sim \text{Po}(72)$ approximated by $N(72,72)$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>$\frac{5000}{60} = 83.33$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$P(A \geq 84) = P\left(Z \geq \frac{83.5 - 72}{\sqrt{72}}\right)$</td>
<td>M1 M1</td>
</tr>
<tr>
<td></td>
<td>= $P(Z \geq 1.355\ldots)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 0.0869 awrt 0.087/0.088</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(16 marks)</td>
<td></td>
</tr>
</tbody>
</table>

### Notes:

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B1:</strong></td>
<td>For writing or using Po(3.2)</td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>$e^{-\lambda} \frac{\lambda^3}{3!}$</td>
</tr>
<tr>
<td><strong>B1:</strong></td>
<td>For writing or using Po(1.6)</td>
</tr>
<tr>
<td><strong>M1:</strong></td>
<td>$1 - P(Y = 0)$ or $1 - e^{-\lambda}$</td>
</tr>
</tbody>
</table>
### Question 1 notes continued

**(c)**

**M1:** Using $P(0.8)$ with $X=1$ or $X=3$ (may be implied by 0.359... or 0.0383...)

**M1:** \[
\left( e^{-\lambda} \times \frac{e^{-\lambda^3}}{3!} \right) \text{ (consistent lambda) awrt 0.0138 implies 1st 2 M marks}
\]

**M1:** Correct use of conditional probability with denominator $= \frac{e^{-1.6^4}}{4!}$

**A1:** Fully correct expression

**A1:** 0.25 (allow awrt 0.250)

**(d)**

**B1:** Writing or using $N(72,72)$

**M1:** For exact fraction or awrt 83.3 (may be implied by 84)

(Note: Use of $N(4320,4320)$ can score B1 and 1st M1)

**M1:** Using $84 \pm 0.5$

**M1:** Standardising using 82.5, 83, 83.3 (awrt 83.3), 83.5, 83.8, 84 or 84.5, ‘their mean’ and ‘their sd’
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(a)</td>
<td>( P(X &gt; 4) = 1 - F(4) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = 1 - \frac{3}{5} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \frac{2}{5} \text{ oe} )</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>1</td>
<td>B1</td>
</tr>
<tr>
<td>(c)</td>
<td>( f(x) = \frac{dF(x)}{dx} = \frac{1}{5} )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( f(x) = \begin{cases} \frac{1}{5} &amp; 1 \leq x \leq 6 \ 0 &amp; \text{otherwise} \end{cases} )</td>
<td>A1</td>
</tr>
<tr>
<td>(d)</td>
<td>( E(X) = 3.5 )</td>
<td>B1</td>
</tr>
<tr>
<td>(e)</td>
<td>Variance = ( \frac{(6-1)^2}{12} ) or ( \int_{1}^{6} \frac{1}{5} x^2 , dx - (3.5)^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = \frac{25}{12} ) awrt 2.08</td>
<td>A1</td>
</tr>
<tr>
<td>(f)</td>
<td>( E(X^2) = \text{Var}(X) + [E(X)]^2 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( = \frac{25}{12} + 3.5^2 ) or ( \int_{1}^{6} \frac{1}{5} x^2 , dx ) or ( \int_{1}^{6} \frac{1}{5} (3x^2 + 1) , dx )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \frac{43}{3} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( E(3X^2 + 1) = 3 E(X^2) + 1 ) = ( \left[ \frac{3x^3 + x}{15} \right]_1^6 )</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>( = 44 )</td>
<td>A1cao</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3)</td>
</tr>
</tbody>
</table>

**Notes:**

(a) **M1:** Writing or using \( 1 - F(4) \) o.e.

(c) **M1:** For differentiating to get \( \frac{1}{5} \)
**Question 2 notes continued**

A1: Both lines correct with ranges

(e)  
M1:  
\[
\frac{(6-1)^2}{12} \text{ or } \int_{1}^{6} \frac{1}{5} x^2 \, dx - \text{their } 3.5^2
\]

(f)  
M1:  
“Their Var(\(X\))” + [“their E(\(X\))”]^2 (which must follow from the 1st method in (e))  
\text{or } \int_{1}^{6} \frac{1}{5} x^2 \, dx \text{ and integrating } x^n \to \frac{x^{n+1}}{n+1} \text{ (may be seen in (e)) or writing } \int_{1}^{6} \frac{1}{5} (3x^2 + 1) \, dx

(May be implied by \(\frac{43}{3}\) seen)

dM1: Using 3 × ‘their E(\(X^2\))’ + 1 or  
\[
\int_{1}^{6} \frac{1}{5} (3x^2 + 1) \, dx \text{ and integrating } x^n \to \frac{x^{n+1}}{n+1}
\]
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(a)</td>
<td>(A random variable) that is a function of a (random) sample involving no unknown quantities/parameters or A quantity calculated solely from a random sample</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>If all possible samples are chosen from a population; then the values of a statistic and the associated probabilities is a sampling distribution or a probability distribution of a statistic</td>
<td>B1</td>
</tr>
<tr>
<td>(c)</td>
<td>Mean ( = 100 \times \frac{4}{7} + 200 \times \frac{3}{7} ) ( = \frac{1000}{7} ) awrt 143</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Variance ( = 100^2 \times \frac{4}{7} + 200^2 \times \frac{3}{7} - \left( \frac{1000}{7} \right)^2 ) ( = \frac{120000}{49} ) awrt 2450 (to 3sf)</td>
<td>M1</td>
</tr>
<tr>
<td>(d)</td>
<td>(100,100,100) (100,200,100) (200,100,100) or 3 x (100,100,200) (100,200,200) (200,100,200) or 3 x (100,200,200) (200,200,200)</td>
<td>B2</td>
</tr>
<tr>
<td>(e)</td>
<td>(100,100,100) ( \left( \frac{4}{7} \right)^3 = \frac{64}{343} ) awrt 0.187</td>
<td>B1 both</td>
</tr>
<tr>
<td></td>
<td>(200,200,200) ( \left( \frac{3}{7} \right)^3 = \frac{27}{343} ) awrt 0.0787</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(100,100,200) ( 3 \times \left( \frac{4}{7} \right)^2 \times \left( \frac{3}{7} \right) = \frac{144}{343} ) awrt 0.420 (allow 0.42)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(100,200,200) ( 3 \times \left( \frac{4}{7} \right) \times \left( \frac{3}{7} \right)^2 = \frac{108}{343} ) awrt 0.315</td>
<td>A1</td>
</tr>
</tbody>
</table>
Question | Scheme | Marks
---|---|---
3(e) continued | | A1

<table>
<thead>
<tr>
<th>$m$</th>
<th>100</th>
<th>400/3 (\text{awrt 133})</th>
<th>500/3 (\text{awrt 167})</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(M = m)$</td>
<td>64/343 or awrt 0.187</td>
<td>144/343 or awrt 0.420 (allow 0.42)</td>
<td>108/343 or awrt 0.315</td>
<td>27/343 or awrt 0.0787</td>
</tr>
</tbody>
</table>

(4) (11 marks)

Notes:

(a)

B1: For a definition which includes each of the following 3 aspects

A function\(^1\) of a (random) sample\(^2\) involving no unknown quantities/parameters\(^3\)

1. function/quantity/calculation/value/random variable
2. sample/observations/data
3. no unknown parameters/no unknown values/solely (from a sample)

(b)

B1: Requires all underlined words: All values of a statistic with their associated probabilities or probability distribution of a statistic

(c)

M1: \(100^2 \times \frac{4}{7} + 200^2 \times \frac{3}{7} - (\text{their mean})^2\)

(d)

B1: Any 2 of (100,100,100), (100,100,200) any order, (100,200,200) any order, (200,200,200)

B1: All correct, allow 3 \(\times (100,100,200)\) and 3 \(\times (100,200,200)\) and (100,100,100) and (200,200,200)

(Note: Allow other notation for 100 and 200 e.g. Small and Large)

(e)

B1: Both probabilities for (100,100,100) and (200,200,200) correct

M1: \(3 \times p^2 \times (1 - p)\)

A1: Either correct

A1: All means correct and all probabilities correct (table not required but means must be associated with correct probabilities)
### Question 4(a)

<table>
<thead>
<tr>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X \sim \text{Po}(6) )</td>
</tr>
</tbody>
</table>

\[
P(5 \leq X < 7) = P(X \leq 6) - P(X \leq 4) \quad \text{or} \quad \frac{e^{-6}6^5}{5!} + \frac{e^{-6}6^6}{6!}
\]

\[
= 0.6063 - 0.2851 \\
= 0.3212 \quad \text{awrt 0.321}
\]

**Marks**

- M1
- M1
- A1

### Question (b)

<table>
<thead>
<tr>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0: \lambda = 9 )  \quad \text{H}_1: \lambda &lt; 9</td>
</tr>
</tbody>
</table>

\( X \sim \text{Po}(9) \) therefore

\[
P(X \leq 4) = 0.05496 \ldots \quad \text{or} \quad \text{CR } X \leq 3
\]

Insufficient evidence to reject \( H_0 \) or Not Significant or 4 does not lie in the critical region.

There is no evidence that the mean number of accidents at the crossroads has reduced/decreased.

**Marks**

- B1
- B1
- dM1
- A1

### Notes:

(a)

- **M1**: Writing or using \( \text{Po}(6) \)

- **M1**: Either \( P(X \leq 6) - P(X \leq 4) \) \text{ or } \frac{e^{-\lambda} \lambda^5}{5!} + \frac{e^{-\lambda} \lambda^6}{6!}

(b)

- **B1**: Both hypotheses correct (\( \lambda \text{ or } \mu \)) allow 0.5 instead of 9

- **B1**: Either awrt 0.055 \text{ or } critical region \( X \leq 3 \)

- **dM1**: For a correct comment (dependent on previous B1)

Contradictory non-contextual statements such as “not significant” so “reject \( H_0 \)” score M0.

(May be implied by a correct contextual statement)

- **A1**: Cso requires correct contextual conclusion with underlined words and all previous marks in (b) to be scored.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5(a)</strong></td>
<td>$\int_{-1}^{2} k(x^2 + a) , dx + \int_{2}^{3} 3k , dx = 1$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$(k \left( \frac{x^3}{3} + ax \right))<em>{-1}^{2} + [3kx]</em>{2}^{3} = 1$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$k \left( \frac{8}{3} + 2a + \frac{1}{3} + a \right) + 9k - 6k = 1$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$6k + 3ak = 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\int_{-1}^{2} k(x^3 + ax) , dx + \int_{2}^{3} 3kx , dx = \frac{17}{12}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$(k \left( \frac{x^4}{4} + \frac{ax^2}{2} \right))<em>{-1}^{2} + [3kx^2]</em>{2}^{3} = \frac{17}{12}$</td>
<td>dM1</td>
</tr>
<tr>
<td></td>
<td>$k \left( \frac{4 + 2a - \frac{1}{4} - \frac{a}{2}}{2} \right) + \frac{27k}{2} - 6k = \frac{17}{12}$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$\frac{45k}{4} + \frac{3ak}{2} = \frac{17}{12}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$135k + 18ak = 17$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$99k = 11$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a = 1, k = \frac{1}{9}$</td>
<td>A1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>2</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1)</td>
</tr>
</tbody>
</table>

(9 marks)

**Notes:**

(a)  

**M1:** Writing or using $\int_{-1}^{2} k(x^2 + a) \, dx + \int_{2}^{3} 3k \, dx = 1$ ignore limits.

**dM1:** Attempting to integrate at least one $x^n \rightarrow \frac{x^{n+1}}{n+1}$ and sight of correct limits (dependent on previous M1).

**A1:** Correct equation – need not be simplified.

**M1:** $\int_{-1}^{2} k(x^3 + ax) \, dx + \int_{2}^{3} 3kx \, dx$ ignore limits.

**dM1:** Setting $= \frac{17}{12}$ and attempting to integrate at least one $x^n \rightarrow \frac{x^{n+1}}{n+1}$ and sight of correct limits (dependent on previous M1).
<table>
<thead>
<tr>
<th>Question 5 notes continued</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A1:</strong> A correct equation – need not be simplified.</td>
</tr>
<tr>
<td><strong>ddM1:</strong> Attempting to solve two simultaneous equations in $a$ and $k$ by eliminating 1 variable (dependent on 1st and 3rd M1s).</td>
</tr>
<tr>
<td><strong>A1:</strong> Both $a$ and $k$ correct.</td>
</tr>
<tr>
<td>Question</td>
</tr>
<tr>
<td>----------</td>
</tr>
</tbody>
</table>
| 6(a) | \[ P(X = 5) = \binom{20}{5}(0.3)^5(0.7)^{15} \quad \text{or} \quad 0.4164 - 0.2375 \]  
\[ = 0.17886\ldots \quad \text{awrt} \ 0.179 \] | M1 | A1 |
| (b) | Mean = 6  
\[ \text{sd} = \sqrt{20 \times 0.7 \times 0.3} \]  
\[ = 2.049\ldots \quad \text{awrt} \ 2.05 \] | B1 | M1 | A1 |
| (c) | \( H_0: p = 0.3 \quad H_1: p > 0.3 \)  
\( X \sim B(20,0.3) \)  
\[ P(X \geq 8) = 0.2277 \quad \text{or} \quad P(X \geq 10) = 0.0480, \text{ so CR } X \geq 10 \] | B1 | M1 | A1 |

Insufficient evidence to reject \( H_0 \) or Not Significant or \( 8 \) does not lie in the critical region.  

There is no evidence to support the Director (of Studies') belief/There is no evidence that the proportion of parents that do not support the new curriculum is greater than 30%.  

| (d) | \( X \sim B(2n, 0.25) \)  
\( X \sim B(8, 0.25) \)  
\[ P(X \geq 4) = 0.1138 \]  
\( X \sim B(10, 0.25) P(X \geq 5) = 0.0781 \]  
2n = 10  
\( n = 5 \) | M1 | A1 | A1 |

<table>
<thead>
<tr>
<th>Notes:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( M1: \quad \binom{20}{5}(p)^5(1-p)^{15} \quad \text{or} \quad P(X \leq 5) - P(X \leq 4) )</td>
</tr>
<tr>
<td>(b)</td>
<td>( M1: \quad \text{Use of } 20 \times 0.7 \times 0.3 \quad \text{(with or without the square root).} )</td>
</tr>
</tbody>
</table>
| (c) | \( B1: \quad \text{Both hypotheses correct (p or \( \pi \)).} \)  
\( M1: \quad \text{Using } X \sim B(20,0.3) \quad \text{(may be implied by 0.7723, 0.2277, 0.8867 or 0.1133)} \)  
\( A1: \quad \text{Awrt 0.228 or CR } X \geq 10 \)  
\( dM1: \quad \text{A correct comment (dependent on previous M1)} \)  
\( A1: \quad \text{Cso requires correct contextual conclusion with underlined words and all previous marks in (c) to be scored.} \) |
(d)
M1: For 0.1138 or 0.0781 or 0.8862 or 0.9219 seen.
A1: $B(10, 0.25)$ selected (may be implied by $n = 10$ or $2n = 10$ or $n = 5$).
    An answer of 5 with no incorrect working seen scores 3 out of 3.
    Special Case: Use of a normal approximation.

M1: For $\frac{(n-0.5)-\frac{n}{2}}{\sqrt{\frac{2}{n}}} = z$ with $1.28 \leq z \leq 1.29$, 1$^{\text{st}}$ A1 for $n=4.2/4.3$, 2$^{\text{nd}}$ A1 for $n=5$
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y \sim N\left( \frac{n}{5}, \frac{4n}{25} \right) )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>( P(Y \geq 30) = P \left( Z &gt; \frac{29.5 - \frac{n}{5}}{\frac{2}{5} \sqrt{n}} \right) )</td>
<td>M1, M1, A1</td>
<td></td>
</tr>
<tr>
<td>( \frac{29.5 - \frac{n}{5}}{\frac{2}{5} \sqrt{n}} = 2 )</td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>( n + 4\sqrt{n} - 147.5 = 0 \quad \text{or} \quad 0.04n^2 - 12.44n + 870.25 = 0 )</td>
<td>dM1</td>
<td></td>
</tr>
<tr>
<td>( \sqrt{n} = 10.3 \ldots \quad n = 106.26 \ldots \quad \text{or} \quad n = 204.73 \ldots )</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>( n = 106 )</td>
<td>A1 cao</td>
<td>(8 marks)</td>
</tr>
</tbody>
</table>

**Notes:**

**B1:** Writing or using \( N\left( \frac{n}{5}, \frac{4n}{25} \right) \)

**M1:** Writing or using 30 +/- 0.5

**M1:** Standardising using 29, 29.5, 30 or 30.5 and their mean and their sd

**A1:** Fully correct standardisation (allow +/-)

**B1:** For \( z = +/- 2 \) or awrt 2.00 must be compatible with their standardisation

**dM1:** (Dependent on 2\textsuperscript{nd} M1) getting quadratic equation and solving leading to a value of \( \sqrt{n} \) or \( n \)

**A1:** Awrt 10.3 \text{ or} awrt (106 \text{ or} 107 \text{ or} 204 \text{ or} 205)

**A1:** For 106 only (must reject other solutions if stated)

(Note: \( \frac{29.5 - \frac{n}{5}}{\frac{2}{5} \sqrt{n}} = -2 \) leading to an answer of 106 may score B1M1M1A1B0M1A1A1)
(Time: 1 hour 30 minutes) Paper Reference **WST03/01**

**Mathematics**

**International Advanced Subsidiary/Advanced Level Statistics S3**

**You must have:**
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

**Instructions**
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – **there may be more space than you need**.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

**Information**
- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – **use this as a guide as to how much time to spend on each question**.

**Advice**
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

---

Please check the examination details below before entering your candidate information

<table>
<thead>
<tr>
<th>Candidate surname</th>
<th>Other names</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Pearson Edexcel**

**International Advanced Level**

Sample Assessment Materials for first teaching September 2018

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

**Instructions**
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- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
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- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Answer ALL questions. Write your answers in the spaces provided.

1. The names of the 720 members of a swimming club are listed alphabetically in the club’s membership book. The chairman of the swimming club wishes to select a systematic sample of 40 names. The names are numbered from 001 to 720 and a number between 001 and \( w \) is selected at random. The corresponding name and every \( x \)th name thereafter are included in the sample.

(a) Find the value of \( w \). 

(b) Find the value of \( x \).

(c) Write down the probability that the sample includes both the first name and the second name in the club’s membership book.

(d) State one advantage and one disadvantage of systematic sampling in this case.
(b) Find the value of $x$.

(c) Write down the probability that the sample includes both the first name and the second name in the club’s membership book. The chairman of the swimming club wishes to select a systematic sample of 40 names. The names are numbered from 001 to 720 and a number between 001 and 720 is selected at random. The corresponding name and every number in the club’s membership book.

(d) State one advantage and one disadvantage of systematic sampling in this case.

(Total for Question 1 is 5 marks)
2. Nine dancers, Adilzhan (A), Bianca (B), Chantelle (C), Lee (L), Nikki (N), Ranjit (R), Sergei (S), Thuy (T) and Yana (Y), perform in a dancing competition.

Two judges rank each dancer according to how well they perform. The table below shows the rankings of each judge starting from the dancer with the strongest performance.

<table>
<thead>
<tr>
<th>Rank</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judge 1</td>
<td>S</td>
<td>N</td>
<td>B</td>
<td>C</td>
<td>T</td>
<td>A</td>
<td>Y</td>
<td>R</td>
<td>L</td>
</tr>
<tr>
<td>Judge 2</td>
<td>S</td>
<td>T</td>
<td>N</td>
<td>B</td>
<td>C</td>
<td>Y</td>
<td>L</td>
<td>A</td>
<td>R</td>
</tr>
</tbody>
</table>

(a) Calculate Spearman’s rank correlation coefficient for these data.  

(5)

(b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not the two judges are generally in agreement.  

(4)
Nine dancers, Adilzhan, Thuy, Sergei, Ranjit, Chantelle, Lee, Bianca, perform in a dancing competition.

(a) Calculate Spearman’s rank correlation coefficient for these data.

(b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not the two judges are generally in agreement.
Question 2 continued

(Total for Question 2 is 9 marks)
3. The number of accidents on a particular stretch of motorway was recorded each day for 200 consecutive days. The results are summarised in the following table.

<table>
<thead>
<tr>
<th>Number of accidents</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>47</td>
<td>57</td>
<td>46</td>
<td>35</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Show that the mean number of accidents per day for these data is 1.6

A motorway supervisor believes that the number of accidents per day on this stretch of motorway can be modelled by a Poisson distribution.

She uses the mean found in part (a) to calculate the expected frequencies for this model. Her results are given in the following table.

<table>
<thead>
<tr>
<th>Number of accidents</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>40.38</td>
<td>64.61</td>
<td>r</td>
<td>27.57</td>
<td>11.03</td>
<td>s</td>
</tr>
</tbody>
</table>

(b) Find the value of \( r \) and the value of \( s \), giving your answers to 2 decimal places.

(c) Stating your hypotheses clearly, use a 10% level of significance to test the motorway supervisor's belief. Show your working clearly.
Question 3 continued
Question 3 continued

(Total for Question 3 is 11 marks)
4. A farm produces potatoes. The potatoes are packed into sacks. The weight of a sack of potatoes is modelled by a normal distribution with mean 25.6 kg and standard deviation 0.24 kg

(a) Find the probability that two randomly chosen sacks of potatoes differ in weight by more than 0.5 kg

(b) Find the probability that the total weight of a randomly chosen full pallet of potatoes is greater than 785 kg
The weight of a sack of potatoes is modelled by a normal distribution with mean 25.6 kg.

A farm produces potatoes. The potatoes are packed into sacks.

(b) Find the probability that the total weight of a randomly chosen full pallet of potatoes is greater than 785 kg.

Each full pallet of potatoes holds 30 sacks of potatoes.

The weight of an empty pallet is modelled by a normal distribution with mean 20.0 kg and standard deviation 0.32 kg.

(s) Sacks of potatoes are randomly selected and packed onto pallets.

(a) Find the probability that two randomly chosen sacks of potatoes differ in weight by more than 0.5 kg.

Question 4 continued
Question 4 continued

(Total for Question 4 is 11 marks)
5. A Head of Department at a large university believes that gender is independent of the grade obtained by students on a Business Foundation course. A random sample was taken of 200 male students and 160 female students who had studied the course.

The results are summarised below.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinction</td>
<td>18.5%</td>
<td>27.5%</td>
</tr>
<tr>
<td>Merit</td>
<td>63.5%</td>
<td>60.0%</td>
</tr>
<tr>
<td>Unsatisfactory</td>
<td>18.0%</td>
<td>12.5%</td>
</tr>
</tbody>
</table>

Stating your hypotheses clearly, test the Head of Department’s belief using a 5% level of significance. Show your working clearly.

(12)
Question 5 continued
Question 5 continued

(Total for Question 5 is 12 marks)
6. As part of an investigation, a random sample was taken of 50 footballers who had completed an obstacle course in the early morning. The time taken by each of these footballers to complete the obstacle course, \( x \) minutes, was recorded and the results are summarised by

\[
\sum x = 1570 \quad \text{and} \quad \sum x^2 = 49467.58
\]

(a) Find unbiased estimates for the mean and variance of the time taken by footballers to complete the obstacle course in the early morning.

(4)

An independent random sample was taken of 50 footballers who had completed the same obstacle course in the late afternoon. The time taken by each of these footballers to complete the obstacle course, \( y \) minutes, was recorded and the results are summarised as

\[
\bar{y} = 30.9 \quad \text{and} \quad s_y^2 = 3.03
\]

(b) Test, at the 5% level of significance, whether or not the mean time taken by footballers to complete the obstacle course in the early morning, is greater than the mean time taken by footballers to complete the obstacle course in the late afternoon. State your hypotheses clearly.

(7)

(c) Explain the relevance of the Central Limit Theorem to the test in part (b).

(1)

(d) State an assumption you have made in carrying out the test in part (b).

(1)
Question 6 continued
Question 6 continued

(Total for Question 6 is 13 marks)
7. A fair six-sided die is labelled with the numbers 1, 2, 3, 4, 5 and 6.
The die is rolled 40 times and the score, $S$, for each roll is recorded.

(a) Find the mean and the variance of $S$.  \hspace{1cm} (2)

(b) Find an approximation for the probability that the mean of the 40 scores is less than 3.  \hspace{1cm} (3)
The die is rolled 40 times and the score 7.
A fair six-sided die is labelled with the numbers 1, 2, 3, 4, 5 and 6.

(b) Find an approximation for the probability that the mean of the 40 scores is less than 3.

(Question 7 continued)

(Total for Question 7 is 5 marks)
8. A factory produces steel sheets whose weights $X$ kg, are such that $X \sim N(\mu, \sigma^2)$

A random sample of these sheets is taken and a 95% confidence interval for $\mu$ is found to be $(29.74, 31.86)$

(a) Find, to 2 decimal places, the standard error of the mean. (3)

(b) Hence, or otherwise, find a 90% confidence interval for $\mu$ based on the same sample of sheets. (3)

Using four different random samples, four 90% confidence intervals for $\mu$ are to be found.

(c) Calculate the probability that at least 3 of these intervals will contain $\mu$. (3)
Question 8 continued
Question 8 continued

(Total for Question 8 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS
## Question 8 continued

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>{w} = 018 or 18</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(b)</td>
<td>{x} = 18</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(c)</td>
<td>{\text{prob}} = 0</td>
<td>B1 (1)</td>
</tr>
<tr>
<td>(d)</td>
<td><strong>Advantage:</strong> Any one of:</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>• Simple or easy to use also allow “quick” or “efficient” (o.e.)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• It is suitable for large samples (or populations)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Gives a good spread of the data</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Disadvantage:</strong> Any one of:</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>• The alphabetical list is (probably) <strong>not random</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Biased since the list is not (truly) random</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Some combinations of names are not possible</td>
<td></td>
</tr>
</tbody>
</table>

(2 marks)

### Notes:

- If no labels are given treat the 1st reason as an advantage and the 2nd as a disadvantage.
- For advantage: B1
- For disadvantage – “it requires a sampling frame” is 2nd B0 since the alphabetical list is given.
- Note: Do not score both B1 marks for opposing advantages and disadvantages.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2(a)</strong></td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td>Judge 1: 6 3 4 9 2 8 1 5 7</td>
<td>Judge 2: 8 4 5 7 3 9 1 2 6</td>
<td></td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Judge 1: 1 2 3 4 5 6 7 8 9</td>
<td>Judge 2: 1 3 4 5 2 8 6 9 7</td>
<td></td>
</tr>
<tr>
<td>$\sum d^2 = 4 + 1 + 1 + 4 + 1 + 0 + 9 + 1$</td>
<td>or $0 + 1 + 1 + 9 + 4 + 1 + 1 + 4 = 22$</td>
<td></td>
</tr>
<tr>
<td>or $0111911411411091$</td>
<td>or $01119111411411091$</td>
<td></td>
</tr>
<tr>
<td>$\text{M1}$</td>
<td>$\text{A1}$</td>
<td></td>
</tr>
<tr>
<td>$r_s = 1 - \frac{6(22)}{9(80)} = 0.8166666...$</td>
<td>$\frac{49}{60}$ or awrt 0.817</td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>(9 marks)</td>
<td></td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>Critical Value = 0.7833 or CR: $r_s \geq 0.7833$</td>
<td></td>
</tr>
<tr>
<td>$H_0 : \rho = 0$, $H_1 : \rho &gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Since $r_s = 0.8166...$ it lies in the CR, or reject $H_0$ (o.e.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The two judges (or “they”) are in agreement or there is a positive correlation between the ranks of the two judges.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)ft</td>
<td>(4)ft</td>
<td></td>
</tr>
<tr>
<td>Notes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(a)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1:</td>
<td>For an attempt to rank at least one row (at least 4 correct)</td>
<td></td>
</tr>
<tr>
<td>M1:</td>
<td>For an attempt at $d^2$ row (may be implied by sight of $\sum d^2 = 22$ or 221 for reverse ranks)</td>
<td></td>
</tr>
<tr>
<td>A1:</td>
<td>For $\sum d^2 = 22$ (or 221 if reverse ranking is used) Can be implied by correct answer.</td>
<td></td>
</tr>
<tr>
<td>M1:</td>
<td>For use of the correct formula with their $\sum d^2$ (if it is clearly stated)</td>
<td></td>
</tr>
<tr>
<td>If the answer is not correct then a correct expression is required</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>False Ranking</strong> - e.g. Alphabetic ranking: Gives</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Judge 1: 7 5 2 3 8 1 9 6 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Judge 2: 7 8 5 2 3 9 4 1 6</td>
<td>$\sum d^2 = 162$ and $r_s = -0.35$</td>
<td></td>
</tr>
</tbody>
</table>
### Question 2 notes continued

Scores: M0 (for ranking), M1 (for attempt at \( d^2 \) row), A0, M1 (for use of their \( \sum d^2 \)), A0 i.e. 2 out of 5. Can follow through their \( r_i \) in (b)

(b)  

<table>
<thead>
<tr>
<th>B1:</th>
</tr>
</thead>
<tbody>
<tr>
<td>For both hypotheses stated correctly in terms of ( \rho ) (allow ( \rho_i )) ( H_1 ) must be compatible with ranking.</td>
</tr>
<tr>
<td>B1:</td>
</tr>
<tr>
<td>For ( cv = 0.7833 ) (independent of their ( H_1 ) (no 2-tail value in tables) but compatible sign with their ( r_s )).</td>
</tr>
<tr>
<td>M1:</td>
</tr>
<tr>
<td>For a correct statement (in words) relating their ( r_i ) with their critical value. E.g. “reject ( H_0 )”, “in critical region”, “significant”, “positive correlation”. May be implied by a correct contextual comment.</td>
</tr>
</tbody>
</table>
| **|cv|>1** - If their \( cv \) is |cv| > 1 (often from using normal tables) award M0A0  
- If |their | > |their cv| then “significant” (o.e.) for M1 and “judges are in agreement” (o.e.) for A1ft  
- If |their | < |their cv| then “not significant” (o.e.) for M1 and “judges don’t agree” (o.e.) for A1ft |
| A1ft: |
| For a correct follow through conclusion in context. “Positive correlation” alone scores M1 A0. For reverse ranking should still say “judges are in agreement” |
Question 3 notes

(b) B1:
For both hypotheses and mentioning Poisson at least once. Allow Poisson is a “good fit/model” but not “good method”. Inclusion of 1.6 for mean in hypotheses is B0 but condone in conclusion.

M1:
For an attempt to pool 4 accidents and 5 accidents or pool when $E_i < 5$. No pooling is M0.

M1:
For an attempt at the test statistic, at least 2 correct expressions/values (to awrt 2 d.p.)

A1:
For awrt 4.65 (score M1M1A1 if awrt 4.65 seen).

No pooling:
If no pooling can allow 2nd M1 if $X^2 = 5.33$ is seen

B1 ft:
For 11

M1:
For a correct ft for their $2(0.10)$, where $11 \frac{n}{n} = \frac{n}{n}$ from their $n$.

(B1B1 may be implied by 6.251 (if pooling) or 7.779 for no pooling)

A1 ft:
(7)
Dep. on the 2nd M1)
For correct comment in context based on their test statistic and their critical value that mentions accidents or supervisor. Condone mention of Po(1.6) in conclusion. Score A0 for inconsistencies e.g. “significant” followed by “supervisor’s belief is justified”

Notes:

(b) Note: Allow A1 for $s = \text{awrt } 4.74$ (fou as a result of using expected values to full accuracy.)
Question 3 notes continued

(c)  
B1: For both hypotheses and mentioning Poisson at least once. Allow Poisson is a “good fit/model” but not “good method”. Inclusion of 1.6 for mean in hypotheses is B0 but condone in conclusion.

M1: For an attempt to pool 4 accidents and ≥ 5 accidents or pool when $E_i < 5$ No pooling is M0

M1: For an attempt at the test statistic, at least 2 correct expressions/values (to awrt 2 d.p.)

A1: For awrt 4.65 (score M1M1A1 if awrt 4.65 seen).

No pooling: If no pooling can allow 2nd M1 if $X^2 = 5.33$ is seen

B1ft: For $n - 1 - 1$ i.e. subtracting 2 from their $n$.

B1ft: For a correct fit for their $\chi^2(0.10)$, where $k = n - 1 - 1$ from their $n$.

(B1B1 may be implied by 6.251 (if pooling) or 7.779 for no pooling)

A1ft: (Dep. on the 2nd M1) For correct comment in context based on their test statistic and their critical value that mentions accidents or supervisor. Condone mention of Po(1.6) in conclusion. Score A0 for inconsistencies e.g. “significant” followed by “supervisor’s belief is justified”

Note: Full accuracy gives a combined expected frequency of 15.76, $\frac{(O - E)^2}{E} = 0.0366$, $\frac{O^2}{E} = 14.2766$, $X^2 = 4.6485...$ and p-value 0.199.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4(a)</strong></td>
<td>Let $X = \text{weight of a sack of potatoes, } X \sim N(25.6, 0.24^2)$</td>
</tr>
<tr>
<td></td>
<td>$D = X_1 - X_2 - N(0, 2(0.24)^2)$ or $D = N(0, 0.1152)$</td>
</tr>
<tr>
<td></td>
<td>${P(</td>
</tr>
<tr>
<td></td>
<td>$2 \times P(D &gt; 0.5)$ can be implied $\quad$ dM1</td>
</tr>
<tr>
<td></td>
<td>$= 2 \times P\left(Z &gt; \frac{0.5}{\sqrt{0.1152}}\right)$ $\quad$ dM1</td>
</tr>
<tr>
<td></td>
<td>$= 2 \times P\left(Z &gt; 1.4731...\right)$ or $2(1-0.9292)$$\quad$= 0.1416 $\quad$ awrt 0.141 or awrt 0.142 $\quad$ A1</td>
</tr>
</tbody>
</table>

| **(b)** | Let $Y = \text{weight of an empty pallet, } Y \sim N(20.0, 0.32^2)$ |
| | $T = X_1 + X_2 + ... + X_{30} + Y$ |
| | $T = N(30(25.6) + 20, 30(0.24)^2 + 0.32^2)$ $\quad$ 30(25.6) + 20 or 788 $\quad$ B1 |
| | $= N(788, 1.8304)$ $\quad$ N and 1.8304 or awrt 1.83 $\quad$ A1 |
| | $\{P(T > 785) = \} P\left(Z > \frac{785 - 788}{1.8304}\right)$ $\quad$ M1 |
| | $= P(Z > -2.2174...)$ $\quad$ = 0.9868 $\quad$ awrt 0.987 $\quad$ A1 |

**Notes:**

(a) M1: For clear definition of $D$ and normal distribution with mean of 0 (Can be implied by 3rd M1).

A1: For correct use of $\text{Var}(X_1 - X_2)$ formula.

A1: For 0.1152

dM1: For realising need $2 \times P(D > 0.5)$ (Dependent on 1st M1 i.e. must be using suitable $D$).

dM1: Dep on 1st M1 for standardising with 0.5, 0 and their s.d.(≠ 0.24) Must lead to $P(Z > +ve)$ (o.e.). $P(Z > 1.47)$ implies 1st M1 1st A1 2nd A1 and 3rd M1. Correct answer only will score 6 out of 6.
Question 4 notes continued

(b)

B1: For a mean of 30(25.6) + 20. Can be implied by 788.

M1: For 30(0.24)² + 0.32². Can be implied by 1.8304 or awrt 1.83

A1: For normal and correct variance of 1.8304 or awrt 1.83. Normality may be implied by standardisation

M1: For standardising with 785 with their mean and st. dev..(≠ 0.24) Must lead to P(Z > −ve)

A1: Awrt 0.987. Correct answer only will score 5 out of 5

Note: Calculator answers are (a) 0.14071..., (b) 0.98670...
**Question 5**

H₀: Grades and gender are independent (or not associated)

H₁: Grades and gender are dependent (or associated)

<table>
<thead>
<tr>
<th>Observed</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinction</td>
<td>37</td>
<td>44</td>
</tr>
<tr>
<td>Merit</td>
<td>127</td>
<td>96</td>
</tr>
<tr>
<td>Unsatisfactory</td>
<td>36</td>
<td>20</td>
</tr>
</tbody>
</table>

“grades” and “gender” mentioned at least once.

<table>
<thead>
<tr>
<th>Expected</th>
<th>Male</th>
<th>Female</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinction</td>
<td>45</td>
<td>36</td>
<td>81</td>
</tr>
<tr>
<td>Merit</td>
<td>123.889</td>
<td>99.111</td>
<td>223</td>
</tr>
<tr>
<td>Unsatisfactory</td>
<td>31.111</td>
<td>24.889</td>
<td>56</td>
</tr>
<tr>
<td>Totals</td>
<td>200</td>
<td>160</td>
<td>360</td>
</tr>
</tbody>
</table>

An attempt to convert percentages to observed frequencies.

All observed frequencies are correct.

<table>
<thead>
<tr>
<th>Observed</th>
<th>Expected</th>
<th>(O – E)²/E</th>
<th>O²/E</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>45</td>
<td>1.422</td>
<td>30.422</td>
</tr>
<tr>
<td>44</td>
<td>36</td>
<td>1.778</td>
<td>53.778</td>
</tr>
<tr>
<td>127</td>
<td>123.889</td>
<td>0.078</td>
<td>130.189</td>
</tr>
<tr>
<td>96</td>
<td>99.111</td>
<td>0.098</td>
<td>92.987</td>
</tr>
<tr>
<td>36</td>
<td>31.111</td>
<td>0.768</td>
<td>41.657</td>
</tr>
<tr>
<td>20</td>
<td>24.889</td>
<td>0.960</td>
<td>16.071</td>
</tr>
<tr>
<td>Totals</td>
<td>5.104</td>
<td>365.104</td>
<td></td>
</tr>
</tbody>
</table>

At least 2 correct terms for \( \frac{(O – E)^2}{E} \) or \( \frac{O^2}{E} \) or correct expressions with their \( E_i \).

Accept 2 sf accuracy for the M1 mark.

All correct \( \frac{(O – E)^2}{E} \) or \( \frac{O^2}{E} \) terms to either 2 dp or better.

Allow truncation. \((\Rightarrow \text{by awrt } 5.1 \text{ if } 3^{rd}\text{ M1 seen})\)

\[
X^2 = \sum \frac{(O – E)^2}{E} \text{ or } \sum \frac{O^2}{E} = 360 \Rightarrow \text{awrt } 5.1
\]

\( \nu = (3 – 1)(2 – 1) = 2 \) (Can be implied by 5.991)

\[\chi^2_{0.05} = 5.991 \Rightarrow \text{CR: } X^2 \geq 5.991 \]

For 5.991 only

Since \( X^2 = 5.1 \) does not lie in the CR, then there is insufficient evidence to reject \( H_0 \)

Marks

- B1
- M1
- A1
### Question 5 (continued)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business Studies grades and gender are independent or There is no association between Business Studies grades and gender or Head of department’s (belief) is correct</td>
<td>A1ft</td>
</tr>
</tbody>
</table>

**Notes:**

- **Final M1:** For a correct statement linking their test statistic and their critical value (> 3.8)
  
  Note: Contradictory statements score M0. E.g. “significant, do not reject $H_0$”.

- **Final A1ft:** For a correct ft statement in context –
  
  must mention “grades” and “gender” or “sex” or “head of department”
  
  Condone “relationship” or “connection” here but **not** “correlation”.
  
  e.g. “There is no evidence of a relationship between grades and gender”

- **5.10 only** Just seeing 5.10... only can imply 1st 3 Ms but loses 1st 3 As so can score 4 out of 7 (Qu says show..”)

**Note:** **Full accuracy gives** $X^2 = 5.104356...$ and p-value 0.0779
**Question 5**

**Mark Scheme for candidates who use percentages instead of observed values.**

- **H₀**: Grades and gender are independent (or not associated)
- **H₁**: Grades and gender are dependent (or associated)

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observed</strong></td>
<td>18.5</td>
<td>27.5</td>
</tr>
<tr>
<td>Distinction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Merit</td>
<td>63.5</td>
<td>60.0</td>
</tr>
<tr>
<td>Unsatisfactory</td>
<td>18.0</td>
<td>12.5</td>
</tr>
</tbody>
</table>

“grades” and “gender” mentioned at least once.  
B1

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected</strong></td>
<td>23</td>
<td>23</td>
<td>46</td>
</tr>
<tr>
<td>Distinction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Merit</td>
<td>61.75</td>
<td>61.75</td>
<td>123.5</td>
</tr>
<tr>
<td>Unsatisfactory</td>
<td>15.25</td>
<td>15.25</td>
<td>30.5</td>
</tr>
<tr>
<td>Totals</td>
<td>100</td>
<td>100</td>
<td>200</td>
</tr>
</tbody>
</table>

These marks cannot be obtained.  
M0 A0

<table>
<thead>
<tr>
<th>Observed</th>
<th>Expected</th>
<th>((O - E)^2) / E</th>
<th>(O^2 / E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.5</td>
<td>23</td>
<td>0.8804</td>
<td>14.8804</td>
</tr>
<tr>
<td>27.5</td>
<td>23</td>
<td>0.8804</td>
<td>32.8804</td>
</tr>
<tr>
<td>63.5</td>
<td>61.75</td>
<td>0.0496</td>
<td>65.2996</td>
</tr>
<tr>
<td>60.0</td>
<td>61.75</td>
<td>0.0496</td>
<td>58.2996</td>
</tr>
<tr>
<td>18.0</td>
<td>15.25</td>
<td>0.4959</td>
<td>21.2459</td>
</tr>
<tr>
<td>12.5</td>
<td>15.25</td>
<td>0.4959</td>
<td>10.2459</td>
</tr>
<tr>
<td>Totals</td>
<td>2.8518</td>
<td>202.8518</td>
<td></td>
</tr>
</tbody>
</table>

Some attempt at \((\text{Row Total})(\text{Column Total})\)  
(M1) Can be implied by one of these \(E_i\)’s

Expected frequencies are not correct.  
A0

\[X^2 = \sum \frac{(O - E)^2}{E} \quad \text{or} \quad \sum \frac{O^2}{E} - 360 = 2.8518\]

This mark cannot be obtained.  
A0

\[\nu = (3 - 1)(2 - 1) = 2\]

\[X^2 = (2)(5.991) \quad \text{(Can be implied by 5.991)}\]

\[\chi^2(0.05) = 5.991 \implies CR: \quad X^2 \geq 5.991\]

For 5.991 only  
B1

\[\chi^2(0.05) = 5.991 \implies CR: \quad X^2 \geq 5.991\]

For 5.991 only  
B1
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5 continued</strong></td>
<td>Since $X^2 = 2.86$ does not lie in the CR, then there is insufficient evidence to reject $H_0$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Not available since comes from incorrect</td>
<td>A0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12)</td>
</tr>
</tbody>
</table>

**Notes:**

If a candidate uses percentages rather than observed values then they can obtain a maximum of **6 marks**. They can get B1 M0A0 M1A0 M1A0A0 B1B1M1A0.
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td>[ \hat{\mu} = \frac{\sum x}{n} = \frac{1570}{50} = 31.4 ] [ \bar{x} = 31.4 ] B1 cao</td>
<td>[ \bar{x} = 31.4 ] B1 cao</td>
</tr>
<tr>
<td></td>
<td>[ \hat{\sigma}^2 = \frac{\sum x^2 - n \bar{x}^2}{n-1} = \frac{49467.58 - 50(31.4)^2}{50 - 1} ] [ \hat{s}^2 = \frac{49467.58 - 50(31.4)^2}{50 - 1} ] M1 A1ft</td>
<td>[ \hat{s}^2 = \frac{49467.58 - 50(31.4)^2}{50 - 1} ] M1 A1ft</td>
</tr>
<tr>
<td></td>
<td>= 3.460816... awrt 3.46 A1</td>
<td>= 3.460816... awrt 3.46 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>[Let ( Y ) = time taken to complete obstacle course in the afternoon.]</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>( H_0: \mu_x = \mu_y, \ H_1: \mu_x &gt; \mu_y ) B1</td>
<td>( H_0: \mu_x = \mu_y, \ H_1: \mu_x &gt; \mu_y ) B1</td>
</tr>
<tr>
<td></td>
<td>( z = \frac{31.4 - 30.9}{\frac{3.46}{\sqrt{50}} + \frac{3.03}{\sqrt{50}}} ) M1 A1ft</td>
<td>( z = \frac{31.4 - 30.9}{\frac{3.46}{\sqrt{50}} + \frac{3.03}{\sqrt{50}}} ) M1 A1ft</td>
</tr>
<tr>
<td></td>
<td>= 1.38781... awrt 1.39 A1</td>
<td>= 1.38781... awrt 1.39 A1</td>
</tr>
<tr>
<td></td>
<td>( CR: \ Z \geq 1.6449 ) or probability = awrt 0.082 or awrt 0.083 1.6449 or better B1</td>
<td>( CR: \ Z \geq 1.6449 ) or probability = awrt 0.082 or awrt 0.083 1.6449 or better B1</td>
</tr>
<tr>
<td></td>
<td>Since ( z = 1.38781... ) does not lie in the CR, then there is insufficient evidence to reject ( H_0 ) M1</td>
<td>Since ( z = 1.38781... ) does not lie in the CR, then there is insufficient evidence to reject ( H_0 ) M1</td>
</tr>
<tr>
<td></td>
<td>Conclude that the mean time to complete the obstacle course is the same for the early morning and late afternoon. A1</td>
<td>Conclude that the mean time to complete the obstacle course is the same for the early morning and late afternoon. A1</td>
</tr>
<tr>
<td>(c)</td>
<td>( \bar{x} ) and ( \bar{y} ) are both approx. normally distributed or ( \bar{x} - \bar{y} ) normal (Condone ( \bar{x} ) and ( \bar{y} )) B1</td>
<td>( \bar{x} ) and ( \bar{y} ) are both approx. normally distributed or ( \bar{x} - \bar{y} ) normal (Condone ( \bar{x} ) and ( \bar{y} )) B1</td>
</tr>
<tr>
<td>(d)</td>
<td>Have assumed ( s^2 = \sigma^2 ) or variance of sample = variance of population B1</td>
<td>Have assumed ( s^2 = \sigma^2 ) or variance of sample = variance of population B1</td>
</tr>
<tr>
<td>Notes:</td>
<td>(13 marks)</td>
<td>(13 marks)</td>
</tr>
</tbody>
</table>

**Notes:**

(a)
- **B1:** 31.4 cao. Allow 31 minutes, 24 seconds.
- **M1:** A correct expression for either \( s \) or \( s^2 \) (ignore label)
- **A1ft:** A correct expression for \( s^2 \) with their ft \( \bar{x} \).
- **A1:** Awrt 3.46 (Correct answer scores 3 out of 3)

(b)
- **B1:** Both hypotheses stated correctly, with some indication of which \( \mu \) is which. E.g: \( \mu_M , \mu_A \)
<table>
<thead>
<tr>
<th>Question 6 notes continued</th>
</tr>
</thead>
</table>
| **M1:** For an attempt at \( \frac{a-b}{\sqrt{\frac{c}{50} + \frac{d}{50}}} \) with at least 3 of \( a, b, c \) or \( d \) correct. Allow ±
| **A1ft:** For ± \( \frac{\text{their } 31.4 - 30.9}{\sqrt{\frac{\text{their } 3.46}{50} + \frac{3.03}{50}}} \)
| Allow \( D = \bar{x} - \bar{y} \) 1.64 ~ 1.65 = \( \frac{D - 0}{\sqrt{\frac{3.46}{50} + \frac{3.03}{50}}} \) [SE = 0.360277..]  
| **A1:** For awrt 1.39 (possibly ±) (Allow for CV \( D = \text{awrt } 0.593 \)) (NB \( d = 0.5 \))  
| Correct answer scores M1A1ftA1 but \( 0 - (31.4 - 30.9) \rightarrow -1.39 \) loses this 2nd A mark
| **B1:** Critical value of 1.6449 or better (seen). Allow for probability = awrt 0.082 or awrt 0.083.  
| Note: p-values are 0.0823 (tables) and 0.0826 (calculator).  
| **M1:** For a correct statement linking their test statistic and their critical value.  
| Note: Contradictory statements score M0. E.g. “significant, do not reject \( H_0 \)”.
| **A1:** For a correct statement in context that accepts \( H_0 \) (no ft) Condone “no difference in mean times”. Must mention “mean time”, “morning” and “afternoon” or “both times of day”
| **(c)**  
| **B1:** E.g. \( \bar{X} \sim N(...) \) need both. Allow in words e.g “sample means are normally distributed”.
| **(d)**  
| **B1:** Condone only mentioning “\( x \)” or “\( y \)” but watch out for \( s_x = s_y \) or \( \sigma_x = \sigma_y \) which scores B0.
### Question 7

#### (a)

Let $X = \text{score on a die}$

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(S) = 3.5$</td>
<td>B1</td>
</tr>
<tr>
<td>$\text{Var}(S) = \frac{35}{12}$</td>
<td>or awrt 2.92</td>
</tr>
</tbody>
</table>

#### (2)

So, $\overline{S} \sim N\left(3.5, \frac{35}{40}\right)$ or awrt $7 \sim N\left(3.5, \frac{7}{96}\right)$

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P\left(\overline{S} &lt; 3\right) = P\left( Z &lt; \frac{3 - 3.5}{\frac{7}{96}}\right) {= P\left(Z &lt; -1.85164\ldots\right)}$</td>
<td>M1</td>
</tr>
<tr>
<td>${= 1 - 0.9678} = 0.0322$</td>
<td>A1</td>
</tr>
<tr>
<td><strong>0.032 to 0.0322</strong></td>
<td></td>
</tr>
</tbody>
</table>

#### (3)

### Notes:

(a)

**B1:** (2nd) allow awrt 2.92

(b)

**B1ft:** For $\overline{S} \sim N\left(3.5, \frac{35}{40}\right)$ seen or implied. Follow through their $E(S)$ and their $\text{Var}(S)$

N.B $\frac{7}{96} = 0.07291696$ accept awrt 0.0729

**M1:** For an attempt to standardise with 3, their mean ($>3$) and $\sqrt{\frac{\text{their Var}(S)}{40}}$. Must lead to $P(Z < -\text{ve})$  

**A1:** For 0.032 ~ 0.0322

### Alternative $\sum S$

**B1ft:** For $\sum S \sim N\left(140, \frac{350}{3}\right)$ where 140 is 40×their $E(S)$ and variance is 40×their $\text{Var}(S)$. 

---

Question 7 notes continued

M1: For \( P\left( Z < \frac{120 - 140}{\sqrt{3}} \right) \) or \( P\left( Z < \frac{119.5 - 140}{\sqrt{3}} \right) \) \( \{= P(Z < -1.8979...)\} \)

A1: for 0.032~0.0322 or (with continuity correction) 0.0287 (tables) or 0.0289 (calculator).
### Question 8(a)

\[
\bar{x} = \frac{29.74 + 31.86}{2} \Rightarrow \bar{x} = 30.8
\]

This can be implied. See note.

M1:

\[
"1.96\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 30.8 \quad \text{or} \quad 2("1.96")\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 29.74
\]

SE\_\bar{x} = \frac{31.86 - 30.8}{1.96} = 0.540816... = 0.54 (2dp) awrt 0.54

A1

\begin{align*}
\text{(b)} & \quad \text{A 90% CI for } \mu \text{ is } \bar{x} \pm 1.6449\left(\frac{\sigma}{\sqrt{n}}\right) \\
& \quad = 30.8 \pm 1.6449(0.54) \quad \text{(their } \bar{x} \text{) } \pm \text{(their } z \text{) (their SE\_\bar{x} from (a))} \\
& \quad = (29.91, 31.69) \quad \text{(awrt 29.9, awrt 31.7)}
\end{align*}

\begin{align*}
\text{(c)} & \quad \text{Let } X = \text{number of confidence intervals containing } \mu \\
& \quad \text{or } Y = \text{number of confidence intervals not containing } \mu \\
& \quad \text{So } X \sim \text{Bin}(4, 0.9) \quad \text{or} \quad Y \sim \text{Bin}(4, 0.1) \\
& \quad P(X \geq 3) \text{ or } P(Y \leq 1) = \sum_{i=3}^{4} (0.9)^i (0.1)^{4-i} \\
& \quad = 0.2916 + 0.6561 = 0.9477 \quad \text{or } 0.9477 \text{ or } 0.948
\end{align*}

\begin{align*}
\text{(9 marls)}
\end{align*}

### Notes:

\begin{align*}
\text{(a)} & \quad \bar{x} = 30.8 \text{ may be implied by } 1.96\left(\frac{\sigma}{\sqrt{n}}\right) = [31.86 - 30.8] = 1.06 \text{ or} \\
& \quad 2(1.96)\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 29.74 \\
\text{M1:} & \quad \text{A correct equation for either a width or a half-width involving a } z\text{-value } 1.5 \leq z \leq 2 \\
& \quad \text{Eg: } "\text{their } z\text{"}\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - "30.8" \text{ fit their } \bar{x} \text{ or} \quad 2("\text{their } z\text{"})\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 29.74 \\
& \quad \text{or } "\text{their } z\text{"}(\text{SE\_\bar{x}}) = 31.86 - "30.8" \text{ or} \quad 2("\text{their } z\text{"})(\text{SE\_\bar{x}}) = 31.86 - 29.74 \text{ are fine for M1.} \\
\text{A1:} & \quad 0.54 \text{ or awrt } 0.54 \text{ Must be seen as final answer to (a) NB } \frac{53}{98} \text{ as final answer is A0} \\
& \quad \text{Condone } \bar{x} \pm 1.96\sigma = \ldots \text{for B1 and M1 but A0 even if they say } "\sigma = \text{standard error} = 0.54". \text{ Otherwise answer only of 0.54 scores 3 out of 3}
\end{align*}
Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- Fill in the boxes at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
Write your answers in the D1 answer book for this paper.

1. The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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</tr>
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<tbody>
<tr>
<td>A</td>
<td>—</td>
<td>122</td>
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<td>D</td>
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<td>F</td>
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<td>204</td>
<td>135</td>
<td>211</td>
<td>113</td>
<td>—</td>
</tr>
</tbody>
</table>

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810 km for Liz’s route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz’s route.

(2)

(c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz’s route.

(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(1)

(Total for Question 1 is 8 marks)
2. Kruskal’s algorithm finds a minimum spanning tree for a connected graph with \( n \) vertices.

(a) Explain the terms

(i) connected graph,

(ii) tree,

(iii) spanning tree.

(b) Write down, in terms of \( n \), the number of arcs in the minimum spanning tree.

The table below shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<td>B</td>
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<td>23</td>
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<td>27</td>
<td>29</td>
<td>31</td>
<td>22</td>
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<tr>
<td>D</td>
<td>19</td>
<td>23</td>
<td>27</td>
<td>–</td>
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<td>–</td>
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<tr>
<td>E</td>
<td>30</td>
<td>–</td>
<td>29</td>
<td>–</td>
<td>–</td>
<td>33</td>
<td>25</td>
</tr>
<tr>
<td>F</td>
<td>–</td>
<td>–</td>
<td>31</td>
<td>40</td>
<td>33</td>
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<td>39</td>
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<tr>
<td>G</td>
<td>–</td>
<td>–</td>
<td>22</td>
<td>–</td>
<td>25</td>
<td>39</td>
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</tr>
</tbody>
</table>

(c) Complete the drawing of the network on Diagram 1 in the answer book by adding the necessary arcs from vertex C together with their weights.

(d) Use Kruskal’s algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(e) State the weight of the minimum spanning tree.

(Total for Question 2 is 10 marks)
3. 12.1  9.3  15.7  10.9  17.4  6.4  20.1  7.9  8.1  14.0

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 33

The list is to be sorted into descending order.

(b) (i) Starting at the left-hand end of the list, perform two passes through the list using a bubble sort. Write down the state of the list that results at the end of each pass.

(ii) Write down the total number of comparisons and the total number of swaps performed during your two passes.

(c) Use a quick sort on the original list to obtain a fully sorted list in descending order. You must make your pivots clear.

(d) Use the first-fit decreasing bin packing algorithm to determine how the numbers listed can be packed into bins of size 33

(e) Determine whether your answer to (d) uses the minimum number of bins. You must justify your answer.

(Total for Question 3 is 15 marks)
4.

Figure 1

[The total weight of the network is 196]

Figure 1 models a network of roads. The number on each edge gives the time, in minutes, taken to travel along that road. Oliver wishes to travel by road from A to K as quickly as possible.

(a) Use Dijkstra’s algorithm to find the shortest time needed to travel from A to K. State the quickest route.

(6)

On a particular day Oliver must travel from B to K via A.

(b) Find a route of minimal time from B to K that includes A, and state its length.

(2)

Oliver needs to travel along each road to check that it is in good repair. He wishes to minimise the total time required to traverse the network.

(c) Use the route inspection algorithm to find the shortest time needed. You must state all combinations of edges that Oliver could repeat, making your method and working clear.

(7)

(Total for Question 4 is 15 marks)
5. A linear programming problem in $x$ and $y$ is described as follows.

Maximise $P = 5x + 3y$

subject to: $x \geq 3$

\[ x + y \leq 9 \]
\[ 15x + 22y \leq 165 \]
\[ 26x - 50y \leq 325 \]

(a) Add lines and shading to Diagram 2 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(b) Use the objective line method to find the optimal vertex, V, of the feasible region. You must draw and label your objective line and label vertex V clearly.

(2)

(c) Calculate the exact coordinates of vertex V and hence calculate the corresponding value of $P$ at V.

(3)

The objective is now to minimise $5x + 3y$, where $x$ and $y$ are integers.

(d) Write down the minimum value of $5x + 3y$ and the corresponding value of $x$ and corresponding value of $y$.

(2)

(Total for Question 5 is 11 marks)
A linear programming problem in $x$ and $y$ is described as follows.

Maximise $P = 5x + 3y$
subject to:

- $x \leq 3$
- $x + y \leq 9$
- $15x + 22y \leq 165$
- $26x - 50y \leq 325$

(a) Add lines and shading to Diagram 2 in the answer book to represent these constraints. Hence determine the feasible region and label it $R$. 

(b) Use the objective line method to find the optimal vertex, $V$, of the feasible region. You must draw and label your objective line and label vertex $V$ clearly.

(c) Calculate the exact coordinates of vertex $V$ and hence calculate the corresponding value of $P$ at $V$.

(d) The objective is now to minimise $5x + 3y$, where $x$ and $y$ are integers.

(e) Write down the minimum value of $5x + 3y$ and the corresponding value of $x$ and corresponding value of $y$.

(Total for Question 5 is 11 marks)

6.

Figure 2

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.

(a) Explain the significance of the dummy activity

(i) from event 5 to event 6
(ii) from event 7 to event 9.

(b) Complete Diagram 3 in the answer book to show the early event times and the late event times.

(c) State the minimum project completion time.

(d) Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time. You must show your working.

(e) On Grid 1 in your answer book, draw a cascade (Gantt) chart for this project.

(f) On Grid 2 in your answer book, construct a scheduling diagram to show that this project can be completed with three workers in just one more hour than the minimum project completion time.

(Total for Question 6 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS
1.

<table>
<thead>
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<td>204</td>
<td>135</td>
<td>211</td>
<td>113</td>
<td>–</td>
</tr>
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</table>

(a) D 98 E 109 A 122 B 110 C

82

F
Question 1 continued

(b)

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & A & B & C & D & E \\
\hline
A & - & 122 & 217 & 137 & 109 \\
B & 122 & - & 110 & 130 & 128 \\
C & 217 & 110 & - & 204 & 238 \\
D & 137 & 130 & 204 & - & 98 \\
E & 109 & 128 & 238 & 98 & - \\
F & 82 & 204 & 135 & 211 & - \\
\hline
\end{array}
\]

(c)

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & A & B & C & D & E \\
\hline
A & - & 122 & 217 & 137 & 109 \\
B & 122 & - & 110 & 130 & 128 \\
C & 217 & 110 & - & 204 & 238 \\
D & 137 & 130 & 204 & - & 98 \\
E & 109 & 128 & 238 & 98 & - \\
F & 82 & 204 & 135 & 211 & - \\
\hline
\end{array}
\]

(Total for Question 1 is 8 marks)
2.

__________________________
__________________________
__________________________
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__________________________

(c) Diagram 1

Diagram 1
Question 2 continued

(Total for Question 2 is 10 marks)
## Question 3

<p>| | | | | | | | | | |</p>
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<tr>
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<td>15.7</td>
<td>10.9</td>
<td>17.4</td>
<td>6.4</td>
<td>20.1</td>
<td>7.9</td>
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### Question 3 continued

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<th>15.7</th>
<th>10.9</th>
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<th>6.4</th>
<th>20.1</th>
<th>7.9</th>
<th>8.1</th>
<th>14.0</th>
</tr>
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</table>


Question 3 continued

(Total for Question 3 is 15 marks)
4.

**Key:**

<table>
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<th>Vertex</th>
<th>Order of labelling</th>
<th>Final value</th>
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<tbody>
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<td></td>
</tr>
<tr>
<td>B</td>
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<td></td>
</tr>
<tr>
<td>A</td>
<td>2</td>
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<td>J</td>
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</tr>
<tr>
<td>K</td>
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<td>F</td>
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<tr>
<td>D</td>
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</tr>
</tbody>
</table>

**Quickest route:**

______________________________

**Shortest time:**

______________________________

---

(Question 4 continued)
Question 4 continued
Diagram 2
Question 5 continued

(Total for Question 5 is 11 marks)
6. (a) _______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________
_______________________________________________________________________

(b) Diagram 3

Key:

Early event time

Late event time
Question 6 continued

(e)

Grid 1

(f)

Grid 2

(Total for Question 6 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS
Question | Scheme | Marks
--- | --- | ---
(a) | E.g. if use CD as shortcut get 807 or if use CF + AD get 793 | M1 A1
(b) | A     F     E     D     B     C     A     B     113  98   130  110  217   = 750 | B1
(c) | length of RMST = 439 | B1
(d) | 439 + 82 + 113 = 634 | M1 A1

Notes:
(a) M1: Their plausible shortcut leading to a value < 810 and a length below 810 stated.
A1: cao
(b) B1: cao
(c) B1:
M1: Adding two least weighted arcs to their RMST length
A1: cao
(d) B1: An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)
### Decision Mathematics D1 Mark scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(a)</td>
<td>E.g. if use CD as shortcut get 807 or if use CF + AD get 793</td>
<td>M1 A1 (2)</td>
</tr>
<tr>
<td>(b)</td>
<td>A F E D B C A</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>82 113 98 130 110 217 = 750</td>
<td>B1</td>
</tr>
<tr>
<td>(c)</td>
<td>length of RMST = 439</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>439 + 82 + 113 = 634</td>
<td>M1 A1 (3)</td>
</tr>
<tr>
<td>(d)</td>
<td>634 &lt; optimal ≤ 750</td>
<td>B1 ≤ (1)</td>
</tr>
</tbody>
</table>

(8 marks)

**Notes:**

(a)  
**M1:** Their plausible shortcut leading to a value < 810 and a length below 810 stated.  
**A1:** cao – shortcut and length must be consistent.  
(Examples shortcuts: CD = 807, CF + AD = 793, CF + BD = 664, AD + EF + FC = 715, DF FC = 785 etc.)

(b)  
**B1:** cao  
**B1:** cao

(c)  
**B1:** cao  
**M1:** Adding two least weighted arcs to their RMST length  
**A1:** cao

(d)  
**B1:** An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2(a)</strong></td>
<td>e.g. accept (i) Every pair of nodes connected by a path (ii) Connected graph with no cycles (iii) All nodes connected</td>
<td>B1 B1 B1</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>$n - 1$</td>
<td>B1</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td><a href="image">Diagram of a graph with nodes A, B, C, D, E, F, G and edges with weights (17, 23, 27, 29, 31, 33, 39, 40)</a></td>
<td>M1 A1</td>
</tr>
<tr>
<td><strong>(d)</strong></td>
<td>Kruskal: AB, AD, BC, CG, reject BD, EG, reject CD, reject CE, reject AE, CF</td>
<td>M1 A1</td>
</tr>
<tr>
<td><strong>(e)</strong></td>
<td>135 (km)</td>
<td>B1</td>
</tr>
</tbody>
</table>

**Notes:**

(a) In (a), all technical language used must be correct – for example, do not accept ‘point’ for node, etc

(i) B1: _every pair_ and _path_ (or clear definition of path) – no bod - not describing complete graph

(ii) B1: _connected_ and _no cycles_ (not ‘loops’, ‘circles’, etc. unless ‘cycle’ seen as well)

(iii) B1: _all nodes_ connected (accept definition of minimum spanning tree)

(b) B1: _cao_

(c) M1: Either all five arcs correct (ignore weights) or at least three arcs correct (including weights)

A1: _cso (arcs and weights)_ – no additional arcs
(d)

<table>
<thead>
<tr>
<th>M1:</th>
<th>Kruskal’s – first three arcs (AB, AD, BC,… or weights 17, 19, 21, …) chosen correctly and at least one rejection seen at some point. For M1 only: follow through from their diagram from (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1:</td>
<td>All six arcs (AB, AD, BC, CG, EG, CF or weights 17, 19, 21, 22, 25, 31) chosen correctly and no additional arcs (no follow through from an incorrect network in (c))</td>
</tr>
<tr>
<td>A1:</td>
<td>cso All selections and rejections correct (in correct order and at the correct time) – do not accept weights or a contradiction between arcs and their weights (e.g. AB (16))</td>
</tr>
<tr>
<td>B1:</td>
<td>cao (ignore lack of units)</td>
</tr>
</tbody>
</table>

Notes:

(a) In (a), all technical language used must be correct – for example, do not accept ‘point’ for node, etc.

(ii) B1: every pair and path (or clear definition of path) – no bod-

(iii) B1: connected and no cycles (not ‘loops’, ‘circles’, etc. unless ‘cycle’ seen as well)

(c) M1: Either all five arcs correct (ignore weights) or at least three arcs correct (including weights) – cso arcs (arcs and weights) – no additional arcs

M1: Kruskal’s – first three arcs (AB, AD, BC,… or weights 17, 19, 21, …) chosen correctly and at least one rejection seen at some point. For M1 only: follow through from their diagram from (c)
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| **3(a)** | Bin 1: 12.1 9.3 10.9  
Bin 2: 15.7 6.4 7.9  
Bin 3: 17.4 8.1  
Bin 4: 20.1 
Bin 5: 14.0 | M1 A1  
A1 | (3) |
| **(b)** | (i) 12.1 15.7 10.9 17.4 9.3 20.1 7.9 8.1 14.0 6.4  
15.7 12.1 17.4 10.9 20.1 9.3 8.1 14.0 7.9 6.4 | M1 A1 | |
| | (ii) Comparisons = 9 + 8 = 17  
Swaps = 7 + 5 = 12 | B1 B1 | (4) |
| **(c)** | e.g. middle right  
12.1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0  
Pivot 6.4  
12.1 9.3 15.7 10.9 17.4 20.1 7.9 8.1 14.0 6.4  
Pivot 17.4  
20.1 17.4 12.1 9.3 15.7 10.9 7.9 8.1 14.0 6.4  
Pivot (20.1) 10.9  
20.1 17.4 12.1 15.7 14.0 10.9 9.3 7.9 8.1 6.4  
Pivots 15.7 7.9  
20.1 17.4 15.7 12.1 14.0 10.9 9.3 8.1 7.9 6.4  
Pivots 14.0 8.1  
20.1 17.4 15.7 14.0 12.1 10.9 9.3 8.1 7.9 6.4  
Sort complete | M1 (quick)  
A1 (1\textsuperscript{st}/2\textsuperscript{nd}  
passes/piv  
for 3\textsuperscript{rd})  
A1ft (3\textsuperscript{rd}/4\textsuperscript{th}  
passes/piv  
for 5\textsuperscript{th})  
A1(cso +  
sort complete") | (4) |
| **(d)** | Bin 1: 20.1 12.1  
Bin 2: 17.4 14.0  
Bin 3: 15.7 10.9 6.4  
Bin 4: 9.3 8.1 7.9 | M1 A1  
A1 | (3) |
| **(e)** | e.g. $\frac{121.9}{33} \approx 3.694$ so yes 4 bins is optimal | B1ft | (1) |

**(15 marks)**
Question 3 continued

Notes:

(a)  
M1: First four numbers placed correctly (therefore Bin 1 correct and 15.7 in Bin 2) and at least seven numbers put in bins – condone cumulative totals here only
A1: First eight numbers placed correctly (therefore Bins 1 and 2 correct and 17.4 in Bin 3 and 20.1 in Bin 4)  
A1: cso All correct

(b)  
(i)M1: Bubble sort – first pass correct
(ii)A1: Bubble sort – first pass correct  
(ii)B1: cso on total number of comparisons
(ii)B1: cso on total number of swaps
SC in b(ii): If B0B0, award B1B0 if correct numbers referred to but not summed

(c)  
M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as pivot is M0) and first pass gives >p, p, <p. So after the first pass the list should read (values greater than the pivot), pivot, (values less than the pivot). If only choosing one pivot per iteration M1 only
A1: First and second passes correct and next pivot(s) chosen correctly for third pass (but third pass does not need to be correct)
A1ft: Third and fourth passes correct (follow through from their second pass and choice of pivots) – and next pivot(s) chosen correctly for the fifth pass
A1: cso (correct solution only – all previous marks in this part must have been awarded) including ‘sort complete’ – this could be shown by the final list being re-written or ‘sorted’ statement or each item being used (not just stated) as a pivot

(d)  
M1: Must be using ‘sorted’ list in decreasing order (independent of (c)). First four numbers placed correctly and at least seven numbers put in bins – condone cumulative totals here only. First-fit increasing is M0
A1: First eight numbers placed correctly
A1: cso – all correct
SC for (d): if the ‘sorted’ list they use in (d) has one ‘error’ from (c) (e.g. a missing number, an extra number or one number incorrectly placed) then M1 only can be awarded in (d) (for the first four numbers). If there is more than one ‘error’ then M0. Allow full marks in (d) if a correct list is used in (d) even if the list is incorrect at the end of (c).

(e)  
B1ft: $\frac{121.9}{33}$ or awrt 3.7 (or 3.6 with correct calculation seen) and 4 together with a correct conclusion based on their answer to (d) (a correct calculation etc. with an answer of 4 with no conclusion (as a minimum accept ‘yes’) scores B0)

<table>
<thead>
<tr>
<th>Pivot</th>
<th>7.9</th>
<th>8.1</th>
<th>14.0</th>
<th>6.4</th>
<th>20.1</th>
<th>17.4</th>
<th>15.7</th>
<th>10.9</th>
<th>6.4</th>
<th>9.3</th>
<th>7.9</th>
<th>8.1</th>
<th>6.4</th>
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<tr>
<td>Pivot</td>
<td>20.1</td>
<td>17.4</td>
<td>12.1</td>
<td>9.3</td>
<td>15.7</td>
<td>10.9</td>
<td>6.4</td>
<td>9.3</td>
<td>7.9</td>
<td>8.1</td>
<td>6.4</td>
<td>14.0</td>
<td>10.9</td>
</tr>
<tr>
<td>Pivot</td>
<td>20.1</td>
<td>17.4</td>
<td>12.1</td>
<td>15.7</td>
<td>14.0</td>
<td>10.9</td>
<td>9.3</td>
<td>6.4</td>
<td>7.9</td>
<td>8.1</td>
<td>6.4</td>
<td>Pivot (14.0)</td>
<td>9.3</td>
</tr>
<tr>
<td>Pivot</td>
<td>20.1</td>
<td>17.4</td>
<td>15.7</td>
<td>14.0</td>
<td>12.1</td>
<td>10.9</td>
<td>9.3</td>
<td>8.1</td>
<td>7.9</td>
<td>6.4</td>
<td>(sort complete (8.1))</td>
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<tr>
<td>Question</td>
<td>Scheme</td>
<td>Marks</td>
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<tr>
<td><strong>4(a)</strong></td>
<td><img src="image" alt="Diagram" /></td>
<td>M1 (JEFD) A1 (BG) A1ft (HK)</td>
<td></td>
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</table>

Quickest route: **A – G – H – K**

Shortest time: 32 (mins)

**4(b)**

Route from **B** to **K** via **A**: **B – D – E – A – G – H – K**

Length: 51 (mins)

**4(c)**

- \[A(ED)B + F(G)H = 19 + 15 = 34\]
- \[AF + B(K)H = 16 + 18 = 34\]
- \[A(G)H + B(DE)F = 29 + 11 = 40\]

Arcs **AF**, **BK**, **KH** or **AE**, **ED**, **DB**, **FG**, **GH** will be traversed twice

Route length = 196 + 34 = 230 (mins)

**Notes:**

**(a)**

**M1**: A larger value replaced by a smaller value at least once in the working values at either **B** or **H** or **K**

**A1**: All values in **J**, **E**, **F** and **D** correct and the working values in the correct order. Penalise order of labelling only once per question. Condone an additional working value at **F** of 22

**A1**: All values in **B** and **G** correct and the working values in the correct order. Penalise order of labelling only once per question (**B** and **G** must be labelled in that order and **B** must be labelled after **J**, **E**, **F**, **D**). Condone an additional working value of 20 at **B** and an additional working value of 26 at **G**

**A1ft**: All values in **H** and **K** correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question (**H** and **K** must be labelled in that order and **H** labelled after all other nodes (excluding **K**))

**A1**: CAO (**AGHK**) **A1ft**: Follow through on their final value at **K** – if their answer is not 32 follow through their final value at **K** (condone lack of units)
<table>
<thead>
<tr>
<th>Question 4 notes continued</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
</tr>
<tr>
<td><strong>B1:</strong> CAO (BDEAGHK)</td>
</tr>
<tr>
<td><strong>B1ft:</strong> 51 or their final value at B + their final value at K (condone lack of units)</td>
</tr>
<tr>
<td>(c)</td>
</tr>
<tr>
<td><strong>M1:</strong> Three distinct pairings of the correct four odd nodes</td>
</tr>
<tr>
<td><strong>A1ft:</strong> One row correct including pairing and total (the ft on the first three A marks in (c) is for using their final values at B, F and H from (a) for the lengths of AB, AF and AH only)</td>
</tr>
<tr>
<td><strong>A1ft:</strong> Two rows correct including pairing and totals</td>
</tr>
<tr>
<td><strong>A1:</strong> CAO one combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)</td>
</tr>
<tr>
<td><strong>A1:</strong> CAO both combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)</td>
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<tr>
<td><strong>A1:</strong> CAO (230)</td>
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<tr>
<td>Question</td>
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<tr>
<td>5(a)(b)</td>
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(c) \[ V \left( \frac{775}{76} - \frac{91}{76} \right) \]

\[ P = \frac{1801}{38} \]

M1 A1

A1

(3)

(d) \( x = 3, y = -4 \)  minimum value is 3

B1 B1

(2)

(11 marks)
**Question 5 continued**

**Notes:**

(a)  
In (a), lines must be long enough to define the correct feasible region **and** pass through one small square of the points stated:  
\[ x + y = 9 \] passes through (5, 4) and (9,0) but in most cases check (0, 9) and (9,0)  
\[ 26x - 50y = 325 \] passes through (5, -3.9) and (10, -1.3) but in most cases check (0, -6.5) and (12.5, 0)  
\[ 15x + 22y = 165 \] passes through \( \left( \frac{3}{11} \right) \) and \( \left( \frac{4}{22} \right) \) but in most cases check (0, 7.5) and (11, 0)  

**B1:** Any two lines correctly drawn  
**B1:** Any three lines correctly drawn  
**B1:** All four lines correctly drawn  
**B1:** Region, R, correctly labelled – not just implied by shading – dependent on scoring the first three marks in (a)

(b)  

**B1:** Drawing the correct objective line on the graph, use line drawing tool to check if necessary.  
Line must not pass outside of a small square if extended from axis to axis  
**B1:** V labelled clearly on their graph. **This mark is dependent on both the correct feasible region (but maybe not labelled) and the correct objective line**

(c)  

**M1:** Candidates **must** have drawn either the correct objective line or its reciprocal. If they have drawn the correct objective line they must be solving \( x + y = 9 \) and \( 26x - 50y = 325 \). If they have drawn the reciprocal objective line they must be solving \( x = 3 \) and \( 15x + 22y = 165 \). Must get to either \( x = ... \) or \( y = ... \) (condone one error in the solving of the simultaneous equations).  
The correct exact answer \( \left( \frac{775}{76}, \frac{-91}{76} \right) \), or for the reciprocal \( \left( \frac{3}{11}, \frac{60}{11} \right) \), can imply this mark  

**A1:** \( \frac{1801}{38} \) or \( \frac{47}{38} \) (must be exact). **This mark is dependent on the correct feasible region (but maybe not labelled)**

(d)  

**B1:** \( x = 3, y = -4 \) or \( (3, -4) \)  
**B1:** cao of 3
<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(a)</td>
<td>(i) The dummy from event 5 to event 6 is needed to show that J depends on F but I depends on D, E and F</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>(ii) The dummy from event 7 to event 9 is because activities G and H must be able to be described uniquely in terms of the events at each end</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td>(2)</td>
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<tr>
<td>(c)</td>
<td>21 (hours)</td>
<td>B1</td>
</tr>
<tr>
<td>(d)</td>
<td>$\frac{64}{21} \approx 3.048$ so at least 4 workers required</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(e)</td>
<td><img src="image" alt="Diagram" /></td>
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</table>
**Question 6(f)**

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<th>Scheme</th>
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<td>e.g.</td>
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<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
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<tbody>
<tr>
<td>6(f)</td>
<td>e.g.</td>
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</table>

0 2 4 6 8 10 12 14 16 18 20 22 24 26

B1: B F J

A D G I K

A1: A1

**Notes:**

(a) **In (a) any use of the terms ‘activity’ and ‘event’ must be correct**

B1: cao dependency - all relevant activities must be referred to - activities I, J, F and either D or E must be mentioned.

B1: cao uniqueness – please note that, for example, ‘so that activities can be defined uniquely’ is not sufficient to earn this mark. There must be some mention of describing activities in terms of the event at each end. However, give bod on statements that imply that an activity begins and ends at the same event.

(b) **M1:** All top boxes complete, values generally increasing in the direction of the arrows (‘left to right’), condone one rogue

**A1:** cao (top boxes)

M1: All bottom boxes complete, values generally decreasing in the opposite direction of the arrows (‘right to left’), condone one rogue

A1: cao (bottom boxes)

(c) B1: cao (21)

(d) **M1:** Attempt to find lower bound: (a value in the interval [55 – 73] / their finish time) or (sum of the activities / their finish time) or (as a minimum) an awrt 3.05 or 3.04 (truncated)

**A1:** cso – either a correct calculation seen or awrt 3.05 (or 3.04) then 4. An answer of 4 with no working scores M0A0

(e) **M1:** At least 8 activities added including 5 floats. Scheduling diagram scores M0

**A1:** Critical activities dealt with correctly and 4 non-critical activities dealt with correctly

M1: All 11 activities including all 8 floats (on the correct non-critical activities)

**A1:** cao (all activities correct and present only once)
### Question 4 notes continued

(f) **M1:** Not a cascade chart. 3 workers used and at least 9 activities placed. The completion time must be no greater than one hour more than the minimum completion time stated in (c) or seen in (b)  
**A1:** 3 workers, All 11 activities present (just once). Condone one error either precedence or activity length. The completion time must be one hour greater than the minimum completion time stated in (c) or seen in (b) 
**A1:** 3 workers. All 11 activities present (just once). No errors. The completion time must be 22

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration</th>
<th>IPA</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td>E</td>
<td>7</td>
<td>B</td>
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<tr>
<td>F</td>
<td>9</td>
<td>B</td>
</tr>
<tr>
<td>G</td>
<td>7</td>
<td>C</td>
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<tr>
<td>H</td>
<td>6</td>
<td>C</td>
</tr>
<tr>
<td>I</td>
<td>2</td>
<td>D, E, F</td>
</tr>
<tr>
<td>J</td>
<td>8</td>
<td>F</td>
</tr>
<tr>
<td>K</td>
<td>5</td>
<td>I</td>
</tr>
</tbody>
</table>
(f) M1: Not a cascade chart. 3 workers used and at least 9 activities placed. The completion time must be no greater than one hour more than the minimum completion time stated in (c) or seen in (b).

A1: 3 workers. All 11 activities present (just once). Condone one error either precedence or activity length. The completion time must be one hour greater than the minimum completion time stated in (c) or seen in (b).

Activity | Duration |
--- | --- |
A | 5 |
B | 4 |
C | 7 |
D | 4 |
E | 7 |
F | 9 |
G | 7 |
H | 6 |
I | 2 |
J | 8 |
K | 5 |