

INTERNATIONAL ADVANCED LEVEL

Mathematics, Further Mathematics and Pure Mathematics

SAMPLE ASSESSMENT MATERIALS

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA01)

Pearson Edexcel International Advanced Level in Mathematics (YMA01)

Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM01)

Pearson Edexcel International Advanced Level in Further Mathematics (YFM01)

Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM01)

Pearson Edexcel International Advanced Level in Pure Mathematics (YPM01)

For first teaching in September 2013

First examination January 2014

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Write your name here

Surname	Other names
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Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Core Mathematics C12

Advanced Subsidiary

Sample Assessment Material
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

--

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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3. Answer this question without the use of a calculator and show all your working.

(i) Show that

$$(5 - \sqrt{8})(1 + \sqrt{2}) \equiv a + b\sqrt{2}$$

giving the values of the integers a and b .

(3)

(ii) Show that

$$\sqrt{80} + \frac{30}{\sqrt{5}} \equiv c\sqrt{5}, \text{ where } c \text{ is an integer.}$$

(3)

5.

$$y = \frac{5}{3x^2 - 2}$$

The table below gives values of y rounded to 3 decimal places where necessary.

x	2	2.25	2.5	2.75	3
y	0.5	0.379	0.299	0.242	0.2

Use the trapezium rule, with all the values of y from the table above, to find an approximate value for

$$\int_2^3 \frac{5}{3x^2 - 2} \, dx \quad (4)$$

6. $f(x) = x^4 + x^3 + 2x^2 + ax + b,$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7

(a) Show that $a + b = 3$ **(2)**

When $f(x)$ is divided by $(x + 2)$, the remainder is -8

(b) Find the value of a and the value of b . **(5)**

8. The equation

$$(k + 3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x .

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0 \quad (4)$$

(b) Hence find the set of possible values of k . (3)

10.

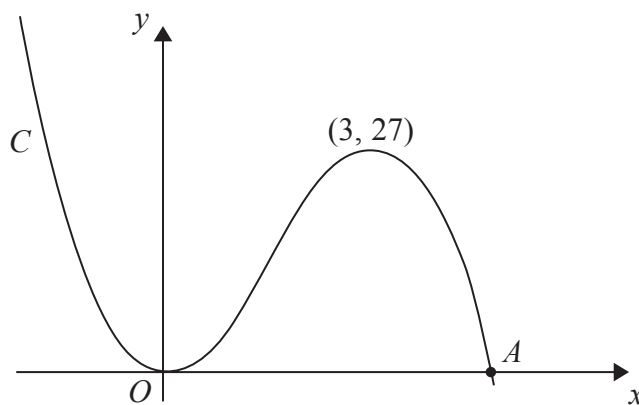


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where

$$f(x) = x^2(9 - 2x).$$

There is a minimum at the origin, a maximum at the point $(3, 27)$ and C cuts the x -axis at the point A .

(a) Write down the coordinates of the point A . (1)

(b) On separate diagrams sketch the curve with equation

(i) $y = f(x + 3)$,

(ii) $y = f(3x)$.

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes. (6)

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at $(3, 10)$.

(c) Write down the value of k . (1)

11.

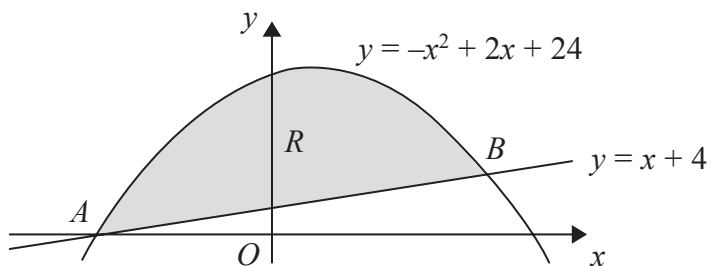


Figure 2

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B , as shown in Figure 2.

- (a) Use algebra to find the coordinates of the points A and B . **(4)**

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 2.

- (b) Use calculus to find the exact area of R . **(7)**

12. The circle C has centre $A(2, 1)$ and passes through the point $B(10, 7)$

(a) Find an equation for C . (4)

The line l_1 is the tangent to C at the point B .

(b) Find an equation for l_1 (4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB .

Given that l_2 intersects C at the points P and Q ,

(c) find the length of PQ , giving your answer in its simplest surd form. (3)

13.

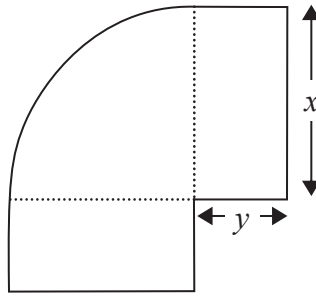


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2 ,

(a) show that

$$y = \frac{16 - \pi x^2}{8x} \tag{3}$$

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x \tag{3}$$

(c) Use calculus to find the minimum value of P .

(5)

15.

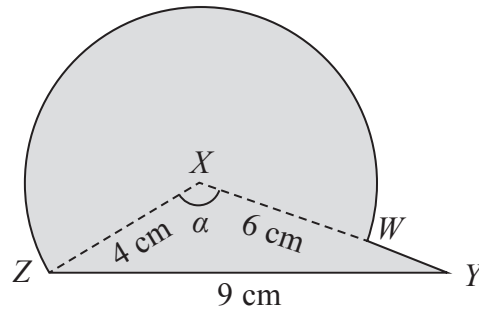


Figure 4

The triangle XYZ in Figure 4 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$. The point W lies on the line XY .

The circular arc ZW , in Figure 4 is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians. **(2)**

(b) Find the area, in cm^2 , of the major sector $XZWX$. **(3)**

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 4.

Calculate

(c) the area of this shaded region, **(3)**

(d) the perimeter $ZWYZ$ of this shaded region. **(4)**

16. Maria trains for a triathlon, which involves swimming, cycling and running. On the first day of training she swims 1.5 km and then she swims 1.5 km on each of the following days.

- (a) Find the **total** distance that Maria swims in the first 17 days of training. **(1)**

Maria also runs 1.5 km on the first day of training and on each of the following days she runs 0.25 km further than on the previous day. So she runs 1.75 km on the second day and 2 km on the third day and so on.

- (b) Find how far Maria runs on the 17th day of training. **(2)**

Maria also cycles 1.5 km on the first day of training and on each of the following days she cycles 5% further than on the previous day.

- (c) Find the **total** distance that Maria cycles in the first 17 days of training. **(3)**

- (d) Find the **total** distance Maria travels by swimming, running and cycling in the first 17 days of training. **(3)**

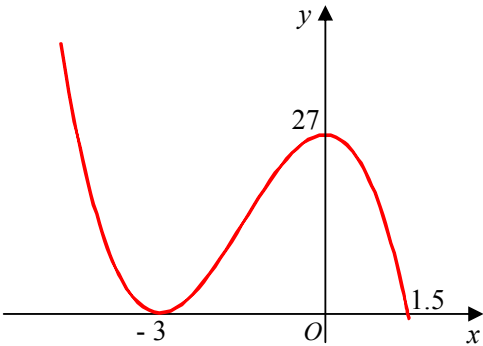
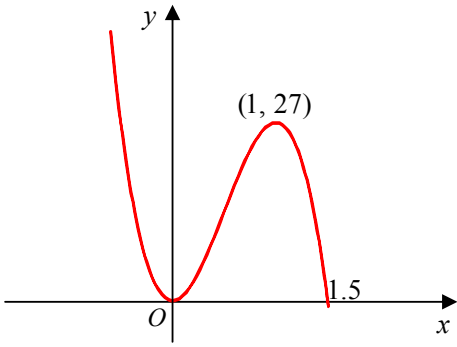
Maria needs to cycle 40 km in the triathlon.

- (e) On which day of training does Maria first cycle more than 40 km? **(4)**

Question Number	Scheme	Marks									
<p>1. (a)</p> <p>(b)</p>	<p>$5x^2$</p> <p>$(25x^4)^{-\frac{3}{2}} = \frac{1}{(25x^4)^{\frac{3}{2}}}$ or $(25x^4)^{\frac{3}{2}} = 125x^6$ or better</p> <p>$\frac{1}{125x^6}$</p>	<p>B1 (1)</p> <p>M1</p> <p>A1 (2)</p> <p>(3 marks)</p>									
<p>2.</p>	<p>$(3-x)^6 = 3^6 + 3^5 \times 6 \times (-x) + 3^4 \times \binom{6}{2} \times (-x)^2$</p> <p>$= 729, \quad -1458x, \quad +1215x^2$</p>	<p>M1</p> <p>B1, A1, A1 (4 marks)</p>									
<p>3. (i)</p> <p>(ii)</p>	<table border="0" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%; vertical-align: top;"> <p>$(5 - \sqrt{8})(1 + \sqrt{2})$</p> <p>$= 5 + 5\sqrt{2} - \sqrt{8} - 4$</p> <p>$= 5 + 5\sqrt{2} - 2\sqrt{2} - 4$</p> <p>$= 1 + 3\sqrt{2}$</p> </td> <td style="width: 33%; vertical-align: top; border-left: 1px solid black;"> <p style="text-align: center;">Multiplies out brackets correctly.</p> <p style="text-align: center;">$\sqrt{8} = 2\sqrt{2}$, seen or implied at any point.</p> <p style="text-align: center;">$1 + 3\sqrt{2}$ or $a = 1$ and $b = 3$</p> </td> <td style="width: 33%;"></td> </tr> <tr> <td style="text-align: center; vertical-align: top;"> <p>Method 1</p> </td> <td style="text-align: center; vertical-align: top;"> <p>Method 2</p> </td> <td style="text-align: center; vertical-align: top;"> <p>Method 3</p> </td> </tr> <tr> <td style="vertical-align: top;"> <p>Either $\sqrt{80} + \frac{30}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right)$</p> <p>$= 4\sqrt{5} + \dots$</p> <p>$= 4\sqrt{5} + 6\sqrt{5}$</p> </td> <td style="vertical-align: top; border-left: 1px solid black;"> <p>Or $\left(\frac{\sqrt{400} + 30}{\sqrt{5}} \right) \frac{\sqrt{5}}{\sqrt{5}}$</p> <p>$= \left(\frac{20 + \dots}{\dots} \right) \dots$</p> <p>$= \left(\frac{50\sqrt{5}}{5} \right)$</p> <p>$= 10\sqrt{5}$</p> </td> <td style="vertical-align: top;"> <p>$\sqrt{80} + \frac{\sqrt{900}}{\sqrt{5}} = \sqrt{80} + \sqrt{180}$</p> <p>$= 4\sqrt{5} + \dots$</p> <p>$= 4\sqrt{5} + 6\sqrt{5}$</p> </td> </tr> </table> <p>As this is a “show that” question – all working should be shown.</p>	<p>$(5 - \sqrt{8})(1 + \sqrt{2})$</p> <p>$= 5 + 5\sqrt{2} - \sqrt{8} - 4$</p> <p>$= 5 + 5\sqrt{2} - 2\sqrt{2} - 4$</p> <p>$= 1 + 3\sqrt{2}$</p>	<p style="text-align: center;">Multiplies out brackets correctly.</p> <p style="text-align: center;">$\sqrt{8} = 2\sqrt{2}$, seen or implied at any point.</p> <p style="text-align: center;">$1 + 3\sqrt{2}$ or $a = 1$ and $b = 3$</p>		<p>Method 1</p>	<p>Method 2</p>	<p>Method 3</p>	<p>Either $\sqrt{80} + \frac{30}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right)$</p> <p>$= 4\sqrt{5} + \dots$</p> <p>$= 4\sqrt{5} + 6\sqrt{5}$</p>	<p>Or $\left(\frac{\sqrt{400} + 30}{\sqrt{5}} \right) \frac{\sqrt{5}}{\sqrt{5}}$</p> <p>$= \left(\frac{20 + \dots}{\dots} \right) \dots$</p> <p>$= \left(\frac{50\sqrt{5}}{5} \right)$</p> <p>$= 10\sqrt{5}$</p>	<p>$\sqrt{80} + \frac{\sqrt{900}}{\sqrt{5}} = \sqrt{80} + \sqrt{180}$</p> <p>$= 4\sqrt{5} + \dots$</p> <p>$= 4\sqrt{5} + 6\sqrt{5}$</p>	<p>M1</p> <p>B1</p> <p>A1 (3)</p> <p>M1</p> <p>B1</p> <p>A1 (3)</p> <p>(6 marks)</p>
<p>$(5 - \sqrt{8})(1 + \sqrt{2})$</p> <p>$= 5 + 5\sqrt{2} - \sqrt{8} - 4$</p> <p>$= 5 + 5\sqrt{2} - 2\sqrt{2} - 4$</p> <p>$= 1 + 3\sqrt{2}$</p>	<p style="text-align: center;">Multiplies out brackets correctly.</p> <p style="text-align: center;">$\sqrt{8} = 2\sqrt{2}$, seen or implied at any point.</p> <p style="text-align: center;">$1 + 3\sqrt{2}$ or $a = 1$ and $b = 3$</p>										
<p>Method 1</p>	<p>Method 2</p>	<p>Method 3</p>									
<p>Either $\sqrt{80} + \frac{30}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right)$</p> <p>$= 4\sqrt{5} + \dots$</p> <p>$= 4\sqrt{5} + 6\sqrt{5}$</p>	<p>Or $\left(\frac{\sqrt{400} + 30}{\sqrt{5}} \right) \frac{\sqrt{5}}{\sqrt{5}}$</p> <p>$= \left(\frac{20 + \dots}{\dots} \right) \dots$</p> <p>$= \left(\frac{50\sqrt{5}}{5} \right)$</p> <p>$= 10\sqrt{5}$</p>	<p>$\sqrt{80} + \frac{\sqrt{900}}{\sqrt{5}} = \sqrt{80} + \sqrt{180}$</p> <p>$= 4\sqrt{5} + \dots$</p> <p>$= 4\sqrt{5} + 6\sqrt{5}$</p>									

Question Number	Scheme	Marks
<p>4. (a)</p> <p>(b)</p>	$\frac{dy}{dx} = 10x^4 - 3x^{-4} \quad \text{or} \quad 10x^4 - \frac{3}{x^4}$ $\left(\int =\right) \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} = \frac{x^6}{3} + 7x - \frac{x^{-2}}{2} + C$	<p>M1 A1 A1 (3)</p> <p>M1 A1 A1 B1 (4) (7 marks)</p>
<p>5.</p>	$\frac{1}{2} \times 0.25 \times \{0.5 + 0.2 + 2(0.379 + 0.299 + 0.242)\}$ $\{= \frac{1}{8}(2.540)\} = 0.3175 \text{ or } 0.318$	<p>Outside brackets $\frac{1}{2} \times 0.25$ or $\frac{1}{8}$ <u>For structure of</u> <u>{.....}</u>; Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant". awrt 0.32</p> <p>B1 aef M1 <u>A1</u>√ A1 4 marks</p>
<p>6. (a)</p> <p>(b)</p>	$f(x) = x^4 + x^3 + 2x^2 + ax + b$ <p>Attempting f(1) or f(-1). $f(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$ (as required) AG</p> <p>Attempting f(-2) or f(2). $f(-2) = 16 - 8 + 8 - 2a + b = -8 \quad \{ \Rightarrow -2a + b = -24 \}$ Solving both equations simultaneously to get as far as $a = \dots$ or $b = \dots$ Any one of $a = 9$ or $b = -6$ Both $a = 9$ and $b = -6$</p>	<p>M1 A1 * cso (2)</p> <p>M1 A1 dM1 A1 A1 cso (5) (7 marks)</p>

Question Number	Scheme	Marks
<p>7. (a)</p> <p>(b)</p>	<p>$(a_2 =) 6 - c$</p> <p>$a_3 = 3(\text{their } a_2) - c \quad (= 18 - 4c)$</p> <p>$a_1 + a_2 + a_3 = 2 + "(6 - c)" + "(18 - 4c)"$</p> <p>$"26 - 5c" = 0$</p> <p>So $c = 5.2$</p>	<p>B1</p> <p>(1)</p> <p>M1</p> <p>M1</p> <p>A1ft</p> <p>A1 o.e</p> <p>(4)</p> <p>(5 marks)</p>
<p>8. (a)</p> <p>(b)</p>	<p>Attempts $b^2 - 4ac$ for $a = (k + 3)$, $b = 6$ and their c. $c \neq k$</p> <p>$b^2 - 4ac = 6^2 - 4(k + 3)(k - 5)$</p> <p>$-4k^2 + 8k + 96$</p> <p>As $b^2 - 4ac > 0$, then $-4k^2 + 8k + 96 > 0$ and so, $k^2 - 2k - 24 < 0$</p> <p>Attempts to solve $k^2 - 2k - 24 = 0$ to give $k =$</p> <p>(\Rightarrow Critical values, $k = 6, -4$.)</p> <p>$k^2 - 2k - 24 < 0$ gives $-4 < k < 6$</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>A1 *</p> <p>(4)</p> <p>M1</p> <p>M1 A1</p> <p>(3)</p> <p>(7 marks)</p>
<p>9. (a)</p> <p>(b)</p>	<p>$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$</p> <p>$\log_3 x^2 = 2 \log_3 x$</p> <p>Using $\log_3 3 = 1$ and deduces answer.</p> <p>$3x^2 = 28x - 9$</p> <p>Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>M1</p> <p>M1 A1</p> <p>(3)</p> <p>(6 marks)</p>

Question Number	Scheme	Marks
<p>10. (a) (b) i</p>	<p>{Coordinates of A are} (4.5, 0)</p>  <p>Horizontal translation -3 and their ft 1.5 on positive x-axis Maximum at 27 marked on the y-axis</p>	<p>B1 M1 A1 ft B1 (3)</p>
<p>(b) ii</p>	 <p>Correct shape, minimum at (0, 0) and a maximum within the first quadrant. 1.5 on x-axis Maximum at (1, 27)</p>	<p>M1 A1 ft B1 (3)</p>
<p>(c)</p>	<p>{k =} -17</p>	<p>B1 (1) (8 marks)</p>

Question Number	Scheme	Marks
<p>11. (a)</p>	<p>Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$ {Curve = Line} $\Rightarrow -x^2 + 2x + 24 = x + 4$ $x^2 - x - 20 \{= 0\} \Rightarrow (x - 5)(x + 4) \{= 0\} \Rightarrow x = \dots$ So, $x = 5, -4$ So corresponding y-values are $y = 9$ and $y = 0$</p>	<p>Eliminating y correctly. Attempt to solve a resulting quadratic to give $x =$ their values. Both $x = 5$ and $x = -4$</p> <p>B1 M1 A1 B1ft</p> <p>(4)</p>
<p>(b)</p>	<p>$\left\{ \int (-x^2 + 2x + 24) dx \right\} = \underline{-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \{+ c\}}$ $\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{-4}^5 = (\dots) - (\dots)$ $\left\{ \left(-\frac{125}{3} + 25 + 120 \right) - \left(\frac{64}{3} + 16 - 96 \right) \right\} = \left(103\frac{1}{3} \right) - \left(-58\frac{2}{3} \right) = 162$ Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$ Uses correct method for finding area of triangle. So area of R is $162 - 40.5 = 121.5$ Area under curve – Area of triangle. 121.5</p>	<p>M1: $x^n \rightarrow x^{n+1}$ for any one term. 1st A1 at least two out of three terms correct. 2nd A1 for <u>correct answer</u>.</p> <p>Substitutes 5 and -4 (or their limits from part (a)) into an “integrated function” and subtracts, either way round.</p> <p>dM1</p> <p>M1 M1 A1 oe cao</p> <p>(7) (11 marks)</p>

Question Number	Scheme	Marks
<p>12. (a)</p>	<p>$(10 - 2)^2 + (7 - 1)^2$ or $\sqrt{(10 - 2)^2 + (7 - 1)^2}$ $(x \pm 2)^2 + (y \pm 1)^2 = k$ (k a positive <u>value</u>) $(x - 2)^2 + (y - 1)^2 = 100$ (Accept 10^2 for 100) (Answer only scores full marks)</p>	<p>M1 A1 M1 A1 (4)</p>
<p>(b)</p>	<p>(Gradient of radius =) $\frac{7-1}{10-2} = \frac{6}{8}$ (or equiv.) Must be seen in part (b) Gradient of tangent = $\frac{-4}{3}$ (Using perpendicular gradient method) $y - 7 = m(x - 10)$ $y - 7 = \frac{-4}{3}(x - 10)$ or equivalent (ft gradient of <u>radius</u>, dep. on <u>both</u> M marks)</p>	<p>B1 M1 M1 A1ft (4)</p>
<p>(c)</p>	<p>$\sqrt{r^2 - \left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag. $= \sqrt{10^2 - 5^2}$ or numerically exact equivalent. $PQ (= 2\sqrt{75}) = 10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark.</p>	<p>M1 A1 A1 (3) (11 marks)</p>

Question Number	Scheme	Marks
<p>13. (a)</p>	$kr^2 + cxy = 4 \quad \text{or} \quad kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$ $\frac{1}{4}\pi x^2 + 2xy = 4$ $y = \frac{4 - \frac{1}{4}\pi x^2}{2x} = \frac{16 - \pi x^2}{8x} \quad *$	<p>M1</p> <p>A1</p> <p>B1 cso</p> <p>(3)</p>
	<p>(b) $P = 2x + cy + k\pi r$ where $c = 2$ or 4 and $k = \frac{1}{4}$ or $\frac{1}{2}$</p> $P = \frac{\pi x}{2} + 2x + 4\left(\frac{4 - \frac{1}{4}\pi x^2}{2x}\right) \text{ or } P = \frac{\pi x}{2} + 2x + 4\left(\frac{16 - \pi x^2}{8x}\right) \text{ o.e.}$ $P = \frac{\pi x}{2} + 2x + \frac{8}{x} - \frac{\pi x}{2} \quad \text{so} \quad P = \frac{8}{x} + 2x \quad *$	<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p>
	<p>(c) $\left(\frac{dP}{dx}\right) - \frac{8}{x^2} + 2$</p> $-\frac{8}{x^2} + 2 = 0 \Rightarrow x^2 = \dots$ <p>and so $x = 2$ o.e. (ignore extra answer $x = -2$)</p> $P = 4 + 4 = 8 \text{ (m)}$	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>(5)</p> <p>(11 marks)</p>

Question Number	Scheme	Marks
<p>14. (a)</p>	<p>$\sin(x + 45^\circ) = \frac{2}{3}$, so $(x + 45^\circ) = 41.8103\dots$ ($\alpha = 41.8103\dots$)</p> <p>$\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73°</p> <p>So, $x + 45^\circ = \{138.1897\dots, 401.8103\dots\}$</p> <p>and $x = \{93.1897\dots, 356.8103\dots\}$</p> <p>$x + 45^\circ =$ either "180 – their α" or "360° + their α"</p> <p>Either awrt 93.2° or awrt 356.8° Both awrt 93.2° and awrt 356.8°</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(4)</p>
<p>(b)</p>	<p>$2(1 - \cos^2 x) + 2 = 7 \cos x$</p> <p>$2 \cos^2 x + 7 \cos x - 4 = 0$</p> <p>$(2 \cos x - 1)(\cos x + 4) = 0$, $\cos x = \dots$</p> <p>$\cos x = \frac{1}{2}$, $\{\cos x = -4\}$</p> <p>$\left(\beta = \frac{\pi}{3}\right)$</p> <p>$x = \frac{\pi}{3}$ or $1.04719\dots^\circ$</p> <p>$x = \frac{5\pi}{3}$ or $5.23598\dots^\circ$</p> <p>Applies $\sin^2 x = 1 - \cos^2 x$</p> <p>Correct 3 term, $2 \cos^2 x + 7 \cos x - 4 = 0$</p> <p>Valid attempt at solving and $\cos x = \dots$</p> <p>$\cos x = \frac{1}{2}$</p> <p>Either $\frac{\pi}{3}$ or awrt 1.05°</p> <p>Either $\frac{5\pi}{3}$ or awrt 5.24° or $2\pi -$ their β</p>	<p>M1</p> <p>A1 oe</p> <p>M1</p> <p>A1 cso</p> <p>B1</p> <p>B1 ft</p> <p>(6)</p> <p>(10 marks)</p>

Question Number	Scheme	Marks
15. (a)	$9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha = \dots$ $\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} \left(= -\frac{29}{48} = -0.604.. \right)$ $\alpha = 2.22 \quad *$ (NB $\alpha = 2.219516005$)	Correct use of cosine rule leading to a value for $\cos \alpha$ M1 cso (2.22 must be seen here) A1
	(2)	
(b)	$2\pi - 2.22 (= 4.06366\dots)$ $\frac{1}{2} \times 4^2 \times "4.06"$ 32.5	$2\pi - 2.22$ or awrt 4.06 B1 Correct method for major sector area. Allow $\pi - 2.22$ for the major sector angle. M1 awrt 32.5 A1
	(3)	
	Or (b)	Alternative method: Circle – Minor sector $\pi \times 4^2$ Correct expression for circle area. B1 $\pi \times 4^2 - \frac{1}{2} \times 4^2 \times 2.22 = 32.5$ Correct method for circle - minor sector area. M1 $= 32.5$ awrt 32.5 A1
(3)		
(c)	Area of triangle = $\frac{1}{2} \times 4 \times 6 \times \sin 2.22 (= 9.56)$ So area required = "9.56" + "32.5" = 42.1 cm ² or 42.0 cm ²	Correct expression for the area of triangle XYZ B1 Their Triangle XYZ (Not triangle ZXW) + (part (b) answer or correct attempt at major sector) M1 awrt 42.1 or 42.0 (Or <u>just</u> 42). A1
	(3)	
	(d)	Arc length = $4 \times 4.06 (= 16.24)$ Or $8\pi - 4 \times 2.22$ Perimeter = ZY + WY + Arc Length Perimeter = 27.2 or 27.3
(4)		
(12 marks)		

Question Number	Scheme	Marks
16. (a)	$17 \times 1.5 = 25.5(\text{km})$	B1 (1)
(b)	Use $l = a + (n-1)d$ with $a = 1.5$, $d = 0.25$ and $n = 17$ So $l = 5.5$	M1 A1 (2)
(c)	Use $S = \frac{a(1-r^n)}{1-r}$ with $a = 1.5$, and $n = 17$ And $r = 1.05$ So $S = 38.76(\text{km})$	M1 A1 A1 (3)
(d)	Total distance running is $S = \frac{n}{2} \{2a + (n-1)d\}$ $= 59.5(\text{km})$ So total in three sports is $123.76(\text{km})$	M1 A1 B1 (3)
(e)	Uses $ar^{n-1} > 40$ so $1.5 \times (1.05)^{n-1} > 40$ with their r $(1.05)^{n-1} > 26.7$ so $(n-1)\log 1.05 > \log 26.7$ $n-1 > 67.297$ So 69th day of training.	M1 M1 M1 A1 (4) (13 marks)

Write your name here

Surname	Other names
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Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Core Mathematics C34

Advanced

Sample Assessment Material
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

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Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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PEARSON

2.

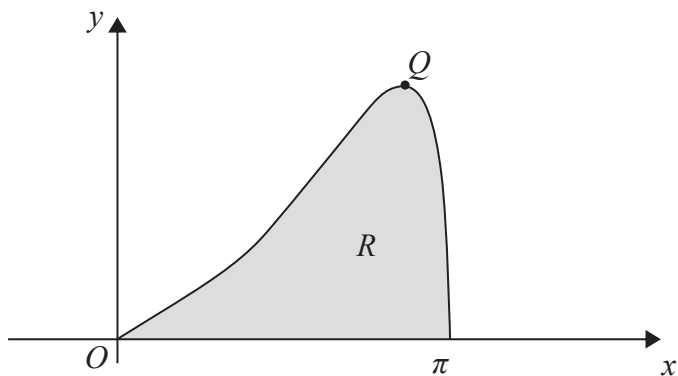


Figure 1

Figure 1 shows a sketch of the curve with equation $y = e^x \sqrt{\sin x}$, $0 \leq x \leq \pi$.

The finite region R , shown shaded in Figure 1, is bounded by the curve and the x -axis.

- (a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of the region R . Give your answer to 4 decimal places.

(3)

The curve $y = e^x \sqrt{\sin x}$, $0 \leq x \leq \pi$, has a maximum turning point at Q , shown in Figure 1.

- (c) Find the x coordinate of Q .

(6)

3. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{(\cos x + 1)} \sin x \, dx = e(e - 1) \quad (6)$$

4. (a) Use the binomial theorem to expand

$$(2 - 3x)^{-2}, \quad |x| < \frac{2}{3}$$

in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a + bx}{(2 - 3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x , the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$

Find

(b) the value of a and the value of b ,

(5)

(c) the coefficient of x^3 , giving your answer as a simplified fraction.

(3)

Question 4 continued

Blank lined area for student response.

6. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y . **(5)**

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$ **(7)**

Question 6 continued

Lined writing area for the answer to Question 6.

(Total 12 marks)

Q6

Two small boxes for marking.

7. (a) Show that

$$\cot x - \cot 2x \equiv \operatorname{cosec} 2x, \quad x \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z} \qquad (5)$$

(b) Hence, or otherwise, solve for $0 \leq \theta \leq \pi$

$$\operatorname{cosec}\left(3\theta + \frac{\pi}{3}\right) + \cot\left(3\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

You must show your working.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

8.
$$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0$$

(a) Show that $h(x) = \frac{2x}{x^2+5}$ (4)

(b) Hence, or otherwise, find $h'(x)$ in its simplest form. (3)

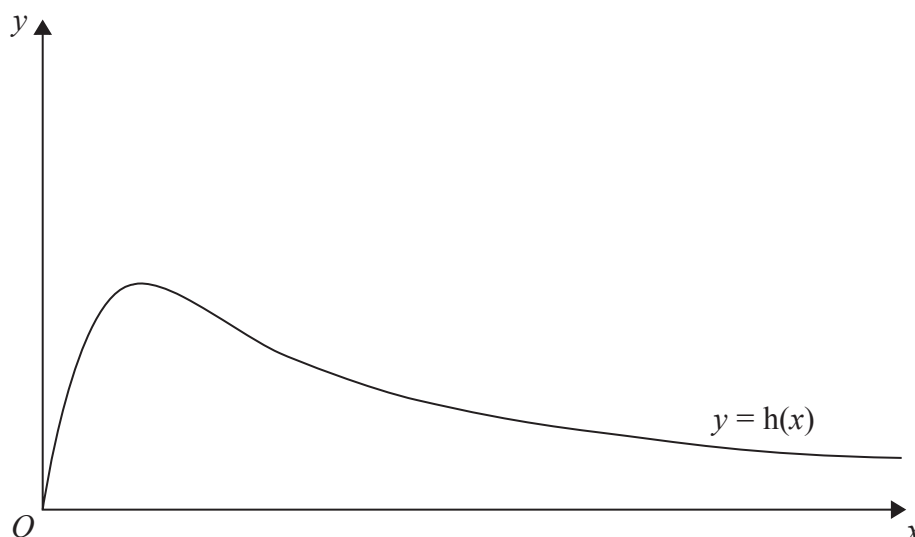


Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(c) Calculate the range of $h(x)$. (5)

Question 8 continued

A large area for handwritten answers, consisting of 25 horizontal lines.

(Total 12 marks)

Q8

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9. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C , find

(a) the coordinates of C . **(3)**

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$

(b) Find the size of the angle ACB . Give your answer in degrees to 2 decimal places. **(4)**

(c) Hence, or otherwise, find the area of the triangle ABC . **(5)**

Question 9 continued

Lined area for student response.

(Total 12 marks)

Q9

10.

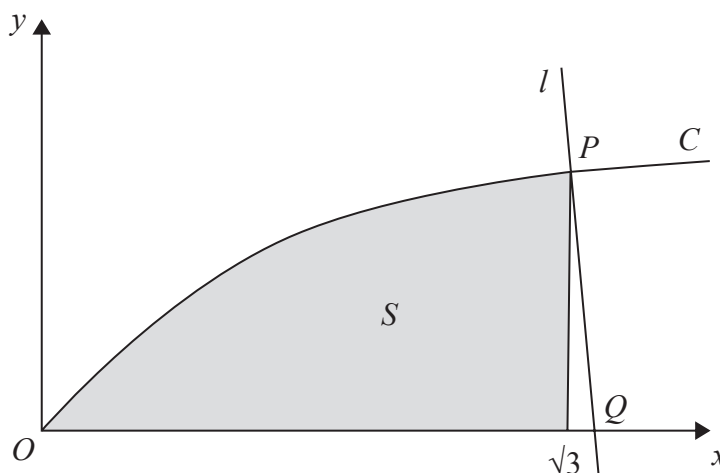


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = \sin \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$

- (a) Find the value of θ at the point P . (2)

The line l is a normal to C at P . The normal cuts the x -axis at the point Q .

- (b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k . (6)

The finite shaded region S shown in Figure 3 is bounded by the curve C , the line $x = \sqrt{3}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

- (c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants. (7)

Question 10 continued

Lined writing area for the question response.

(Total 15 marks)

Q10

- 11.** A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15} P(5 - P), \quad t \geq 0$$

where P , in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

- (a) solve the differential equation, giving your answer in the form

$$P = \frac{a}{b + ce^{\frac{-1}{3}t}}$$

where a , b and c are integers.

(11)

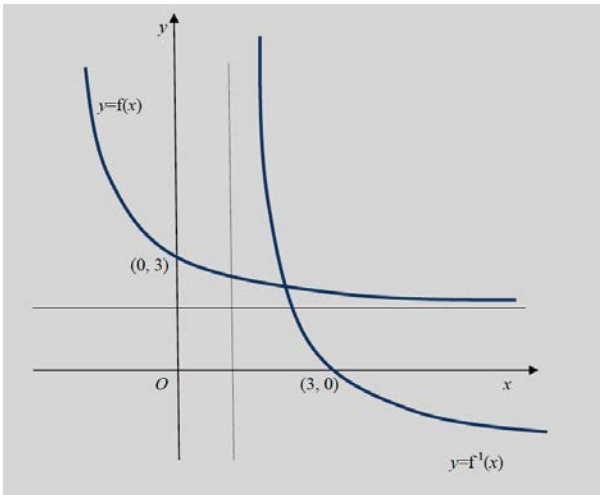
- (b) Hence show that the population cannot exceed 5000

(1)

WMA02/01: Core Mathematics C34

Question Number	Scheme	Marks										
<p>1. (a)</p> <p>(b)</p>	$R = 13$ $\tan \alpha = \frac{12}{5} \Rightarrow \alpha = 67.38^\circ$ $13 \cos(2\theta + 67.4^\circ) = 10 \Rightarrow \cos(2\theta + 67.4^\circ) = \frac{10}{13}$ $2\theta + 67.38^\circ = 39.715^\circ, (320.285^\circ, 399.715^\circ)$ $\theta = 126.5^\circ$ $\theta = 166.2^\circ$	<p>B1</p> <p>M1 A1</p> <p>(3)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1 A1</p> <p>(5)</p> <p>(8 marks)</p>										
<p>2. (a)</p> <p>(b)</p> <p>(c)</p>	<table border="1" data-bbox="395 757 1209 837"> <tr> <td>x</td> <td>$\frac{\pi}{4}$</td> <td>$\frac{\pi}{2}$</td> <td>$\frac{3\pi}{4}$</td> <td>π</td> </tr> <tr> <td>y</td> <td>1.844321332</td> <td>4.810477381</td> <td>8.87207</td> <td>0</td> </tr> </table> $\frac{1}{2} \times \frac{\pi}{4} \text{ or } \frac{\pi}{8}$ $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} \times \{0 + 2(1.84432 + 4.81048 + 8.87207) + 0\}$ 12.1948 <p>Uses $vu' + uv'$</p> $\frac{dy}{dx} = e^x \times \frac{1}{2} (\sin x)^{-\frac{1}{2}} (\cos x) + e^x (\sin x)^{\frac{1}{2}}$ $\frac{dy}{dx} = 0 \Rightarrow e^x \times \frac{1}{2} (\sin x)^{-\frac{1}{2}} (\cos x) + e^x (\sin x)^{\frac{1}{2}} = 0$ $\cos x = -2 \sin x$ $\tan x = -\frac{1}{2} \Rightarrow x = 2.68$	x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	y	1.844321332	4.810477381	8.87207	0	<p>awrt 1.84432 B1</p> <p>awrt 4.81048 or 4.81047 B1</p> <p>(2)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>M1 A1 A1</p> <p>M1</p> <p>M1 A1</p> <p>(6)</p> <p>(11 marks)</p>
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π								
y	1.844321332	4.810477381	8.87207	0								

Question Number	Scheme	Marks
3.	$\frac{du}{dx} = -\sin x$ $\int e^{\cos x+1} \sin x dx = -\int e^u du$ $= -e^u (+c)$ $= -e^{(\cos x+1)} (+c)$ $\left[-e^{(\cos x+1)} \right]_0^{\frac{\pi}{2}} = (-e) - (-e^2) = e(e-1)$	B1 M1 A1 A1 M1 A1* (6 marks)
4. (a)	$(2-3x)^{-2} = 2^{-2} \left(1 - \frac{3x}{2}\right)^{-2}$ $\left(1 - \frac{3x}{2}\right)^{-2} = 1 + (-2)\left(-\frac{3x}{2}\right) + \frac{(-2)(-3)}{2 \times 1} \left(-\frac{3x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3 \times 2 \times 1} \left(-\frac{3x}{2}\right)^3 + \dots$ $= 1 + 3x + \frac{27}{4}x^2 + \frac{27}{2}x^3 + \dots$ $(2-3x)^{-2} = \frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots$	B1 M1 A1 M1 A1 (5)
(b)	$f(x) = (a+bx) \left(\frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots \right)$ <p>Coefficient of x: $\frac{3a}{4} + \frac{b}{4} = 0 \quad (3a+b=0)$</p> <p>Coefficient of x^2: $\frac{27a}{16} + \frac{3b}{4} = \frac{9}{16} \quad (9a+4b=3)$</p> <p>A1 (either correct)</p> <p>Leading to $a = -1, b = 3$</p>	M1 M1 A1 dM1 A1 (5)
(c)	<p>Coefficient of x^3: $\frac{27a}{8} + \frac{27b}{16} = \frac{27}{8} \times -1 + \frac{27}{16} \times 3$</p> $= \frac{27}{16} = \left(1 \frac{11}{16}\right)$	M1 A1ft A1 (3) (13 marks)

Question Number	Scheme	Marks
5. (a)	$fg(x) = e^{-2\ln x} + 2,$ $= e^{\ln x^{-2}} + 2 = x^{-2} + 2 = \left(\frac{1}{x^2} + 2\right)$	M1 M1 A1 (3)
(b)	$e^{-(2x+3)} + 2 = 6 \Rightarrow e^{-(2x+3)} = 4$ $\Rightarrow -(2x+3) = \ln 4$ $\Rightarrow x = \frac{-3 - \ln 4}{2}$	M1 A1 M1 A1 (4)
(c)	Let $y = e^{-x} + 2 \Rightarrow y - 2 = e^{-x} \Rightarrow \ln(y - 2) = -x$ $\Rightarrow x = -\ln(y - 2)$ $f^{-1}(x) = -\ln(x - 2), \quad x > 2.$	M1 A1 B1 (3)
(d)		Shape for $f(x)$ B1 (0, 3) B1 Shape for $f^{-1}(x)$ B1 (3, 0) B1 (4) (14 marks)

Question Number	Scheme	Marks
<p>6. (a)</p> <p>(b)</p>	<p>Differentiating implicitly to obtain $\pm ay^2 \frac{dy}{dx}$ and/or $\pm bx^2 \frac{dy}{dx}$</p> $48y^2 \frac{dy}{dx} + \dots - 54 \dots$ $9x^2y \rightarrow 9x^2 \frac{dy}{dx} + 18xy \quad \text{or equivalent}$ $(48y^2 + 9x^2) \frac{dy}{dx} + 18xy - 54 = 0$ $\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2} \quad \left(= \frac{18 - 6xy}{16y^2 + 3x^2} \right)$ <p>18 - 6xy = 0</p> <p>Using $x = \frac{3}{y}$ or $y = \frac{3}{x}$</p> $16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0 \quad \text{or} \quad 16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0$ <p>Leading to</p> $16y^4 + 81 - 162 = 0 \quad \text{or} \quad 16 + x^4 - 2x^4 = 0$ $y^4 = \frac{81}{16} \quad \text{or} \quad x^4 = 16$ $y = \frac{3}{2}, -\frac{3}{2} \quad \text{or} \quad x = 2, -2$ <p>Subs either of their values into $xy = 3$ to obtain a value of other variable.</p> $\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right) \quad \text{both}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>M1</p> <p>M1</p> <p>A1, A1</p> <p>M1</p> <p>A1</p> <p>(7)</p> <p>(12 marks)</p>

Question Number	Scheme	Marks
7. (a)	$\cot x - \cot 2x \equiv \frac{\cos x}{\sin x} - \frac{\cos 2x}{\sin 2x}$ $\equiv \frac{\sin 2x \cos x - \cos 2x \sin x}{\sin x \sin 2x}$ $\equiv \frac{\sin(2x - x)}{\sin x \sin 2x}$ $\equiv \frac{\sin x}{\sin x \sin 2x} \equiv \frac{1}{\sin 2x} \equiv \operatorname{cosec} 2x$	B1 M1 M1 M1 A1* (5)
(b)	$2x = 3\theta + \frac{\pi}{3} \Rightarrow x = 1.5\theta + \frac{\pi}{6}$ $\cot\left(1.5\theta + \frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \Rightarrow \tan\left(1.5\theta + \frac{\pi}{6}\right) = \sqrt{3}$ $\left(1.5\theta + \frac{\pi}{6}\right) = \frac{\pi}{3}, \frac{4\pi}{3}$ $\theta = \frac{\pi}{9}, \frac{7\pi}{9}$	B1 M1 M1 A1, A1 (5) (10 marks)

Question Number	Scheme	Marks
8. (a)	$\frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x+2)(x^2+5)} = \frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$ $= \frac{2x(x+2)}{(x+2)(x^2+5)}$ $= \frac{2x}{(x^2+5)}$	M1 A1 M1 A1* (4)
(b)	$h'(x) = \frac{(x^2+5) \times 2 - 2x \times 2x}{(x^2+5)^2}$ $h'(x) = \frac{10-2x^2}{(x^2+5)^2}$	M1 A1 A1 (3)
(c)	<p>Maximum occurs when $h'(x) = 0 \Rightarrow 10 - 2x^2 = 0 \Rightarrow x = ..$</p> $\Rightarrow x = \sqrt{5}$ <p>When $x = \sqrt{5} \Rightarrow h(x) = \frac{\sqrt{5}}{5}$</p> <p>Range of $h(x)$ is $0 \leq h(x) \leq \frac{\sqrt{5}}{5}$</p>	M1 A1 M1 A1 A1ft (5) (12 marks)

Question Number	Scheme	Marks
9. (a)	Equate j components $3 + 2\lambda = 9 \Rightarrow \lambda = 3$ Leading to $C = (5, 9, -1)$	M1 A1 A1 (3)
(b)	Choosing correct directions or finding \overline{AC} and \overline{BC} $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 5 + 2 = \sqrt{6} \times \sqrt{29} \times \cos \angle ABC$ Use of scalar product. $\angle ACB = 57.95^\circ$	M1 M1 A1 A1 (4)
(c)	$A = (2, 3, -4) \quad B = (-5, 9, -5)$ $\overline{AC} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \quad \text{AND} \quad \overline{BC} = \begin{pmatrix} 10 \\ 0 \\ 4 \end{pmatrix}$ $AC^2 = 3^2 + 6^2 + 3^2 = (3\sqrt{6}) \quad BC^2 = 10^2 + 4^2 = (2\sqrt{29})$ $\text{Area triangle } ABC = \frac{1}{2} AC BC \sin \angle ACB = \frac{1}{2} \times '3\sqrt{6}' \times '2\sqrt{29}' \times \sin 57.95^\circ$ $= 33.5$	M1 A1 A1 M1 A1 (5) (12 marks)

Question Number	Scheme	Marks
10. (a)	$\tan \theta = \sqrt{3} \text{ or } \sin \theta = \frac{\sqrt{3}}{2}$ $\theta = \frac{\pi}{3}$	M1 awrt 1.05 A1 (2)
(b)	$\frac{dx}{d\theta} = \sec^2 \theta, \quad \frac{dy}{d\theta} = \cos \theta$ $\frac{dy}{dx} = \frac{\cos \theta}{\sec^2 \theta} \quad (= \cos^3 \theta)$ <p>At P, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$ Can be implied.</p> <p>Using $mm' = -1$, $m' = -8$</p> <p>For normal $y - \frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$</p> <p>At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$</p> <p>leading to $x = \frac{17}{16}\sqrt{3}$ $(k = \frac{17}{16})$ 1.0625</p>	M1 A1 A1 M1 dM1 A1 (6)
(c)	$\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta = \int \sin^2 \theta \sec^2 \theta d\theta$ $= \int \tan^2 \theta d\theta$ $= \int (\sec^2 \theta - 1) d\theta$ $= \tan \theta - \theta \quad (+C)$ $V = \pi \int_0^{\frac{\pi}{3}} y^2 dx = [\tan \theta - \theta]_0^{\frac{\pi}{3}} = \pi \left[\left(\sqrt{3} - \frac{\pi}{3} \right) - (0 - 0) \right]$ $= \sqrt{3}\pi - \frac{1}{3}\pi^2 \quad (p = 1, q = -\frac{1}{3})$	M1 A1 A1 dM1 A1 dM1 A1 (7) (15 marks)

Question Number	Scheme	Marks
<p>11. (a)</p> <p>(b)</p>	$\int \frac{1}{P(5-P)} dP = \int \frac{1}{15} dt$ $1 = A(5-P) + BP$ $A = \frac{1}{5}, B = \frac{1}{5}$ <p>giving $\int \frac{1}{P(5-P)} dP = \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)} dP$</p> <p>Hence $\int \frac{1}{P(5-P)} dP = \int \frac{1}{15} dt$</p> $\Rightarrow \frac{1}{5} \ln P - \frac{1}{5} \ln(5-P) = \frac{1}{15} t (+c)$ $\{t=0, P=1 \Rightarrow\} \quad \frac{1}{5} \ln 1 - \frac{1}{5} \ln(4) = 0 + c \quad \left\{ \Rightarrow c = -\frac{1}{5} \ln 4 \right\}$ <p>eg: $\frac{1}{5} \ln \left(\frac{P}{5-P} \right) = \frac{1}{15} t - \frac{1}{5} \ln 4$</p> $\ln \left(\frac{4P}{5-P} \right) = \frac{1}{3} t$ <p>Using any of the subtraction (or addition) laws for logarithms CORRECTLY.</p> <p>eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$ Eliminate ln's correctly.</p> <p>gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$</p> $P = \frac{5e^{\frac{1}{3}t}}{(4 + e^{\frac{1}{3}t})} \quad \left\{ \begin{array}{l} (+ e^{\frac{1}{3}t}) \\ (+ e^{\frac{1}{3}t}) \end{array} \right\} \quad \text{Make } P \text{ the subject.}$ $P = \frac{5}{(1 + 4e^{-\frac{1}{3}t})} \quad \text{or} \quad P = \frac{25}{(5 + 20e^{-\frac{1}{3}t})} \text{ etc.}$ <p>Note that the 'dM' marks are dependent upon the first two M marks.</p> <p>$1 + 4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1 A1ft</p> <p>dM1</p> <p>dM1</p> <p>dM1</p> <p>A1</p> <p>(11)</p> <p>B1</p> <p>(1)</p> <p>(12 marks)</p>

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Surname	Other names
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**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference

WFM01/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

--

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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S 4 5 0 0 1 A 0 1 2 0

PEARSON



1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 8i \quad \text{and} \quad z_2 = 1 - i$$

Find, showing your working,

(a) $\frac{z_1}{z_2}$ in the form $a + bi$, where a and b are real, (3)

(b) the value of $\left| \frac{z_1}{z_2} \right|$, (2)

(c) the value of $\arg \frac{z_1}{z_2}$, giving your answer in radians to 2 decimal places. (2)

2.

$$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0$$

The root α of the equation $f(x) = 0$ lies in the interval $[1.6, 1.8]$

(a) Use linear interpolation once on the interval $[1.6, 1.8]$ to find an approximation to α . Give your answer to 3 decimal places.

(4)

(b) Taking 1.7 as a first approximation to α , apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α . Give your answer to 3 decimal places.

(6)

3.

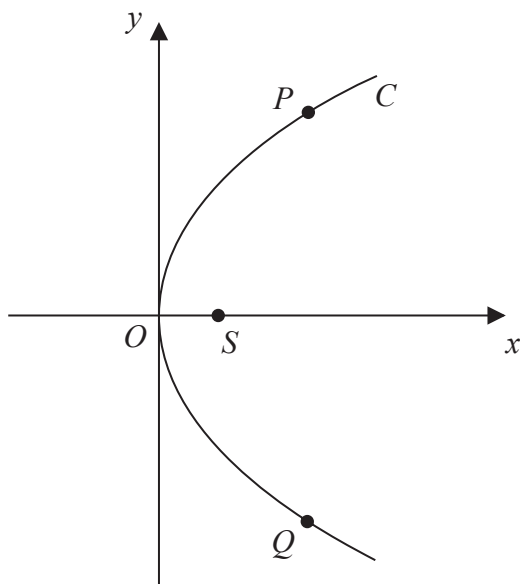


Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 8x$.
 The point P lies on C , where $y > 0$, and the point Q lies on C , where $y < 0$.
 The line segment PQ is parallel to the y -axis.

Given that the distance PQ is 12,

(a) write down the y coordinate of P , **(1)**

(b) find the x coordinate of P . **(2)**

Figure 1 shows the point S which is the focus of C .

The line l passes through the point P and the point S .

(c) Find an equation for l in the form $ax + by + c = 0$, where a , b and c are integers. **(4)**

4. The quadratic equation

$$5x^2 - 4x + 1 = 0$$

has roots α and β .

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. **(2)**

(b) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{6}{5}$ **(4)**

(c) Find a quadratic equation with integer coefficients, which has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$ **(6)**

7. The point $P\left(6t, \frac{6}{t}\right)$, $t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 36$

(a) Show that an equation for the tangent to H at P is

$$y = -\frac{1}{t^2}x + \frac{12}{t} \tag{5}$$

The tangent to H at the point A and the tangent to H at the point B meet at the point $(-9, 12)$.

(b) Find the coordinates of A and B . (7)

8. (i) The transformation U is represented by the matrix \mathbf{P} where,

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

(a) Describe fully the transformation U . (2)

The transformation V , represented by the matrix \mathbf{Q} , is a stretch scale factor 3 parallel to the x -axis.

(b) Write down the matrix \mathbf{Q} . (1)

Transformation U followed by transformation V is a transformation which is represented by matrix \mathbf{R} .

(c) Find the matrix \mathbf{R} . (3)

(ii)

$$\mathbf{S} = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}$$

Given that the matrix \mathbf{S} represents an enlargement, with a positive scale factor and centre $(0, 0)$, followed by a rotation with centre $(0, 0)$,

(a) find the scale factor of the enlargement, (2)

(b) find the angle and direction of rotation, giving your answer in degrees to 1 decimal place. (3)

Question 8 continued

A series of horizontal lines for writing the answer to Question 8.

WFM01/01: Further Pure Mathematics F1

Question Number	Scheme	Marks
<p>1. (a)</p>	$\frac{z_1}{z_2} = \frac{2+8i}{1-i} \times \frac{1+i}{1+i}$ $= \frac{2+2i+8i-8}{2} = -3+5i$	<p>M1</p> <p>A1 A1</p> <p>(3)</p>
<p>(b)</p>	$\left \frac{z_1}{z_2} \right = \sqrt{(-3)^2 + 5^2} = \sqrt{34}$ <p>(or awrt 5.83)</p>	<p>M1 A1ft</p> <p>(2)</p>
<p>(c)</p>	$\tan \alpha = -\frac{5}{3} \text{ or } \frac{5}{3}$	<p>M1</p>
	$\arg \frac{z_1}{z_2} = \pi - 1.03\dots = 2.11$	<p>A1</p> <p>(2)</p>
		<p>(7 marks)</p>
<p>2. (a)</p>	$f(1.6) = -1.29543081\dots$ $f(1.8) = 0.5401863372$ $\frac{\alpha - 1.6}{"1.29543081\dots"} = \frac{1.8 - \alpha}{"0.5401863372"}$ $\alpha = 1.741143899$	<p>awrt -1.30</p> <p>awrt 0.54</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>awrt 1.741</p> <p>A1</p> <p>(4)</p>
<p>(b)</p>	$f'(x) = 10x - 6x^{\frac{1}{2}}$ $f(1.7) = -0.4161152711\dots$ $f'(1.7) = 9.176957114\dots$ $\alpha_2 = 1.7 - \frac{f(1.7)}{f'(1.7)}$ $\alpha_2 = 1.745$	<p>awrt -0.42</p> <p>awrt 9.18</p> <p>M1 A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>cao</p> <p>(6)</p> <p>(10 marks)</p>

WFM01/01: Further Pure Mathematics F1

Question Number	Scheme	Marks
<p>3. (a)</p> <p>(b)</p> <p>(c)</p>	<p>$PQ = 12 \Rightarrow$ By symmetry $y_p = \frac{12}{2} = 6$</p> <p>$y^2 = 8x \Rightarrow 6^2 = 8x$ $\Rightarrow x = \frac{36}{8} = \frac{9}{2}$</p> <p>Focus $S(2, 0)$</p> <p>Gradient $PS = \frac{6-0}{\frac{9}{2}-2} = \left(\frac{6-0}{\frac{9}{2}-2} = \frac{12}{5} \right)$</p> <p>Either $y-0 = \frac{12}{5}(x-2)$ or $y-6 = \frac{12}{5}(x-\frac{9}{2})$</p> <p>Or $y = \frac{12}{5}x + c$ and $0 = \frac{12}{5}(2) + c \Rightarrow c = -\frac{24}{5}$</p> <p>$l: \quad \underline{12x - 5y - 24 = 0}$</p>	<p>B1</p> <p>(1)</p> <p>M1</p> <p>A1</p> <p>(2)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>(7 marks)</p>
<p>4. (a)</p> <p>(b)</p> <p>(c)</p>	<p>$\alpha + \beta = \frac{4}{5}, \quad \alpha\beta = \frac{1}{5}$</p> <p>$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$</p> <p>$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$</p> <p>$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\frac{16}{25} - \frac{2}{5}}{\frac{1}{5}}$</p> <p>$= \frac{6}{5} *$</p> <p>$\alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta} = \frac{24}{5}$</p> <p>$\left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right) = \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta} = \frac{1}{5} + \frac{6}{5} + 5 = \frac{32}{5}$</p> <p>$x^2 - \frac{24}{5}x + \frac{32}{5} = 0 \Rightarrow 5x^2 - 24x + 32 = 0$</p>	<p>B1, B1</p> <p>(2)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(6)</p> <p>(12 marks)</p>

WFM01/01: Further Pure Mathematics F1

Question Number	Scheme	Marks
5.	<p>$f(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). (\therefore True for $n = 1$).</p> <p>Assume true for $f(k)$</p> <p>Using the formula to write down $f(k + 1)$, $f(k + 1) = 5^{k+1} + 8(k + 1) + 3$</p> $f(k + 1) - f(k) = 5^{k+1} + 8(k + 1) + 3 - 5^k - 8k - 3$ $= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$ <p>$f(k + 1) = 4(5^k + 2) + f(k)$, which is divisible by 4</p> <p>\therefore True for $n = k + 1$ if true for $n = k$. True for $n = 1$, \therefore true for all n.</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>(6 marks)</p>
6. (a)	<p>$r(r + 1)(r + 3) = r^3 + 4r^2 + 3r$, so use $\sum r^3 + 4\sum r^2 + 3\sum r$</p> $= \frac{1}{4}n^2(n + 1)^2 + 4\left(\frac{1}{6}n(n + 1)(2n + 1)\right) + 3\left(\frac{1}{2}n(n + 1)\right)$ $= \frac{1}{12}n(n + 1)\{3n(n + 1) + 8(2n + 1) + 18\}$ <p>or $= \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$</p> <p>or $= \frac{1}{12}(n + 1)\{3n^3 + 19n^2 + 26n\}$</p> $= \frac{1}{12}n(n + 1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n + 1)(n + 2)(3n + 13) \quad (k = 13)$	<p>M1</p> <p>A1, A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>(7)</p>
(b)	$\sum_{21}^{40} = \sum_1^{40} - \sum_1^{20}$ $= \frac{1}{12}(40 \times 41 \times 42 \times 133) - \frac{1}{12}(20 \times 21 \times 22 \times 73), = 707210$	<p>M1</p> <p>A1, A1</p> <p>(3)</p> <p>(10 marks)</p>

Question Number	Scheme	Marks
<p>7. (a)</p> <p>(b)</p>	$y = \frac{36}{x} \Rightarrow \frac{dy}{dx} = -36x^{-2}$ $\text{At } \left(6t, \frac{6}{t}\right), \frac{dy}{dx} = -\frac{c^2}{(6t)^2} = -\frac{1}{t^2}$ $y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t) \Rightarrow y = -\frac{1}{t^2}x + \frac{12}{t} \quad (*)$ <p>Substitute $(-9, 12)$: $12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$</p> $12t^2 - 12t - 9 = 0$ $(2t - 3)(2t + 1) = 0 \Rightarrow t = \frac{3}{2} \quad t = -\frac{1}{2}$ $t = \frac{3}{2} \quad t = -\frac{1}{2} \Rightarrow \text{Points are } (9, 4) \text{ and } (-3, -12)$	<p>M1</p> <p>M1 A1</p> <p>M1 A1 cso</p> <p>(5)</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1 A1</p> <p>(7)</p> <p>(12 marks)</p>
<p>8. (i)</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(ii)</p> <p>(a)</p> <p>(b)</p>	<p>120° or $\frac{2\pi}{3}$ rotation about the origin, anticlockwise.</p> $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{R} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} & -\frac{3\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ <p>$\det \mathbf{S} = 1 \times 1 - 3 \times -3 (= 10)$ or $3^2 + 1^2 (= 10) \Rightarrow$ Enlargement scale factor $= \sqrt{10}$</p> <p>$\tan \theta = \frac{3}{1} \Rightarrow \theta = 71.6^\circ$, anticlockwise.</p>	<p>B1, B1</p> <p>(2)</p> <p>B1</p> <p>(1)</p> <p>M1 A1 A1 (-1 each error)</p> <p>(3)</p> <p>M1 A1</p> <p>(2)</p> <p>M1 A1, A1</p> <p>(3)</p> <p>(11 marks)</p>

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Surname	Other names
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**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Further Pure Mathematics F2

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference

WFM02/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

--

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
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- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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PEARSON

1. (a) Express $\frac{3}{(3r-1)(3r+2)}$ in partial fractions. (2)

(b) Using your answer to part (a) and the method of differences, show that

$$\sum_{r=1}^n \frac{3}{(3r-1)(3r+2)} = \frac{3n}{2(3n+2)} \quad (3)$$

(c) Evaluate $\sum_{r=100}^{1000} \frac{3}{(3r-1)(3r+2)}$, giving your answer to 3 significant figures. (2)

Question 1 continued

Lined area for student response

(Total 7 marks)

Q1

4.
$$z = -8 + (8\sqrt{3})i$$

- (a) Find the modulus of z and the argument of z . **(3)**

Using de Moivre's theorem,

- (b) find z^3 , **(2)**

- (c) find the values of w such that $w^4 = z$, giving your answers in the form $a + ib$, where $a, b \in \mathbb{R}$. **(5)**

5.

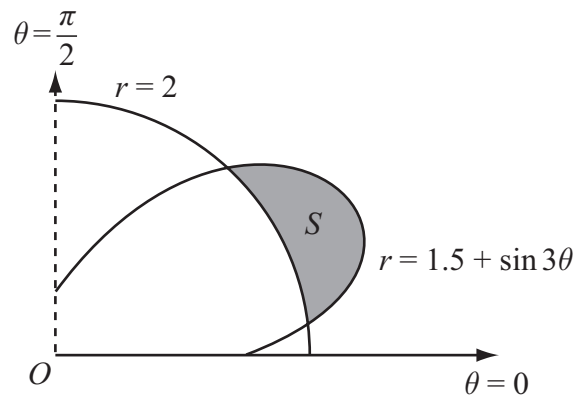


Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \quad 0 \leq \theta \leq \frac{\pi}{2},$$

$$\text{and } r = 1.5 + \sin 3\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

(a) Find the coordinates of the points where the curves intersect. (3)

The region S , between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S , giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions. (7)

Question 5 continued

Lined area for writing the answer to Question 5 continued.

6. A complex number z is represented by the point P in the Argand diagram.

(a) Given that $|z - 6| = |z|$, sketch the locus of P . (2)

(b) Find the complex numbers z which satisfy both $|z - 6| = |z|$ and $|z - 3 - 4i| = 5$. (3)

The transformation T from the z -plane to the w -plane is given by $w = \frac{30}{z}$.

(c) Show that T maps $|z - 6| = |z|$ onto a circle in the w -plane and give the cartesian equation of this circle. (5)

7. (a) Show that the transformation $z = y^{\frac{1}{2}}$ transforms the differential equation

$$\frac{dy}{dx} - 4y \tan x = 2y^{\frac{1}{2}} \quad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 2z \tan x = 1 \quad \text{(II)} \quad \textbf{(5)}$$

(b) Solve the differential equation (II) to find z as a function of x . **(6)**

(c) Hence obtain the general solution of the differential equation (I). **(1)**

Question 7 continued

Lined writing area for the answer to Question 7.

8. (a) Find the value of λ for which $y = \lambda x \sin 5x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3 \cos 5x \quad (4)$$

- (b) Using your answer to part (a), find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3 \cos 5x \quad (3)$$

Given that at $x = 0$, $y = 0$ and $\frac{dy}{dx} = 5$,

- (c) find the particular solution of this differential equation, giving your solution in the form $y = f(x)$. (5)

- (d) Sketch the curve with equation $y = f(x)$ for $0 \leq x \leq \pi$. (2)

Question 8 continued

(The main content area of this page is intentionally left blank for student answers.)

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

WFM02/01: Further Pure Mathematics F2

Question Number	Scheme	Marks
1(a)	$\frac{1}{3r-1} - \frac{1}{3r+2}$	M1 A1 (2)
(b)	$\sum_{r=1}^n \frac{3}{(3r-1)(3r+2)} = \frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \dots - \frac{1}{3n-1} + \frac{1}{3n+2}$ $= \frac{1}{2} - \frac{1}{3n+2} = \frac{3n}{2(3n+2)} *$	M1 A1ft A1 (3)
(c)	<p>Sum = $f(1000) - f(99)$</p> $\frac{3000}{6004} - \frac{297}{598} = 0.00301 \quad \text{or } 3.01 \times 10^{-3}$	M1 A1 (2) 7

WFM02/01: Further Pure Mathematics F2

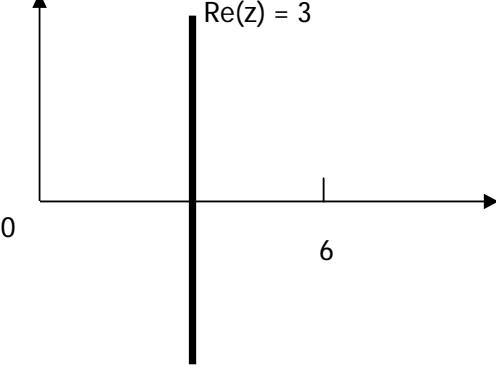
Question Number	Scheme	Marks
2	$f''(t) = -x - \cos x, \quad f''(0) = -1$ $f'''(t) = (-1 + \sin x) \frac{dx}{dt}, \quad f'''(0) = -0.5$ $f(t) = f(0) + tf'(0) + \frac{t^2}{2} f''(0) + \frac{t^3}{3!} f'''(0) + \dots$ $= 0.5t - 0.5t^2 - \frac{1}{12}t^3 + \dots$	B1 M1A1 M1 A1 5

WFM02/01: Further Pure Mathematics F2

Question Number	Scheme	Marks
<p>3(a)</p>	<p>$(x+4)(x+3)^2 - 2(x+3) = 0$, $(x+3)(x^2 + 7x + 10) = 0$ so $(x+2)(x+3)(x+5) = 0$ or alternative method including calculator</p> <p>Finds critical values -2 and -5</p> <p>Establishes $x > -2$</p> <p>Finds and uses critical value -3 to give $-5 < x < -3$</p>	<p>M1</p> <p>A1 A1</p> <p>A1ft</p> <p>M1A1</p> <p>(6)</p>
<p>(b)</p>	<p>$x > -2$</p>	<p>B1ft</p> <p>(1)</p> <p>7</p>

Question Number	Scheme	Marks
4(a)	Modulus = 16 Argument = $\arctan(-\sqrt{3}) = \frac{2\pi}{3}$	B1 M1 A1 (3)
(b)	$z^3 = 16^3 \left(\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right) \right)^3 = 16^3 (\cos 2\pi + i \sin 2\pi) = 4096 \text{ or } 16^3$	M1 A1 (2)
(c)	$w = 16^{\frac{1}{4}} \left(\cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right) \right)^{\frac{1}{4}} = 2 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right) (= \sqrt{3} + i)$ OR $-1 + \sqrt{3}i$ OR $-\sqrt{3} - i$ OR $1 - \sqrt{3}i$	M1 A1ft M1A2 (1,0) (5) 10

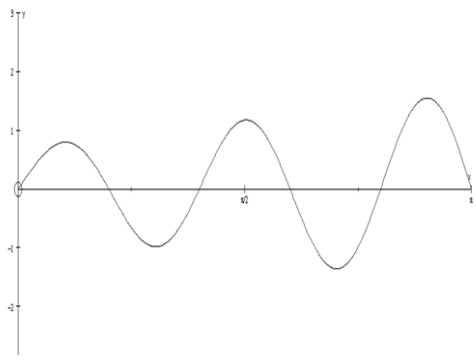
Question Number	Scheme	Marks
5(a)	$1.5 + \sin 3\theta = 2 \rightarrow \sin 3\theta = 0.5 \therefore 3\theta = \frac{\pi}{6} \left(\text{or } \frac{5\pi}{6} \right),$ $\text{and } \therefore \theta = \frac{\pi}{18} \text{ or } \frac{5\pi}{18}$	M1 A1, A1 (3)
5(b)	$\text{Area} = \frac{1}{2} \left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (1.5 + \sin 3\theta)^2 d\theta \right], -\frac{1}{9}\pi \times 2^2$ $= \frac{1}{2} \left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (2.25 + 3\sin 3\theta + \frac{1}{2}(1 - \cos 6\theta)) d\theta \right] - \frac{1}{9}\pi \times 2^2$ $= \frac{1}{2} \left[(2.25\theta - \cos 3\theta + \frac{1}{2}(\theta - \frac{1}{6}\sin 6\theta)) \right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} - \frac{1}{9}\pi \times 2^2$ $= \frac{13\sqrt{3}}{24} - \frac{5\pi}{36}$	M1, M1 M1 M1 A1 M1 A1 (7) 10

Question Number	Scheme	Marks
6(a)	<p>Imaginary Axis</p>  <p>Real axis</p> <p>Vertical Straight line Through 3 on real axis</p>	<p>B1 B1</p> <p>(2)</p>
(b)	<p>These are points where line $x = 3$ meets the circle centre $(3, 4)$ with radius 5.</p> <p>The complex numbers are $3 + 9i$ and $3 - i$.</p>	<p>M1</p> <p>A1 A1</p> <p>(3)</p>
(c)	$ z - 6 = z \Rightarrow \left \frac{30}{w} - 6 \right = \left \frac{30}{w} \right $ $\therefore 30 - 6w = 30 \Rightarrow \therefore 5 - w = 5 $ <p>This is a circle with Cartesian equation $(u - 5)^2 + v^2 = 25$</p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(5)</p> <p>10</p>

WFM02/01: Further Pure Mathematics F2

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} \text{ and } \frac{dy}{dz} = 2z \text{ so } \frac{dy}{dx} = 2z \cdot \frac{dz}{dx}$ Substituting to get $2z \cdot \frac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\frac{dz}{dx} - 2z \tan x = 1$ *	M1 M1 A1 M1 A1 (5)
(b)	$\text{I.F.} = e^{\int -2 \tan x dx} = e^{2 \ln \cos x} = \cos^2 x$ $\therefore \frac{d}{dx}(z \cos^2 x) = \cos^2 x \therefore z \cos^2 x = \int \cos^2 x dx$ $\therefore z \cos^2 x = \int \frac{1}{2}(\cos 2x + 1) dx = \frac{1}{4} \sin 2x + \frac{1}{2} x + c$ $\therefore z = \frac{1}{2} \tan x + \frac{1}{2} x \sec^2 x + c \sec^2 x$	M1 A1 M1 M1 A1 A1 (6)
(c)	$\therefore y = \left(\frac{1}{2} \tan x + \frac{1}{2} x \sec^2 x + c \sec^2 x\right)^2$	B1ft (1) 12

WFM02/01: Further Pure Mathematics F2

Question Number	Scheme	Marks
8(a)	Differentiate twice and obtaining $\frac{dy}{dx} = \lambda \sin 5x + 5\lambda x \cos 5x$ and $\frac{d^2y}{dx^2} = 10\lambda \cos 5x - 25\lambda x \sin 5x$	M1 A1
	Substitute to give $\lambda = \frac{3}{10}$	M1 A1 (4)
(b)	Complementary function is $y = A \cos 5x + B \sin 5x$ or $Pe^{5ix} + Qe^{-5ix}$	M1 A1
	So general solution is $y = A \cos 5x + B \sin 5x + \frac{3}{10}x \sin 5x$ or in exponential form	A1ft (3)
(c)	$y=0$ when $x=0$ means $A=0$	B1
	$\frac{dy}{dx} = 5B \cos 5x + \frac{3}{10} \sin 5x + \frac{3}{2}x \cos 5x$ and at $x=0$ $\frac{dy}{dx} = 5$ and so $5 = 5A$	M1 M1
	So $B=1$	A1
	So $y = \sin 5x + \frac{3}{10}x \sin 5x$	A1 (5)
(d)	 <p>"Sinusoidal" through O amplitude becoming larger</p> <p>Crosses x axis at $\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$</p>	B1 B1 (2) 14

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Surname	Other names
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**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Further Pure Mathematics F3

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference

WFM03/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

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Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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PEARSON

3. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2 \sinh^2 x \tag{3}$$

(b) Solve the equation

$$\cosh 2x - 3 \sinh x = 15,$$

giving your answers as exact logarithms. (5)

4. $I_n = \int_0^a (a-x)^n \cos x \, dx, \quad a > 0, \quad n \geq 0$

(a) Show that, for $n \geq 2$,

$$I_n = na^{n-1} - n(n-1)I_{n-2} \quad (5)$$

(b) Hence evaluate $\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \, dx.$ (3)

Question 5 continued

Lined area for writing the answer to Question 5.

(Total 9 marks)

Q5

6.
$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix}, \text{ where } k \text{ is a constant.}$$

Given that $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ is an eigenvector of \mathbf{M} ,

(a) find the eigenvalue of \mathbf{M} corresponding to $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$, (2)

(b) show that $k = 3$, (2)

(c) show that \mathbf{M} has exactly two eigenvalues. (4)

A transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by \mathbf{M} .

The transformation T maps the line l_1 , with cartesian equations $\frac{x-2}{1} = \frac{y}{-3} = \frac{z+1}{4}$, onto the line l_2 .

(d) Taking $k = 3$, find cartesian equations of l_2 . (5)

Question 6 continued

Lined writing area for question 6.

(Total 13 marks)

Q6

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Question 7 continued

8. The hyperbola H has equation $\frac{x^2}{16} - \frac{y^2}{4} = 1$.

The line l_1 is the tangent to H at the point $P(4 \sec t, 2 \tan t)$.

(a) Use calculus to show that an equation of l_1 is

$$2y \sin t = x - 4 \cos t \tag{5}$$

The line l_2 passes through the origin and is perpendicular to l_1 .

The lines l_1 and l_2 intersect at the point Q .

(b) Show that, as t varies, an equation of the locus of Q is

$$(x^2 + y^2)^2 = 16x^2 - 4y^2 \tag{8}$$

WFM03/01: Further Pure Mathematics F3

Question Number	Scheme	Marks
1.	$\pm \frac{a}{e} = 8, \quad \pm ae = 2$ $\frac{a}{e} \times ae = a^2 = 16$ $a = 4$ $b^2 = a^2(1 - e^2) = a^2 - a^2 e^2$ $\Rightarrow b^2 = 16 - 4 = 12$ $\Rightarrow b = \sqrt{12} = 2\sqrt{3}$	B1, B1 B1 M1 A1 (5) 5

WFM03/01: Further Pure Mathematics F3

Question Number	Scheme	Marks
2.	$x^2 + 4x + 13 = (x + 2)^2 + 9$ $\int \frac{1}{(x + 2)^2 + 9} dx = \frac{1}{3} \arctan\left(\frac{x + 2}{3}\right)$ $\left[\frac{1}{3} \arctan\left(\frac{x + 2}{3}\right)\right]_{-2}^1 = \frac{1}{3} (\arctan 1 - \arctan 0)$ $= \frac{\pi}{12}$	B1 M1 A1 M1 A1 (5) 5

Question Number	Scheme	Marks
3(a)	$rhs = 1 + 2 \sinh^2 x = 1 + 2 \left(\frac{e^x - e^{-x}}{2} \right)^2$ $= \frac{2 + e^{2x} - 2 + e^{-2x}}{2}$ $= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x = lhs \quad *$	M1 M1 A1 (3)
(b)	$1 + 2 \sinh^2 x - 3 \sinh x = 15$ $2 \sinh^2 x - 3 \sinh x - 14 = 0$ $(\sinh x + 2)(2 \sinh x - 7) = 0$ $\sinh x = -2, \frac{7}{2}$ $x = \ln \left(-2 + \sqrt{(-2)^2 + 1} \right) = \ln \left(-2 + \sqrt{5} \right)$ $x = \ln \left(\frac{7}{2} + \sqrt{\left(\frac{7}{2} \right)^2 + 1} \right) = \ln \left(\frac{7 + \sqrt{53}}{2} \right)$	M1 M1 A1 M1 A1 (5) 8

Question Number	Scheme	Marks
<p>5(a)</p> <p>(b)</p>	$\frac{dy}{dx} = 2 \operatorname{ar} \cosh(3x) \times \frac{3}{\sqrt{9x^2 - 1}}$ $\sqrt{9x^2 - 1} \frac{dy}{dx} = 6 \operatorname{ar} \cosh(3x)$ $(9x^2 - 1) \left(\frac{dy}{dx} \right)^2 = 36 (\operatorname{ar} \cosh(3x))^2$ $(9x^2 - 1) \left(\frac{dy}{dx} \right)^2 = 36y \quad *$ $\left\{ 18x \left(\frac{dy}{dx} \right)^2 + (9x^2 - 1) \times 2 \frac{dy}{dx} \times \frac{d^2y}{dx^2} \right\} = 36 \frac{dy}{dx}$ $(9x^2 - 1) \frac{d^2y}{dx^2} + 9x \frac{dy}{dx} = 18 \quad *$	<p>M1A1A1</p> <p>dM1</p> <p>A1 (5)</p> <p>M1 {A1} A1</p> <p>A1 (4)</p> <p style="text-align: right;">9</p>

Question Number	Scheme	Marks
6(a)	$\begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix} = \lambda \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ $\begin{pmatrix} 24 \\ 4 \\ 6k+6 \end{pmatrix} = \begin{pmatrix} 6\lambda \\ \lambda \\ 6\lambda \end{pmatrix}$ <p>Uses the first or second row to obtain $\lambda = 4$</p>	M1A1 (2)
(b)	<p>Uses the third row and their $\lambda = 4$ to obtain</p> $6k+6=24 \Rightarrow k=3 \quad *$	M1 A1 (2)
(c)	$\begin{vmatrix} 1-\lambda & 0 & 3 \\ 0 & -2-\lambda & 1 \\ 3 & 0 & 1-\lambda \end{vmatrix} = 0$ $\Rightarrow (1-\lambda)((-2-\lambda)(1-\lambda)-0)-0(0(1-\lambda)-3)+3(0-3(-2-\lambda))=0$ $\Rightarrow (1-\lambda)(-2-\lambda)(1-\lambda)+9(2+\lambda)=(2+\lambda)(9-(1-\lambda)^2)=0 \quad (\lambda^3-12\lambda-16=0)$ $\Rightarrow (\lambda+2)(\lambda^2-2\lambda-8)=0$ $\Rightarrow (\lambda+2)(\lambda+2)(\lambda-4)=0$ $\lambda = -2, 4$	M1 A1 M1 A1 (4)
(d)	<p>Parametric form of $l_1 : (t+2, -3t, 4t-1)$</p> $\begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} t+2 \\ -3t \\ 4t-1 \end{pmatrix} = \begin{pmatrix} 13t-1 \\ 10t-1 \\ 7t+5 \end{pmatrix}$ <p>Cartesian equations of $l_2 : \frac{x+1}{13} = \frac{y+1}{10} = \frac{z-5}{7}$</p>	M1 M1 A1 ddM1A1(5)
		13

Question Number	Scheme	Marks
7(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 1 & 0 \\ 6 & -2 & 1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = 5$ $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$	M1 A2(1,0)
(b)	<p>Equation of l is $\mathbf{r} = \begin{pmatrix} 6 \\ 13 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$</p> <p>At intersection $\begin{pmatrix} 6+t \\ 13+4t \\ 5+2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$</p> $\Rightarrow 6+t+4(13+4t)+2(5+2t)=5 \Rightarrow t=-3$ <p>N is $(3,1,-1)$ *</p>	M1 M1 M1 A1 (4)
(c)	$\overline{PN} \cdot \overline{PR} = (-3\mathbf{i} - 12\mathbf{j} - 6\mathbf{k}) \cdot (-5\mathbf{i} - 13\mathbf{j} - 3\mathbf{k}) = 189$ $\sqrt{9+144+36} \sqrt{25+169+9} \cos NPR = 189$ $NX = NP \sin NPR = \sqrt{189} \sin NPR = 3.61$	M1 A1ft A1 M1A1 (5)
		14

Question Number	Scheme	Marks
8(a)	$\frac{dx}{dt} = 4 \sec t \tan t \quad \frac{dy}{dt} = 2 \sec^2 t$ $\frac{dy}{dx} = \frac{2 \sec^2 t}{4 \sec t \tan t} \quad \left(= \frac{1}{2 \sin t} \right)$ $y - 2 \tan t = \frac{1}{2 \sin t} (x - 4 \sec t)$ $2y \sin t - \frac{4 \sin^2 t}{\cos t} = x - \frac{4}{\cos t}$ $2y \sin t = x - \frac{4 - 4 \sin^2 t}{\cos t} = x - 4 \cos t \quad *$	B1 (both) M1 M1 A1 A1 (5)
(b)	<p>Gradient of l_2 is $-2 \sin t$</p> $y = -2x \sin t \quad (2)$ $2(-2x \sin t) \sin t = x - 4 \cos t \Rightarrow x = \frac{4 \cos t}{1 + 4 \sin^2 t} \quad (1)$ $y = \frac{-8 \sin t \cos t}{1 + 4 \sin^2 t}$ $(x^2 + y^2)^2 = \left(\frac{16 \cos^2 t}{(1 + 4 \sin^2 t)^2} + \frac{64 \sin^2 t \cos^2 t}{(1 + 4 \sin^2 t)^2} \right)^2$ $= \frac{256 \cos^4 t}{(1 + 4 \sin^2 t)^4} (1 + 4 \sin^2 t)^2 = \frac{256 \cos^4 t}{(1 + 4 \sin^2 t)^2}$ $16x^2 - 4y^2 = \frac{256 \cos^2 t}{(1 + 4 \sin^2 t)^2} - \frac{256 \sin^2 t \cos^2 t}{(1 + 4 \sin^2 t)^2} = \frac{256 \cos^4 t}{(1 + 4 \sin^2 t)^2}$	M1 A1 M1 A1 M1 A1 M1 A1 (8) 13

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Surname	Other names
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Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Mechanics M1

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference

WME01/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

--

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
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- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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PEARSON

2. Particle P has mass m kg and particle Q has mass $3m$ kg. The particles are moving in opposite directions along a smooth horizontal plane when they collide directly. Immediately before the collision P has speed $4u$ m s^{-1} and Q has speed ku m s^{-1} , where k is a constant. As a result of the collision the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of k . (4)

(b) Find, in terms of m and u , the magnitude of the impulse exerted on P by Q . (3)

3.

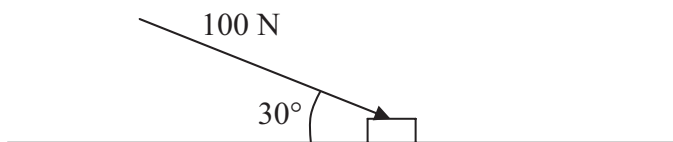


Figure 1

A small box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is $\frac{1}{2}$. The box is pushed by a force of magnitude 100 N which acts at an angle of 30° with the floor, as shown in Figure 1.

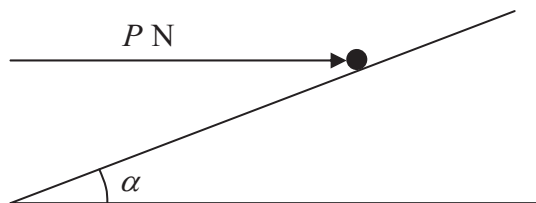
Given that the box moves with constant speed, find the mass of the box.

(7)

- 4. A beam AB has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points C and D , where $AC = 1$ m and $DB = 1$ m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C . By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B .

(7)

7.

**Figure 2**

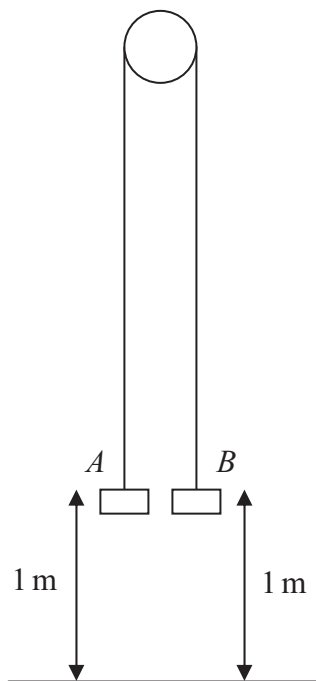
A particle of mass 0.4 kg is held at rest on a fixed rough plane by a horizontal force of magnitude P newtons. The force acts in the vertical plane containing the line of greatest slope of the inclined plane which passes through the particle. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 2.

The coefficient of friction between the particle and the plane is $\frac{1}{3}$.

Given that the particle is on the point of sliding up the plane, find

- (a) the magnitude of the normal reaction between the particle and the plane, (5)
- (b) the value of P . (5)

8.

**Figure 3**

Two particles A and B have mass 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor, as shown in Figure 3. The particles are released from rest and in the subsequent motion B does not reach the pulley.

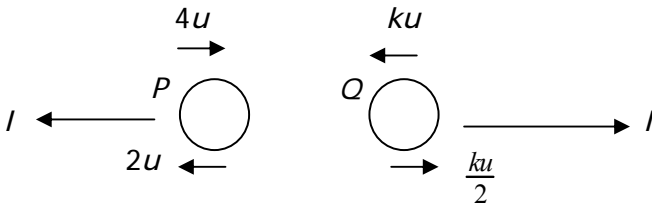
(a) Find the tension in the string immediately after the particles are released. (6)

(b) Find the acceleration of A immediately after the particles are released. (2)

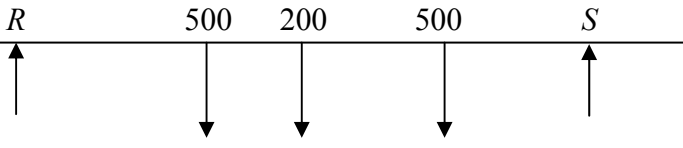
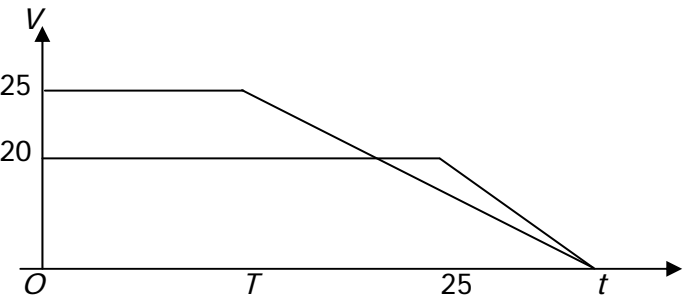
When the particles have been moving for 0.5 s , the string breaks.

(c) Find the further time that elapses until B hits the floor. (9)

WME01/01: Mechanics M1

Question Number	Scheme	Marks
Q1	$(-4\mathbf{i} - 7\mathbf{j}) = \mathbf{r} + 4(-3\mathbf{i} + 2\mathbf{j})$ $\mathbf{r} = (8\mathbf{i} - 15\mathbf{j})$ $ \mathbf{r} = \sqrt{8^2 + (-15)^2} = 17 \text{ m}$	M1 A1 A1 M1 A1 ft [5]
Q2 (a) (b)	 $4mu - 3mku = -2mu + 3mk \frac{u}{2}$ $k = \frac{4}{3}$ <p>For P, $I = m(2u - -4u)$ $= 6mu$</p> <p>OR For Q, $I = 3m(\frac{ku}{2} - -ku)$</p>	M1 A1 M1 A1cso (4) M1 A1 A1 (3) (M1A1) [7]
Q3	$(\rightarrow) 100\cos 30 = F$ $F = 0.5 R \text{ seen}$ $(\downarrow) mg + 100\cos 60 = R$ $m = 13 \text{ kg or } 12.6 \text{ kg}$	M1 A1 A1 (B1) M1 A1 DM1 A1 [7]

WME01/01: Mechanics M1

Question Number	Scheme	Marks
Q4	 <p> $M(B),$ $500x + 500 \cdot 2x + 200 \cdot 3 = Rx5 + Sx1$ (or any valid moments equation) $(\downarrow) R + S = 500 + 500 + 200 = 1200$ (or a moments equation) solving for $x; x = 1.2$ m </p>	M1 A1 A1 M1 A1 M1 A1 cso [7]
Q5 (a)	 <p> Shape (both) Cross Meet on t-axis 25,20,T,25 Figures </p>	B1 B1 B1 B1 (4)
(b)	<p> For $Q: 20\left(\frac{t+25}{2}\right) = 800$ $t = 55$ </p> <p> For $P: 25\left(\frac{T+55}{2}\right) = 800$ solving for $T: T = 9$ </p>	M1 A1 DM1 A1 M1 A1 DM1 A1 (8) [12]

WME01/01: Mechanics M1

Question Number	Scheme	Marks
Q6	(a) $(\uparrow)v^2 = u^2 + 2as$ $0 = 14.7^2 - 2 \times 9.8 \times s$ $s = 11.025$ (or 11 or 11.0 or 11.03) m Height is 60 m or 60.0 m ft	M1A1 A1 A1ft (4)
	(b) $(\downarrow)v^2 = u^2 + 2as$ $v^2 = (-14.7)^2 + 2 \times 9.8 \times 49$ $v = 34.3$ or 34 m s^{-1}	M1 A1 A1 (3)
	(c) $(\downarrow)v = u + at$ OR $(\downarrow)s = ut + \frac{1}{2}at^2$ $34.3 = -14.7 + 9.8t$ $49 = -14.7t + 4.9t^2$ $t = 5$ $t = 5$	M1 A1 A1 (3) [10]
Q7	(a) $F = \frac{1}{3}R$ $(\uparrow) R \cos \alpha - F \sin \alpha = 0.4g$ $R = \frac{2}{3}g = 6.53$ or 6.5	B1 M1 A1 M1 A1 (5)
	(b) $(\rightarrow)P - F \cos \alpha - R \sin \alpha = 0$ $P = \frac{26}{45}g = 5.66$ or 5.7	M1 A2 M1 A1 (5) [10]

WME01/01: Mechanics M1

Question Number	Scheme	Marks
<p>Q8 (a) Mark together</p> <p>(b)</p> <p>(c)</p>	$(\downarrow)0.4g - T = 0.4a$ $(\uparrow)T - 0.3g = 0.3a$ <p>solving for T</p> $T = 3.36 \text{ or } 3.4 \text{ or } 12g/35 \text{ (N)}$	<p>M1 A1</p> <p>M1 A1</p> <p>DM1</p> <p>A1 (6)</p>
	$0.4g - 0.3g = 0.7a$ $a = 1.4 \text{ m s}^{-2}, g/7$	<p>DM1</p> <p>A1 (2)</p>
	$(\uparrow)v = u + at$ $v = 0.5 \times 1.4$ $= 0.7$ $(\uparrow)s = ut + \frac{1}{2}at^2$ $s = 0.5 \times 1.4 \times 0.5^2$ $= 0.175$ $(\downarrow)s = ut + \frac{1}{2}at^2$ $1.175 = -0.7t + 4.9t^2$ $4.9t^2 - 0.7t - 1.175 = 0$ $t = \frac{0.7 \pm \sqrt{0.7^2 + 19.6 \times 1.175}}{9.8}$ $= 0.5663 \text{ or } \dots$ <p>Ans 0.57 or 0.566 s</p>	<p>M1</p> <p>A1 ft on a</p> <p>M1</p> <p>A1 ft on a</p> <p>DM1 A1 ft</p> <p>DM1 A1 cao</p> <p>A1 cao (9)</p> <p>[17]</p>

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Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Mechanics M2

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference

WME02/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

--

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
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Information

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Advice

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- Try to answer every question.
- Check your answers if you have time at the end.

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Question 1 continued

Lined writing area for the question.

(Total 6 marks)

Q1

2. A particle P of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When P has moved 12 m , its speed is 4 m s^{-1} . Given that friction is the only non-gravitational resistive force acting on P , find

(a) the work done against friction as the speed of P increases from 0 m s^{-1} to 4 m s^{-1} , **(4)**

(b) the coefficient of friction between the particle and the plane. **(4)**

3.

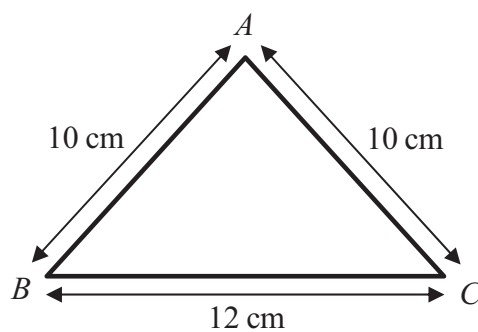


Figure 1

A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle ABC , where $AB = AC = 10$ cm and $BC = 12$ cm, as shown in Figure 1.

(a) Find the distance of the centre of mass of the frame from BC .

(5)

The frame has total mass M . A particle of mass M is attached to the frame at the mid-point of BC . The frame is then freely suspended from B and hangs in equilibrium.

(b) Find the size of the angle between BC and the vertical.

(4)

4. A car of mass 750 kg is moving up a straight road inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{15}$. The resistance to motion of the car from non-gravitational forces has constant magnitude R newtons. The power developed by the car's engine is 15 kW and the car is moving at a constant speed of 20 m s^{-1} .

(a) Show that $R = 260$. (4)

The power developed by the car's engine is now increased to 18 kW. The magnitude of the resistance to motion from non-gravitational forces remains at 260 N. At the instant when the car is moving up the road at 20 m s^{-1} the car's acceleration is $a \text{ m s}^{-2}$.

(b) Find the value of a . (4)

Question 4 continued

Lined area for writing the answer to Question 4.

(Total 8 marks)

Q4

5. [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A ball of mass 0.5 kg is moving with velocity $(10\mathbf{i} + 24\mathbf{j}) \text{ m s}^{-1}$ when it is struck by a bat. Immediately after the impact the ball is moving with velocity $20\mathbf{i} \text{ m s}^{-1}$.

Find

- (a) the magnitude of the impulse of the bat on the ball, (4)

- (b) the size of the angle between the vector **i** and the impulse exerted by the bat on the ball, (2)

- (c) the kinetic energy lost by the ball in the impact. (3)

Question 5 continued

A large rectangular area containing 28 horizontal lines for writing.

(Total 9 marks)

Q5

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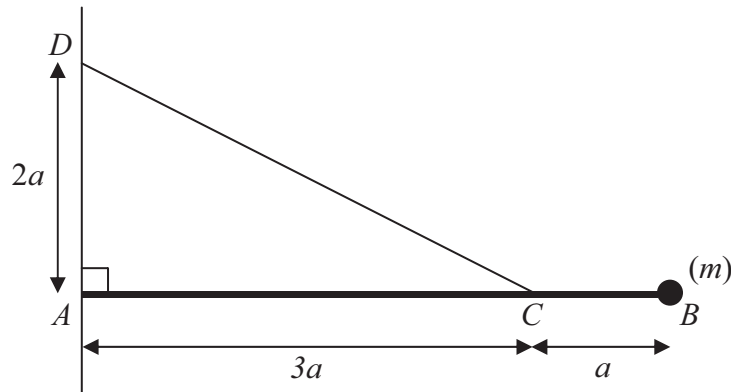


Figure 2

Figure 2 shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B . One end of a light inextensible string is attached to the rod at C , where $AC = 3a$. The other end of the string is attached to the wall at D , where $AD = 2a$ and D is vertically above A . The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T .

(a) Show that $T = mg\sqrt{13}$. (5)

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B . The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leq \frac{5}{2}m$. (3)

7.

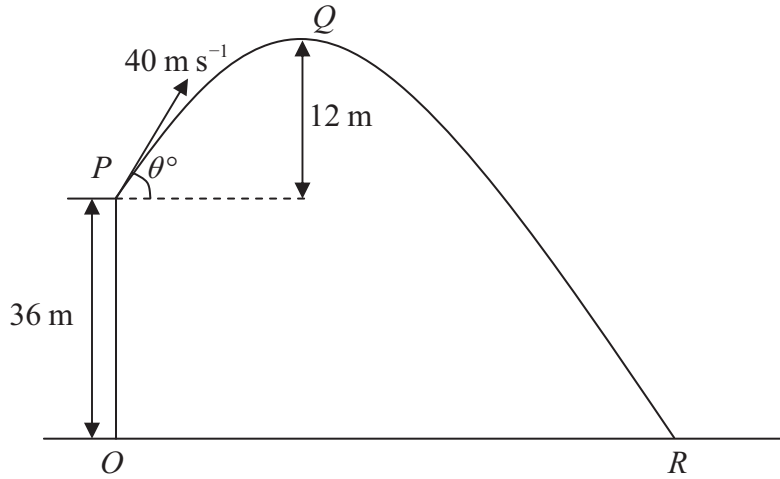


Figure 3

A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m . The ball is projected at an angle θ° to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P . The ball moves freely under gravity and hits the ground at the point R , as shown in Figure 3. Find

- (a) the value of θ , **(3)**
- (b) the distance OR , **(6)**
- (c) the speed of the ball as it hits the ground at R . **(3)**

8. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(a) Find

- (i) the speed of A immediately after the collision,
- (ii) the speed of B immediately after the collision.

(7)

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B . The coefficient of restitution between B and the wall is $\frac{2}{5}$.

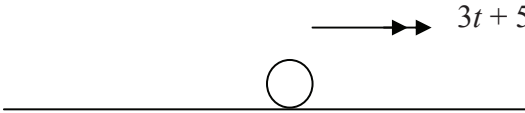
(b) Find the speed of B immediately after hitting the wall.

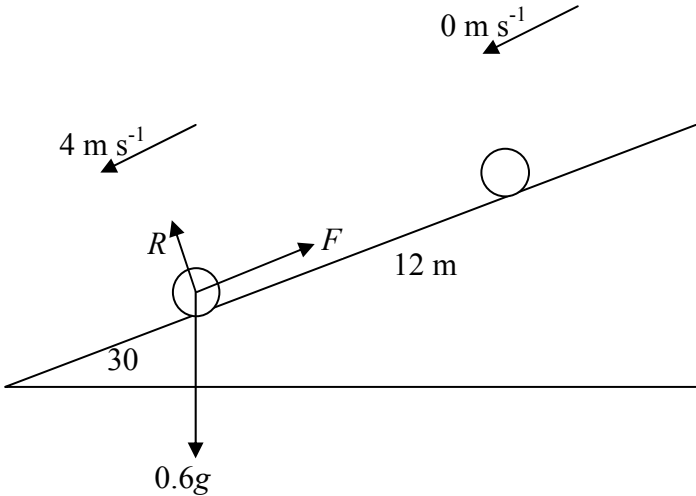
(2)

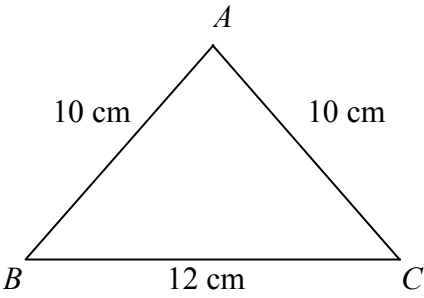
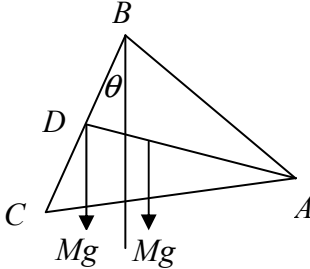
The first collision between A and B occurred at a distance $4a$ from the wall. The balls collide again T seconds after the first collision.

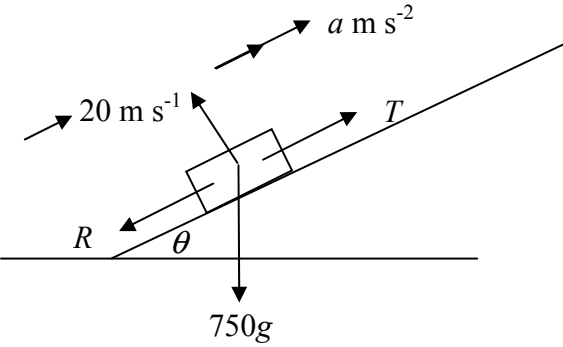
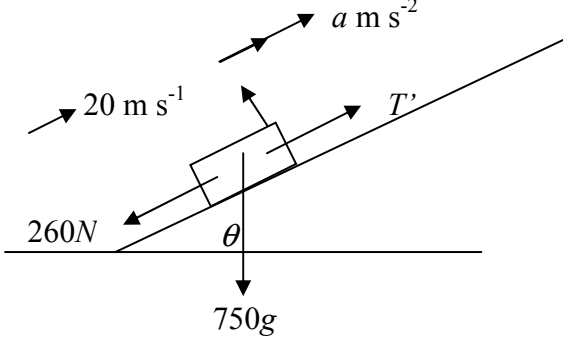
(c) Show that $T = \frac{112a}{15u}$.

(6)

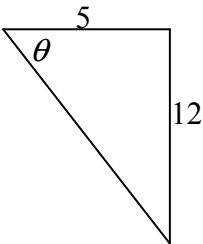
Question Number	Scheme	Marks
Q1	<div style="text-align: center;">  </div> $\frac{dv}{dt} = 3t + 5$ $v = \int (3t + 5) dt$ $v = \frac{3}{2}t^2 + 5t \quad (+c)$ $t = 0 \quad v = 2 \Rightarrow c = 2$ $v = \frac{3}{2}t^2 + 5t + 2$ $t = T \quad 6 = \frac{3}{2}T^2 + 5T + 2$ $12 = 3T^2 + 10T + 4$ $3T^2 + 10T - 8 = 0$ $(3T - 2)(T + 4) = 0$ $T = \frac{2}{3} \quad (T = -4)$ $\therefore T = \frac{2}{3} \quad (\text{or } 0.67)$	<p>M1*</p> <p>A1</p> <p>B1</p> <p>DM1*</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">[6]</p>

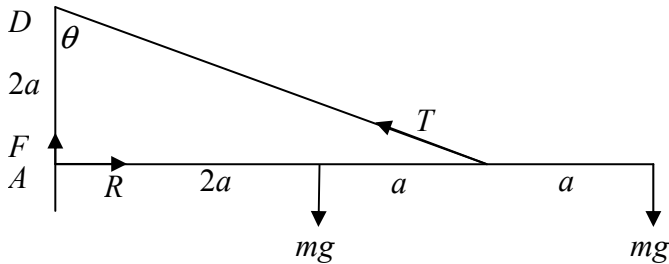
Question Number	Scheme	Marks
<p>Q2</p>	 <p>(a) K.E. gained = $\frac{1}{2} \times 0.6 \times 4^2$ P.E. lost = $0.6 \times g \times (12 \sin 30)$ Change in energy = P.E. lost - K.E. gained $= 0.6 \times g \times 12 \sin 30 - \frac{1}{2} \times 0.6 \times 4^2$ $= 30.48$ Work done against friction = 30 or 30.5 J</p>	<p>M1 A1 A1 A1 (4)</p>
<p>(b)</p>	<p>R (\uparrow) $R = 0.6g \cos 30$ $F = \frac{30.48}{12}$ $F = \mu R$ $\mu = \frac{30.48}{12 \times 0.6g \cos 30}$ $\mu = 0.4987$ $\mu = 0.499$ or 0.50</p>	<p>B1 B1ft M1 A1 (4) [8]</p>

Question Number	Scheme					Marks
Q3						
(a)		AB	AC	BC	frame	
	mass ratio	10	10	12	32	B1
	dist. from BC	4	4	0	\bar{x}	B1
	<p>Moments about BC:</p> $10 \times 4 + 10 \times 4 + 0 = 32\bar{x}$ $\bar{x} = \frac{80}{32}$ $\bar{x} = 2\frac{1}{2} \quad (2.5)$					<p>M1 A1</p> <p>A1 (5)</p>
(b)	 <p>Moments about B:</p> $Mg \times 6 \sin \theta = Mg \times (\bar{x} \cos \theta - 6 \sin \theta)$ $12 \sin \theta = \bar{x} \cos \theta$ $\tan \theta = \frac{\bar{x}}{12}$ $\theta = 11.768\dots = 11.8^\circ$ <p>Alternative method : C of M of loaded frame at distance $\frac{1}{2}\bar{x}$ from D along DA</p> $\tan \theta = \frac{\frac{1}{2}\bar{x}}{6}$ $\theta = 11.768\dots = 11.8^\circ$					<p>M1 A1 A1</p> <p>A1 (4)</p> <p>B1</p> <p>M1 A1</p> <p>A1</p> <p>[9]</p>

Question Number	Scheme	Marks
<p>Q4</p> <p>(a)</p>	 <p> $T = \frac{15000}{20} = 750$ R(parallel to road) $T = R + 750g \sin \theta$ $R = 750 - 750 \times 9.8 \times \frac{1}{15}$ $R = 260 *$ </p>	<p>M1</p> <p>M1 A1</p> <p>A1 (4)</p>
<p>(b)</p>	 <p> $T' = \frac{18000}{20} = 900$ $T' - 260 - 750g \times \sin \theta = 750a$ $a = \frac{900 - 260 - 750 \times 9.8 \times \frac{1}{15}}{750}$ $a = 0.2$ </p>	<p>M1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>[8]</p>

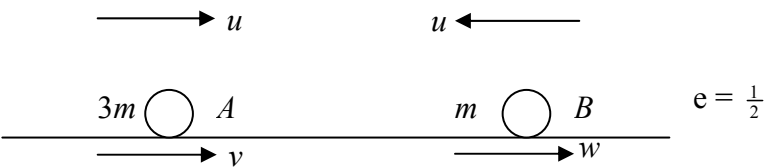
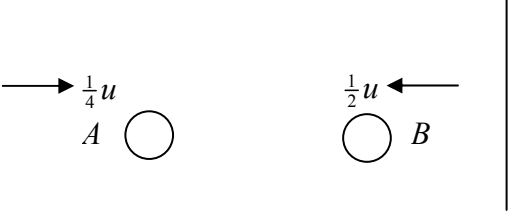
WME02/01: Mechanics 2

Question Number	Scheme	Marks
Q5 (a)	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $= 0.5 \times 20\mathbf{i} - 0.5(10\mathbf{i} + 24\mathbf{j})$ $= 5\mathbf{i} - 12\mathbf{j}$ $ 5\mathbf{i} - 12\mathbf{j} = 13 \text{ Ns}$	M1 A1 M1 A1 (4)
(b)	 $\tan \theta = \frac{12}{5}$ $\theta = 67.38$ $\theta = 67.4^\circ$	M1 A1 (2)
(c)	$\text{K.E. lost} = \frac{1}{2} \times 0.5(10^2 + 24^2) - \frac{1}{2} \times 0.5 \times 20^2$ $= 69 \text{ J}$	M1 A1 A1 (3) [9]

Question Number	Scheme	Marks
<p>Q6</p> <p>(a)</p>	 <p>M(A) $3a \times T \cos \theta = 2amg + 4amg$</p> $\cos \theta = \frac{2}{\sqrt{9+4}} = \frac{2}{\sqrt{13}}$ $\frac{6}{\sqrt{13}}T = 6mg$ $T = mg\sqrt{13} \quad *$	<p>M1 A1 A1 B1 A1 (5)</p>
<p>(b)</p>	$3a \times T \times \cos \theta = 2amg + 4aMg$ $T = \frac{(2mg + 4Mg)\sqrt{13}}{6} \leq 2mg\sqrt{13}$ $mg + 2Mg \leq 6mg$ $M \leq \frac{5}{2}m \quad *$	<p>M1 A1 CSO A1 (3) [8]</p>

WME02/01: Mechanics 2

Question Number	Scheme	Marks
<p>Q7</p> <p>(a)</p>	<p>Vertical motion: $v^2 = u^2 + 2as$ $(40 \sin \theta)^2 = 2 \times g \times 12$ $(\sin \theta)^2 = \frac{2 \times g \times 12}{40^2}$ $\theta = 22.54 = 22.5^\circ$ (accept 23)</p>	<p>M1 A1</p> <p>A1 (3)</p>
<p>(b)</p>	<p>Vert motion $P \rightarrow R$: $s = ut + \frac{1}{2}at^2$ $-36 = 40 \sin \theta t - \frac{g}{2}t^2$ $\frac{g}{2}t^2 - 40 \sin \theta t - 36 = 0$ $t = \frac{40 \sin 22.54 \pm \sqrt{(40 \sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$ $t = 4.694\dots$</p> <p>Horizontal P to R: $s = 40 \cos \theta t$ $= 173 \text{ m}$ (or 170 m)</p>	<p>M1 A1 A1</p> <p>A1</p> <p>M1 A1 (6)</p>
<p>(c)</p>	<p>Using Energy: $\frac{1}{2}mv^2 - \frac{1}{2}m \times 40^2 = m \times g \times 36$ $v^2 = 2(9.8 \times 36 + \frac{1}{2} \times 40^2)$ $v = 48.0\dots\dots$ $v = 48 \text{ m s}^{-1}$ (accept 48.0)</p>	<p>M1 A1</p> <p>A1 (3)</p> <p>[12]</p>

Question Number	Scheme	Marks
<p>Q8</p> <p>(a)</p> <p>(i)</p>	 <p>Con. of Mom: $3mu - mu = 3mv + mw$ $2u = 3v + w$ (1)</p> <p>N.L.R: $\frac{1}{2}(u + u) = w - v$ $u = w - v$ (2)</p> <p>(1) - (2) $u = 4v$ $v = \frac{1}{4}u$</p> <p>(ii) In (2) $u = w - \frac{1}{4}u$ $w = \frac{5}{4}u$</p>	<p>M1# A1 M1# A1 DM1# A1 A1 (7)</p>
<p>(b)</p>	<p>B to wall: N.L.R: $\frac{5}{4}u \times \frac{2}{5} = V$ $V = \frac{1}{2}u$</p>	<p>M1 A1ft (2)</p>
<p>(c)</p>	 <p>B to wall: $\text{time} = 4a \div \frac{5}{4}u = \frac{16a}{5u}$</p> <p>Dist. Travelled by A = $\frac{1}{4}u \times \frac{16a}{5u} = \frac{4}{5}a$</p> <p>In t secs, A travels $\frac{1}{4}ut$, B travels $\frac{1}{2}ut$</p> <p>Collide when speed of approach = $\frac{1}{2}ut + \frac{1}{4}ut$, distance to cover = $4a - \frac{4}{5}a$</p> <p>$\therefore t = \frac{4a - \frac{4}{5}a}{\frac{3}{4}u} = \frac{16a}{5} \times \frac{4}{3u} = \frac{64a}{15u}$</p> <p>Total time = $\frac{16a}{5u} + \frac{64a}{15u} = \frac{112a}{15u}$ *</p>	<p>B1ft B1ft M1\$ DM1\$ A1 A1 (6)</p> <p style="text-align: right;">15</p>

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**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Mechanics M3

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference

WME03/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

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1.

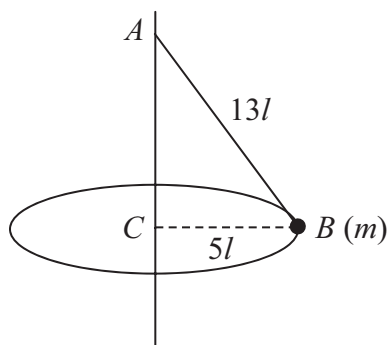


Figure 1

A garden game is played with a small ball B of mass m attached to one end of a light inextensible string of length $13l$. The other end of the string is fixed to a point A on a vertical pole as shown in Figure 1. The ball is hit and moves with constant speed in a horizontal circle of radius $5l$ and centre C , where C is vertically below A . Modelling the ball as a particle, find

(a) the tension in the string, (3)

(b) the speed of the ball. (4)

2. A particle P of mass m is above the surface of the Earth at distance x from the centre of the Earth. The Earth exerts a gravitational force on P . The magnitude of this force is inversely proportional to x^2 .

At the surface of the Earth the acceleration due to gravity is g . The Earth is modelled as a sphere of radius R .

- (a) Prove that the magnitude of the gravitational force on P is $\frac{mgR^2}{x^2}$. (3)

A particle is fired vertically upwards from the surface of the Earth with initial speed $3U$. At a height R above the surface of the Earth the speed of the particle is U .

- (b) Find U in terms of g and R . (7)

Question 2 continued

3.

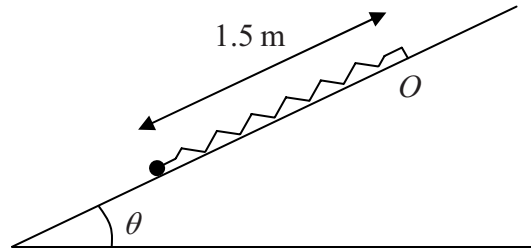


Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15 . The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O , as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ .

(9)

4.

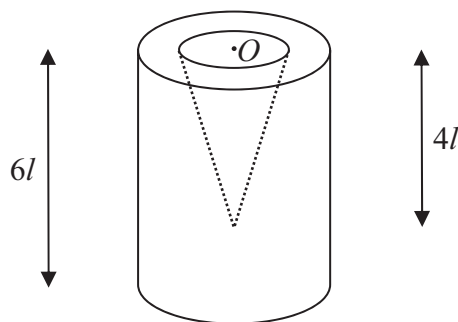


Figure 3

A container is formed by removing a right circular solid cone of height $4l$ from a uniform solid right circular cylinder of height $6l$. The centre O of the plane face of the cone coincides with the centre of a plane face of the cylinder and the axis of the cone coincides with the axis of the cylinder, as shown in Figure 3. The cylinder has radius $2l$ and the base of the cone has radius l .

(a) Find the distance of the centre of mass of the container from O .

(6)

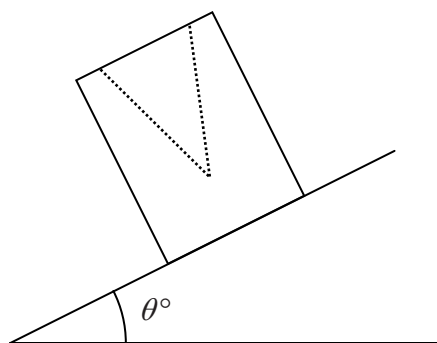


Figure 4

The container is placed on a plane which is inclined at an angle θ° to the horizontal. The open face is uppermost, as shown in Figure 4. The plane is sufficiently rough to prevent the container from sliding. The container is on the point of toppling.

(b) Find the value of θ .

(4)

Question 4 continued

A series of horizontal lines for writing, starting from the top of the main area and extending to the bottom, providing space for the answer to Question 4.

Question 4 continued

[Lined area for writing]

(Total 10 marks)

Q4

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5.

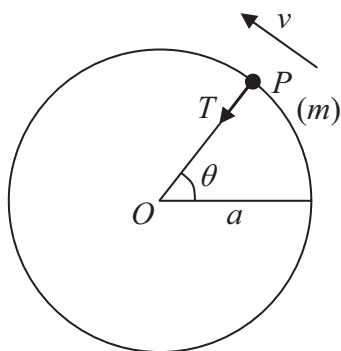


Figure 5

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is fixed at the point O . The particle is initially held with OP horizontal and the string taut. It is then projected vertically upwards with speed u , where $u^2 = 5ag$. When OP has turned through an angle θ the speed of P is v and the tension in the string is T , as shown in Figure 5.

- (a) Find, in terms of a , g and θ , an expression for v^2 . (3)
- (b) Find, in terms of m , g and θ , an expression for T . (4)
- (c) Prove that P moves in a complete circle. (3)
- (d) Find the maximum speed of P . (2)

Question 6 continued

7. A light elastic string, of natural length $3a$ and modulus of elasticity $6mg$, has one end attached to a fixed point A . A particle P of mass $2m$ is attached to the other end of the string and hangs in equilibrium at the point O , vertically below A .

(a) Find the distance AO .

(3)

The particle is now raised to point C vertically below A , where $AC > 3a$, and is released from rest.

(b) Show that P moves with simple harmonic motion of period $2\pi\sqrt{\frac{a}{g}}$.

(5)

It is given that $OC = \frac{1}{4}a$.

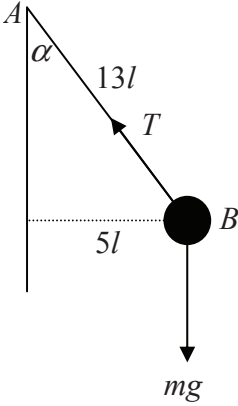
(c) Find the greatest speed of P during the motion.

(3)

The point D is vertically above O and $OD = \frac{1}{8}a$. The string is cut as P passes through D , moving upwards.

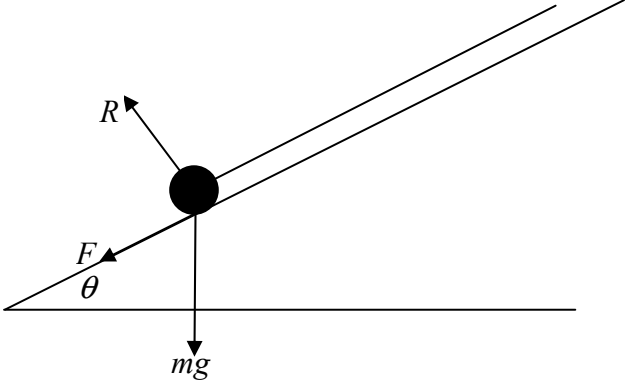
(d) Find the greatest height of P above O in the subsequent motion.

(4)

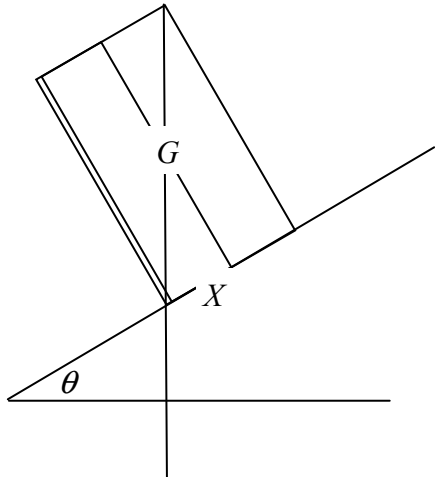
Question Number	Scheme	Marks
<p>Q1</p> <p>(a)</p>	 <p style="text-align: center;"> $\cos \alpha = \frac{12}{13}$ $R(\uparrow) \quad T \cos \alpha = mg$ $T \times \frac{12}{13} = mg$ $T = \frac{13}{12} mg \quad \text{oe}$ </p>	<p>B1</p> <p>M1</p> <p>A1 (3)</p>
<p>(b)</p>	<p>Eqn of motion $T \sin \alpha = m \frac{v^2}{5l}$</p> <p>$\frac{13mg}{12} \times \frac{5}{13} = m \frac{v^2}{5l}$</p> <p>$v^2 = \frac{25gl}{12}$</p> <p>$v = \frac{5}{2} \sqrt{\frac{gl}{3}} \quad \left(\text{accept } 5\sqrt{\frac{gl}{12}} \text{ or } \sqrt{\frac{25gl}{12}} \text{ or any other equiv} \right)$</p>	<p>M1 A1</p> <p>M1 dep</p> <p>A1 (4)</p> <p>[7]</p>

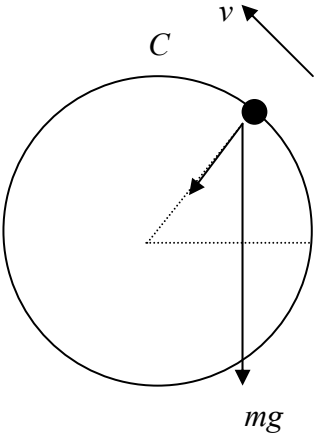
WME03/01: Mechanics M3

Question Number	Scheme	Marks
Q2 (a)	$F = (-)\frac{k}{x^2}$ $mg = (-)\frac{k}{R^2}$ $F = \frac{mgR^2}{x^2} *$	M1 M1 A1 (3)
(b)	$m\ddot{x} = -\frac{mgR^2}{x^2}$ $v \frac{dv}{dx} = -\frac{gR^2}{x^2}$ $\frac{1}{2}v^2 = \int \left(-\frac{gR^2}{x^2}\right) dx$ $\frac{1}{2}v^2 = \frac{gR^2}{x} (+c)$ $x = R, v = 3U \quad \frac{9U^2}{2} = gR + c$ $\frac{1}{2}v^2 = \frac{gR^2}{x} + \frac{9U^2}{2} - gR$ $x = 2R, v = U \quad \frac{1}{2}U^2 = \frac{gR^2}{2R} + \frac{9U^2}{2} - gR$ $U^2 = \frac{gR}{8}$ $U = \sqrt{\frac{gR}{8}}$	M1 M1 M1 dep on 1st M mark A1 M1 dep on 3rd M mark M1 dep on 3rd M mark A1 (7) [10]

Question Number	Scheme	Marks
Q3	 <p> $\text{EPE lost} = \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} \left(= \frac{7}{36} \lambda \right)$ $R(\uparrow) \quad R = mg \cos \theta$ $= 0.5g \times \frac{4}{5} = 0.4g$ $F = \mu R = 0.15 \times 0.4g$ $\text{P.E. gained} = \text{E.P.E. lost} - \text{work done against friction}$ $0.5g \times 0.7 \sin \theta = \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} - 0.15 \times 0.4g \times 0.7$ $0.1944\lambda = 0.5 \times 9.8 \times 0.7 \times \frac{3}{5} + 0.15 \times 0.4 \times 9.8 \times 0.7$ $\lambda = 12.70 \dots\dots$ $\lambda = 13 \text{ N} \quad \text{or } 12.7$ </p>	<p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1 A1 A1</p> <p>A1</p> <p style="text-align: right;">[9]</p>

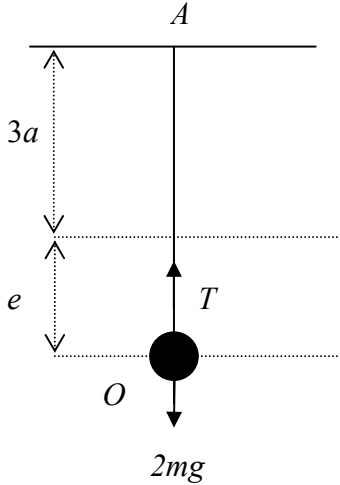
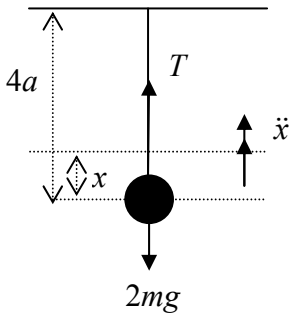
WME03/01: Mechanics M3

Question Number	Scheme			Marks	
Q4 (a)		cone	container	cylinder	M1 A1 B1 M1 A1ft A1 (6)
	mass ratio	$\frac{4\pi l^3}{3}$	$\frac{68\pi l^3}{3}$	$24\pi l^3$	
		4	68	72	
	dist from O	l	\bar{x}	$3l$	
Moments: $4l + 68\bar{x} = 72 \times 3l$ $\bar{x} = \frac{212l}{68} = \frac{53}{17}l$ accept $3.12l$					
(b)	 <p>$GX = 6l - \bar{x}$ seen</p> $\tan \theta = \frac{2l}{6l - \bar{x}}$ $= \frac{2 \times 17}{49}$ $\theta = 34.75\dots = 34.8 \text{ or } 35$			M1 M1 A1 A1 (4) [10]	

Question Number	Scheme	Marks
Q5		
(a)	Energy: $mga \sin \theta = \frac{1}{2} m \times 5ag - \frac{1}{2} mv^2$ $v^2 = 5ag - 2ag \sin \theta$	M1 A1 A1 (3)
(b)	Eqn of motion along radius: $T + mg \sin \theta = \frac{mv^2}{a}$ $T = \frac{m}{a}(5ag - 2ag \sin \theta) - mg \sin \theta$ $T = mg(5 - 3 \sin \theta)$	M1 A1 M1 A1 (4)
(c)	At C, $\theta = 90^\circ$ $T = mg(5 - 3) = 2mg$ $T > 0 \therefore P$ reaches C	M1 A1 A1 (3)
(d)	Max speed at lowest point $(\theta = 270^\circ; \quad v^2 = 5ag - 2ag \sin 270)$ $v^2 = 5ag + 2ag$ $v = \sqrt{7ag}$	M1 A1 (2) [12]

WME03/01: Mechanics M3

Question Number	Scheme	Marks
Q6 (a)	$\frac{d^2x}{dt^2} = -\frac{3}{(t+1)^2}$ $\frac{dx}{dt} = \int -3(t+1)^{-2} dt$ $= 3(t+1)^{-1} (+c)$ <p>$t=0, v=2 \quad 2=3+c \quad c=-1$</p> $\frac{dx}{dt} = \frac{3}{t+1} - 1 \quad *$	<p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (5)</p>
(b)	$x = \int \left(\frac{3}{t+1} - 1 \right) dt$ $= 3\ln(t+1) - t \quad (+c')$ <p>$t=0, x=0 \Rightarrow c'=0$</p> $x = 3\ln(t+1) - t$ $v=0 \Rightarrow \frac{3}{t+1} = 1$ $t=2$ $x = 3\ln 3 - 2$ $= 1.295\dots$ $= 1.30 \text{ m (Allow 1.3)}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (7)</p> <p>[12]</p>

Question Number	Scheme	Marks
<p>Q7</p> <p>(a)</p>	 <p> $R(\uparrow) \quad T = 2mg$ Hooke's law: $T = \frac{6mge}{3a}$ $2mg = \frac{6mge}{3a}$ $e = a$ $AO = 4a$ </p>	<p>B1</p> <p>M1</p> <p>A1 (3)</p>
<p>(b)</p>	 <p> H.L. Eqn. of motion $T = \frac{6mg(a-x)}{3a} = \frac{2mg(a-x)}{a}$ $-2mg + T = 2m\ddot{x}$ $-2mg + \frac{2mg(a-x)}{a} = 2m\ddot{x}$ $-\frac{2mgx}{a} = 2m\ddot{x}$ $\ddot{x} = -\frac{g}{a}x$ period $2\pi\sqrt{\frac{a}{g}}$ * </p>	<p>B1ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (5)</p>

WME03/01: Mechanics M3

Question Number	Scheme	Marks
(c)	$v^2 = \omega^2 (a^2 - x^2)$ $v_{\max}^2 = \frac{g}{a} \left(\left(\frac{a}{4} \right)^2 - 0 \right)$ $v_{\max} = \frac{1}{4} \sqrt{ga}$	<p>M1 A1</p> <p>A1 (3)</p>
(d)	$x = -\frac{a}{8}$ $v^2 = \frac{g}{a} \left(\frac{a^2}{16} - \frac{a^2}{64} \right)$ $= \frac{3ag}{64}$ $v^2 = u^2 + 2as$ $0 = \frac{3ag}{64} - 2gh$ $h = \frac{3a}{128}$ <p>Total height above $O = \frac{a}{8} + \frac{3a}{128} = \frac{19a}{128}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (4)</p> <p>[15]</p>

Write your name here

Surname	Other names
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**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Statistics S1

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference
WST01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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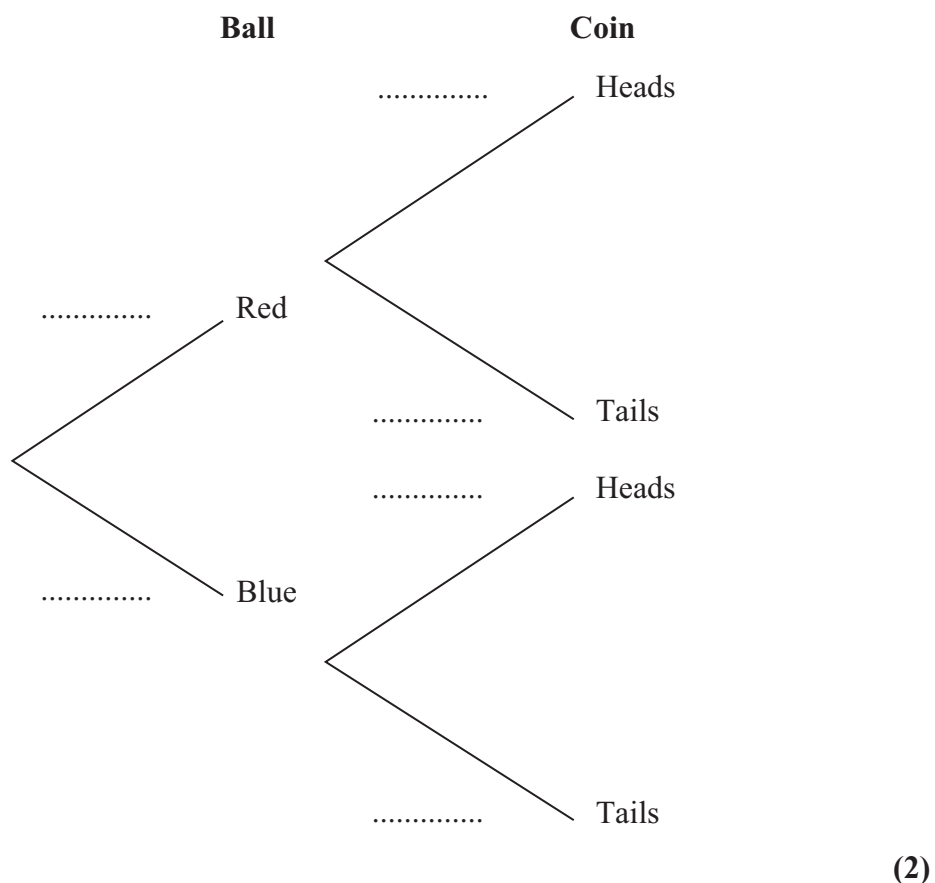
PEARSON

2. An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its colour is noted and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun.

When a blue ball is selected a fair coin is spun.

- (a) Complete the tree diagram below to show the possible outcomes and associated probabilities.



Shivani selects a ball and spins the appropriate coin.

- (b) Find the probability that she obtains a head. **(2)**

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

- (c) find the probability that Tom selected a red ball. **(3)**

Shivani and Tom each repeat this experiment.

- (d) Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects. **(3)**

3. The discrete random variable X has probability distribution given by

x	-1	0	1	2	3
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{10}$	a	$\frac{1}{5}$

where a is a constant.

(a) Find the value of a . **(2)**

(b) Write down $E(X)$. **(1)**

(c) Find $\text{Var}(X)$. **(3)**

The random variable $Y = 6 - 2X$

(d) Find $\text{Var}(Y)$. **(2)**

(e) Calculate $P(X \geq Y)$. **(3)**

Question 3 continued

Lined area for student response.

4. The Venn diagram in Figure 1 shows the number of students in a class who read any of 3 popular magazines *A*, *B* and *C*.

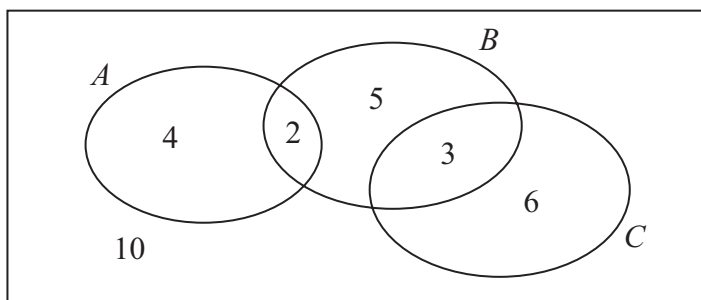


Figure 1

One of these students is selected at random.

- (a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$. **(2)**
- (b) Find the probability that the student reads *A* or *B* (or both). **(2)**
- (c) Write down the probability that the student reads both *A* and *C*. **(1)**

Given that the student reads at least one of the magazines,

- (d) find the probability that the student reads *C*. **(2)**
- (e) Determine whether or not reading magazine *B* and reading magazine *C* are statistically independent. **(3)**

5. A teacher selects a random sample of 56 students and records, to the nearest hour, the time spent watching television in a particular week.

Hours	1–10	11–20	21–25	26–30	31–40	41–59
Frequency	6	15	11	13	8	3
Mid-point	5.5	15.5		28		50

- (a) Find the mid-points of the 21–25 hour and 31–40 hour groups. **(2)**

A histogram was drawn to represent these data. The 11–20 group was represented by a bar of width 4 cm and height 6 cm.

- (b) Find the width and height of the 26–30 group. **(3)**

- (c) Estimate the mean and standard deviation of the time spent watching television by these students. **(5)**

- (d) Use linear interpolation to estimate the median length of time spent watching television by these students. **(2)**

The teacher estimated the lower quartile and the upper quartile of the time spent watching television to be 15.8 and 29.3 respectively.

- (e) State, giving a reason, the skewness of these data. **(2)**

6. A travel agent sells flights to different destinations from *Beerow* airport. The distance d , measured in 100 km, of the destination from the airport and the fare $\pounds f$ are recorded for a random sample of 6 destinations.

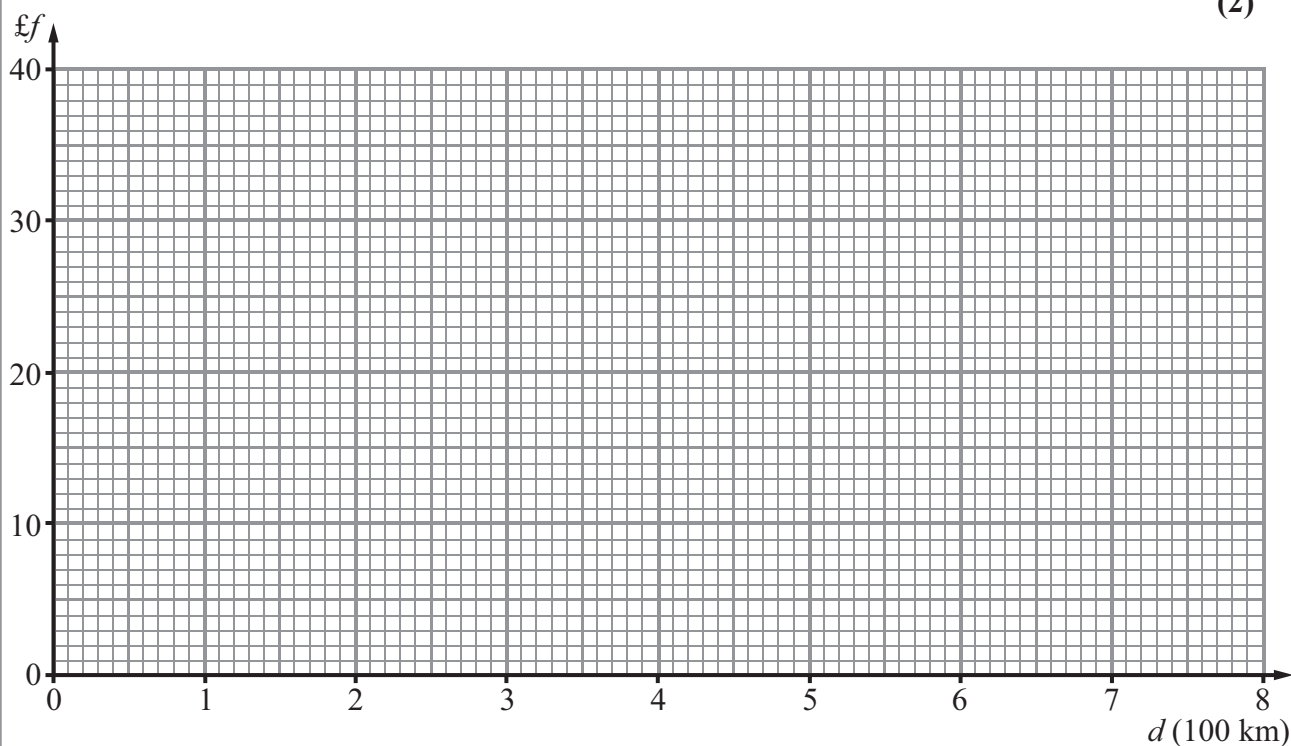
Destination	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
d	2.2	4.0	6.0	2.5	8.0	5.0
f	18	20	25	23	32	28

[You may use $\sum d^2 = 152.09$ $\sum f^2 = 3686$ $\sum fd = 723.1$]

- (a) Using the axes below, complete a scatter diagram to illustrate this information. (2)
- (b) Explain why a linear regression model may be appropriate to describe the relationship between f and d . (1)
- (c) Calculate S_{dd} and S_{fd} (4)
- (d) Calculate the equation of the regression line of f on d giving your answer in the form $f = a + bd$. (4)
- (e) Give an interpretation of the value of b . (1)

Jane is planning her holiday and wishes to fly from *Beerow* airport to a destination t km away. A rival travel agent charges 5p per km.

- (f) Find the range of values of t for which the first travel agent is cheaper than the rival. (2)



Question 6 continued

7. The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim N(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km. **(3)**

(b) Find the upper quartile, Q_3 , of D . **(3)**

(c) Write down the lower quartile, Q_1 , of D . **(1)**

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5 \times (Q_3 - Q_1) \quad \text{and} \quad k = Q_3 + 1.5 \times (Q_3 - Q_1)$$

(d) Find the value of h and the value of k . **(2)**

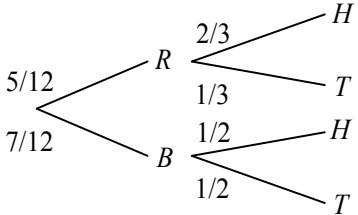
An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier. **(3)**

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WST01/01: Statistics S1

Question Number	Scheme	Marks
<p>Q1</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	$r = \frac{8825}{\sqrt{1022500 \times 130.9}}, \quad = \text{awrt } \underline{\underline{0.763}}$ <p>Teams with high attendance scored more goals (oe, statement in context)</p> <p>0.76(3)</p>	<p>M1 A1 (2)</p> <p>B1 (1)</p> <p>B1ft (1)</p> <p>Total 4</p>
	<p>(a) M1 for a correct expression, square root required Correct answer award 2/2</p> <p>(b) Context required (attendance and goals). Condone causality. B0 for 'strong positive correlation between attendance and goals' on its own oe</p> <p>(c) Value required. Must be a correlation coefficient between -1 and +1 inclusive. B1ft for 0.76 or better or same answer as their value from part (a) to at least 2 d.p.</p>	

Question Number	Scheme	Marks
<p>Q2 (a)</p> <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin-left: 20px;"> <p>P(R) and P(B)</p> <p>2nd set of probabilities</p> </div> </div> <p>(b) $P(H) = \frac{5}{12} \times \frac{2}{3} + \frac{7}{12} \times \frac{1}{2} = \frac{41}{72}$ or awrt 0.569</p> <p>(c) $P(R H) = \frac{\frac{5}{12} \times \frac{2}{3}}{\frac{41}{72}} = \frac{20}{41}$ or awrt 0.488</p> <p>(d) $\left(\frac{5}{12}\right)^2 + \left(\frac{7}{12}\right)^2 = \frac{25}{144} + \frac{49}{144} = \frac{74}{144}$ or $\frac{37}{72}$ or awrt 0.514</p>	<p>B1</p> <p>B1</p> <p>(2)</p> <p>M1 A1</p> <p>(2)</p> <p>M1 A1ft A1</p> <p>(3)</p> <p>M1 A1ft</p> <p>A1</p> <p>(3)</p> <p style="text-align: right;">Total 10</p>	
<p>(a)</p> <p>1st B1 for the probabilities on the first 2 branches. Accept 0.416̇ and 0.583̇</p> <p>2nd B1 for probabilities on the second set of branches. Accept 0.6̇, 0.3̇, 0.5 and $\frac{1.5}{3}$</p> <p>Allow exact decimal equivalents using clear recurring notation if required.</p> <p>(b)</p> <p>M1 for an expression for P(H) that follows through their sum of two products of probabilities from their tree diagram</p> <p>(c)</p> <p>Formula seen M1 for $\frac{P(R \cap H)}{P(H)}$ with denominator their (b) substituted e.g. $\frac{P(R \cap H)}{P(H)} = \frac{\frac{5}{12}}{\text{(their (b))}}$ award M1.</p> <p>Formula not seen M1 for $\frac{\text{probability} \times \text{probability}}{\text{their } b}$ but M0 if fraction repeated e.g. $\frac{\frac{5}{12} \times \frac{2}{3}}{\frac{2}{3}}$.</p> <p>1st A1ft for a fully correct expression or correct follow through</p> <p>2nd A1 for $\frac{20}{41}$ o.e.</p> <p>(d)</p> <p>M1 for $\left(\frac{5}{12}\right)^2$ or $\left(\frac{7}{12}\right)^2$ can follow through their equivalent values from tree diagram</p> <p>1st A1 for both values correct or follow through from their original tree and +</p> <p>2nd A1 for a correct answer</p> <p>Special Case $\frac{5}{12} \times \frac{4}{11}$ or $\frac{7}{12} \times \frac{6}{11}$ seen award M1A0A0</p>		

WST01/01: Statistics S1

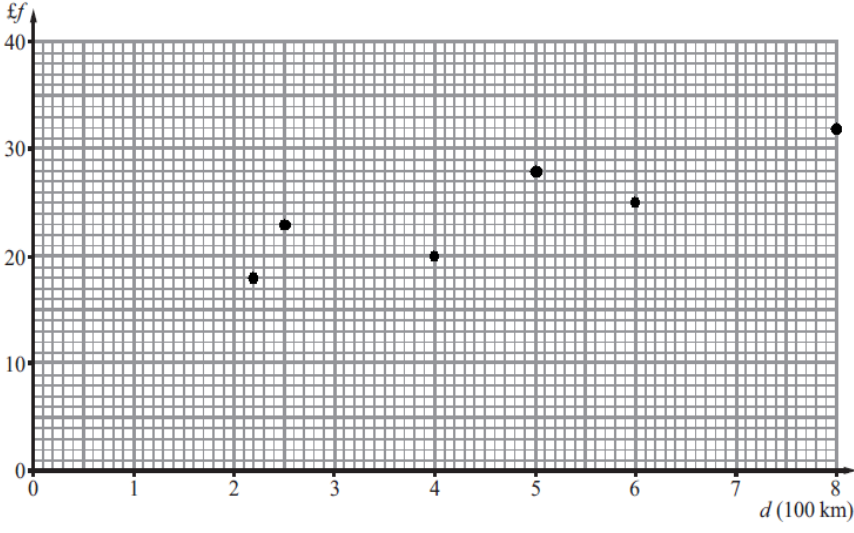
Question Number	Scheme	Marks
<p>Q3 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	$2a + \frac{2}{5} + \frac{1}{10} = 1$ <p style="text-align: right;">(or equivalent)</p> $a = \frac{1}{4} \text{ or } \underline{\underline{0.25}}$ <p>$E(X) = \underline{1}$</p> <p>$E(X^2) = 1 \times \frac{1}{5} + 1 \times \frac{1}{10} + 4 \times \frac{1}{4} + 9 \times \frac{1}{5} \quad (= 3.1)$</p> <p>$\text{Var}(X) = 3.1 - 1^2, \quad = \underline{\underline{2.1}} \text{ or } \frac{21}{10} \text{ oe}$</p> <p>$\text{Var}(Y) = (-2)^2 \text{Var}(X), \quad = \underline{\underline{8.4}} \text{ or } \frac{42}{5} \text{ oe}$</p> <p>$X \geq Y$ when $X = 3$ or 2, so probability = $\frac{1}{4} + \frac{1}{5}$</p> <p>$= \underline{\underline{\frac{9}{20}}}$ oe</p>	<p>M1</p> <p>A1 (2)</p> <p>B1 (1)</p> <p>M1</p> <p>M1 A1 (3)</p> <p>M1 A1 (2)</p> <p>M1 A1ft</p> <p>A1 (3)</p> <p>Total 11</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>M1 for a clear attempt to use $\sum P(X = x) = 1$ Correct answer only 2/2. NB Division by 5 in parts (b), (c) and (d) seen scores 0. Do not apply ISW.</p> <p>B1 for 1</p> <p>1st M1 for attempting $\sum x^2 P(X = x)$ at least two terms correct. Can follow through. 2nd M1 for attempting $E(X^2) - [E(X)]^2$ or allow subtracting 1 from their attempt at $E(X^2)$ provided no incorrect formula seen. Correct answer only 3/3.</p> <p>M1 for $(-2)^2 \text{Var}(X)$ or $4\text{Var}(X)$ Condone missing brackets provided final answer correct for their $\text{Var}(X)$. Correct answer only 2/2.</p> <p>Allow M1 for distribution of $Y = 6 - 2X$ and correct attempt at $E(Y^2) - [E(Y)]^2$ M1 for identifying $X = 2, 3$ 1st A1ft for attempting to find their $P(X=2) + P(X=3)$ 2nd A1 for $\frac{9}{20}$ or 0.45</p>	

WST01/01: Statistics S1

Question Number	Scheme	Marks
<p>Q4</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>$\frac{2+3}{\text{their total}} = \frac{5}{6} = \frac{1}{6}$ (** given answer**)</p> <p>$\frac{4+2+5+3}{\text{total}}, = \frac{14}{30}$ or $\frac{7}{15}$ or 0.46</p> <p>$P(A \cap C) = 0$</p> <p>$P(C \text{reads at least one magazine}) = \frac{6+3}{20} = \frac{9}{20}$</p> <p>$P(B) = \frac{10}{30} = \frac{1}{3}, P(C) = \frac{9}{30} = \frac{3}{10}, P(B \cap C) = \frac{3}{30} = \frac{1}{10}$ or $P(B C) = \frac{3}{9}$</p> <p>$P(B) \times P(C) = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = P(B \cap C)$ or $P(B C) = \frac{3}{9} = \frac{1}{3} = P(B)$</p> <p>So yes they are statistically independent</p>	<p>M1 A1cso (2)</p> <p>M1 A1 (2)</p> <p>B1 (1)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1cso (3)</p> <p>Total 10</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>M1 for $\frac{2+3}{\text{their total}}$ or $\frac{5}{30}$</p> <p>M1 for adding at least 3 of “4, 2, 5, 3” and dividing by their total to give a probability Can be written as separate fractions substituted into the completely correct Addition Rule</p> <p>B1 for 0 or 0/30</p> <p>M1 for a denominator of 20 or $\frac{20}{30}$ leading to an answer with denominator of 20 $\frac{9}{20}$ only, 2/2</p> <p>1st M1 for attempting all the required probabilities for a suitable test 2nd M1 for use of a correct test - must have attempted all the correct probabilities. Equality can be implied in line 2. A1 for fully correct test carried out with a comment</p>	

WST01/01: Statistics S1

Question Number	Scheme	Marks
<p>Q5 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>23, 35.5 (may be in the table)</p> <p>Width of 10 units is 4 cm so width of 5 units is 2 cm</p> <p>Height = $2.6 \times 4 = \mathbf{10.4 \text{ cm}}$</p> <p>$\sum fx = 1316.5 \Rightarrow \bar{x} = \frac{1316.5}{56} =$ awrt 23.5</p> <p>$\sum fx^2 = 37378.25$ can be implied</p> <p>So $\sigma = \sqrt{\frac{37378.25}{56} - \bar{x}^2} =$ awrt 10.7 allow $s = 10.8$</p> <p>$Q_2 = (20.5) + \frac{(28-21)}{11} \times 5 = 23.68\dots$ awrt 23.7 or 23.9</p> <p>$Q_3 - Q_2 = 5.6, Q_2 - Q_1 = 7.9$ (or $\bar{x} < Q_2$)</p> <p>[7.9 > 5.6 so] negative skew</p>	<p>B1 B1 (2)</p> <p>B1</p> <p>M1 A1 (3)</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1 (5)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>Total 14</p>
<p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>M1 for their width x their height=20.8. Without labels assume width first, height second and award marks accordingly.</p> <p>1st M1 for reasonable attempt at $\sum x$ and /56 2nd M1 for a method for σ or s, $\sqrt{\quad}$ is required Typical errors $\sum (fx)^2 = 354806.3$ M0, $\sum f^2x = 13922.5$ M0 and $(\sum fx)^2 = 1733172$ M0 Correct answers only, award full marks.</p> <p>Use of $\sum f(x - \bar{x})^2 =$ awrt 6428.75 for B1 lcb can be 20, 20.5 or 21, width can be 4 or 5 and the fraction part of the formula correct for M1 - Allow 28.5 in fraction that gives awrt 23.9 for M1A1</p> <p>M1 for attempting a test for skewness using quartiles or mean and median. Provided median greater than 22.55 and less than 29.3 award for M1 for $Q_3 - Q_2 < Q_2 - Q_1$ without values as a valid reason. SC Accept mean close to median and no skew oe for M1A1</p>	

Question Number	Scheme	Marks
<p>Q6 (a)</p>		<p>B1 B1 (2)</p>
<p>(b)</p>	<p>The points lie reasonably close to a straight line (o.e.)</p>	<p>B1 (1)</p>
<p>(c)</p>	<p>$\sum d = 27.7, \quad \sum f = 146$ (both, may be implied)</p>	<p>B1</p>
<p>(d)</p>	<p>$S_{dd} = 152.09 - \frac{(27.7)^2}{6} = 24.208\dots$ awrt <u>24.2</u> $S_{fd} = 723.1 - \frac{27.7 \times 146}{6} = 49.06\dots$ awrt <u>49.1</u></p>	<p>M1 A1 A1 (4)</p>
<p>(d)</p>	<p>$b = \frac{S_{fd}}{S_{dd}} = 2.026\dots$ awrt <u>2.03</u> $a = \frac{146}{6} - b \times \frac{27.7}{6} = 14.97\dots$ so <u>$f = 15.0 + 2.03d$</u></p>	<p>M1 A1 (4)</p>
<p>(e)</p>	<p>A flight costs £2.03 (or about £2) for every extra 100km or about 2p per km.</p>	<p>B1ft (1)</p>
<p>(f)</p>	<p>$15.0 + 2.03d < 5d$ so $d > \frac{15.0}{(5 - 2.03)} = 5.00 \sim 5.05$ So $t > 500 \sim 505$</p>	<p>M1 A1 (2)</p>
		<p>Total 14</p>

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(a)	1 st B1 for at least 4 points correct (allow \pm one 2mm square) 2 nd B1 for all points correct (allow \pm one 2 mm square)
(b)	Ignore extra points and lines Require reference to points and line for B1.
(c)	M1 for a correct method seen for either - a correct expression 1 st A1 for S_{dd} awrt 24.2 2 nd A1 for S_{fd} awrt 49.1
(d)	1 st M1 for a correct expression for b - can follow through their answers from (c) 2 nd M1 for a correct method to find a - follow through their b and their means 2 nd A1 for $f = \dots$ in terms of d and all values awrt given expressions. Accept 15 as rounding from correct answer only.
(e)	Context of cost and distance required. Follow through their value of b
(f)	M1 for an attempt to find the intersection of the 2 lines. Value of t in range 500 to 505 seen award M1. Value of d in range 5 to 5.05 award M1. Accept t greater than 500 to 505 inclusive to include graphical solution for M 1A1

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Question Number	Scheme	Marks
<p>Q7 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	$P(D > 20) = P\left(Z > \frac{20-30}{8}\right)$ $= P(Z > -1.25)$ $= \underline{\underline{0.8944}} \quad \text{awrt } \underline{\underline{0.894}}$ $P(D < Q_3) = 0.75 \text{ so } \frac{Q_3 - 30}{8} = 0.67$ $Q_3 = \text{awrt } \underline{\underline{35.4}}$ $35.4 - 30 = 5.4 \text{ so } Q_1 = 30 - 5.4 = \text{awrt } \underline{\underline{24.6}}$ $Q_3 - Q_1 = 10.8 \text{ so } 1.5(Q_3 - Q_1) = 16.2 \text{ so } Q_1 - 16.2 = h \text{ or } Q_3 + 16.2 = k$ $h = \underline{\underline{8.4 \text{ to } 8.6}} \text{ and } k = \underline{\underline{51.4 \text{ to } 51.6}} \quad \text{both}$ $2P(D > 51.6) = 2P(Z > 2.7)$ $= 2[1 - 0.9965] = \text{awrt } \underline{\underline{0.007}}$	<p>M1</p> <p>A1</p> <p>A1 (3)</p> <p>M1 B1</p> <p>A1 (3)</p> <p>B1ft (1)</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>M1 A1 (3)</p> <p>Total 12</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>M1 for an attempt to standardise 20 or 40 using 30 and 8.</p> <p>1st A1 for $z = \pm 1.25$</p> <p>2nd A1 for awrt 0.894</p> <p>M1 for $\frac{Q_3 - 30}{8} = \text{to a } z \text{ value}$</p> <p>M0 for 0.7734 on RHS.</p> <p>B1 for (z value) between 0.67~0.675 seen.</p> <p>M1B0A1 for use of $z = 0.68$ in correct expression with awrt 35.4</p> <p>Follow through using their of quartile values.</p> <p>M1 for an attempt to calculate 1.5(IQR) and attempt to add or subtract using one of the formulae given in the question - follow through their quartiles</p> <p>1st M1 for attempting $2P(D > \text{their } k)$ or $(P(D > \text{their } k) + P(D < \text{their } h))$</p> <p>2nd M1 for standardising their h or k (may have missed the 2) so allow for standardising $P(D > 51.6)$ or $P(D < 8.4)$</p> <p>Require boths Ms to award A mark.</p>	

Write your name here

Surname	Other names
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**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Statistics S2

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference
WST02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

S45008A

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PEARSON

2. Bhim and Joe play each other at badminton and for each game, independently of all others, the probability that Bhim loses is 0.2

Find the probability that, in 9 games, Bhim loses

(a) exactly 3 of the games, (3)

(b) fewer than half of the games. (2)

Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05

Bhim and Joe agree to play a further 60 games.

(c) Calculate the mean and variance for the number of these 60 games that Bhim loses. (2)

(d) Using a suitable approximation calculate the probability that Bhim loses more than 4 games. (3)

Question 3 continued

Lined area for writing the answer to Question 3.

(Total 5 marks)

Q3

4. The lifetime, X , in tens of hours, of a battery has a cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{4}{9}(x^2 + 2x - 3) & 1 \leq x \leq 1.5 \\ 1 & x > 1.5 \end{cases}$$

(a) Find the median of X , giving your answer to 3 significant figures. (3)

(b) Find, in full, the probability density function of the random variable X . (3)

(c) Find $P(X \geq 1.2)$ (2)

A camping lantern runs on 4 batteries, all of which must be working. Four new batteries are put into the lantern.

(d) Find the probability that the lantern will still be working after 12 hours. (2)

5. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case. (1)

Find the probability that, in a randomly chosen **2 hour** period,

(b) (i) all users connect at their first attempt,
 (ii) at least 4 users fail to connect at their first attempt. (5)

The company suffered from a virus infecting its computer system. During this infection it was found that the number of users failing to connect at their first attempt, over a 12 hour period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users per hour who failed to connect at their first attempt had increased. Use a 5% level of significance and state your hypotheses clearly. (9)

6. A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

(a) Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample. (2)

(b) Using a 5% significance level, find the critical region for a two-tailed test of the hypothesis that the probability of a bolt being faulty is $\frac{1}{4}$. The probability of rejection in either tail should be as close as possible to 0.025 (3)

(c) Find the actual significance level of this test. (2)

In the sample of 50 the actual number of faulty bolts was 8.

(d) Comment on the company’s claim in the light of this value. Justify your answer. (2)

The machine making the bolts was reset and another sample of 50 bolts was taken. Only 5 were found to be faulty.

(e) Test at the 1% level of significance whether or not the probability of a faulty bolt has decreased. State your hypotheses clearly. (6)

7. The random variable Y has probability density function $f(y)$ given by

$$f(y) = \begin{cases} ky(a-y) & 0 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are positive constants.

(a) (i) Explain why $a \geq 3$

(ii) Show that $k = \frac{2}{9(a-2)}$

(6)

Given that $E(Y) = 1.75$

(b) show that $a = 4$ and write down the value of k .

(6)

For these values of a and k ,

(c) sketch the probability density function,

(2)

(d) write down the mode of Y .

(1)

Question 7 continued

Lined area for writing answers to Question 7.

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

Q7

Question Number	Scheme	Marks
Q1	<p>(a) A population is collection of all items</p> <p>(b) (A random variable) that is a function of the sample which contains no unknown quantities/parameters.</p> <p>(c) The voters in the town Percentage/proportion voting for Dr Smith</p> <p>(d) Probability Distribution of those voting for Dr Smith from all possible samples (of size 100)</p>	<p>B1 (1)</p> <p>B1 (1)</p> <p>B1</p> <p>B1 (2)</p> <p>B1 (1)</p> <p>[5]</p>
	<p>Notes</p> <p>(a) B1 – collection/group all items – need to have /imply all eg entire/complete/every</p> <p>(b) B1 – needs <u>function/calculation</u>(o.e.) <u>of the sample/random variables/observations</u> and no unknown quantities/parameters(o.e.) NB do not allow unknown variables e.g. “A calculation based <u>solely</u> on observations from a given sample.” B1 “A calculation based <u>only</u> on known data from a sample” B1 “A calculation based on known observations from a sample” B0</p> <p>(c) B1 – Voters Do not allow 100 voters. B1 – percentage/ proportion voting (for Dr Smith) the number of people voting (for Dr Smith) Allow 35% of people voting (for Dr Smith) Allow 35 people voting (for Dr Smith) Do not allow 35% or 35 alone</p> <p>(d) B1 – answers must include all three of these features (i) All possible samples, (ii) their associated probabilities, (iii) context of voting for Dr Smith. e.g. “It is all possible values of the percentage and their associated probabilities.” B0 no context</p>	<p>Solely/only imply no unknown quantities</p>

Question Number	Scheme	Marks
<p>Q2 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>Let X be the random variable the number of games Bhim loses. $X \sim B(9, 0.2)$</p> <p>$P(X \leq 3) - P(X \leq 2) = 0.9144 - 0.7382$ or $(0.2)^3 (0.8)^6 \frac{9!}{3!6!}$ $= 0.1762$ or $= 0.1762$ awrt 0.176</p> <p>$P(X \leq 4) = 0.9804$ awrt 0.98</p> <p>Mean = 3 variance = $2.85, \frac{57}{20}$</p> <p>Po(3) poisson</p> <p>$P(X > 4) = 1 - P(X \leq 4)$ $= 1 - 0.8153$ $= 0.1847$</p>	<p>B1</p> <p>M1 A1 (3)</p> <p>M1A1 (2)</p> <p>B1 B1 (2)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>[10]</p>
	<p>Notes</p> <p>(a) B1 – writing or use of B(9, 0.2) M1 for writing/ using $P(X \leq 3) - P(X \leq 2)$ or $(p)^3 (1 - p)^6 \frac{9!}{3!6!}$ A1 awrt 0.176</p> <p>(b) M1 for writing or using $P(X \leq 4)$ A1 awrt 0.98</p> <p>(c) B1 3 B1 2.85, or exact equivalent</p> <p>(d) M1 for using Poisson M1 for writing or using $1 - P(X \leq 4)$ NB $P(X \leq 4)$ is 0.7254 Po(3.5) and 0.8912 Po(2.5) A1 awrt 0.185</p> <p>Special case: Use of Po(1.8) in (a) and (b)</p> <p>(a) can get B1 M1 A0 – B1 if written B(9, 0.2), M1 for $\frac{e^{-1.8} 1.8^3}{3!}$ or awrt to 0.161 If B(9, 0.2) is not seen then the only mark available for using Poisson is M1. (b) can get M1 A0 - M1 for writing or using $P(X \leq 4)$ or may be implied by awrt 0.964</p> <p>Use of Normal in (d) Can get M0 M1 A0.- for M1 they must write $1 - P(X \leq 4)$ or get awrt 0.187</p>	

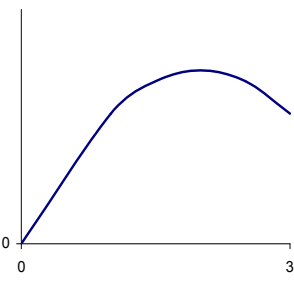
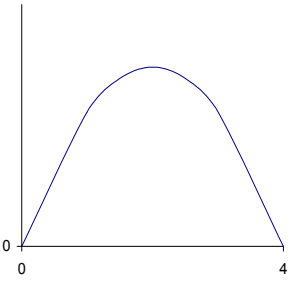
Question Number	Scheme			Marks
Q3	<p>Method 1</p> $P(X > 6) = \frac{1}{6}$ $P(X < 4) = \frac{1}{2}$ $\text{total} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	<p>Method 2</p> $P(4 < X < 6) = \frac{1}{3}$ $1 - \frac{1}{3} = \frac{2}{3}$	<p>Method 3</p> $P(X > 6) = \frac{1}{6}$ $Y \sim U[3,9] \quad P(Y > 6) = \frac{1}{2}$ $\text{total} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	<p>B1 M1 A1 M1dep B A1 (5) [5]</p>
<p>Notes</p> <p>Methods 1 and 2</p> <p>B1 for 6 and 4 (allow if seen on a diagram on x-axis)</p> <p>M1 for $P(X > 6)$ or $P(6 < X < 7)$; or $P(X < 4)$ or $P(1 < X < 4)$; or $P(4 < X < 6)$</p> <p>Allow \leq and \geq signs</p> <p>A1 $\frac{1}{6}$; or $\frac{1}{2}$; $\frac{1}{3}$ must match the probability statement</p> <p>M1 for adding their “$P(X > 6)$” and their “$P(X < 4)$” or 1 - their “$P(4 < X < 6)$” dep on getting first B mark</p> <p>A1 cao $\frac{2}{3}$</p> <p>Method 3 $Y \sim U[3, 9]$</p> <p>B1 for 6 with $U[1,7]$ and 6 with $U[3,9]$</p> <p>M1 for $P(X > 6)$ or $P(6 < X < 7)$ or $P(6 < Y < 9)$</p> <p>A1 $\frac{1}{6}$; or $\frac{1}{2}$; must match the probability statement</p> <p>M1 for adding their “$P(X > 6)$” and their “$P(Y > 6)$” dep on getting first B mark</p> <p>A1 cao $\frac{2}{3}$</p>				

Question Number	Scheme	Marks
Q4 (a)	$\frac{4}{9}(m^2 + 2m - 3) = 0.5$ $m^2 + 2m - 4.125 = 0$ $m = \frac{-2 \pm \sqrt{4 + 16.5}}{2}$ $m = 1.26, -3.264$ (median =) 1.26	M1 M1 A1 (3)
(b)	Differentiating $\frac{d\left(\frac{4}{9}(x^2 + 2x - 3)\right)}{dx} = \frac{4}{9}(2x + 2)$ $f(x) = \begin{cases} \frac{8}{9}(x+1) & 1 \leq x \leq 1.5 \\ 0 & \text{otherwise} \end{cases}$	M1 A1 B1ft (3)
(c)	$P(X \geq 1.2) = 1 - F(1.2)$ $= 1 - 0.3733$ $= \frac{47}{75}, 0.6267$	M1 awrt 0.627 A1 (2)
(d)	$(0.6267)^4 = 0.154$	awrt 0.154 or 0.155 M1 A1 (2)
		[10]
	<u>Notes</u>	
(a)	M1 putting $F(x) = 0.5$ M1 using correct quadratic formula. If use calc need to get 1.26 (384...) A1 cao 1.26 must reject the other root. If they use Trial and improvement they have to get the correct answer to gain the second M mark.	
(b)	M1 attempt to differentiate. At least one $x^n \rightarrow x^{n-1}$ A1 correct differentiation B1 must have both parts- follow through their $F'(x)$ Condone <	
(c)	M1 finding/writing $1 - F(1.2)$ may use/write $\int_{1.2}^{1.5} \frac{8}{9}(x+1)dx$ or $1 - \int_1^{1.2} \frac{8}{9}(x+1)dx$ or $\int_{1.2}^{1.5}$ "their $f(x)$ " dx. Condone missing dx A1 awrt 0.627	
(d)	M1 (c) ⁴ If expressions are not given you need to check the calculation is correct to 2sf. A1 awrt 0.154 or 0.155	

Question Number	Scheme	Marks
Q5 (a) (b) (i) (ii) (c)	Connecting occurs at random/independently, singly or at a constant rate $Po(8)$ $P(X=0) = 0.0003$ $P(X \geq 4) = 1 - P(X \leq 3)$ $= 1 - 0.0424$ $= 0.9576$ $H_0: \lambda = 4$ (48) $H_1: \lambda > 4$ (48) $N(48,48)$ Method 1 $P(X \geq 59.5) = P\left(Z \geq \frac{59.5 - 48}{\sqrt{48}}\right)$ $= P(Z \geq 1.66)$ $= 1 - 0.9515$ $= 0.0485$ $0.0485 < 0.05$ Reject H_0 . Significant. 60 lies in the Critical region The number of failed connections at the first attempt has increased. Method 2 $\frac{x - 0.5 - 48}{\sqrt{48}} = 1.6449$ $x = 59.9$	B1 (1) B1 M1A1 M1 A1 (5) B1 M1 A1 M1 M1 A1 A1 M1 A1 ft (9) [15]
	Notes (a) B1 Any one of randomly/independently/singly/constant rate. Must have context of connection/logging on/fail (b) B1 Writing or using $Po(8)$ in (i) or (ii) (i) M1 for writing or finding $P(X=0)$ A1 awrt 0.0003 (ii) M1 for writing or finding $1 - P(X \leq 3)$ A1 awrt 0.958 (c) B1 both hypotheses correct. Must use λ or μ M1 identifying normal A1 using or seeing mean and variance of 48 These first two marks may be given if the following are seen in the standardisation formula : 48 and $\sqrt{48}$ or awrt 6.93 M1 for attempting a continuity correction (Method 1: 60 ± 0.5 / Method 2: $x \pm 0.5$) M1 for standardising using their mean and their standard deviation and using either Method 1 [$59.5, 60$ or 60.5 . accept $\pm z$.] Method 2 [$(x \pm 0.5)$ and equal to a $\pm z$ value] A1 correct z value awrt ± 1.66 or $\pm \frac{59.5 - 48}{\sqrt{48}}$, or $\frac{x - 0.5 - 48}{\sqrt{48}} = 1.6449$ A1 awrt 3 sig fig in range 0.0484 – 0.0485, awrt 59.9 M1 for “reject H_0 ” or “significant” maybe implied by “correct contextual comment” If one tail hypotheses given follow through “their prob” and 0.05, $p < 0.5$ If two tail hypotheses given follow through “their prob” with 0.025, $p < 0.5$ If one tail hypotheses given follow through “their prob” and 0.95, $p > 0.5$ If two tail hypotheses given follow through “their prob” with 0.975, $p > 0.5$ If no H_1 given they get M0 A1 ft correct contextual statement followed through from their prob and H_1 . need the words <u>number of failed connections/log ons has increased</u> o.e. Allow “there are more failed connections” NB A correct contextual statement alone followed through from their prob and H_1 gets M1 A1	

Question Number	Scheme	Marks
<p>Q6 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>2 outcomes/faulty or not faulty/success or fail A constant probability Independence Fixed number of trials (fixed n)</p> <p>$X \sim B(50, 0.25)$ $P(X \leq 6) = 0.0194$ $P(X \leq 7) = 0.0453$ $P(X \geq 18) = 0.0551$ $P(X \geq 19) = 0.0287$</p> <p>CR $X \leq 6$ and $X \geq 19$</p> <p>$0.0194 + 0.0287 = 0.0481$</p> <p>8(It) is not in the Critical region or 8(It) is not significant or $0.0916 > 0.025$; There is evidence that the probability of a faulty bolt is 0.25 or the company's claim is correct.</p> <p>$H_0: p = 0.25$ $H_1: p < 0.25$ $P(X \leq 5) = 0.0070$ or CR $X \leq 5$ $0.007 < 0.01$, 5 is in the critical region, reject H_0, significant. There is evidence that the probability of faulty bolts has decreased</p>	<p>B1 B1 (2)</p> <p>M1</p> <p>A1 A1 (3)</p> <p>M1A1 (2)</p> <p>M1; A1ft (2)</p> <p>B1B1 M1A1</p> <p>M1 A1ft (6)</p> <p>[15]</p>
	<p>Notes</p> <p>(a) B1 B1 one mark for each of any of the four statements. Give first B1 if only one correct statement given. No context needed.</p> <p>(b) M1 for writing or using $B(50, 0.25)$ also may be implied by both CR being correct. Condone use of P in critical region for the method mark. A1 $(X) \leq 6$ o.e. $[0, 6]$ DO NOT accept $P(X \leq 6)$ A1 $(X) \geq 19$ o.e. $[19, 50]$ DO NOT accept $P(X \geq 19)$</p> <p>(c) M1 Adding two probabilities for two tails. Both probabilities must be less than 0.5 A1 awrt 0.0481</p> <p>(d) M1 one of the given statements followed through from their CR. A1 contextual comment followed through from their CR. NB A correct contextual comment alone followed through from their CR will get M1 A1</p> <p>(e) B1 for H_0 must use p or π (pi) B1 for H_1 must use p or π (pi) M1 for finding or writing $P(X \leq 5)$ or attempting to find a critical region or a correct critical region A1 awrt 0.007/CR $X \leq 5$ M1 correct statement using their Probability and 0.01 if one tail test or a correct statement using their Probability and 0.005 if two tail test. The 0.01 or 0.005 needn't be explicitly seen but implied by correct statement compatible with their H_1. If no H_1 given M0 A1 correct contextual statement follow through from their prob and H_1. Need faulty bolts and decreased. NB A correct contextual statement alone followed through from their prob and H_1 get M1 A1</p>	

Question Number	Scheme	Marks
Q7 (ai)	$f(y) \geq 0$ or $f(3) \geq 0$ $ky(a-y) \geq 0$ or $3k(a-3) \geq 0$ or $(a-y) \geq 0$ or $(a-3) \geq 0$ $a \geq 3$	M1 A1 cso
(ii)	$\int_0^3 k(ay - y^2) dy = 1$ $\left[k \left(\frac{ay^2}{2} - \frac{y^3}{3} \right) \right]_0^3 = 1$ $k \left(\frac{9a}{2} - 9 \right) = 1$ $k \left[\frac{9a-18}{2} \right] = 1$ $k = \frac{2}{9(a-2)}$ *	integration answer correct answer = 1 A1 cso (6)
(b)	$\int_0^3 k(ay^2 - y^3) dy = 1.75$ $\left[k \left(\frac{ay^3}{3} - \frac{y^4}{4} \right) \right]_0^3 = 1.75$ $k \left(9a - \frac{81}{4} \right) = 1.75$ $2 \left(9a - \frac{81}{4} \right) = 15.75(a-2)$ $2.25a = -31.5 + \frac{81}{2}$ $a = 4$ * $k = \frac{1}{9}$	Int $\int xf(x)$ Correct integration $\int xf(x) = 1.75$ and limits 0,3 subst k A1 M1dep M1dep A1cso B1 (6)

Question Number	Scheme	Marks
(c)		<p>B1 B1 (2)</p>
(d)	mode = 2	B1 (1)
[15]		
(a) (i)	<p>Notes</p> <p>M1 for putting $f(y) \geq 0$ or $f(3) \geq 0$ or $ky(a-y) \geq 0$ or $3k(a-3) \geq 0$ or $(a-y) \geq 0$ or $(a-3) \geq 0$ or state in words the probability can not be negative o.e. A1 need one of $ky(a-y) \geq 0$ or $3k(a-3) \geq 0$ or $(a-y) \geq 0$ or $(a-3) \geq 0$ and $a \geq 3$</p> <p>(ii) M1 attempting to integrate (at least one $y^n \rightarrow y^{n+1}$) (ignore limits) A1 Correct integration. Limits not needed. And equals 1 not needed. M1 dependent on the previous M being awarded. Putting equal to 1 and have the correct limits. Limits do not need to be substituted. A1 cso</p> <p>(b) M1 for attempting to find $\int yf(y) dy$ (at least one $y^n \rightarrow y^{n+1}$) (ignore limits) A1 correct Integration M1 $\int yf(y) = 1.75$ and limits 0,3 dependent on previous M being awarded M1 subst in for k. dependent on previous M being awarded A1 cso 4 B1 cao 1/9</p> <p>(c) B1 correct shape. No straight lines. No need for patios. B1 completely correct graph. Needs to go through origin and the curve ends at 3. <u>Special case:</u> If draw full parabola from 0 to 4 get B1 B0 Allow full marks if the portion between $x = 3$ and $x = 4$ is dotted and the rest of the curve solid.</p> 	
(d)	B1 cao 2	

Write your name here

Surname	Other names
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**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Statistics S3

Advanced/Advanced Subsidiary

Sample Assessment Material

Time: 1 hour 30 minutes

Paper Reference

WST03/01

You must have:

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

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Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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S 4 5 0 0 9 A 0 1 2 4

PEARSON

2. Philip and James are racing car drivers. Philip's lap times, in seconds, are normally distributed with mean 90 and variance 9. James' lap times, in seconds, are normally distributed with mean 91 and variance 12. The lap times of Philip and James are independent. Before a race, they each take a qualifying lap.

- (a) Find the probability that James' time for the qualifying lap is less than Philip's. (4)

The race is made up of 60 laps. Assuming that they both start from the same starting line and lap times are independent,

- (b) find the probability that Philip beats James in the race by more than 2 minutes. (5)

3. A woodwork teacher measures the width, w mm, of a board. The measured width, X mm, is normally distributed with mean w mm and standard deviation 0.5 mm.

(a) Find the probability that X is within 0.6 mm of w . (2)

The same board is measured 16 times and the results are recorded.

(b) Find the probability that the mean of these results is within 0.3 mm of w . (4)

Given that the mean of these 16 measurements is 35.6 mm,

(c) find a 98% confidence interval for w . (4)

4. A researcher claims that, at a river bend, the water gradually gets deeper as the distance from the inner bank increases. He measures the distance from the inner bank, b cm, and the depth of a river, s cm, at seven positions. The results are shown in the table below.

Position	A	B	C	D	E	F	G
Distance from inner bank b cm	100	200	300	400	500	600	700
Depth s cm	60	75	85	76	110	120	104

- (a) Calculate Spearman's rank correlation coefficient between b and s . **(6)**

- (b) Stating your hypotheses clearly, test whether or not the data provides support for the researcher's claim. Use a 1% level of significance. **(4)**

Question 4 continued

Horizontal lines for writing answers.

(Total 10 marks)

Q4

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Question 5 continued

Lined writing area for the answer to Question 5 continued.

- 6. A total of 228 items are collected from an archaeological site. The distance from the centre of the site is recorded for each item. The results are summarised in the table below.

Distance from the centre of the site (m)	0–1	1–2	2–4	4–6	6–9	9–12
Number of items	22	15	44	37	52	58

Test, at the 5% level of significance, whether or not the data can be modelled by a continuous uniform distribution. State your hypotheses clearly.

(12)

7. A large company surveyed its staff to investigate the awareness of company policy. The company employs 6000 full-time staff and 4000 part-time staff.

(a) Describe how a stratified sample of 200 staff could be taken. (3)

(b) Explain an advantage of using a stratified sample rather than a simple random sample. (1)

A random sample of 80 full-time staff and an independent random sample of 80 part-time staff were given a test of policy awareness. The results are summarised in the table below.

	Mean score (\bar{x})	Variance of scores (s^2)
Full-time staff	52	21
Part-time staff	50	19

(c) Stating your hypotheses clearly, test, at the 1% level of significance, whether or not the mean policy awareness scores for full-time and part-time staff are different. (7)

(d) Explain the significance of the Central Limit Theorem to the test in part (c). (2)

(e) State an assumption you have made in carrying out the test in part (c). (1)

After all the staff had completed a training course the 80 full-time staff and the 80 part-time staff were given another test of policy awareness. The value of the test statistic z was 2.53

(f) Comment on the awareness of company policy for the full-time and part-time staff in light of this result. Use a 1% level of significance. (2)

(g) Interpret your answers to part (c) and part (f). (1)

WST03/01: Statistics S3

Hypothesis Tests (Final M1A1)

For an incorrect comparison (e.g. probability with z value) even with a correct statement and/or comment award M0A0

For a correct or no comparison with more than one statement one of which is false
Award M0A0 (This is compatible with the principle above of contradictory statements being penalised)

Apply these rules to all questions

Question Number	Scheme	Marks
Q1	$H_0: \mu = 80, \quad H_1: \mu > 80$ $z = \frac{83 - 80}{\frac{15}{\sqrt{100}}} = 2$ $2 > 1.6449 \quad \text{(accept 1.645 or better)}$ Reject H_0 <u>or</u> significant result <u>or</u> in the critical region Managing director's claim is supported.	B1,B1 M1A1 B1 M1 A1
	<p>1st B1 for H_0. They must use μ not x, p, λ or \bar{x} etc</p> <p>2nd B1 for H_1 (must be > 80). Same rules about μ.</p> <p>1st M1 for attempt at standardising using 83, 80 and $\frac{15}{\sqrt{100}}$. Can accept \pm.</p> <p>May be implied by $z = \pm 2$</p> <p>1st A1 for + 2 only</p> <p>3rd B1 for ± 1.6449 seen (or probability of 0.0228 or better)</p> <p>2nd M1 for a correct statement about "significance" or rejecting H_0 (or H_1) based on their z value and their 1.6449 (provided it is a recognizable critical value from normal tables) <u>or</u> their probability (< 0.5) and significance level of 0.05. Condone their probability > 0.5 compared with 0.95 for the 2nd M1</p> <p>2nd A1 for a correct contextualised comment. Must mention "director" and "claim" <u>or</u> "time" and "use of Internet". No follow through.</p> <p>2nd M1A1 If no comparison or statement is made but a correct contextualised comment is given the M1 can be implied. If a comparison is made it must be <u>compatible</u> with statement otherwise M0 e.g. comparing 0.0228 with 1.6449 is M0 or comparing probability 0.9772 with 0.05 is M0 comparing -2 with - 1.6449 is OK provided a correct statement accompanies it condone -2 $>$ -1.6449 provided their statement correctly rejects H_0.</p> <p>Critical Region They may find a critical region for \bar{X}: $\bar{X} > 80 + \frac{15}{\sqrt{100}} \times 1.6449 = \text{awrt } 82.5$</p> <p>1st M1 for $80 + \frac{15}{\sqrt{100}} \times (z \text{ value})$</p> <p>3rd B1 for 1.645 or better</p> <p>1st A1 for awrt 82.5</p> <p>The rest of the marks are as per the scheme.</p>	<p style="text-align: right;">7</p>

Question Number	Scheme	Marks
<p>Q2</p> <p>(a)</p> <p>(b)</p>	<p>[$P \sim N(90,9)$ and $J \sim N(91,12)$]</p> <p>$(J - P) \sim N(1,21)$ $P(J < P) = P(J - P < 0)$ $= P\left(Z < \frac{0-1}{\sqrt{21}}\right)$ $= P(Z < -0.2182\dots)$ $= 1 - 0.5871 = 0.4129$ awrt (0.413 ~ 0.414)</p> <p>calculator (0.4136....)</p> <p>$X = (J_1 + J_2 + \dots + J_{60}) - (P_1 + P_2 + \dots + P_{60})$ $E(X) = 60 \times 91 - 60 \times 90 = 60$ [stated as $E(X) = 60$ or $X \sim N(60, \dots)$] $\text{Var}(X) = 60 \times 9 + 60 \times 12 = 1260$ $P(X > 120) = P\left(Z > \frac{120-60}{\sqrt{1260}}\right)$ $= P(Z > 1.69030\dots)$ $= 1 - 0.9545 = 0.0455$ awrt (0.0455)</p>	<p>M1, A1</p> <p>dM1</p> <p>A1</p> <p>(4)</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>9</p>
<p>(a)</p> <p>(b)</p> <p>Use of means</p>	<p>1st M1 for attempting $J - P$ and $E(J - P)$ or $P - J$ and $E(P - J)$ 1st A1 for variance of 21 (Accept $9 + 12$). Ignore any slip in μ here. 2nd dM1 for attempting the correct probability and standardising with their mean and sd. This mark is dependent on previous M so if $J - P$ (or $P - J$) is not being used score M0 If their method is not crystal clear then they must be attempting $P(Z < -ve \text{ value})$ or $P(Z > +ve \text{ value})$ i.e. their probability <u>after</u> standardisation should lead to a prob. < 0.5 so e.g. $P(J - P < 0)$ leading to 0.5871 is M0A0 unless the M1 is clearly earned. 2nd A1 for awrt 0.413 or 0.414</p> <p style="text-align: center;">The first 3 marks may be implied by a correct answer</p> <p>1st M1 for a clear attempt to identify a correct form for X. This may be implied by correct variance of 1260 B1 for $E(X) = 60$. Can be awarded even if they are using $X = 60J - 60P$. Allow $P - J$ and -60 1st A1 for a correct variance. If 1260 is given the M1 is scored by implication. 2nd M1 for attempting a correct probability and standardising with 120 and their 60 and 1260 If the answer is incorrect a full <u>expression</u> must be seen following through their values for M1 e.g. $P\left(Z > \frac{120 - \text{their } 60}{\sqrt{\text{their variance}}}\right)$. If using -60, should get $P\left(Z < \frac{-120 - -60}{\sqrt{\text{their variance}}}\right)$</p> <p>Attempt to use $\bar{J} - \bar{P}$ for 1st M1, $E(\bar{J} - \bar{P}) = 1$ for B1 and $\text{Var}(\bar{J} - \bar{P}) = 0.35$ for A1 Then 2nd M1 for standardisation with 2, and their 1 and 0.35</p>	

WST03/01: Statistics S3

Question Number	Scheme	Marks
Q3 (a)	$E \sim N(0, 0.5^2)$ or $X \sim N(w, 0.5^2)$ $P(E < 0.6) = P\left(Z < \frac{0.6}{0.5}\right)$ or $P(X - w < 0.6) = P\left(Z < \frac{0.6}{0.5}\right)$ $= P(Z < 1.2)$ $= 2 \times 0.8849 - 1 = 0.7698$ awrt 0.770	M1 A1 (2)
(b)	$\bar{E} \sim N\left(0, \frac{1}{64}\right)$ or $\bar{X} \sim N\left(w, \frac{0.5^2}{16}\right)$ $P(\bar{E} < 0.3) = P\left(Z < \frac{0.3}{\frac{1}{8}}\right)$ or $P(\bar{X} - w < 0.3) = P\left(Z < \frac{0.3}{\frac{1}{8}}\right)$ $= P(Z < 2.4)$ $= 2 \times 0.9918 - 1 = 0.9836$ awrt 0.984	M1 M1, A1 A1 (4)
(c)	$35.6 \pm 2.3263 \times \frac{1}{8}$ (35.3, 35.9)	M1 B1 A1, A1 (4) 10
(a)	1 st M1 for identifying a correct probability (they must have the 0.6) and attempting to standardise. Need . This mark can be given for 0.8849 - 0.1151 seen as final answer. 1 st A1 for awrt 0.770. NB an answer of 0.3849 or 0.8849 scores M0A0 (since it implies no) M1 may be implied by a correct answer	
(b)	1 st M1 for a correct attempt to define \bar{E} or \bar{X} but must attempt $\frac{\sigma^2}{n}$. Condone labelling as E or X This mark may be implied by standardisation in the next line. 2 nd M1 for identifying a correct probability statement using \bar{E} or \bar{X} . Must have 0.3 and 1 st A1 for correct standardisation as printed or better 2 nd A1 for awrt 0.984 The M marks may be implied by a correct answer.	
Sum of 16, not means	1 st M1 for correct attempt at suitable sum distribution with correct variance ($= 16 \times \frac{1}{4}$) 2 nd M1 for identifying a correct probability. Must have 4.8 and 1 st A1 for correct standardisation i.e. need to see $\frac{4.8}{\sqrt{4}}$ or better	
(c)	M1 for $35.6 \pm z \times \frac{0.5}{\sqrt{16}}$ B1 for 2.3263 or better. Use of 2.33 will lose this mark but can still score $\frac{3}{4}$ 1 st A1 for awrt 35.3 2 nd A1 for awrt 35.9	

Question Number	Scheme	Marks																																
<p>Q4</p> <p>(a)</p> <table border="1" data-bbox="344 286 1198 528"> <tr> <td>Distance rank</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>Depth rank</td> <td>1</td> <td>2</td> <td>4</td> <td>3</td> <td>6</td> <td>7</td> <td>5</td> </tr> <tr> <td>d</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>2</td> </tr> <tr> <td>d^2</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>4</td> </tr> </table> <p>$\sum d^2 = 8$</p> <p>$r_s = 1 - \frac{6 \times 8}{7 \times 48}$</p> <p>$= \frac{6}{7} = 0.857142$</p> <p>(b) $H_0 : \rho = 0, H_1 : \rho > 0$ Critical value at 1% level is 0.8929 $r_s < 0.8929$ so not significant evidence to reject H_0, The researcher's claim is not correct (at 1% level). <u>or</u> insufficient evidence for researcher's claim <u>or</u> there is insufficient evidence that water gets deeper further from inner bank. <u>or</u> no (positive) correlation between depth of water and distance from inner bank</p>	Distance rank	1	2	3	4	5	6	7	Depth rank	1	2	4	3	6	7	5	$ d $	0	0	1	1	1	1	2	d^2	0	0	1	1	1	1	4	<p>M1</p> <p>M1</p> <p>M1A1</p> <p>M1</p> <p>awrt 0.857</p> <p>(6)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1ft</p> <p>(4)</p> <p>10</p>	
Distance rank	1	2	3	4	5	6	7																											
Depth rank	1	2	4	3	6	7	5																											
$ d $	0	0	1	1	1	1	2																											
d^2	0	0	1	1	1	1	4																											
<p>(a)</p> <p>1st M1 for an attempt to rank the depths against the distances</p> <p>2nd M1 for attempting d for their ranks. Must be using ranks.</p> <p>3rd M1 for attempting $\sum d^2$ (must be using ranks)</p> <p>1st A1 for sum of 8 (or 104 for reverse ranking)</p> <p>4th M1 for use of the correct formula with their $\sum d^2$. If answer is not correct an expression is required.</p> <p>2nd A1 for awrt (\pm) 0.857. Sign should correspond to ranking (so use of 104 should get -0.857)</p> <p>(b)</p> <p>1st B1 for both hypotheses in terms of ρ, H_1 must be one tail and compatible with their ranking</p> <p>2nd B1 for cv of 0.8929 (accept \pm)</p> <p>M1 for a correct statement relating their r_s with their cv but cv must be such that $cv < 1$</p> <p>A1ft for a correct contextualised comment. Must mention "researcher" and "claim" <u>or</u> "distance (from bank)" and "depth (of water)" Follow through their r_s and their cv (provided it is $cv < 1$) Use of "association" is A0</p>																																		

Question Number	Scheme				Marks																												
Q5	<table border="1"> <thead> <tr> <th>Finances</th> <th>Worse</th> <th>Same</th> <th>Better</th> <th></th> </tr> </thead> <tbody> <tr> <td>Income</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>Under £15 000</td> <td>10.54</td> <td>10.54</td> <td>12.92</td> <td>34</td> </tr> <tr> <td>£15 000 and above</td> <td>20.46</td> <td>20.46</td> <td>25.08</td> <td>66</td> </tr> <tr> <td></td> <td>31</td> <td>31</td> <td>38</td> <td>100</td> </tr> </tbody> </table>				Finances	Worse	Same	Better		Income					Under £15 000	10.54	10.54	12.92	34	£15 000 and above	20.46	20.46	25.08	66		31	31	38	100	M1 A1			
	Finances	Worse	Same	Better																													
	Income																																
	Under £15 000	10.54	10.54	12.92	34																												
	£15 000 and above	20.46	20.46	25.08	66																												
		31	31	38	100																												
	<p>H_0 : State of finances and income are independent (not associated) H_1 : State of finances and income are not independent (associated)</p>				B1																												
	<table border="1"> <thead> <tr> <th>O_i</th> <th>E_i</th> <th>$\frac{(O_i - E_i)^2}{E_i}$</th> <th>$\frac{O_i^2}{E_i}$</th> </tr> </thead> <tbody> <tr> <td>14</td> <td>10.54</td> <td>1.1358....</td> <td>18.59..</td> </tr> <tr> <td>11</td> <td>10.54</td> <td>0.0200....</td> <td>11.48..</td> </tr> <tr> <td>9</td> <td>12.92</td> <td>1.1893...</td> <td>6.269..</td> </tr> <tr> <td>17</td> <td>20.46</td> <td>0.5851...</td> <td>14.12..</td> </tr> <tr> <td>20</td> <td>20.46</td> <td>0.0103...</td> <td>19.55..</td> </tr> <tr> <td>29</td> <td>25.08</td> <td>0.6126...</td> <td>33.53..</td> </tr> </tbody> </table>				O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$	14	10.54	1.1358....	18.59..	11	10.54	0.0200....	11.48..	9	12.92	1.1893...	6.269..	17	20.46	0.5851...	14.12..	20	20.46	0.0103...	19.55..	29	25.08	0.6126...	33.53..	M1 A1
	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$																													
	14	10.54	1.1358....	18.59..																													
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20	20.46	0.0103...	19.55..																														
29	25.08	0.6126...	33.53..																														
$\sum \frac{(O_i - E_i)^2}{E_i} = 3.553... \quad \text{or} \quad \sum \frac{O_i^2}{E_i} - 100 = 103.553... - 100 = 3.553... \quad (\text{awrt } 3.55)$				A1																													
$\nu = (3 - 1)(2 - 1) = 2$				B1																													
cv is 5.991				B1																													
3.553 < 5.991 so insufficient evidence to reject H_0 <u>or</u> not significant				M1																													
There is no evidence of association between state of finances and income.				A1																													
10																																	
<p>1st M1 for some use of $\frac{\text{Row Total} \times \text{Col. Total}}{\text{Grand Total}}$. May be implied by correct E_i</p> <p>1st A1 for all expected frequencies correct</p> <p>B1 for both hypotheses. Must mention “state” or “finances” and “income” at least once Use of “relationship” or “correlation” or “connection” is B0</p> <p>2nd M1 for at least two correct terms (as in 3rd or 4th column) or correct expressions with their E_i</p> <p>2nd A1 for all correct terms. May be implied by a correct answer. (2 dp or better - allow eg 1.13...)</p> <p>3rd M1 for a correct statement linking their test statistic and their cv. Must be χ^2 not normal.</p> <p>4th A1 for a correct comment in context - must mention “state” or “finances” and “income” condone “relationship” or “connection” here but not “correlation”. No follow through. e.g. “There is no evidence of a relationship between finances and income”</p>																																	

Question Number	Scheme						Marks	
Q6	Distance from centre of site (m)	0-1	1-2	2-4	4-6	6-9	9-12	
	$b-a$	1	1	2	2	3	3	M1
	No of artefacts	22	15	44	37	52	58	A1
	$P(a \leq X < b)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{4}$	A1
	$228 \times P(a \leq X < b)$	19	19	38	38	57	57	
	Class	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$	$\frac{O_i^2}{E_i}$			
	0-1	22	19	$\frac{9}{19} = 0.4736\dots$	25.57...			M1
	1-2	15	19	$\frac{16}{19} = 0.8421\dots$	11.84...			
	2-4	44	38	$\frac{36}{38} = 0.9473\dots$	50.94...			
	4-6	37	38	$\frac{1}{38} = 0.0263\dots$	36.02...			
6-9	52	57	$\frac{25}{57} = 0.4385\dots$	47.43...			A1	
9-12	58	57	$\frac{1}{57} = 0.0175\dots$	59.01...				
	<p>H_0: <u>continuous uniform</u> distribution <u>is</u> a good fit</p> <p>H_1: <u>continuous uniform</u> distribution <u>is not</u> a good fit</p> <p>$\sum \frac{(O_i - E_i)^2}{E_i} = \frac{313}{114} = 2.75$ or $\sum \frac{O_i^2}{E_i} - 228 = 230.745\dots - 228 = \dots$ (awrt 2.75)</p> <p>$\nu = 6 - 1 = 5$</p> <p>$\chi^2_5(0.05) = 11.070$ (ft their ν i.e. $\chi^2_\nu(0.05)$)</p> <p>$2.75 < 11.070$, insufficient evidence to reject H_0</p> <p>Continuous uniform distribution is a suitable model</p>							
								B1
								dM1A1
								B1
								B1ft
								M1
								A1
								12
	1 st M1	for calculation of at least 3 widths and attempting proportions/probs. <u>or</u> for 1:2:3 ratio seen						
	1 st A1	for correct probabilities						
	2 nd A1	for all correct expected frequencies						
	2 nd M1	for attempting $\frac{(O - E)^2}{E}$ or $\frac{O^2}{E}$, at least 3 correct expressions or values.						
		Follow through their E_i provided they are not all = 38						
	3 rd A1	for a correct set of calcs - 3 rd or 4 th column. (2 dp or better and allow e.g. 0.94...)						
	3 rd dM1	dependent on 2nd M1 for attempting a correct sum or calculation (must see at least 3 terms and +)						
		The first three Ms and As can be implied by a test statistic of awrt 2.75						
	4 th M1	for a correct statement based on their test statistic (> 1) and their cv (> 3.8)						
		Contradictory statements score M0 e.g. "significant" do not reject H_0 .						
	5 th A1	for a correct comment suggesting that continuous uniform model is suitable. No ft						

Question Number	Scheme	Marks
Q7	<p>(a) Label full time staff 1-6000, part time staff 1-4000 Use random numbers to select Simple random sample of 120 full time staff and 80 part time staff</p> <p>(b) Enables estimation of statistics / errors for each strata <u>or</u> “reduce variability” <u>or</u> “more representative” <u>or</u> “reflects population structure” NOT “more accurate”</p> <p>(c) $H_0: \mu_f = \mu_p, \quad H_1: \mu_f \neq \mu_p$ (accept μ_1, μ_2) $\text{s.e.} = \sqrt{\frac{21}{80} + \frac{19}{80}}, \quad z = \frac{52 - 50}{\sqrt{\frac{21}{80} + \frac{19}{80}}} = (2\sqrt{2})$ $= 2.828\dots$ (awrt 2.83) Two tailed critical value $z = 2.5758$ (or prob of awrt 0.002 (<0.005) or 0.004 (<0.01)) [2.828 > 2.5758 so] significant evidence to reject H_0 There is evidence of a difference in policy awareness between full time and part time staff</p> <p>(d) Can use mean full time and mean part time ~ Normal</p> <p>(e) Have assumed $s^2 = \sigma^2$ or variance of sample = variance of population</p> <p>(f) $2.53 < 2.5758$, not significant <u>or</u> do not reject H_0 So there is insufficient evidence of a difference in mean awareness</p> <p>(g) Training course has closed the gap between full time staff and part time staff’s mean awareness of company policy.</p>	<p>M1 M1 A1 (3) B1 B1 (1) M1, M1 A1 B1 dM1 A1ft (7) B1 B1 (2) B1 (1) M1 A1ft (2) B1 (1) 17</p>
	<p>(a) 1st M1 for attempt at labelling full-time and part-time staff. One set of correct numbers. 2nd M1 for mentioning use of random numbers 1st A1 for s.r.s. of 120 full-time and 80 part-time</p> <p>(c) 1st M1 for attempt at s.e. - condone one number wrong . NB correct s.e. = $\sqrt{\frac{1}{2}}$ 2nd M1 for using their s.e. in correct formula for test statistic. Must be $\frac{\pm(52 - 50)}{\sqrt{\frac{p}{q} + \frac{r}{s}}}$ 3rd dM1 dep. on 2nd M1 for a correct statement based on their normal cv and their test statistic 2nd A1 for correct comment in context. Must mention “scores” or “policy awareness” and types of “staff”. Award A0 for a one-tailed comment. Allow ft</p> <p>(d) 1st B1 for mention of mean(s) <u>or</u> use of \bar{X}, provided \bar{X} clearly refers to full-time or part-time 2nd B1 for stating that distribution can be assumed normal e.g. “mean score of the test is normally distributed” gets B1B1</p> <p>(f) M1 for correct statement (may be implied by correct contextualised comment) A1 for correct contextualised comment. Accept “no difference in mean scores”. Allow ft</p> <p>(g) B1 for correct comment in context that implies training was effective. This must be supported by their (c) and (f). Condone one-tailed comment here.</p>	

Pearson Edexcel International Advanced Level

Decision Mathematics D1

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

You must have:
D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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PEARSON

Write your answers in the D1 answer book for this paper.

1.

Hajra (H)	Vicky (V)	Leisham (L)	Alice (A)	Nicky (N)	June (J)	Sharon (S)	Tom (T)	Paul (P)
--------------	--------------	----------------	--------------	--------------	-------------	---------------	------------	-------------

The table shows the names of nine people.

(a) Use a quick sort to produce the list of names in ascending alphabetical order.

You must make your pivots clear.

(4)

(b) Use the binary search algorithm on your list to locate the name Paul.

(4)

(Total 8 marks)

2.

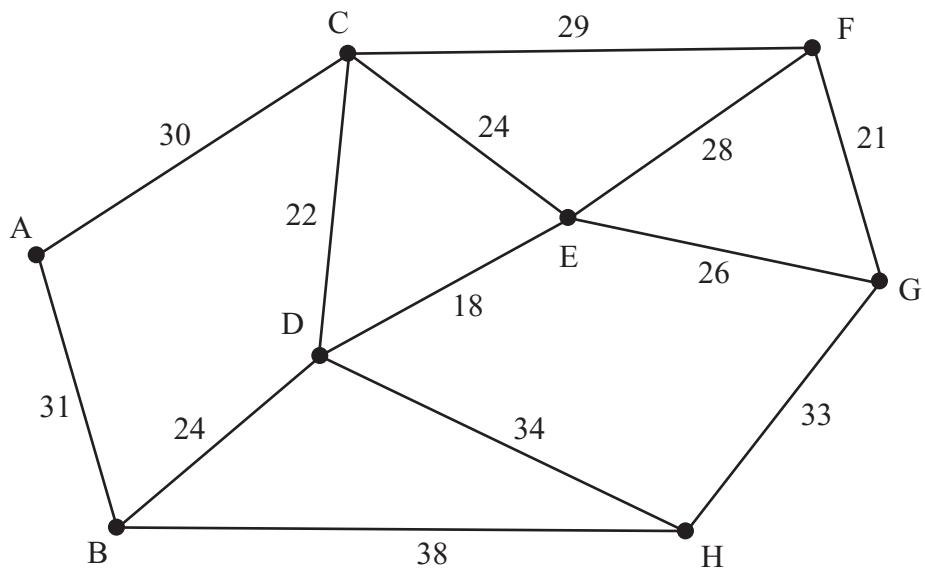


Figure 1

Figure 1 represents the distances, in metres, between eight vertices, A, B, C, D, E, F, G and H, in a network.

- (a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree. (3)
- (b) Complete Matrix 1 in your answer book, to represent the network. (2)
- (c) **Starting at A, use Prim's algorithm** to determine a minimum spanning tree. You must clearly state the order in which you considered the vertices and the order in which you included the arcs. (3)
- (d) State the weight of the minimum spanning tree. (1)

(Total 9 marks)

3.

41 28 42 31 36 32 29

The numbers in the list represent the weights, in kilograms, of seven statues. They are to be transported in crates that will each hold a maximum weight of 60 kilograms.

- (a) Calculate a lower bound for the number of crates that will be needed to transport the statues. (2)
- (b) Use the first-fit bin packing algorithm to allocate the statues to the crates. (3)
- (c) Use the full bin algorithm to allocate the statues to the crates. (2)
- (d) Explain why it is not possible to transport the statues using fewer crates than the number needed for part (c). (2)

(Total 9 marks)

4.

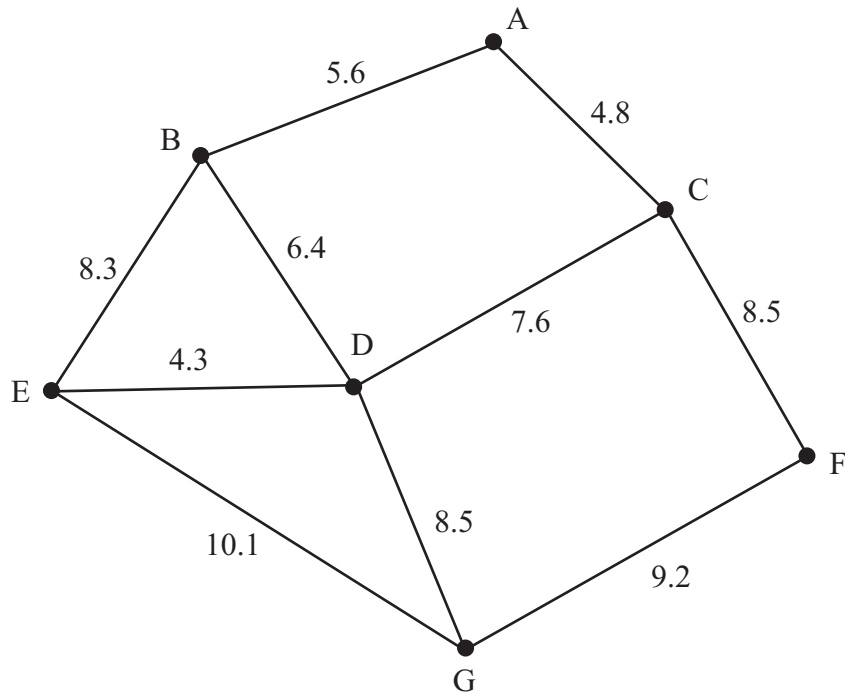


Figure 2

[The total weight of the network is 73.3 km]

Figure 2 models a network of tunnels that have to be inspected. The number on each arc represents the length, in km, of that tunnel.

Malcolm needs to travel through each tunnel at least once and wishes to minimise the length of his inspection route.

He must start and finish at A.

- (a) Use the route inspection algorithm to find the tunnels that will need to be traversed twice. You should make your method and working clear. (5)

- (b) Find a route of minimum length, starting and finishing at A. State the length of your route. (3)

A new tunnel, CG, is under construction. It will be 10 km long. Malcolm will have to include the new tunnel in his inspection route.

- (c) What effect will the new tunnel have on the total length of his route? Justify your answer. (2)

(Total 10 marks)

5.

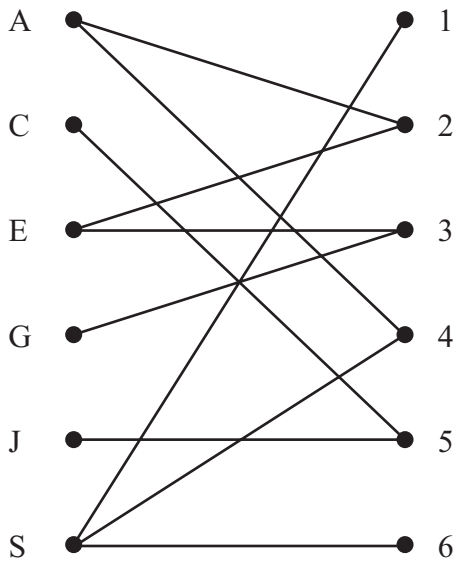


Figure 3

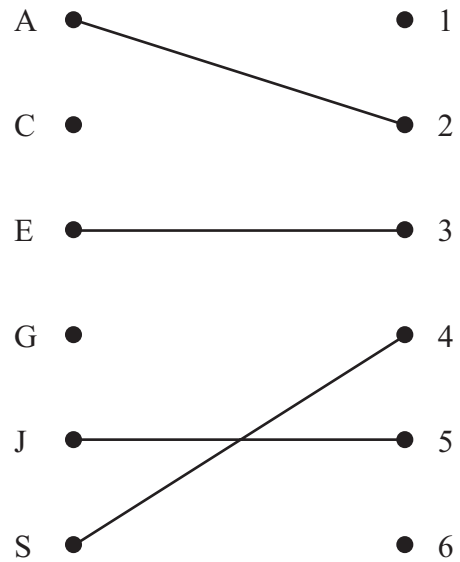


Figure 4

Figure 3 shows the possible allocations of six people, Amelia, Charlie, Ellie, Gemma, Jimmy and Saskia, to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 4 shows an initial matching.

- (a) Use the maximum matching algorithm once to find an improved matching. You must state the alternating path used and your improved matching. (3)
- (b) Explain why a complete matching is not possible. (2)

After training, Jimmy can be assigned to tasks 4 or 5 and Ellie to tasks 2, 3, 5 or 6.

- (c) **Starting with your current maximal matching**, use the maximum matching algorithm to obtain a complete matching. You must state the alternating path used and your final matching. (3)

(Total 8 marks)

6.

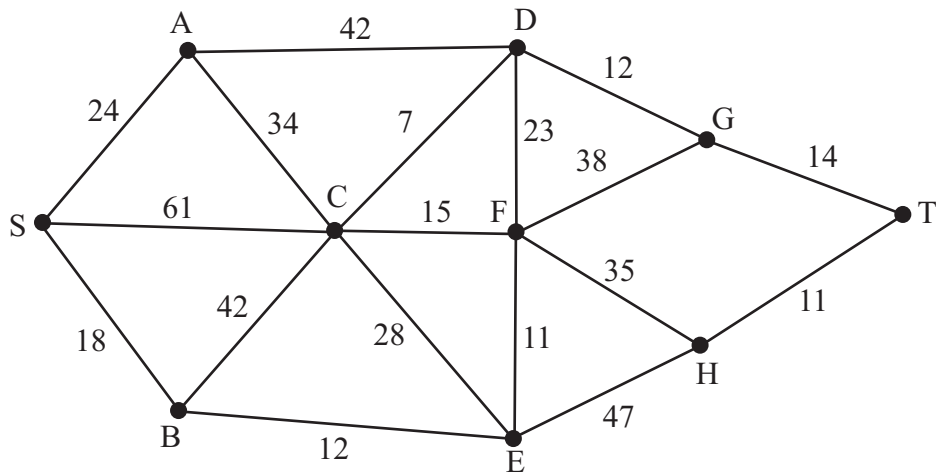


Figure 5

Figure 5 shows a network of cycle tracks within a national park. The number on each arc represents the time taken, in minutes, to cycle along the corresponding track.

- (a) Use Dijkstra's algorithm to find the quickest route from S to T. State your quickest route and the time it takes. (6)
- (b) Explain how you determined your quickest route from your labelled diagram. (2)
- (c) Write down the quickest route from E to T. (1)

(Total 9 marks)

7.

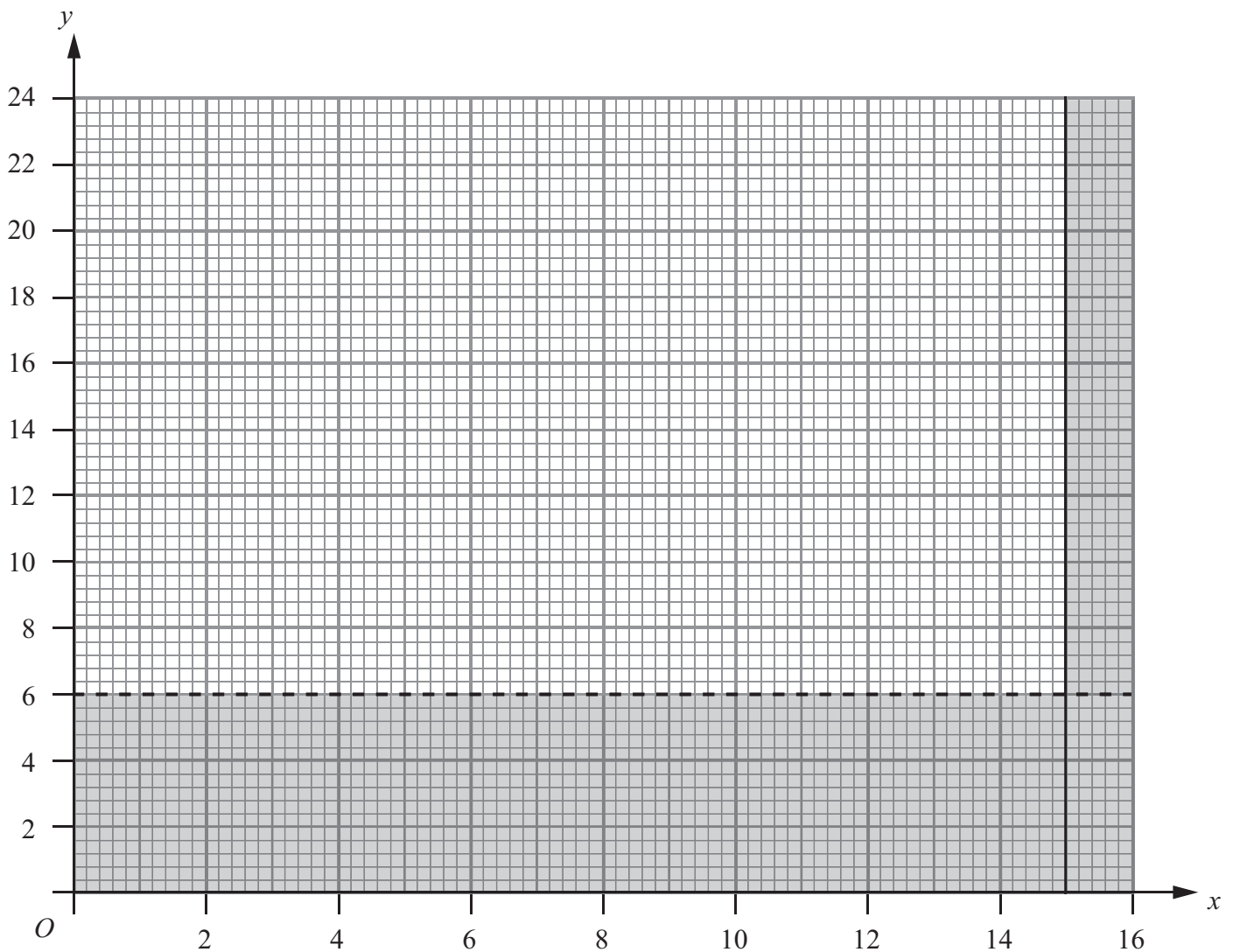


Figure 6

Keith organises two types of children’s activity, ‘Sports Mad’ and ‘Circus Fun’. He needs to determine the number of times each type of activity is to be offered.

Let x be the number of times he offers the ‘Sports Mad’ activity. Let y be the number of times he offers the ‘Circus Fun’ activity.

Two constraints are

$$\begin{aligned} x &\leq 15 \\ \text{and } y &> 6 \end{aligned}$$

These constraints are shown on the graph in Figure 6, where the rejected regions are shaded out.

(a) Explain why $y = 6$ is shown as a dotted line.

(1)

Two further constraints are

$$\begin{aligned} & 3x \geq 2y \\ \text{and } & 5x + 4y \geq 80 \end{aligned}$$

- (b) Add two lines and shading to Diagram 1 in the answer book to represent these inequalities. Hence determine the feasible region and label it R.

(3)

Each 'Sports Mad' activity costs £500.

Each 'Circus Fun' activity costs £800.

Keith wishes to minimise the total cost.

- (c) Write down the objective function, C , in terms of x and y .

(2)

- (d) Use your graph to determine the number of times each type of activity should be offered and the total cost. You must show sufficient working to make your method clear.

(5)

(Total 11 marks)

8.

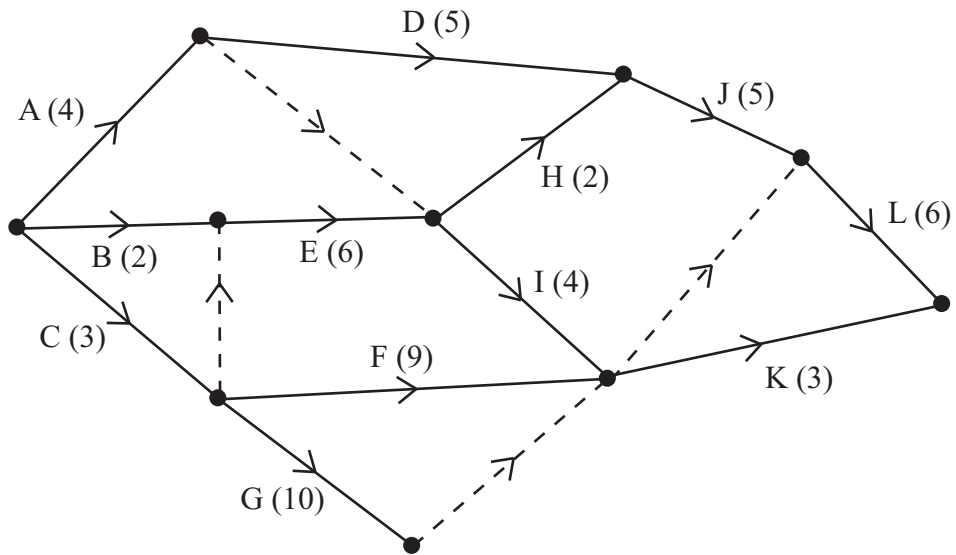


Figure 7

A project is modelled by the activity network shown in Figure 7. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

- (a) Complete Diagram 2 in the answer book to show the early and late event times. (4)
- (b) State the critical activities. (1)
- (c) On Grid 1 in the answer book, draw a cascade (Gantt) chart for this project. (4)
- (d) Use your cascade chart to determine a lower bound for the number of workers needed. You must justify your answer. (2)

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

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Write your name here

Surname	Other names
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Pearson Edexcel
International
Advanced Level

Centre Number

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Candidate Number

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Decision Mathematics D1

Advanced/Advanced Subsidiary

Sample Assessment Material
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

Answer Book

Do not return the question paper with the answer book.

Total Marks

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Turn over ►

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2.

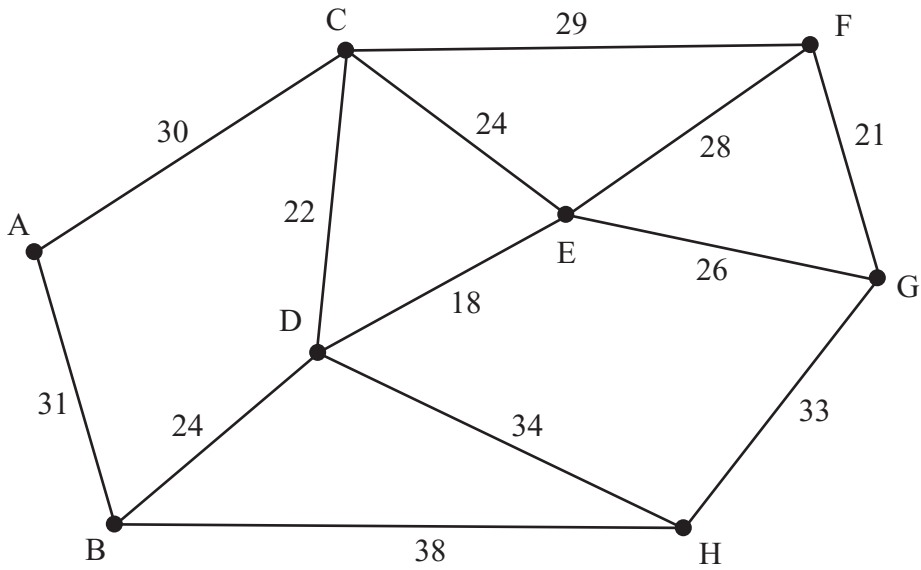


Figure 1

(a)

(b)

	A	B	C	D	E	F	G	H
A	-	31	30	-	-	-	-	-
B	31	-	-		-	-	-	38
C	30	-	-		24		-	-
D	-			-	18	-	-	
E	-	-	24	18	-	28		-
F	-	-		-	28	-	21	-
G	-	-	-	-		21	-	
H	-	38	-		-	-		-

Matrix 1

(c)

(d) Weight of minimum spanning tree: _____

(Total 9 marks)

Q2

Question 3 continued

Lined area for writing answers, consisting of 30 horizontal lines.

Q3

(Total 9 marks)

4.

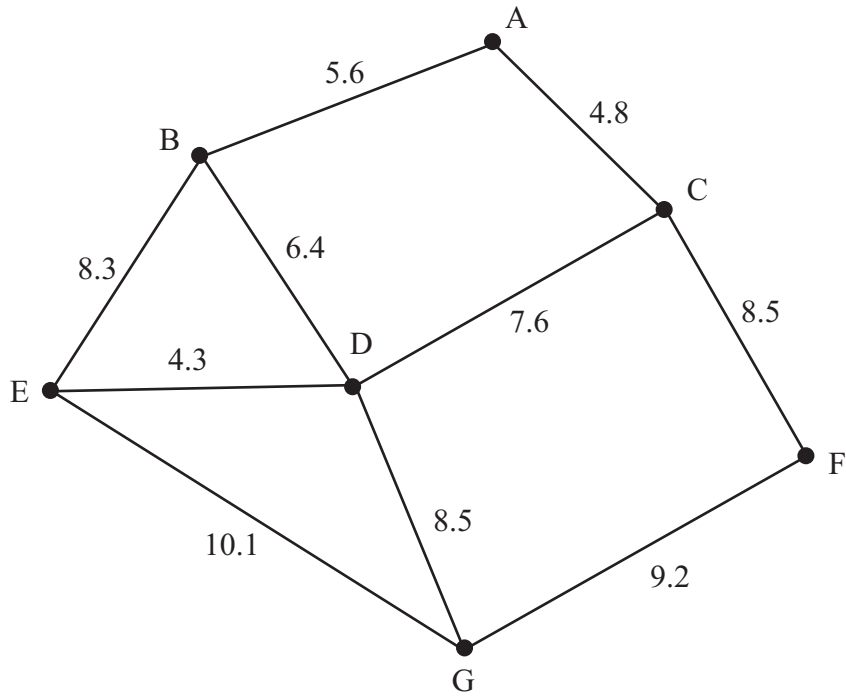
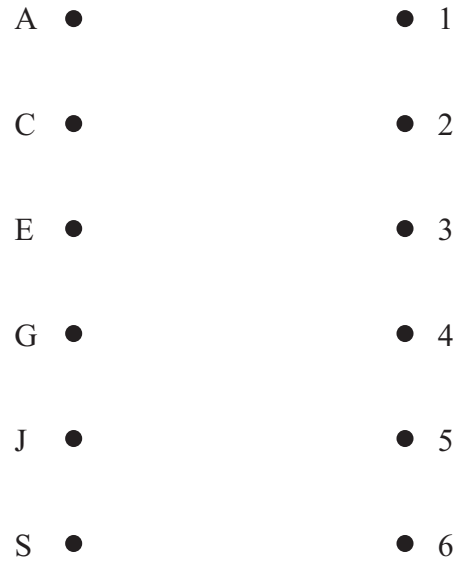
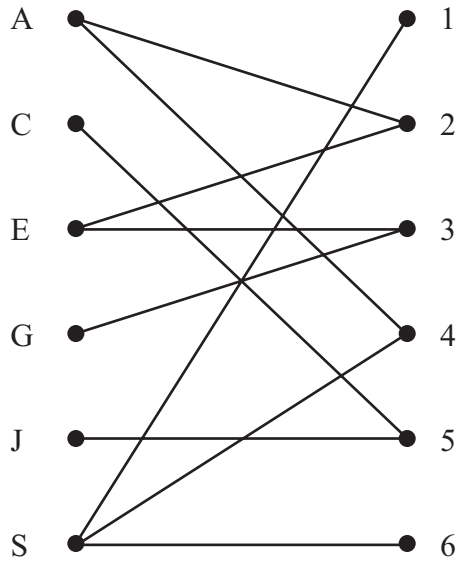


Figure 2

[The total weight of the network is 73.3 km]

Question 5 continued

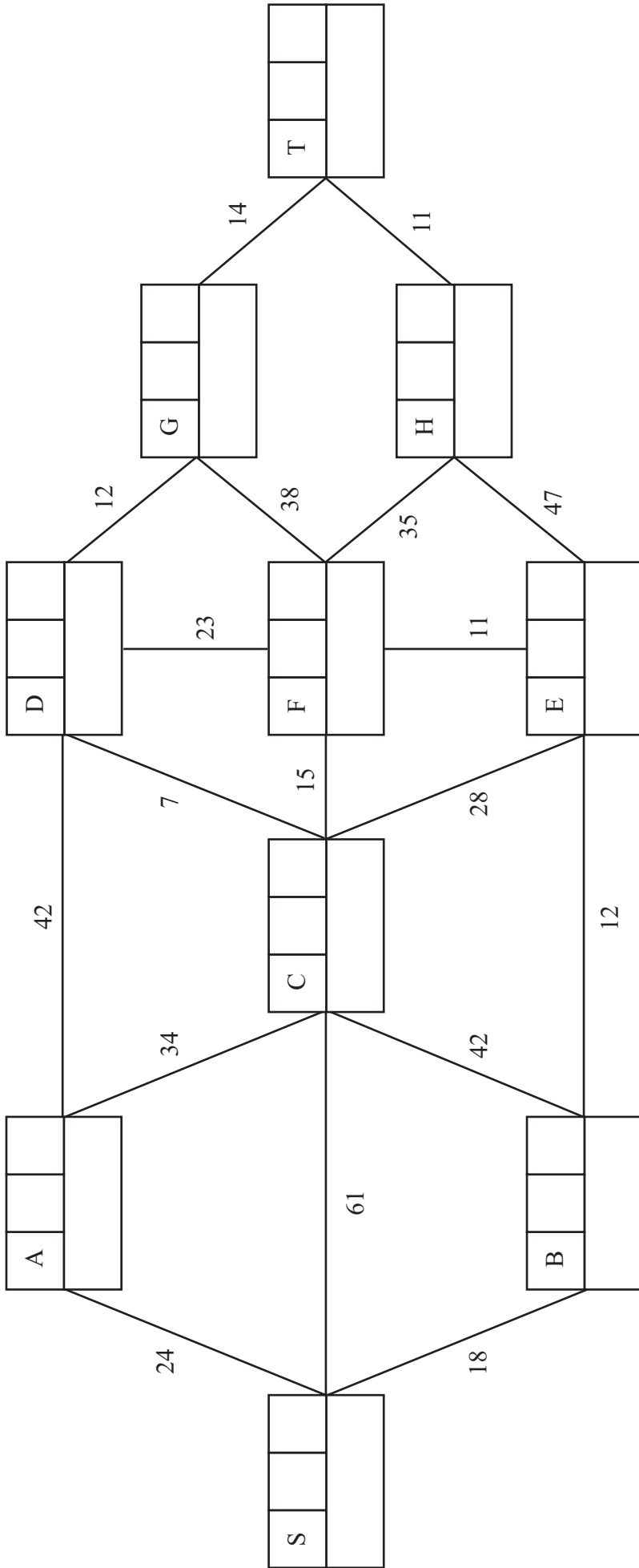


(Total 8 marks)

Q5

6.

(a)



Route S to T:

Time taken for route:

Question 6 continued

(b)

Blank lined area for writing answers to question 6(b).

(c) Route E to T:

(Total 9 marks)

Q6

Small empty box for marking.

7.

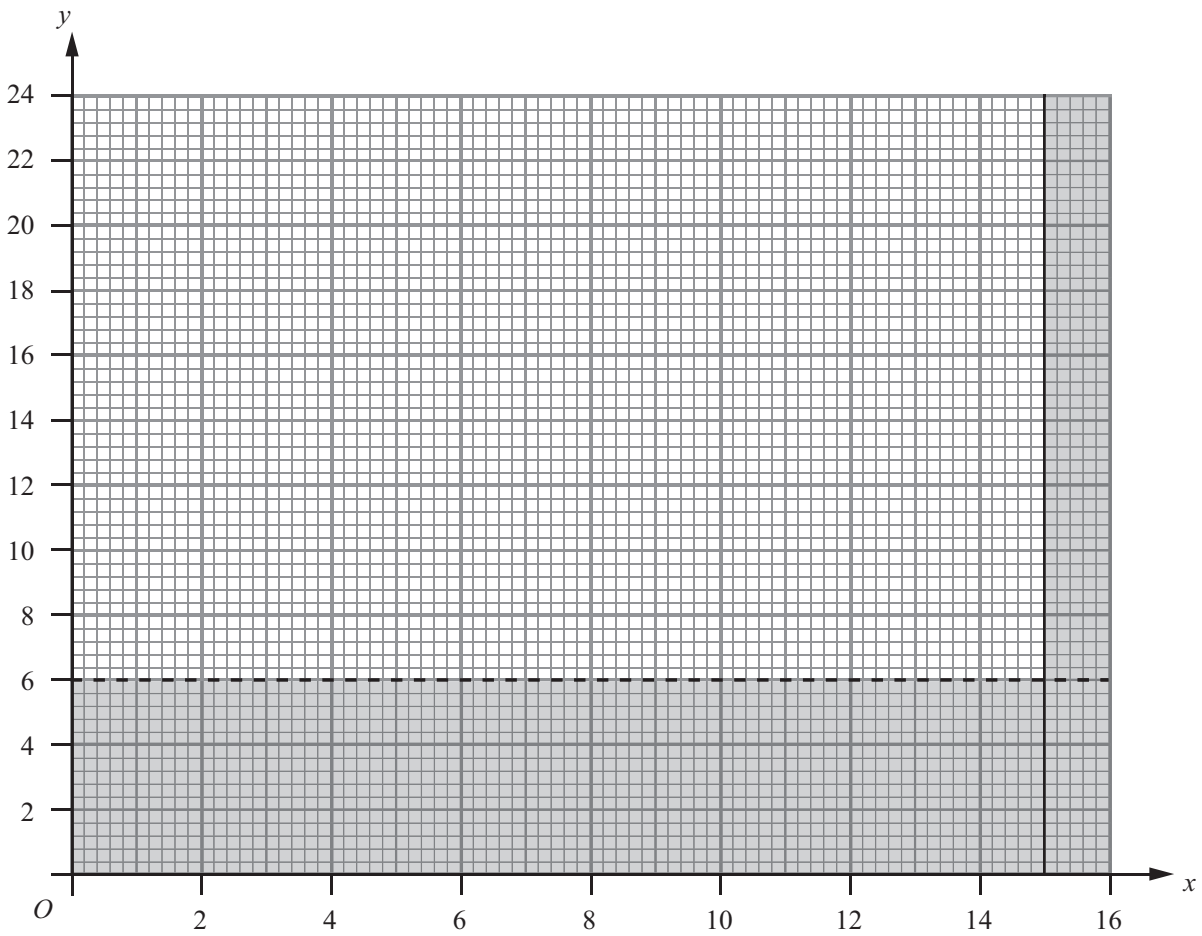


Diagram 1

8. (a)

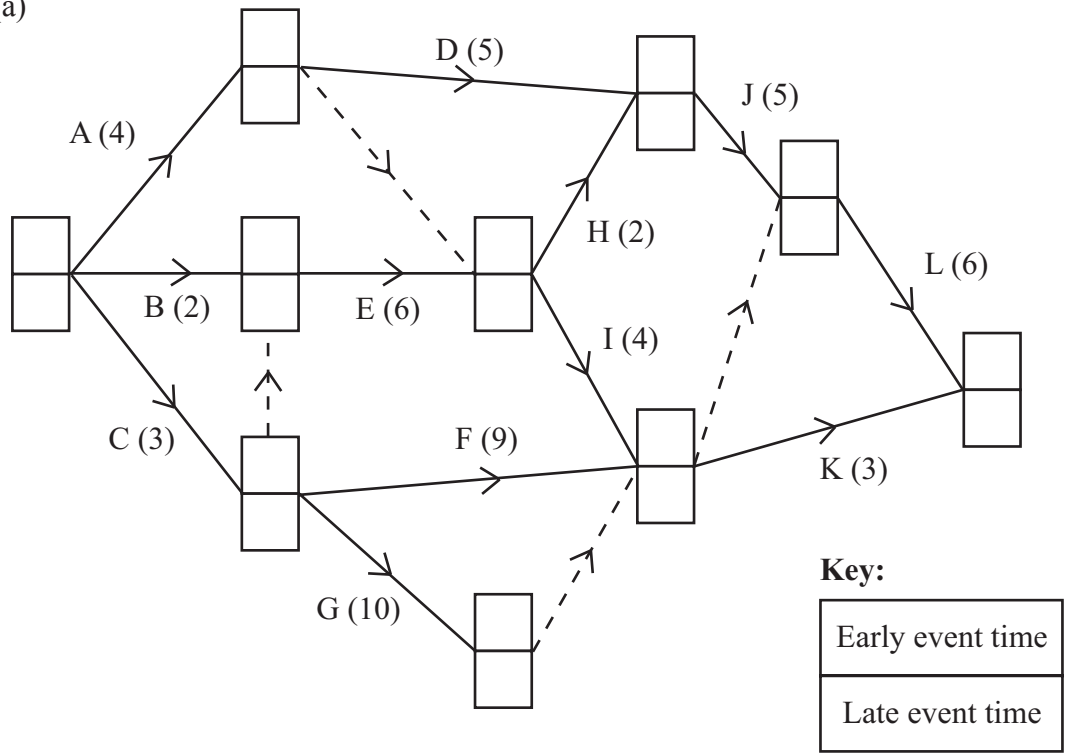
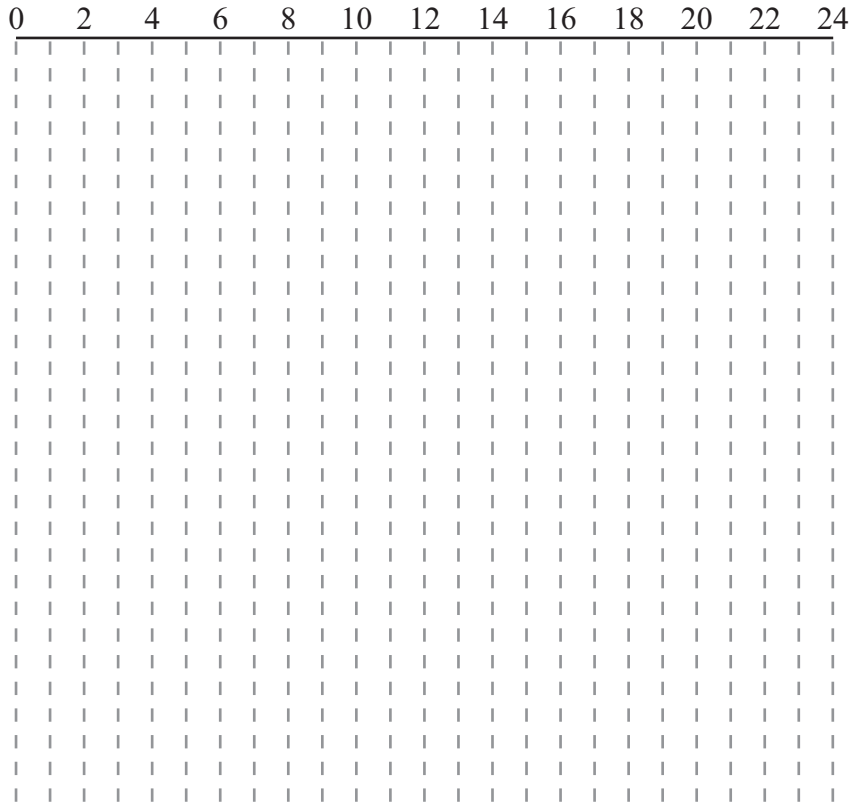


Diagram 2

(b) Critical activities: _____

(c)



Grid 1

(d)

(Total 11 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END

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WDM01/01: Decision Mathematics D1

Question Number	Scheme	Marks																																																		
<p>Q1</p> <p>(a)</p> <table border="1" data-bbox="268 246 798 436"> <tr><td>H</td><td>V</td><td>L</td><td>A</td><td>N</td><td>J</td><td>S</td><td>T</td><td>P</td><td>(N)</td></tr> <tr><td>H</td><td>L</td><td>A</td><td>J</td><td>N</td><td>V</td><td>S</td><td>T</td><td>P</td><td>(A, T)</td></tr> <tr><td>A</td><td>H</td><td>L</td><td>J</td><td>N</td><td>S</td><td>P</td><td>T</td><td>V</td><td>(L, P)</td></tr> <tr><td>A</td><td>H</td><td>J</td><td>L</td><td>N</td><td>P</td><td>S</td><td>T</td><td>V</td><td>(J)</td></tr> <tr><td>A</td><td>H</td><td>J</td><td>L</td><td>N</td><td>P</td><td>S</td><td>T</td><td>V</td><td></td></tr> </table> <p>(b)</p> <p>1st choice $\left[\frac{1+9}{2} \right] = 5$ Nicky, reject 1 - 5</p> <p>2nd choice $\left[\frac{6+9}{2} \right] = [7.5] = 8$ Tom, reject 8 - 9</p> <p>3rd choice $\left[\frac{6+7}{2} \right] = [6.5] = 7$ Sharon, reject 7</p> <p>4th choice 6 Paul name found</p> <p>Notes:</p> <p>(a) 1M1: quick sort, pivots, p, chosen and two sublists one <p one >p. 1A1: first pass correct and next pivots chosen correctly/consistently. 2A1ft: second pass correct, next pivots correctly/consistently chosen. 3A1: all correct, cso.</p> <p>(b) 1M1: binary search on what they think is a alphabetical list, choosing pivot, rejecting half list. 1A1: first pass correct, condone 'sticky' pivot here, bod generous 2A1: second pass correct, pivot rejected. 3A1: cso.</p> <p>Note: If incorrect list in (a) mark (b) as a misread.</p>	H	V	L	A	N	J	S	T	P	(N)	H	L	A	J	N	V	S	T	P	(A, T)	A	H	L	J	N	S	P	T	V	(L, P)	A	H	J	L	N	P	S	T	V	(J)	A	H	J	L	N	P	S	T	V		<p>M1 A1 A1ft A1cso</p> <p>M1A1 A1 A1cso</p> <p>Total 8</p>	<p>4</p> <p>4</p>
H	V	L	A	N	J	S	T	P	(N)																																											
H	L	A	J	N	V	S	T	P	(A, T)																																											
A	H	L	J	N	S	P	T	V	(L, P)																																											
A	H	J	L	N	P	S	T	V	(J)																																											
A	H	J	L	N	P	S	T	V																																												

WDM01/01: Decision Mathematics D1

Q1 Alternative solutions

Middle right

H	V	L	A	N	J	S	T	P	(N)	M1
H	L	A	J	N	V	S	T	P	(A T)	A1
A	H	L	J	N	S	P	T	V	(L P)	A1ft
A	H	J	L	N	P	S	T	V	(J)	
A	H	J	L	N	P	S	T	V		A1 cso

list sorted

Middle left

H	V	L	A	N	J	S	T	P	(N)	M1
H	L	A	J	N	V	S	T	P	(L S)	A1
H	A	J	L	N	P	S	V	T	(A V)	A1ft
A	H	J	L	N	P	S	T	V	(H)	
A	H	J	L	N	P	S	T	V		A1 cso

First

H	V	L	A	N	J	S	T	P	(H)	M1
A	H	V	L	N	J	S	T	P	(V)	A1
A	H	L	N	J	S	T	P	V	(L)	
A	H	J	L	N	S	T	P	V	(N)	A1ft
A	H	J	L	N	P	S	T	V	(S)	
A	H	J	L	N	P	S	T	V		A1 cso

Question Number	Scheme	Marks																																																																																	
<p>Q2</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>Notes:</p> <p>(a) 1M1: Kruskal’s algorithm – first 4 arcs selected chosen correctly. 1A1: All seven non-rejected arcs chosen correctly. 2A1: All rejections correct and in correct order and at correct time.</p> <p>(b) 1B1: condone two (double) errors 2B1: cao</p> <p>(c) 1M1: Prim’s algorithm – first four arcs chosen correctly, in order, or first five nodes chosen correctly, in order. {A,C,D,E,B....} 1A1: First six arcs chosen correctly or all 8 nodes chosen correctly, in order. {A,C,D,E,B,G,F,H} 2A1: All correct and arcs chosen in correct order.</p> <p>(d) 1B1: cao</p>	<p>DE GF DC $\left\{ \begin{matrix} \text{not CE} \\ \text{BD} \end{matrix} \right\}$ EG (not EF not CF) AC (not AB) GH</p> <table border="1" data-bbox="550 472 1043 801"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> </tr> </thead> <tbody> <tr> <th>A</th> <td>-</td> <td>31</td> <td>30</td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>-</td> </tr> <tr> <th>B</th> <td>31</td> <td>-</td> <td>-</td> <td>24</td> <td>-</td> <td>-</td> <td>-</td> <td>38</td> </tr> <tr> <th>C</th> <td>30</td> <td>-</td> <td>-</td> <td>22</td> <td>24</td> <td>29</td> <td>-</td> <td>-</td> </tr> <tr> <th>D</th> <td>-</td> <td>24</td> <td>22</td> <td>-</td> <td>18</td> <td>-</td> <td>-</td> <td>34</td> </tr> <tr> <th>E</th> <td>-</td> <td>-</td> <td>24</td> <td>18</td> <td>-</td> <td>28</td> <td>26</td> <td>-</td> </tr> <tr> <th>F</th> <td>-</td> <td>-</td> <td>29</td> <td>-</td> <td>28</td> <td>-</td> <td>21</td> <td>-</td> </tr> <tr> <th>G</th> <td>-</td> <td>-</td> <td>-</td> <td>-</td> <td>26</td> <td>21</td> <td>-</td> <td>33</td> </tr> <tr> <th>H</th> <td>-</td> <td>38</td> <td>-</td> <td>34</td> <td>-</td> <td>-</td> <td>33</td> <td>-</td> </tr> </tbody> </table> <p>AC CD DE BD GE GF GH</p> <p>Weight: 174</p>		A	B	C	D	E	F	G	H	A	-	31	30	-	-	-	-	-	B	31	-	-	24	-	-	-	38	C	30	-	-	22	24	29	-	-	D	-	24	22	-	18	-	-	34	E	-	-	24	18	-	28	26	-	F	-	-	29	-	28	-	21	-	G	-	-	-	-	26	21	-	33	H	-	38	-	34	-	-	33	-	<p>M1 A1 A1 3</p> <p>B2, 1, 0 2</p> <p>M1 A1 A1 3</p> <p>B1 1</p> <p>Total 9</p>
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Question Number	Scheme	Marks
Q3		
(a)	e.g. total weight is 239, lower bound is $\frac{239}{60} = 3.98$ so 4 bins.	M1 A1 2
(b)	Bin 1 : 41 Bin 4 : 36 Bin 2 : 28 + 31 Bin 5 : 32 Bin 3 : 42 Bin 6 : 29	M1 A1 A1 3
(c)	Full Bins : 28 + 32 31 + 29 The other 3 items (42, 41, 36) require 3 separate bins	M1 A1 2
(d)	There are 5 items over 30. No two of these 5 can be paired in a bin, so at least 5 bins will be required.	B2, 1, 0 2
Total 9		
Notes:		
(a) 1M1: Any correct statement, must involve calculation		
1A1: cao (accept 4 for both marks)		
(b) 1M1: Bins 1 and 2 correct and at least 6 values put in bins		
1A1: Bins 1,2,3 and 4 correct.		
2A1: All correct		
(c) 1M1: Attempt to find two full bins and allocate at least 6 values		
1A1: cao		
(d) 1B1: Correct argument may be imprecise or muddled		
(bod gets B1)		
2B1: A good, clear, correct argument.		
(They have answered the question ‘why?’)		
Misread in (b) First Fit Decreasing		
Bin 1: 42 Bin 2: 41 Bin 3: 36 Bin 4: 32 28 Bin 5: 31 29		
(Remove up to two A marks if earned – so M1 max in (b) if first 4 bins correct.)		

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Question Number	Scheme	Marks
<p>Q4</p> <p>(a) $BC + EG = 10.4 + 10.1 = 20.5$ smallest $BE + CG = 8.3 + 16.1 = 24.4$ $BG + CE = 14.9 + 11.9 = 26.8$</p> <p>So repeat tunnels BA, AC and EG</p> <p>(b) Any route e.g. ACFGDCABDEGEBA Length = $73.3 + \text{their } 20.5 = 93.8\text{km}$</p> <p>(c) The new tunnel would make C and G even. So only BE would need to be repeated. Extra distance would be $10 + 8.3 = 18.3 < 20.5$ [$91.6 < 93.8$] So it would decrease the total distance.</p> <p>Notes:</p> <p>(a) 1M1: Three pairings of their four odd nodes 1A1: one row correct 2A1: two rows correct 3A1: all correct 4A1: correct arcs identified</p> <p>(b) 1B1: Any correct route (14 nodes) 1M1: $73.3 + \text{ft their least, from a choice of at least two.}$ 1A1: cao</p> <p>(c) 1B1: A correct explanation, referring to BE and relevant numbers (8.3, 12.2, 2.2, 18.3, 81.3, 91.6) maybe confused, incomplete or lack conclusion –bod gets B1 2B1D: A correct, clear explanation all there + conclusion (ft on their numbers.)</p>		<p>M1 A1 A1 A1</p> <p>A1</p> <p style="text-align: right;">5</p> <p>B1 M1 A1</p> <p style="text-align: right;">3</p> <p>B1</p> <p>DB1</p> <p style="text-align: right;">2</p> <p style="text-align: right;">Total 10</p>

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Question Number	Scheme	Marks
<p>Q5</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>Notes:</p> <p>(a) 1M1: Path from G to 6 or 1 1A1: CAO including change status (stated or shown), chosen path clear. 2A1: CAO must fit from stated path, diagram ok</p> <p>(b) 1B1: Correct answer, may be imprecise or muddled (bod gets B1) all relevant nodes should be referred to and must be correct, but condone one (genuine) slip. 2B1: Good, clear, correct answer.</p> <p>(c) 1M1: Path from C to 1 or 6 [whichever they didn't use before.] 1A1: CAO including change status (stated or shown), chosen path clear. (Don't penalise change status twice.) 2A1: CAO must fit from stated path, diagram ok</p> <p>Alt</p> <p>(a) $G - 3 = E - 2 = A - 4 = S - 1$ c.s. $G = 3 - E = 2 - A = 4 - S = 1$ $A = 4$, (C unmatched), $E = 2$, $G = 3$, $J = 5$, $S = 1$</p> <p>(c) $C - 5 = J - 4 = A - 2 = E - 6$ c.s. $C = 5 - J = 4 - A = 2 - E = 6$ $A = 2$, $C = 5$, $E = 6$, $G = 3$, $J = 4$, $S = 1$</p>	<p>e.g.</p> <p>$G - 3 = E - 2 = A - 4 = S - 6$ Change status $G = 3 - E = 2 - A = 4 - S = 6$</p> <p>Improved matching $A = 4$ (C unmatched) $E = 2$ $G = 3$ $J = 5$ $S = 6$</p> <p>e.g. Both C and J can only be matched to 5 Both 1 and 6 can only be done by S</p> <p>$C - 5 = J - 4 = A - 2 = E - 6 = S - 1$ Change status $C = 5 - J = 4 - A = 2 - E = 6 - S = 1$</p> <p>Complete matching $A = 2$ $C = 5$ $E = 6$ $G = 3$ $J = 4$ $S = 1$</p>	<p>M1 A1 A1 3</p> <p>B2, 1, 0 2</p> <p>M1 A1 A1 3 Total 8</p>

Question Number	Scheme	Marks
<p>Q6</p> <p>(a)</p> <div data-bbox="292 286 1270 703" data-label="Diagram"> </div> <p>Route: SBEFHT Time: 87 minutes</p>	<p>(b) Accept demonstration of relevant subtractions, or general explanation.</p> <p>(c) Route: EFHT</p> <p>Notes:</p> <p>(a) 1M1: Smaller number replacing larger number in the working values at C or D or G or H or T. (generous – give bod) 1A1: All values in boxes S, A, B, E and F correct 2A1ft: All values in boxes C and D (ft) correct. Penalise order of labelling errors just once. 3A1: All values in boxes G, H and T correct 1B1: CAO (not ft) 2B1ft: Follow through from their T value, condone lack of units here.</p> <p>(b) 1B1ft: Partially complete account, maybe muddled, bod gets B1 2B1ft: Complete, clear account.</p> <p>(c) 1B1: CAO</p>	<p>M1 A1 A1ft A1</p> <p>B1 B1ft</p> <p style="text-align: right;">6</p> <p>B2ft, 1ft, 0</p> <p style="text-align: right;">2</p> <p>B1</p> <p style="text-align: right;">1</p> <p style="text-align: right;">Total 9</p>

Question Number	Scheme	Marks
<p>Q7</p> <p>(a) To indicate the strict inequality</p> <p>(b) $3x = 2y$ and $5x + 4y = 80$ added to the diagram. R correctly labelled.</p> <div data-bbox="316 436 1268 1220" style="text-align: center;"> <p>Diagram 1</p> </div> <p>(c) [Minimise $C =$] $500x + 800y$</p> <p>(d) Point testing or Profit line Seeking integer solutions (11, 7) at a cost of £ 11 100.</p>		<p>B1</p> <p style="text-align: right;">1</p> <p>B1, B1</p> <p>B1</p> <p style="text-align: right;">3</p> <p>B1, B1</p> <p style="text-align: right;">2</p> <p>M1 A1</p> <p>M1</p> <p>B1, B1</p> <p style="text-align: right;">5</p> <p>Total 11</p>

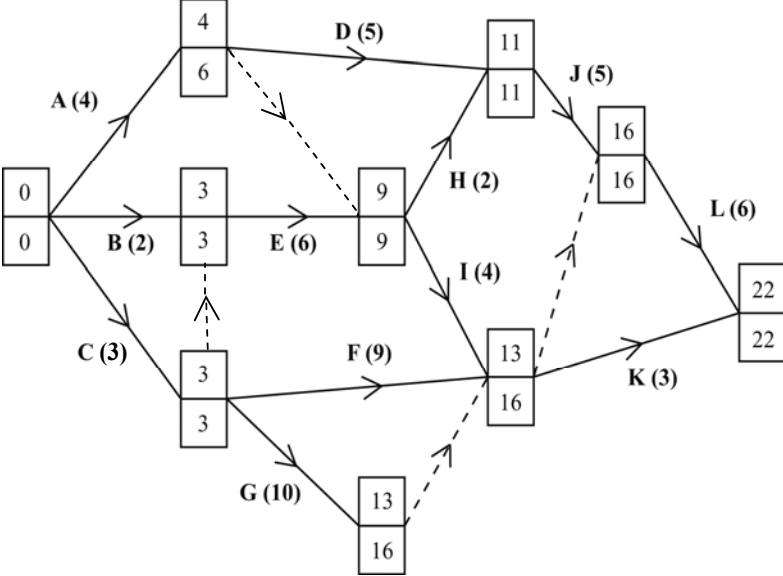
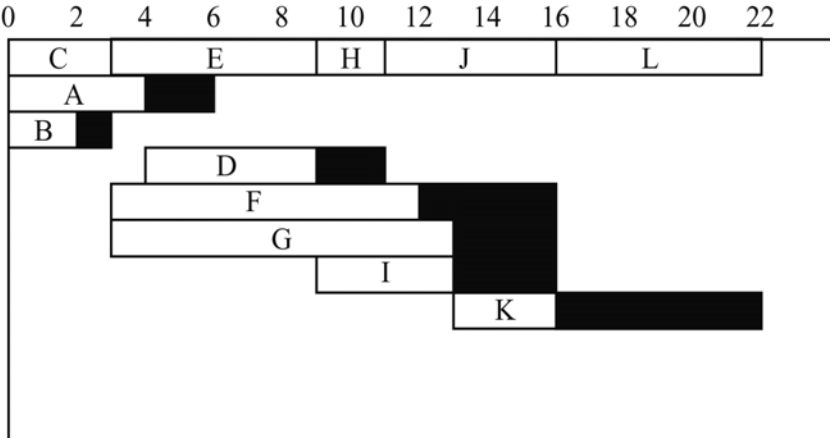
Notes:

- (a) 1B1: CAO
- (b) 1B1: $3x = 2y$ passing through 1 small square of (0,0) and (12, 18), but must reach $x = 15$
 2B1: $5x + 4y = 80$ passing through 1 small square of (0, 20) and (16, 0) (extended if necessary) but must reach $y = 6$
 3B1: R CAO (condoning slight line inaccuracy as above.)
- (c) 1B1: Accept expression and swapped coefficients. Accept $5x + 8y$ for 1 mark
 2B1: CAO (expression still ok here)
- (d) 1M1: Profit line [gradient accept reciprocal, minimum length line passes through (0, 2.5) (4, 0)] **OR** testing 2 points in their FR near two different vertices.
 1A1: Correct profit line **OR** 2 points correctly tested in correct FR (my points)

e.g

$(7\frac{3}{11}, 10\frac{10}{11}) = 12\,363\frac{7}{11}$	or	$(7, 11) = 12\,300$
		$(8, 10) = 12\,000$
		$(8, 11) = 12\,800$
$(11\frac{1}{5}, 6) = 10\,400$	or	$(11, 6) = 10\,300$
$(15, 6) = 12\,300$	or	$(15, 7) = 13\,100$
$(15, 22\frac{1}{2}) = 25\,500$	or	$(15, 22) = 25\,100$
		$(11, 7) = 11\,100$

- 2M1: Seeking integer solution in correct FR (so therefore no $y = 6$ points)
 1B1: (11,7) CAO
 2B1: £11 100 CAO

Question Number	Scheme	Marks
<p>Q8</p> <p>(a)</p>  <p>(b)</p> <p>Critical activities: C E H J L</p> <p>(c)</p>  <p>(d)</p> <p>4 workers needed e.g. at time $8 \frac{1}{2}$ (noon on day 9) activities E, D, F and G must be happening.</p>	<p>M1 A1 M1 A1</p> <p>4</p> <p>B1</p> <p>1</p> <p>M1 A1 A1 A1</p> <p>4</p> <p>B2, 1, 0</p> <p>2</p> <p>Total 11</p>	

Notes for Q8

- (a) 1M1: Top boxes completed generally increasing left to right.
1A1: CAO.
2M1: Bottom boxes completed generally decreasing right to left.
2A1: CAO.
- (b) 1B1: Critical activities cao.
- (c) 1M1: At least 10 activities placed, at least five floats. Scheduling diagram gets M0.
1A1: my critical activities correct.
2A1: condone one error on my non-critical activities.
3A1: my non-critical activities correct.
- (d) 1B1: A correct statement, details of either time ($7 < \text{time} < 9$, $8 < \text{day} < 10$), or activities, bod gets B1. Allow 1 B mark (only) on ft from their 12 activity, 7 float diagram.
2B1: A correct, complete full statement details of time and activities.

Notes on Marking Principles for all Mark Schemes

1. Types of mark

M marks: method marks

A marks: accuracy marks

B marks: unconditional accuracy marks (independent of M marks)

2. Abbreviations

cao - correct answer only

ft - follow through

isw - ignore subsequent working

SC: special case

oe - or equivalent (and appropriate)

dep - dependent

indep - independent

3. No working

If no working is shown then correct answers normally score full marks.

If no working is shown then incorrect (even though nearly correct) answers score no marks.

4. With working

If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

Any case of suspected misread loses A (and B) marks on that part, but can gain the M marks.

If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

If there is no answer on the answer line then check the working for an obvious answer.

5. Follow through marks

Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

6. Ignoring subsequent work

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

7. Probability

Probability answers must be given as fractions, percentages or decimals. If a candidate gives a decimal equivalent to a probability, this should be written to at least 2 decimal places (unless tenths).

Incorrect notation should lose the accuracy marks, but be awarded any implied method marks.

If a probability answer is given on the answer line using both incorrect and correct notation, award the marks.

If a probability fraction is given then cancelled incorrectly, ignore the incorrectly cancelled answer.

8. Linear equations

Full marks can be gained if the solution alone is given on the answer line, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.


9. Parts of questions

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

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