Pearson Edexcel

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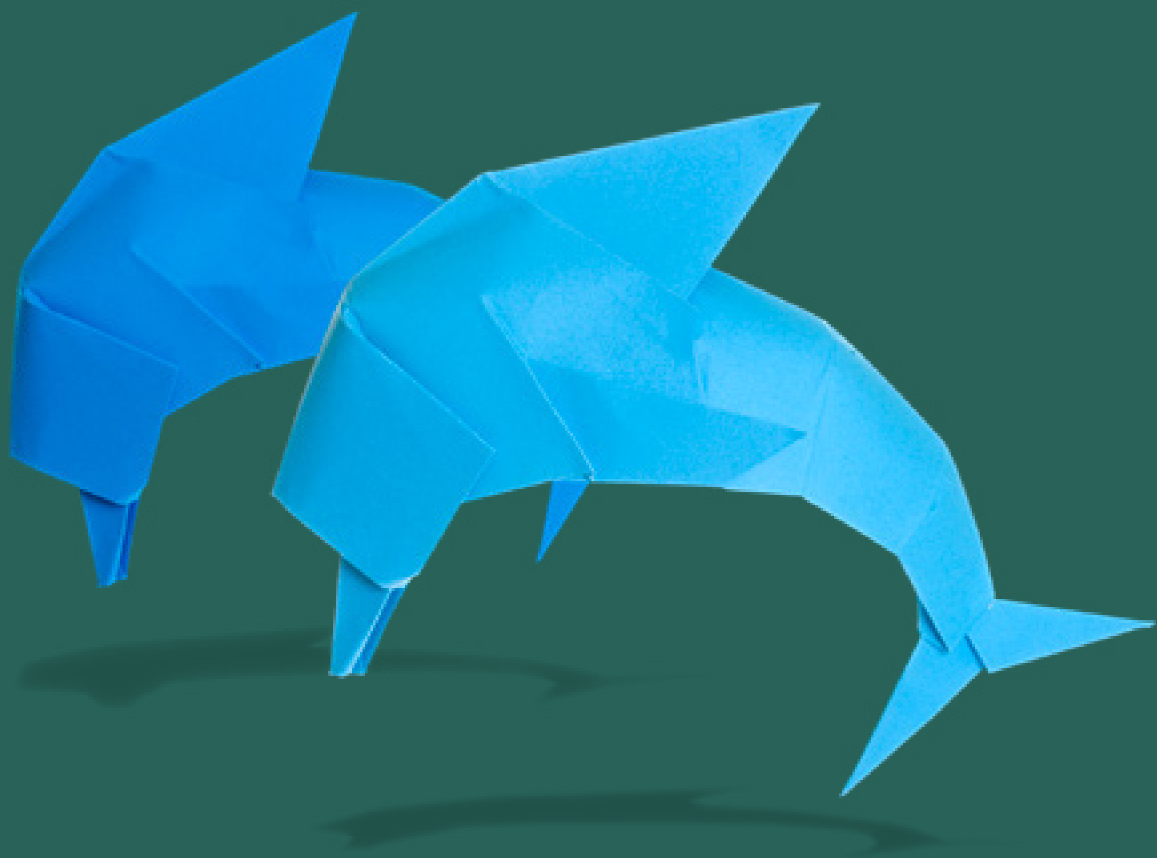
Level 1/Level 2 GCSE (9-1)

Mathematics (1MA1)



June 2019 - Exemplar

Student answers with examiner comments

Higher

First certification 2017

Contents

About this booklet [back to Contents page]

This booklet has been produced to support mathematics teachers delivering the new GCSE (9–1) Mathematics specification.

The booklet looks at a selection of questions from the Summer 2019 GCSE (9–1) Mathematics Higher tier examination. It shows real student responses to selected questions and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

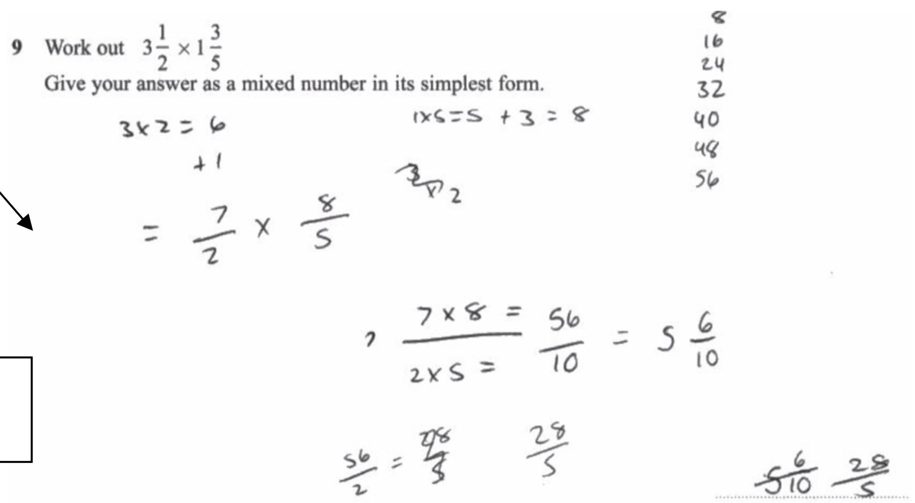
How to use this booklet

Our examining team have selected student responses to higher tier questions from the Summer 2019 examination. **Exemplification of common questions can be found in the Foundation tier exemplar document.**

Following each question, you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

Student response

Student response B



Mean score for each question

2/3

Mean score: 2.34

Examiner comment

Both fractions were converted to improper fractions and multiplied correctly to give $\frac{56}{10}$, so the two M1 marks were awarded. The fraction $\frac{56}{10}$ was written in its simplest form and also as the mixed number $5\frac{6}{10}$ but it was not written as a mixed number in its simplest form, so A1 could not be awarded.

Examiner commentary on the student response

Marks awarded for the question or question parts

Paper 1H (non-calculator) [back to Contents page]

Exemplar question 1

Higher tier Question 9

9 Work out $3\frac{1}{2} \times 1\frac{3}{5}$

Give your answer as a mixed number in its simplest form.

....

(Total for Question 9 is 3 marks)

Mean score: 2.34

Examiner comment

This was a routine question that assessed the ability of students to multiply two mixed numbers.

Students were expected to start by writing the mixed numbers as improper fractions, then multiplying correctly and giving the answer as a mixed number in its simplest form.

The requirement for the answer to be given as a mixed number in its simplest form meant that the final accuracy mark was often not awarded. Answers of $5\frac{6}{10}$ and $\frac{28}{5}$ were common.

Students needed to remember that it is not necessary to have a common denominator in order to multiply two fractions. When students chose to use a common denominator of 10 the calculation $\frac{35}{10} \times \frac{16}{10}$ was often followed by $\frac{560}{10}$ or by $\frac{51}{10}$ rather than by $\frac{560}{100}$.

Mark scheme

Question	Answer	Mark	Mark scheme
9	$5\frac{3}{5}$	M1	for writing as improper fractions with at least one correct, e.g. $\frac{7}{2} \times \frac{8}{5}$ oe
		M1	(dep) for multiplying improper fractions, e.g. $\frac{\text{“56”}}{\text{“10”}}$ **or** $5\frac{6}{10}$ **or** $\frac{28}{5}$ oe
		A1	cao

Examiner comment

The second M1 was dependent on the first M1, i.e. the mixed numbers in the question must have been written as improper fractions with at least one of them correct.

In the first example given for the second M1, “56” needed to be the product of the numerators of the improper fractions and “10” the product of the denominators.

The A1 is ‘cao’, so the only acceptable answer was $5\frac{3}{5}$.

Student response A

9 Work out $3\frac{1}{2} \times 1\frac{3}{5}$

Give your answer as a mixed number in its simplest form.

$$\frac{7}{2} \times \frac{8}{5} = \frac{56}{10} = 5\frac{6}{10}$$

$5\frac{3}{5}$

3/3

Examiner comment
The student showed a complete method, giving the final answer as a mixed number in its simplest form.

Student response B

9 Work out $3\frac{1}{2} \times 1\frac{3}{5}$

Give your answer as a mixed number in its simplest form.

8
16
24
32
40
48
56

$3 \times 2 = 6$
$+1$

$1 \times 5 = 5 + 3 = 8$

$= \frac{7}{2} \times \frac{8}{5}$

$\frac{7 \times 8 =}{2 \times 5 =} \frac{56}{10} = 5\frac{6}{10}$

$\frac{56}{2} = \frac{28}{5}$ $\frac{28}{5}$

$\cancel{5\frac{6}{10}}$ $\frac{28}{5}$

2/3

Examiner comment
Both fractions were converted to improper fractions and multiplied correctly to give $\frac{56}{10}$, so the two M1 marks were awarded. The fraction $\frac{56}{10}$ was written in its simplest form and also as the mixed number $5\frac{6}{10}$ but it was not written as a mixed number in its simplest form, so A1 could not be awarded.

Student response C

9 Work out $3\frac{1}{2} \times 1\frac{3}{5}$

Give your answer as a mixed number in its simplest form.

$$\frac{7}{2}_{\times 5} \times \frac{8}{5}_{\times 2} = \frac{16}{10} \ast \frac{35}{10} = \frac{51}{10}$$

$$5\frac{1}{10}$$

$5\frac{1}{10}$

1/3

Examiner comment

The two fractions were written as improper fractions. For the first M1 at least one must be correct and in this response both are correct. The student then decided to use a common denominator but unfortunately added the two fractions instead of multiplying them, so no further marks were awarded.

Exemplar question 2

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Higher tier Question 11

11 A bus company recorded the ages, in years, of the people on coach A and the people on coach B.

Here are the ages of the 23 people on coach A.

41	42	44	48	52	53	53	53	56	57	57	59
60	61	63	64	64	66	67	69	74	77	79	

(a) Complete the table below to show information about the ages of the people on coach A.

Median	
Lower quartile	
Upper quartile	
Least age	41
Greatest age	79

(2)

Here is some information about the ages of the people on coach B.

Median	70
Lower quartile	54
Upper quartile	73
Least age	42
Greatest age	85

Richard says that the people on coach A are younger than the people on coach B.

(b) Is Richard correct?
You must give a reason for your answer.

....

....

....

(1)

Richard says that the people on coach A vary more in age than the people on coach B.

(c) Is Richard correct?
You must give a reason for your answer.

....

....

....

(1)

(Total for Question 11 is 4 marks)

Mean score: 2.75

Examiner comment

Note that exemplar responses are given for parts (b) and (c) only.

Parts (b) and (c) assessed the ability of students to compare distributions and make inferences, using appropriate measures of central tendency and spread.

Students needed to decide whether Richard is correct and give a reason for their decision that included a comparison of relevant statistical values. A simple comparison that one value is higher or lower than the other was sufficient. When students were attempting to compare statistical values, they needed to avoid ambiguous statements using words such as 'but' or 'whereas'.

A common mistake was to state the correct statistical values, e.g. the medians in part (b), but fail to provide a comparison between them.

Some students selected the wrong statistical values to compare. In part (b), for example, the greatest ages and the least ages were often used.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
11(b)	Yes, with reason	C1	for Yes and comment comparing median ages, ft from (a) **Acceptable examples** "59" < 70 All statistics/values are lower for coach A (so they are younger) Median is lower The middle age is lower on coach A **Not acceptable examples** Median is higher Median for coach A is "59" and coach B is 70 The oldest on coach A is 79 and the oldest on coach B is 85 There are people on coach B that are older than on coach A	

Question	Answer	Mark	Mark scheme	Additional guidance
(c)	No, with reason	C1	for No and comment comparing spreads of ages from ranges or IQRs, ft from (a) **Acceptable examples** 38 < 43 **or** "13" < 19 Greater difference between greatest and least age for coach B Range for coach B is larger than coach A The range of ages is wider on coach B than on coach A The range is 5 greater on coach B There is a smaller difference between the lower and upper quantiles on coach A than on coach B The IQR is shorter for coach A **Not acceptable examples** Quartiles are less for coach A 53 < 54 **or** 79 < 85 (oe) Range for coach A is 38 and range for coach B is 43 Coach A ranges from 41–79 but coach B ranges from 42–85	Working A: Range = 38, IQR = "13" B: Range = 43, IQR = 19

Examiner comment

In both parts (b) and part (c), a correct decision and a correct reason were needed. Decisions that differed from the mark scheme could be correct following through from the values the student wrote in the table in part (a).

The reason needed include a relevant comparison. The mark scheme gave examples of reasons that were acceptable and reasons that were not acceptable. Values did not need to be given as part of a reason but any values quoted needed to be correct or follow through correctly from part (a) to be awarded the mark.

Student response A

Richard says that the people on coach A are younger than the people on coach B.

(b) Is Richard correct?
You must give a reason for your answer.

He is correct because the median for the ages of coach B is higher.

(1)

1/1

Richard says that the people on coach A vary more in age than the people on coach B.

(c) Is Richard correct?
You must give a reason for your answer.

~~the~~ 73 − 54 = 19 → IQR for coach B 66 − 53 = 13 → IQR for coach A. he's wrong because the ~~coa~~ ~~the~~ inter quartile ranges for coach B is higher.

(1)

1/1

Examiner comment

Part (b): The student gave the correct decision and valid comparison of the medians.

Part (c): The student gave the correct decision and a comparison of the interquartile ranges with values given. Values were not required but any values given needed to be correct to gain the mark.

Student response B

Richard says that the people on coach A are younger than the people on coach B.

(b) Is Richard correct?
You must give a reason for your answer.

He is ~~in~~correct as the median age for the people on coach A is lower than on coach B and the lowest and greatest age are lower on coach A.

(1)

1/1

Richard says that the people on coach A vary more in age than the people on coach B.

(c) Is Richard correct?
You must give a reason for your answer.

No he is incorrect as the range of ages for coach B which means there is a greater variation in age.

(1)

0/1

Examiner comment
Part (b): The student gave the correct decision and a valid comparison of the medians. Any reference to other comparative values (in this case, the least ages and greatest ages) could be ignored unless there was a contradiction.
Part (c): The student gave the correct decision, but the reason was incomplete. Although they mentioned the range of ages for coach B, they did not compare the ranges.

Student response C

Richard says that the people on coach A are younger than the people on coach B.

(b) Is Richard correct?
You must give a reason for your answer.

No because the median for coach A is 70 and the median for coach B is 59

(1)

0/1

Richard says that the people on coach A vary more in age than the people on coach B.

(c) Is Richard correct?
You must give a reason for your answer.

Yes because they have got people in their ~~50s, 60~~ 40, 50, 60, 76 whereas Coach B doesnt have 60.

(1)

0/1

Examiner comment
Part (b): The student gave the incorrect decision. Even if the correct decision had been made the mark would still not have been awarded because the reason was incomplete. The student gave the correct values for the medians but did not compare them by stating or implying that one was smaller or greater.
Part (c): The student gave the incorrect decision and the reason did not compare the spreads of the ages.

Exemplar question 3

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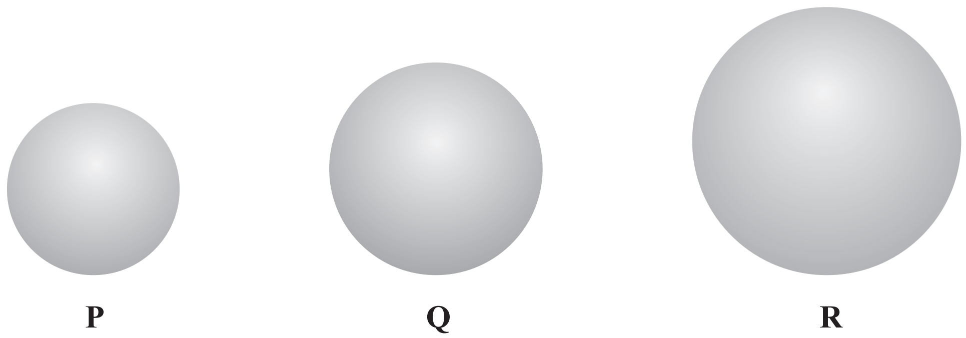
Higher tier Question 12

12 Here are three spheres.



The volume of sphere **Q** is 50% more than the volume of sphere **P**.
The volume of sphere **R** is 50% more than the volume of sphere **Q**.

Find the volume of sphere **P** as a fraction of the volume of sphere **R**.

....

(Total for Question 12 is 3 marks)

Mean score: 1.27

Examiner comment

In this question the ability of students to make deductions from mathematical information was assessed. This was within the context of percentage change. Students were required to express a multiplicative relationship between two quantities as a fraction.

A typical approach was to assign a value to the volume of **P** and use it to work out the volumes of **Q** and **R**, for example **P** = 100, **Q** = 150, **R** = 225. A different approach was to work out that the multiplier from **P** to **R** is 1.5×1.5. Whichever method was used, the final challenge was to write the volume of **P** as a fraction of the volume of **R**. Students should not forget to write down a fraction in which both the numerator and denominator are integers.

A common misconception was to assume that the volume of **Q** is 50 % less than the volume of **R** and the volume of **P** is 50 % less than the volume of **Q** (writing, for example, **R** = 100, **Q** = 50, **P** = 25), giving an answer of $\frac{1}{4}$.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
12	$\frac{4}{9}$	P1	for process to find link between volume of Q and volume of P **or** between volume of R and volume of Q, e.g. ratio 1.5 : 1 **or** Q = 1.5P **or** P = $\frac{2}{3}$Q **or** two values in the ratio 1 : 1.5 such as 100 and 150	
		P1	for process to find link between volume of R and volume of P e.g. $1.5^2 : 1$ **or** two values in the ratio 1 : 2.25 such as 100 and 225	1.5^2 (=$\frac{9}{4}$) is enough for this mark, award P1 P1
		A1	for $\frac{4}{9}$ oe fraction e.g. $\frac{100}{225}$	Accept P = $\frac{4}{9}$R

Examiner comment

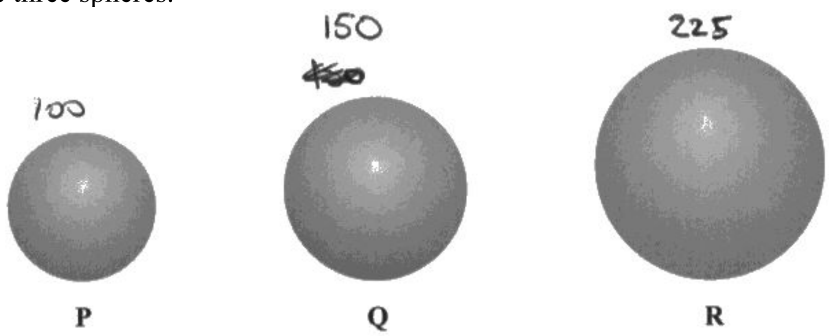
The first P1 was awarded for a link between the volume of **Q** and the volume of **P**, or between the volume of **R** and the volume of **Q**. This could be a statement such as **Q** = 1.5 **P** or two values in the ratio 1 : 1.5, such as 100 and 150 given for the volumes of sphere **P** and sphere **Q** respectively. These values could be written on the spheres in the diagram.

The second P1 was awarded for finding a link between volume of **R** and volume of **P**. Sight of 1.5^2 or 2.25 or $\frac{9}{4}$ was sufficient for this mark. Two values in the ratio 1 : 2.25, such as 100 and 225, could be given for the volumes of sphere **P** and sphere **R** respectively.

A1 was awarded for an answer of $\frac{4}{9}$ or an equivalent fraction such as $\frac{100}{225}$ or $\frac{20}{45}$. Fractions with decimals in the numerator or denominator, for example $\frac{1}{2.25}$, were not acceptable. If an acceptable fraction such as $\frac{100}{225}$ was seen then any subsequent incorrect attempt at simplifying it was ignored.

Student response A

12 Here are three spheres.



The volume of sphere **Q** is 50% more than the volume of sphere **P**.
The volume of sphere **R** is 50% more than the volume of sphere **Q**.

Find the volume of sphere **P** as a fraction of the volume of sphere **R**.

P = 100% × 1.5
Q = 150% × 1.5
R = 225%

$\frac{20}{45} = \frac{4}{9}$

$\frac{100}{225} = \frac{4}{9}$

$\frac{4}{9}$

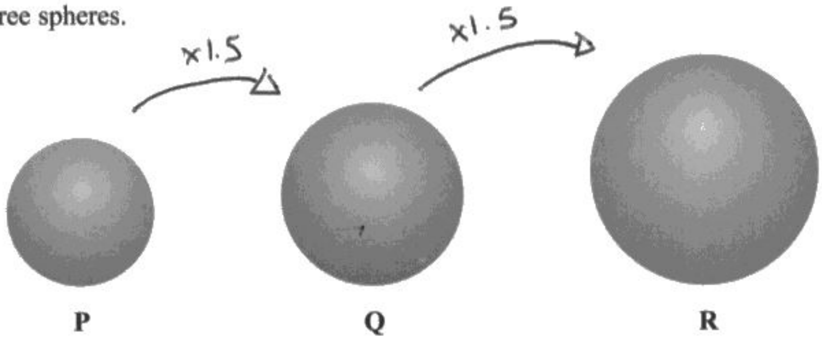
3/3

Examiner comment

The student used the common approach of assigning a value to the volume of **P** and using it to work out the volumes of **Q** and **R**. Not only were the values 150 and 225 given, but the student also showed the working that led to them. The fraction $\frac{100}{225}$ would have been sufficient for A1, although in this response the student simplified it correctly to $\frac{4}{9}$.

Student response B

12 Here are three spheres.



The volume of sphere **Q** is 50% more than the volume of sphere **P**.
The volume of sphere **R** is 50% more than the volume of sphere **Q**.

Find the volume of sphere **P** as a fraction of the volume of sphere **R**.

$1\frac{1}{2} \times 1\frac{1}{2}$

$\frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$

$\frac{9}{4}$

2/3

Examiner comment

The student found a link between the volume of **P** and the volume of **R**. Either 1.5×1.5 or $\frac{9}{4}$ was sufficient for the two P1 marks. The final answer was incorrect because the student gave the volume of **R** as a fraction of the volume of **P**.

Student response C

12 Here are three spheres.

P **Q** **R**

The volume of sphere **Q** is 50% more than the volume of sphere **P**.
The volume of sphere **R** is 50% more than the volume of sphere **Q**.

Find the volume of sphere **P** as a fraction of the volume of sphere **R**.

$Q = P \times 1.5$

$R = Q \times 1.5$

$\therefore$

$R = 3 \times P$

$\frac{P}{0.3} = R$

$\frac{P}{0.3} = R$

1/3

Examiner comment

The first P1 was awarded for the process to find a link between the volume of **Q** and volume of **P** (**Q** = **P** × 1.5) or between the volume of **R** and the volume of **Q** (**R** = **Q** × 1.5).

There was insufficient evidence of a process to find the link between the volume of **R** and the volume of **P**. If the student had written **R** = 1.5 × 1.5 × **P** before writing **R** = 3 × **P** then the second P1 could have been awarded.

Exemplar question 4

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Higher tier Question 13

13 Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

(Total for Question 13 is 2 marks)

Mean score: 0.34

Examiner comment

This question assessed the ability of students to present a proof, an assessment objective strand that is tested in higher tier papers.

Students were expected to approach the proof in one of two ways. One possible strategy was to factorise $n^2 - n$ to $n(n - 1)$ and explain that $n(n - 1)$ must be even because either n or $n - 1$ must be even. The other approach was to give correct reasoning for both n even, e.g. even × even = even and even – even = even, and n odd, e.g. odd × odd = odd and odd – odd = even. In this approach, students needed to remember to give reasoning for both n even and n odd.

Some students chose to use general expressions for even numbers and odd numbers such as $2n$ and $2n + 1$. This was not a wise choice because this approach was unnecessarily time consuming for a 2-mark question and also tended to be error prone. For n odd, for example, it was necessary to simplify $(2n + 1)^2 - (2n + 1)$ to $4n^2 + 2n$ and then show that $4n^2 + 2n$ is always even. Mistakes in simplifying and factorising were common. Reasoning for n even often went wrong at the first step, with students writing $2n^2 - 2n$ rather than $(2n)^2 - 2n$.

A common misconception was that this type of problem could be solved using a numerical approach. Many students simply substituted different values of n into $n^2 - n$ and stated that the result is never odd. Such responses gained no marks.

Mark scheme

Question	Answer	Mark	Mark scheme
13	Proof shown	C2 (C1	for complete argument, e.g. $n(n-1)$ is the product of two consecutive integers and must be even as either n or $n-1$ must be even **or** gives correct reasoning for n odd **and** n even n odd: odd × odd = odd and odd – odd = even n even: even × even = even and even – even = even **or** n odd: $(2n+1)^2-(2n+1)=4n^2+2n=2(2n^2+n)$ n even: $(2n)^2-(2n)=4n^2-2n=2(2n^2-n)$ for factorising, e.g. $n(n-1)$ **or** gives correct reasoning for n odd **or** n even **or** gives a partial explanation using n odd **and** n even, e.g. odd^2 – odd = even and even^2 – even = even)

Examiner comment

For C2 to be awarded, the argument needed to be complete. If n^2-n was factorised to $n(n-1)$ then there also needed be an explanation as to why $n(n-1)$ is always even. Alternatively, the student could have given correct reasoning for both n even and n odd as exemplified in the mark scheme. When general expressions such as $2n$ and $2n+1$ were used, the algebra needed to be fully correct.

If the argument was not complete, then C1 could be awarded for factorising to $n(n-1)$ or for correct reasoning for either n even or n odd. A student who used both n even and n odd but only gave a partial explanation such as odd^2 – odd = even and even^2 – even = even would be awarded C1.

The substitution of values into n^2-n gained no credit and was ignored.

Student response A

13 Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

n = odd

odd number squared = odd number

odd number - odd number = even

n = even

even number squared = even number

even number - even number = even

∴ Always even, never odd

2/2

Examiner comment
The student gave a complete argument, with correct reasoning for both n odd and n even.

Student response B

13 Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

$n^2 - n =$

$n(n-1) =$

$n \times (n-1)$ ∴ odd since, any integer multiplied by it's previous number is always even.
e.g $8 \times 7 = 56$ and $10 \times 9 = 90$ . . .

1/2

Examiner comment

Factorising $n^2 - n$ to $n(n-1)$ WAS sufficient for C1. The argument that followed was incomplete. Stating that ‘any integer multiplied by its previous number is always even’ did not explain **why** it is always even, i.e. because one of the numbers must be even.

Student response C

13 Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

$$(2n+1)^2 - 2n + 1 = 4n^2 + 1 + 4n - 2n + 1$$
$$= 4n^2 + 2n + 2$$
$$= 2(2n^2 + n + 1)$$

~~$(2n)^2 - 2n = 4n^2 - 2n$~~

~~$= 2(\ldots)$~~

Multiplied by 2 so ~~even~~ never odd number

~~2n + 1 always is~~

0/2

Examiner comment

The student made a correct start by using a general expression for an odd number, but the result of simplifying $(2n + 1)^2 - (2n + 1)$ was incorrect, probably because of the absence of brackets around the $2n + 1$ that was being subtracted. No reasoning was given for n even.

Exemplar question 5

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Higher tier Question 14

14 Find the exact value of tan 30° × sin 60°
Give your answer in its simplest form.

....

(Total for Question 14 is 2 marks)

Mean score: 0.70

Examiner comment

This question assessed knowledge and use of exact trigonometric values. Students needed to recall (or be able to work out) the exact values of tan 30º and sin 60º. They were expected to be able to use surds in a simple calculation and give the answer in its simplest form.

Different strategies were used to find the values of tan 30º and sin 60º. Some students worked them out by drawing an appropriate triangle; some used patterns in a table with angles of 0º, 30º, 45º, 60º and 90º; and some simply recalled them from memory. Students need to remember to label their values so it was clear which was tan 30º and which was sin 60º. The strategies used were not always successful and incorrect values were very common.

Some of the students who wrote $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$ did not go on to get full marks because they gave $\frac{\sqrt{3}}{2\sqrt{3}}$ as the final answer or simplifed it incorrectly to 2.

Mark scheme

Question	Answer	Mark	Mark scheme
14	$\frac{1}{2}$	M1	for $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$ **or** $\frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2}$ **or** $\left(\frac{1}{2} \div \frac{\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{2}$ **OR** tan 30 = $\frac{1}{\sqrt{3}}$ oe **or** sin 60 = $\frac{\sqrt{3}}{2}$
		A1	for $\frac{1}{2}$ or 0.5

Examiner comment

A correct value for either tan 30º or sin 60º was sufficient for M1. The value of tan 30º could be given as $\frac{1}{\sqrt{3}}$ or equivalent, for example $\frac{\sqrt{3}}{3}$. If only one of the values, $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{2}$, was correct and the values were not labelled then it was assumed that the order in any calculation shown was tan 30º × sin 60º.

The final answer needed to be $\frac{1}{2}$ or 0.5 since the question asked for the answer to be given in its simplest form. If the 'correct' answer was the result of using two incorrect values such as tan 30º = 1 and sin 60º = 0.5 then A1 was not awarded.

Student response A

14 Find the exact value of tan 30° × sin 60°
Give your answer in its simplest form.

TOA SOH

$\tan 30 = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{3}{6} = \frac{1}{2}$

$\frac{1}{2}$

2/2

Examiner comment

The student worked out the exact values of tan 30º and sin 60º by drawing an appropriate triangle and used these values correctly. The answer was given in its simplest form.

Student response B

14 Find the exact value of $\tan 30° \times \sin 60°$
Give your answer in its simplest form.

	0	30	45	60	90
Sin	0	1	2	3	4
Cos	4	3	2	1	0

$$\frac{\sqrt{}}{2}$$

$\tan 30 = \frac{1}{\sqrt{3}}$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{2}{3}$

$\sqrt{3} \times \sqrt{3} = 3$

$1 \times 2 = 2$

$\frac{2}{3}$

1/2

Examiner comment

The student used a table to find the exact values of tan 30º and sin 60º. Just one of these values was sufficient for M1. The multiplication was then carried out incorrectly.

Student response C

14 Find the exact value of tan 30° × sin 60°
Give your answer in its simplest form.

$\tan(30°)$ $\quad\quad$ $\sin(60)°$

$$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\frac{\sqrt{3}}{2\sqrt{2}}$$

0/2

Examiner comment

Neither the value of tan 30º nor the value of sin 60º was correct. The student did use $\frac{\sqrt{3}}{2}$ but linked this value to tan 30º, not to sin 60º.

Exemplar question 6

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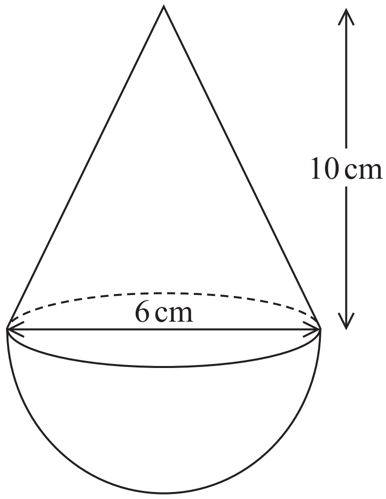
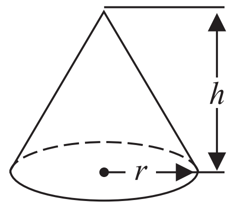
Higher tier Question 15

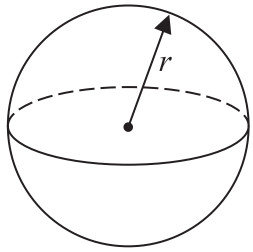
15 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.



Volume of a cone = $\frac{1}{3}\pi r^2 h$



Volume of a sphere = $\frac{4}{3}\pi r^3$



The height of the cone is 10 cm.
The base of the cone has a diameter of 6 cm.
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

k = ...

(Total for Question 15 is 4 marks)

Mean score: 2.29

Examiner comment

This question assessed the use of volume formulae to find the volume of a composite solid. The question also assessed the ability of students to calculate exactly with multiples of π.

Students were expected to start by substituting correct values into the given formulae for the volume of a cone and the volume of a sphere. Some students made the mistake at this stage of using a radius of 6 cm instead of 3 cm for one solid or both solids. It was also not uncommon to see 3^2 instead of 3^3 substituted into the formula for the volume of a sphere.

Students often struggled to simplify their expressions for volume correctly. Some had difficulty dealing with the fractions in calculations such as $\frac{4}{3} \times 27$ or $\frac{1}{2} \times \frac{4}{3} \times 27$ and arithmetic errors, e.g. $3^3 = 9$, were also frequent.

A very common error was for students to work out the total volume of a cone and a sphere, not the total volume of a cone and a hemisphere.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
15	48	M1	for method to use a volume formula with correct substitution for the cone, sphere or hemisphere e.g. $\frac{1}{3} \times \pi \times 3^2 \times 10$ **or** $\frac{4}{3} \times \pi \times 3^3$ **or** $\frac{2}{3} \times \pi \times 3^3$ oe	May work without π or with an approximation of π; must use the correct radius of 3 (and 10) in substitution
		M1	for complete method to find total volume e.g. $\frac{1}{3} \times \pi \times 3^2 \times 10 + \frac{2}{3} \times \pi \times 3^3$	
		M1	(dep first M1) for correct partial simplification, e.g. 30π **or** 18π	Must be cone or hemisphere
		A1	cao SC B2 for answer of 264 or 264π	Accept 48π

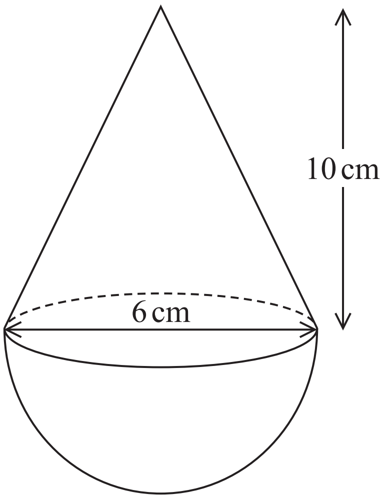
Examiner comment

The first M1 was for substituting correctly into a volume formula for a cone, a sphere or a hemisphere. The student could work without π or with an approximation of π. A complete method to find the total volume of the cone and the hemisphere was needed for the second M1. The third M1 was dependent on the first M1 and was awarded for simplifying the volume of the cone to 30π or the volume of the hemisphere to 18π. Either 48 or 48π was acceptable for A1.

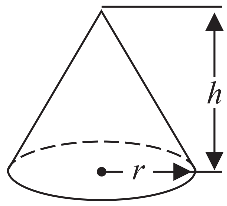
Students who used a diameter of 6 cm rather than 3 cm for both the cone and the hemisphere were awarded 2 marks for an answer of 264 or 264π.

Student response A

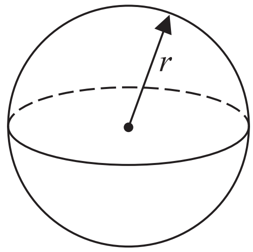
15 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.



Volume of a cone $= \frac{1}{3}\pi r^2 h$



Volume of a sphere $= \frac{4}{3}\pi r^3$



The height of the cone is 10 cm.
The base of the cone has a diameter of 6 cm.
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

cone vol $= \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3} \times \pi \times 9 \times 10$

$= \frac{1}{3} \times \pi \times 90$

$= 30\pi$ cm^3

sphere vol $= \frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3} \times \pi \times 27$

$27 \div 3 = 9$

$9 \times 4 = 36$

$= 36\pi$

$\div 2$

$= 18\pi$

$30\pi + 18\pi = 48\pi$

$k =$ 48

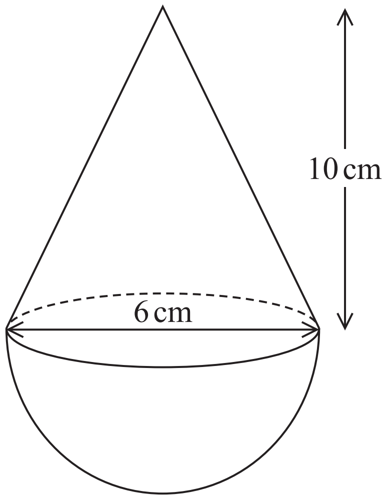
4/4

Examiner comment

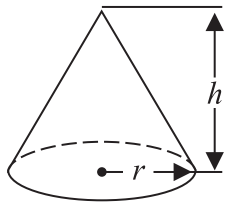
This was a fully correct and well-presented solution.

Student response B

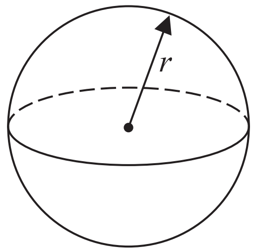
15 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.



Volume of a cone $= \frac{1}{3}\pi r^2 h$



Volume of a sphere $= \frac{4}{3}\pi r^3$



The height of the cone is 10 cm.
The base of the cone has a diameter of 6 cm.
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

$\frac{1}{3}\pi r^2 h$ $\quad$ $\frac{1}{3} \times \pi \times 3^2 \times 10$

$\frac{1}{3} \times \pi \times 90$

30π

volume of cone $= 30\pi$

$\frac{4}{3} \times \pi \times 3^3$

$\frac{4}{3} \times \pi \times 27$ $\quad$ 36π

$30\pi + 36\pi = 66\pi$ cm^3

$k =$ 66

2/4

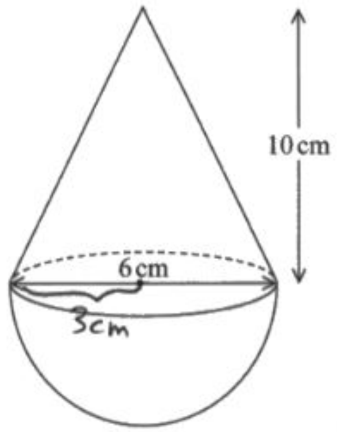
Examiner comment

The student correctly substituted values into the formulae for the volume of a cone and the volume of a sphere. Either of these was sufficient for the first M1. The method was not complete, since they found the total volume of a cone and sphere not the total volume of a cone and hemisphere, so the second M1 was not awarded. They correctly simplified the volume of the cone to 30π, so were awarded the third M1. For the total volume, they used the volume of a sphere instead of the volume of a hemisphere, so the final answer was incorrect.

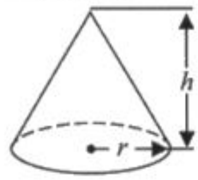
Student response C

15 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.

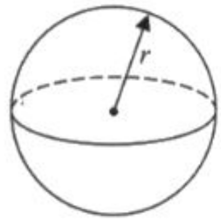
$6 \div 2 = 3$



Volume of a cone $= \frac{1}{3}\pi r^2 h$



Volume of a sphere $= \frac{4}{3}\pi r^3$



The height of the cone is 10 cm.
The base of the cone has a diameter of 6 cm.
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

Volume of cone =

$\frac{1}{3}\pi \times 3 \times 10 =$

$\frac{1}{3}\pi \times 30 =$

10π

Volume of hemisphere =

$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 3^3$

$= \frac{2}{3}\pi \times 9$

$= 6\pi$

$10\pi + 6\pi = 16\pi$

$k = 16$

$k =$ 16

1/4

Examiner comment

The student substituted correctly into the volume formula to find the volume of the hemisphere, so was awarded the first M1. The volume of the cone was incorrect and therefore there was not a complete method to find the total volume of the cone and hemisphere, so the second M1 was not given. The student simplified the volume of the hemisphere incorrectly to 6π, so did not gain the third M1.

Exemplar question 7

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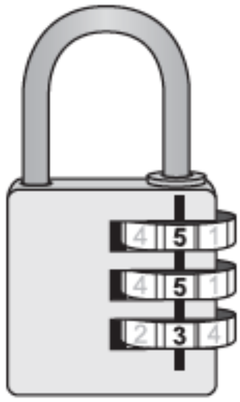
Higher tier Question 16

16 There are three dials on a combination lock.
Each dial can be set to one of the numbers 1, 2, 3, 4, 5
The three digit number 553 is one way the dials can be set, as shown in the diagram.



(a) Work out the number of different three digit numbers that can be set for the combination lock.

....
(2)

(b) How many of the possible three digit numbers have three different digits?

....
(2)

(Total for Question 16 is 4 marks)

Mean score: 1.39

Examiner comment

This question assessed a standard technique – the use of the product rule for counting.

Part (a): Students were expected to use a method such as $5 \times 5 \times 5$ to work out the number of different three digit numbers. A surprising number of arithmetic errors were made in evaluating 5^3. A common mistake was to include extra working with 5^3, for example $5^3 \times 3$ or $5^3 \times 5$ or $5^3 - 20$. Other common incorrect methods included $5 \times 4 \times 3 \times 2 \times 1$ and $5 \times 3 \times 5$.

Part (b): Students were expected to be able to work out that 60 of the possible three digit numbers have three different digits. The most common method was $5 \times 4 \times 3$. Another strategy was to subtract the number of combinations that do not have three different digits from the answer to part (a).

Mark scheme

Question	Answer	Mark	Mark scheme
16(a)	125	M1	for method to find the number of 3 digit combinations, e.g. 5^3 **or** $5^3 - 1$
		A1	for 125 **or** 124
(b)	60	M1	for method to find the number of combinations with 3 different digits e.g. $5 \times 4 \times 3$ **or** finds there are 65 combinations that do not have 3 different digits
		A1	cao

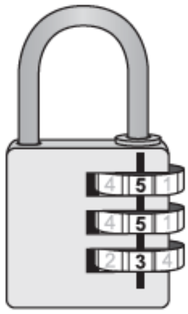
Examiner comment

Part (a): M1 was awarded for a correct method, e.g. 5^3. Extra working with 5^3, for example $5^3 - 20$ or $5^3 \times 3$ or $5^3 \times 5$, invalidated the method and the method mark could not be awarded. A method of $5^3 - 1$ was acceptable because there are 124 combinations that are different to the one in the diagram (553). A1 was awarded for a final answer of either 125 or 124.

Part (b): M1 was awarded for either a correct method, such as $5 \times 4 \times 3$, to find the number of combinations with three different digits or for finding that there are 65 combinations that do not have three different digits. A1 was only available for the correct answer of 60.

Student response A

16 There are three dials on a combination lock.
Each dial can be set to one of the numbers 1, 2, 3, 4, 5
The three digit number 553 is one way the dials can be set, as shown in the diagram.



(a) Work out the number of different three digit numbers that can be set for the combination lock.

$5^3 = 125$

125

(2)

2/2

(b) How many of the possible three digit numbers have three different digits?

$5 \times 4 \times 3 = 60$

60

(2)

2/2

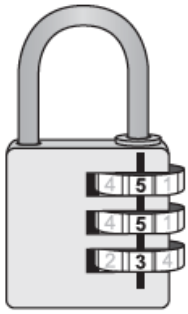
Examiner comment

Part (a): The student used the method of 5^3 to obtain the correct answer of 125.

Part (b): The student used the method of $5 \times 4 \times 3$ to obtain the correct answer of 60.

Student response B

16 There are three dials on a combination lock.
Each dial can be set to one of the numbers 1, 2, 3, 4, 5
The three digit number 553 is one way the dials can be set, as shown in the diagram.



(a) Work out the number of different three digit numbers that can be set for the combination lock.

111 131 151
112 132 152
113 133 153
114 134 154
115 135 155
121 141
122 142
123 143
124 144
125 145

25 × 5 = 100

100

(2)

1/2

(b) How many of the possible three digit numbers have three different digits?

111 121 144
112 122 151
113 131 155
114 133
115 141

13 × 5 = 65

100 − 65 = 35

35

(2)

1/2

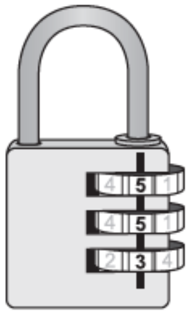
Examiner comment

Part (a): The student found by listing that there are 25 possible three digit numbers that start with 1 and then multiplied this number by 5. This was a correct method to find the number of different three digit numbers and gets M1. Unfortunately, 25×5 was evaluated incorrectly.

Part (b): M1 was awarded for finding that 65 of the possible three digit numbers do not have three different digits. The student again used listing as part of the method, finding that there were 13 possibilities that start with 1 and then multiplying by 5. Since the answer to part (a) was incorrect, the final answer to this part was also incorrect.

Student response C

16 There are three dials on a combination lock.
Each dial can be set to one of the numbers 1, 2, 3, 4, 5
The three digit number 553 is one way the dials can be set, as shown in the diagram.



(a) Work out the number of different three digit numbers that can be set for the combination lock.

$5 \times 5 = 25$

$25 \times 3 = 75$

75

(2)

0/2

(b) How many of the possible three digit numbers have three different digits?

$75 - 5 = 70$

70

(2)

0/2

Examiner comment

Part (a): Multiplying 5 by 5 was a promising start, but the method then went wrong when 25 was multiplied by 3.

Part (b): The student's strategy was to subtract the number of combinations that do not have three different digits from the answer to part (a). The number subtracted, however, was not 65 and so M1 could not be awarded. Subtracting 5, not 65, from the answer to part (a) was a common error.

Exemplar question 8

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Higher tier Question 17

17 Given that

$$x^2 : (3x + 5) = 1 : 2$$

find the possible values of x.

....

(Total for Question 17 is 4 marks)

Mean score: 0.92

Examiner comment

This problem required students to relate ratios to fractions. Students were expected to use the given ratios to form an equation in x and to then demonstrate the ability to solve a quadratic equation that requires rearrangement.

The biggest difficulty for many students was setting up the initial equation; the ability to rearrange and solve quadratic equations was a limiting factor of many solutions. Some students gained only the first P1 because they could not write their equation (which was often $2x^2 = 3x + 5$) in a suitable form ready for solution. Many attempts at forming an equation were unsuccessful. Common incorrect equations were $x^2 = 2(3x + 5)$ and $x^2 + 3x + 5 = 3$, and some students used $x^2 = 1$ and $3x + 5 = 2$.

Mark scheme

Question	Answer	Mark	Mark scheme
17	−1, 2.5	P1	for process to form an equation, e.g. $\frac{x^2}{3x+5} = \frac{1}{2}$ **or** $2x^2 = 3x + 5$
		P1	for writing in a suitable form ready for solution, e.g. $2x^2 - 3x - 5$ (= 0) **or** $-2x^2 + 3x + 5$ (= 0)
		P1	(dep 1st P1) for process to solve quadratic equation of form $ax^2 + bx + c$ (= 0) e.g. $(2x - 5)(x + 1)$ (= 0) **or** $\frac{--3 \pm \sqrt{(-3)^2 - 4\times2\times-5}}{2\times2}$
		A1	for −1, 2.5 oe

Examiner comment

The first P1 was awarded for a process to form an equation. Two suitable equations were given on the mark scheme; other possibilities included $x^2 = \frac{1}{2(3x+5)}$ and $\frac{x^2}{x^2 + 3x+5} = \frac{1}{3}$. If the first P1 was not awarded then no further marks were available (unless the student went straight to the second step and gave an equation that was already in a suitable form ready for solution, in which case the first two P1 marks were awarded). The third P1 was dependent on the first P1 and was awarded for factorising correctly or for substituting correctly into the quadratic equation formula. Both solutions were needed for A1.

Substituting different values of x into the given statement and finding one solution gained no marks.

Student response A

17 Given that

$$x^2 : (3x + 5) = 1 : 2$$

find the possible values of x.

$$\frac{x^2}{3x+5} = \frac{1}{2}$$

$2x^2 = 3x + 5$

$2x^2 - 3x - 5 = 0$

$(2x - 5)(x + 1) = 0$

$x = -1$

&

$x = 2.5$

2.5
2.5
1.25
5.0
6.25

$6.25 : 3 \times 2.5 + 5$

$7.5 + 5 = 12.5$

$x =$ -1 & 2.5

4/4

Examiner comment

This was a fully correct solution. The student rearranged their equation to give $2x^2 - 3x - 5 = 0$ and then factorised correctly to find the two possible values of x.

The working in the bottom left-hand corner was a check using one of the values found.

Student response B

17 Given that

$$x^2 : (3x + 5) = 1 : 2$$

find the possible values of x.

$x^2 : (3x+5) = \cancel{2:1}$ $1:2$

$\frac{x^2}{1} = \frac{3x+5}{2}$

$x^2 = \frac{3x+5}{2}$

$2x^2 = 3x+5$

$2x^2 - 3x + 5 = 0$

$(2x-5)(x+1) = 0$

$x = -1$
or
$x = 5$

$x = 5$ or -1

3/4

Examiner comment

The student showed all the processes required for the award of the three P1 marks. Although the quadratic was factorised correctly, one of the values of x was incorrect so A1 could not be awarded.

Student response C

17 Given that

$$x^2 : (3x + 5) = 1 : 2$$

find the possible values of x.

$x^2 : (3x + 5)$
$1 : 2$

$\cancel{\frac{3x+5}{\sqrt{2}} = x^2}$

$x^2 \times 2 = 3x + 5$
$\hookrightarrow 2x^2$

$2x^2 = 3x + 5$

$2x^2 - 3x = 5$

$x(2x - 3) = 5$

$x = 5$

or

$2x - 3 = 5$

$2x = 5 + 3$

$2x = 8$

$x = 4$

5 or 4

1/4

Examiner comment

The student made a good start by forming the equation $2x^2 = 3x + 5$, gaining the first P1. They rearranged their equation, but not into the form $ax^2 + bx + c = 0$, and so no further marks were awarded.

Exemplar question 9

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Higher tier Question 18

18 (a) Express $\sqrt{3}+\sqrt{12}$ in the form $a\sqrt{3}$ where a is an integer.

....

(2)

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

....

(3)

(Total for Question 18 is 5 marks)

Mean score: (a) 1.11, (b) 0.85

Examiner comment

This question assessed the ability of students to manipulate surds. Students were expected to be able to simplify surd expressions involving squares and rationalise denominators in order to express the given expressions in the required form.

Part (a): It was not unusual to see $\sqrt{12}$ written as $2\sqrt{3}$ and for $2\sqrt{3}$ to be given as the final answer. A common mistake was to write $\sqrt{12}$ as $4\sqrt{3}$.

Part (b): Students did not always use the most efficient method to rationalise the denominator. Multiplying the numerator and denominator of $\frac{1}{27\sqrt{3}}$ by $27\sqrt{3}$ instead of by $\sqrt{3}$, for example, often led to arithmetic errors or to a fraction that had to be simplified to get it into the required form.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18(a)	$3\sqrt{3}$	M1	for working unambiguously with $\sqrt{12}$, e.g. $\sqrt{4\times 3}$ **or** $\sqrt{4}\times\sqrt{3}$ **or** $2\sqrt{3}$	
		A1	cao	
(b)	$\frac{\sqrt{3}}{81}$	M1	for simplifying the power e.g. $(\sqrt{3})^7 = 27\sqrt{3}$	
		M1	for method to rationalise the denominator e.g. multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$	May be seen as the first step
		A1	for $\frac{\sqrt{3}}{81}$ **or** equivalent fraction in form $\frac{\sqrt{b}}{c}$, e.g. $\frac{\sqrt{2\,187}}{2\,187}$	

Examiner comment

Part (a): M1 was awarded for writing $\sqrt{12}$ as $\sqrt{4\times 3}$ or as $\sqrt{4}\times\sqrt{3}$ or as $2\sqrt{3}$.

Part (b): The two M1 marks were independent. They could be awarded in either order and the second M1 for rationalising the denominator could be awarded even if the first M1 was not awarded. A1 was awarded for an answer of $\frac{\sqrt{3}}{81}$ or an equivalent fraction in the form $\frac{\sqrt{b}}{c}$ such as $\frac{\sqrt{2\,187}}{2\,187}$ or $\frac{\sqrt{27}}{243}$.

Student response A

18 (a) Express $\sqrt{3}+\sqrt{12}$ in the form $a\sqrt{3}$ where a is an integer.

$\sqrt{3}+\sqrt{12}$

$\sqrt{3}+\sqrt{4}\times\sqrt{3}$

$\sqrt{3}+2\sqrt{3} = 3\sqrt{3}$

$3\sqrt{3}$

(2)

2/2

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

$\left(\frac{1}{\sqrt{3}}\right)^7$

$=\left(\frac{1}{\sqrt{3}\sqrt{3}\sqrt{3}\sqrt{3}\sqrt{3}\sqrt{3}\sqrt{3}}\right)$

$=\frac{1}{3\times3\times3\sqrt{3}}$

$=\frac{1}{27\sqrt{3}}$

$=\frac{1}{27\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{\sqrt{3}}{81}$

$\frac{\sqrt{3}}{81}$

(3)

3/3

Examiner comment

Part (a): A correct method led to the correct answer and the award of M1 A1. Had the final answer been incorrect then M1 could have be awarded for either $\sqrt{4}\times\sqrt{3}$ or $2\sqrt{3}$.

Part (b): The student started by simplifying the power and completes the method by rationalising the denominator. The final answer was correct and each stage of working was clearly shown.

Student response B

18 (a) Express $\sqrt{3}+\sqrt{12}$ in the form $a\sqrt{3}$ where a is an integer.

$\sqrt{12}$

$\sqrt{4}\ \sqrt{3}$

$2\sqrt{3} + \sqrt{3}$

$2\sqrt{3}$

(2)

1/2

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

$\left(\frac{1}{\sqrt{3}}\right)^7 = \left(\frac{\sqrt{3}}{3}\right)^7$

$= \frac{27\sqrt{3}}{2187}$

$\sqrt{3}^7$

$= \sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}$

$= \sqrt{9}\times\sqrt{9}\times\sqrt{9}\times\sqrt{3}$

$= 3\times3\times3\times\sqrt{3}$

$= 27\sqrt{3}$

$3^7 =$

$3\times3\times3\times3\times3\times3\times3$

$9\times9\times9\times3$

81×27

$= 2187$

$\frac{27\sqrt{3}}{2187}$

(3)

2/3

Examiner comment

Part (a): M1 was awarded for writing $\sqrt{12}$ as $2\sqrt{3}$. The final answer was incorrect because $2\sqrt{3} + \sqrt{3}$ was evaluated as $2\sqrt{3}$. This was a surprisingly common incorrect answer.

Part (b): The student started by rationalising the denominator to give $\frac{\sqrt{3}}{3}$, gaining the second M1. They then simplified the power to $27\sqrt{3}$ for the first M1. The final answer was equivalent to $\frac{\sqrt{3}}{81}$ but A1 could not be awarded because the fraction was not in the form $\frac{\sqrt{b}}{c}$.

Student response C

18 (a) Express $\sqrt{3}+\sqrt{12}$ in the form $a\sqrt{3}$ where a is an integer.

whole number.

$12 \to 2,\ 6$; $6 \to 3,\ 3$

$\sqrt{3} + \sqrt{2\times 6}$

$\sqrt{3} + 2\sqrt{6} = 2\sqrt{9}$

$2\sqrt{9}$ ....

(2)

0/2

(b) Express $\left(\dfrac{1}{\sqrt{3}}\right)^7$ in the form $\dfrac{\sqrt{b}}{c}$ where b and c are integers.

$\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3} =$

$3 \quad 3\sqrt{3} \quad 4\sqrt{3} \quad 5\sqrt{3} \quad 6\sqrt{3} \quad 7\sqrt{3}$

$\left(\dfrac{1}{\sqrt{3}}\right)^7$

$7\sqrt{3}\times ?$

$\dfrac{1}{7\sqrt{3}} \times \dfrac{7\sqrt{3}}{7\sqrt{3}}$

$\dfrac{7\sqrt{3}}{49\sqrt{3}} = \dfrac{\sqrt{3}}{7\sqrt{3}}$

$\cancel{\dfrac{\sqrt{3}}{49\sqrt{3}}}\ \dfrac{\sqrt{3}}{7}$ ....

(3)

1/3

Examiner comment

Part (a): The student wrote $\sqrt{12}$ as $\sqrt{2\times 6}$. This was correct, but not sufficient for M1. The subsequent working was incorrect.

Part (b): The student started by attempting to deal with the power of 7. This stage of working resulted in $\dfrac{1}{7\sqrt{3}}$ and is incorrect, so the first M1 could not be awarded. However, a correct method to rationalise the denominator of their expression, multiplying by $\dfrac{7\sqrt{3}}{7\sqrt{3}}$, gained the second M1.

Exemplar question 10

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Higher tier Question 10

20 h is inversely proportional to p
p is directly proportional to $\sqrt{t}$

Given that $h = 10$ and $t = 144$ when $p = 6$
find a formula for h in terms of t

....

(Total for Question 20 is 4 marks)

Mean score: 1.95

Examiner comment

This question assessed the ability of students to construct and use equations that describe direct and inverse proportion.

The first part of the solution was routine. Students were expected to use the statements given to set up two proportional relationships and to find the values of the constants of proportionality. To complete the solution, students needed to use their two proportional relationships to find a formula for h in terms of t.

Problems arose from dealing with two constants. Some students found the value of one constant, but used it incorrectly in their second equation. Another error was to use k as the constant of proportionality in both relationships and to assume that it had the same value in both equations. Another common mistake was to use direct proportion instead of inverse proportion for the relationship between h and p.

Students needed to remember to use the values of the constants in their original equations. It was very common for students to find the values of both constants but fail to use them in their equations. Those that did write down $h = \dfrac{60}{p}$ and $p = 0.5\sqrt{t}$ gain the third P1 and frequently went on to find a formula for h in terms of t.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
20	$h = \frac{120}{\sqrt{t}}$	P1	for setting up a proportional relationship between h and p, e.g. $h \propto \frac{1}{p}$ **or** $h = \frac{k}{p}$ **OR** a proportional relationship between p and t, e.g. $p \propto \sqrt{t}$ **or** $p = K\sqrt{t}$	Condone the use of '$\propto$' instead of '=' for the first two P marks Relationship may be implied by substitution
		P1	for process to substitute at least 2 values, e.g. $10 = \frac{k}{6}$ $(k = 60)$ **or** $6 = K\sqrt{144}$ $(K = 0.5)$	
		P1	for full process leading to $h = \frac{\text{“60”}}{p}$ oe **and** $p = \text{“0.5”}\sqrt{t}$ oe	Both constants must come from a correct process
		A1	$h = \frac{120}{\sqrt{t}}$ oe e.g. $h = \frac{120\sqrt{t}}{t}$ **or** $h = \frac{60}{0.5\sqrt{t}}$	Formula for h in terms of t Does not need to be in simplest form

Examiner comment

The first P1 was for setting up a proportional relationship between h and p or between p and t. The use of '$\propto$' instead of '=' was condoned for the first two P1 marks. For example, $h \propto \frac{k}{p}$ was acceptable for the first P1. The proportional relationship could be implied by a correct substitution such as $10 = \frac{k}{6}$. For the third P1, the two constants needed to come from a correct process, for example “60” from $10 = \frac{k}{6}$ and “0.5” from $6 = K \times \sqrt{144}$.

A1 was awarded for a correct formula for h in terms of t. There was no requirement to give the formula in its simplest form.

Student response A

20 h is inversely proportional to p
p is directly proportional to $\sqrt{t}$

Given that $h = 10$ and $t = 144$ when $p = 6$
find a formula for h in terms of t

$\because$ h is inversely proportional to p

$\therefore \quad h = \frac{k}{p}$

$\because$ when $p = 6, \quad h = 10$

$\therefore \quad 10 = \frac{k}{6}$

$k = 10 \times 6$

$k = 60$

$\therefore \quad h = \frac{60}{p}$

$\because$ p is directly proportional to $\sqrt{t}$

$\therefore \quad p = m\sqrt{t}$

when $\quad p = 6, \quad t = 144$

So $\quad 6 = m\sqrt{144}$

$6 = 12m$

$m = \frac{6}{12}$

$m = 0.5$

$\therefore \quad p = 0.5\sqrt{t}$

$\therefore \quad h = \frac{60}{0.5\sqrt{t}}$

$h = \frac{120}{\sqrt{t}}$

$h = \frac{120}{\sqrt{t}}$

(Total for Question 20 is 4 marks)

4/4

Examiner comment

This was a well-presented and fully correct solution. If the student had given $h = \dfrac{60}{0.5\sqrt{t}}$ as their final answer then full marks would still have been awarded. The formula did not have to be given in its simplest form.

Student response B

20 h is inversely proportional to p
p is directly proportional to $\sqrt{t}$

Given that $h = 10$ and $t = 144$ when $p = 6$
find a formula for h in terms of t

$h \propto \frac{k}{p}$ $\quad p \propto k\sqrt{t}$

~~6 = 460~~

$h = \frac{k}{p}$

$\cancel{p}$ $6 = k\sqrt{144}$

$10 = \frac{k}{6}$

$6 = 12k$

$\frac{1}{2} = k$

$60 = k$

$h = \frac{60}{p}$ $\quad p = \frac{1}{2}\sqrt{t}$

$h = \frac{k}{p}$ $\quad p = 60h$ $\quad 60h = \frac{1}{2}\sqrt{t}$

$30h = \sqrt{t}$

$p = k\sqrt{t}$

$30h = \sqrt{t}$

3/4

Examiner comment

The three P1 marks were awarded for a full process that led to $h = \frac{60}{p}$ and $p = \frac{1}{2}\sqrt{t}$. The two formulae were combined but an error was made in the unnecessary attempt to rearrange $h = \frac{60}{p}$.

Student response C

20 h is inversely proportional to p
p is directly proportional to $\sqrt{t}$

Given that $h = 10$ and $t = 144$ when $p = 6$
find a formula for h in terms of t

$h \propto p$

$p \propto \sqrt{t}$

$h = 10$
$t = 144$
$p = 6$

$10 \propto 6$
$6 \propto \sqrt{144}$
$\sqrt{144} = 12$

$6 \propto 12$

1/4

Examiner comment

The first P1 was awarded for setting up the proportional relationship $p \propto \sqrt{t}$ but there was no valid substitution. Instead of forming the equation $p = k\sqrt{t}$ and finding the value of k the student simply substituted the values $p = 6$ and $t = 144$ into $p \propto \sqrt{t}$.

For the relationship between h and p, direct proportion was used instead of inverse proportion.

Exemplar question 11

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Higher tier Question 21

21 The functions f and g are such that

$$f(x) = 3x - 1 \qquad \text{and} \qquad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$f^{-1}(x) = ...$

(2)

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

(5)

(Total for Question 21 is 7 marks)

Mean score: 2.76

Examiner comment

This question assessed knowledge and the use of formal function notation.

Part (a): Students were expected to recognise that $f^{-1}(x)$ is the inverse function and to use an appropriate method to find it. Changing the subject of either $y = 3x - 1$ or $x = 3y - 1$ was the most common approach. Mistakes were sometimes made in the first step with $y = 3x - 1$ being followed by, for example, $y - 1 = 3x$. A common incorrect answer was $\frac{1}{3x-1}$ because some students interpreted $f^{-1}(x)$ as $\frac{1}{f(x)}$, confusing the notation for inverse functions with the notation for a negative power.

Part (b): Students needed to find the composite functions $fg(x)$ and $gf(x)$ and use them to construct a chain of reasoning to achieve a given result. Students needed to demonstrate the ability to set their working out in a formal manner, showing the steps that were needed to get to the required statement. They needed to remember to label $fg(x)$ and $gf(x)$. If correct labelling was not seen or implied, the answer scored no marks. Most errors occurred in the simplification of $gf(x)$ or $2gf(x)$. Incorrect terms in the expansion and simplification of $(3x - 1)^2 + 4$ were quite common and sometimes only $(3x - 1)^2$ was multiplied by 2.

Mark scheme

Question	Answer	Mark	Mark scheme
21(a)	$\frac{x+1}{3}$	M1	first step to change the subject of $y = 3x - 1$ **or** $x = 3y - 1$, e.g. $y + 1 = 3x$
		A1	oe
(b)	Shown	M1	for method to find fg(x), e.g. fg$(x) = 3(x^2 + 4) - 1$
		M1	for method to find gf(x), e.g. gf$(x) = (3x - 1)^2 + 4$
		M1	(dep on previous two M marks) for setting up equation, e.g. $3(x^2 + 4) - 1 = 2[(3x - 1)^2 + 4]$
		M1	(dep 2nd M1) for correct expansion of $(3x - 1)^2$ e.g. $9x^2 - 3x - 3x + 1$
		C1	for $15x^2 - 12x - 1 = 0$ from correct working

Examiner comment

Part (a): M1 was awarded for a correct first step to change the subject of $y = 3x - 1$ or $x = 3y - 1$. A correct flow diagram was also acceptable for this mark.

Part (b): The first two M1 marks was awarded for methods to find fg(x) and gf(x). If fg(x) and gf(x) were not labelled, then this could be implied by the setting up of the equation fg(x) = 2gf(x) or by sight of 2gf(x).

The third M1 was dependent on the previous two M1 marks. Errors in the simplification of either (or both) fg(x) and gf(x) were not penalised in this mark, which was awarded for equating fg(x) with 2gf(x). The fourth M1 for the correct expansion of $(3x - 1)^2$ was dependent on the second M1. If the student had not been awarded the mark for finding gf(x) then this mark could not be awarded.

For C1, $15x^2 - 12x - 1 = 0$ needed to be obtained from correct working. Students needed to check their solution thoroughly since the result that they were working towards was given in the question.

Student response A

21 The functions f and g are such that

$$f(x) = 3x - 1 \qquad \text{and} \qquad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$$x \xrightarrow{\times 3} \xrightarrow{-1}$$
$$\xleftarrow{\div 3} \xleftarrow{+1}$$

$$f^{-1}(x) = \frac{x+1}{3}$$

$f^{-1}(x) = \frac{x+1}{3}$

(2)

2/2

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

$$fg(x) = 3(x^2+4) - 1$$

$$2gf(x) = 2((3x-1)^2 + 4)$$

$$3(x^2+4)-1 = 2((3x-1)^2+4)$$
$$3x^2+12-1 = 2(9x^2-6x+1+4)$$
$$3x^2+11 = 2(9x^2-6x+5)$$
$$3x^2+11 = 18x^2-12x+10$$
$$3x^2 = 18x^2-12x-1$$
$$0 = \underline{15x^2-12x-1}$$

5/5

Examiner comment
Part (a): The student found the inverse function using the flow diagram method.
Part (b): The student achieved the required result from a fully correct chain of reasoning. The solution was well presented, with each line of work clearly following on from the previous line.

Student response B

21 The functions f and g are such that

$$f(x) = 3x - 1 \qquad \text{and} \qquad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$y = 3x - 1$

$y + 1 = 3x$

$\frac{y+1}{3} = x \qquad f^{-1}(x) = \frac{y+1}{3}$

$f^{-1}(x) = \frac{y+1}{3}$

(2)

1/2

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

$3(x^2+4) - 1$

$3x^2 + 12 - 1 \qquad = 2((3x-1)^2 + 4)$

$3x^2 - 11 = 2(9x^2 - 6x + 5)$

$3x^2 - 11 = 18x^2 - 12x + 10$

$3x^2 = 18x^2 - 12x - 1$

$0 = 15x^2 - 12x - 1$

4/5

Examiner comment

Part (a): M1 was awarded for the first step to change the subject of $y = 3x - 1$. The rearrangement was completed correctly but A1 could not be awarded as the inverse function was not a function of x.

Part (b): Both $3(x^2 + 4) - 1$ and $(3x - 1)^2 + 4$ were given. These were not labelled as $fg(x)$ and $gf(x)$, but this was implied by the multiplication of $(3x - 1)^2 + 4$ by 2 and the first two M1 marks could be awarded. Setting up the equation $fg(x) = 2gf(x)$ gained the third M1. Note that $3x^2 - 11$ is incorrect at this stage, but errors in the simplification of $fg(x)$ and $gf(x)$ were not penalised in the third M1. The correct expansion of $(3x - 1)^2$ was implied by the simplification of $gf(x)$ to $9x^2 - 6x + 5$ and so the fourth M1 was awarded. C1 could not be awarded because $15x^2 - 12x - 1 = 0$ did not come from correct working; there was an error in the third line of working.

Student response C

21 The functions f and g are such that

$$f(x) = 3x - 1 \qquad \text{and} \qquad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$-3x+1$

$f^{-1}(x) =$ $-3x+1$

(2)

0/2

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

$fg(x) = x^2+4$

$= (3x-1)^2+4$

$(3x-1)(3x-1)+4$

$9x^2-3x-3x+1$

$9x^2-6x+1$

$9x^2-6x+5 = 9x^2+22$

$2gf(x) = 3x-1$

$3(x^2+4)-1$

$3x^2+12-1$

$2(3x^2+11)$

$9x^2+22$

~~18x2-12x-1=0~~

~~16x2-~~

4.5

1.49

3.5

a b c

$15x^2-12x-1=0$

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\dfrac{12 \pm \sqrt{(-12)^2-(4\times15\times-1)}}{2\times15} = \dfrac{12 \pm \sqrt{144+60}}{30}$

$\dfrac{12 \pm \sqrt{204}}{30}$

(5)

0/5

Examiner comment

Part (a): The answer was incorrect and no correct method is shown. The student simply changed the signs of $3x - 1$ to get $-3x + 1$.

Part (b): Both $3(x^2 + 4) - 1$ and $(3x - 1)^2 + 4$ were given but the student wrote $fg(x) = (3x - 1)^2 + 4$ and $gf(x) = 3(x^2 + 4) - 1$, i.e. labelled them the wrong way round. Since both $fg(x)$ and $gf(x)$ were incorrect, neither of the first two M1 marks could be awarded. The correct expansion of $(3x - 1)^2$ was shown but the fourth M1 could not be awarded because it was dependent on the second M1. The student also attempted to solve the equation $15x^2 - 12x - 1 = 0$. This was unwise. Not only was it an incorrect strategy but it also presented the examiner with a choice of methods.

Exemplar question 12

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Higher tier Question 22

22 There are only r red counters and g green counters in a bag.

A counter is taken at random from the bag.

The probability that the counter is green is $\frac{3}{7}$

The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.

The probability that the counter is green is $\frac{6}{13}$

Find the number of red counters and the number of green counters that were in the bag originally.

red counters...

green counters...

(Total for Question 22 is 5 marks)

Mean score: 0.62

Examiner comment

This question assessed the ability of students to make and use connections between different parts of mathematics, unprompted, in order to solve a problem. The two areas that were linked in this question were probability and algebra, specifically the ability to derive and solve equations.

Students were expected to use the given information to derive an equation (or two simultaneous equations) and solve the equation(s) to find the number of red counters and the number of green counters.

Some students that attempted an algebraic approach had difficulty deriving the necessary equation(s). Those that did derive two correct equations often made mistakes when dealing with the fractions or eliminating one of the variables to solve their equations.

The main obstacle to a successful outcome was the inability to find a suitable strategy. Many students jumped straight to probability tree diagrams, which were generally unhelpful, and had no other strategies. Unsuccessful attempts were often based on using fractions with a denominator of 91 (from 7 × 13).

Mark scheme

Question	Answer	Mark	Mark scheme
22	12 red, 9 green	P1	for process to find a relationship between r and g e.g. $\frac{g}{r+g}=\frac{3}{7}$ **or** $\frac{g}{r}=\frac{3}{4}$
		P1	for process to find a second relationship between r and g e.g. $\frac{g+3}{r+2+g+3}=\frac{6}{13}$ **or** $\frac{g+3}{r+2}=\frac{6}{7}$
		P1	(dep P2) for start to process of solving pair of equations, e.g. eliminates one variable from the equations **or** removes fractions from both equations
		P1	(dep P3) for complete process to solve equations to find g **or** r
		A1	cao **OR**
		P1	for two of $3x + 3$, $4x + 2$ and $7x + 5$
		P1	for $\frac{3x+3}{7x+5}=\frac{6}{13}$
		P1	(dep P2) for removing fractions from the equation, e.g. $13(3x + 3) = 6(7x + 5)$ **or** $39x + 39 = 42x + 30$
		P1	(dep P3) for complete process to solve $13(3x + 3) = 6(7x + 5)$
		A1	cao

Examiner comment

Two possible methods were shown in the mark scheme. These covered the majority of the algebraic responses seen, but they were only examples; there were other algebraic methods used which were marked in a similar way. Students using the second method were generally more successful than those using the first method.

The third P1 was dependent on the first two P1 marks being awarded. If the first two P1 marks had not been awarded, then no further marks were possible. The fourth P1 dependent on the previous P1 mark.

Student response A

22 There are only r red counters and g green counters in a bag.

A counter is taken at random from the bag.

The probability that the counter is green is $\frac{3}{7}$

The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.

The probability that the counter is green is $\frac{6}{13}$

Find the number of red counters and the number of green counters that were in the bag originally.

$P(G) = \frac{3}{7}$

$P(G) = \frac{3x}{7x}$

$\frac{3x+3}{7x+5} = \frac{6}{13}$

$13(3x+3) = 6(7x+5)$

$39x + 39 = 42x + 30$

$9 = 3x$

$x = 3$

$3 \times 3 = 9 \rightarrow$ 9 green

$7 \times 3 = 21 \rightarrow$ 21 altogether

$21 - 9 = 12 \rightarrow$ 12 red.

~~9 red~~,

9 green, 12 red.

red counters ~~9~~ 12

green counters ~~12~~ 9

5/5

Examiner comment

The student used the second method shown on the mark scheme.

The first P1 was awarded for $3x + 3$ and $7x + 5$ and the second P1 for forming the equation $\frac{3x+3}{7x+5} = \frac{6}{13}$. A complete process to solve the equation gained the final two P1 marks. The value $x = 3$ was used to find that there are 12 red counters and 9 green counters, gaining A1.

Student response B

22 There are only r red counters and g green counters in a bag.

A counter is taken at random from the bag.

The probability that the counter is green is $\frac{3}{7}$

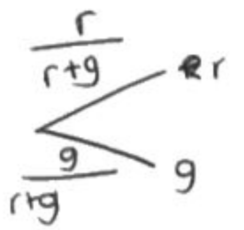
The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.

The probability that the counter is green is $\frac{6}{13}$

Find the number of red counters and the number of green counters that were in the bag originally.

red = r green = g total = r+g

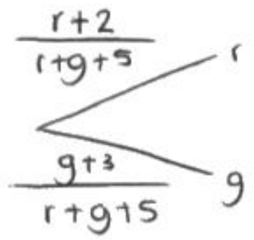


$\frac{g}{r+g} = \frac{3}{7}$

$7g = 3r + 3g$

$4g - 3r = 0$

red = r + 2 green = g + 3 total = r+g+5



$\frac{g+3}{r+g+5} = \frac{6}{13}$

$13g + 39 = 6r + 6g + 30$

$7g - 6r = -9$

$7g - 6r = -9$
$4g - 3r = 0$

$7g - 6r = -9$
$8g - 6r = 0$
$15g = -9$

red counters ....8........

green counters ....6........

3/5

Examiner comment

The student used the first method shown on the mark scheme.

The first two P1 marks were awarded for forming the equations $\frac{g}{r+g}=\frac{3}{7}$ and $\frac{g+3}{r+g+5}=\frac{6}{13}$ respectively. The student removed the fractions from both equations giving $7g-6r=-9$ and $4g-3r=0$, gaining the third P1. Their process to solve the simultaneous equations by eliminating r was not correct, since they added the two equations, so the fourth P1 was not awarded.

Student response C

22 There are only r red counters and g green counters in a bag.

A counter is taken at random from the bag.

The probability that the counter is green is $\frac{3}{7}$

The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.

The probability that the counter is green is $\frac{6}{13}$

Find the number of red counters and the number of green counters that were in the bag originally.

$r+2+g+7$

$\frac{g+3}{r+g+5} = \frac{6}{13}$

$13g+39 = 6r+6g+30$

$13g+9 = 6r+6g$

$7g+9 = 6r$

$r = \frac{7g+9}{6}$

$\frac{7}{13} = \frac{7g+9}{6}$

$42 = 13(7g+9)$

$42 = 91g+9$

$33 = 91g$

$g = \frac{91}{33}$

1/5

Examiner comment

Forming the equation $\frac{g+3}{r+g+5} = \frac{6}{13}$ gained the first P1. The student did not use either $\frac{3}{7}$ or $\frac{4}{7}$ to find a second relationship between r and g and so the second P1 was not awarded. Since the student did not gain both of the first P1 marks no further marks could be awarded.

Paper 2H (calculator)

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Exemplar question 1

Higher tier Question 9

9 The circumference of circle **B** is 90% of the circumference of circle **A**.

(a) Find the ratio of the area of circle **A** to the area of circle **B**.

....
(2)

Square **E** has sides of length *e* cm.
Square **F** has sides of length *f* cm.

The area of square **E** is 44% greater than the area of square **F**.

(b) Work out the ratio $e : f$

....
(2)

(Total for Question 9 is 4 marks)

Mean score: (a) 0.72, (b) 0.74

Examiner comment

This question combined the standard writing of a ratio, with scale factors of similar shapes.

Part (a): Students had to write the ratio of the circumferences of the two circles, and then apply knowledge of length and area scale factors to find the ratio of the areas. Many students tried to actually find areas from a created length, and often got lost in the numbers, while the more successful students simply worked in terms of scale factors and squared to go from length to area.

Part (b): This was the opposite process from part (a), starting with area scale factors and square rooting to find length scale factor. Many students struggled to generate the correct area scale factor, often working with 44 or 0.44 rather than 1.44.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
9(a)	100 : 81	M1	for a scale factor of 0.9 oe used; **OR** for 10 : 9 oe **OR** 81 : 100 oe **OR** 81%	
		A1	for 100 : 81 oe	e.g. 1 : 0.81, accept 1.23(4…) : 1
(b)	6 : 5	P1	for 1.44 oe used as the scale factor **or** 1.2 oe **OR** for 144 : 100 oe **or** $\sqrt{144} : \sqrt{100}$ oe **OR** 5 : 6 oe	
		A1	for 6 : 5 oe	e.g. 1.2 : 1, Accept 1 : 0.83(3…)

Examiner comment

In both parts, M1 was for either using the correct scale factor, 0.9 and 1.44 respectively, or for writing a correct starting ratio. This means the ratio of the circumferences in part (a) or the areas in part (b). These ratios had to be written in the correct order to gain credit, and a number of students lost the mark when it wasn't.

For A1, it is worth noting that the ratios do not have to be in a particular form and all equivalent ratios gain credit.

Student response A

9 The circumference of circle **B** is 90% of the circumference of circle **A**.

(a) Find the ratio of the area of circle **A** to the area of circle **B**.

πr^2

$\pi \times 10^2 = 100\pi$

$\pi \times 9^2 = 81\pi$

10 : 9

~~$\pi \times 9^2 = 81$~~

~~$\pi \times 10^2 = 100\pi$~~

100 : 81

100 : 81

10 : 9

(2)

2/2

Square **E** has sides of length e cm.
Square **F** has sides of length f cm.

The area of square **E** is 44% greater than the area of square **F**.

(b) Work out the ratio $e : f$

$\boxed{E}$ $\boxed{F}$

144 100

Area of E : Area of F

144 : 100

$\sqrt{\ }$

12 : 10

6 : 5

6 : 5

(2)

2/2

Examiner comment

Part (a): The student worked with the area of circles of radius 10 and 9. In fact the ratio 10 : 9 stated on the left-hand side would have score M1. The student continued to work with areas, obtaining to 100π and 81π. They then correctly cancel the 'π' to give the answer 100 : 81. It is worth noting that an answer of $100\pi : 81\pi$ would also have scored A1.

Part (b): The student correctly started with the ratio of areas 144 : 100, gaining M1. They then correctly square-rooted both values to get 12 : 10, which they cancel to 6 : 5. Either of these would score A1.

Student response B

9 The circumference of circle **B** is 90% of the circumference of circle **A**.

(a) Find the ratio of the area of circle **A** to the area of circle **B**.

$$\left(\frac{90}{100}\right)^2 = \frac{81}{100}$$

~~100:80~~

~~81:100~~ 100:81

(2)

2/2

Square **E** has sides of length *e* cm.
Square **F** has sides of length *f* cm.

The area of square **E** is 44% greater than the area of square **F**.

(b) Work out the ratio $e : f$

E F

144 : 100

36 : 25

36 : 25

(2)

1/2

Examiner comment

Part (a): The student started to work with the scale factor of $\frac{90}{100}$, so M1 was awarded. The student correctly squared the scale factor to get the area scale factor of $\frac{81}{100}$. On its own, this could not be awarded A1 as the question asks for a ratio. However, this student completed the response by writing the correct ratio from the fraction.

Part (b): The student started well by writing the ratio of the areas, gaining M1. They then went on to simplify, but failed to square-root and so were not awarded A1.

Student response C

9 The circumference of circle **B** is 90% of the circumference of circle **A**.

(a) Find the ratio of the area of circle **A** to the area of circle **B**.

9π 10π

$r = 4.5$ $r = 5$

25

20.25

$81 : 100$

(2)

1/2

Square **E** has sides of length e cm.
Square **F** has sides of length f cm.

The area of square **E** is 44% greater than the area of square **F**.

(b) Work out the ratio $e : f$

$100 : 144$

(2)

0/2

Examiner comment

Part (a): The student correctly started to work with the ratio and reached the correct two values. However, they gave the ratio of **B** : **A**, rather than **A** : **B**, and so only M1 was awarded.

Part (b): The student stated the ratio of the areas. However, the ratio was in the wrong order and they did not take the square root so no marks could be awarded.

Exemplar question 2

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Higher tier Question 11

11 The grouped frequency table gives information about the times, in minutes, that 80 office workers take to get to work.

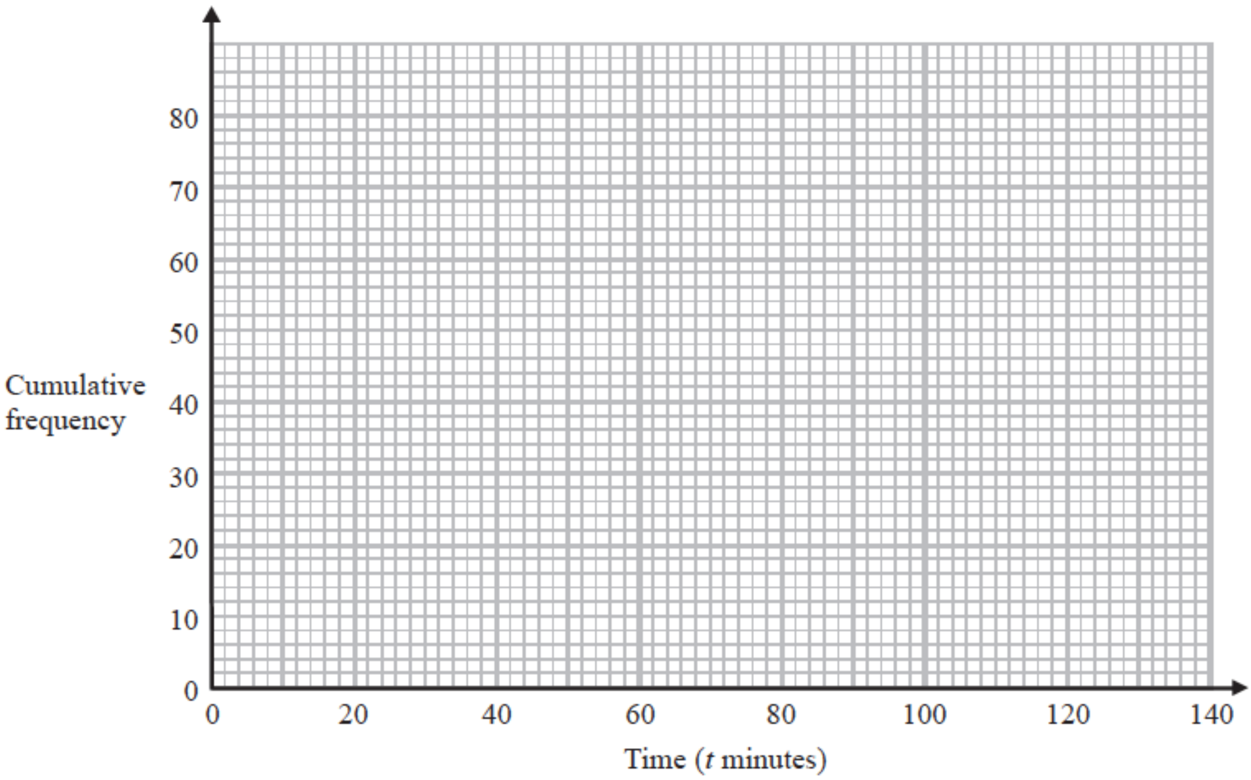
Time (t minutes)	**Frequency**
$0 < t \leqslant 20$	5
$20 < t \leqslant 40$	30
$40 < t \leqslant 60$	20
$60 < t \leqslant 80$	15
$80 < t \leqslant 100$	8
$100 < t \leqslant 120$	2

(a) Complete the cumulative frequency table.

Time (t minutes)	**Cumulative frequency**
$0 < t \leqslant 20$	
$0 < t \leqslant 40$	
$0 < t \leqslant 60$	
$0 < t \leqslant 80$	
$0 < t \leqslant 100$	
$0 < t \leqslant 120$	

(1)

(b) On the grid, draw the cumulative frequency graph for this information.



(2)

(c) Use your graph to find an estimate for the percentage of these office workers who take more than 90 minutes to get to work.

....%

(3)

(Total for Question 11 is 6 marks)

Mean score: 3.87

Examiner comment

Parts (a) and (b) were a very familiar cumulative frequency question, with part (a) completing a table and (b) drawing the graph. However, part (c) was different to many questions set previously, with students needing to interpret information from the graph and then calculate a percentage of the total cohort.

The majority of students dealt with the first two parts very well. However, some made the typical mistake of not plotting the end interval points for the graph.

Part (c) was generally answered well, with the vast majority able to gain at least one mark for a method to read from the graph. Some then failed to subtract this value from 80 and therefore found the percentage of office workers that took up to 90 minutes, rather than over 90 minutes.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
11(a)	5, 35, 55, 70, 78, 80	B1	cao	
(b)	cf graph	M1	for 5 or 6 of their points plotted correctly from a cf table	Ignore to the left of the first point and right of the last point
		A1	for a fully correct graph	Accept a smooth curve or line segments
			SC B1 if 5 or 6 of their points plotted not at end but consistent within each interval and joined by a curve or line segments providing no gradient is negative	
(c)	7.5	M1	for a clear method to read off the cf graph at 90	Sight of 74 or 6 implies M1
		M1	for a full method to find the percentage e.g. (80 – "74") ÷ 80 × 100 (= 7.5)	The following readings give the following percentages
		A1	for 7.5 **or** ft cf graph	72 = 10% 73 = 8.75% 74 = 7.5% 75 = 6.25% 76 = 5%

Examiner comment

Part (a): This was a simple B1 for correctly completing the cumulative frequency table.

Part (b): Students were able to gain M1 (or B1) for either plotting their points correctly but failing to join them, or for plotting consistently within the interval and joining. This consistent plotting was for the students who typically plotted on the mid-interval values rather than the end points, and was catered for in the special case in the mark scheme. If an error was made in the table then these were followed through for (b), providing values in the cumulative frequency column were increasing. The graph needed to be fully correct for A1.

Part (c): The first M1 was for reading from 90 on their cumulative frequency graph. If students had drawn a step diagram or had a graph with some negative gradient, they were unable to score this mark. The second M1 was for subtracting their value from 80 and finding the result as a percentage out of 80. At this stage some students didn't use the graph and used the table. A1 was for 7.5 or a correct follow through value from their cumulative frequency graph.

Student response A

11 The grouped frequency table gives information about the times, in minutes, that 80 office workers take to get to work.

Time (t minutes)	**Frequency**
$0 < t \leqslant 20$	5
$20 < t \leqslant 40$	30
$40 < t \leqslant 60$	20
$60 < t \leqslant 80$	15
$80 < t \leqslant 100$	8
$100 < t \leqslant 120$	2

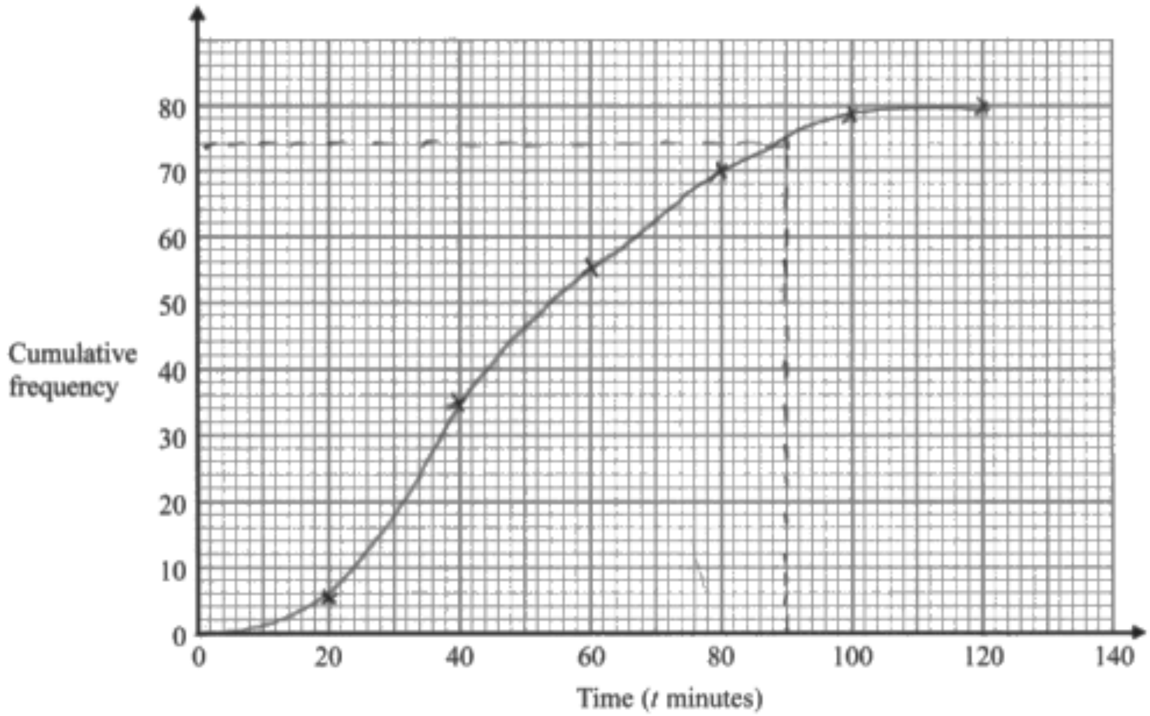
(a) Complete the cumulative frequency table.

Time (t minutes)	**Cumulative frequency**
$0 < t \leqslant 20$	~~4~~ 5
$0 < t \leqslant 40$	35
$0 < t \leqslant 60$	55
$0 < t \leqslant 80$	70
$0 < t \leqslant 100$	78
$0 < t \leqslant 120$	80

(1)

1/1

(b) On the grid, draw the cumulative frequency graph for this information.



(2)

2/2

(c) Use your graph to find an estimate for the percentage of these office workers who take more than 90 minutes to get to work.

6 ~~74~~ workes tak more than 90 minutes to get to work

74 workers take 90 minuets or less to get to work.

$\frac{6}{80} \times 100$

7.5 %

(3)

3/3

Examiner comment

Part (a): The student correctly completed the table, so B1 was awarded.

Part (b): The student plotted the points correctly and joined them with a curve (line segments were equally acceptable), and so scored M1 A1.

Part (c): The student correctly read from the graph at a time of 90, for the first M1. They then completed their solution by subtracting this value from 80 and then finding a percentage, so the second M1 was awarded. Their final answer was correct for A1.

Student response B

11 The grouped frequency table gives information about the times, in minutes, that 80 office workers take to get to work.

Time (t minutes)	Frequency
$0 < t \leqslant 20$	5
$20 < t \leqslant 40$	30
$40 < t \leqslant 60$	20
$60 < t \leqslant 80$	15
$80 < t \leqslant 100$	8
$100 < t \leqslant 120$	2

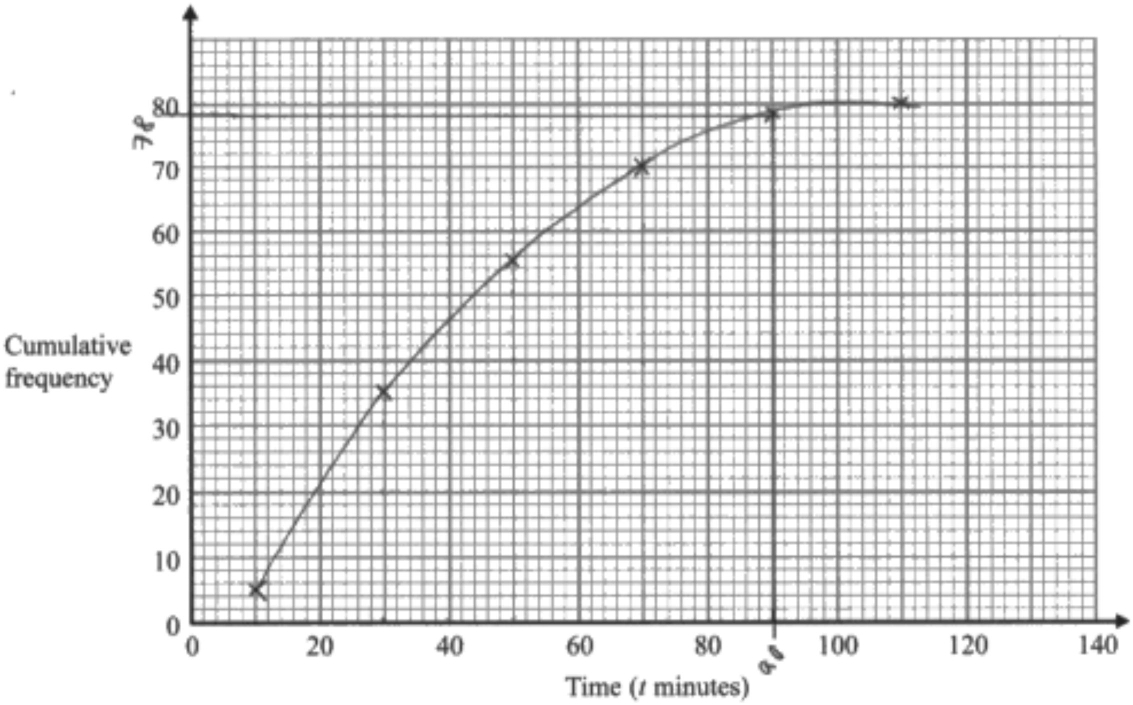
(a) Complete the cumulative frequency table.

Time (t minutes)	Cumulative frequency
$0 < t \leqslant 20$	5
$0 < t \leqslant 40$	35
$0 < t \leqslant 60$	55
$0 < t \leqslant 80$	70
$0 < t \leqslant 100$	78
$0 < t \leqslant 120$	80

(1)

1/1

(b) On the grid, draw the cumulative frequency graph for this information.



(2)

1/2

(c) Use your graph to find an estimate for the percentage of these office workers who take more than 90 minutes to get to work.

90 mins = 78 $\qquad \frac{2}{80} = \frac{1}{40}$

$\frac{2}{80} \times 100 = 2.5$

2.5% %

(3)

3/3

Examiner comment

Part (a): The student completed the table correctly for B1.

Part (b): The student made the common error of plotting at mid-interval values. Their plotting was consistent within each interval, at the correct heights and joined, so using the special case B1 was awarded.

Part (c): This could be marked using follow through from their graph. Their graph was classed as a cumulative frequency graph for the sake of the follow through, as it was consistently rising and had no negative gradient.

The method here was fully correct for their graph and the follow though could apply to all three marks, so M1 M1 A1 was awarded.

Student response C

11 The grouped frequency table gives information about the times, in minutes, that 80 office workers take to get to work.

Time (t minutes)	**Frequency**
$0 < t \leqslant 20$	5
$20 < t \leqslant 40$	30
$40 < t \leqslant 60$	20
$60 < t \leqslant 80$	15
$80 < t \leqslant 100$	8
$100 < t \leqslant 120$	2

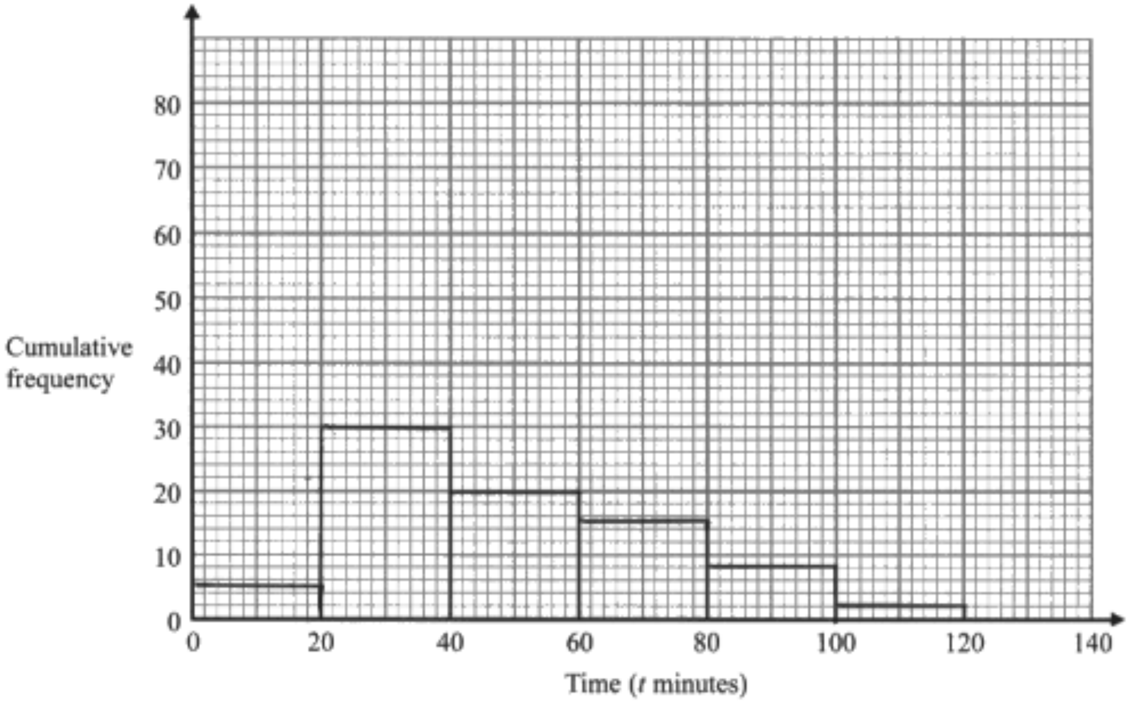
(a) Complete the [redacted]

Time (t minutes)	**Cumulative frequency**
$0 < t \leqslant 20$	5
$0 < t \leqslant 40$	35
$0 < t \leqslant 60$	55
$0 < t \leqslant 80$	70
$0 < t \leqslant 100$	78
$0 < t \leqslant 120$	80

(1)

1/1

(b) On the grid, [redacted]



(2)

0/2

(c) Use your graph to find an [redacted] of these office workers who take more than [redacted]

$\frac{8}{80} = 0.1$

$90 = 8$

10 %

(3)

0/3

Examiner comment
Part (a): The student correctly completed the table for B1.
Part (b): The student attempted to draw a frequency diagram rather than a cumulative frequency graph. There were no correct points plotted and so no marks were awarded.
Part (c): The student stated an incorrect answer with no indication where the 8 came from. Since there was no cumulative frequency graph, follow through cannot apply and so no marks were awarded.

Exemplar question 3

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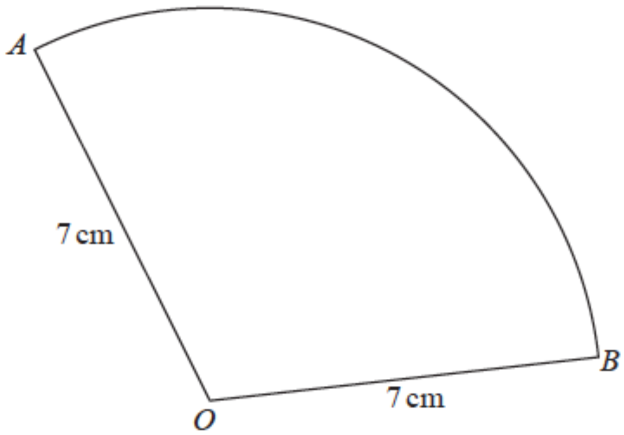
Higher tier Question 12

12 *OAB* is a sector of a circle with centre *O* and radius 7 cm.



The area of the sector is 40 cm^2

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

.... cm

(Total for Question 12 is 4 marks)

Mean score: 2.20

Examiner comment

This question linked the two main aspects of sectors, namely area and arc length, in the context of a problem. There were numerous ways for students to approach the problem, but the most common of the successful approaches was to use the area to find the angle in the sector. Students then used this to find the arc length and subsequently the perimeter.

Another approach was to work in terms of proportion, using the area of the sector and the total area of the circle. This method avoided the need to find the angle.

The most efficient method seen was those students that realised to get from the area to the arc length they needed to only divide by the radius and multiply by 2.

A significant number of students struggled to access the problem as it seemed they were not confident in using the correct formulae for area and arc lengths of sectors. Of these many gained the first mark but were unable to progress further.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
12	25.4	P2	for finding the size of the angle e.g. $\frac{40\times360}{\pi\times7^2}$ (= 93.5(4…)) **OR** for working with proportion, e.g. $\frac{40}{49\pi}$ (= 0.259(8…) or 0.26) **or** $\frac{49\pi}{40}$ (= 3.84(8…) or 3.85)	
		(P1	for finding the area of the circle e.g. $\pi\times7^2$ (= 153(.938…) or 154))	May be embedded
		P1	(dep on P2) for a process to find the arc length, e.g. $\frac{\text{“93.5(4...)”}}{360}\times\pi\times2\times7$ (= 11.4(28…)) **or** $\frac{40}{49\pi}\times\pi\times2\times7$ (= 11.4(28…)) **or** $\pi\times2\times7\div\frac{49\pi}{40}$ (= 11.4(28…))	
		A1	for answer in the range 25 to 25.44	If an answer is shown in the range in working and then incorrectly rounded award full marks. Accept $\frac{178}{7}$

Examiner comment

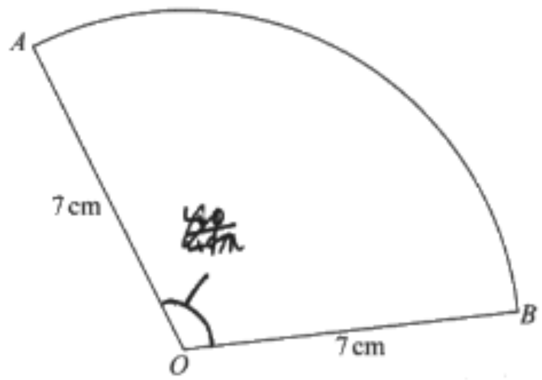
The start to the mark scheme was split into a P2/P1. So those students that either completed the process to find the angle, or wrote a suitable statement for proportion were awarded 2 marks. If students were unable to gain these marks, they could gain P1 for finding the area of the circle.

The third P1 was for the process to find the arc length, either by applying the found angle to the correct formula, or by correctly using their proportion statement with the formula for the circumference of the circle. This was slightly different to the mark scheme on many questions, where the last method or process mark was for a complete method or process. This mark stops short of that, as the student who got as far as finding the arc length but forgot to add 2×7 was worthy of 3 marks rather than 2 marks. It is important that students do read the question carefully.

As with a number of other questions on this paper, some students lost marks due to premature rounding in their working. It is important that students realise the effect premature rounding can have on the accuracy of final answers and retain accuracy throughout.

Student response A

12 *OAB* is a sector of a circle with centre *O* and radius 7 cm.



The area of the sector is 40 cm^2

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

Circumference = $\pi d = 14\pi$

πr^2

$49\pi \times x = 40$

$\frac{40}{49\pi} = x = 0.259844805$

$14\pi \times 0.259844805 = 11.\dot{4}2857\dot{1}$

$11.\dot{4}2857\dot{1} + 7 + 7 = 25.\dot{4}2857\dot{1}$

25.4 cm

4/4

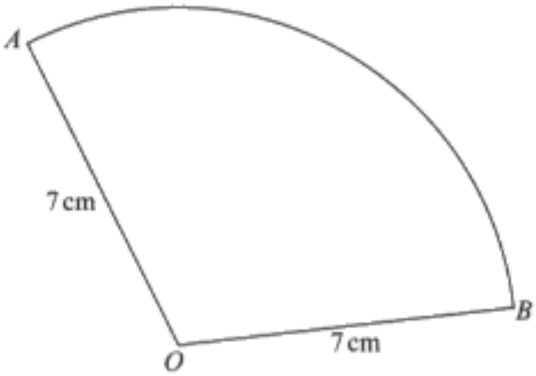
Examiner comment
The student worked in terms of proportion, rather than finding the angle. $\frac{40}{49}\pi$ scored the P2 mark. The student then correctly multiplied this fraction by 14π to calculate the arc length, which was sufficient for the third P1, and finally added on 14 to complete the response. The answer in range gains A1.

Student response B

12 *OAB* is a sector of a circle with centre *O* and radius 7 cm.

$C = \pi d$

$A = \pi r^2$



The area of the sector is 40 cm^2

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

$40 = (\pi \times 7^2) \times x$

$40 = 153.93804 \times x$

~~153.93804 ÷~~

$40 \div 153.93804 = 0.259844805$

$C = \pi d$

$C = \pi \times 14$

$C = 14\pi$

$= 43.98229715$

$43.98229715 \times 0.259844805 = 11.42857143$

$= 11.4$

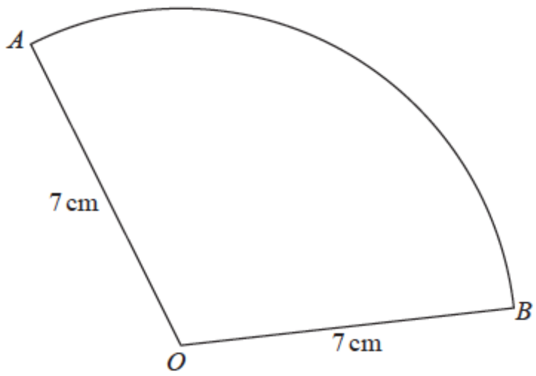
11.4 cm

3/4

Examiner comment
This student followed a similar method to Student A. They set up an equation to find the proportion factor, here called *x*, which they evaluated correctly to gain the P2 marks. As with Student A, they then multiply their proportion factor by 14π, although this student calculated that figure as a decimal. It was acceptable to use either form. Unfortunately, in this case the student stopped at finding the arc length and failed to add on the two radii to complete the perimeter. Since this final step was included in the accuracy mark, only A1 was lost.

Student response C

12 OAB is a sector of a circle with centre O and radius 7 cm.



The area of the sector is 40 cm^2

Calculate the perimeter of the sector.
Give your answer correct to 3 significant figures.

$A = \pi r^2$

$= \pi \times 7^2$

$= 153.93804 \text{ cm}^2$

$153.93804 - 40 = 113.93804$

.... cm

1/4

Examiner comment

The student started like many did, by finding the area of the circle, and gained the first P1. Unfortunately, the student then incorrectly subtracted 40, and was unable to proceed any further.

Exemplar question 4

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Higher tier Question 13

13 Show that $6 + \left[(x+5) \div \dfrac{x^2+3x-10}{x-1} \right]$ simplifies to $\dfrac{ax-b}{cx-d}$ where a, b, c and d are integers.

(Total for Question 13 is 4 marks)

Mean score: 1.29

Examiner comment

This question assessed students' understanding of algebraic fractions, as it required them to factorise, divide and then add algebraic fractions using a common denominator.

Many students were able to access the question in some way and it differentiated well between the higher-achieving students.

The three method marks were for the three distinct steps of the process: factorising the quadratic expression, the method to divide (typically involving multiplying by the reciprocal); and addition of algebraic fractions using a common denominator. No one mark was dependent on the others, meaning the steps could be carried out in differing orders depending on how the student approached the problem.

Many students dropped marks because they didn't know how to divide fractions, quite often crossing out common expressions with no real understanding of what they were doing. In a good number of cases, students struggled with the addition, and multiplyed their fraction by 6 rather than adding.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
13	$\dfrac{7x-13}{x-2}$	B1	for factorising e.g. $(x+5)(x-2)$	
		M1	for a method to divide $(x+5)$ by the algebraic fraction e.g. $(x+5)\times\dfrac{(x-1)}{x^2+3x-10}$	Condone incorrect factorising
		M1	for finding 2 fractions with a common denominator or a single fraction e.g. $\dfrac{6(x-2)}{x-2}+\dfrac{(x-1)}{x-2}$ **or** $\dfrac{6(x-2)+(x-1)}{x-2}$ **or** $\dfrac{6(x^2+3x-10)}{x^2+3x-10}+\dfrac{(x+5)(x-1)}{x^2+3x-10}$ **or** $\dfrac{6(x^2+3x-10)+(x+5)(x-1)}{x^2+3x-10}$	Condone incorrect factorising
		A1	$\dfrac{7x-13}{x-2}$	

Examiner comment

The mark scheme was a simple one. The mark for factorising was an independent B1, as it could be done at any point in the process and this meant that students would pick it up no matter when they completed that step.

The two M1 marks were for dealing with the division and the addition. The first M1 was often dropped as students changed the sign from divide to multiply but didn't exchange the second fraction for its reciprocal. The second M1 was the one least often awarded. The majority of students were unable to write the 6 with a common denominator and hence were unable to get beyond 2 marks.

Sadly, quite a number of students who had worked perfectly with the algebra then lost the A1 as they were unable to correctly simplify '$6x - 12 + x - 1$' and therefore finished with an incorrect numerator.

Student response A

13 Show that $6 + \left[(x+5) \div \dfrac{x^2+3x-10}{x-1} \right]$ simplifies to $\dfrac{ax-b}{cx-d}$ where a, b, c and d are integers.

$$6 + \left[\frac{x+5}{1} \div \frac{(x+5)(x-2)}{x-1} \right]$$

$$= 6 + \left[\frac{\cancel{x+5}}{1} \times \frac{x-1}{(\cancel{x+5})(x-2)} \right]$$

$$= 6 + \frac{x-1}{x-2} =$$

$$= \frac{6(x-2)+x-1}{x-2} =$$

$$= \frac{6x-12+x-1}{1x-2} =$$

$$= \frac{7x-13}{1x-2}$$

4/4

Examiner comment
This response was taken almost perfectly from the mark scheme itself! The student started by correctly factorising the quadratic to gain B1. They then showed a correct method for division, multiplying by the reciprocal, gaining the first M1. From here, they correctly simplified the product to leave them with $6 + \dfrac{x-1}{x-2}$. The final M1 could be awarded for either a correct method to find two fractions with a common denominator, or, as this student did, a single fraction. The student then correctly simplified the numerator to obtain the correct expression for A1.

Student response B

13 Show that $6 + \left[(x+5) \div \dfrac{x^2+3x-10}{x-1}\right]$ simplifies to $\dfrac{ax-b}{cx-d}$ where a, b, c and d are integers.

$$6+\left[x+5 \div \frac{x^2+3x-10}{(x-1)}\right. \qquad 5 \quad -2$$

$$\cancel{6+\left[x\right.}$$

$$6+\left[\frac{(x+5)}{1} \div \frac{(x+5)\ (x-2)}{(x-1)}\right]$$

$$6+\left[\frac{(x+5)}{1} \times \frac{(x-1)}{(x+5)(x-2)}\right]$$

$$6+\left[\frac{(x+5)(x-2)}{(x+5)(x-2)}\right]$$

$$6+\frac{x-1}{x-2}$$

$$\frac{6}{1}+\frac{x-1}{x-2}$$

$$\frac{6(x-2)}{x-2}+\frac{+x-1}{x-2}$$

$$\frac{6x-12+x-1}{(x-2+x-2} = \frac{7x-13}{2x-4}$$

3/4

Examiner comment
In the case of the first two marks, this response was almost identical to that of Student A, gaining B1 M1. For the third M1, this student chose to work with two fractions with a common denominator. Their fractions were correct and so the second M1 was awarded. Unfortunately, when adding the two fractions the student added the denominators, so they lost the A1. It was very common to see this mistake, or to see the numerator incorrectly simplified.

Student response C

13 Show that $6 + \left[(x+5) \div \dfrac{x^2+3x-10}{x-1}\right]$ simplifies to $\dfrac{ax-b}{cx-d}$ where a, b, c and d are integers.

$(x+5)(x-2)$

$x^2 \quad -2x + 5x$

$$6 + (x+5) \div \frac{x^2 + 3x - 10}{x - 1} = \frac{(x+5)\ (x-2)}{x-1}$$

$$6 + \{(x+5) \div \frac{(x+5)(x-2)}{x-1}$$

1/4

Examiner comment

The student correctly factorised the quadratic expression to gain B1. However, they did not know how to deal with the division of the fractions and no further work of credit was seen.

Exemplar question 5

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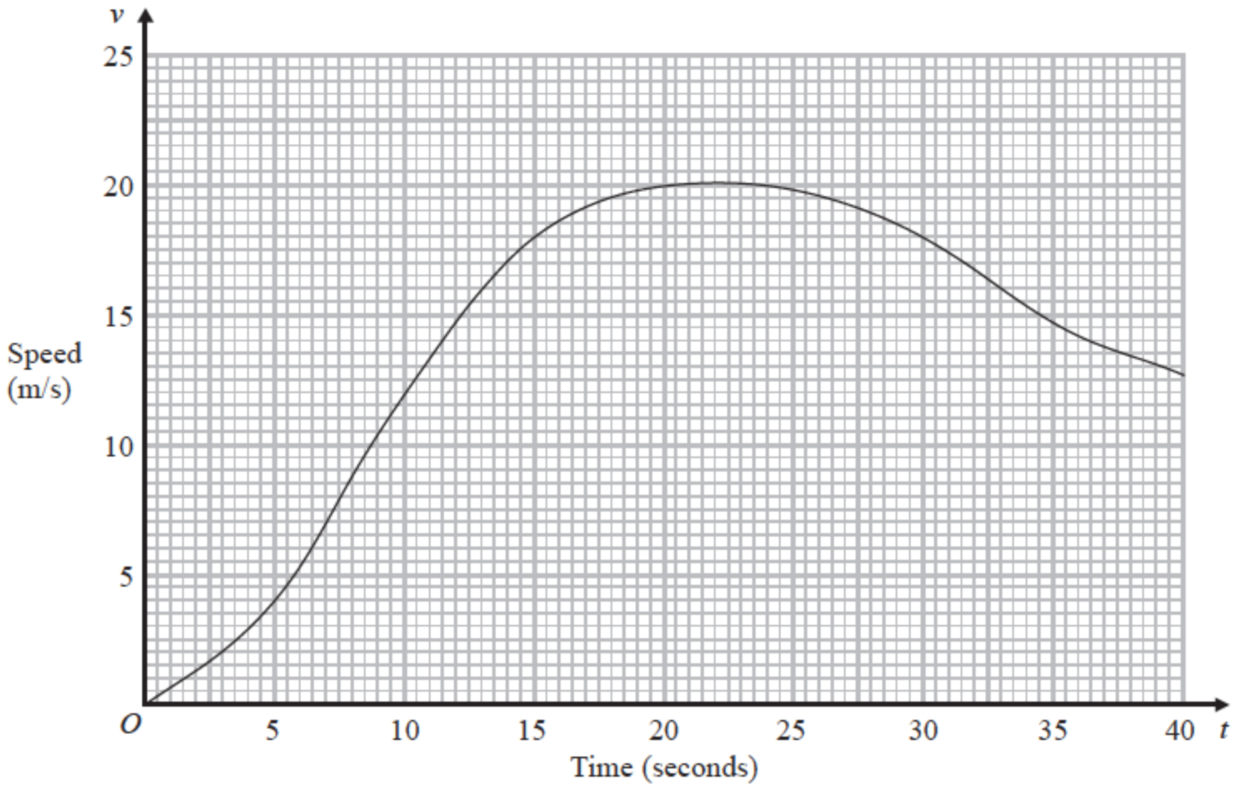
Higher tier Question 14

14 A car moves from rest.

The graph gives information about the speed, v metres per second, of the car t seconds after it starts to move.



(a) (i) Calculate an estimate of the gradient of the graph at $t = 15$

....
(3)

(ii) Describe what your answer to part (i) represents.

....
(1)

(b) Work out an estimate for the distance the car travels in the first 20 seconds of its journey.
Use 4 strips of equal width.

....m
(3)

(Total for Question 14 is 7 marks)

Mean score: 3.21

Examiner comment

This question addressed two new topics to the specification: finding and interpreting the gradient of a speed–time graph, and area under a graph. Both topics have been on previous papers and it was therefore surprising to see so many students not prepared for this topic.

Part (a): Students were required to draw a tangent to the curve at a time of 15 seconds and then find the gradient, while part (a)(ii) required them to interpret the gradient of this graph as rate of change of speed, or acceleration. A large number of students didn't draw a tangent at all and therefore were unable to gain any credit in part (a)(i). Of these students, most simply read off the speed at a time of 15 seconds and divided by 15.

Many of the students who did draw a tangent had very poor accuracy in reading values from the axes, which often resulted in losing accuracy marks. Some credit was given to students who misinterpreted the scale, but for many it appeared to be a lack of care and attention.

In part (b)Part (b): Students were prompted to use 4 strips to find the area under the curve in the first 20 seconds. Many didn't read the question properly and tried to find the area under the whole of the curve. While credit was given to students who used rectangles under or above the curve, it should be noted that this did not provide a very good estimate for the area and almost invariably resulted in an answer outside the permitted range.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
14(a)(i)	0.83	B1	for a tangent drawn at $t = 15$	Working: $7.5 \div 9 = 0.83\ldots$ No tangent scores 0 marks
		M1	full method to use the tangent to find the gradient (e.g. $7.5 \div 9$)	This mark can be awarded if the tangent is drawn at $t \neq 15$ Working may be seen on the diagram
		A1	for answer in the range 0.6 to 1.0	
(a)(ii)	Statement	C1	statement **Acceptable examples** acceleration rate of change of speed increase in speed over time	
14(a)(ii) *continued*			**Not acceptable examples** rate of change m/s/s increase in speed	

Question	Answer	Mark	Mark scheme	Additional guidance
(b)	220	P1	for splitting the area into strips and correct process to find the area of one strip, e.g. $\frac{5\times4}{2}$ (= 10) **or** $\frac{(4+12)}{2}\times5(=40)$ **or** $\frac{(12+18)}{2}\times5(=75)$ **or** $\frac{(18+20)}{2}\times5\ (=95)$	Working 4, 12, 18, 20
		P1	for a complete process using at least 4 strips to find the area under the curve e.g. "10" + "40" + "75" + "95"	Allow one error in the reading of speeds
		A1	for answer in the range 215 to 225 from correct working using at least 4 strips	

Examiner comment

Part (a)(i): It is important to note that without a tangent drawn, no marks could be awarded. If a tangent was drawn, but it was at the wrong time, M1 could still be awarded if there was a suitable method shown to find its gradient.

Students needed to take real care when reading values from the axes. Many misinterpreted the scale altogether, and while this was taken into account in the awarding of M1, it meant A1 could not be awarded. However, many students just appeared to read the axes wrong, resulting in incorrect values used in the calculation. Where this was the case M1 was not awarded, and neither was A1, even if an answer in range was achieved.

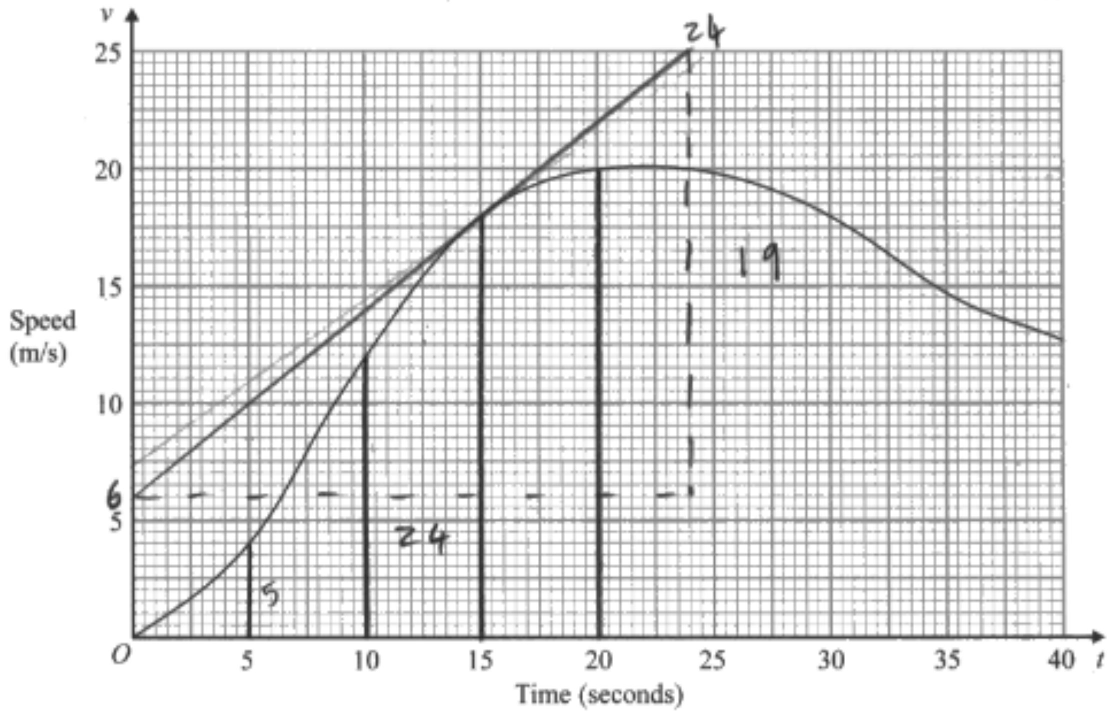
Part (a)(ii): This part was looking for students to realise that the rate of change of speed was in fact acceleration, but either answer (or a similar statement) was awarded C1.

Part (b): The first P1 was for splitting into strips and for a correct process to find the area of one strip. Students who used fewer than 4 strips for the first 20 seconds were able to access this mark. Students using rectangles could gain both this and the next P1, providing the rectangles were clearly seen on the diagram and a correct process was shown. The second P1 was for using at least 4 strips and having a correct method to find each area and add. Some students used more than 4 strips; while this should not to be encouraged from a time perspective, it was not penalised as it would provide a better estimate. One error in reading speed could be condoned in the award of this mark. The answer needed to be in the range 215 and 225 for A1.

Student response A

14 A car moves from rest.

The graph gives information about the speed, v metres per second, of the car t seconds after it starts to move.



(a) (i) Calculate an estimate of the gradient of the graph at $t = 15$

$\frac{19}{24}$

$\frac{19}{24}$

(3)

3/3

(ii) Describe what your answer to part (i) represents.

the accelaration of the car at that point.

(1)

1/1

(b) Work out an estimate for the distance the car travels in the first 20 seconds of its journey.
Use 4 strips of equal width.

$\cancel{\frac{1}{2}(0+4)5}$

$\cancel{\frac{1}{2}(46+}$

$\cancel{\frac{1}{2}}$

$\cancel{\frac{1}{2}}$

$\frac{1}{2}(0+4)5 = 10$

$\frac{1}{2}(4+12)5 = 40$

$\frac{1}{2}(12+18)5 = 75$

$\frac{1}{2}(18+20)5 = 95$

220

220 m

(3)

3/3

Examiner comment

Part (a)(i): The student clearly drew a tangent at 15 seconds. Using dashed lines, they drew in a right-angled triangle, demonstrating a correct method to find the gradient, and their answer of $\frac{19}{24}$ was in range.

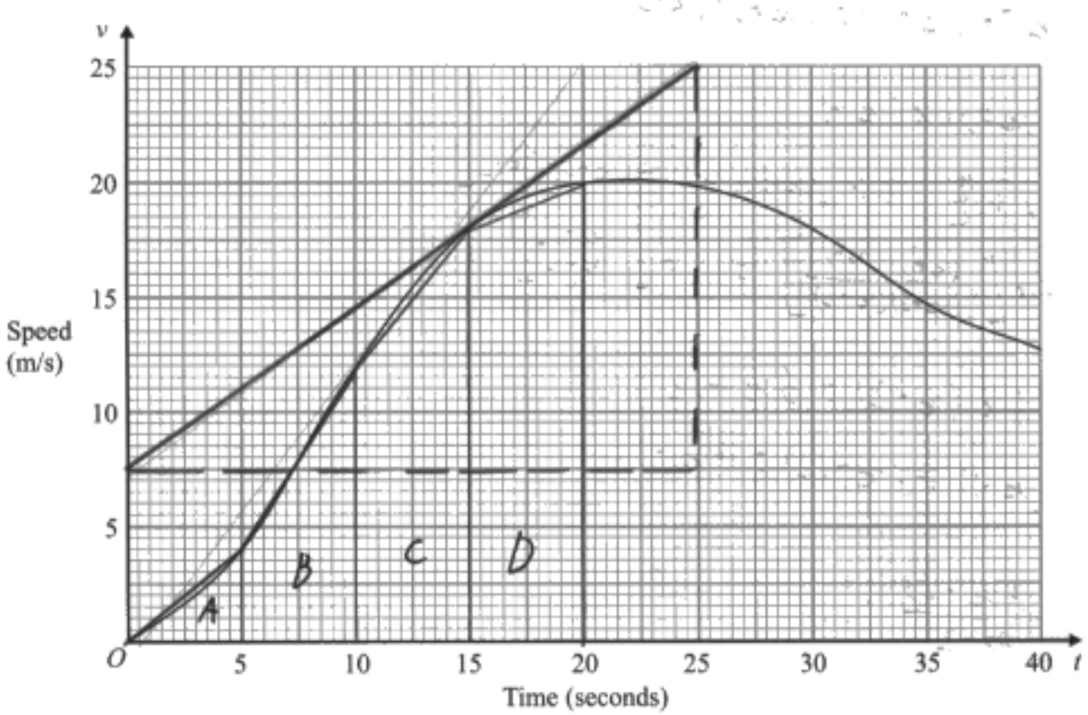
Part (a)(ii): C1 was awarded for 'acceleration'.

Part (b): The student showed strips drawn onto the diagram, and although they did not draw the chords their working showed that they worked with trapezia. If they had been using the less accurate method of using rectangles, the rectangles would have needed to be clearly drawn on the grid for the M1 marks. The fully correct working led to the correct answer, and so all 3 marks were awarded.

Student response B

14 A car moves from rest.

The graph gives information about the speed, v metres per second, of the car t seconds after it starts to move.



(a) (i) Calculate an estimate of the gradient of the graph at $t = 15$

$25 - 7.5 = 17.5$

$25 - 0 = 25$

$\frac{17.5}{25} = 0.7$

0.7

(3)

3/3

(ii) Describe what your answer to part (i) represents.

~~than~~ The car's accelaration at t =15

(1)

1/1

(b) Work out an estimate for the distance the car travels in the first 20 seconds of its journey.
Use 4 strips of equal width.

A = 0.5 × 5 × 4 = 10m
B = 0.5 × (4 + 11.75) × 5 = 39.375m
C = 0.5 × (11.75 + 16) × 5 = 69.375m
D = 0.5 × (16 + 20) × 5 = 90m

10 + 39.375 + 69.375 + 90 = 208.75m

208.75 m
(3)

2/3

Examiner comment

Part (a)(i): The student drew a tangent at 15 seconds for B1. They drew in the right-angled triangle and showed correct working to find the gradient using this tangent, gaining M1. Their answer was within the given range, so 3 A1.

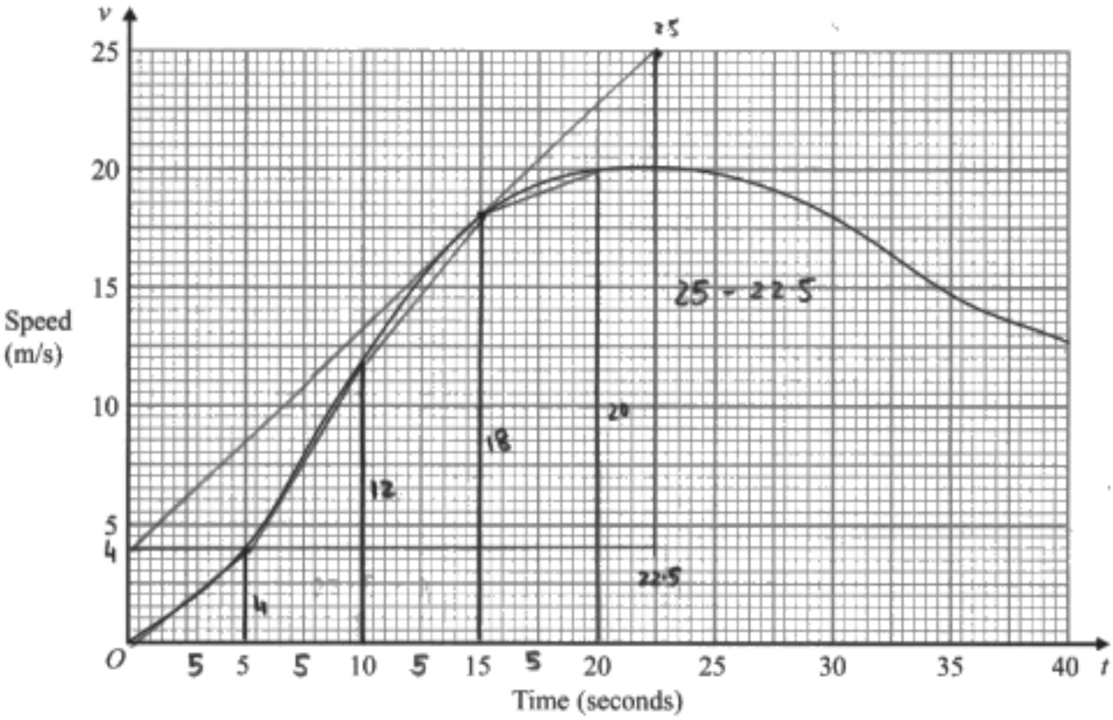
Part (a)(ii): C1 was awarded for 'acceleration'.

Part (b): This was a nearly perfect response. The student split the diagram into strips, including the chords, and showed the method to find the area of each. In areas B and C, a value of 11.75 was used. Professional judgement was used when looking at whether the values are correct, and this was perfectly acceptable instead of the correct value of 12. However, in areas C and D the value 16 should be 18, which is too far away to be taken as correct. This constituted one error in reading speeds, which the additional guidance stated was acceptable for P2. Unfortunately, because of this error, the answer was outside the acceptable range, so A1 could not be awarded.

Student response C

14 A car moves from rest.

The graph gives information about the speed, v metres per second, of the car t seconds after it starts to move.



(a) (i) Calculate an estimate of the gradient of the graph at $t = 15$

Gradient = $\frac{\text{Change in } y}{\text{Change in } x}$

$= \frac{25 - 22.5}{22.5 - 4} = \frac{2.5}{18.5} = 0.135 \text{ m/s}^2.$

0.135 m/s².

(3)

1/3

(ii) Describe what your answer to part (i) represents.

It represents the acceleration of the car

(1)

1/1

(b) Work out an estimate for the distance the car travels in the first 20 seconds of its journey.
Use 4 strips of equal width.

Distance = area under the graph.

$$\left(\frac{1}{2} \times \overset{2}{\cancel{4}} \times 5\right) + \left(\frac{1}{2} \times (12+4) \times 5\right) + \left(\frac{1}{2} \times (18+12) \times 5\right) + \left(\frac{1}{2} \times (18+20) \times 20\right)$$

$$= 10 + 40 + 75 + 380$$

$$= \underline{\underline{505\text{m}}}$$

505. m
(3)

1/3

Examiner comment

Part (a)(i): The student drew a tangent at 15 seconds for B1. In this case the student confused their '*x*' and '*y*' values making their method incorrect, so no further marks were awarded.

Part (a)(ii): C1 was awarded for 'acceleration'.

Part (b): The student started well, correctly split the area and showing the method to find the area of a strip, gaining the first P1. However, in the fourth strip they used a value of 20 in place of 5. For the second P1, one mistake in reading speeds could be condoned, but not in reading times. This means the second P1 could not be awarded. The answer is outside the range so was not awarded A1.

Exemplar question 6 [back to Contents page]

Higher tier Question 16

16 The straight line **L** has the equation $3y = 4x + 7$
The point A has coordinates $(3, -5)$

Find an equation of the straight line that is perpendicular to **L** and passes through A.

....

(Total for Question 16 is 3 marks)

Mean score: 0.93

Examiner comment

This question assessed understanding of $y = mx + c$ and the relationship of the gradients of perpendicular lines. Students needed to extract a gradient from the given equation, which for most students involved rearranging into the form $y = m \ + c$, then apply the knowledge of $-\frac{1}{m}$ to find the gradient of the perpendicular line. The accuracy mark then included the step of substitution to find the y-intercept.

Many students struggled to find the gradient of the line **L** as they didn't understand that the coefficient of x is only the gradient when the equation is in the correct form.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
16	$y=-\frac{3}{4}x-\frac{11}{4}$	M1	for identifying gradient of $\frac{4}{3}$	Ignore constant term
		M1	for beginning a method to find the gradient of the perpendicular line e.g. $\frac{4}{3}\times m=-1$ **or** identifies gradient of perpendicular line as $-\frac{3}{4}$	Can ft providing gradient is clearly stated
		A1	for $y=-\frac{3}{4}x-\frac{11}{4}$ or any equivalent equation	$4y+3x=-11$ $y+5=-\frac{3}{4}(x-3)$

Examiner comment

The first M1 was for correctly identifying the gradient of **L**. The second M1 was for a method to find the gradient of the perpendicular line. For many students, this was use of $-\frac{1}{m}$. This mark could be gained even if the value of m was incorrect, provided the gradient was clearly stated. The A1 mark included the substitution to find the y-axis intercept, leading to the final equation. Any equivalent equation was acceptable, but the one in the answer column, and its decimal equivalent, were the ones most commonly seen.

Student response A

16 The straight line **L** has the equation $3y = 4x + 7$
The point A has coordinates $(3, -5)$

Find an equation of the straight line that is perpendicular to **L** and passes through A.

$$3y = 4x + 7$$

$$y = \frac{4}{3}x + \frac{7}{3} \longrightarrow y = -\frac{3}{4}x + \frac{7}{3}$$

$x \quad y$
$(3, -5)$

$$-5 = \left(-\frac{3}{4} \times 3\right) + c$$

$$-5 = \frac{-9}{4} + c$$

$$-\frac{11}{4} = c$$

$$y = -\frac{3}{4}x - \frac{11}{4}$$

3/3

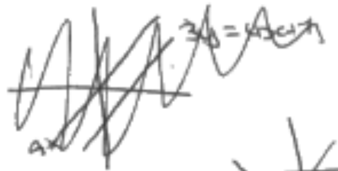
Examiner comment

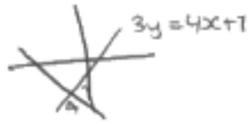
The student started, like most successful students, by rearranging the equation into the form $y = mx + c$. However, this alone was not enough to gain the first M1, for which the gradient needed to be identified. Although the student did not explicitly state the gradient, using it to find the perpendicular gradient was sufficient for the first M1. The second M1 was awarded for using the correct gradient of the perpendicular line in the working to get the equation of the perpendicular line. The student then used substitution to find the y-axis intercept, and hence the correct equation, gaining A1.

Student response B

16 The straight line **L** has the equation $3y = 4x + 7$
The point A has coordinates $(3, -5)$

Find an equation of the straight line that is perpendicular to **L** and passes through A.





$y = \frac{4x+7}{3}$ $g = \frac{4}{3}$

$pg = -\frac{3}{4}$

$y = -\frac{3}{4}x + c$

$-5 = -\frac{3}{4}(3) + c$

$-5 = -4 + c$

$c = -1$

$y = -\frac{3}{4}x + -1$

$y = -\frac{3}{4}x - 1$

2/3

Examiner comment

The student correctly rearranged the equation and stated $g = \frac{4}{3}$ to gain the first M1 mark. They then stated $pg = -\frac{3}{4}$ (for perpendicular gradient), gaining the second M1 mark. The substitution to find the intercept was correct, but the evaluation was incorrect, leading to a value of -1 rather than -5.25 and so A1 could not be awarded.

Student response C

16 The straight line **L** has the equation $3y = 4x + 7$
The point A has coordinates $(3, -5)$

Find an equation of the straight line that is perpendicular to **L** and passes through A.

$3y = 4x + 7$ $\quad \cancel{(3, -5)}$

$m = 4$

$y = mx + c$

$y \cancel{m} = -\frac{1}{4}x + c$ $\quad (3, -5)$

$m = -\frac{1}{4}$

$-5 = -\frac{1}{4}(3) + c$

$-5 = -0.75 + c$

$c = -4.25$

$y = -\frac{1}{4}x \cancel{+} -4.25$

(Total for Question 16 is 3 marks)

1/3

Examiner comment

As with a large number of students, this student did not rearrange the equation and they stated the gradient as 4, so the first M1 could not be awarded. However, since the gradient was explicitly stated, and the method to find the perpendicular gradient (here $m = -\frac{1}{4}$) was correct, the second M1 mark could be awarded. Because of the early mistake the answer was wrong, so A1 was not awarded.

Exemplar question 7

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Higher tier Question 17

17 There are some small cubes and some large cubes in a bag.
The cubes are red or the cubes are yellow.

The ratio of the number of small cubes to the number of large cubes is 4 : 7

The ratio of the number of red cubes to the number of yellow cubes is 3 : 5

(a) Explain why the least possible number of cubes in the bag is 88

....

....

....

(1)

All the small cubes are yellow.

(b) Work out the least possible number of large yellow cubes in the bag.

....

(3)

(Total for Question 17 is 4 marks)

Mean score: 1.39

Examiner comment

Part (a): The question assessed students' understanding of ratio. If the total can be divided in two different ratios and the values need to be integers then the smallest total must be the lowest common multiple of the sum of both ratios.

Part (b): This part could be accessed in a number of ways. The most common successful routes were to use the ratios to either find the total number of yellows and the number of small cubes and subtract, or to find the number of large cubes and subtract the number of red cubes. However, some students used two-way tables or probability trees and could be just as successful.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
17(a)	Explanation	C1	for stating the LCM of (4 + 7) and (5 + 3) is 88 **or** there is no smaller multiple of 8 and 11 (than 88)	
(b)	23	P1	for using a scale factor appropriately e.g. 4×8 (= 32) **or** 3×11 (= 33) **or** 7×8 (= 56) **or** 5×11 (= 55) **OR** for writing a pair of suitable fractions, e.g. $\frac{7}{11}$ and $\frac{3}{8}$ **or** $\frac{4}{11}$ and $\frac{5}{8}$ **or** $\frac{3}{8}$ and $\frac{4}{11}$	May be seen in a two-way table or probability tree
		P1	for finding the number of large cubes and red cubes **or** small and yellow **or** small and red e.g. 7×8 (= 56) and 3×11 (= 33) **or** 4×8 (= 32) and 5×11 (= 55) **or** 4×8 (= 32) and 3×11 (= 33) **OR** a suitable fractional equation, e.g. $\frac{7}{11} - x = \frac{3}{8}$ **or** $\frac{5}{8} - x = \frac{4}{11}$ **or** $x = 1 - \frac{3}{8} - \frac{4}{11}$ **OR** a suitable pair of probabilities with a common denominator, e.g. $\frac{56}{88}$ and $\frac{33}{88}$ **or** $\frac{32}{88}$ and $\frac{55}{88}$ **or** $\frac{33}{88}$ and $\frac{32}{88}$	May be seen in a two-way table or probability tree $\frac{23}{88}$ scores P2 A0
		A1	cao	

Examiner comment

Part (a): To gain this single mark, students needed to realise that 88 was the LCM of 11 and 8, although they didn't need to specifically quote those words. Many students did not gain this mark and that was because they stated that $8 \times 11 = 88$, but did not address the lowest common multiple element.

Part (b): Students first needed to find a path through the problem by finding a suitable pair of values that would lead to the number of large yellow cubes. The first P1 was for finding one suitable value and the second P1 for finding two that together could lead to the correct answer. So by finding, for example, 56 and 32 a student would only score the first P1; even though both are suitable starting values, together they cannot lead directly to the correct answer.

For those who worked with fractions or probabilities, to gain the first P1 a suitable pair was needed for the P1 mark and then a suitable equation for the second P1. For those who dealt with fractions, $\frac{23}{88}$ was a common answer, but it failed to score the final mark as it is not the answer to the question.

Student response A

17 There are some small cubes and some large cubes in a bag.
The cubes are red or the cubes are yellow.

The ratio of the number of small cubes to the number of large cubes is 4 : 7

The ratio of the number of red cubes to the number of yellow cubes is 3 : 5

(a) Explain why the least possible number of cubes in the bag is 88

small to large = 4:7 so total = 11, red to yellow = 3:5 so total = 8. The lowest common multiple of 11 and 8 is 88, so this is the least possible number of cubes.

(1)

1/1

All the small cubes are yellow.

(b) Work out the least possible number of large yellow cubes in the bag.

S : L
4 : 7 = 11

R : Y
3 : 5 = 8

least cubes = 88

S : L : R : Y
32 : 56 : 33 : 55

smallest no yellow cubes = 55

all 32 small are yellow,
55 − 32 = 23 yellow cubes left
These 23 can only be large cubes, since all small are yellow

23

(3)

3/3

Examiner comment

Part (a): The student correctly stated that the LCM of 11 and 8 is 88, gaining C1.

Part (b): The student correctly used both ratios to find the number of red cubes and yellow cubes, and the number of small cubes and large cubes, gaining the first P1. They then interpreted the values correctly, realising that all small cubes are yellow for the second P1. They therefore worked out the correct number of large yellow cubes as the total yellow cubes (55) minus the number of small cubes (32), gaining A1.

Student response B

17 There are some small cubes and some large cubes in a bag.
The cubes are red or the cubes are yellow.

The ratio of the number of small cubes to the number of large cubes is 4 : 7

The ratio of the number of red cubes to the number of yellow cubes is 3 : 5

(a) Explain why the least possible number of cubes in the bag is 88

4+7=11, and 3+5 is 8. ~~the~~ The Lowest Common Multiple of these numbers is 88. If the least possible number was lower than 88, You wouldn't have a whole number.

(1)

1/1

All the small cubes are yellow.

(b) Work out the least possible number of large yellow cubes in the bag.

S : L   R : Y
4 : 7   3 : 5

$\frac{88}{11} = 8$ $\frac{88}{8} = 11$

8 × 4 = 32   11 × 3 = 33
8 × 7 = 56   11 × 5 = 55

S : L   R : Y
32 : 56   33 : 56

56 − 32 = 24

24

(3)

2/3

Examiner comment
Part (a): The correct statement about LCM was awarded C1.
Part (b): The student worked with both ratios to find the number of small cubes and large cubes and the number of red cubes and yellow cubes. Unfortunately, they were unable to interpret the information correctly and chose an incorrect pair to subtract, so no further marks were awarded.

Student response C

17 There are some small cubes and some large cubes in a bag.
The cubes are red or the cubes are yellow.

The ratio of the number of small cubes to the number of large cubes is 4 : 7 = 11

The ratio of the number of red cubes to the number of yellow cubes is 3 : 5 = 8

(a) Explain why the least possible number of cubes in the bag is 88

because 4 + 7 = 11 and 3 + 5 = 8, 11 × 8 = 88

~~88 is the smallest ratio~~

(1)

0/1

(b) Work out the least possible number of large yellow cubes in the bag.

small : large
4 : 7 = 11 → $\frac{4}{11}$ of the cubes are yellow

yellow : red
5 : 3 = 8 → more yellow than red $\frac{5}{8}$

~~$\frac{4}{11} \times \frac{5}{8} = \frac{20}{88} = \frac{10}{44} = \frac{5}{22}$~~

4 : 7 × 5
5 : 3 × 4

20 : 35 → total large cubes
20 : 12
→ total yellow

$\frac{20}{35} = \frac{4}{7}$

20

(3)

1/3

Examiner comment

Part (a): Although the student showed that 88 is a multiple of 11 and 8, they made no reference to the LCM and so no mark was awarded.

Part (b): Like many students, this student started to work with fractions. They gained the first P1 mark for finding a suitable pair, here $\frac{4}{11}$ and $\frac{5}{8}$. However, no correct equation was set up with these fractions and so no further marks were awarded.

Exemplar question 8

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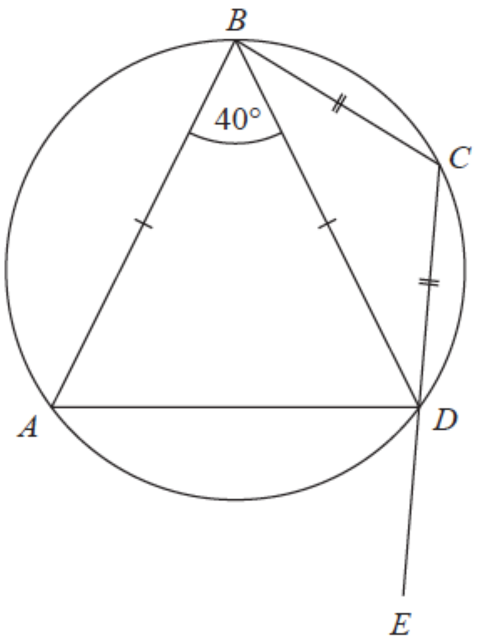
Higher tier Question 18

18 The points A, B, C and D lie on a circle.
CDE is a straight line.



$BA = BD$
$CB = CD$
Angle $ABD = 40°$

Work out the size of angle ADE.
You must give a reason for each stage of your working.

(Total for Question 18 is 5 marks)

Mean score: 2.62

Examiner comment

This circle theorem question only required the knowledge of cyclic quadrilaterals alongside the other standard geometric properties. However, without the knowledge of cyclic quadrilaterals students could not score more than 1 mark.

The question stated that reasons must be stated, and it was disappointing to see so many students either failing to give reasons, or not stating them fully, and therefore dropping marks.

Many students mistook CDE as a tangent and then stated, incorrectly, that angle ADE was 90°.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18	75° with reasons	M1	for finding angle $BAD = \frac{180-40}{2}$ (= 70) **or** angle $BDA = \frac{180-40}{2}$ (= 70)	Could be shown on the diagram or in working
		M1	for finding angle BCD = 180 – "70" (= 110) **or** 40 + x + 70 + x = 180	
		A1	for finding angle ADE = 75	
		C2	(dep M2) for Opposite angles of a cyclic quadrilateral add up to 180 **and** one other reason; all reasons given must be appropriate for their working Base angles of an isosceles triangle are equal, Angles in a triangle add up to 180 Angles on a straight line add up to 180 [**or** exterior angle of a cyclic quadrilateral is equal to the interior opposite angle]	Underlined words need to be shown; reasons need to be linked to their method
		(C1	(dep M2) for Opposite angles of a cyclic quadrilateral add up to 180, **or** all other reasons given appropriate for their working)	Apply the above criteria

Examiner comment

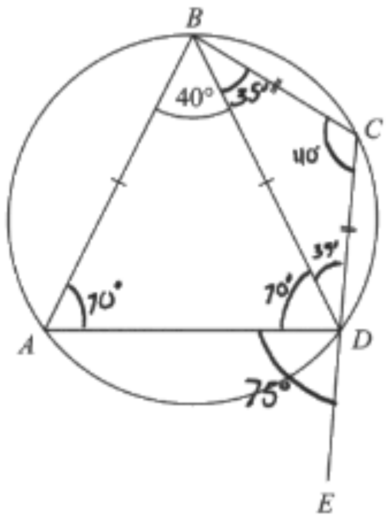
The two M1 marks were for firstly finding angle BAD or angle BDA using isosceles triangles, and then for finding angle BCD using the cyclic quadrilateral. A1 included the final two steps, without a separate method mark, for finding angle BDC using isosceles triangles and then angle ADE using angles on a straight line.

In terms of the C marks, it was 2 marks for the correct circle theorem plus one other reason, relevant to the method used.

If the C2 mark was not scored, C1 could be awarded, either for the correct circle theorem, or if this was missing or not complete, for all other reasons appropriate for their method stated and correct.

Student response A

18 The points A, B, C and D lie on a circle.
CDE is a straight line.



$BA = BD$
$CB = CD$
Angle $ABD = 40°$

Work out the size of angle ADE.
You must give a reason for each stage of your working.

180 - 40 = 140
140 ÷ 2 = 70°
Angles BAD and BDA = 70° because triangle ABD is an isoceles.

180 - 70 = 110
Angle BCD = 110° because opposite angles in a cyclic quadrilateral add to 180°

180 - 110 = 70°
70 ÷ 2 = 35°
Angles CBD and CDB = 35° because triangle BCD is an isoceles.

70 + 35 = 105
180 - 105 = 75 ∴ angle ADE = 75° because angles on a straight line add to 180°.

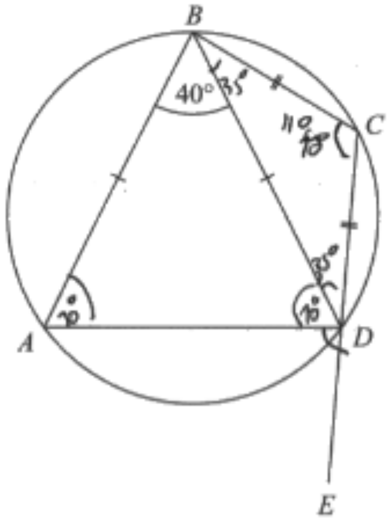
5/5

Examiner comment

This was the most common correct approach to the question, and in each case the property used was stated, and was complete enough to gain credit. However, it is worth noting that one reason, namely angles in a triangle sum to 180, was missing. The mark scheme for the C2 mark stated that they needed to state the circle theorem and one other correct reason, which the student clearly did, so full marks were awarded.

Student response B

18 The points A, B, C and D lie on a circle.
CDE is a straight line.



$BA = BD$
$CB = CD$
Angle $ABD = 40°$

Work out the size of angle ADE.
You must give a reason for each stage of your working.

Angles BAD and BDA are the same,
two angles in an isosceles triangle are same,
all angles add to 180.
180 - 40 = 140 ÷ 2 = 70°

~~BAD = BAD~~ Angle BCD = 110°, angles opposite each other in a quadrilateral add to 180°

CBD = CDB as two angles in an isosceles triangle are same, all angles add to 180.
180 - 110 = 70 ÷ 2 = 35

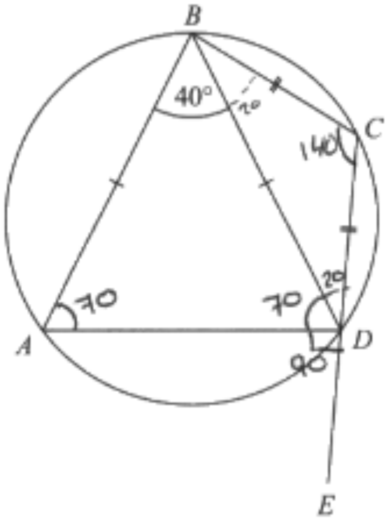
70 + 35 = 105, so ADE = 75°, as angles in a straight line add to 180°.

4/5

Examiner comment
At first glance, this looked like a complete and correct response. Angle *ADE* was correctly found using a correct method and all the reasons appeared to be stated and correct. However, in the circle theorem the word 'cyclic' was missing, meaning that C2 could not be awarded. Since all other reasons were appropriate and correct, C1 could be awarded.

Student response C

18 The points *A*, *B*, *C* and *D* lie on a circle.
CDE is a straight line.



BA = *BD*
CB = *CD*
Angle *ABD* = 40°

Work out the size of angle *ADE*.
You must give a reason for each stage of your working.

BA = BD

∴ $\frac{180-40}{2}$ = 70 (Isosceles triangle base angles are equal)

Angle ADC = 20 ∴ (CB = CD)
BCD = 140 (Angles in triangle = 180)

70+20 = 90 180 − 90 = 90 (Angles on straight line = 180)

∴ CDE = ~~7~~90°

1/5

Examiner comment
The student started well by finding angle *BAD* and angle *BCD*, gaining the first M1. However, they incorrectly assumed that angle *ADE* is 90° and worked from there, meaning neither the second M1 nor A1 could be awarded. Since the C marks were dependent on M2, no C marks could be awarded, even if the reasons given were correct and appropriate.

Exemplar question 9

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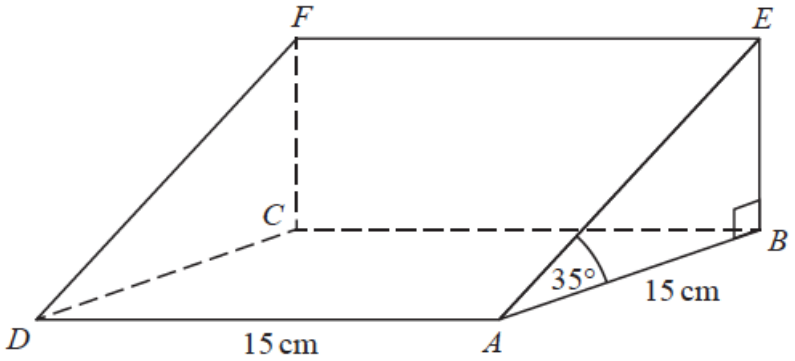
Higher tier Question 19

19 The diagram shows a triangular prism.



The base, *ABCD*, of the prism is a square of side length 15 cm.
Angle *ABE* and angle *CBE* are right angles.
Angle $EAB = 35°$

M is the point on *DA* such that

$$DM : MA = 2 : 3$$

Calculate the size of the angle between *EM* and the base of the prism.
Give your answer correct to 1 decimal place.

.... °

(Total for Question 19 is 4 marks)

Mean score: 1.60

Examiner comment

This was a challenging question on 3D trigonometry and Pythagoras' theorem. Students first had to split an edge in a given ratio to find the point *M*. Once this was done there were at least three steps to finding the correct angle, and some students used more. The most common route through the problem was to find *EB*, then *MB* and then the angle *EMB*, but it was not the only method.

Students who rounded prematurely often ended with an answer outside the acceptable range; premature rounding should be discouraged.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19	31.0	P1	for tan 35 = BE ÷ 15 **or** BE =10.5(0…) **OR** finding the length $DM = \frac{2}{5} \times 15\ (=6)$ **or** $MA = \frac{3}{5} \times 15\ (=9)$ **or** 6 : 9 **OR** showing the required angle on a diagram e.g. with an arc	$MB = \sqrt{9^2 + 15^2} = \sqrt{306}$ (= 17.4(9…) or 17.5) BE = 15 × tan 35 (= 10.5(0…)) AE = 15 ÷ cos 35 (= 18.3(1…)) $ME = \sqrt{9^2 + 18.31...^2}$ $= \sqrt{416.(3...)}$ (= 20.4(0…))
		P1	for $MB = \sqrt{15^2 + \text{“}9\text{”}^2}$ **or** $\sqrt{306}$ or 17.4(9....) **OR** $ME = \sqrt{\text{“}9\text{”}^2 + \text{“}18.3(1...)\text{”}^2}$ **or** $\sqrt{416.(3...)}$ or 20.4(0…)	Check diagram for working
		P1	for using appropriate trigonometry ratio to set up an equation in angle EMB e.g. tan θ= “10.5(0…)” ÷ “17.4(9…)” **or** cos θ= “17.4(9…)” ÷ “20.4(0…)” **or** sin θ= “10.5(0…)” ÷ “20.4(0…)”	
		A1	for answer in the range 30.9 to 31	If an answer is shown in the range in working and then incorrectly rounded award full marks.

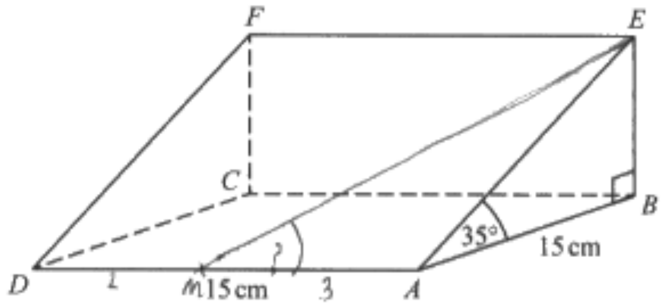
Examiner comment

The majority of students who attempted this question were able to gain one mark, typically the first P1 for either splitting the edge DA in the correct ratio or for finding the length of BE. A good proportion of students then gained the second P1 for correctly finding MB or ME.

Gaining the third P1 proved challenging, with only a few students able to put together a correct trigonometric statement including angle EMB. Most who got this far gained full credit, apart from those who had made arithmetic errors in their working. This meant, providing steps and calculations were clearly shown, that they could gain all three P1 marks, and only lose a1.

Student response A

19 The diagram shows a triangular prism.



The base, *ABCD*, of the prism is a square of side length 15 cm.
Angle *ABE* and angle *CBE* are right angles.
Angle *EAB* = 35°

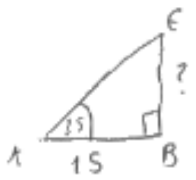
M is the point on *DA* such that

SOHCAHTOA

$$DM : MA = 2 : 3$$

Calculate the size of the angle between *EM* and the base of the prism.
Give your answer correct to 1 decimal place.

$3\sqrt{34}$



$EB = \tan(35) \times 15$
$= 10.503...$

$\sqrt{15^2 + 9^2} = 3\sqrt{34}$

$BM = 3\sqrt{34}$

$EMB = \tan^{-1}\left(\frac{10.503...}{3\sqrt{34}}\right) = 30.98157$

31.0 °

4/4

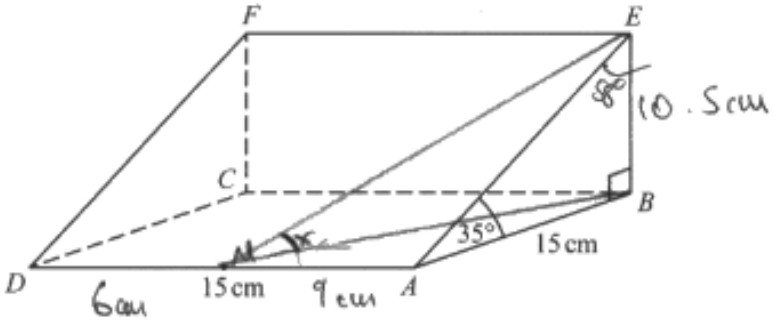
Examiner comment

This response follows the same route as the majority of successful students.

First, the student divided the front edge in the ratio to find the lengths of *DM* and *MA*, gaining the first P1. They then correctly found both *BE* and *MB*, the second of which gained the second P1. The third P1 was awarded for a correct trigonometric statement in angle *EMB*. The final answer was in range for A1.

Student response B

19 The diagram shows a triangular prism.



The base, *ABCD*, of the prism is a square of side length 15 cm.
Angle *ABE* and angle *CBE* are right angles.
Angle *EAB* = 35°

M is the point on *DA* such that

$$DM : MA = 2 : 3$$

Calculate the size of the angle between *EM* and the base of the prism.
Give your answer correct to 1 decimal place.

$\frac{2}{5} \times 15 = 6$

$\frac{3}{5} \times 15 = 9$

$\text{Tan}(35) = \frac{\text{opp}}{15}$

$\text{Tan}(35) \times 15 = 10.503$

$90 + 35 = 125$

$180 - 125 = 55$

$15^2 + 9^2 = 306$

$\sqrt{306} = 17.49$

$\cancel{\frac{10.5}{\sin(x)} = 17.49}$

$\text{Tan}^{-1}\left(\frac{10.5}{17.5}\right) = 7.9$

7.9 °

3/4

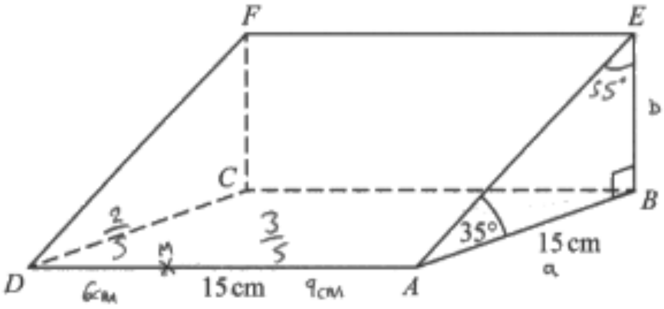
Examiner comment

This response follows exactly the same path as Student A.

The student correctly used the given ratio, for the first P1, and then found *BE* and *MB*, for the second P1. They then gave a correct trigonometric statement for angle *EMB*, gaining the third P1. However, they must have made a mistake in entering the calculation into the calculator, as the final answer was incorrect.

Student response C

19 The diagram shows a triangular prism.

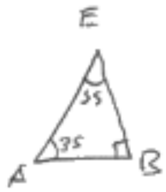


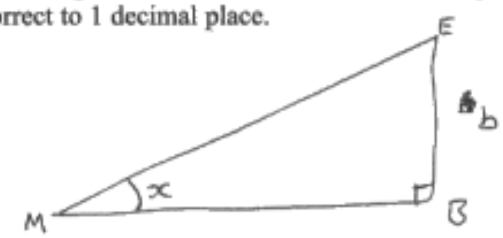
The base, *ABCD*, of the prism is a square of side length 15 cm.
Angle *ABE* and angle *CBE* are right angles.
Angle *EAB* = 35°

M is the point on *DA* such that

$$DM : MA = 2 : 3$$

Calculate the size of the angle between *EM* and the base of the prism.
Give your answer correct to 1 decimal place.





$90 + 35 = 125$
$180 - 125 = 55$

$$\frac{15}{\sin 55} = \frac{b}{\sin 35}$$

$$b = 10.50311307$$

1/4

Examiner comment
The student started correctly, and the first P1 mark could be awarded for either the division of *DA* in the correct ratio, or for finding *BE*. They were unable to progress any further and so no further marks could be awarded.

Exemplar question 10

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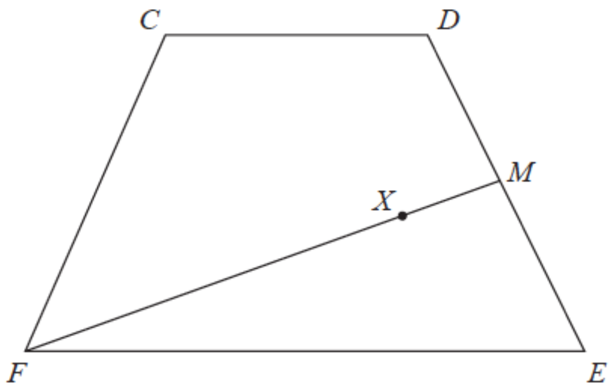
Higher tier Question 20

20 *CDEF* is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
Give your answer in its simplest form.

....

(2)

M is the midpoint of *DE*.
X is the point on *FM* such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of *n*.

$n =$ ....

(4)

(Total for Question 20 is 6 marks)

Mean score: 2.09

Examiner comment

This question assessed students' ability to firstly understand vector algebra in relation to a diagram, and then in part (b) to apply this to tackle a significant problem involving fractions of vectors without values and equating of coefficients to set up an equation to solve.

Part (a): This gave all students the opportunity to show their understanding of vectors, and it is important that they remember to write out expressions before they simplify. A good number of students did, and were rewarded with the M1 mark even when they then simplified incorrectly.

Part (b): This part was written as a discriminator at grade 9, and proved to be one. The algebra involved was complex and it was vital that students were comfortable expanding brackets and simplifying expressions involving fractions and/or decimals. The question also required students to connect two expressions to equate coefficients and set up an equation. This proved to be something that only the most able students were able to do.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
20(a)	2a	M1	for **a** – **b** + **a** + **b** (= 2**a**)	
		A1	cao	
(b)	4	P1	for a process to find $\overrightarrow{MF}$ = –0.5**b** – **a** – (**a** – **b**) (= 0.5**b** – 2**a**) **or** $\overrightarrow{CE}$ = **a** + **b** **or** $\overrightarrow{FM}$ = **a** – **b** + **a** + 0.5**b** (= 2**a** – 0.5**b**)	Accept ft from (a) providing vectors are clearly stated
		P1	For finding a suitable vector expression for **two** of ($\overrightarrow{CE}$ or $\overrightarrow{EC}$), ($\overrightarrow{CX}$ or $\overrightarrow{XC}$) **or** ($\overrightarrow{EX}$ or $\overrightarrow{XE}$) e.g. $\overrightarrow{CX} = \mathbf{a} + 0.5\mathbf{b} + \frac{1}{n+1}(0.5\mathbf{b} - 2\mathbf{a})$ **or** $\overrightarrow{CX} = -\mathbf{a} + \mathbf{b} + \frac{n}{n+1}(2\mathbf{a} - 0.5\mathbf{b})$ **or** $\overrightarrow{XE} = \frac{1}{n+1}(2\mathbf{a} - 0.5\mathbf{b}) + 0.5\mathbf{b}$ **or** $\overrightarrow{XE} = \frac{n}{n+1}(0.5\mathbf{b} - 2\mathbf{a}) + 2\mathbf{a}$ **or** $\overrightarrow{XC} = \frac{n}{n+1}(0.5\mathbf{b} - 2\mathbf{a}) + \mathbf{a} - \mathbf{b}$ **or** $\overrightarrow{XC} = \frac{1}{n+1}(2\mathbf{a} - 0.5\mathbf{b}) - 0.5\mathbf{b} - \mathbf{a}$ **or** $\overrightarrow{EX} = -0.5\mathbf{b} + \frac{1}{n+1}(0.5\mathbf{b} - 2\mathbf{a})$ **or** $\overrightarrow{EX} = -2\mathbf{a} + \frac{n}{n+1}(2\mathbf{a} - 0.5\mathbf{b})$	$\overrightarrow{CX} = \frac{n-1}{n+1}\mathbf{a} + \frac{n+2}{2(n+1)}\mathbf{b}$ $\overrightarrow{XE} = \frac{2}{n+1}\mathbf{a} + \frac{n}{2(n+1)}\mathbf{b}$ $\overrightarrow{XC} = \frac{1-n}{n+1}\mathbf{a} + \frac{-n-2}{2(n+1)}\mathbf{b}$ $\overrightarrow{EX} = \frac{-2}{n+1}\mathbf{a} - \frac{n}{2(n+1)}\mathbf{b}$
		P1	for complete process to equate the coefficients of **a** and **b** e.g. $\frac{n-1}{n+1} = \frac{n+2}{2(n+1)}$	
		A1	cao	

Question	Answer	Mark	Mark scheme	Additional guidance
20(b) *continued*			**ALTERNATIVE**	
		P1	for a process to find $\overrightarrow{MF} = -0.5\mathbf{b} - \mathbf{a} - (\mathbf{a} - \mathbf{b})$ $(= 0.5\mathbf{b} - 2\mathbf{a})$ **or** $\overrightarrow{CE} = \mathbf{a} + \mathbf{b}$ **or** $\overrightarrow{FM} = \mathbf{a} - \mathbf{b} + \mathbf{a} + 0.5\mathbf{b}$ $(= 2\mathbf{a} - 0.5\mathbf{b})$	Accept ft from (a) providing vectors are clearly stated
		P1	for finding two suitable vector expressions for $\overrightarrow{FX}$ e.g. $\overrightarrow{FX} = \frac{n}{n+1}(2\mathbf{a} - 0.5\mathbf{b})$ **and** $\overrightarrow{FX} = \mathbf{a} - \mathbf{b} + k\mathbf{a} + k\mathbf{b}$	
		P1	for complete process to equate the coefficients of **a** and **b** e.g. $\frac{2n}{n+1} - 1 = 1 - \frac{n}{2(n+1)}$	
		A1	cao	

Examiner comment

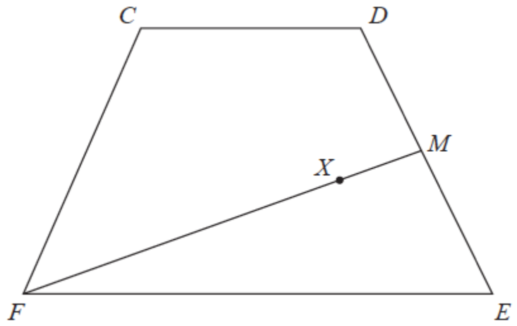
Part (a): M1 was awarded for the correct vector addition and A1 for a correct simplification of this addition.

Part (b): The first P1 was awarded for obtaining a correct expression for either $\overrightarrow{CE}$ or $\overrightarrow{FM}$, which most students achieved, normally simplified correctly. This mark could be awarded using the value obtained in part (a). For the second P1, students needed to set up expressions using the correct fractions, which very few were able to do.

Hardly any students gained the third P1 mark, most having no idea how to then set up an equation equating coefficients. Only a very limited number of students were able to then carry the algebra through to reach the correct answer, and gain A1.

Student response A

20 *CDEF* is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
Give your answer in its simplest form.

$a - b + a + b$

$2a$

(2)

2/2

M is the midpoint of *DE*.
X is the point on *FM* such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of *n*.

$\overrightarrow{FM} = a - b + a + \frac{1}{2}b$

$= 2a - \frac{1}{2}b$

$\overrightarrow{CE} = a + b$

$\overrightarrow{CX} = k\overrightarrow{CE}$

~~$\overrightarrow{CX} = a - b + \frac{n}{n+1}(2a - \frac{1}{2}b)$~~

$\overrightarrow{XE} = \frac{1}{n+1}(2a - \frac{1}{2}b) + \frac{1}{2}b$

$= \frac{2a}{n+1} - \frac{\frac{1}{2}b}{n+1} + \frac{\frac{1}{2}bn + \frac{1}{2}b}{n+1} \quad \frac{\frac{1}{2}b(n+1)}{n+1}$

$2a - \frac{1}{2}b + \frac{1}{2}bn + \frac{1}{2}b = k(a+b)$

$2a + \frac{1}{2}bn = k(a+b)$

$\frac{1}{2}n = 2$

$n = 4$

$4:1$

$\frac{4}{5}(2a - \frac{1}{2}b)$

$\frac{8}{5}a - \frac{2}{5}b - a + b$

$\frac{3}{5}a + \frac{3}{5}b$

$\frac{3}{5}(a+b)$

$n = 4$

(4)

4/4

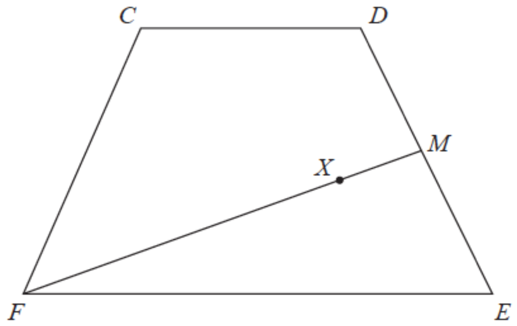
Examiner comment

Part (a): The student wrote the correct expression for $\overrightarrow{FE}$, gaining M1, and then correctly simplified it for A1.

Part (b): The student gave correct expressions for both $\overrightarrow{FM}$ and $\overrightarrow{CE}$, either of which was sufficient for the first P1. They then gave a correct statement for $\overrightarrow{XE}$, so were awarded the second P1. Unusually, they managed to equate the coefficients, gaining the third P1. They correctly solved this equation to get $n = 4$ for A1.

Student response B

20 *CDEF* is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
Give your answer in its simplest form.

$a - b + a + b$

$2a$

$2a$

(2)

2/2

M is the midpoint of *DE*.
X is the point on *FM* such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of *n*.

$\cancel{\overrightarrow{CX} = a - b + 2a}$

$\overrightarrow{FM} = a - b + a + \frac{1}{2}b$

$\overrightarrow{FM} = 2a - \frac{1}{2}b$

$\overrightarrow{FX} = \frac{n}{n+1} \times \frac{2a - \frac{1}{2}b}{1}$

$\overrightarrow{MF} = -\frac{1}{2}b - a - (a - b)$

$\overrightarrow{MF} = -\frac{1}{2}b - a - a + b$

$\overrightarrow{MF} = -2a + \frac{1}{2}b$

$\overrightarrow{Mx} = \frac{1}{n+1} \times \frac{-2a + \frac{1}{2}b}{1} = \frac{-2a + \frac{1}{2}b}{n+1}$

$\overrightarrow{Cx} = a + \frac{1}{2}b + \frac{-2a + \frac{1}{2}b}{n+1}$

$-a + b + \frac{2an - \frac{1}{2}bn}{n+1} =$

$-a + b + \frac{2an - \frac{1}{2}bn}{n+1} = a + \frac{1}{2}b + \frac{-2a + \frac{1}{2}b}{n+1}$

$\frac{2an - \frac{1}{2}bn}{n+1} = 2a - \frac{1}{2}b + \frac{-2a + \frac{1}{2}b}{n+1}$

$\frac{2an - \frac{1}{2}bn - (-2a + \frac{1}{2}b)}{n+1} = 2a$

$\frac{2an - \frac{1}{2}bn + 2a - \frac{1}{2}b}{n+1}$

$= 2a - \frac{1}{2}b$

$n = 2a - \frac{1}{2}b$

(4)

1/4

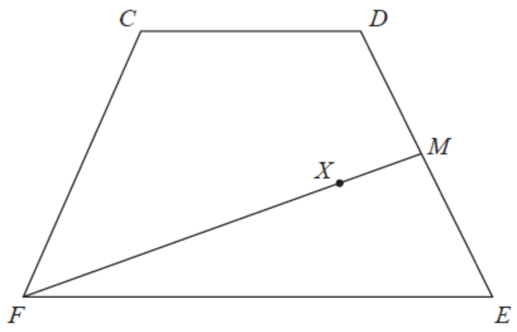
Examiner comment

Part (a): The student wrote the correct expression for $\overrightarrow{FE}$, gaining M1, and then correctly simplified it for A1.

Part (b): The student wrote a correct statement for $\overrightarrow{FM}$ for the first P1. They also wrote the correct expression for $\overrightarrow{CX}$, but $\overrightarrow{CX}$ alone does not score the second P1. Since there were no other acceptable vectors stated, no further marks could be awarded.

Student response C

20 $CDEF$ is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
Give your answer in its simplest form.

$a - b + a + b$

$2a$

.... $2a$ ....
(2)

2/2

M is the midpoint of DE.
X is the point on FM such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of n.

$\overrightarrow{FM} = a - b + a + \frac{1}{2}b$

$= 2a - \frac{1}{2}b$

$2a - \frac{1}{2}b \div 4$

$\frac{3}{4}(2a - \frac{1}{2}b)$

$\frac{3}{2}a - \frac{3}{8}b$

$n =$ .... 3 ....
(4)

1/4

Examiner comment

Part (a): The student wrote the correct expression for $\overrightarrow{FE}$, gaining M1, and then correctly simplified it for A1.

Part (b): The student gave a correct expression for $\overrightarrow{FM}$ and so scored the first P1. There was no further correct working and so no further marks were awarded.

Paper 3H (calculator)

Exemplar question 1

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Higher tier Question 9

9 A company has to make a large number of boxes.

The company has 6 machines.
All the machines work at the same rate.
When all the machines are working, they can make all the boxes in 9 days.

The table gives the number of machines working each day.

	day 1	day 2	day 3	all other days
Number of machines working	3	4	5	6

Work out the total number of days taken to make all the boxes.

....

(Total for Question 9 is 3 marks)

Mean score: 1.03

Examiner comments

This question assessed solving a problem involving inverse proportion. Students were expected to translate this real-life context into a series of mathematical processes. In particular, most students should have considered the total number of 'machine days' needed (54) together with either the number of machine days used in the first three days (12) or the number of machine days used in the first nine days (48) and worked from there.

Students needed to set their working out in a logical manner with some words to explain what they were doing.

The concepts involved in this question were found difficult by many students, although there was no single common error seen.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
9	10	P1	for a process to start to solve the problem e.g. 6 × 9 (= 54) machine days needed **or** 12 (machine days used in first 3 days) **or** 42 (machine days needed after first 3 days) **or** 6 (machine days not used in first 3 days) **or** 3 + 4 + 5 equivalent to 2 days with 6 machines **or** has used 48 machine days in first 9 days	 e.g. 3 + 4 + 5 (= 12) e.g. 6 × 9 − 12 (= 42) e.g. 3 + 2 + 1 = 6 e.g. 12 ÷ 6 = 2
		P1	for "42" ÷ 6 (= 7) (more days needed) or 3 days – 2 (equivalent) days (= 1) extra day needed to make up for the days not used	
		A1	cao	

Examiner comments

The first P1 was awarded for a process to start the question, i.e. a first step that could lead to a successful solution. The second P1 was for an almost complete process, for example for finding the number of days after the first three that all machines would need to work to complete making the boxes (42 ÷ 6) or for finding the number of days after the first nine that all machines would need to work ((54 – 48) ÷ 6) to complete making all the boxes.

The use of quotation marks in the mark scheme showed that although the value in quotes does not need to be correct, it does need to come from a correct process.

Student response A

9 A company has to make a large number of boxes.

The company has 6 machines.
All the machines work at the same rate.
When all the machines are working, they can make all the boxes in 9 days.

The table gives the number of machines working each day.

	day 1	day 2	day 3	all other days
Number of machines working	3	4	5	6

Work out the total number of days taken to make all the boxes.

6 machines.
9 days so 9×6 = 54 total needed.

3+4+5 = 12
54 − 12 = 42
rest of days = 6.
6)42
= 7.

day 3 + 7 more days
= 10

10 days

3/3

Examiner comments

The student showed clearly how they got their answer. They worked with machine days and worked out that 54 machine days were needed in total, gaining the first P1 They then used the data in the table to find out that in the first 3 days 12 machine days have been used, and calculated that an extra 42 machine days are needed, which was equivalent to 7 days with all the machines working, which was awarded the second P1. Adding this value to the 3 days already worked gained them A1.

Student response B

9 A company has to make a large number of boxes.

The company has 6 machines.
All the machines work at the same rate.
When all the machines are working, they can make all the boxes in 9 days.

The table gives the number of machines working each day.

	day 1	day 2	day 3	all other days
Number of machines working	3	4	5	6

Work out the total number of days taken to make all the boxes.

$6 \times 9 = 54$

$3 + 4 + 5 + 6x = 54$

$12 + 6x = 54$

$-12 \qquad -12$

$6x = \frac{42}{6} = x$

$x = 7$

7 days

2/3

Examiner comments

The student formulated a correct equation, for the first P1, and solved it accurately to find that 7 more days are needed with all the machines working, for the second P1. However, they then failed to add the first 3 days to get the correct answer, 10, and so were not awarded A1.

Student response C

9 A company has to make a large number of boxes.

The company has 6 machines.
All the machines work at the same rate.
When all the machines are working, they can make all the boxes in 9 days.

The table gives the number of machines working each day.

	day 1	day 2	day 3	all other days
Number of machines working	3	4	5	6

Work out the total number of days taken to make all the boxes.

6 machines = ~~a~~ x boxes in 9 days

3 machines = ~~27~~ 18 days

~~4~~ 4 machines = ~~6~~ 13.5 days

5 machines = ~~7.5~~ 10.8 days

1 machine = ~~4~~ 54 days

rate = ~~...~~

17

1/3

Examiner comments

The student realised that this question was based on the concept of inverse proportion and showed that it would take 1 machine 54 days to make all the boxes, gaining the first P1. However, the information in the table was not used to find out the number of machine days used by the machines working in the first 3 days or more so no further marks could be awarded.

Exemplar question 2

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Higher tier Question 10

10 Marie invests £8000 in an account for one year.
At the end of the year, interest is added to her account.

Marie pays tax on this interest at a rate of 20%
She pays £28.80 tax.

Work out the percentage interest rate for the account.

.... %

(Total for Question 10 is 3 marks)

Mean score: 1.70

Examiner comments

This question tested percentages in the context of paying tax and proved to be straightforward for many students. Students were expected to calculate the total amount of interest gained on the account before converting this to a percentage rate. The first stage of the calculation was usually carried out successfully and about a half of all students went on to get a correct answer.

A common error was for students to work out 144 as a percentage of 8144 instead of as a percentage of 8000 or to misinterpret a multiplier, for example 0.018 or 1.018

It was noticeable that an increasing proportion of students were using a multiplier approach, for example showing $\frac{144}{8000} = 0.018$ or $\frac{8144}{8000} = 1.018$ before interpreting this as a 1.8% interest rate.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
10	1.8	P1	process to find the amount of interest before tax e.g. $28.80 \div 20 \times 100$ (= 144) **OR** for equation which would lead to (x =) 0.018, 1.8 **or** 1.018 e.g. $0.2 \times 8000 \times x = 28.8$ **or** $\frac{8000\,(100 + x)}{100} = 8144$	
		P1	process to find the interest rate e.g. $\frac{\text{“144”}}{8000}$ (= 0.018) **or** $\frac{\text{“8144”}}{8000}$ (= 1.018)	These numerical expressions may be seen multiplied by 100, e.g. $\frac{144}{8000} \times 100$
		A1	cao	

Examiner comments

The first P1 was for showing a correct process that would lead to the amount of interest before tax or for an equation which when solved would lead to this. This was usually evidenced by the calculation to get £144 or sight of £144 itself. The second P1 was for a process that would enable the interest rate to be identified. This could be shown by the calculation needed to find a multiplier, for example $\frac{144}{8000}$ or $\frac{8144}{8000}$ or by the complete process to find the percentage interest rate, for example $\frac{144}{8000} \times 100$. The second P1 could only be awarded if the first P1 had been given. The final mark, A1, was for a correct answer.

Student response A

10 Marie invests £8000 in an account for one year.
At the end of the year, interest is added to her account.

Marie pays tax on this interest at a rate of 20%
She pays £28.80 tax.

Work out the percentage interest rate for the account.

£8000 year

interest = £28.80 ÷ 0.2 = £144

$\frac{144}{8000} \times 100 = 1.8$

∴ 8000 x 1.018 = £8144

1.8 %

3/3

Examiner comments

This fully correct solution was clearly and concisely presented, with a check to show the answer was correct. The student divided 28.80 by 0.2 to find the total interest added before tax, gaining the first P1, and then used this to work out 144 as a percentage of 8000, for the second P2. The correct answer was awarded A1. The last line in the working space was a check using a multiplier to show that the answer is correct.

Student response B

10 Marie invests £8000 in an account for one year.
At the end of the year, interest is added to her account.

Marie pays tax on this interest at a rate of 20%
She pays £28.80 tax.

Work out the percentage interest rate for the account.

~~£8000 +x~~

$28.8 \times 5 = 144$

$8000 + 144 = 8144$

$\cancel{\frac{144}{8000} \times 100 = 1.8}$

$\frac{8144}{8000} = 1.018$

1.018
~~1.8~~ %

2/3

Examiner comments
The student showed a correct and complete process to find the multiplier (1.018) for the percentage interest rate and so scored both P1 marks. However, they were unable to interpret this correctly to give the answer, 1.8, and so were not awarded A1.

Student response C

10 Marie invests £8000 in an account for one year.
At the end of the year, interest is added to her account.

Marie pays tax on this interest at a rate of 20%
She pays £28.80 tax.

Work out the percentage interest rate for the account.

£28.80 = 20%
100% = £144

£8144

$28.8/8144 \times 100 = 0.3536$

0.35 %

1/3

Examiner comments
The student made a good start and gained the first P1 for the 144, although the process was not fully shown. No further marks could be awarded because the candidate used 8144 instead of 8000 in the calculation to find the interest rate.

Exemplar question 3

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Higher tier Question 11

11 In May 2019, the distance between Earth and Mars was 3.9×10^7 km.

In May 2019, a signal was sent from Earth to Mars.
Assuming that the signal sent from Earth to Mars travelled at a speed of 3×10^5 km per second,

(a) how long did the signal take to get to Mars?

.... seconds
(2)

The speed of the signal sent from Earth to Mars in May 2019 was actually less than 3×10^5 km per second.

(b) How will this affect your answer to part (a)?

....

....

....
(1)

(Total for Question 11 is 3 marks)

Mean score: 2.59

Examiner comments

Part (a): This part primarily assessed calculation with numbers in standard form in the context of a routine distance/speed/time problem. Students were expected to show the division of 3.9×10^7 by 3×10^5 and use their calculator to evaluate it. Most students were able to do this successfully but unfortunately a good proportion of students who wrote down $3.9 \times 10^7 \div 3 \times 10^5$ without brackets did not use the correct order of operations in evaluating it with their calculator, and gave the incorrect answer 1.3×10^{12}.

Some students converted the distance and speed to ordinary numbers before carrying out the calculation. Though unnecessary, this was acceptable.

Part (b): This was generally answered well, with most students writing that their answer to part (a) would be increased.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
11(a)	130	P1	for process to divide e.g. $(3.9 \times 10^7) \div (3 \times 10^5)$	Condone missing brackets
		A1	cao	Accept 1.3×10^2
(b)	Explanation	C1	Explanation referring to the time **Acceptable examples** The time will be more It will take longer The answer will be bigger **Not acceptable examples** The answer will be wrong The answer will be different	

Examiner comments

Part (a): Though the division of distance by speed should be shown with brackets, examiners condoned their absence for P1. Answers written in ordinary number form (130) or in standard form (1.3×10^2) were accepted for A1.

Part (b): Any answer where the intention was clear and correct was awarded C1.

Student response A

11 In May 2019, the distance between Earth and Mars was 3.9×10^7 km.

In May 2019, a signal was sent from Earth to Mars.
Assuming that the signal sent from Earth to Mars travelled at a speed of 3×10^5 km per second,

(a) how long did the signal take to get to Mars?

$$\frac{3.9 \times 10^7}{3 \times 10^5} = 130$$

130 seconds
(2)

2/2

The speed of the signal sent from Earth to Mars in May 2019 was actually less than 3×10^5 km per second.

(b) How will this affect your answer to part (a)?

It will increase the time it took for ~~all of~~ the message to reach Mars.

(1)

1/1

Examiner comments

Part (a): The student used fraction notation to show a correct division for P1 and then evaluated it correctly for A1.

Part (b): This clear and correct statement was awarded C1.

Student response B

11 In May 2019, the distance between Earth and Mars was 3.9×10^7 km.

In May 2019, a signal was sent from Earth to Mars.
Assuming that the signal sent from Earth to Mars travelled at a speed of 3×10^5 km per second,

(a) how long did the signal take to get to Mars?

time

$$\frac{D}{S \quad T}$$

$$3.9 \times 10^7$$
$$\div\ 3 \times 10^5$$
$$=$$

1.3×10^{12} seconds

(2)

1/2

The speed of the signal sent from Earth to Mars in May 2019 was actually less than 3×10^5 km per second.

(b) How will this affect your answer to part (a)?

It will ~~[illegible]~~ increase. my answer.

(1)

1/1

Examiner comments

Part (a): The student correctly identified that they needed to divide the distance by the speed by writing $3.9 \times 10^7 \div 3 \times 10^5$, gaining P1. However, the absence of brackets led to an incorrect order of operations being used to get the final answer, so did not score A1. The student did not seem to have carried out a common-sense check to see if the size of their answer was reasonable.

Part (b): This short but direct and correct response was awarded C1.

Student response C

11 In May 2019, the distance between Earth and Mars was 3.9×10^7 km.

In May 2019, a signal was sent from Earth to Mars.
Assuming that the signal sent from Earth to Mars travelled at a speed of 3×10^5 km per second,

(a) how long did the signal take to get to Mars?

39000,000km × 300000 km/s = 1.17×10^{13}

$3.9 \times 10^7 \times 3 \times 10^5 = 1.17 \times 10^{13}$

1.17×10^{13} seconds
(2)

0/2

The speed of the signal sent from Earth to Mars in May 2019 was actually less than 3×10^5 km per second.

(b) How will this affect your answer to part (a)?

It will have a larger number as it would take longer

(1)

1/1

Examiner comments

Part (a): The student decided to change from standard form to ordinary numbers. They did this correctly but then multiplied the distance by the speed so could not be awarded any marks for their working.

Part (b): The student stated that a smaller speed would lead to a longer travel time, gaining C1. The award of C1 in part (b) was independent of the working and answer in part (a).

Exemplar question 4

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Higher tier Question 13

13 The density of ethanol is 1.09 g/cm^3
The density of propylene is 0.97 g/cm^3

60 litres of ethanol are mixed with 128 litres of propylene to make 188 litres of antifreeze.

Work out the density of the antifreeze.
Give your answer correct to 2 decimal places.

.... g/cm^3

(Total for Question 13 is 4 marks)

Mean score: 2.40

Examiner comments

This question involving mass, density and volume assessed the use of compound units in a real-life context.

Students could convert litres to millilitres before carrying out the calculation to find the density of the antifreeze but because the mix depended on proportions, it was not necessary. Some candidates who did try to convert to millilitres used an incorrect conversion factor.

Students generally seemed more familiar with this context than in the past and it was usual to see correct calculations. However, there were more cases than expected where students used 188 for the number of litres of propylene.

A significant number of students presented the incorrect calculation $188 \div \left(\frac{60}{1.09} + \frac{128}{0.97}\right)$, which resulted in a value (1.005…). Although this value was within the interval given on the mark scheme, it could not be given any credit as it came from an incorrect method.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
13	1.01	P1	for 1.09 × 60 (= 65.4 or $\frac{327}{5}$) **or** 0.97 × 128 (= 124.16 or $\frac{3104}{25}$)	Note that the volumes may be converted to ml, e.g. 1.09 × 60 000 (= 65 400)
		P1	for 1.09 × 60 (= 65.4 or $\frac{327}{5}$) **and** 0.97 × 128 (= 124.16 or $\frac{3104}{25}$) **or** "65.4" + "124.16" (= 189.56 **or** $\frac{4739}{25}$)	
		P1	for a complete process to find the density of antifreeze e.g. ("65.4" + "124.16") ÷ 188 **or** 189.56 ÷ 188 **or** $\frac{4739}{25}$ ÷ 188	Candidates working in ml must use 188 000
		A1	for answer in the range 1.00 to 1.01	If an answer within the range is seen in working but then rounded incorrectly award full marks. Accept 1 for 1.00 Note that the correct value is 1.008…

Examiner comments

The first P1 was for multiplying a density by a volume for either ethanol or propylene. The second P1 was given if this was done for both ethanol and propylene. Both these marks could be given if an attempt was made to use consistent units by changing litres to ml by multiplying 60 and 128 by 1000 or any multiple of 10, provided this was is consistently used. The third P1 was for a complete process that would lead to the correct answer if no arithmetic errors were made, and was therefore dependent on the award of the first two P1 marks. A1 was given for an answer, from correct processes, within the range 1.00 to 1.01. Examiners were careful to check that correct processes had been used, particularly because an answer of 1.005… from incorrect working was seen fairly often.

Student response A

13 The density of ethanol is 1.09 g/cm^3
The density of propylene is 0.97 g/cm^3

60 litres of ethanol are mixed with 128 litres of propylene to make 188 litres of antifreeze.

60000 128000

Work out the density of the antifreeze.
Give your answer correct to 2 decimal places.

$D = \frac{M}{V}$

$60000 \times 1.09 = 65400$

$128000 \times 0.97 = 124160$

$124160 + 65400 = 189560$

$188 \times 1000 = 188000$

$\frac{189560}{188000} = 1.01$

1.01 g/cm^3

4/4

Examiner comments

This was a fully correct answer from clear, accurate working and where the student converted litres to millilitres. The answer was given correct to 2 decimal places and no more accuracy is seen in the working space, but the student used correct processes so full marks were awarded.

Student response B

13 The density of ethanol is 1.09 g/cm³
The density of propylene is 0.97 g/cm³

density = $\frac{\text{mass}}{\text{volume}}$

60 litres of ethanol are mixed with 128 litres of propylene to make 188 litres of antifreeze.
→ ~~60 000 ml~~

Work out the density of the antifreeze.
Give your answer correct to 2 decimal places.

Ethanol:

mass = 1.09 × 60 litres
= 65.4 g

propylene = 128 × 0.97 = 124.16 g

124.16 + 65.4 = ~~189~~ 189.56 g

Density = $\frac{189.56}{188}$ = 1.02

1.02 g/cm³

3/4

Examiner comments

The student used the processes outlined in the mark scheme, recording them accurately as far as the statement Density = $\frac{189.56}{188}$. This was awarded the three P1 marks. However, on evaluating the fraction, the student did not write down the answer from the calculator but incorrectly rounded it so cannot be awarded A1. Note that if they had written down 1.008… but then rounded incorrectly they could have been awarded full marks.

Student response C

13 The density of ethanol is 1.09 g/cm^3
The density of propylene is 0.97 g/cm^3

60 litres of ethanol are mixed with 128 litres of propylene to make 188 litres of antifreeze.

Work out the density of the antifreeze.
Give your answer correct to 2 decimal places.

$d = \frac{m}{V}$

$1.09 \times 60 = 65.4$

$0.97 \times 188 = 182.36$

$65.4 + 182.36 = 247.76$

247.76

.... g/cm^3

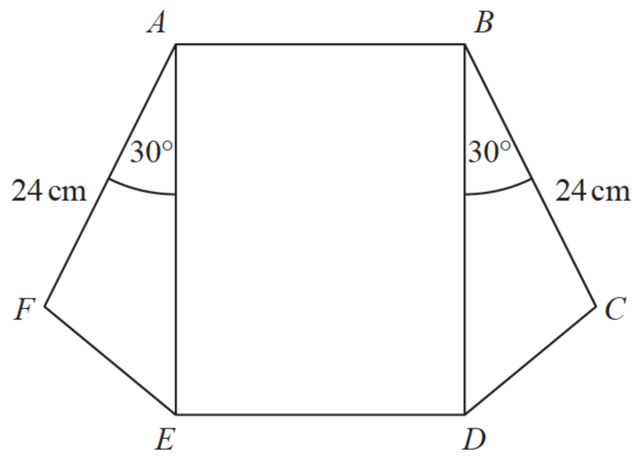
1/4

Examiner comments
The first P1 was awarded for the correct process 1.09 × 60, for ethanol. The student then used 188, not 128 for propylene. This is an incorrect figure and cannot be treated as a misread, so the second P1 could not be given. A1 could not be awarded as it was dependent on the two P1 marks.

Exemplar question 5 [back to Contents page]

Higher tier Question 14

14 The diagram shows a rectangle, *ABDE*, and two congruent triangles, *AFE* and *BCD*.



area of rectangle *ABDE* = area of triangle *AFE* + area of triangle *BCD*

$$AB : AE = 1 : 3$$

Work out the length of *AE*.

.... cm

(Total for Question 14 is 4 marks)

Mean score: 0.65

Examiner comments

This question assessed the ability of students to make and use connections between different parts of mathematics. Students needed to interpret and use ratio in the context of a problem involving the areas of non-right-angled triangles and a rectangle. They were expected to use the formula $\frac{1}{2}ab\sin C$ for the area of the triangle, together with the given relationships between the area of a triangle and the area of the rectangle and between the lengths of the sides of the rectangle, in order to find the length of the longest side of the rectangle.

Students needed to show all their working and could annotate the diagram in order to show how they were using the ratio 1 : 3 for the lengths of the sides of the rectangle. This may be done, for example, by labelling *AE* as $3x$ and *AB* as x.

A common error was to use right-angled trigonometry on triangle *AFE*, assuming angle *AFE* was 90°.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
14	36	P1	for process to find an expression for the area of triangle e.g. $\frac{1}{2} \times 24 \times AE \times \sin 30$ (= $6AE$)	Accept any correct expression, e.g. $\frac{1}{2} \times 24 \times y \times \sin 30$
		P1	(dep P1) for process to link the area of rectangle with the area of the triangle e.g. $2 \times \frac{1}{2} \times 24 \times AE \times \sin 30$ (= $12AE$) **or** for $AB = 12$	
		P1	(indep) for use of given ratio e.g. $AE = 3AB$ oe, e.g. area of rectangle = $AE \times AB$ = $3x \times x$	May be shown on the diagram by labelling AE and AB with, for example, $3x$, x or x, $\frac{1}{3}x$ or $\frac{3}{4}x$, $\frac{1}{4}x$ Do not accept 3, 1 or 1, $\frac{1}{3}$ or $\frac{3}{4}$, $\frac{1}{4}$ for this mark.
		A1	cao	

Examiner comments

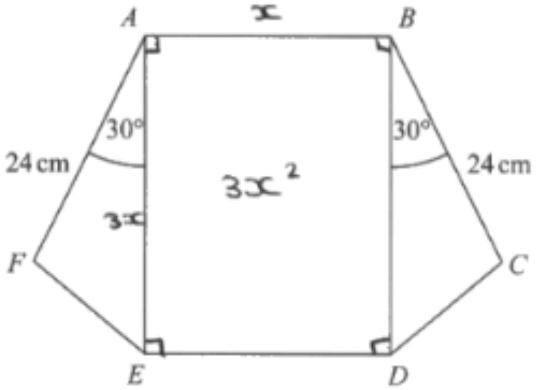
The most common strategy seen in this question was to use $AE = 3x$, $AB = x$ to write down the area of triangle AFE as $\frac{1}{2} \times 24 \times 3x \times \sin 30°$ and the area of the rectangle $ABDE$ as $3x \times x$ before using the given relationship 'area of rectangle $ABDE$ = area of triangle AFE + area of triangle BCD' to form an equation in x. This equation could then be solved and the length of AE ($3x$) found.

Either the first P1 or the third P1 may be awarded first. The second P1 was dependent on the first P1 having been awarded. However, some candidates took a shortcut and wrote down the area of the rectangle straight away as $24 \times 3x \times \sin 30°$. This was awarded the first two P1 marks.

The third P1 was an independent mark, given for using the 3 : 1 ratio, and could be awarded for labelling AE and AB with, for example, $3x$, x or $\frac{3}{4}x$, $\frac{1}{4}x$ but not for 3, 1 or $\frac{3}{4}$, $\frac{1}{4}$. Students needed to remember to multiply the value for x by 3 to find the required length for A1 to be awarded.

Student response A

14 The diagram shows a rectangle, *ABDE*, and two congruent triangles, *AFE* and *BCD*.



area of rectangle *ABDE* = area of triangle *AFE* + area of triangle *BCD*

$AB : AE = 1 : 3$

Work out the length of *AE*.

$\frac{1}{2}ab\sin C = \text{Area}$

$\frac{1}{2} \times 24 \times b \times \sin 30 = \frac{3x^2}{2}$

$12b \times \sin 30 = \frac{3x^2}{2}$

$24b \times \sin 30 = 3x^2$

$24(3x) \times \sin 30 = 3x^2$

$72x \times \sin 30 = 3x^2$

$\sin 30 = \frac{3x^2}{72x}$

$\sin 30 = \frac{x}{24}$

$x = \sin 30 \times 24$

~~[illegible]~~

$x = 12$

$3x = 36$

......36...... cm

(Total for Question 14 is 4 marks)

4/4

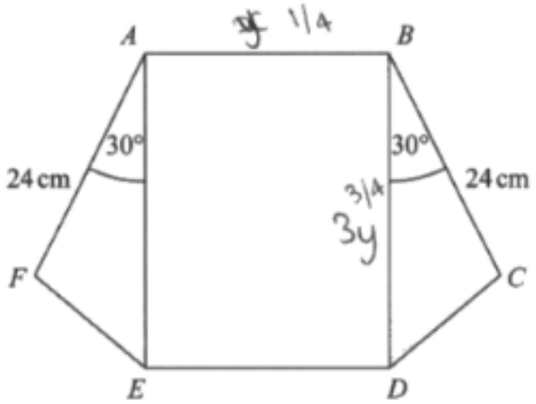
Examiner comments

The student equated the area of one of the triangles (using $\frac{1}{2}ab\sin C$) with a half of the area of the rectangle (using $\frac{1}{2} \times 3x \times x$) to obtain a fully correct equation, so scoring the three P1 marks. They solved the equation to get $x = 12$ then multiplied by 3 to get the correct answer, 36, for A1. The division by $72x$ was condoned on the basis that $x \neq 0$.

This solution was well presented and accurately executed.

Student response B

14 The diagram shows a rectangle, *ABDE*, and two congruent triangles, *AFE* and *BCD*.



area of rectangle *ABDE* = area of triangle *AFE* + area of triangle *BCD*

$AB : AE = 1 : 3$

Work out the length of *AE*.

area rectangle = $3y^2$ = absinC because $2 \times \frac{1}{2}$ absinC = absinC

$3y^2 = 24 \times 3y \times \sin 30$

$3y = 24 \times \sin 30$

$3y = 12$

12 cm

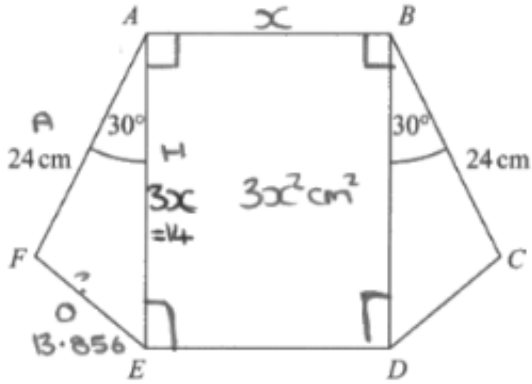
3/4

Examiner comments

The student wrote the sides of the rectangle in the correct ratio as $3y$ and y, gaining the third P1. They also wrote a correct expression for the area of the rectangle as $2 \times$ area of a triangle, using $\frac{1}{2}ab\sin C$ for the area of a triangle, and so were awarded the first tow P1 marks. A mistake in simplifying their correct equation led to an incorrect value for y and thus an incorrect answer, so A1 could not be given.

Student response C

14 The diagram shows a rectangle, *ABDE*, and two congruent triangles, *AFE* and *BCD*.



area of rectangle *ABDE* = area of triangle *AFE* + area of triangle *BCD*

$AB : AE = 1 : 3$

Work out the length of *AE*.

T O A

?

tan30 24

$\tan 30 \times 24 = 13 \cdot 85640646$

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 24^2 + 13 \cdot 856^2 - 2 \times 24 \times 13 \cdot 856 \times \cos 30$

$a^2 = 192$

$a = \sqrt{192} = 14.$

$3x = 14$

$x = 4 \cdot 6$

14 cm

1/4

Examiner comments

The student incorrectly applied the cosine rule (with the intention of finding the length of *AE*) and wrote an incorrect expression involving tan 30, scoring neither of the first two P1 marks. However, they correctly used the ratio of the lengths of the sides *AE* and *AB* to label them $3x$ and x and used this to write an expression for the area of the rectangle, $3x^2$, gaining the third P1.

Exemplar question 6

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Higher tier Question 15

15 The graph of the curve C with equation $y = f(x)$ is transformed to give the graph of the curve S with equation $y = f(-x) - 3$

The point on C with coordinates (7, 2) is mapped to the point Q on S.

Find the coordinates of Q.

(........................... , ...)

(Total for Question 15 is 2 marks)

Mean score: 0.56

Examiner comments

This question assessed a combination of a reflection and translation of the graph of a function. Students needed to understand these transformations in the context of function notation. Students were expected to consider the two transformations of reflection in the y-axis and translation of $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$, probably with the aid of a sketch, to find the image of the point (7, 2) after the application of these two transformations. Candidates needed to identify and give a brief description of the two transformations in order to give themselves the best opportunity to gain one mark if their answer was not correct. For the same reason, it would also be wise for students to write down the coordinates of the original point after one of the two transformations has been applied.

Students generally find the concept of transformations applied to a general function difficult. Some weaker students merely substituted $x = 7$ into the equation $y = f(-x) - 3$ but made no further progress. This scored no marks.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
15	(–7, –1)	M1 A1	for a method which shows understanding of the type of transformation e.g. reflection in the y axis or translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ **or** "(0 units right and) 3 units down" **or** for x coordinate as –7 **or** y coordinate as –1 for (–7, –1)	"Reflection" or "Translation" alone is insufficient. Note that the –7 or the –1 may appear in the working space, not necessarily in the final answer.

Examiner comments

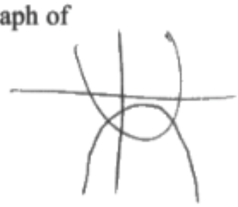
M1 was awarded to students who showed that they had some understanding of the transformations involved in mapping the curve with equation $y = f(x)$ onto the graph of the curve with equation $y = f(-x) - 3$. This could be demonstrated by a full description of at least one of the transformations or one correct coordinate shown in intermediate working or in the final answer. Students who changed the sign of and subtracted 3 from both the 7 and 2 to get (–10, –5) received no marks. A1 was given for a fully correct answer.

Student response A

15 The graph of the curve C with equation $y = \mathrm{f}(x)$ is transformed to give the graph of the curve S with equation $y = \mathrm{f}(-x) - 3$

The point on C with coordinates (7, 2) is mapped to the point Q on S.

Find the coordinates of Q.



(7, 2)

ref in y

(−7, 2)

down 3

(−7, −1)

(.....−7....., ...−1.....)

2/2

Examiner comments

This correct response scored full marks. The student identified the transformations, although they could perhaps have described them more fully and, for example, have made it clear that the reflection was in the y axis and not in $y = 0$. Nevertheless, the student applied the transformations correctly in succession to the original point to find the coordinates of Q.

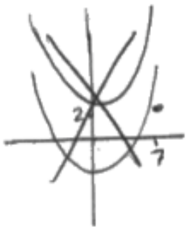
Student response B

15 The graph of the curve C with equation $y = \mathrm{f}(x)$ is transformed to give the graph of the curve S with equation $y = \mathrm{f}(-x) - 3$

reflection of y + 3 down

The point on C with coordinates (7, 2) is mapped to the point Q on S.

Find the coordinates of Q.



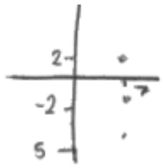
reflection on the y axis

+

3 spaces down

(7, -2)

(7, -5)



(.......7......., ...-5.......)

1/2

Examiner comments
Neither coordinate of the point given on the answer line was correct and examiners looked to see if they could award M1. Either of the descriptions given for the transformations qualified for this.

Student response C

15 The graph of the curve C with equation $y = f(x)$ is transformed to give the graph of the curve S with equation $y = f(-x) - 3$

The point on C with coordinates (7, 2) is mapped to the point Q on S.

Find the coordinates of Q.

$(-x) = y$ axis

$(7, 2)$

$\downarrow$

-3 $(7, -2)$

$\downarrow$

$(10, -2)$

(10 , −2)

0/2

Examiner comments

The student made a start on identifying the significance of '$(-x)$'. However, 'y axis' was not enough to describe the transformation and the candidate changed the sign of the y coordinate not the x coordinate in their working, so M1 could not be awarded. The candidate also added 3 to the x coordinate but this was also incorrect.

Exemplar question 7

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Higher tier Question 18

18 (a) Show that $(2x + 1)(x + 3)(3x + 7)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

(3)

(b) Solve $(1 - x)^2 < \dfrac{9}{25}$

....

(3)

(Total for Question 18 is 6 marks)

Mean score: (a) 0.2.32, (b) 0.56

Examiner comments

Part (a): Students were assessed on their ability to expand and simplify the product of three linear expressions in x. Students were expected to expand and simplify the product of two of the expressions first before multiplying their answer by the third linear expression and simplifying. Some students did not simplify their first expansion before multiplying again so they got 8 terms instead of the 6 terms expected. Students needed to take care in checking that they wrote down all 6 (or 8) terms before collecting these terms to get their final answer.

Part (b): This assessed the solution of a quadratic inequality. Students were expected to take the square root of each side of the inequality, not forgetting the ± nature of the square root of $\dfrac{9}{25}$. They then needed to find the critical value for each of the two cases before writing down the inequalities needed for a complete solution. In fact, only a small proportion of students followed this strategy; of those who did, many omitted the negative square root. Most students expanded $(1 - x)^2$ but then made errors when dealing with the mixture of fractions and integers in the resulting inequality.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18(a)	$6x^3 + 35x^2 + 58x + 21$	M1	for a method to find the product of two linear expressions, 3 correct terms out of 4 terms e.g. $2x^2 + x + 6x + 3$ **or** $3x^2 + 7x + 9x + 21$ **or** $6x^2 + 14x + 3x + 7$	Note that, for example, $7x + 3$ is regarded as three terms in the expansion of $(2x + 1)(x + 3)$
		M1	for a complete method to obtain all terms, at least half of which are correct (ft their first product) e.g. $6x^3 + 32x^2 + 42x + 3x^2 + 16x + 21$	First product must be a 3 or 4 term quadratic but need not be simplified or may be simplified incorrectly
		A1	cao	Accept $a = 6$, $b = 35$, $c = 58$, $d = 21$
(b)	$\frac{2}{5} < x < 1\frac{3}{5}$	M1	for first step of finding the square root of both sides e.g. $1 - x < \pm\frac{3}{5}$ **OR** for writing in the form $ax^2 + bx + c$ (< 0) e.g. $x^2 - 2x + \frac{16}{25}$ (< 0) **or** $25x^2 - 50x + 16$ (< 0)	Condone use of an "=" sign; accept one square root (e.g. $\frac{3}{5}$) only shown.
		M1	for showing critical values $\frac{2}{5}$ (= 0.4) and $1\frac{3}{5}$ (= 1.6) oe	Critical values can be stated, or shown in an expression (which may have incorrect inequality symbols)
		A1	for $\frac{2}{5} < x < 1\frac{3}{5}$	Could be written as two separate expressions e.g. $x > \frac{2}{5}$ and $x < 1\frac{3}{5}$ oe

Examiner comments

Part (a): The first M1 was awarded for 3 out of 4 terms correct in the expansion of the product of two of the three brackets. Provided the student obtained a three- or four-term quadratic expression, the second M1 was awarded for a complete method to get all 6 (or 8) terms when multiplying by the third bracket. Students were expected to get at least half of their terms correct when carrying out this expansion, but this was a follow through mark and so could be earned even if the first M1 had not been awarded provided they had a three- or four-term quadratic from their earlier expansion. A1 is for a fully correct answer.

Part (b): For the first M1, students needed to either take the square root of each side of the inequality or expand the brackets and write the inequality in the form $ax^2 + bx + c$ (< 0). In the former case, omission of the ± was condoned for the award of the first M1 but the square root of $\frac{9}{25}$ needed to be evaluated as $\frac{2}{5}$ or equivalent. In the latter case, all terms needed to be taken to one side of the inequality, but absence of the '< 0' was condoned. The replacement of the inequality sign with an incorrect inequality sign or with an '=' sign was also accepted for the award of the first M1. The second M1 was for the correct critical values of $\frac{2}{5}$ and $1\frac{3}{5}$ or equivalent. These could be seen as part of a final answer which had incorrect inequality signs. A1 was for a fully correct answer, which could be expressed as two separate expressions but needed to be correct and unambiguous.

Student response A

FOIL

18 (a) Show that $(2x + 1)(x + 3)(3x + 7)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

$(2x+1)(x+3)(3x+7)$

$(2x^2+6x+x+3)(3x+7)$

$(2x^2+7x+3)(3x+7)$

×	$2x^2$	$7x$	3
$3x$	$6x^3$	$21x^2$	$9x$
7	$14x^2$	$49x$	21

$6x^3+21x^2+9x+14x^2+49x+21$

$6x^3+35x^2+58x+21$

3/3

(b) Solve $(1 - x)^2 < \dfrac{9}{25}$

$1-x < 0.6$

$1-x > -0.6$

$x > 0.4$

$x < 1.6$

$x > 0.4$

$x < 1.6$

(3)

3/3

Examiner comments

Part (a): The student showed a correct expansion and simplification of the product of two brackets before using a table used to show the multiplication of the resulting quadratic by the third linear expression. The terms were then collected to give a final correct cubic expression.

Part (b): The student used a concise and correct strategy using inequalities. This was rarely seen and demonstrated an excellent understanding of the content being assessed.

Student response B

18 (a) Show that $(2x + 1)(x + 3)(3x + 7)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

$(2x+1)(x+3)$

$2x^2 + 1x + 6x + 3$

$= 2x^2 + 7x + 3$

$(2x^2 + 7x + 3)(3x + 7)$

$9x + 21x + 6x^3 + 21 + 49x + 14x^2$

$= 6x^3 + 14x^2 + 79x + 21$

2/3

(b) Solve $(1 - x)^2 < \dfrac{9}{25}$

$(1-x)(1-x)$

$1^2 - 1x - 1x + x^2$

$x^2 \quad 2x -$

$(-x+1)(-x+1)$

$x^2 - 1x - 1x + 1$

$x^2 - 2x + 1$

$x^2 - 2x + 1 < \dfrac{9}{25}$

$25(x^2 - 2x + 1) < 9$

$25x^2 - 50x + 25 < 9$

$-9 \quad -9$

$25x^2 - 50x + 16 < 0$

$\dfrac{-(-50) \pm \sqrt{(-50)^2 - 4(25)(16)}}{2(25)}$

$x = 1.6$

$x = 0.4$

$x = 1.6,\ x = 0.4$

(3)

2/3

Examiner comments

Part (a): The student produced a correct quadratic expression, for the first M1. They then attempted to find the product of the quadratic expression and $3x + 7$. Five of the six terms were correct so the second M1 could be awarded for the unsimplified cubic expression. A1 was for a correct (and fully simplified) answer, so could not be awarded.

Part (b): The student's strategy was to expand $(1 - x)^2$ and write the resultant inequality with all terms on one side. This was carried out accurately and the first M1 could be awarded for $25x^2 - 50x + 16 < 0$. Correct critical values were found, so the second M1 could also be given, but they were not used to give a final answer in the form of an inequality or inequalities, so A1 could not be awarded.

Student response C

18 (a) Show that $(2x + 1)(x + 3)(3x + 7)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

$(2x+1)(x+3) = 2x^2 + 5x + 1x + 3$

$(2x^2 + 5x + 1x + 3)(3x+7)$

$\boxed{6x^3 + 15x^2 + 7x + 10}$

1/3

(b) Solve $(1 - x)^2 < \dfrac{9}{25}$

$\cancel{(1-x)(1-x) < \frac{9}{25}}$

$\cancel{1 + x^2 - 2x < \frac{9}{}}$

$(1-x)^2 < \frac{9}{25}$

$1 - x < \frac{3}{5}$

$x - 1 < -0.6$

$x < 0.4$

$x < 0.4$

(3)

1/3

Examiner comments

Part (a): 3 out of the 4 terms of the quadratic expansion were correct, gaining the first M1. However, the student lacked a clear strategy to multiply the resultant quadratic by the remaining linear expression. They wrote down a cubic expression, but it was incorrect and there was no evidence that it came from a correct and complete expansion. There are only 4 terms and although it seems that 3 of these are correct there is only 1 other term and so the second M1 for a complete method to obtain all terms could not be awarded.

Part (b): The first M1 was given for the first operation of taking the square root of both sides, but no further marks could be awarded as the student did not consider the negative square root.

Exemplar question 8

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Higher tier Question 19

19 $D = \dfrac{u^2}{2a}$

$u = 26.2$ correct to 3 significant figures

$a = 4.3$ correct to 2 significant figures

(a) Calculate the upper bound for the value of D.
Give your answer correct to 6 significant figures.
You must show all your working.

....

(3)

The lower bound for the value of D is 78.6003 correct to 6 significant figures.

(b) By considering bounds, write down the value of D to a suitable degree of accuracy.
You must give a reason for your answer.

....

....

(2)

(Total for Question 19 is 5 marks)

Mean score: 2.07

Examiner comments

Part (a): Students needed to decide how to find the upper bound of a quantity linked to two other variables by a formula. They were expected to identify that the upper bound for D would be given by using the upper bound for u together with the lower bound for a, write these down then substitute them into the formula. A common mistake was to use the upper bound for both u and a.

Part (b): Students were given the lower bound for D and needed to decide, given this and their answer to part (a), what degree of accuracy they could guarantee in the context of the particular measurements given in the question. Many students mistakenly calculated the mean of the upper and lower bounds.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19(a)	81.0662	M1	for one of 26.15 **or** 26.25 **or** 4.25 **or** 4.35	Accept $26.24\dot{9}$ for 26.25 and $4.34\dot{9}$ for 4.35
		M1	for a correct process to find the upper bound for D [UB of u]^2 ÷ [2 × LB of a] e.g. $\dfrac{26.25^2}{2\times 4.25}$ where $26.2 <$ UB of $u \leq 26.25$ and $4.25 \leq$ LB of $a < 4.3$	Award for $\dfrac{26.25^2}{4.25}$
		A1	for answer given in the range 81.0661 to 81.0662 from correct working	
(b)	80	B1	for 80 ft answer to (a) with 78.6003	
	explanation	C1	for explanation relating to the upper bound found in (a) **Acceptable examples** bounds agree when rounded to 80 bounds agree to nearest 10 **Not acceptable examples** 80 79.83325 rounded to nearest tenth	

Examiner comments

Part (a): The first M1 was awarded for correctly identifying at least one correct upper or lower bound for either of the quantities u or a. The second M1 was awarded for substituting their upper bound for u and their lower bound for a into the formula, provided their bounds were values within the intervals stated on the mark scheme. The second M1 was not dependent on the award of the first M1. A1 was awarded for a correct answer written to at least 6 significant figures.

Part (b): B1 was awarded for stating that 80 is the suitable value of D from a consideration of the bounds of D. This was a follow through mark, so could be awarded for a correct decision following from a consideration of their (incorrect) answer to (a) together with 78.6003. C1 was awarded for the reason that 'the bounds agree when rounded to 80' or equivalent.

Student response A

19 $D = \dfrac{u^2}{2a}$

u = 26.2 correct to 3 significant figures 25.15 ≤ u < 26.25

a = 4.3 correct to 2 significant figures 4.25 ≤ a < 4.35

(a) Calculate the upper bound for the value of D.
Give your answer correct to 6 significant figures.
You must show all your working.

$UB = \dfrac{UB}{LB}$ $\dfrac{(26.25)^2}{2 \times 4.25}$

$= 81.0662$

81.0662
(3)

3/3

The lower bound for the value of D is 78.6003 correct to 6 significant figures.

(b) By considering bounds, write down the value of D to a suitable degree of accuracy.
You must give a reason for your answer.

80 because 78.6003 and 81.0662

are both 80 to 1 significant figure
(2)

2/2

Examiner comments

This was a model answer, showing sufficient detail, and was awarded full marks.

Part (a): The student wrote down correct lower and upper bounds for both u and a before using appropriate bounds to calculate the correct value of the upper bound of D.

Part (b): The student gave a correct answer together with a clear explanation concerning accuracy to one significant figure.

Student response B

19 $D = \dfrac{u^2}{2a}$

$u = 26.2$ correct to 3 significant figures

$a = 4.3$ correct to 2 significant figures

(a) Calculate the upper bound for the value of D.
Give your answer correct to 6 significant figures.
You must show all your working.

~~2~~ 26.24

4.325

$$\frac{26.24^2}{2(4.25)}$$

81.0044
(3)

2/3

The lower bound for the value of D is 78.6003 correct to 6 significant figures.

(b) By considering bounds, write down the value of D to a suitable degree of accuracy.
You must give a reason for your answer.

80 ~~81 0 Suit~~ because its closest to lower bound ~~there is no suitable degree~~ and upper bound.
(2)

1/2

Examiner comments

Part (a): The student attempted to identify an upper bound for u and a lower bound for a. The lower bound for a was correct, so the first M1 was awarded. Both values lay within the acceptable limits stated on the mark scheme and the student made a correct substitution into the formula for D, so the second M1 could also be awarded. Since the student did not use the correct upper bound for u, they did not get a correct final answer, so did not gain A1.

Part (b): B1 was given for the correct answer, 80, but the reason given is insufficient for the award of C1.

Student response C

19 $D = \dfrac{u^2}{2a}$

$u = 26.2$ correct to 3 significant figures

$a = 4.3$ correct to 2 significant figures

(a) Calculate the upper bound for the value of *D*.
Give your answer correct to 6 significant figures.
You must show all your working.

$$\frac{26.25^2}{2 \times 4.35}$$

4
80.718~~3~~ **(3)**

1/3

The lower bound for the value of *D* is 78.6003 correct to 6 significant figures.

(b) By considering bounds, write down the value of *D* to a suitable degree of accuracy.
You must give a reason for your answer.

78 · 600325

(2)

0/2

Examiner comments

Part (a): The student substituted two upper bounds into the expression for *D*. This was a commonly seen error that was awarded the first M1, but no further marks.

Part (b): An incorrect answer with no reason scored no marks. Note that an appropriate value for *D* following from the student's answer to (a) and 78.6003 would have been 80.

Exemplar question 9

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Higher tier Question 20

20 Solve algebraically the simultaneous equations

$$x^2 - 4y^2 = 9$$
$$3x + 4y = 7$$

....

(Total for Question 20 is 5 marks)

Mean score: 0.60

Examiner comments

This high-grade algebra question tested a student's ability to solve simultaneous equations in the case where one of the equations has squared terms. To gain full marks, students needed to accurately carry out a routine procedure comprising several steps. They were expected to rearrange the equation $3x + 4y = 7$ and make a substitution into $x^2 - 4y^2 = 9$ in order to get a quadratic equation in one of the variables. This could then be solved and the pairs of values for x and y found. Students needed to make sure that their final answer clearly showed the values paired appropriately.

Students found the expansion of the square of an algebraic fraction difficult to deal with and many errors were seen. A significant number of students forgot to multiply all terms of the quadratic equation when multiplying through by a constant to eradicate the fraction.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
20	$x = 3\frac{2}{5}$, $y = -\frac{4}{5}$ $x = 5$, $y = -2$	M1	for substitution of a rearrangement e.g. $x = \frac{7-4y}{3}$ **or** $y = \frac{7-3x}{4}$ into $x^2 - 4y^2 = 9$ **OR** expansion of $\left(\frac{7-4y}{3}\right)^2 = \frac{49-56y+16y^2}{9}$ **or** $\left(\frac{7-3x}{4}\right)^2 = \frac{49-42x+9x^2}{16}$	Expansion may not be in simplest form but must be correct
		M1	for correct expansion **and** substitution e.g. $\frac{49-56y+16y^2}{9} - 4y^2 = 9$ **or** $x^2 - 4\left(\frac{49-42x+9x^2}{16}\right) = 9$	
		A1	for forming quadratic ready for solving e.g. $-20y^2 - 56y - 32$ (= 0) **or** $5y^2 + 14y + 8$ (= 0) oe **or** $5x^2 - 42x + 85$ (= 0) oe	Note we do not need to see "= 0"; just the LHS is sufficient.
		M1	ft a 3 term quadratic, factorising e.g. $(5y + 4)(y + 2)$ (= 0) **or** $(5x - 17)(x - 5)$ (= 0) **OR** correct use of formula e.g. $(y =) \frac{-14 \pm \sqrt{14^2 - 4\times5\times8}}{2\times5}$ **or** $(x =) \frac{--42 \pm \sqrt{42^2 - 4\times5\times85}}{2\times5}$ **OR** completing the square, e.g. $(y + \frac{7}{5})^2 - \frac{9}{25}$ (= 0) **or** $(x - \frac{7}{5})^2 - \frac{16}{25}$ (= 0)	Can be implied by both x values correct or both y values correct.
		A1	correctly pairs x and y values: $x = 3\frac{2}{5}, y = -\frac{4}{5}$ oe, $x = 5$, $y = -2$	Answers must be correctly paired. Accept coordinate pairs

Examiner comments

Students were expected to both rearrange and substitute in order to score the first M1. The second M1 was awarded when the substitution had been made, together with the expansion of the square of the algebraic fraction. The first A1 was an accuracy mark given for a fully correct quadratic equation in the form $ax^2 + bx + c$ (= 0) or in the form $ay^2 + by + c$ (= 0). The '(= 0)' showed that the mark could be awarded for a correct expression where the '= 0' was omitted. The third M1 was awarded for a correct method to solve the student's quadratic equation (provided it had 3 terms) and was independent of the award of the first 3 marks. To score full marks, correctly paired values needed to be seen, not just 4 correct values listed. It was acceptable for students to give either of the forms $x = 3\frac{2}{5}, y = -\frac{4}{5}$; $x = 5$, $y = -2$ or $(3\frac{2}{5}, -\frac{4}{5})$, $(5, -2)$. Correct decimal equivalents were also acceptable.

Student response A

20 Solve algebraically the simultaneous equations

$$x^2 - 4y^2 = 9$$
$$3x + 4y = 7$$

$$3x = 7 - 4y$$
$$x = \frac{7}{3} - \frac{4y}{3}$$

$$\left(\frac{7}{3} - \frac{4}{3}y\right)^2 - 4y^2 = 9$$
$$\frac{49}{9} + \frac{16}{9}y^2 - \frac{56}{9}y - 4y^2 = 9$$
$$-\frac{20}{9}y^2 - \frac{56}{9}y + \frac{49}{9} = 9$$
$$-20y^2 - 56y + 49 = 81$$
$$20y^2 + 56y + 32 = 0$$
$$20y^2 + 40y + 16y + 32 = 0$$
$$20y(y + 2) + 16(y + 2) = 0$$
$$(20y + 16)(y + 2) = 0$$
$$y = -\frac{4}{5} \qquad y = -2$$

$$3x + 4(-2) = 7$$
$$3x = 15$$
$$x = 5$$

$$3x + 4\left(\frac{-4}{5}\right) = 7$$
$$3x = \frac{51}{5}$$
$$x = 3.4$$

$x = 5$ and $y = -2$, $x = 3.4$ and $y = -0.8$

5/5

Examiner comments
This response showed an accurate and concise solution, scoring full marks. Every stage was clearly presented and the answers given in appropriate pairs.

Student response B

20 Solve algebraically the simultaneous equations

$$x^2 - 4y^2 = 9$$
$$3x + 4y = 7$$

$3x + 4y = 7$

$3x = 7 - 4y$

$x = \frac{7-4y}{3}$

$\left(\frac{7-4y}{3}\right)^2 - 4y^2 = 9$

	7	$-4y$
7	49	$-28y$
$-4y$	$-28y$	$16y^2$

$\frac{16y^2 - 56y + 49}{9} - 4y^2 = 9$

$\frac{16y^2 - 56y + 49}{9} = 9 + 4y^2$

$20 \times 32 = 640$

$16y^2 - 56y + 49 = 9(9 + 4y^2)$

$16y^2 - 56y + 49 = 81 + 36y^2$

$20y^2 + 8y + 40y$

$4y(5y+4)$ $8(5y+4)$

$36y^2 - 16y^2 + 56y + 81 - 49 = 0$

$20y^2 + 56y + 32 = 0$

$(5y + 4)(4y + 8)$

$y = -4/5$ $y = -2$

When $y = 4/5$, ~~$x = 3x + (7 \times 4/5) = 7$~~

~~$3x + 16/5 = 7$~~

~~$3x = 7 - 16/5$~~

~~$3x = 19/5$~~

~~$x = 19/15$~~

$3x + (4 \times -4/5) =$

$$\boxed{\begin{array}{ll} x = 5 & y = -2 \\ x = 19/15 & y = 4/5 \end{array}}$$

When $y = -2$, $3x + (4 \times -2) = 7$

$3x - 8 = 7$

$3x = 15$

$x = 5$

(Total for Question 20 is 5 marks)

4/5

Examiner comments

The student gave a near-perfect response. They found correct values for *y* but had one incorrect value for *x* so could not be awarded the final A1.

Student response C

20 Solve algebraically the simultaneous equations

$$x^2 - 4y^2 = 9$$
$$3x + 4y = 7$$

$3x + 4y = 7.$

$3x = 7 - 4y.$

$x = \dfrac{7-4y}{3}.$

$\left(\dfrac{7-4y}{3}\right)^2 - 4y^2 = 9.$

$\left(\dfrac{7-4y}{3}\right)\left(\dfrac{7-4y}{3}\right) = \dfrac{49-16y^2}{9} - 4y^2 = 9$

$49 - 16y^2 - 4y^2 = 81.$

$-20y^2 = 32.$

$y^2 = -1.6.$

$x^2 - 4(-1.6) = 9.$

$x^2 = 2.6.$

$x = 1.612. \quad y = \sqrt{-1.6}.$

1/5

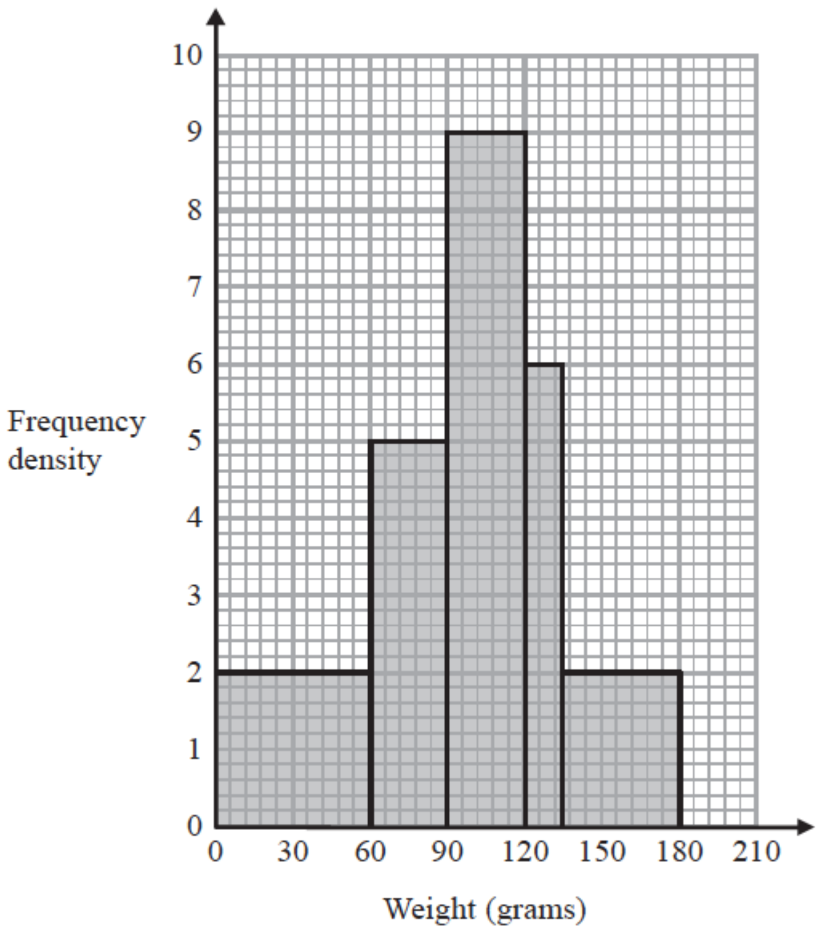
Examiner comments

The student had a correct rearrangement for x in terms of y and substituted it into the first equation, gaining the first M1. However, the expansion of $\left(\dfrac{7-4y}{3}\right)^2$ was not correct and the candidate did not get a three-term quadratic equation, so no further marks could be awarded.

Exemplar question 10 [back to Contents page]

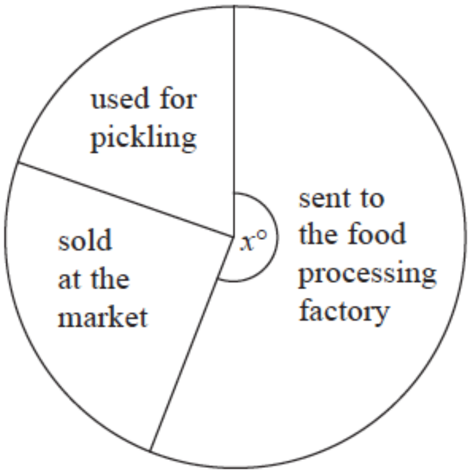
Higher tier Question 21

21 The histogram gives information about the distribution of the weights of some onions grown by a farmer.



Onions less than 60 grams in weight are used for pickling.
Onions greater than 120 grams in weight are sold at the market.
The rest of the onions are sent to a food processing factory.

A pie chart is drawn using the information opposite to show what the farmer does with the onions he grows.



The angle of the sector for the onions sent to the food processing factory is $x°$.

Work out the value of x.

x = ...

(Total for Question 21 is 4 marks)

Mean score: 1.72

Examiner comments

This question assessed a student's ability to interpret a histogram and to draw a pie chart. Students were expected to find the proportion of onions sent to the food processing factory by either considering the frequencies represented in the histogram or by considering the relative areas of the bars. These frequencies or areas needed to be clearly shown. Most students who followed a successful strategy used the former approach. Students were then required to work out the angle in a pie chart that would represent these onions.

Weaker students often made the mistake of using the heights of the bars. This incorrect method could not be awarded any marks.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
21	210	M1	for method to find total frequency, 60×2 (= 120) + 30×5 (= 150) + 30×9 (= 270) + 15×6 (= 90) + 45×2 (= 90) **or** 720 **OR** for method to find the total area, $4 + 5 + 9 + 3 + 3$ (= 24 cm^2)	Accept one error in total for the award of the method marks 24 must be from adding areas of bars not heights of bars
		M1	for finding the number of onions less than 60 g or greater than 120 g = 120 + 90 + 90 (= 300), **OR** for finding the number of onions between 60 g and 120 g = 150 + 270 (= 420) **OR** for finding the area under the graph less than 60 or greater than 120 = 4 + 3 + 3 (= 10 cm^2) **OR** for finding the area under the graph between 60 and 120 = 5 + 9 (= 14 cm^2)	 14 must be from adding areas of bars not heights of bars
		M1	(dep M2) for $1 - \frac{\text{“300”}}{\text{“720”}} \left(= \frac{7}{12}\right)$ oe **OR** for $\frac{\text{“420”}}{\text{“720”}} \left(= \frac{7}{12}\right)$ oe **OR** for $\frac{\text{“14”}}{\text{“24”}} \left(= \frac{7}{12}\right)$ oe	Accept 58.3…%
		A1	cao	

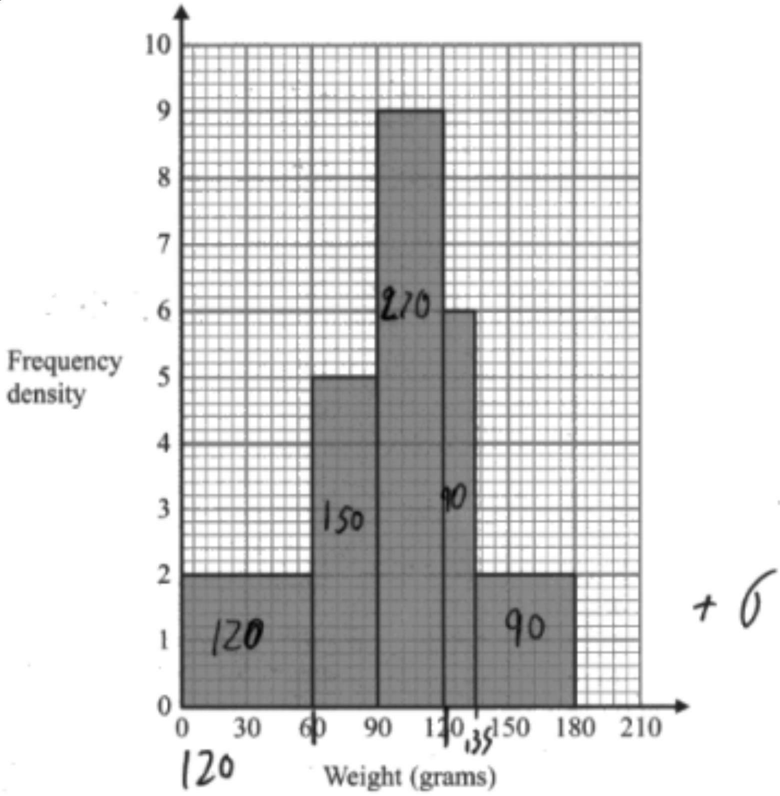
Examiner comments

The first M1 was for a correct method to find the total number of onions represented on the histogram. This was usually awarded for seeing the frequencies represented by each bar added together. Examiners condoned one incorrect frequency when awarding this mark. Alternatively, students could work out the area of each bar, 4 (cm^2), 5 (cm^2) etc. and add these together. The second M1 was awarded for working out the total number of onions less than 60 g or greater than 120 g or the number of onions between 60 g and 120 g (or total area under the graph in each case), as this would enable them to make further progress. Provided both of the first two M1 marks were given, students could earn the third M1 for a complete method to find the proportion of onions sent to the factory. A1 given for a correct answer supported by working and from a correct method.

Some candidates could opt to work with percentages, either giving their final answer as a percentage, or rounding a recurring decimal to one decimal place and hence introducing a rounding error. In these cases, A1 was likely to be lost.

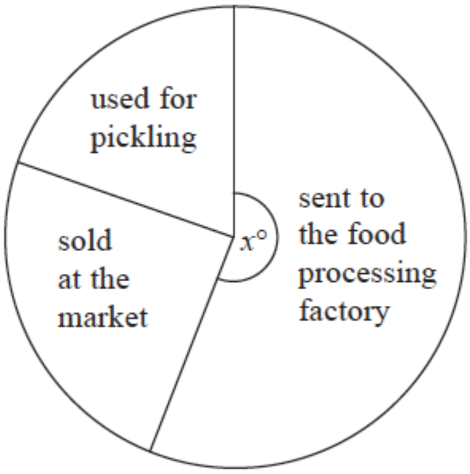
Student response A

21 The histogram gives information about the distribution of the weights of some onions grown by a farmer.



Onions less than 60 grams in weight are used for pickling. 120
Onions greater than 120 grams in weight are sold at the market. 180
The rest of the onions are sent to a food processing factory. 420

A pie chart is drawn using the information opposite to show what the farmer does with the onions he grows.



The angle of the sector for the onions sent to the food processing factory is $x°$.
Work out the value of x.

$$\frac{420}{120+180+420} = \frac{7}{12}$$

$$\frac{7}{12} \times 360 = 210$$

$x =$ 210

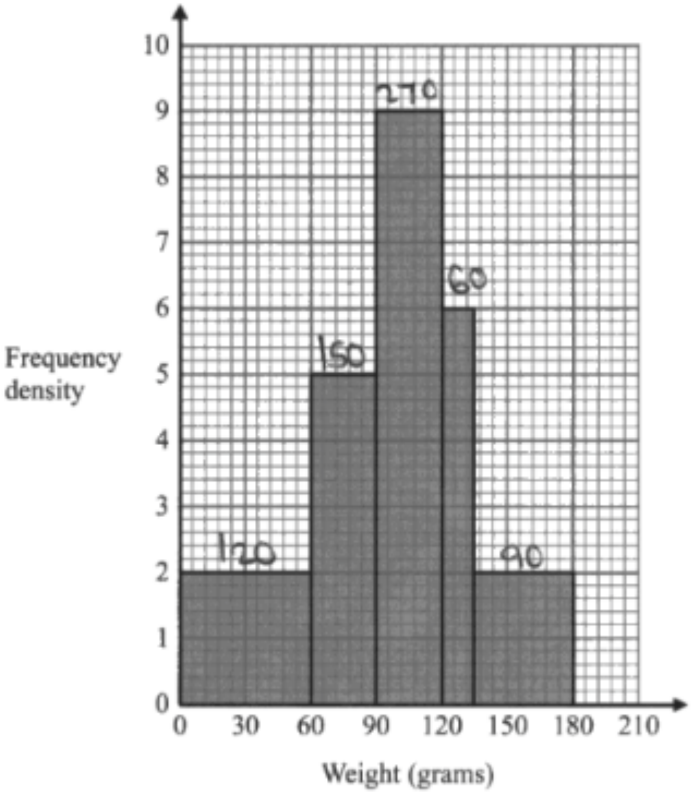
4/4

Examiner comments

This was a concise and accurate response, with enough detail to make the method clear. The student showed the frequency represented by each bar, writing them on the bars themselves. Students often did this. The first M1, for a method to find the total frequency, was awarded on the second page for 120 + 180 + 420 in the denominator of the fraction. The second M1 could be given either for the 420 seen on the first page next to the statement ‘The rest of the onions are sent to the food processing factory’ or for sight of 420 in the fraction on the second page. The student showed a correct proportion as a fraction, for the third M1. They simplified their fraction and used it to work out the size of the angle of the sector in the pie chart for A1.

Student response B

21 The histogram gives information about the distribution of the weights of some onions grown by a farmer.



Onions less than 60 grams in weight are used for pickling.
Onions greater than 120 grams in weight are sold at the market.
The rest of the onions are sent to a food processing factory.

30x5

45x2
= 90

120+15
= 135

30x90

6x15 = 60

A pie chart is drawn using the information opposite to show what the farmer does with the onions he grows.

120

$\frac{120}{690} \times 360$

$= 62.\dot{6}$

150

$\frac{150}{690} \times 360$

$= 78.\dot{2}6$

The angle of the sector for the onions sent to the food processing factory is x°.
Work out the value of x.

$150 + 270 = 420$

Frequency = FD × CW
= 2 × 60
= 120 onions

$\frac{420}{690} \times 360$

$= 219.\dot{1}3$

Total = 120 + 150 + 270 + 60 + 90
= 690 onions

$x =$ $219.\dot{1}3$

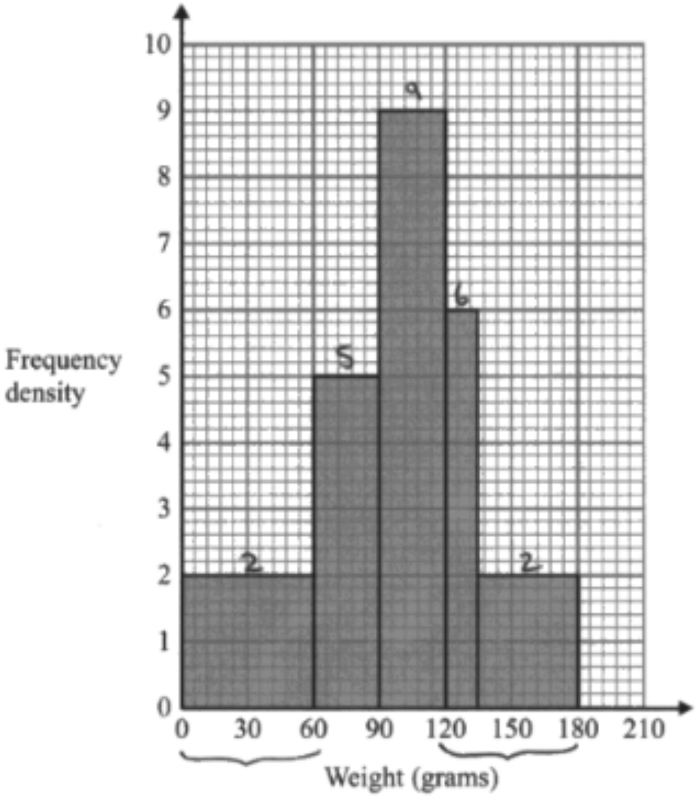
3/4

Examiner comments

The student made a good attempt, using and showing a correct method throughout their response. The only error they made was ‘6 × 15 = 60’, seen on the first page in the working space so the candidate was awarded all three M1 marks but not A1.

Student response C

21 The histogram gives information about the distribution of the weights of some onions grown by a farmer.



Onions less than 60 grams in weight are used for pickling.
Onions greater than 120 grams in weight are sold at the market.
The rest of the onions are sent to a food processing factory.

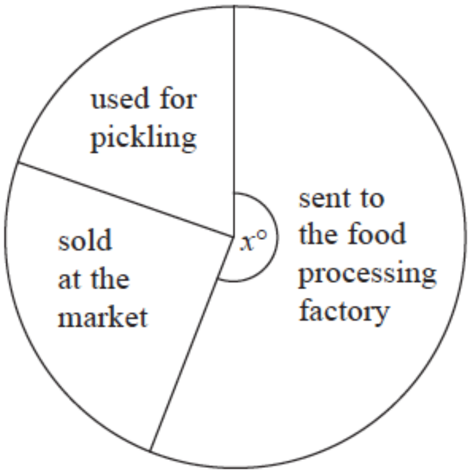
60 → pickling → 2

40 → Market → $6 + \cancel{8}^{2} = 8$

$2 + 2 + 6 + 5 + 9 = 24$

~~foo~~ factory = $24 - 10 = 14$

A pie chart is drawn using the information opposite to show what the farmer does with the onions he grows.



The angle of the sector for the onions sent to the food processing factory is $x°$.
Work out the value of x.

Frequency = 24

$100 \div 24 = 4.16$

$14 \times 4.16 = 58.24$

$2 \times 4.16 = 8.32$

$8 \times 4.16 = 33.28$

$x =$ ....

0/4

Examiner comments

The student realised that a possible method was to work out the number of onions weighing less than 60 g or more than 120 g, then subtract this from the total number of onions to find the number of onions sent to the processing factory. However, they made the common error of using the heights of the bars instead of using the areas of the bars and so could not be awarded any marks.

Exemplar question 11

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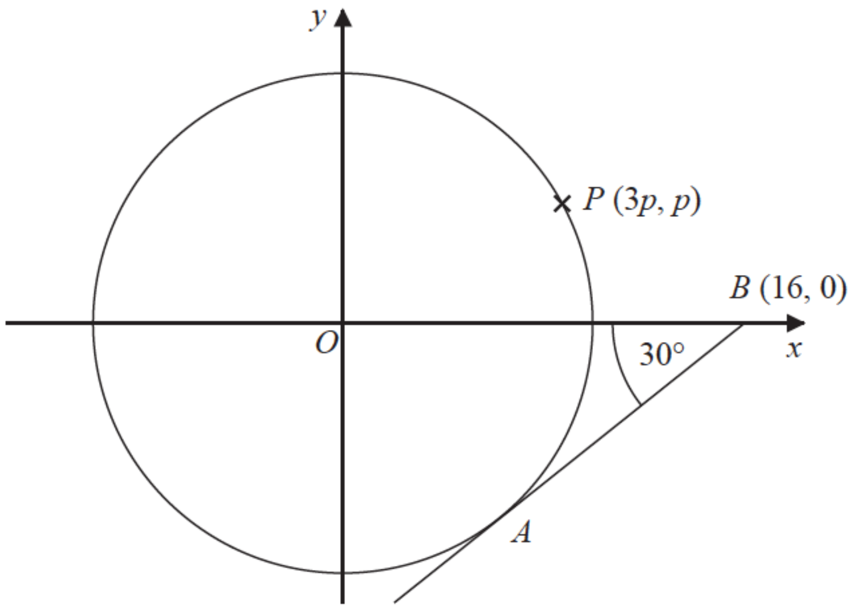
Higher tier Question 22

22 The diagram shows a circle, centre *O*.



AB is the tangent to the circle at the point *A*.
Angle *OBA* = 30°

Point *B* has coordinates (16, 0)
Point *P* has coordinates (3*p*, *p*)

Find the value of *p*.
Give your answer correct to 1 decimal place.
You must show all your working.

p = ...

(Total for Question 22 is 4 marks)

Mean score: 0.42

Examiner comments

Recognising and using the equation of a circle with centre at the origin, knowing that the angle between a tangent and radius at a point is 90°, using Pythagoras' theorem and being able to apply trigonometry to right-angled triangles were all relevant to this problem, which connects different parts of mathematics through a series of mathematical processes. Students needed to identify the right-angled triangle OAB, use trigonometry to find the radius of the circle, then substitute the coordinates $(3p, p)$ into the equation of the circle or use Pythagoras' theorem to obtain an equation in p. They then needed to solve this to find the value of p.

The most common error made by students who used a correct strategy occurred when they substituted the coordinates $(3p, p)$ and the radius of the circle into the equation $x^2 + y^2 = r^2$. A significant proportion of students wrote $3p^2 + p^2 = 8^2$ instead of the correct $(3p)^2 + p^2 = 8^2$, before solving the equation $4p^2 = 64$ instead of $10p^2 = 64$. These students scored 2 of the 4 marks available.

Some students tried to use equations of lines and gradients but this proved fruitless.

Mark scheme

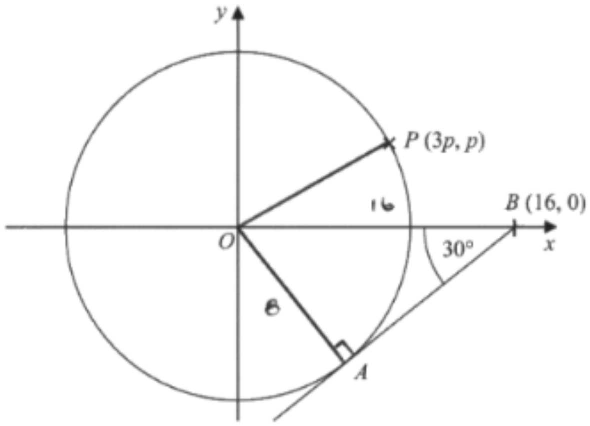
Question	Answer	Mark	Mark scheme	Additional guidance
22	2.5	P1	use of $\sin 30 = \frac{1}{2}$ to find OA (= 8) **or** $OAB = 90°$ e.g. $OA = 16 \sin 30°$ **or** right angle marked on diagram	
		P1	recognition that equation of circle is $x^2 + y^2 = r^2$	Accept $3p^2 + p^2 = r^2$ for the award of this mark
		P1	Correct substitution of p, $3p$ and r in $x^2 + y^2 = r^2$ e.g. $9p^2 + p^2 = OA^2$ **or** $(3p)^2 + p^2 =$ "8^2"	Do not accept $3p^2 + p^2 = 8^2$ for the award of this mark
		A1	for answer in the range 2.5 to 2.53	Accept $\sqrt{6.4}$ or $\frac{4\sqrt{10}}{5}$ If an answer within the given range is seen in working and rounded incorrectly award full marks. Award 0 marks for the answer without supportive working.

Examiner comments

The first P1 was for a process to find the radius of the circle or to identify the right angle OAB. This could be given for either of these marked clearly on the diagram. The second P1 was for writing down the equation of a circle, $x^2 + y^2 = r^2$ and could be awarded if this was shown with $x = 3p$, $y = p$ substituted, i.e. for $(3p)^2 + p^2 = r^2$. Examiners also accepted $3p^2 + p^2 = r^2$ as evidence for the award of this mark. The third P1 was awarded for a correct equation that could lead to a correct value for p. This would usually be $(3p)^2 + p^2 = 8^2$ or equivalent, but examiners also accepted $(3p)^2 + p^2 = OA^2$ for the award of this mark. A1 was awarded for an answer in the range 2.5 to 2.53. Students who gave a correct value but did not support it with correct working scored no marks, as the instruction given in the question is to show all working.

Student response A

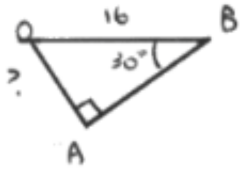
22 The diagram shows a circle, centre O.



AB is the tangent to the circle at the point A.
Angle $OBA = 30°$

Point B has coordinates (16, 0)
Point P has coordinates $(3p, p)$

Find the value of p.
Give your answer correct to 1 decimal place.
You must show all your working.



$\angle OAB = 90°$ (radius and tangent meet at 90°)

$\sin 30° = \frac{OA}{16}$

$OA = 8$

OP = radius

$OP = 8$

$(3p)^2 + p^2 = 8^2$ (Pythagoras theorem)

$9p^2 + p^2 = 64$

$10p^2 = \cancel{6.4}\ 64$

$p = 6.4$

$p = 2.5298...$

$= 2.5$ (1.d.p)//

$p =$ 2.5

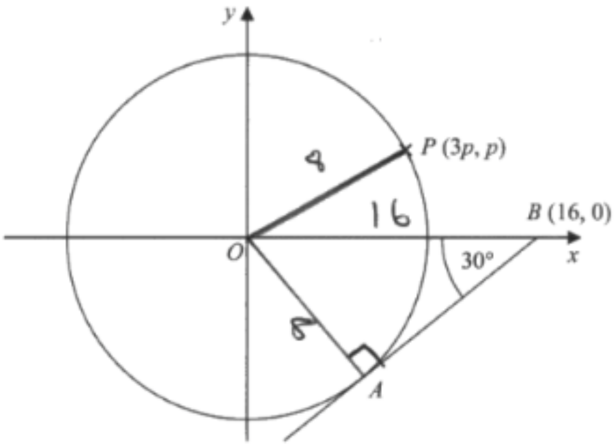
4/4

Examiner comments

The first P1 could be awarded either for the right angle OAB marked on the diagram or for a correct method to find the radius as seen in the working space. $(3p)^2 + p^2 = 8^2$ gave evidence for the award of both the second and third P1 marks. The student viewed this as use of Pythagoras' theorem rather than using the equation of the circle which is, of course, equivalent. The equation obtained was successfully solved and full marks awarded for this complete and accurate solution.

Student response B

22 The diagram shows a circle, centre O.



AB is the tangent to the circle at the point A.
Angle $OBA = 30°$

Point B has coordinates (16, 0)
Point P has coordinates ($3p$, p)

Find the value of p.
Give your answer correct to 1 decimal place.
You must show all your working.

$$\frac{\sin 90}{16} = \frac{\sin 30}{x}$$

$$0.0625 = \frac{\sin 30}{x}$$

$$0.0625x = \sin 30$$

$$x = 8$$

$$x^2 + y^2 = 8^2$$

$$3p^2 + p^2 = 64$$

$$\cancel{10p^2 = 64}$$

$$4p^2 = 64 \quad \cancel{p^2 = 64}$$

$$p^2 = 16$$

$$p = 4$$

$p =$ ~~value~~ 4

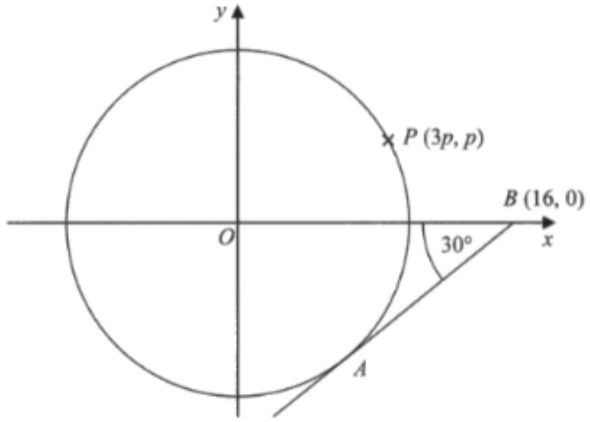
2/4

Examiner comments

The student found the radius of the circle, albeit by using the sine rule. They also identified angle OAB as 90° so scored the first P1 for either of these. $x^2 + y^2 = 8^2$ led to the award of the second M1, but unfortunately the student wrote $3p^2 + p^2 = 64$ rather than the correct $(3p)^2 + p^2 = 64$ or $9p^2 + p^2 = 64$ and went on to solve $4p^2 = 64$ so could not be awarded any subsequent marks. This was a frequently seen error in student responses.

Student response C

22 The diagram shows a circle, centre O.



AB is the tangent to the circle at the point A.
Angle $OBA = 30°$

Point B has coordinates (16, 0)
Point P has coordinates $(3p, p)$

Find the value of p.
Give your answer correct to 1 decimal place.
You must show all your working.

equation of AB :

$y = +mx - c$

equation of circle :

$x^2 + y^2 = r^2$

substitute point P :

$(3p)^2 + p^2 = r^2$

$\therefore 9p^2 + p^2 = r^2$

$\therefore 10p^2 = r^2$

$\therefore r = \sqrt{10p^2}$

(1 d.p.)

$p =$ ....

1/4

Examiner comments
The student had neither a correct method for finding the radius of the circle nor did they recognise that angle OAB was 90° so they could not be given the first P1. They did, however, recognise that they could use the equation of the circle and so satisfied the requirement for the second P1. No further marks could be awarded because they required use of the radius, which had not been found.