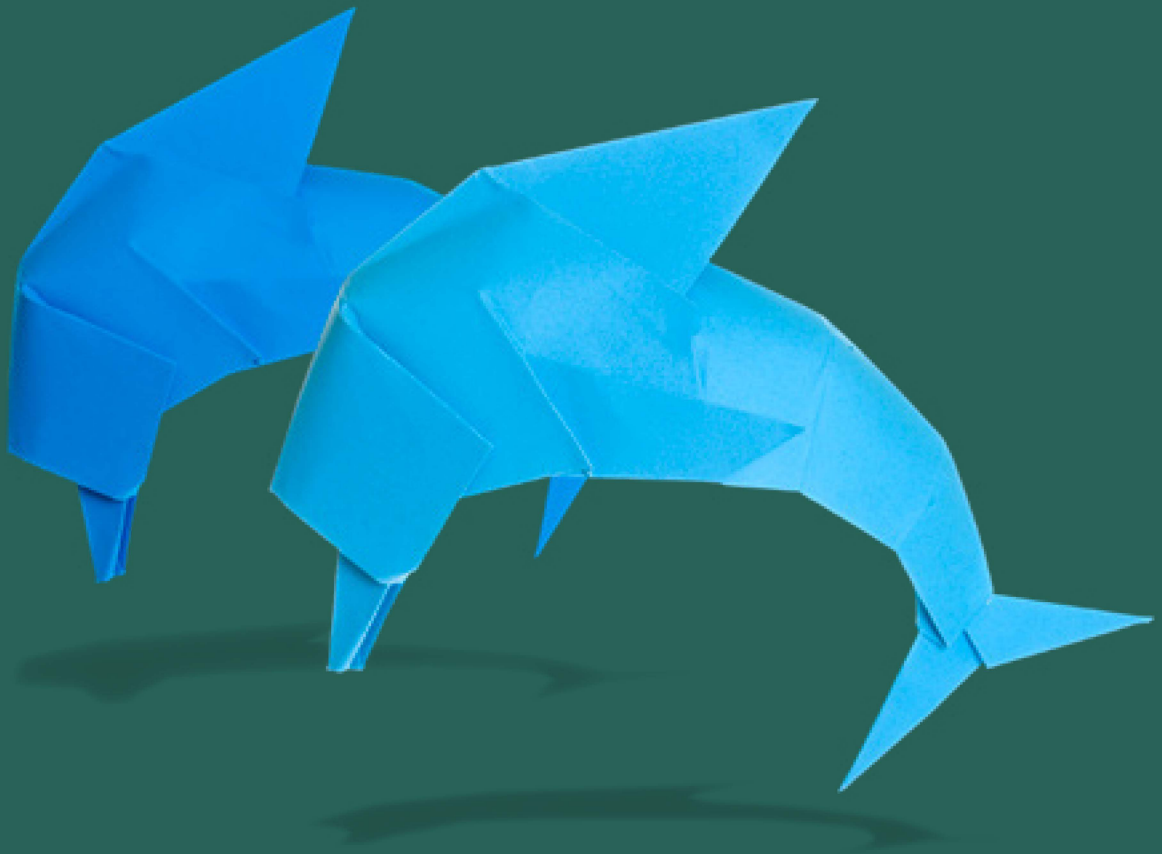


Pearson Edexcel
Level 1/Level 2 GCSE (9-1)

Mathematics (1MA1)



November 2021 - Exemplar

Student answers with examiner comments

Higher

First certification 2017

Contents

About this Booklet	5
Paper 1H (non-calculator)	7
Exemplar question 1 – Paper 1H Question 8	7
Exemplar question 2 – Paper 1H Question 10	12
Exemplar question 3 – Paper 1H Question 11	18
Exemplar question 4 – Paper 1H Question 12	23
Exemplar question 5 – Paper 1H Question 13	28
Exemplar question 6 – Paper 1H Question 14	33
Exemplar question 7 – Paper 1H Question 15	38
Exemplar question 8 – Paper 1H Question 16	43
Exemplar question 9 – Paper 1H Question 17	48
Exemplar question 10 – Paper 1H Question 18	53
Exemplar question 11 – Paper 1H Question 19	58
Paper 2H (calculator)	63
Exemplar question 1 – Paper 2H Question 7	63
Exemplar question 2 – Paper 2H Question 8	68
Exemplar question 3 – Paper 2H Question 9	74
Exemplar question 4 – Paper 2H Question 10	79
Exemplar question 5 – Paper 2H Question 11	84
Exemplar question 6 – Paper 2H Question 14	89
Exemplar question 7 – Paper 2H Question 17	94
Exemplar question 8 – Paper 2H Question 19	99
Exemplar question 9 – Paper 2H Question 20	104
Exemplar question 10 – Paper 2H Question 21	109
Paper 3H (calculator)	114
Exemplar question 1 – Paper 3H Question 8	114
Exemplar question 2 – Paper 3H Question 10	122
Exemplar question 3 – Paper 3H Question 13	127
Exemplar question 4 – Paper 3H Question 14	132
Exemplar question 5 – Paper 3H Question 15	137
Exemplar question 6 – Paper 3H Question 16	142
Exemplar question 7 – Paper 3H Question 18	147
Exemplar question 8 – Paper 3H Question 19	152
Exemplar question 9 – Paper 3H Question 21	157



About this booklet

This booklet has been produced to support mathematics teachers delivering the new GCSE (9–1) Mathematics specification.

The booklet looks at a selection of questions from the November 2021 GCSE (9 – 1) Mathematics Higher tier examination. It shows real student responses to selected questions and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

How to use this booklet

Our examining team have selected student responses to higher tier questions from the November 2021 examination. Following each question, you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

Student Response C

- 9 Danny buys,
1 loaf of bread for £1.20
1 bottle of milk for 70p
2 packets of cheese for £2.30 each packet

Danny pays with a £10 note.

He says,

"I should get £3.30 change."

Is Danny correct?

You must show how you get your answer.

$$\begin{array}{r} 11.20 \\ 70 \\ 2.30 \\ \hline 4.20 \end{array} \quad \begin{array}{r} 10.00 \\ 4.20 \\ \hline 5.80 \end{array}$$

Danny isn't correct, the right amount of change he'll receive is £5.80.

Student Response

Mean score for the question

Marks awarded for the question or question part

1/3

Examiner Comments

P1 for adding just three of the four items.

P0 Even though their total cost of the three items is subtracted from £10, this is clearly not a complete correct method.

A0 The figures are correct for their working, but the final accuracy mark is dependent upon fully correct figures as shown in the mark scheme.

Examiner commentary on how marks have been awarded

Paper 1H (non – calculator)

Exemplar Question 1

Higher tier Paper 1

8 Solve $x^2 = 5x + 24$

.....
(Total for Question 8 is 3 marks)

Mean Score: 1.04

Examiner Comments

This question assessed the ability of students to solve a quadratic equation, albeit one that needed to be rearranged. Students were expected to rearrange $x^2 = 5x + 24$ to form a quadratic equation with 0 on its own and then solve the equation by factorising.

Students who rearranged the given equation to get $x^2 - 5x - 24 = 0$ often went on to give a fully correct answer. Some, however, lost marks because they were unable to factorise correctly or they factorised correctly but then wrote $(x - 8)(x + 3)$ as the final answer. A surprising number of students did not realise that a sensible first step would be to rearrange the equation and attempted to factorise straight away.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
8	8 and -3	M1	for rearranging to get $x^2 - 5x - 24 (= 0)$ or $-x^2 + 5x + 24 (= 0)$	Can be implied by $(x - 8)(x + 3)$ or $(-x + 8)(x + 3)$
		M1	for $(x \pm 8)(x \pm 3)$ or $(x + a)(x + b)$ where $ab = -24$ or $a + b = -5$ or substitution into formula, condoning one sign error eg $(x =)$ $\frac{- - 5 \pm \sqrt{(-5)^2 - 4 \times 1 \times -24}}{2 \times 1}$	
		A1	for 8 and -3	

Mark Scheme

Examiner Comments

Those students who rearranged the equation incorrectly or made no attempt to rearrange it could still be awarded the second M1 for a factorisation of the form $(x \pm 8)(x \pm 3)$.

The correct factorisation $(x - 8)(x + 3)$ gained the first two marks.

One correct value of x seen without supporting working did not gain any marks.



Student Response A

8 Solve $x^2 = 5x + 24$

$$x^2 - 5x - 24 = 0$$

$$(\rightarrow x - 8)$$

$$(x - 8)(x + 3)$$

$$x = 8, x = -3$$

$$x = 8, x = -3$$

(Total for Question 8 is 3 marks)

3/3

Examiner Comments**M1** for rearranging the equation to get $x^2 - 5x - 24 = 0$ **M1** for the correct factorisation**A1** for the correct values, 8 and -3



Student Response B

8 Solve $x^2 = 5x + 24$

$$\begin{aligned} x^2 &= 5x + 24 \\ x^2 - 5x - 24 &= 0 \\ (x - 8)(x + 3) \\ x^2 + 3x - 8x - 24 \\ x^2 - 5x - 24 \end{aligned}$$

$$(x - 8)(x + 3)$$

(Total for Question 8 is 3 marks)

2/3

Examiner Comments**M1** for a correct rearrangement of the equation to $x^2 - 5x - 24 = 0$ **M1** for the correct factorisation**A0** no values of x were given



Student Response C

8 Solve $x^2 = 5x + 24$

$$(x+8)(x-3)$$

$$\begin{array}{l} 24 \\ 4 \times 6 \\ 2 \times 12 \\ 3 \times 8 \quad \leftarrow \end{array}$$

$$(x+8)(x-3)$$

(Total for Question 8 is 3 marks)

1/3

Examiner Comments

M0 no attempt at rearranging the equation

M1 for a factorisation of the form $(x \pm 8)(x \pm 3)$

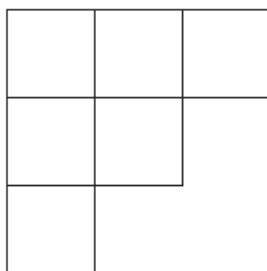
A0 no values of x were given



Exemplar Question 2

Higher tier Paper 1

- 10 The diagram shows a shape made from 6 identical squares.



The total area of the shape is 5406 cm^2

- (a) Find an estimate for the length of one side of each square.
Give your answer correct to the nearest whole number.

..... cm
(3)

- (b) Is your answer to part (a) an underestimate or an overestimate?
You must give a reason for your answer.

.....
.....
.....
(1)

(Total for Question 10 is 4 marks)

Mean Scores: Part (a) 1.36

Part (b) 0.41

**Examiner Comments**

Part (a) assessed the ability of students to devise a suitable strategy to solve a problem. This required students to combine their knowledge of geometry with number work in the form of estimation. It was expected that students would divide the total area by 6 to work out the area of one square and then find the square root of the result, using an appropriate rounded value at some stage of the process.

Since students were asked to find an estimate they should have recognised that they would need to use a rounded value. Some students did this at the start of the process, rounding the total area to 5400 before dividing by 6. Others started by dividing 5406 by 6 and then rounded 901 to 900 before attempting to find the square root. Many students gained the first mark for dividing by 6 but the next stage in the process proved to be beyond a considerable number of these students. Those who recognised that they needed to find the square root often went on to get full marks for an answer of 30 but the incorrect processes seen were many and varied. Often these involved division by 4 or by 2.

Part (b) assessed a specific requirement to evaluate results obtained. It was expected that students would make a comparison between their estimate and the actual length of one side of the square by considering the method they had used and explain why their estimate was an underestimate or an overestimate.

Many students did state that their answer was an underestimate and gave a suitable reason; for example they had rounded the total area down or they had rounded 901 to 900. Some students gave no reason for their choice. Overall, part (b) was not answered particularly well.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
10 (a)	30	P1	for a start to the process, eg $5406 \div 6 (= 901)$ or $5400 \div 6 (= 900)$ or $5000 \div 6 (= 833.33..)$ or $5410 \div 6 (= 901.66..)$	
		P1	for a process to find the length of one side, eg $\sqrt{"901"}$ or $\sqrt{"900"}$ or $\sqrt{"833.33.."}$ or $\sqrt{"901.66.."}$	
		A1	for 30	
(b)	Explanation	C1	for a correct explanation based on their working in (a), eg underestimate because I rounded the total area down	Must be based on the use of a rounded value in a calculation

Examiner Comments

Part (a)

The first P1 was awarded for the first step in the process, dividing the total area of the shape by 6, and the second P1 was for completing the process to find the length of one side by showing the intention to find the square root. The value in quotation marks did not need to be correct but it did have to come from a correct process.

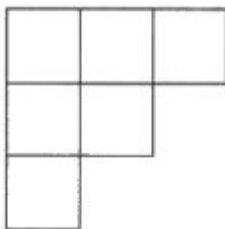
Part (b)

The C mark was not dependent on a complete process in part (a) but to be in a position to say whether their answer was an underestimate or an overestimate, students must have used a rounded value in a calculation.



Student Response A

10 The diagram shows a shape made from 6 identical squares.



The total area of the shape is 5406 cm^2

- (a) Find an estimate for the length of one side of each square.
Give your answer correct to the nearest whole number.

$$(x^2) + (x^2) + (x^2) + (x^2) + (x^2) + (x^2)$$

$$6x^2 = 5406$$

$$\begin{array}{r} 0901 \\ 6 \overline{)5406} \end{array}$$

$$\sqrt{900} \quad 30 \times 30 = 900$$

30 cm
(3)

- (b) Is your answer to part (a) an underestimate or an overestimate?
You must give a reason for your answer.

Underestimate as I used the value 900 instead
of 901. $\sqrt{901}$ would produce a larger value
than $\sqrt{900}$.

(1)

(Total for Question 10 is 4 marks)

4/4

Examiner Comments

Part (a)

P1 for a start to the process, $5406 \div 6$

P1 for a process to find the length of one side, $\sqrt{900}$

A1 for the correct answer

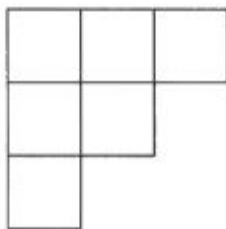
Part (b)

C1 for explaining that the answer to part (a) is an underestimate because 901 was rounded down to 900



Student Response B

10 The diagram shows a shape made from 6 identical squares.



The total area of the shape is 5406 cm^2

- (a) Find an estimate for the length of one side of each square.
Give your answer correct to the nearest whole number.

$$5406 = 5000$$

6 squares

$$6 \overline{) 5006.4}$$

$$834.4$$

$$166.4$$

$$166$$

$$\underline{167}$$

(3) cm

- (b) Is your answer to part (a) an underestimate or an overestimate?
You must give a reason for your answer.

It's an underestimate as the area is rounded down.

(1)

(Total for Question 10 is 4 marks)

2/4

Examiner Comments

Part (a)

P1 for a start to the process, dividing 5000 by 6

P0 as there is no process to find the length of one side

A0 as the final answer is incorrect

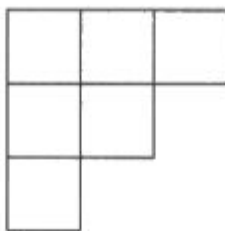
Part (b)

C1 a correct explanation based on rounding down the total area in part (a)



Student Response C

10 The diagram shows a shape made from 6 identical squares.



The total area of the shape is 5406 cm^2

- (a) Find an estimate for the length of one side of each square.
Give your answer correct to the nearest whole number.

$$\begin{array}{r} 0901 \\ 6 \overline{) 5406} \end{array}$$

$$901 \text{ cm}^2 = 1 \text{ square}$$

$$901 \div 2 = 450.5 \text{ cm}$$

$$\text{length of 1 side} = 450.5 \text{ cm}$$

$$\begin{array}{r} 451 \\ \hline \end{array} \text{ cm} \quad (3)$$

- (b) Is your answer to part (a) an underestimate or an overestimate?
You must give a reason for your answer.

It is an overestimate because I rounded up to get my answer.

(1)

(Total for Question 10 is 4 marks)

1/4

Examiner Comments**Part (a)**

P1 for a start to the process, dividing 5406 by 6

P0 because the process to find the length of one side is incorrect

A0 as the final answer is incorrect

Part (b)

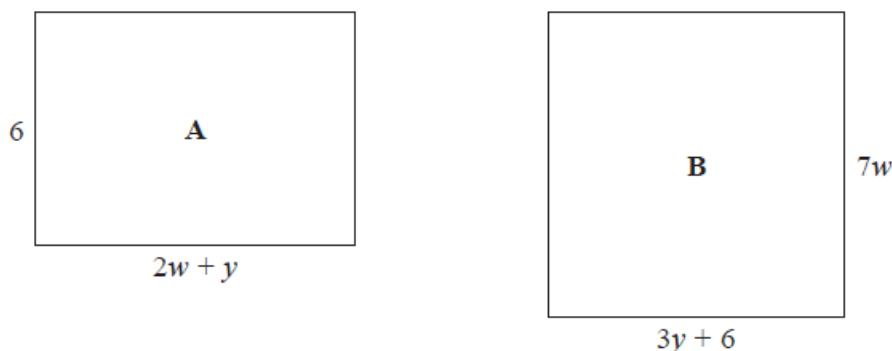
C0 as a rounded value was not used in either calculation in part (a). An explanation based solely on rounding their final answer in (a) is not acceptable. A rounded value must have been used in a calculation.



Exemplar Question 3

Higher tier Paper 1

- 11 The diagram shows two rectangles, **A** and **B**.



All measurements are in centimetres.

The area of rectangle **A** is equal to the area of rectangle **B**.

Find an expression for y in terms of w .

.....
(Total for Question 11 is 4 marks)

Mean Score: 1.00

Examiner Comments

This question assessed a number of different algebraic skills. Students were expected to start by writing an expression for the area of each rectangle and forming an equation. They then needed to demonstrate their skills of algebraic manipulation by expanding brackets, simplifying, rearranging and factorising in order to find an expression for y in terms of w .

Forming an equation ought to have been a straightforward process for the first mark, particularly as missing brackets were condoned. Some students wrote two expressions and failed to form an equation but many students were unable to write two correct expressions. A significant number of students worked with perimeter rather than with area. Having gained the first mark, many students were not able to make any further correct progress. Those without brackets in their equation usually failed to expand correctly. Some with brackets made errors expanding and some expanded correctly but then simplified incorrectly. Many students did not know how to deal with the fact that y appeared on both sides of the equation. Those who isolated the terms in y often went on to factorise correctly and were able to find a correct expression for y in terms of w .



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
11	$\frac{30w}{6-21w}$	P1	for forming an equation, eg $6(2w + y) = 7w(3y + 6)$ or $12w + 6y = 21wy + 42w$ oe	Condone missing brackets for this mark
		P1	for expanding brackets correctly and gathering w terms or isolating y terms in a correct equation, eg $6y = 21wy + 30w$ or $6y - 21wy = 42w - 12w$ or $6y - 21wy = 30w$	
		P1	(dep on two terms in y) for factorising out the y , eg $y(6 - 21w) = 42w - 12w$ or $y(6 - 21w) = 30w$ or $3y(2 - 7w) = 30w$	
		A1	for $(y =) \frac{30w}{6-21w}$ oe	

Examiner Comments

The first P1 was for forming an equation. For this mark, missing brackets were condoned.

For the second P1 students needed to expand the brackets correctly and gather the w terms or isolate the terms in y in a correct equation.

The third P1 for factorising was dependent on having two terms in y on one side of an equation.

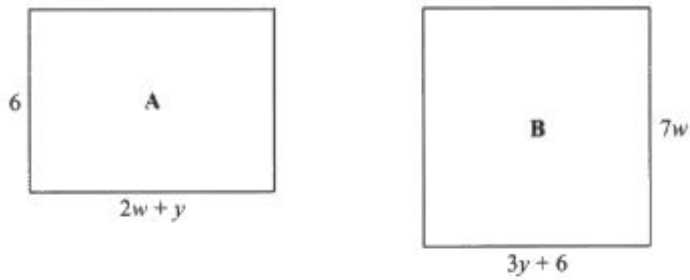
The A mark was awarded for $\frac{30w}{6-21w}$ or an equivalent expression such as $\frac{-30w}{21w-6}$ or $\frac{10w}{2-7w}$.

If $\frac{30w}{6-21w}$ was simplified incorrectly then the A mark was lost as it is not appropriate to ignore subsequent work in an algebra question.



Student Response A

11 The diagram shows two rectangles, A and B.



All measurements are in centimetres.

The area of rectangle A is equal to the area of rectangle B.

Find an expression for y in terms of w .

$$6 \times (2w + y) = 7w(3y + 6) \quad \text{①}$$

$$\text{①} \quad 12w + 6y = 21wy + 42w \quad \text{②}$$

$$\text{②} \quad 6y - 21wy = 42w - 12w \quad \text{③}$$

$$\text{③} \quad y(6 - 21w) = 30w \quad \text{④}$$

$$\text{④} \quad y = \frac{30w}{6 - 21w} \quad \text{⑤} \quad y = \frac{10w}{2 - 7w}$$

$$y = \frac{10w}{2 - 7w}$$

(Total for Question 11 is 4 marks)

4/4

Examiner Comments

P1 for forming an equation $6(2w + y) = 7w(3y + 6)$

P1 for expanding brackets and isolating the terms in y in a correct equation

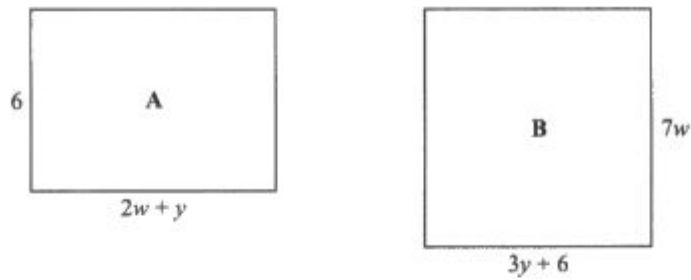
P1 for factorising out the y to get $y(6 - 21w) = 30w$

A1 for a correct expression; both $\frac{30w}{6 - 21w}$ and $\frac{10w}{2 - 7w}$ are correct.



Student Response B

11 The diagram shows two rectangles, A and B.



All measurements are in centimetres.

The area of rectangle A is equal to the area of rectangle B.

Find an expression for y in terms of w .

$$6 \times (2w + y) = 7w \times (3y + 6)$$

$$12w + 6y = 21wy + 42w$$

$$-12w \quad -12w$$

$$6y = 21wy + 30w$$

$$-21w$$

$$6y - 21w = 21y + 30w$$

$$+21w \quad +21w$$

$$6y = 21y + 30w$$

$$-21y \quad -21y$$

$$-15y = 30w$$

$$+6y \quad +6y$$

$$-21w = 15y + 30w$$

$$-30w \quad -30w$$

$$-21w = 15y$$

$$-21w = 15y$$

$$9w = 15y$$

(Total for Question 11 is 4 marks)

2/4

Examiner Comments

P1 for forming an equation $6(2w + y) = 7w(3y + 6)$

P1 for expanding brackets and collecting the w terms in a correct equation, $6y = 21wy + 30w$

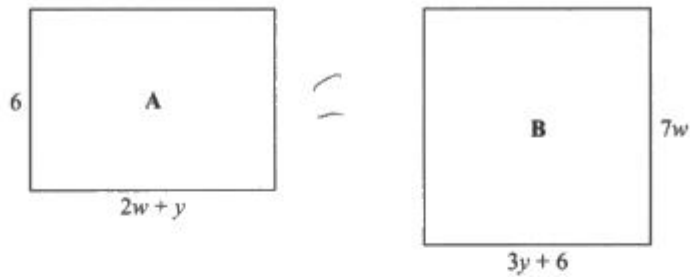
P0 there is no attempt to factorise (the two terms in y are dealt with incorrectly)

A0 incorrect final answer



Student Response C

11 The diagram shows two rectangles, A and B.



All measurements are in centimetres.

The area of rectangle A is equal to the area of rectangle B.

Find an expression for y in terms of w .

$$2w + y \quad \text{A}$$

$$12w + 6y$$

$$12w + 6y = 21wy + 42$$

$$21wy + 42$$

B

$$3y + 6 \quad 7w \quad 3y + 6$$

$$21wy + 42w$$

(Total for Question 11 is 4 marks)

1/4

Examiner Comments

P1 for forming an equation, $12w + 6y = 21wy + 42w$

P0 no attempt at gathering w terms or isolating terms in y following a correct expansion

P0 the terms in y are not isolated so it is not possible to factorise out the y

A0 no expression for y in terms of w



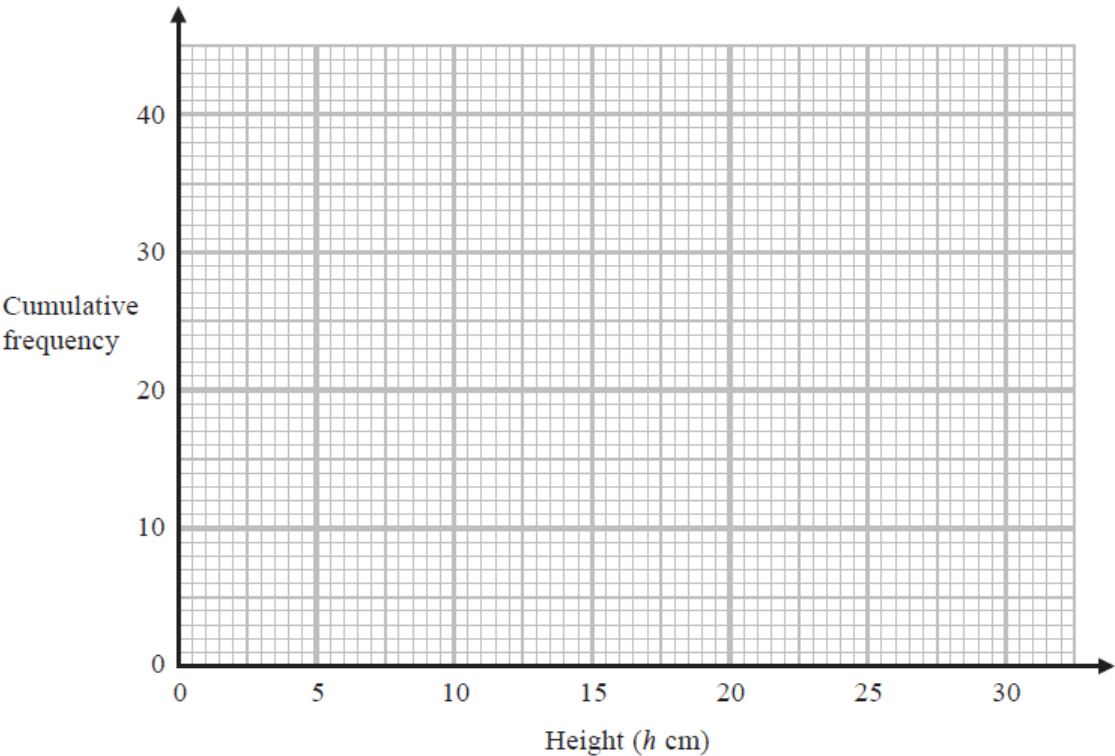
Exemplar Question 4

Higher tier Paper 1

12 The cumulative frequency table gives information about the heights, in cm, of 40 plants.

Height (h cm)	Cumulative Frequency
$0 < h \leq 5$	4
$0 < h \leq 10$	11
$0 < h \leq 15$	24
$0 < h \leq 20$	34
$0 < h \leq 25$	38
$0 < h \leq 30$	40

(a) On the grid, draw a cumulative frequency graph for this information.



(2)

(b) Use the graph to find an estimate for the median height of the plants.

..... cm
(1)

(Total for Question 12 is 3 marks)

**Mean Scores: Part (a) 1.12****Part (b) 0.29****Examiner Comments**

This familiar type of question assessed the ability of students to draw a cumulative frequency graph and to use the graph to find an estimate for the median height.

Many students plotted the points correctly and joined them with a curve or with line segments and gained both marks. A common mistake was not joining the points and some attempts at a line of best fit were seen. When graphs were drawn with the points plotted at the midpoints of the intervals one mark was awarded if the points were plotted at the correct heights and joined.

Some students plotted at the midpoints of the intervals in the cumulative frequency table and this resulted in graphs that were squashed into the region from $h = 2.5$ to $h = 15$. These graphs gained no marks. Histograms were often drawn and these gained no marks unless at least 5 of the 6 points were identified and plotted correctly.

In part (b) many students gained one mark for finding the median. Some errors were made when reading from the graph, most often with the scale on the horizontal axis. A surprising number of students used the graph incorrectly or made no attempt to use the graph.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
12 (a)	cf graph	M1	for 5 or 6 points plotted correctly	If histograms drawn, points must be identified
		A1	for a fully correct graph	Accept a smooth curve or line segments Ignore to the left of the first point and right of the last point
(b)	13 to 14	B1	SC B1 if 5 or 6 of their points plotted not at the end but consistent within each interval and joined by a curve or line segments providing no gradient is negative for answer in the range 13 to 14 or ft their cf graph	ft only from a cf graph

Examiner Comments

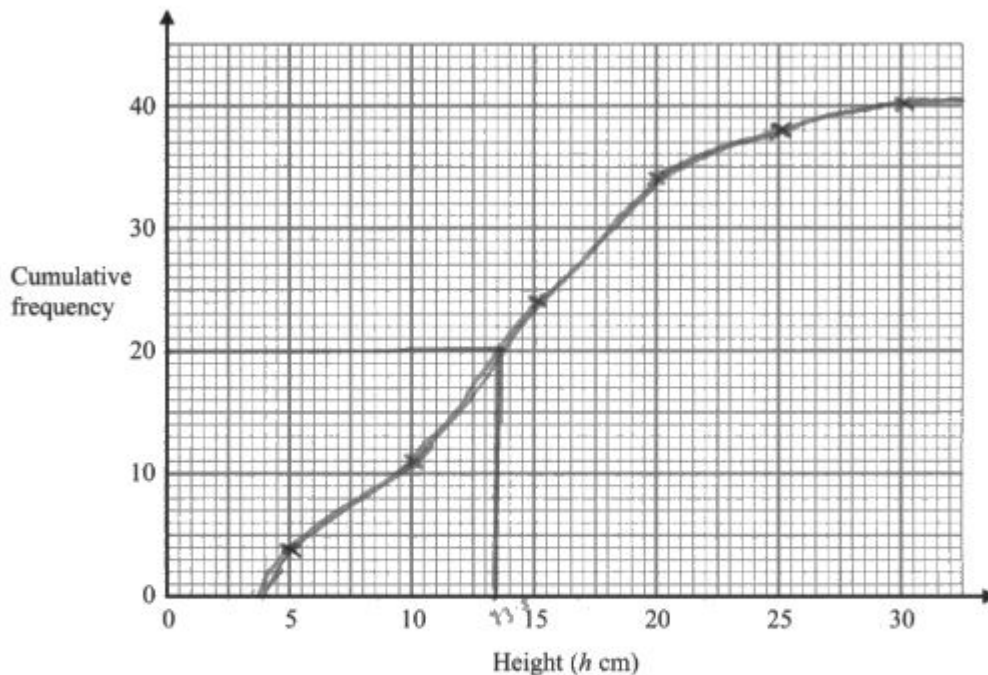
In part (a) students were able to gain M1 for plotting 5 or 6 points correctly. To gain the accuracy mark students needed to have drawn a fully correct graph. Those students who typically plotted at the mid-interval values rather than at the end points were catered for by the special case in the mark scheme.

In part (b) students were awarded B1 for an answer in the range 13 to 14 or for an answer that followed through correctly from their cumulative frequency graph.



Student Response A

(a) On the grid, draw a cumulative frequency graph for this information.



(2)

(b) Use the graph to find an estimate for the median height of the plants.

13.5 cm
(1)

(Total for Question 12 is 3 marks)

3/3

Examiner Comments

Part (a)

M1 all 6 points have been plotted correctly.

A1 was awarded for a fully correct graph. The parts of the graph to the left of the first point and to the right of the last point were ignored (additional guidance).

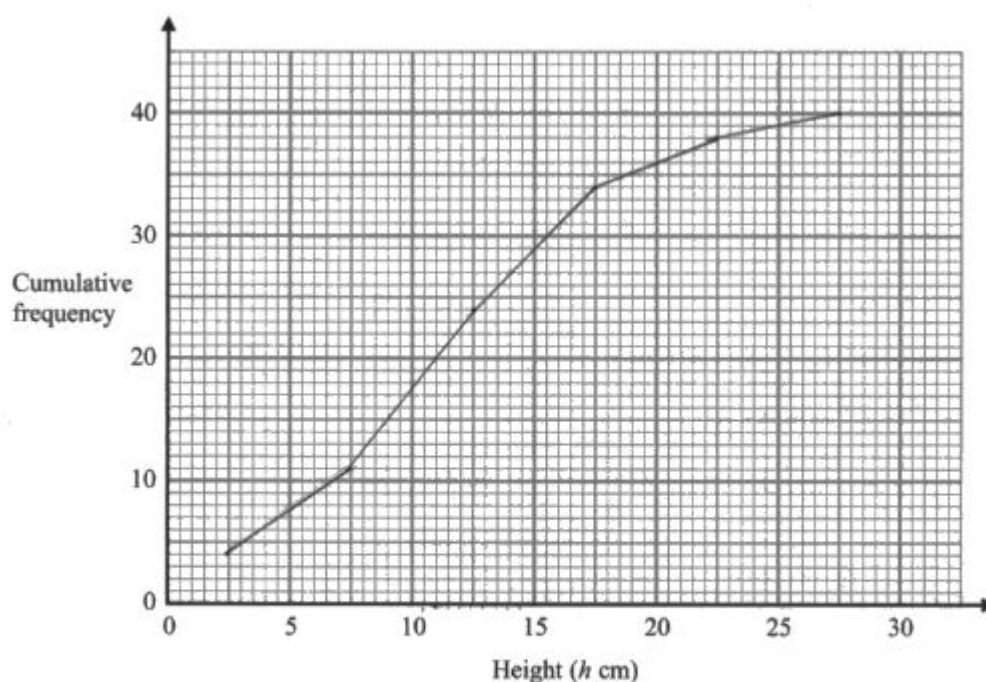
Part (b)

B1 was awarded as the answer is in the range 13 to 14



Student Response B

(a) On the grid, draw a cumulative frequency graph for this information.



(2)

(b) Use the graph to find an estimate for the median height of the plants.

20

11 26 12 cm
(1)

(Total for Question 12 is 3 marks)

2/3

Examiner Comments

Part (a)

Special Case B1 was awarded because at least 5 points have been plotted consistently (at the midpoints) within each interval, at the correct heights, and joined.

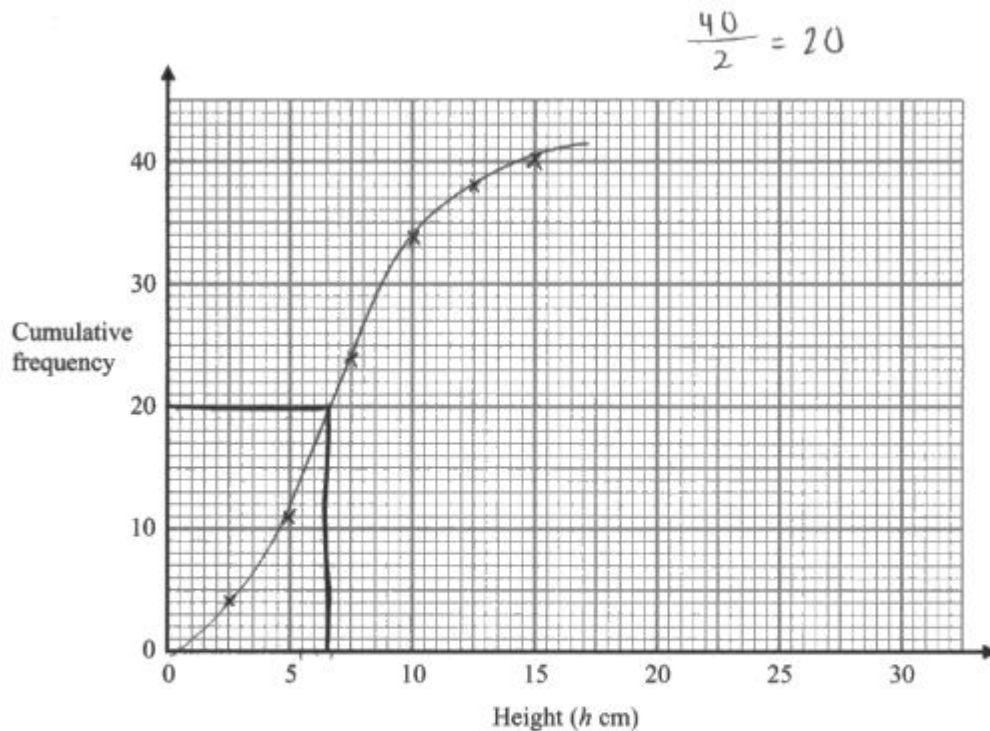
Part (b)

B1 was awarded as the answer follows though correctly from the cumulative frequency graph.



Student Response C

(a) On the grid, draw a cumulative frequency graph for this information.



(b) Use the graph to find an estimate for the median height of the plants.

8 cm
(1)

(Total for Question 12 is 3 marks)

0/3

Examiner Comments**Part (a)**

M0 most of the points have not been plotted in the correct intervals. Instead of plotting in the intervals 0-5, 5-10, etc the student used the midpoints of the intervals 0-5, 0-10, 0-15, etc.

A0 as M0 awarded

Part (b)

B0 was awarded as the answer is not in the range 13 to 14 and it does not follow though correctly from the cumulative frequency graph. The student misread the scale on the horizontal axis.



Exemplar Question 5

Higher tier Paper 1

- 13 Ted is trying to change $0.\dot{4}\dot{3}$ to a fraction.

Here is the start of his method.

$$\begin{aligned}x &= 0.\dot{4}\dot{3} \\10x &= 4.\dot{4}\dot{3} \\10x - x &= 4.\dot{4}\dot{3} - 0.\dot{4}\dot{3}\end{aligned}$$

Evaluate Ted's method so far.

.....

.....

.....

(Total for Question 13 is 1 mark)

Mean Score: 0.35

Examiner Comments

This question assessed a specific requirement to evaluate methods used. Students were expected to review Ted's method and explain what was wrong with it or why it wouldn't work. Explaining what Ted should have done instead was acceptable.

Successful students often explained that Ted should have used $100x$ and x or that he should have used $1000x$ and $10x$. Some students gave acceptable explanations that focused on the recurring numbers after the decimal points. Any numerical examples given as part of an explanation needed to be correct for the mark to be awarded. This was often not the case. Some answers, such as "he should have used $100x$ " or "he needs to find $1000x$ ", did not go quite far enough. Many answers were simply incorrect. A common misconception was that Ted had made a multiplication error because the value he found for $10x$ was wrong. A significant number of answers consisted of a commentary of what Ted was doing with no evaluation of his method.



Mark Scheme

Question	Answer	Mark	Mark scheme
13	Explanation	C1	explanation Acceptable examples he should have used $100(x)$ rather than $10(x)$ he should have used $1000x$ and $10x$ Ted's working does not eliminate the recurring decimals the recurring numbers after the decimal point have to be in the same sequence he should have multiplied by 100 to subtract easier after the decimal point he should have multiplied by 100 because two numbers are recurring Not acceptable examples it is not correct the method is not complete he should have used $1000x$ he should have multiplied by 100 he should have multiplied by 100 and then done $100x - 10x$ to give $43/90$

Examiner Comments

Explanations were expected to be clear and concise. Students who gave conflicting information did not score the mark.

Any numerical examples given as part of an explanation needed to be correct for the mark to be awarded.

Answers that consisted only of a correct numerical method showing that the recurring decimal could be written as the fraction $43/99$ were accepted.

Some exemplar statements were provided in the mark scheme. However, these are only examples.

Note that a C mark is a communication mark awarded for a fully correct statement with no contradiction or ambiguity.



Student Response A

13 Ted is trying to change $0.\dot{4}\dot{3}$ to a fraction.

Here is the start of his method.

$$x = 0.\dot{4}\dot{3}$$

$$10x = 4.\dot{3}\dot{4}$$

$$10x - x = 4.\dot{3}\dot{4} - 0.\dot{4}\dot{3}$$

Evaluate Ted's method so far.

He should've ~~been~~ multiplied $0.\dot{4}\dot{3}$ by a 100 then
he should've taken away x from $100x$, ~~then~~

(Total for Question 13 is 1 mark)

1/1

Examiner Comments

C1 was awarded for a correct explanation that is comparable to the first of the acceptable examples in the mark scheme. This student stated what Ted should have done.

Note that "he should have used $100x$ " is not sufficient on its own (see the not acceptable examples in the mark scheme).



Student Response B

13 Ted is trying to change $0.\dot{4}\dot{3}$ to a fraction.

Here is the start of his method.

$$x = 0.\dot{4}\dot{3}$$

$$10x = 4.\dot{3}\dot{4}$$

$$10x - x = 4.\dot{3}\dot{4} - 0.\dot{4}\dot{3}$$

Evaluate Ted's method so far.

His method is incorrect because both numbers are meant to end in 43 before subtracting.

(Total for Question 13 is 1 mark)

1/1

Examiner Comments

C1 was awarded for a correct explanation that is equivalent to the fourth of the acceptable responses in the mark scheme. This student identified what is wrong with Ted's method.



Student Response C

13 Ted is trying to change $0.\dot{4}\dot{3}$ to a fraction.

Here is the start of his method.

$$x = 0.\dot{4}\dot{3}$$

$$10x = 4.\dot{3}\dot{4}$$

$$10x - x = 4.\dot{3}\dot{4} - 0.\dot{4}\dot{3}$$

Evaluate Ted's method so far.

He needs to change to $100x$ and $40.\dot{3}\dot{4}$ so dividing will be easier.

(Total for Question 13 is 1 mark)

0/1

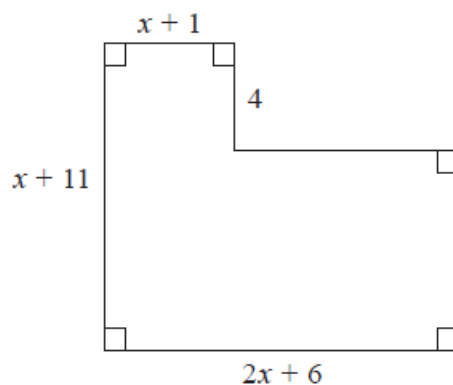
Examiner Comments

C0 was awarded. The explanation gets off to a good start with "needs to change to $100x$ ". However, the numerical value is incorrect and "dividing will be easier" is at best ambiguous so the explanation was not acceptable.

Exemplar Question 6

Higher tier Paper 1

- 14 Here is a shape with all its measurements in centimetres.



The area of the shape is $A \text{ cm}^2$

Show that $A = 2x^2 + 24x + 46$

(Total for Question 14 is 3 marks)

Mean Score: 1.43

Examiner Comments

This question assessed the ability of students to construct a chain of reasoning to achieve a given result. This was within the context of algebraic manipulation and geometry.

Students needed to write down algebraic statements to represent areas and were expected to set their working out in a formal manner, showing the steps needed to get to the required statement.

There were many fully correct answers with clear accurate algebra but there were also very many responses where algebraic weaknesses were apparent. Most of the students who indicated that they were going to find the area of the L-shape by splitting it up into two rectangles were able to score at least one mark for a correct expression for the area of one rectangle. A complete expression for the total area was needed for the second mark; most commonly this was given as $4(x + 1) + (x + 7)(2x + 6)$, with $(x + 1)(x + 11) + (x + 7)(x + 5)$ and $(x + 11)(2x + 6) - 4(x + 5)$ used less often. Incorrect expressions for the total area were often the result of working out missing lengths incorrectly. Many students could not handle the algebra accurately enough to show a complete chain of reasoning leading to $A = 2x^2 + 24x + 46$. Omitting necessary brackets and expanding brackets incorrectly were common errors.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
14	Shown	M1	for a start to the method, eg finds one correct area $4(x + 1)$ or $(x + 7)(2x + 6)$ or $(x + 1)(x + 11)$ or $(x + 7)(x + 5)$ or $4(x + 5)$ or $(x + 11)(2x + 6)$	
		M1	for a complete expression for the total area, eg $4(x + 1) + (x + 7)(2x + 6)$ or $4x + 4 + 2x^2 + 14x + 6x + 42$ OR $(x + 1)(x + 11) + (x + 7)(x + 5)$ or $x^2 + x + 11x + 11 + x^2 + 7x + 5x + 35$ OR $(x + 11)(2x + 6) - 4(x + 5)$ or $2x^2 + 22x + 6x + 66 - 4x - 20$	
		A1	for a complete chain of reasoning with fully correct algebra leading to $2x^2 + 24x + 46$	$2x^2 + 24x + 46$ is given so need to see brackets expanded correctly

Examiner Comments

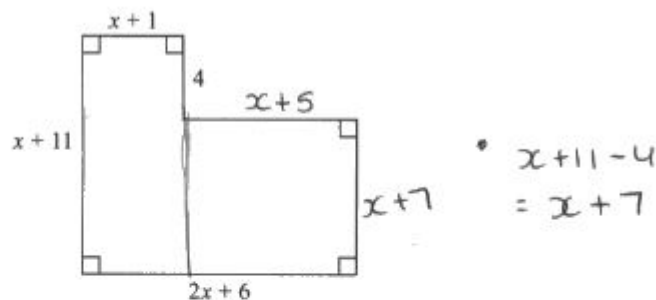
The first M1 was awarded for one correct area and the second M1 was for a complete expression for the total area. For these two marks the expressions did not have to be simplified but expressions without necessary brackets gained no credit.

The A mark was awarded for a complete chain of reasoning with fully correct algebra leading to $2x^2 + 24x + 46$. For this mark it was necessary to see the brackets expanded correctly since the result was given in the question.



Student Response A

14 Here is a shape with all its measurements in centimetres.



The area of the shape is $A \text{ cm}^2$

Show that $A = 2x^2 + 24x + 46$

• $2x+6 - x+1$
 $= x+5$

$$A = (x+1)(x+11) + (x+7)(x+5)$$

$$A = x^2 + 11x + x + 11 + x^2 + 5x + 7x + 35$$

$$A = 2x^2 + 24x + 46$$

3/3

Examiner Comments

M1 for one correct area, either $(x+1)(x+11)$ or $(x+7)(x+5)$ was sufficient for this mark.

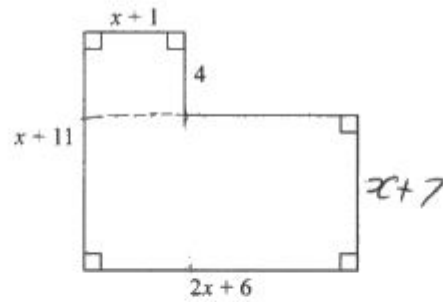
M1 for a complete expression for the total area $(x+1)(x+11) + (x+7)(x+5)$

A1 was awarded because a complete chain of reasoning with fully correct algebra leads to the required result.



Student Response B

14 Here is a shape with all its measurements in centimetres.



The area of the shape is $A \text{ cm}^2$

Show that $A = 2x^2 + 24x + 46$

$$\begin{aligned} & \cancel{2x+6} \times \cancel{x+7} = \cancel{2x^2} \\ & (2x+6)(x+7) = 2x^2 + 14x + 6x + 42 \\ & \quad = 2x^2 + 20x + 42 \\ & x+1 \times 4 = 4x + 4 \\ & 2x^2 + 20x + 42 + 4x + 4 = \\ & \quad 2x^2 + 24x + 46 \end{aligned}$$

2/3

Examiner Comments

M1 for one correct area, either $(2x+6)(x+7)$ or $4x+4$ was sufficient for this mark.

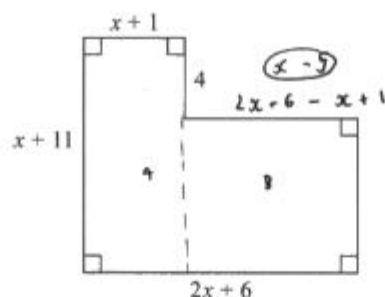
M1 for a correct expression for the total area $2x^2 + 20x + 42 + 4x + 4$

A0 an addition error means that the final expression is not fully correct.



Student Response C

14 Here is a shape with all its measurements in centimetres.



$$\begin{aligned} 2x+6 - x+1 \\ x-5 \\ x+5 \end{aligned}$$

The area of the shape is $A \text{ cm}^2$

Show that $A = 2x^2 + 24x + 46$

$$\begin{aligned} A &= (x+11)(x+1) \\ &= x^2 + x + 11x + 11 \\ &= x^2 + 12x + 11 \end{aligned}$$

$$\begin{aligned} B &= (2x+6)(x+5) \\ &= 2x^2 + 10x + 6x + 30 \\ &= 2x^2 + 16x + 30 \end{aligned}$$

$$\begin{aligned} (x+5)(x+5) \\ x^2 + 8x + 5x + 25 \\ x^2 + 10x + 25 \end{aligned}$$

$$\begin{aligned} (x+6)(x+6) \\ x^2 + 6x + 6x + 36 \\ x^2 + 12x + 36 \end{aligned}$$

$$\begin{aligned} x^2 + 10x + 25 \\ x^2 + 12x + 11 \\ \hline 2x^2 + 22x + 36 \end{aligned}$$

(Total for Question 14 is 3 marks)

$$\begin{aligned} x^2 + 12x + 36 \\ x^2 + 12x + 10 \\ \hline 2x^2 + 24x + 46 \end{aligned}$$

1/3

Examiner Comments

M1 for one correct area, $(x+11)(x+1)$

M0 was awarded because the expression for the total area is incorrect. The student used $(x+6)(x+6)$ instead of $(x+7)(x+5)$.

A0 the required result appears in the final line of working but it does not come from a correct chain of reasoning.



Exemplar Question 7

Higher tier Paper 1

- 15 Show that $\frac{4x+3}{2x} + \frac{3}{5}$ can be written in the form $\frac{ax+b}{cx}$ where a , b and c are integers.

(Total for Question 15 is 3 marks)

Mean Score: 1.15

Examiner Comments

This question assessed the ability of students to manipulate algebraic fractions. Students were expected to add the two fractions by using a common denominator.

Those students who used $10x$ as a common denominator often went on to score full marks. Exceptions to this were those who made errors in expanding brackets, for example $5(4x + 3) = 20x + 3$, and those who wrote $3/5$ as $6/10x$. Some students wrote only one of the fractions with a suitable denominator, often this was $(20x + 15)/10x$, and gained the first mark only. Many students did not know how to tackle this question.



Mark Scheme

Question	Answer	Mark	Mark scheme
15	$\frac{26x+15}{10x}$	M1	for method to write at least one of the fractions with a suitable denominator, eg $\frac{4x+3}{2x} \times \frac{5}{5} (= \frac{20x+15}{10x})$ or $\frac{3}{5} \times \frac{2x}{2x} (= \frac{6x}{10x})$
		M1	for method to combine the fractions, eg $\frac{5(4x+3)}{5 \times 2x} + \frac{3 \times 2x}{5 \times 2x}$ or $\frac{5(4x+3)+3 \times 2x}{5 \times 2x}$ or $\frac{20x+15}{10x} + \frac{6x}{10x}$
		A1	for correct algebra leading to $\frac{26x+15}{10x}$ or in form $\frac{ax+b}{cx}$

Examiner Comments

The first M1 was awarded for a method to write at least one of the fractions with a suitable denominator and the second M1 was awarded for a method to combine the fractions.

For the A mark it was necessary to see correct algebra leading to $\frac{26x+15}{10x}$ or to an equivalent expression in the form $\frac{ax+b}{cx}$, e.g. $\frac{52x+30}{20x}$. If $\frac{26x+15}{10x}$ was shown but then simplified incorrectly the A mark was lost as it is not appropriate to ignore subsequent work in an algebra question.



Student Response A

15 Show that $\frac{4x+3}{2x} + \frac{3}{5}$ can be written in the form $\frac{ax+b}{cx}$ where a , b and c are integers.

$$5(4x+3)$$

$$\frac{4x+3 \times 5}{2x \times 5} + \frac{3 \times 2x}{5 \times 2x}$$

$$\frac{20x+15}{10x} + \frac{6x}{10x} = \frac{26x+15}{10x}$$

3/3

Examiner Comments

M1 for a method to write one of the fractions with a suitable denominator

M1 for a method to combine the fractions

A1 was awarded as correct algebra leads to a correct final answer.



Student Response B

15 Show that $\frac{4x+3}{2x} + \frac{3}{5}$ can be written in the form $\frac{ax+b}{cx}$ where a , b and c are integers.

$$\frac{4x+3}{2x} + \frac{3}{5} = 0$$

$$5(4x+3)$$

$$45x+4$$

$$\frac{4x+3}{2x} + \frac{3}{5}$$

$$\frac{5(4x+3)}{10x} + \frac{6x}{10x}$$

$$\frac{20x+3}{10x} + \frac{6x}{10x} = \frac{26x+3}{10x}$$

2/3

Examiner Comments

M1 for either $\frac{5(4x+3)}{10x}$ or $\frac{6x}{10x}$

M1 for a method to combine the fractions, $\frac{5(4x+3)}{10x} + \frac{6x}{10x}$

A0 the final answer is incorrect because an error was made when expanding $5(4x+3)$



Student Response C

15 Show that $\frac{4x+3}{2x} + \frac{3}{5}$ can be written in the form $\frac{ax+b}{cx}$ where a , b and c are integers.

$$\begin{array}{l} \frac{4x+3}{2x} + \frac{3}{5} = \\ \times 5 \quad \times 2 \\ \frac{20x+15}{10x} + \frac{6}{10} = \frac{35x+6}{10x} \end{array}$$

$$\frac{35x+6}{10x}$$

1/3

Examiner Comments

M1 for a correct method to write the first fraction with a denominator of $10x$

M0 the method to combine the fractions is not correct because the second fraction was not written with a denominator of $10x$

A0 as the final answer is incorrect



Exemplar Question 8

Higher tier Paper 1

- 16 There are only 3 red counters and 5 yellow counters in a bag.

Jude takes at random 3 counters from the bag.

Work out the probability that he takes exactly one red counter.

.....
(Total for Question 16 is 4 marks)

Mean Score: 0.67

Examiner Comments

This question assessed the ability of students to calculate the probability of dependent combined events. Students were expected to identify the three ways that Jude could take exactly one red counter, RYY, YRY and YYR, and work out the probability of each of these happening.

Tree diagrams were very common and those students that attempted to draw a tree diagram often achieved the first mark for use of $\frac{3}{7}$, $\frac{4}{7}$ or $\frac{5}{7}$, showing understanding that the first counter was not replaced. Those who wrote at least one correct product gained the second mark but often these students were not able to go on and show a complete process. Sometimes this was because they did not consider all three ways that Jude could take exactly one red counter and sometimes it was because they used incorrect probabilities in their products. The majority of those that did show a complete process were able to complete the arithmetic and give a correct final answer.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
16	$\frac{180}{336}$	P1	for $\frac{3}{7}$ or $\frac{4}{7}$ or $\frac{5}{7}$ as probability for second counter	May be seen in a calculation or diagram
		P1	for one correct product eg $\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} (= \frac{60}{336})$ or $\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6}$ $(= \frac{60}{336})$ or $\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} (= \frac{60}{336})$	
		P1	for a complete process eg $\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$	
		A1	oe, eg $\frac{15}{28}$ SC B1 for answer of $\frac{225}{512}$ (replacement)	Accept equivalent fractions, decimals (0.53... or 0.54) or percentages (53% or 54%)

Examiner Comments

The first P1 was awarded when students showed understanding that the first counter was not replaced by writing $\frac{3}{7}$ or $\frac{4}{7}$ or $\frac{5}{7}$ as the probability for the second counter. Often this was seen on a probability tree diagram.

The second P1 was awarded for one correct product. A complete process, e.g. the addition of three correct products, was needed for the third P1.

The A mark was awarded for an answer of $\frac{180}{336}$ or equivalent. If an acceptable answer such as $\frac{180}{336}$ was seen, then any subsequent incorrect attempt at simplifying it was ignored.

Student Response A

16 There are only 3 red counters and 5 yellow counters in a bag.

Jude takes at random 3 counters from the bag.

Work out the probability that he takes exactly one red counter.

The student's work shows three probability tree diagrams for drawing 3 counters from a bag containing 3 red (r) and 5 yellow (y) counters.

- Tree 1 (Red first):**
 - First draw: $\frac{3}{8}$ (r)
 - Second draw: $\frac{2}{7}$ (r), $\frac{5}{7}$ (y)
 - Third draw: From $\frac{2}{7}$ (r), $\frac{1}{6}$ (r), $\frac{5}{6}$ (y); From $\frac{5}{7}$ (y), $\frac{2}{6}$ (r), $\frac{4}{6}$ (y)
- Tree 2 (Yellow first):**
 - First draw: $\frac{5}{8}$ (y)
 - Second draw: $\frac{4}{7}$ (y), $\frac{3}{7}$ (r)
 - Third draw: From $\frac{4}{7}$ (y), $\frac{3}{6}$ (y), $\frac{2}{6}$ (r); From $\frac{3}{7}$ (r), $\frac{2}{6}$ (r), $\frac{4}{6}$ (y)
- Tree 3 (Red first, different path):**
 - First draw: $\frac{3}{8}$ (r)
 - Second draw: $\frac{5}{7}$ (y), $\frac{2}{7}$ (r)
 - Third draw: From $\frac{5}{7}$ (y), $\frac{4}{6}$ (y), $\frac{2}{6}$ (r); From $\frac{2}{7}$ (r), $\frac{3}{6}$ (r), $\frac{3}{6}$ (y)

Calculations shown:

$$\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} = \frac{60}{336}$$

$$\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} = \frac{60}{336}$$

$$\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{60}{336}$$

$$= \frac{180}{336}$$

Final calculation:

$$\begin{array}{r} 56 \\ \times 6 \\ \hline 336 \end{array}$$

Small calculation at bottom right:

$$\begin{array}{r} 180 \\ \hline 336 \end{array}$$

(Total for Question 16 is 4 marks)

4/4

Examiner Comments

P1 for $\frac{3}{7}$ or $\frac{4}{7}$ or $\frac{5}{7}$ as probability for second counter, seen on diagram and in working

P1 for at least one correct product

P1 for a complete process, consisting of the addition of three correct products

A1 was awarded for the correct answer

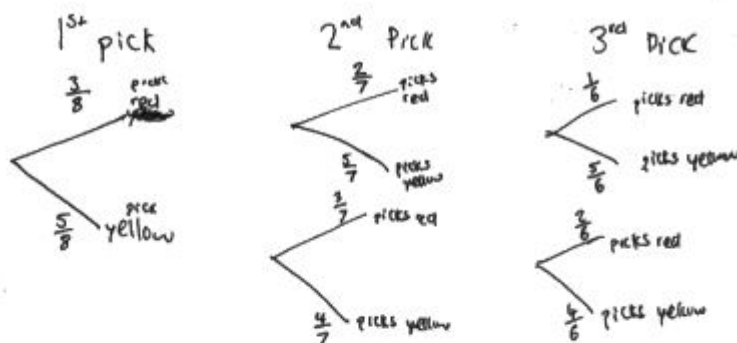


Student Response B

16 There are only 3 red counters and 5 yellow counters in a bag. ~~50~~ 1 in 20+41

Jude takes at random 3 counters from the bag. — doesn't take it back, now 7 counters

Work out the probability that he takes exactly one red counter.



$$P(RYY) = \frac{3}{8} \times \frac{5}{7} \times \frac{5}{6} = \frac{75}{336}$$

$$P(YRY) = \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} = \frac{60}{336}$$

$$P(YYR) = \frac{5}{8} \times \frac{4}{7} \times \frac{2}{6} = \frac{40}{336}$$

$$\text{Probability of exactly one} = \frac{75}{336} + \frac{60}{336} + \frac{40}{336} = \frac{175}{336}$$

$$\begin{array}{r} 48 \\ \times 7 \\ \hline 336 \\ \hline 280 \\ \hline 336 \end{array}$$

$$\begin{array}{r} 15 \\ \times 5 \\ \hline 25 \\ 50 \\ \hline 75 \end{array}$$

$$\begin{array}{r} 15 \\ \times 4 \\ \hline 20 \\ 60 \\ \hline 60 \end{array}$$

$$20 \times 2 = 40$$

$$\frac{175}{336}$$

(Total for Question 16 is 4 marks)

2/4

Examiner Comments

P1 for $\frac{3}{7}$ or $\frac{4}{7}$ or $\frac{5}{7}$ as probability for second counter, seen on diagram and in working

P1 one correct product seen

P0 was awarded as there is no complete process. Three products are added but only one of the products is correct.

A0 as the final answer is incorrect

Student Response C

16 There are only 3 red counters and 5 yellow counters in a bag.

Jude takes at random 3 counters from the bag.

Work out the probability that he takes exactly one red counter.

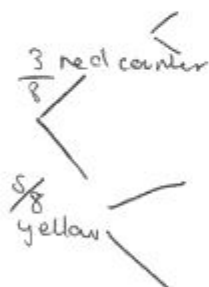


$$5 + 3 = 8$$

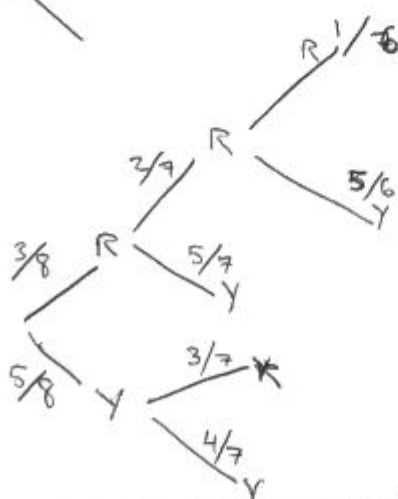
$$5Y + 3R = 8 \text{RY}$$

$$\frac{3}{8} = \text{chance of red counter}$$

$$\frac{5}{8} = \text{chance of yellow counter}$$



$$\frac{3}{8} \times 5 \quad \frac{5}{8} \times 5 \quad \frac{25}{40}$$



$$\frac{3}{8} + \frac{2}{7} + \frac{1}{6}$$

$$\frac{6}{21}$$

$$\frac{6 \div 3}{21 \div 3} = \frac{2}{7}$$

$$\frac{1}{7}$$

(Total for Question 16 is 4 marks)

1/4

Examiner Comments

P1 for 3/7 or 4/7 or 5/7 as probability for second counter (seen on diagram)

P0 no correct products seen

P0 no complete process

A0 as the final answer is incorrect

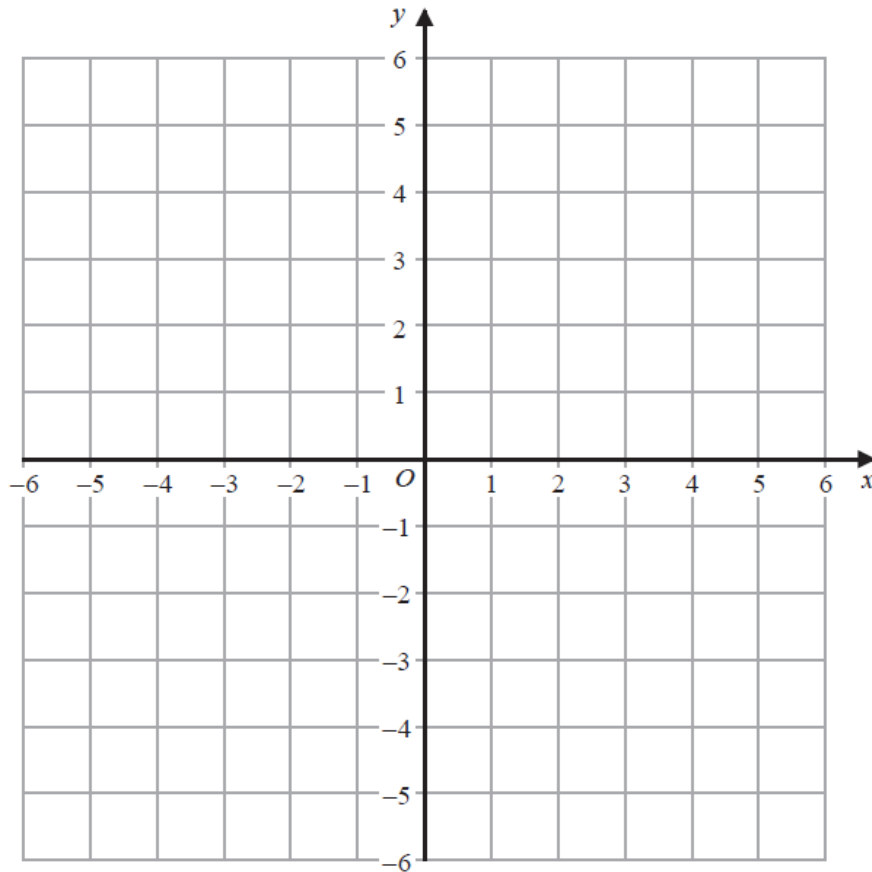
Exemplar Question 9

Higher tier Paper 1

- 17 On the grid show, by shading, the region that satisfies all of these inequalities.

$$2y + 4 < x \quad x < 3 \quad y < 6 - 3x$$

Label the region **R**.



(Total for Question 17 is 3 marks)

Mean Score: 0.37

Examiner Comments

This question assessed the ability of students to represent the solution set of several simultaneous inequalities on a coordinate grid. Students were expected to draw the three lines $2y + 4 = x$, $x = 3$, $y = 6 - 3x$ and then indicate, by shading, the region that satisfies the three inequalities $2y + 4 < x$, $x < 3$, $y < 6 - 3x$.

Many students gained no marks because they were not able to draw two of the three lines correctly. When only two of the three lines were drawn correctly it was most commonly the lines $x = 3$ and $y = 6 - 3x$. The graph of $2y + 4 = x$ caused the most problems. A common error was to draw this line with the correct gradient but the wrong y -axis intercept. Some students confused the line $x = 3$ with the line $y = 3$. After drawing all three lines correctly many students identified the wrong region – often this was the region enclosed by the three lines. Students should check whether points in the region they choose satisfy all three inequalities.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
17	Region shaded	M1	for two of the lines $2y + 4 = x$, $x = 3$, $y = 6 - 3x$ correctly drawn	Accept full or broken lines for all marks
		M1	for all three correct lines correctly drawn	
		A1	for a fully correct region indicated with all lines correct	Award for clear intention, shading not needed Diagram at end of mark scheme

Examiner Comments

Two lines needed to be correct for the award of the first M1. The second M1 was awarded for drawing the three lines $2y + 4 = x$, $x = 3$, $y = 6 - 3x$ correctly. To gain these marks, the lines did not have to be of any particular minimum length, though students are advised to draw lines which extend over the whole grid. It was not expected that students would draw broken lines to illustrate the strict inequality signs. Full or broken lines were accepted for all marks.

To score the A mark, students needed to have drawn three correct lines and identified the correct region. Some students shaded the region but did not label it. This was acceptable. Any clear way of indicating the region was accepted.

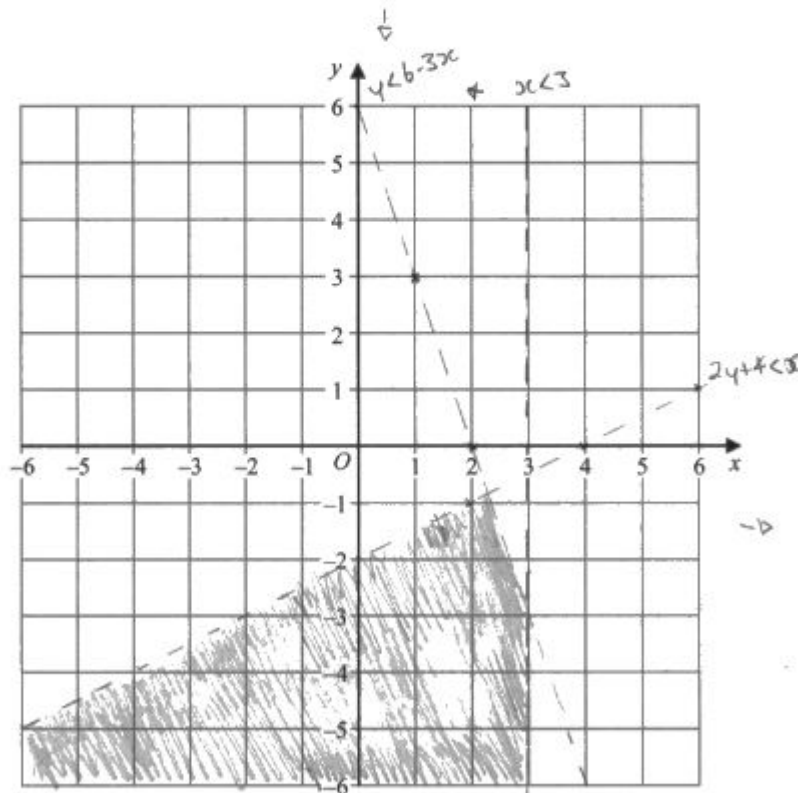


Student Response A

17 On the grid show, by shading, the region that satisfies all of these inequalities.

$$2y + 4 < x \quad x < 3 \quad y < 6 - 3x$$

Label the region **R**.



(Total for Question 17 is 3 marks)

3/3

Examiner Comments

M1 for two lines drawn correctly

M1 for all three lines drawn correctly

A1 was awarded as all three lines are correct and the correct region is indicated by shading.



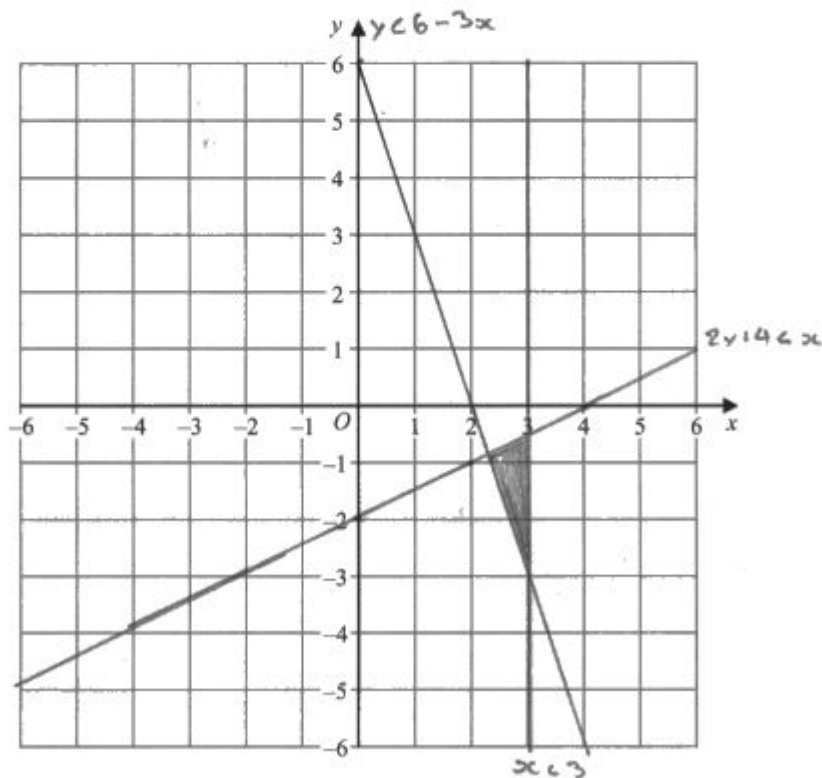
Student Response B

17 On the grid show, by shading, the region that satisfies all of these inequalities.

Label the region R.

$$\begin{array}{lll} 2y + 4 < x & x < 3 & y < 6 - 3x \\ 2y < x - 4 & x < 3 & y < 6 - 3x \\ y < \frac{1}{2}x - 2 & & \end{array}$$

$$\begin{array}{l} 0.5 - 2 \\ -2.5 \end{array}$$



(Total for Question 17 is 3 marks)

2/3

Examiner Comments

M1 for two lines drawn correctly

M1 for all three lines drawn correctly

A0 was awarded as an incorrect region has been shaded. Shading the enclosed region was a common error.



Student Response C

17 On the grid show, by shading, the region that satisfies all of these inequalities.

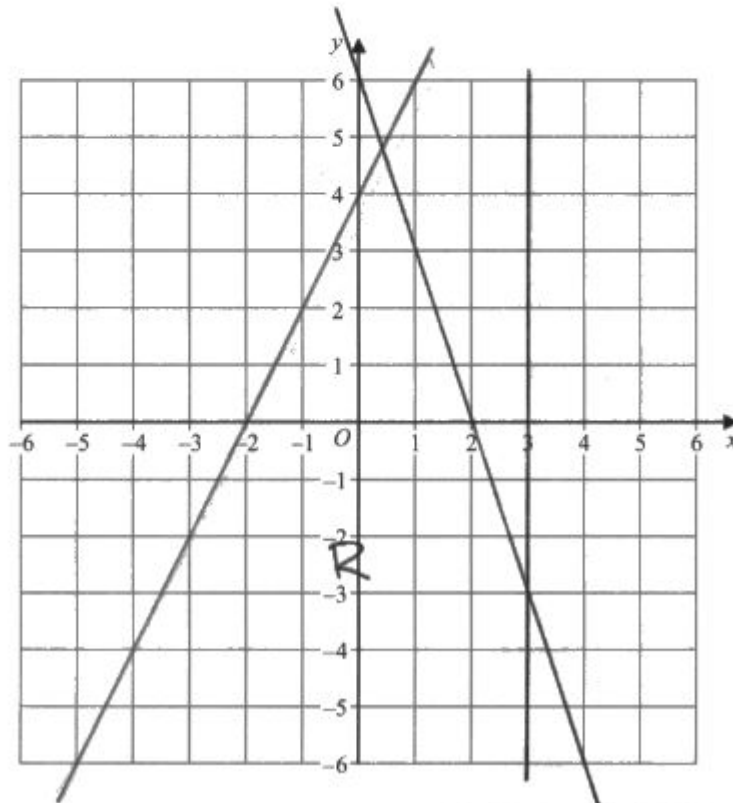
$$2y + 4 < x$$

$$x < 3$$

$$y < 6 - 3x$$

Label the region **R**.

$$y = mx + c$$



(Total for Question 17 is 3 marks)

1/3

Examiner Comments

M1 for two lines drawn correctly, $x = 3$ and $y = 6 - 3x$.

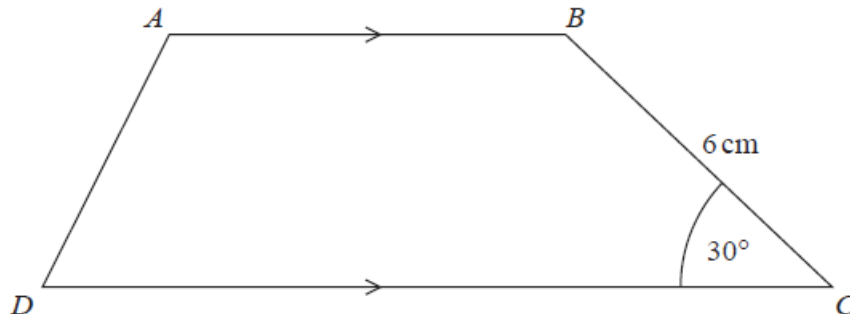
M0 as not all three lines have been drawn correctly.

A0 as the region is not fully correct.

Exemplar Question 10

Higher tier Paper 1

18 Here is trapezium $ABCD$.



The area of the trapezium is 66 cm^2

the length of AB : the length of $CD = 2 : 3$

Find the length of AB .

..... cm

(Total for Question 18 is 5 marks)

Mean Score: 0.61

Examiner Comments

This question assessed the ability of students to solve a problem by breaking it down into a series of mathematical processes. This was within the context of geometry and ratio. Students were expected to start by using trigonometry and their knowledge of exact trigonometric values to find the perpendicular height of the trapezium and then use the formula for the area of a trapezium and the given ratio in order to find the length of AB .

Some of those who attempted the question identified that they need to find the perpendicular height of the trapezium and gained the first mark for writing a correct trigonometric statement such as $\sin 30 = h/6$. Some students could not recall the value of $\sin 30$ and did not complete the process to find the height. After finding the height many students struggled to make any further progress, failing to realise that they should use the formula for the area of a trapezium. It was common to see the trapezium split into two triangles and a rectangle and attempts at using Pythagoras. A good proportion of those that did substitute correctly into the formula went on to show a complete process to find the length of AB . A small number of students gave $88/5$ as the final answer and lost the accuracy mark.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18	17.6	P1	for correct trig statement, eg $\sin 30 = \frac{h}{6}$	
		P1	for complete process to find h , eg $6 \times \frac{1}{2} (= 3)$	
		P1	for correct substitution into the area of a trapezium formula, eg $\frac{1}{2}(a+b) \times "3" = 66$ or $a+b = 44$ or $\frac{1}{2}(2x+3x) \times h = 66$	
		P1	for complete correct process to find the length of AB , eg $\left[\frac{66 \times 2}{3} \div (2 + "3") \right] \times 2$	
		A1	cao	An answer of $\frac{88}{5}$ gets P4 A0

Examiner Comments

The first P1 was awarded for starting the process of finding the perpendicular height of the trapezium by writing a correct trig statement. Completing the process to find the height gained the second P1. If 3 cm was shown marked on the diagram as the perpendicular height then the first two marks could be awarded.

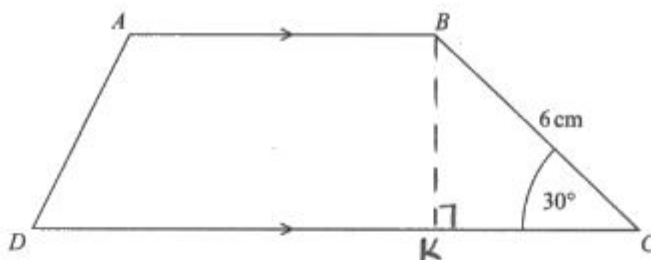
The third P1 was awarded for a correct substitution, e.g. $h = "3"$, into the formula for the area of a trapezium. Showing an understanding of ratio by substituting $2x + 3x$ for $a + b$ also gained this mark. Hence it was possible for students to get the third P1 even if one or both of the first two marks had not been awarded.

A complete process to find the length of AB was needed for the fourth P1.



Student Response A

18 Here is trapezium $ABCD$.



The area of the trapezium is 66 cm^2

the length of AB : the length of $CD = 2:3$

Find the length of AB .

Add a line $BK \perp DC$,

Let the length of AB be $2x \text{ cm}$,

$$\sin 30^\circ = \frac{BK}{6}$$
$$BK = 6 \left(\frac{1}{2} \right) = 3 \text{ cm}$$

$$\frac{(2x + 3x)(3)}{2} = 66$$

$$3(5x) = 132$$

$$5x = 44$$

$$x = \frac{44}{5}$$

$$x = 8.8$$

$\therefore$ The length of AB is

$$8.8 \times 2$$

$$= 17.6 \text{ cm}$$

$$\frac{66}{132}$$

$$\begin{array}{r} 0.88 \\ 5 \overline{) 44.0} \\ \underline{40} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\frac{8.8}{5} = 1.76$$

$$17.6 \text{ cm}$$

(Total for Question 18 is 5 marks)

5/5

Examiner Comments

P1 for a correct trig statement, $\sin 30 = BK/6$

P1 for a complete process to find $h = 3$. Note that sight of 3cm in the correct place on the diagram was enough for the first two marks.

P1 for a correct substitution into the area of a trapezium formula, $\frac{(2x + 3x)(3)}{2} = 66$

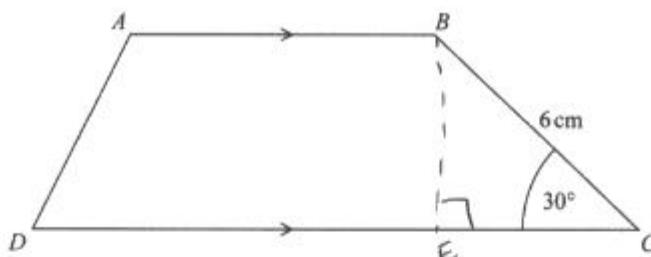
P1 the process was completed by dividing 44 in the ratio 2:3 to find the required length

A1 correct final answer



Student Response B

18 Here is trapezium $ABCD$.



The area of the trapezium is 66 cm^2

the length of AB : the length of $CD = 2:3$

Find the length of AB .

Area = 66 cm^2 .

$AB:CD$
 $2:3$

$S = \frac{O}{H}$
 $\frac{1}{2} = \frac{O}{6}$
 $6 = 3$
 $n = 3$

$h(AB+CD)$
 2

$66 = \frac{h(AB+CD)}{2}$

$132 = h(AB+CD)$

$132 = 3(AB+CD)$

$44 = AB+CD$

$16 + 24 = 40$

16 cm

(Total for Question 18 is 5 marks)

3/5

Examiner Comments

P1 for a correct trig statement $1/2 = O/6$

P1 for a complete process to find $h = 3$

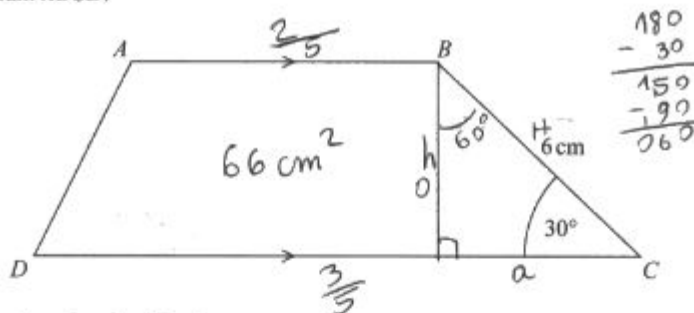
P1 for a correct substitution into the area of a trapezium formula, leading to $132 = 3(AB + CD)$ followed by $44 = AB + CD$

P0 was awarded because the process to find the length of AB is incomplete. The student worked with ratios equivalent to $2:3$ but did not reach 44

A0 the final answer is incorrect



Student Response C

18 Here is trapezium $ABCD$.The area of the trapezium is 66 cm^2 the length of AB : the length of $CD = 2:3 \rightarrow \textcircled{5}$ Find the length of AB .

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{1}{2} ([AB] + [DC]) h = 66 \text{ cm}^2$$

$$\frac{6}{\sin(90)} = \frac{h}{\sin(30)}$$

$$\frac{6}{1} = \frac{h}{\frac{1}{2}}$$

cm

(Total for Question 18 is 5 marks)

1/5

Examiner Comments

P1 for a correct trig statement $6/\sin 90 = h/\sin 30$ **P0** no complete process to find h **P0** the area of a trapezium formula was used but to award this mark we would have needed to have seen h replaced by "3" or $AB + DC$ replaced by $2x + 3x$ **P0** follows from previous P0**A0** no final answer was given

Exemplar Question 11

Higher tier Paper 1

- 19 Show that $\frac{8+\sqrt{12}}{5+\sqrt{3}}$ can be written in the form $\frac{a+\sqrt{3}}{b}$, where a and b are integers.

(Total for Question 19 is 4 marks)

Mean Score: 0.82

Examiner Comments

This question assessed the ability of students to manipulate surds and to construct a chain of reasoning to show that an expression could be written in a particular form. Students needed to be able to simplify surd expressions involving squares and rationalise denominators.

Writing $\sqrt{12}$ as $2\sqrt{3}$ was sometimes seen as the first step and sometimes it was seen later in the process. Some of the students who started by writing $\sqrt{12}$ as $2\sqrt{3}$ and simplified the numerator of the fraction to $8+2\sqrt{3}$ made no further progress. Those students who realised that they needed to rationalise the denominator were not always able to show a correct method to do so. Attempts at multiplying both the numerator and the denominator by $5+\sqrt{3}$ or by $\sqrt{3}$ were often seen and gained no credit. Those who did appreciate the need to multiply numerator and denominator by $5-\sqrt{3}$ often went on to expand the terms correctly although some students made sign errors. Having got as far as $\frac{34+2\sqrt{3}}{22}$ some students gave this as the final answer and did not score the final mark because the question asked for an answer in the form $\frac{a+\sqrt{3}}{b}$.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19	Result shown	M1	(indep) for writing $\sqrt{12}$ as $2\sqrt{3}$	This mark can be awarded whenever this is seen, which might be later in the process.
		M1	for method to rationalise the denominator eg $\frac{8+\sqrt{12}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$ or $\frac{8+2\sqrt{3}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$ oe	
		M1	(dep on previous M1) for expanding terms, condone one error in numerator or denominator eg $\frac{40-8\sqrt{3}+5\sqrt{12}-\sqrt{12}\sqrt{3}}{25-5\sqrt{3}+5\sqrt{3}-\sqrt{3}\sqrt{3}}$ or $\frac{40-8\sqrt{3}+10\sqrt{3}-2\sqrt{3}\sqrt{3}}{25-5\sqrt{3}+5\sqrt{3}-\sqrt{3}\sqrt{3}}$ or $\frac{34+2\sqrt{3}}{22}$ oe	
		A1	for a complete chain of reasoning leading to $\frac{17+\sqrt{3}}{11}$	

Examiner Comments

The first M1 was awarded for writing $\sqrt{12}$ as $2\sqrt{3}$. This did not have to be the first step – it could appear later in the working and it could be implied by seeing $5\sqrt{12}$ written as $10\sqrt{3}$.

The third M1 for expanding terms was dependent on a correct method to rationalise the denominator. For this mark, one error in expanding the numerator or denominator was condoned.

The A mark was awarded for a complete chain of reasoning leading to $\frac{17+\sqrt{3}}{11}$. An answer of $\frac{34+2\sqrt{3}}{22}$ did not get the final mark because it is not in the required form.



Student Response A

19 Show that $\frac{8 + \sqrt{12}}{5 + \sqrt{3}}$ can be written in the form $\frac{a + \sqrt{3}}{b}$, where a and b are integers.

$$\begin{aligned}\frac{8 + \sqrt{12}}{5 + \sqrt{3}} &= \frac{(8 + \sqrt{12})(5 - \sqrt{3})}{(5 + \sqrt{3})(5 - \sqrt{3})} = \frac{40 - 8\sqrt{3} + 5\sqrt{12} - 6}{25 - 5\sqrt{3} + 5\sqrt{3} - 3} \\&= \frac{40 - 8\sqrt{3} + 10\sqrt{3} - 6}{22} = \frac{34 + 2\sqrt{3}}{22} = \frac{17 + \sqrt{3}}{11} \\&\quad a = 17 \\&\quad b = 11\end{aligned}$$

4/4

Examiner Comments

M1 was awarded for writing $\sqrt{12}$ as $2\sqrt{3}$. After the brackets have been expanded $5\sqrt{12}$ is written as $10\sqrt{3}$.

M1 for a method to rationalise the denominator

M1 was awarded as the terms have been expanded correctly

A1 for a complete chain of reasoning leading to $\frac{17 + \sqrt{3}}{11}$

Student Response B

19 Show that $\frac{8+\sqrt{12}}{5+\sqrt{3}}$ can be written in the form $\frac{a+\sqrt{3}}{b}$, where a and b are integers. $\frac{3}{4}$

$$\begin{aligned} & \frac{8+\sqrt{12}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}} \\ &= \frac{(8+\sqrt{12})(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} \\ &= \frac{40 - 8\sqrt{3} + 5\sqrt{12} - \sqrt{36}}{25 - 3} \\ &= \frac{40 - 8\sqrt{3} + 5\sqrt{4}\sqrt{3} - 6}{22} \\ &= \frac{34 - 8\sqrt{3} + 10\sqrt{3}}{22} \\ &= \frac{2\sqrt{3} + 34}{22} \end{aligned}$$

$$\begin{aligned} & (5+\sqrt{3})(5-\sqrt{3}) \\ &= 25 - 3 \\ &= 22 \end{aligned}$$

$$\begin{aligned} & \frac{8+\sqrt{12}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}} \\ &= \frac{2\sqrt{3} + 34}{22} \\ &= \frac{17+\sqrt{3}}{11} \end{aligned}$$

$$\begin{aligned} a &= 17 \\ b &= 11 \end{aligned}$$

(Total for Question 19 is 4 marks)

3/4

Examiner Comments

M1 was awarded for writing $\sqrt{12}$ as $2\sqrt{3}$. After the brackets have been expanded $5\sqrt{12}$ is written as $5\sqrt{4}\sqrt{3}$ and then as $10\sqrt{3}$.

M1 for a correct method to rationalise the denominator

M1 was awarded for expanding the terms. The one error (+3 instead of -3 in the denominator) was condoned for this mark.

A0 as the chain of reasoning did not lead to $\frac{17+\sqrt{3}}{11}$

Student Response C

19 Show that $\frac{8 + \sqrt{12}}{5 + \sqrt{3}}$ can be written in the form $\frac{a + \sqrt{3}}{b}$, where a and b are integers.

$$\begin{array}{l} \text{8+12} \\ \frac{8 + \sqrt{12}}{5 + \sqrt{3}} \end{array}$$

$$\begin{array}{l} \sqrt{12} \\ 4 \end{array}$$

$$\textcircled{2} (8 + 2\sqrt{3})(5 + \sqrt{3})$$

$$\frac{8 + 2\sqrt{3}}{5 + \sqrt{3}}$$

$$40 + 8\sqrt{3} + 10\sqrt{3} + 6$$

$$46 + 18\sqrt{3}$$

$$\textcircled{1} (5 + \sqrt{3})(5 - \sqrt{3})$$

$$25 - 5\sqrt{3} + 5\sqrt{3} - 3$$

$$22$$

$$\frac{46 + 18\sqrt{3}}{22}$$

(Total for Question 19 is 4 marks)

1/4

Examiner Comments

M1 for writing $\sqrt{12}$ as $2\sqrt{3}$

M0 was awarded as the method to rationalise the denominator is not correct. The numerator was multiplied by $5 + \sqrt{3}$ and the denominator was multiplied by $5 - \sqrt{3}$.

M0 dependent on the previous mark

A0 no chain of reasoning leading to $\frac{17 + \sqrt{3}}{11}$



Paper 2H (calculator)

Exemplar Question 1

Higher tier Paper 2

- 7 Liquid **A** has a density of 1.8 g/cm^3
Liquid **B** has a density of 1.2 g/cm^3

80 cm³ of liquid **A** is mixed with 40 cm³ of liquid **B** to make 120 cm³ of liquid **C**.

Work out the density of liquid **C**.

..... g/cm³

(Total for Question 7 is 3 marks)

Mean Score: 1.38

Examiner Comments

This question is looking to assess students understanding of the relationship between mass, volume and density. Candidates needed to recall the formula, and use it to find both mass and density. To gain full marks they needed to understand that the mass of **C** was equal to the total mass of **A** and **B** combined.



Mark Scheme

Question	Answer	Mark	Mark scheme
7	1.6	P1	for $1.8 \times 80 (= 144)$ or $1.2 \times 40 (= 48)$ or for 192 or for $80 : 40 = 2 : 1$
		P1	for $(“144” + “48”) \div (80 + 40)$ or $192 \div 120$ or for $(1.8 \times 2 + 1.2) \div 3$ or $4.8 \div 3$
		A1	oe

Mark Scheme

Examiner Comments

The first process mark was for finding either the mass of **A** or **B**, or the total mass. It is worth noticing on this question, just like on many others, that the values for the mass, eg 144 and 48, are in brackets. This indicates that they need not be seen to award the mark, and likewise they may be calculated incorrectly, providing the process is seen and is correct. Candidates could also gain this mark if they started to work with the ratios of the volumes.

The second process is to divide the total mass by the total volume. This time it is key to notice the inverted commas over the values 144 and 48. This tells us that although these values need not be correct, they must have come from a correct calculation. This mark could also be awarded if a candidate applied the ratio of the volumes to the densities and then divided by 3.

The final mark is an accuracy mark is could only be awarded for 1.6 or an equivalent value, such as $\frac{8}{5}$



Student Response A

- 7 Liquid A has a density of 1.8 g/cm^3
Liquid B has a density of 1.2 g/cm^3

$$D = \frac{m}{V}$$

80 cm^3 of liquid A is mixed with 40 cm^3 of liquid B to make 120 cm^3 of liquid C.

Work out the density of liquid C.

$$D = \frac{192}{120} = 1.6$$

	A	B	C
D	1.8	1.2	? 1.6
M	144	48	192
V	80	40	120

$$144 + 48 = 192$$

$$D = \frac{m}{V}$$

$$m = D \times V$$

$$1.8 \times 80 = 144$$

$$1.2 \times 40 = 48$$

1.6

g/cm³

(Total for Question 7 is 3 marks)

3/3

Examiner Comments

P1 Here we could award the first mark for either the 144 or the 48 in the table, or for the processes seen to find these values, or for the 192 in the table.

P1 The second mark is awarded for the $\frac{192}{120}$ seen at the top of the working space

A1 Correct answer stated

It is worth noting that on this question the correct answer given will gain full marks providing it has not come from incorrect working.



Student Response B

- 7 Liquid A has a density of 1.8 g/cm^3
Liquid B has a density of 1.2 g/cm^3

80 cm^3 of liquid A is mixed with 40 cm^3 of liquid B to make 120 cm^3 of liquid C.

Work out the density of liquid C.



1.8

$$\text{Liquid A} = \text{Mass} = \text{density} \times \text{volume} \quad 1.8 \times 80 = 144 \text{ g}$$
$$\text{Liquid B} = \text{Mass} = \text{density} \times \text{volume} = 1.2 \times 40 = 48 \text{ g}$$
$$\text{density} = \frac{\text{Mass}}{\text{volume}} \quad 144 + 48 = 192$$

$$\frac{192}{120} = 1.58\bar{3}$$

$$1.58\bar{3} \text{ g/cm}^3$$

(Total for Question 7 is 3 marks)

2/3

Examiner Comments

P1 for $1.8 \times 80 (=144)$ or for $1.2 \times 40 (=48)$ or for 192

P1 for $\frac{192}{120}$

A0 answer is incorrect



Student Response C

- 7 Liquid A has a density of 1.8 g/cm^3
Liquid B has a density of 1.2 g/cm^3

80 cm^3 of liquid A is mixed with 40 cm^3 of liquid B to make 120 cm^3 of liquid C.

Work out the density of liquid C.

$$80 \text{ cm}^3 = 80 \times 1.8 = 144 \text{ g}$$

$$40 \text{ cm}^3 = 40 \times 1.2 = 48 \text{ g}$$

$$144 + 48 = 192$$

$$192 \div 120 = 1.6$$

$$2.92 \text{ g/cm}^3$$

(Total for Question 7 is 3 marks)

1/3

Examiner Comments

P1 for $1.8 \times 80 (=144)$ or for $1.2 \times 40 (=48)$ or for 192

P0 the candidate appears to have got confused and tried to work with percentages, rather than dividing by 120

A0 the answer is incorrect



Exemplar Question 2

Higher tier Paper 2

- 8 The grouped frequency table gives information about the time, in minutes, taken by 50 people to solve a puzzle.

Time (t minutes)	Frequency
$0 < t \leq 10$	5
$10 < t \leq 20$	8
$20 < t \leq 30$	12
$30 < t \leq 40$	15
$40 < t \leq 50$	7
$50 < t \leq 60$	3

Brian was asked to draw a cumulative frequency table for this information.

This is the table that Brian drew.

Time (t minutes)	Cumulative frequency
$0 < t \leq 10$	5
$10 < t \leq 20$	13
$20 < t \leq 30$	25
$30 < t \leq 40$	40
$40 < t \leq 50$	47
$50 < t \leq 60$	50

Write down **one** thing that is wrong with this cumulative frequency table.

.....

.....

(Total for Question 8 is 1 mark)

Mean Score: 0.19

**Examiner Comments**

This question is testing if students genuinely understand the meaning of ‘cumulative frequency’ and that the inequalities are different in the cumulative frequency table compared to the frequency.

Mark Scheme

Question	Answer	Mark	Mark scheme
8	Error in inequalities	C1	for identifying incorrect inequalities Acceptable examples gives at least one correct inequality eg $(10 < t \leq 20)$ should be $0 < t \leq 20$ it should be $t \leq 20$ (all) inequalities should start with 0 should start with 0 Not acceptable examples $10 < t \leq 20$ is wrong the numbers have been added wrong

Examiner Comments

To award the C1 in this question examiners were looking for candidates to recognise that the lower bound of the inequality was wrong, without doing that candidates could not gain any credit.



Student Response A

Time (t minutes)	Cumulative frequency
$0 < t \leq 10$	5
$10 < t \leq 20$	13
$20 < t \leq 30$	25
$30 < t \leq 40$	40
$40 < t \leq 50$	47
$50 < t \leq 60$	50

Write down **one** thing that is wrong with this cumulative frequency table.

The number ~~on the left~~ on the left should be zero for all of them. i.e.

$0 < t \leq 20$, $0 < t \leq 40$, $0 < t \leq 40$, $0 < t \leq 50$
 $0 < t \leq 60$.

(Total for Question 8 is 1 mark)

1/1

Examiner Comments

C1 the candidate has clearly identified the error in the cumulative frequency table.



Student Response B

Brian was asked to draw a cumulative frequency table for this information.

This is the table that Brian drew.

Time (t minutes)	Cumulative frequency
$0 < t \leq 10$	5
$10 < t \leq 20$	13
$20 < t \leq 30$	25
$30 < t \leq 40$	40
$40 < t \leq 50$	47
$50 < t \leq 60$	50

+8 ✓
+12 ✓
+15 ✓
+7 ✓
✓

Write down **one** thing that is wrong with this cumulative frequency table.

→ This should all be
 $10 < t \leq 20, 10 < t \leq 30$ etc
because if you have to make
all equal 10 is $< t$ to more than
(Total for Question 8 is 1 mark)
and equal to 20.

0/1

Examiner Comments

C0 Here the candidate has realised that the lower bound is wrong, but then have stated it should be 10 not 0, and hence do not get the mark



Student Response C

Brian was asked to draw a cumulative frequency table for this information.

This is the table that Brian drew.

Time (t minutes)	Cumulative frequency
$0 < t \leq 10$	5
$10 < t \leq 20$	13
$20 < t \leq 30$	25
$30 < t \leq 40$	40
$40 < t \leq 50$	47
$50 < t \leq 60$	50

Write down **one** thing that is wrong with this cumulative frequency table.

incorrect signs on time

(Total for Question 8 is 1 mark)

0/1

Examiner Comments

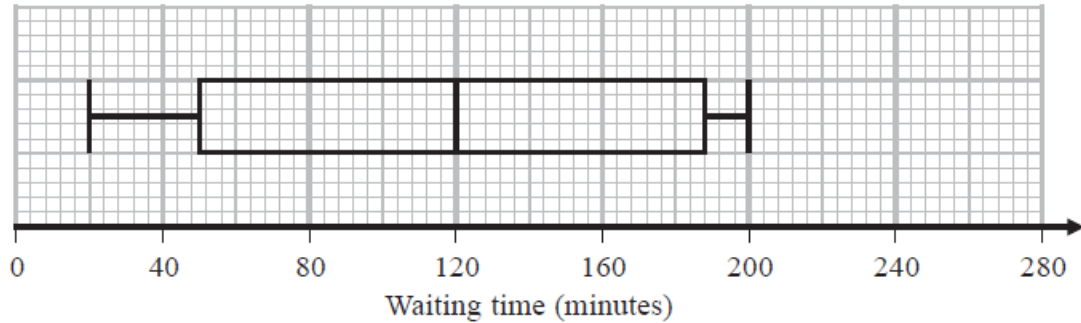
C0 clearly an incorrect reason and so no marks are given.



Exemplar Question 3

Higher tier Paper 2

- 9 The box plot shows information about the length of time, in minutes, some people waited to see a doctor at a hospital on Monday.



- (a) Work out the interquartile range of the information in the box plot.

S..... minutes

(2)

Becky says,

“50% of the people waited for at least 2 hours.”

- (b) Is Becky correct?

Explain why.

.....

.....

.....

(1)



The table gives information about the length of time, in minutes, some people waited to see a doctor at the same hospital on Tuesday.

	Waiting time (minutes)
Shortest time	20
Lower quartile	50
Median	100
Upper quartile	140
Longest time	210

Becky was asked to compare the distribution of the lengths of times people waited on Monday with the distribution of the lengths of times people waited on Tuesday. She wrote,

“People had to wait longer on Tuesday than on Monday.”

(c) Give **one** reason why Becky may be wrong.

.....

.....

.....

(1)

(Total for Question 9 is 4 marks)

Mean Scores: Part (a) 0.96

Part (b) 0.38

Part (c) 0.48

Examiner Comments

This question is assessing students understanding of box plots and not just their ability to draw or use them.

Part (a) focusses on finding the IQR from the completed box plot, testing if students understand this term and how to use the box plot to find the value.

Part (b) is testing whether candidates understand the fundamental of box plots and the proportion of the distribution that each section of the graph represents.

The final part is assessing whether students understand comparisons of box plots and can assess the validity of a response.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance																								
9 (a)	138	M1	for upper quartile = 188 or lower quartile = 50 or an indication that they are trying UQ – LQ	Could be written on the grid																								
		A1	cao																									
(b)	Yes, with reason	C1	Yes, with reason Acceptable examples Yes, because the median is at 2 hour (120 min) Yes, since 50% is at the 2 hour mark Yes, because the middle is at 2 hours Not acceptable examples No The median is at the 2 hour mark Yes, because 50% is exactly half way between “188” and “50”																									
(c)	statement	C1	Acceptable examples The median is lower on Tuesday (higher on Monday) The upper quartile is lower on Tuesday (higher on Monday) There may just have been one person waiting for 210 mins We don’t know how many people were waiting for each time Not acceptable examples The range is bigger for Tuesday (smaller for Monday) The IQR is smaller for Tuesday (bigger for Monday)	<table><tr><td></td><td>M</td><td>T</td></tr><tr><td>Shortest time</td><td>20</td><td>20</td></tr><tr><td>Lower quartile</td><td>50</td><td>50</td></tr><tr><td>Median</td><td>120</td><td>100</td></tr><tr><td>Upper quartile</td><td>188</td><td>140</td></tr><tr><td>Longest time</td><td>200</td><td>210</td></tr><tr><td>Range</td><td>180</td><td>190</td></tr><tr><td>IQR</td><td>138</td><td>90</td></tr></table>		M	T	Shortest time	20	20	Lower quartile	50	50	Median	120	100	Upper quartile	188	140	Longest time	200	210	Range	180	190	IQR	138	90
	M	T																										
Shortest time	20	20																										
Lower quartile	50	50																										
Median	120	100																										
Upper quartile	188	140																										
Longest time	200	210																										
Range	180	190																										
IQR	138	90																										

Examiner Comments

Part (a) carried 2 marks, the first mark was for a correct value found for either the UQ or the LQ. If both these were wrong, candidates could gain a mark if it was clear that they were reading from the right places on the diagram and then subtracting these values. The second mark was for a value of 138 only.

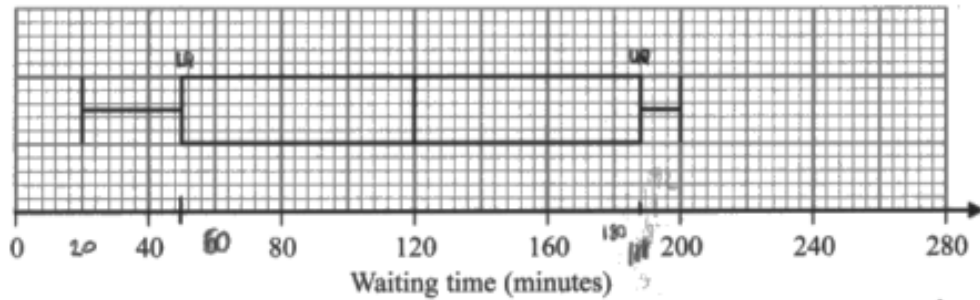
Part (b) carried a single C mark for recognising that the median was at 2 hours and therefore half the population came below that value. It was key that candidates identified the median or middle in their response, **and** 2 hours.

The final part again carried a single C mark for a response that related to an aspect of the distributions that would indicate why Becky may be wrong. Most focused on the mean values on both days, but equally it could be given for a response relating to not knowing how many people were at each time, or realising that 210 may just be an outlier for Tuesday.



Student Response A

- 9 The box plot shows information about the length of time, in minutes, some people waited to see a doctor at a hospital on Monday.



- (a) Work out the interquartile range of the information in the box plot.

$$\begin{aligned} \text{interquartile range} &= \text{UQ} - \text{LQ} \\ &= 188 - 50 \\ &= 138 \end{aligned}$$

$$180 \mid 184 \mid 188 \mid 192 \mid 196 \mid 200$$

$$138 \text{ minutes} \quad (2)$$

Becky says,

"50% of the people waited for at least 2 hours."

- (b) Is Becky correct?

Explain why.

Becky is correct as the median is at 120 min (≈ 2 hours).

(1)

- (c) Give one reason why Becky may be wrong.

We don't know exactly how many people waited there might have been less people.

(1)

(Total for Question 9 is 4 marks)

4/4

Examiner Comments

Part (a)

M1 for UQ = 188 or LQ = 50

A1 correct answer

Part (b)

C1 correct answer linking the median to 120 minutes.

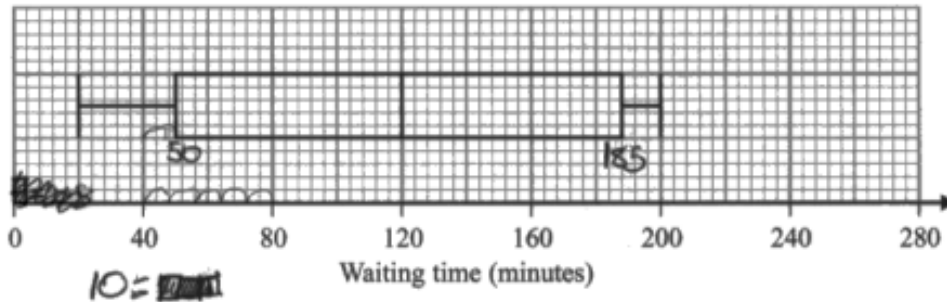
Part (c)

C1 a correct response relating to not knowing how many people are at each time.



Student Response B

- 9 The box plot shows information about the length of time, in minutes, some people waited to see a doctor at a hospital on Monday.



- (a) Work out the interquartile range of the information in the box plot.

$$185 - 50 = 135$$

135 minutes
(2)

Becky says,

"50% of the people waited for at least 2 hours."

- (b) Is Becky correct?
Explain why.

Yes, the ~~median~~ ^{median} is 120 minutes

(1)

- (c) Give one reason why Becky may be wrong.

The ~~area~~ ^{median} is lower

(1)

(Total for Question 9 is 4 marks)

2/4

Examiner Comments

Part (a)

M1 for a lower quartile of 50, either seen in the calculation or on the diagram
A0 because they have read the UQ incorrectly they have got the wrong answer

Part (b)

C1 correct answer linking the median to 120 minutes.

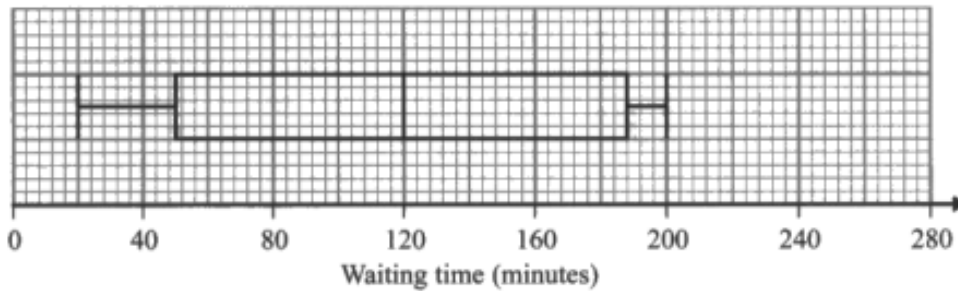
Part (c)

C0 This answer is incomplete as it doesn't say which day the median is lower on, and so cannot gain the mark.



Student Response C

- 9 The box plot shows information about the length of time, in minutes, some people waited to see a doctor at a hospital on Monday.



- (a) Work out the interquartile range of the information in the box plot.

$$185 - 50 = 135$$

135 minutes
(2)

Becky says,

"50% of the people waited for at least 2 hours."

- (b) Is Becky correct?

Explain why.

NO BECAUSE THE QUESTION SAYS ~~AND~~ SOME PEOPLE IT DOESN'T SAY 50% OF THE PEOPLE WAITED
(1)

- (c) Give one reason why Becky may be wrong.

BECAUSE THERE'S NO DIFFERENCE BOTH TABLES SHOWS ~~EXACTLY~~ SIMILAR TIME WAITING TIME
(1)

(Total for Question 9 is 4 marks)

1/4

Examiner Comments

Part (a)

M1 for a lower quartile of 50

A0 because they have read the UQ incorrectly they have got the wrong answer

Part (b)

C0 Any response starting with no would score C0

Part (c)

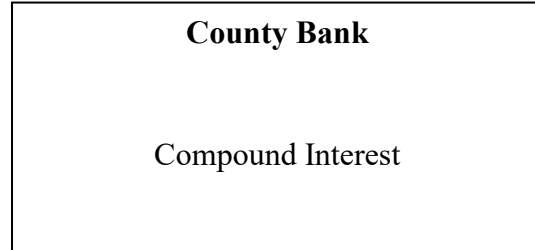
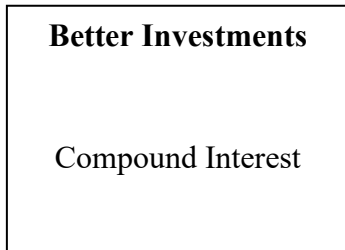
C0 the candidate has not identified a characteristic that would indicate that Becky is wrong.



Exemplar Question 4

Higher tier Paper 2

- 10 Louise invests £x in Better Investments for 3 years.
Sadiq invests £x in County Bank for 3 years.



At the end of the 3 years, the value of Louise's investment is £344 605

Work out the value of Sadiq's investment at the end of the 3 years.

£.....

(Total for Question 10 is 4 marks)

Mean Score: 1.41

Examiner Comments

This is a different compound interest question compared to what we often see. Candidates need to fully understand the processes of compound interest to first work backwards to find the initial investment, in this case dividing by 1.025^3 , rather than the usual question where they multiply to find the end value.

Once the initial investment had been found candidates had to then find the value of Sadiq's investment, using 2 different interest rates.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
10	344 580.48	P1	for a start to the process to find the initial investment eg $344\,605 \div 1.025$ oe (= 336 200) or for 1.025^3 (= 1.07689....)	
		P1	for complete process to find original investment, eg $344\,605 \div 1.025^3$ oe (= 319 078 to 320 265)	
		P1	for [initial investment] $\times 1.02^2 \times 1.035$ oe	[initial investment] must be clearly what they believe to be that and cannot be 344605
		A1	for answer in the range 343 587 to 344 581	

Examiner Comments

The first process mark could be awarded for either starting to work backward by dividing by 1.025, or for finding the multiplier for the full three years. This was often awarded together with the second mark which required a complete process to find the initial investment.

The third mark could be awarded even if P0P0 had been awarded for the first two. This was providing the candidate applied the correct process to what they believed to be the initial investment. This can be seen by the use of the [] brackets and the note in additional guidance. This allowed candidates who could use compound interest to find the value of an investment, to gain some credit even if they were unable to work backward to find the initial investment.

The A mark included a range to allow candidates who did some premature rounding to still get full marks.



Student Response A

Work out the value of Sadiq's investment at the end of the 3 years.

$$I = x \times 1.025^3 = 344605$$

$$x = 320000$$

$$S = 320000 \times 1.02^2 = 332928$$

$$332928 \times 1.035 = 344580.48$$

£ 344580.48

(Total for Question 10 is 4 marks)

4/4

Examiner Comments

P1 for 1.025^3

P1 for an initial investment of 320 000 found

P1 for a correct process to find the value of Sadiq's investment. Here it is seen in 2 stages, but could be seen in one or even 3 or more.

A1 for an answer in range



Student Response B

Work out the value of Sadiq's investment at the end of the 3 years.

	<u>Better hv</u>		<u>County Bank</u>	
100% + 2.5% = 1.025	$x \times 1.025^3 = £344605$		~~322~~	100% + 2% = 1.02 100% + 3.5% = 1.035
	$x = \frac{£344605}{1.025^3}$		$x \times 1.02^2 \times 1.035 =$	
	$x = 320000$			
	$320000 \times 1.025^3 = 344605$			

£

(Total for Question 10 is 4 marks)

3/4

Examiner Comments

P1 for 1.025^3

P1 for an initial investment of 320 000 found

P1 for a correct process to find the value of Sadiq's investment.

A0 no answer stated



Student Response C

Work out the value of Sadiq's investment at the end of the 3 years.

~~3446x~~

~~£344605 x 0.75~~

$$£ 344605 \times 0.975 = 335989.875$$

$$£ 335989.875 \times 0.975 = 327590.1281$$

$$£ 327590.1281 \times 0.975 = \underline{\underline{319900.37}}$$

$$£ 319,900.37 \times 1.02 = 325788.3774$$

$$£ 325788.38 \times 1.02 = 332304.15$$

$$£ 332304.15 \times 1.035 = \underline{\underline{£ 343,934.79}}$$

£ 343,934.79

(Total for Question 10 is 4 marks)

1/4

Examiner Comments

P0 / P0 incorrect process to find the initial investment. Candidate is using a multiplier of 0.975 rather than 1.025

P1 correct process to find the value of Sadiq's investment using their initial investment

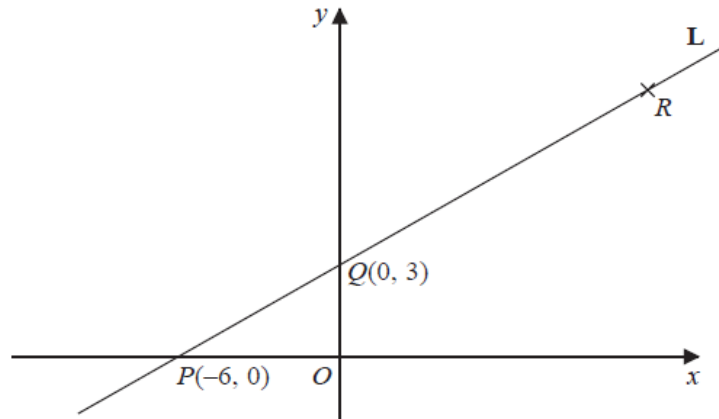
A0 incorrect answer



Exemplar Question 5

Higher tier Paper 2

- 11 Here is a sketch of the line **L**.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line **L**.

The point R is such that PQR is a straight line and $PQ : QR = 2 : 3$

- (a) Find the coordinates of R .

(....., ...)
(2)

- (b) Find an equation of the line that is perpendicular to **L** and passes through Q .

.....
(3)

(Total for Question 11 is 5 marks)

Mean Scores: Part (a) 0.38

Part (b) 0.68

Examiner Comments

This question is assessing coordinate geometry in two ways. In part (a) the focus of the question is to deal with a ratio of distances between points, and using that ratio, along with the given coordinates, to find a third.

Part (b) is a relatively familiar style of question looking for the equation of a perpendicular line. This time we are not given the equation of the initial line, **L**, and so have to find that gradient using the coordinates on the diagram first.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
11 (a)	(9, 7.5)	M1	for x coordinate = $PO(6) \times \frac{3}{2}$ (=9) or y coordinate = $OQ(3) \times \frac{5}{2}$ (=7.5) or $PO(6) \times \frac{5}{2}$ (=15) or $OQ(3) \times \frac{3}{2}$ (=4.5)	
		A1	cao	
(b)	$y = -2x + 3$	P1	for process to find the gradient of the line, eg $3 \div 6$ (=0.5) or $y = mx + 3$	Could use P and R or Q and R as ft from (a)
		P1	for process to find gradient of perpendicular eg $-1 \div [\text{gradient of } PQ]$ (= -2)	
		A1	for $y = -2x + 3$ oe	

Examiner Comments

Part (a) is looking for candidates to use the given ratio. To gain the M1 candidates needed to use a scale factor of $\frac{3}{2}$ or $\frac{5}{2}$ along with either the horizontal or vertical distance between P and Q . The A mark is for the correct coordinate found.

In part (b) the first mark is awarded for a correct method to find the gradient of L , and could be done using the coordinates of P , Q or even R . If using R and incorrect coordinate found in part (a) could be followed through for both this mark and the next. We could also award this mark if the candidate gave an answer in the form $y = mx + 3$

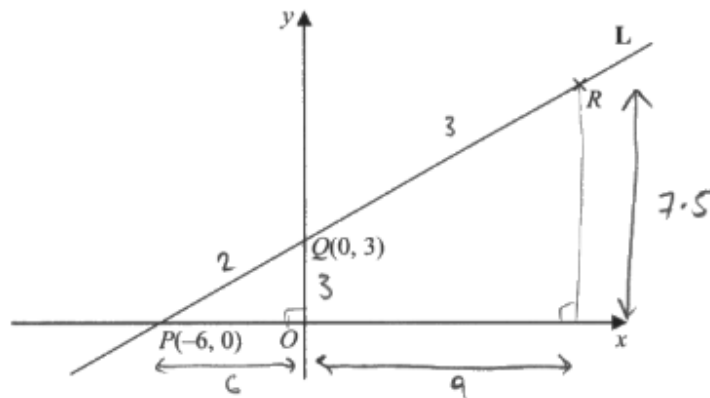
The second P1 is awarded for a process to find the perpendicular gradient. Notice again, the use of [] brackets. This tells us that the gradient of L need not be correct, must be clearly what the candidate believes to be the gradient of L . The mark is for the process.

The final mark is for a correct equation. It must be a full equation, eg include ' $y =$ ' but could be an equivalent one to the one given in the scheme, eg $y = \frac{-x}{0.5} + 3$



Student Response A

11 Here is a sketch of the line L.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line L.

The point R is such that PQR is a straight line and $PQ:QR = 2:3$

(a) Find the coordinates of R.

~~length of PQ = 5~~
~~length of QR = 7.5~~
~~length of PR = 12.5~~
~~length of PQ = 5~~
~~length of QR = 7.5~~
~~length of PR = 12.5~~

$$SF = \frac{5}{2}$$

$$6 \times \frac{5}{2} = 15$$

$$3 \times \frac{5}{2} = 7.5$$

$$\left(\frac{9}{2}, \frac{7.5}{2} \right)$$

(b) Find an equation of the line that is perpendicular to L and passes through Q.

$$m_L = \frac{3}{6} = \frac{1}{2}$$

m of perpendicular line = -2

$$y\text{-intercept} = 3$$

$$y = -2x + 3$$

(3)

(Total for Question 11 is 5 marks)

5/5

Examiner Comments

Part (a)

M1 for $6 \times \frac{5}{2}$ or for $3 \times \frac{5}{2}$

A1 correct answer given

Part (b)

P1 correct process to find gradient of L

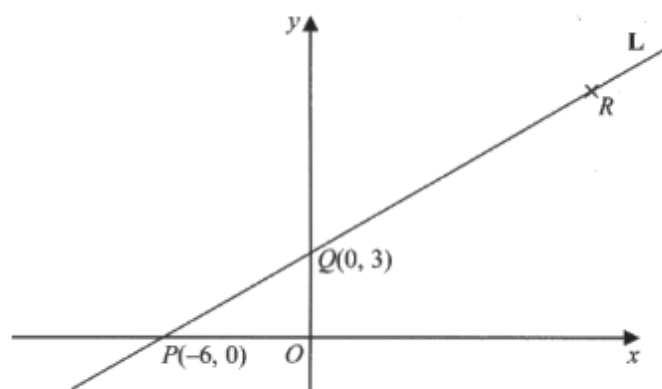
P1 correct gradient of perpendicular stated (we can therefore imply the correct process)

A1 a correct equation given.



Student Response B

11 Here is a sketch of the line L.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line L.

The point R is such that PQR is a straight line and $PQ:QR = 2:3$

(a) Find the coordinates of R.

$$PQ = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$\frac{3}{2} \times \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 4.5 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} -6 \\ 0 \end{pmatrix} + \begin{pmatrix} 4.5 \\ 9 \end{pmatrix} = \begin{pmatrix} -1.5 \\ 9 \end{pmatrix} \quad (2)$$

(b) Find an equation of the line that is perpendicular to L and passes through Q.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{0 - (-6)} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{1}{2} \times m_2 = -1 \Rightarrow m_2 = -2$$

$$y = 2x + c$$

$$3 = 2(0) + c$$

$$3 = c$$

$$y = 2x + 3$$

$$y = 2x + 3 \quad (3)$$

(Total for Question 11 is 5 marks)

3/5

Examiner Comments

Part (a)

M1 for $\frac{3}{2} \times 3$ or for $\frac{3}{2} \times 6$

A0 answer is incorrect

Part (b)

P1 correct process to find the gradient of L

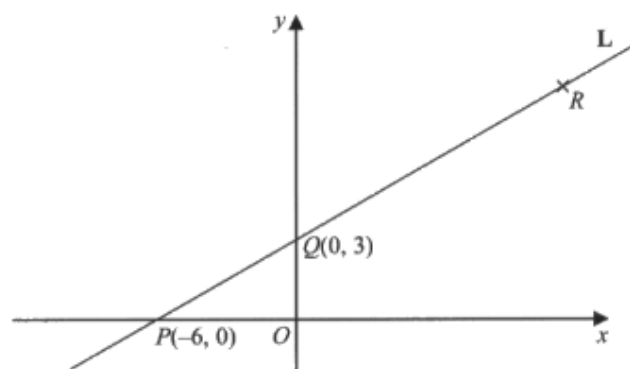
P1 for [gradient of PQ] $\times$ $M_2 = -1$

A0 answer is incorrect



Student Response C

11 Here is a sketch of the line L.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line L.

The point R is such that PQR is a straight line and $PQ:QR = 2:3$

(a) Find the coordinates of R.

$$\frac{0 - (-6)}{-6} = \frac{1}{2}$$

$$\frac{1.5}{3} \quad x = 9$$

$$y = 3 + 1.5 = 4.5$$

$$(9, 4.5)$$

(b) Find an equation of the line that is perpendicular to L and passes through Q.

$$y = mx + 3$$

$$y = \frac{1}{2}x + 3$$

$$y = \frac{1}{2}x + 3$$

(Total for Question 11 is 5 marks)

2/5

Examiner Comments

Part (a)

M1 for a correct x coordinate, or for getting as far as 4.5 for y

A0 Answer is not fully correct

Part (b)

P1 for an equation in the form $y = mx + 3$. Note, we cannot award for the gradient of $\frac{1}{2}$ as this is given as the gradient of the perpendicular, not the gradient of L

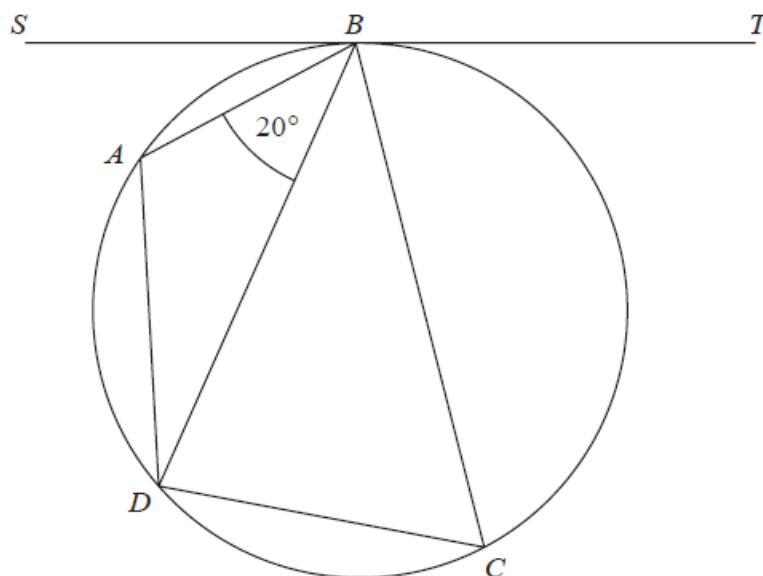
P0 No proves to find a perpendicular gradient seen

A0 Equation is incorrect.

Exemplar Question 6

Higher tier Paper 2

14



A , B , C and D are four points on a circle.

SBT is a tangent to the circle.

Angle $ABD = 20^\circ$

the size of angle BAD : the size of angle $BCD = 3 : 1$

Find the size of angle SBA .

Give a reason for each stage of your working.

.....^o
(Total for Question 14 is 4 marks)

Mean Score: 0.43

Examiner Comments

This problem is testing two different circle theorems, but also the application of ratios. Candidates need to know that ‘opposite angles in a cyclic quadrilateral sum to 180° ’ and combine this with a ratio to find BAD and BCD . Candidates then have a choice whether to use the triangle ABD and the ‘alternate segment theorem’, or to jump straight from angle BCD to angle SBD using the alternate segment theorem.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
14	25 with reasons	M1	for method to find angle BCD eg $180 \div (3 + 1) (=45)$ or $BAD = 180 \div (3 + 1) \times 3 (=135)$	Could be shown on the diagram or in working
		M1	for method to find angle BDA eg $180 - 20 - (180 - "45") (=25)$ or method to find angle SBD eg $SBD = BCD (=45)$	Do not award if it ambiguous as to which angle is being found
		C2	for finding $SBA (=25)$ and both reasons given, eg <u>Opposite angles</u> of a <u>cyclic quadrilateral</u> add up to 180 for angle $SBD = 45$ because <u>alternate segment</u> theorem	
		(C1	(dep M1) for one reason given <u>Opposite angles</u> of a <u>cyclic quadrilateral</u> add up to 180 for angle $SBD = 45$ because <u>alternate segment</u> theorem)	Underlined words need to be shown; reasons need to be linked to their method

Examiner Comments

The first method mark is for using the ratio and ‘opposite angles in a cyclic quadrilateral sum to 180° ’ to find either BAD or BCD . The second method mark is then to use BAD along with angles in a triangle to find the value of BDA or to link SBD and BCD .

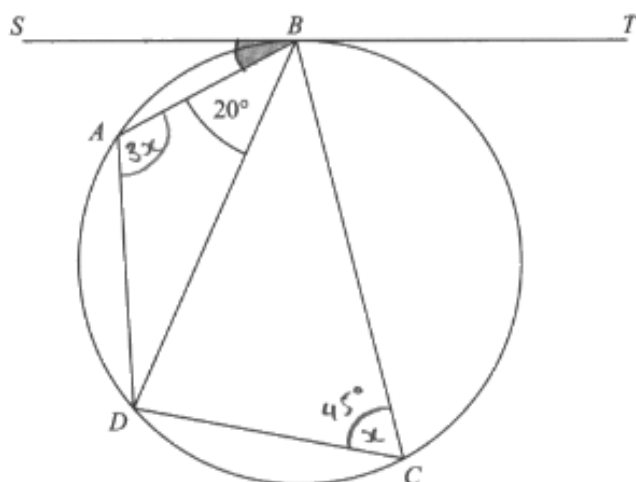
There are then two communication marks. To gain both, candidates must complete the problem to find the value of SBA and provide both circle theorem reasons.

If they were unable to do this, they could gain one of the C marks (providing they have scored at least M1) for one of the correct circle theorems stated.



Student Response A

14



A, B, C and D are four points on a circle.
 SBT is a tangent to the circle.
 Angle $ABD = 20^\circ$

the size of angle BAD : the size of angle $BCD = 3:1$

Find the size of angle SBA .

Give a reason for each stage of your working.

Let $\angle BCD = x$

$\therefore \angle BAD = 3x$

$$3x + x = 180$$

$$3x + x = 180 \quad (\text{opposite angles in a cyclic quadrilateral add up to } 180^\circ)$$

$$4x = 180$$

$$x = 45^\circ$$

$$\angle BCD = 45^\circ$$

$$\angle SBD = \angle SBA + 20^\circ$$

$$\angle SBD = \angle BCD \quad (\text{alternate segment theorem})$$

$$\therefore \angle SBA + 20^\circ = 45^\circ$$

$$\angle SBA = 45 - 20$$

$$= 25^\circ$$

25

4/4

Examiner Comments

M1 for a correct process to find angle BCD

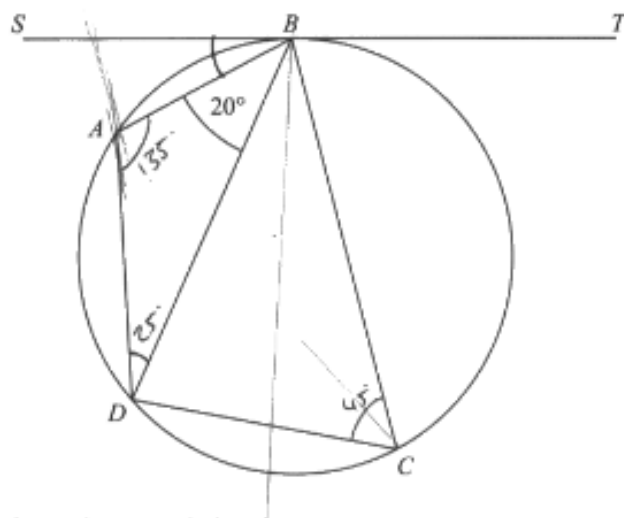
M1 for correctly linking SBD and BCD

C2 for a correct angle found and both correct reasons stated.



Student Response B

14



A, B, C and D are four points on a circle.
 SBT is a tangent to the circle.
 Angle $ABD = 20^\circ$

the size of angle BAD : the size of angle $BCD = 3 : 1$

Find the size of angle SBA .

Give a reason for each stage of your working.

$$\begin{array}{l} BAD : BCD \\ 3 : 1 \end{array}$$

$$BAD + BCD = 180^\circ \text{ opposite angles in a cyclic quadrilateral add up to } 180^\circ$$

$$\frac{180}{4} = 45$$

$$BCD = 45^\circ$$

$$BAD = 135^\circ$$

$$\text{angle } BDA = 25^\circ \text{ angles in a triangle add up to } 180^\circ$$

(Total for Question 14 is 4 marks)

3/4

Examiner Comments

M1 for a correct method to find BCD or BAD

M1 for correct method to find BDA

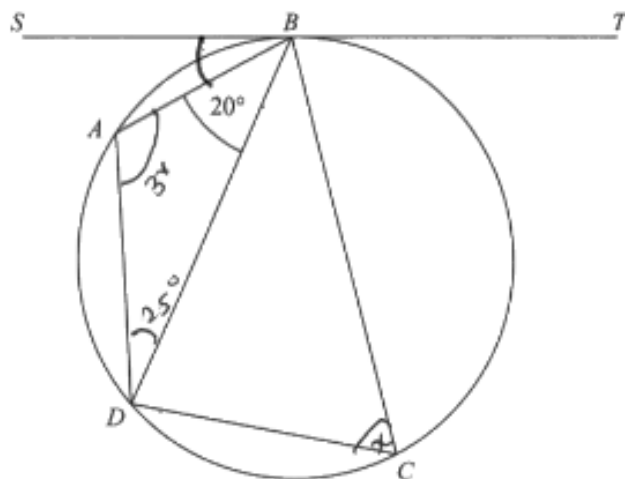
C1 only as SBA not found, but one correct reason given

92



Student Response C

14



A, B, C and D are four points on a circle.
 SBT is a tangent to the circle.
 Angle $ABD = 20^\circ$

the size of angle BAD : the size of angle $BCD = 3:1$

Find the size of angle SBA .
 Give a reason for each stage of your working.

$$3x + x = 180 \quad (\text{sum})$$

$$4x = 180$$

$$x = 45^\circ$$

$$\angle BAD = 3 \times 45$$

$$= 135^\circ$$

$$\text{angle of triangle} = 135 + 20 + BDA$$

$$= 180^\circ$$

$$\text{So } BDA = 180 - 155$$

$$= 25^\circ$$

(Total for Question 14 is 4 marks)

2/4

Examiner Comments

M1 for a correct method to find BCD or BAD

M1 for correct method to find BDA

C0 as SBA not found and no reasons given

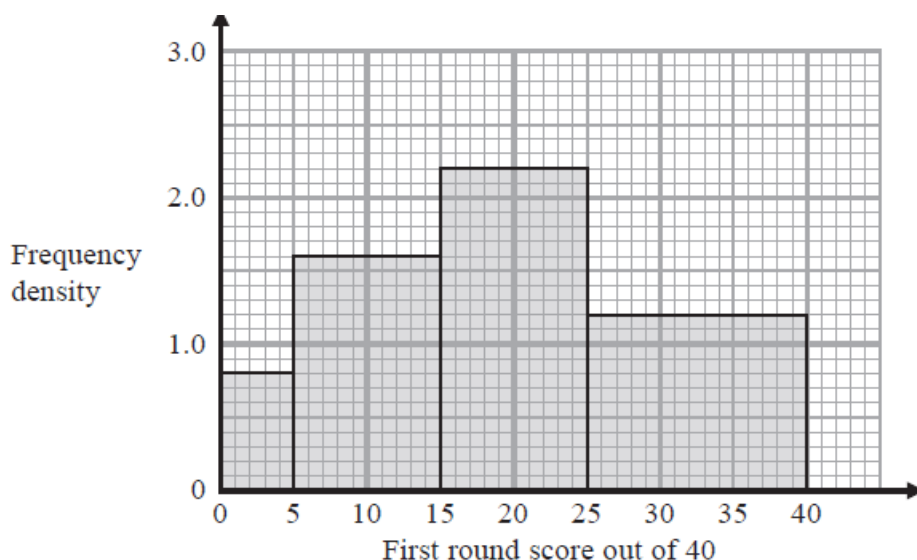


Exemplar Question 7

Higher tier Paper 2

- 17 Some people took part in the first round of a competition.

The histogram gives information about the scores of these people in the first round.



20% of the people got a score high enough for them to qualify for the second round.

Work out an estimate for the score needed to qualify for the second round.
You must show all your working.

.....
(Total for Question 17 is 4 marks)

Mean Score: 0.93

Examiner Comments

This histogram question is quite different to ones many pupils have seen before. From the completed histogram candidates need to find the total frequency (or the total area) represented by the graph. Once they do that they need to calculate 20%, and then use areas to work out the point on the graph that contains 80 of the frequencies (or 80% of the total area).



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
17	30	P1	for process to find one correct frequency, eg. $0.8 \times 5 (= 4)$ or $1.6 \times 10 (= 16)$ or $2.2 \times 10 (= 22)$ or $1.2 \times 15 (= 18)$ or to find one correct area eg $5 \times 8 (=40)$ or $10 \times 16 (=160)$ or $10 \times 22 (=220)$ or $15 \times 12 (=180)$	Accept equivalent methods proportional to those shown.
		P1	for process to find total number of people, eg. “4” + “16” + “22” + “18” (= 60) or for process to find total area eg “40” + “160” + “220” + “180” (= 600)	Condone 1 error in reading from the graph for 2 nd and 3 rd P marks
		P1	for process to find 20% of the total number of people, eg. “60” $\times$ 0.2 oe (= 12) or for process to find 20% of the total area “600” $\times$ 0.2 oe (=120)	
		A1	cao	NB: correct answer without supportive working gets 0 marks

Examiner Comments

The first P1 is for finding a single correct frequency using the frequency density and class width, or to find a correct area simply working with squares.

To gain the second mark the candidates had to calculate either the total frequency or the total area. In both this mark, and the next one, examiners were allowed to condone a single error in reading from the graph.

The third mark was for finding 20% of the total frequency or area. This had to have come from a correct method, hence the inverted commas on the values. Then the final mark was for an answer of 30 only.

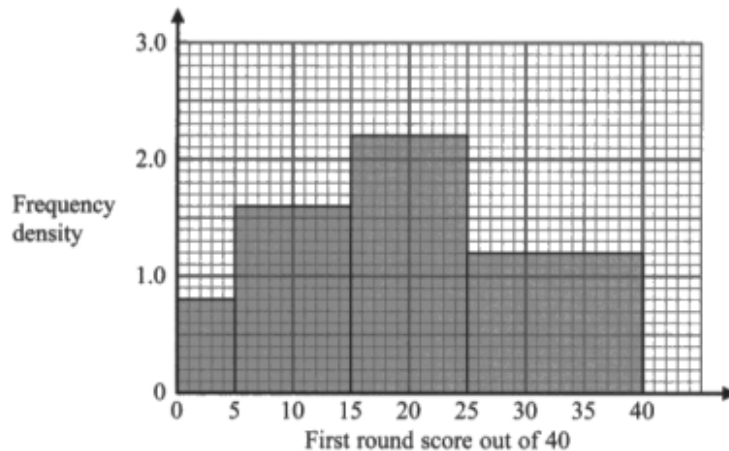
It is worth noting the comment in additional guidance, a correct answer with no working scores zero marks.



Student Response A

17 Some people took part in the first round of a competition.

The histogram gives information about the scores of these people in the first round.



20% of the people got a score high enough for them to qualify for the second round.

Work out an estimate for the score needed to qualify for the second round.

You must show all your working.

$$5 \times 0.8 = 4$$

$$10 \times 1.6 = 16$$

$$10 \times 2.2 = 22$$

$$15 \times 1.2 = 18$$

$$18 + 22 + 16 + 4 = 60$$

$$20\% \text{ of } 60 = 12$$

$$10\% = 6$$

$$20\% = 12$$

$$\begin{array}{r} 54 \\ 106 \\ 22 \\ 18 \end{array}$$

$$0 \times 5$$

$$60$$

$$10\% = 6$$

$$20\% = 12$$

$$1 =$$

$$\text{or } 10 \times 1.2 = 12$$

$$40 - 30 = 10$$

30

(Total for Question 17 is 4 marks)

4/4

Examiner Comments

P1 for process to find one correct frequency

P1 for a correct process to find the total frequency

P1 for correct process to find 20% of the total frequency

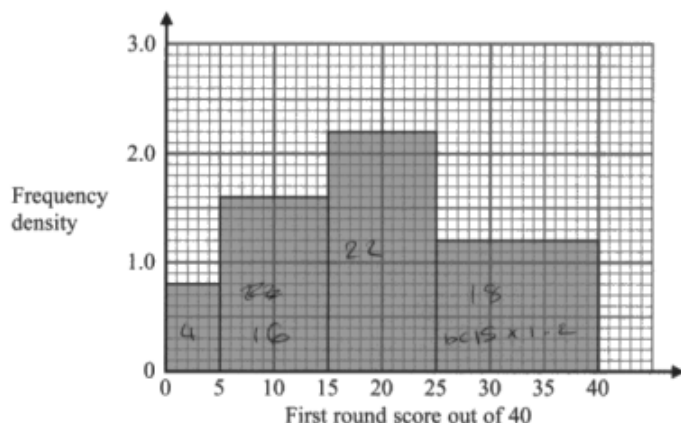
A1 for a correct answer



Student Response B

17 Some people took part in the first round of a competition.

The histogram gives information about the scores of these people in the first round.



20% of the people got a score high enough for them to qualify for the second round.

Work out an estimate for the score needed to qualify for the second round.

You must show all your working.

$$4 + 16 + 22 + 18 = 58 \text{ ppl}$$

$$20\% \text{ of } 58 = 11.6 \text{ ppl}$$

9.8 to 11.2

11.6 is closest to 12 ppl
which

15

(Total for Question 17 is 4 marks)

3/4

Examiner Comments

P1 for process to find one correct frequency

P1 for a correct process to find the total frequency. There is an arithmetic error in this step, but the process is correct.

P1 for correct process to find 20% of the total frequency. We can award this mark as the '58' has come from correct working

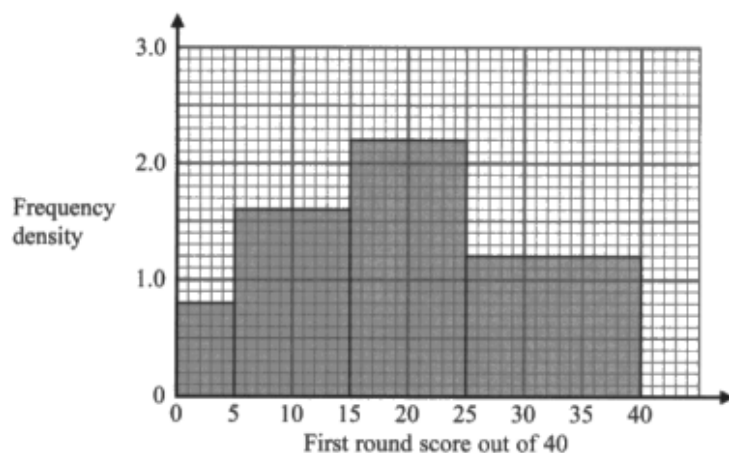
A0 the answer is wrong



Student Response C

17 Some people took part in the first round of a competition.

The histogram gives information about the scores of these people in the first round.



20% of the people got a score high enough for them to qualify for the second round.

Work out an estimate for the score needed to qualify for the second round.

You must show all your working.

$$5 \times 0.8 = 4$$

$$10 \times 1.6 = 16$$

$$10 \times 2.2 = 22$$

$$15 \times 1.2 = 18$$

(Total for Question 17 is 4 marks)

1/4

Examiner Comments

P1 for process to find one correct frequency

P0 no process to find the total frequency

P0 this mark cannot be awarded without the previous mark

A0 no answer given



Exemplar Question 8

Higher tier Paper 2

- 19 **A, B and C** are three spheres.

The volume of sphere **A** is 125 cm^3

The volume of sphere **B** is 27 cm^3

The ratio of the radius of sphere **B** to the radius of sphere **C** is $1 : 2$

Work out the ratio of the surface area of sphere **A** to the surface area of sphere **C**.

.....
(Total for Question 19 is 3 marks)

Mean Score: 0.38

Examiner Comments

This similar shapes question requires candidates to work with volume scale factors between **A** and **B** to find the length scale factors. They then need to use the given ratio to link the length scale factors of **A** and **C** before finding the ratios of their surface areas.

This question was easiest done working entirely in ratios, but could equally be done working with the formulae for volume and surface area and using actual values.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19	25 : 36	P1	for $\sqrt[3]{125}$ (= 5) and $\sqrt[3]{27}$ (= 3) oe OR for correct process to find the radius of A and radius of B (3.10... and 1.86...)	Accept scale factors expressed as fractions or decimals eg 1.66, 1.67, 0.6 or better Ignore units throughout
		P1	for method to find values in ratio of length between A and C eg 5 and 2×3 (= 6) oe or “3.10...” and “1.86...” $\times 2$ (=3.72...) OR 25 and 36 OR for correct process to find SA of A and SA of C (120.(8...)) and (174.(0...))	For both P marks the lengths need not be written as a ratio
		A1	for 25 : 36 oe eg 1: 1.44	

Examiner Comments

The first P1 is for cube rooting both 125 and 27, or for a correct process to find the radius of both **A** and **B**.

The second mark is for then using the ratio between the radii of **B** and **C** eg finding both 5 (for **A**) and 6 (for **C**), or for finding the surface area of both **A** and **C**.

The final mark was for a correct ratio. Note it does say ‘oe’ so any equivalent ratio would gain this mark.



Student Response A

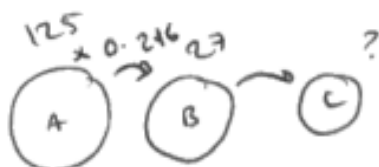
19 A, B and C are three spheres.

The volume of sphere A is 125 cm^3

The volume of sphere B is 27 cm^3

The ratio of the radius of sphere B to the radius of sphere C is $1:2$

Work out the ratio of the surface area of sphere A to the surface area of sphere C.



radius

B C
1 : 2

$$4\pi r^3 = 125$$

$\div 4\pi$

$$r^3 = 2.148$$

$\sqrt[3]{}$

$$r = 1.4658$$

length of B is 3

B : C
1 : 2
3 : 6

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Volume of C} = 5.832$$

St L 2

St A 2

St V 2

$$SH = 5^2$$

$$SA = 25$$

$$SV = \sqrt[3]{125}$$

A B

$$SV = \sqrt[3]{125}$$

$$SH = 3$$

St length for C

$$SA = 6^2 =$$

$$SA = 36$$

for C

$$25 : 36$$

$$25 : 36$$

(Total for Question 19 is 3 marks)

3/3

Examiner Comments

P1 for 5 and 3

P1 for 5 and 6

A1 for a correct answer



Student Response B

19 A, B and C are three spheres.

The volume of sphere A is 125 cm^3

The volume of sphere B is 27 cm^3

The ratio of the radius of sphere B to the radius of sphere C is $1:2$

Work out the ratio of the surface area of sphere A to the surface area of sphere C.

$2\pi r^2$

$\sqrt[3]{125} = 5$

$\sqrt[3]{27} = 3$

A : B : C

5 : 3 : 2

5 : 3

A : C

5 : 6

5 : 6

(Total for Question 19 is 3 marks)

2/3

Examiner Comments

P1 for 5 and 3

P1 for 5 and 6

A0 for an incorrect answer (they have not worked with surface areas)



Student Response C

19 A, B and C are three spheres.

The volume of sphere A is 125 cm^3

The volume of sphere B is 27 cm^3

The ratio of the radius of sphere B to the radius of sphere C is $1:2$

Work out the ratio of the surface area of sphere A to the surface area of sphere C.

$$\begin{array}{c} V \\ A \\ 125 \text{ cm}^3 : 27 \text{ cm}^3 \\ 5 : 3 \end{array}$$

V

$$\begin{array}{c} A : B \\ 125 \text{ cm}^3 : 27 \text{ cm}^3 \\ 5 : 3 \end{array}$$

$$\begin{array}{c} B : C \\ 1 : 2 \end{array}$$

$$\begin{array}{l} 4\pi r^2 = 100\pi \\ 4\pi(5)^2 = 100\pi \end{array}$$

$$100\pi : 99\pi$$

(Total for Question 19 is 3 marks)

1/3

Examiner Comments

P1 for 5 and 3

P0 candidate has not used the ratio with the 3 for B, to have 6

A0 incorrect answer



Exemplar Question 9

Higher tier Paper 2

20 In a village,

if it rains on one day, the probability that it will rain on the next day is 0.8

if it does **not** rain on one day, the probability that it will rain on the next day is 0.6

A weather forecaster says,

“There is a 70% chance that it will rain in the village on Monday.”

Work out an estimate for the probability that it will rain in the village on Wednesday.

You must show all your working.

.....
(Total for Question 20 is 4 marks)

Mean Score: 0.73

Examiner Comments

This conditional probability question was entirely unstructured and also required to work over 3 events rather than the more common 2. Candidates need to work out what combination of events would lead to it raining on Wednesday (there are 4 of them) and then be able to calculate a correct triple probability product for each and add them together.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
20	0.748	P1	for a process to find a correct probability product for 2 consecutive days, eg. 0.7×0.8 (rain M + T) or 0.7×0.2 (rain M + no rain T) or 0.3×0.6 (no rain M + rain on T) or 0.3×0.4 (no rain M + T)	Throughout accept probabilities given as fractions or percentages Could be for Tuesday and Wednesday also
		P1	for process to find a correct triple probability product for it raining on Wednesday, eg. $0.7 \times 0.8 \times 0.8$ (rain M + T + W) (= 0.448 or $\frac{56}{125}$ oe) or $0.7 \times 0.2 \times 0.6$ (rain M + no rain T + rain W) (= 0.084 or $\frac{21}{250}$ oe) or $0.3 \times 0.6 \times 0.8$ (no rain M + rain T + rain W) (= 0.144 or $\frac{18}{125}$ oe) or $0.3 \times 0.4 \times 0.6$ (no rain M + no rain T + rain W) (= 0.072 or $\frac{9}{125}$ oe)	
		P1	for complete process, eg. “0.448” + “0.084” + “0.144” + “0.072”	
		A1	oe eg, $\frac{187}{250}$	
				NB: correct answer without supportive working gets 0 marks

Examiner Comments

The first mark was for a correct probability for consecutive days. It is important to note that it could be for Tuesday and Wednesday as well as Monday and Tuesday, and providing the products are labelled they could be reversed. However, without correct annotation a reversed product for Tuesday and Wednesday would not gain credit.

The second mark is for a correct product for a triple probability that gives the event that it rains on Wednesday. There are 4 possible products that could gain the mark.

The final process mark is for a fully correct process. This means we need to see all 4 correct products and then these products added. Arithmetic errors can be found, but the process must be seen.

The final mark is for 0.748 or any equivalent fraction or percentage.

Just like with Question 17, a correct answer with no working scores zero.



Student Response A

20 In a village,

if it rains on one day, the probability that it will rain on the next day is 0.8

if it does **not** rain on one day, the probability that it will rain on the next day is 0.6

A weather forecaster says,

"There is a 70% chance that it will rain in the village on Monday."

Work out an estimate for the probability that it will rain in the village on Wednesday.

You must show all your working.

Monday rain 70% The rain - yes 0.8 / no 0.2

wednes The rain - yes 0.8 / no 0.2

no rain - 30% The rain - yes 0.6 / no 0.4

wednes The rain - yes 0.8 / no 0.2

$$\begin{aligned} 0.7 \times 0.8 \times 0.8 &= 0.448 \\ 0.7 \times 0.2 \times 0.6 &= 0.084 \\ 0.3 \times 0.6 \times 0.8 &= 0.144 \\ 0.3 \times 0.4 \times 0.6 &= 0.072 \\ \hline * 0.748 \end{aligned}$$

0.748

(Total for Question 20 is 4 marks)

4/4

Examiner Comments

P1 / P1 for a correct triple probability product

P1 for a complete method (all 4 correct products added)

A1 for a correct answer

106



Student Response B

20 In a village,

if it rains on one day, the probability that it will rain on the next day is 0.8

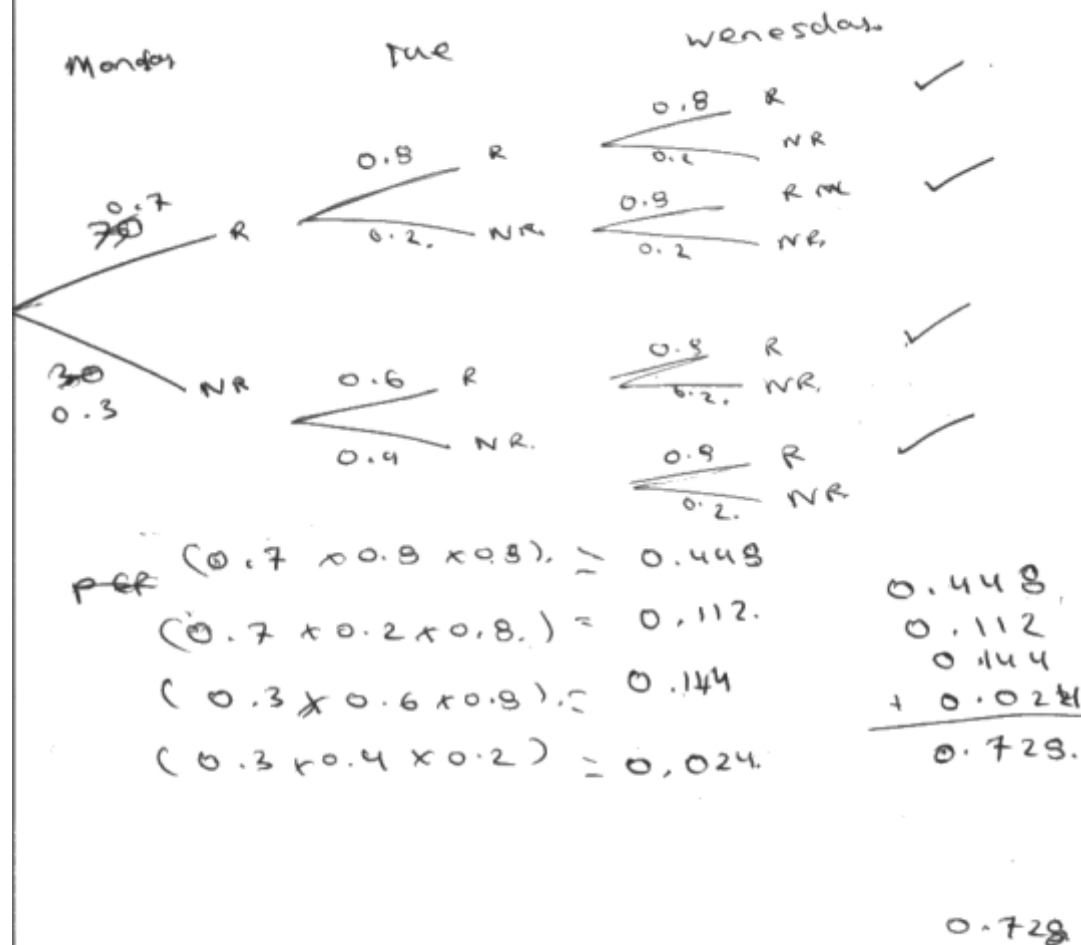
if it does **not** rain on one day, the probability that it will rain on the next day is 0.6

A weather forecaster says,

"There is a 70% chance that it will rain in the village on Monday."

Work out an estimate for the probability that it will rain in the village on Wednesday.

You must show all your working.



(Total for Question 20 is 4 marks)

2/4

Examiner Comments

P1 / P1 for a correct triple probability product

P0 this mark cannot be awarded as 2 of the products are incorrect. The second product should have 0.6 rather than 0.8 and the final product should have 0.6 rather than 0.2

A0 incorrect answer



Student Response C

20 In a village,

if it rains on one day, the probability that it will rain on the next day is 0.8

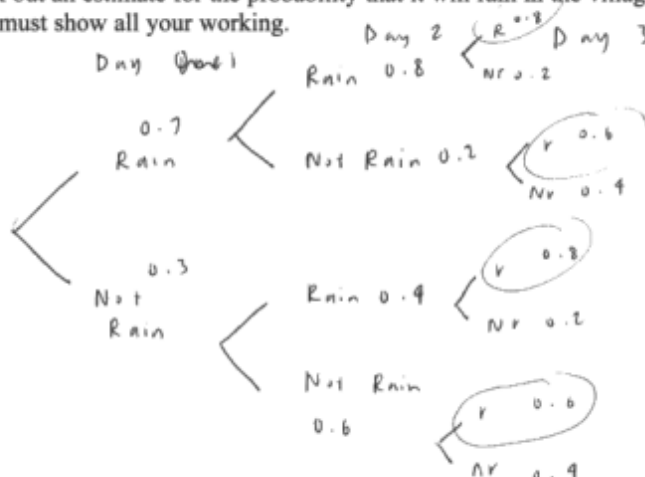
if it does **not** rain on one day, the probability that it will rain on the next day is 0.6

A weather forecaster says,

"There is a 70% chance that it will rain in the village on Monday."

Work out an estimate for the probability that it will rain in the village on Wednesday.

You must show all your working.



$$\frac{92}{125} = 0.736$$

0.736

(Total for Question 20 is 4 marks)

0/4

Examiner Comments

P0 / P0 / P0 this candidate has produced a partially correct probability tree, which has branches which could lead to correct products. However, there are no products seen and therefore none of the P marks can be awarded

A0 incorrect answer



Exemplar Question 10

Higher tier Paper 2

- 21 The time period, T seconds, of a simple pendulum of length l cm is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Katie uses a simple pendulum in an experiment to find an estimate for the value of g .

Here are her results.

$l = 52.0$ correct to 3 significant figures.

$T = 1.45$ correct to 3 significant figures.

Work out the upper bound and the lower bound for the value of g .

Use $\pi = 3.142$

You must show all your working.

upper bound = ...

lower bound = ...

(Total for Question 21 is 4 marks)

Mean Score: 0.38

Examiner Comments

This bounds question is familiar in some senses, but is made much more challenging by the need to rearrange the formula, which in itself is tricky.

Candidates needed to carry out 3 main steps. Find the correct bounds for l and T . Rearrange the formula to make g the subject and then finally choose the correct pairs of upper and lower bounds of l and T to find the upper and lower bounds of g .



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
	984.(3677853) and 969.(0181643)	B1	stating bound of 51.95 or 52.05 or 1.445 or 1.455	Accept 52.049 or 52.0499... for 52.05 Accept 1.4549 or 1.4549... for 1.455
		P1	for process to rearrange formula to give g as the subject, eg $g = \frac{4\pi^2 l}{T^2}$ oe	Rearrangement may occur after substitution, in this case correct bounds are not needed for this mark
		P1	for process to use LB of l and UB of T in formula for g or T or process to use UB of l and LB of T in formula for g or T eg $\frac{4\pi^2 [\text{LB of } l]}{[\text{UB of } T]^2}$ or $\frac{4\pi^2 [\text{UB of } l]}{[\text{LB of } T]^2}$	$51.95 \leq [\text{LB of } l] < 52.0$ $1.45 < [\text{UB of } T] \leq 1.455$ $52.0 < [\text{UB of } l] \leq 52.05$ $1.445 \leq [\text{LB of } T] < 1.45$ Rearrangement may not be correct
		A1	for upper bound = 984.(3677853) or 984.(1125639..) and lower bound = 969.(0181643) or 968.(7669227..)	NB: correct answer without supportive working gets 0 marks Accept answers rounded or truncated to 3sf or better

Examiner Comments

The first mark is an independent accuracy mark for one correct bound of l or T and was the most common mark awarded.

The second and third mark are interchangeable in the order they are awarded. The second is for the rearrangement of the formula. This may be seen purely as an algebraic rearrangement, or it may be seen after substitution of bounds has occurred. In the latter case, these bounds may not be correct, as long as the steps in the rearrangement (or evaluation of g) are correct.

The third is for correctly choosing the right pair of values that would lead to a correct bound. These could be seen prior to rearrangement, or after, but need to meet the range given in the additional guidance. This means that if an incorrect set of bounds is given for l say, such as 51.5 and 52.5, then these values could not gain this mark as they are outside the given range.

The final mark is for one of the given pairs. There are 2 pairs of acceptable answers as some candidates used a value of 3.14 for π as given in the question, and some still used the π button on their calculator.



Student Response A

21 The time period, T seconds, of a simple pendulum of length l cm is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Katie uses a simple pendulum in an experiment to find an estimate for the value of g .

Here are her results.

$l = 52.0$ correct to 3 significant figures.

$T = 1.45$ correct to 3 significant figures.

Work out the upper bound and the lower bound for the value of g .

Use $\pi = 3.142$

You must show all your working.

$$\begin{aligned}
 &1.45 = 2\pi\sqrt{\frac{52.0}{g}} \\
 &\frac{1.45}{2\pi} = \sqrt{\frac{52.0}{g}} \\
 &\frac{1.45}{2\pi} = 0.2299 \dots \\
 &0.2299 \dots = \sqrt{\frac{52.0}{g}} \\
 &0.05289 = \frac{52.0}{g} \\
 &g = 984.11256039 \\
 &\text{upper bound} = 984.11256039 \\
 &\text{lower bound} = 968.7669227
 \end{aligned}$$

(Total for Question 21 is 4 marks)

4/4

Examiner Comments

B1 all four correct bounds given

P1 we award the 3rd mark here for the correct substitution into the formula for T

P1 we then award the 2nd mark for correct rearrangement to evaluate g

A1 both correct bounds given



Student Response B

21 The time period, T seconds, of a simple pendulum of length l cm is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Katie uses a simple pendulum in an experiment to find an estimate for the value of g .

Here are her results.

$l = 52.0$ correct to 3 significant figures.

$T = 1.45$ correct to 3 significant figures.

Work out the upper bound and the lower bound for the value of g .

Use $\pi = 3.142$

You must show all your working.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}$$

$$\frac{T^2}{4\pi^2} = \frac{l}{g}$$

$$g = l \div \frac{T^2}{4\pi^2}$$

$$g = \frac{4\pi^2 l}{T^2}$$

Handwritten calculations for bounds:

$$UB_l = 52.5$$

$$LB_l = 51.5$$

$$UB_T = 1.455$$

$$LB_T = 1.445$$

$$UB_g = \frac{4\pi^2 UB_l}{(LB_T)^2} = 992.8781696$$

$$LB_g = \frac{4\pi^2 LB_l}{(UB_T)^2} = 960.6243592$$

$$\text{upper bound} = 992.8781696$$

$$\text{lower bound} = 960.6243592$$

(Total for Question 21 is 4 marks)

2/4

Examiner Comments

B1 both correct bounds for T given

P1 we award the 2nd mark for a correct rearrangement to give a formula for g

P0 we cannot award the 3rd mark as even though they choose the correct pairs of bounds, their values for l are outside the given range

A0 incorrect answer



Student Response C

21 The time period, T seconds, of a simple pendulum of length l cm is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Katie uses a simple pendulum in an experiment to find an estimate for the value of g .

Here are her results.

$l = 52.0$ correct to 3 significant figures.

$T = 1.45$ correct to 3 significant figures.

Work out the upper bound and the lower bound for the value of g .

Use $\pi = 3.142$

You must show all your working.

$$\begin{aligned}\text{Upper bound} &= 2\pi \times 3.142 \sqrt{\frac{52.05}{1.445}} \\ &= 37.7 \\ \text{Lower bound} &= 2 \times 3.142 \sqrt{\frac{51.95}{1.455}} \\ &= 37.5\end{aligned}$$

$$\text{upper bound} = 37.7$$

$$\text{lower bound} = 37.5$$

(Total for Question 21 is 4 marks)

1/4

Examiner Comments

B1 all four correct bounds given

P0 no attempt to rearrange

P0 although correct bounds have been chosen, they have been substituted for l and g rather than l and T

A0 incorrect answer



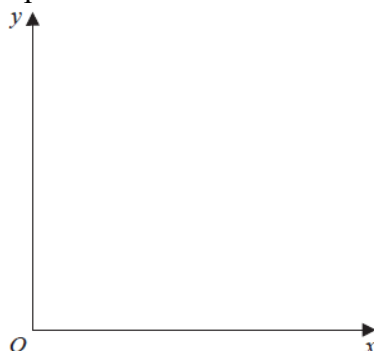
Paper 3H (calculator)

Exemplar Question 1

Higher tier Paper 3

- 8 (a) Using the axes below, sketch a graph to represent the statement

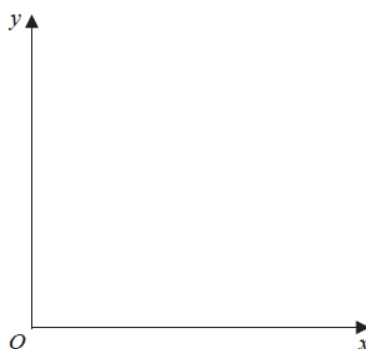
y is directly proportional to x



(1)

- (b) Using the axes below, sketch a graph to represent the statement

y is inversely proportional to x



(1)

(Total for Question 8 is 2 marks)

Mean Scores: Part (a) 0.69

Part (b) 0.22

Examiner Comments

This question is designed to assess understanding of the graphical representation of direct and inverse proportion. Students are expected to show the general shape of the graph for each relationship. In part (a) we expect to see a straight line from the origin and in part (b) a curve which ideally approaches but does not touch the axes.

For part (b) a common error is for students to draw a straight line with a negative gradient. This does relay the idea that as the value of x increases then the value of y decreases. However, it does not accurately represent the inverse relationship



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
8	Graph sketched	C1	Sketch	Accept freehand provided intention is clear
	Graph sketched	C1	Sketch	

Examiner Comments

For the graph representing direct proportionality, we can condone a freehand drawing provided the intention to draw a straight line is clear. Similarly if the line does not include the origin, we can accept it provided it would go through the origin if extended.

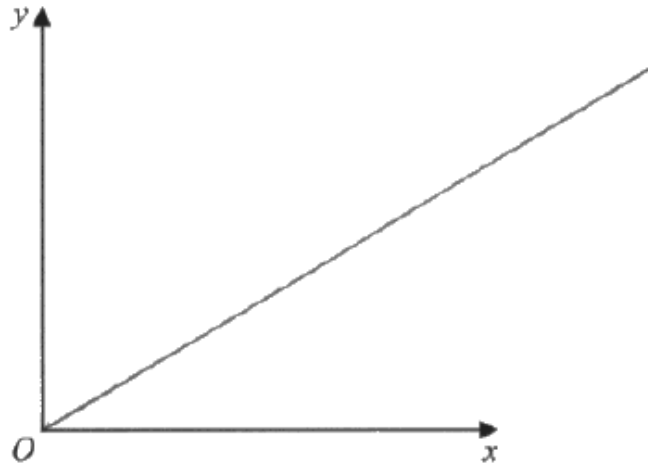
For the graph representing inverse proportionality, it is the general shape that is required for the mark to be awarded and so we can ignore it if the curve does touch either or both axes.



Student Response A

(a) Using the axes below, sketch a graph to represent the statement

y is directly proportional to x



(1)

1/1

Examiner Comments

Part (a)

C1 Acceptable sketch, straight line from the origin with a positive gradient

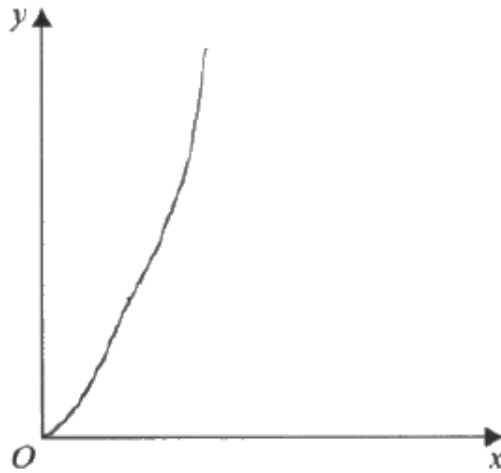


Student Response B

(a) Using the axes below, sketch a graph to represent the statement

y is directly proportional to x

4 marks



(1)

0/1

Examiner Comments

Part (a)

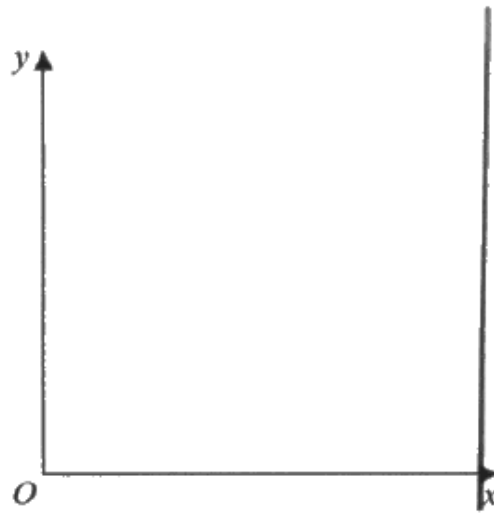
C0 Although freehand sketches are allowed, this diagram suggests a curve rather than a straight line so the mark is not given.



Student Response C

(a) Using the axes below, sketch a graph to represent the statement

y is directly proportional to x



(1)

0/1

Examiner Comments

Part (a)

C0 Incorrect graph sketch as the line does not include the origin and does not have a gradient between 0 and 1

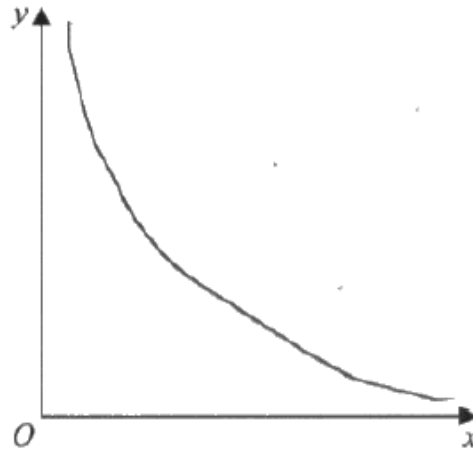
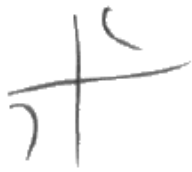


Student Response A

(b) Using the axes below, sketch a graph to represent the statement

y is inversely proportional to x

$$y = \frac{1}{x}$$



(1)

1/1

Examiner Comments

Part (b)

C1 Acceptable sketch. The general shape is correct and the curve approaches the axes.

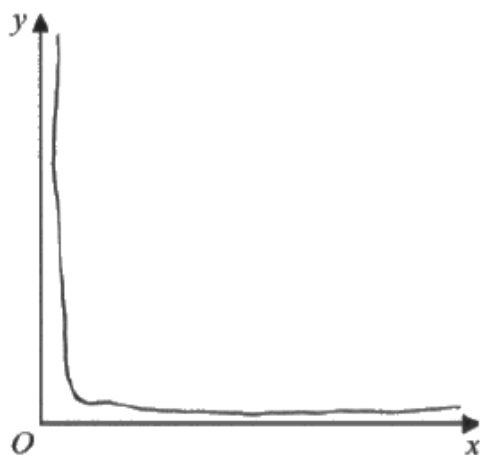


Student Response B

(b) Using the axes below, sketch a graph to represent the statement

y is inversely proportional to x

$$y = \frac{k}{x}$$



(1)

1/1

Examiner Comments

Part (b)

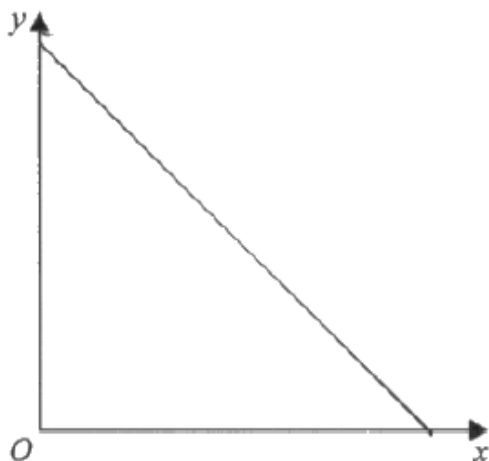
C1 Acceptable sketch. Intention is clear.



Student Response C

(b) Using the axes below, sketch a graph to represent the statement

y is inversely proportional to x



(1)

0/1

Examiner Comments

Part (b)

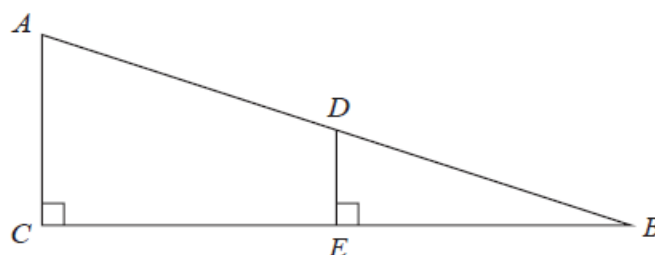
C0 Incorrect sketch, a common error



Exemplar Question 2

Higher tier Paper 3

- 10 The diagram shows two right-angled triangles ACB and DEB .



$$AD = 9 \text{ cm}$$

$$DE = 2 \text{ cm}$$

$$DB = 6 \text{ cm}$$

Calculate the length of CB .

Give your answer correct to 2 decimal places.

..... cm

(Total for Question 10 is 4 marks)

Mean Score: 1.30

Examiner Comments

This question testing the concept of similarity together with Pythagoras' rule and/or trigonometry proved to be a good discriminator.

Students were expected to use the scale factor linking two similar triangles together with Pythagoras' rule to find the length of a side of a triangle, CB . Alternatively they could use trigonometry on the right-angled triangles to solve the problem.

A common error was for students to use a scale factor of $\frac{9}{6}$ ($= 1.5$) instead of $\frac{9+6}{6}$ ($= 2.5$) between triangle DBE and triangle ABC or to apply Pythagoras' rule incorrectly and for example, write $6^2 + 2^2$ instead of $6^2 - 2^2$ to find the length of EB .



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
10	14.14	P1	works out scale factor eg $(9 + 6) \div 6 (= 2.5)$ OR for start of process to find angle DBE eg $\sin B = \frac{2}{6}$ oe	Note method can be carried out in either order
		P1	uses Pythagoras eg $6^2 - 2^2 (= 32)$ or $\sqrt{32} (= 5.6...)$ OR calculates AC eg $2 \times "2.5" (= 5)$ OR for complete process to find angle DBE eg $\sin^{-1} \left(\frac{2}{6} \right) (= 19.4...)$	May be seen on diagram
		P1	complete process to find CB eg $"2.5" \times \sqrt{32} (= 10\sqrt{2})$ or $\sqrt{(9 + 6)^2 - "5" ^2} (= 10\sqrt{2})$ OR uses trigonometry, eg $15 \times \cos "19.4..."$	
		A1	14.1 to 14.15	If the answer is given within the range but then rounded incorrectly award full marks.

Examiner Comments

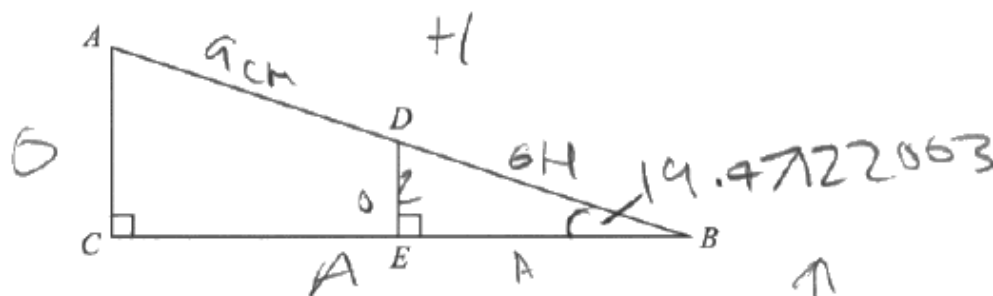
The first mark is awarded for a process to start the question. This could be either a method to find the scale factor between triangles DBE and ABC or the beginning of the process to find the size of angle DBE . The second mark may be awarded for sight of the length of AC – this may be seen as a 5 on side AC on the diagram. So $AC = 5$ gains the first 2 process marks because the scale factor has been used. Alternatively the second mark can be earned for the final stage of working out the size of angle DBE ($\sin^{-1} \left(\frac{2}{6} \right)$ or $19.47...^\circ$). So, again, sight of $19.47...^\circ$ gains the first 2 process marks. As a third possibility, students may earn 1 mark for a correct application of Pythagoras' rule on triangle DBE . The third process mark is for a correct and complete process to find the length of CB . The accuracy mark is given for any answer in the range 14.1 to 14.15. The request for candidates to give their answer correct to 2 decimal places is for guidance only so an answer such as 14.1 or 14.142 would gain full marks.

As always, the use of quotation marks in the mark scheme show that although the value in quotes does not need to be correct, it does need to come from a correct process.



Student Response A

The diagram shows two right-angled triangles ACB and DEB .



$AD = 9$ cm

$DE = 2$ cm

$DB = 6$ cm

Calculate the length of CB .

Give your answer correct to 2 decimal places.

$$\sin^{-1}\left(\frac{2}{6}\right) =$$

$$9 + 6 = 15$$

$$\cos$$

$$\cos(19.4722063) \times 15 =$$

$$14.14 \text{ cm}$$

4/4

Examiner Comments

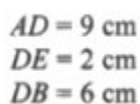
P1 P1 Uses right-angled triangle trigonometry to find angle DBE . The process seen here is complete. The expression $\sin^{-1}\frac{2}{6}$ or sight of 19.4...for angle B is enough evidence to award the first 2 process marks.

P1 Uses trigonometry on triangle ABC with $9 + 6 = 15$ and $19.47...^\circ$ to give a correct expression for the length of CB , $\cos(19.47) \times 15$

A1 Correct answer



The diagram shows two right-angled triangles ACB and DEB .


$$6^2 - 2^2 = 32$$
$$\checkmark \quad \checkmark$$
$$= 5 \cdot 7$$
$$18 \div 6 = 5 \frac{1}{2}$$
$$5 \cdot 7 \times \frac{5}{2} = \frac{57}{4} / 14.25$$

14.25 cm

3/4

P1 Works out the scale factor, $15 \div 6$ or $\frac{5}{2}$

P1 Uses Pythagoras' rule correctly, $6^2 - 2^2$

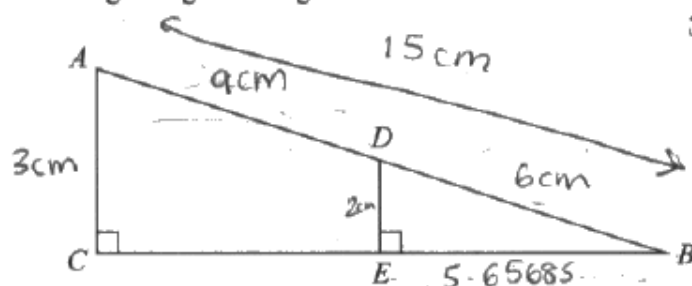
P1 Although the evaluation of $\sqrt{32}$ is rounded prematurely, the candidate uses the scale factor to multiply their $\sqrt{32}$ by 2.5, a complete process.

A0 The answer is not within the acceptable range due to premature rounding of $\sqrt{32}$



Student Response C

The diagram shows two right-angled triangles ACB and DEB .



$$SF \rightarrow 9 \div 6 = 1.5$$

SO

$$AC = 2 \times 1.5 = 3$$

$$AD = 9 \text{ cm}$$

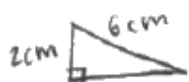
$$DE = 2 \text{ cm}$$

$$DB = 6 \text{ cm}$$

$$CB = 5.656854249 \times 1.5 = 8.485281374 = 8.49$$

Calculate the length of CB .

Give your answer correct to 2 decimal places.



$$a^2 + b^2 = c^2$$

$$2^2 + b^2 = 6^2$$

$$4 + b^2 = 36$$

$$-4 \quad -4$$

$$b^2 = 32$$

$$b = \sqrt{32}$$

$$b = 5.656854249$$



$$a^2 + b^2 = c^2$$

$$3^2 + b^2 = 15^2$$

$$9 + b^2 = 225$$

$$-9 \quad -9$$

$$b^2 = 216$$

$$b = \sqrt{216}$$

$$b = 14.69693846$$

$$CB = 5.656854249 + 14.69693846 = 20.35379271 = 20.35$$

$$8.49$$

$$20.35 \text{ cm}$$

1/4

Examiner Comments

P0 No scale factor equivalent to 2.5 or start to a process using trigonometry.

P1 Correct process to find the length of EB by Pythagoras' rule.

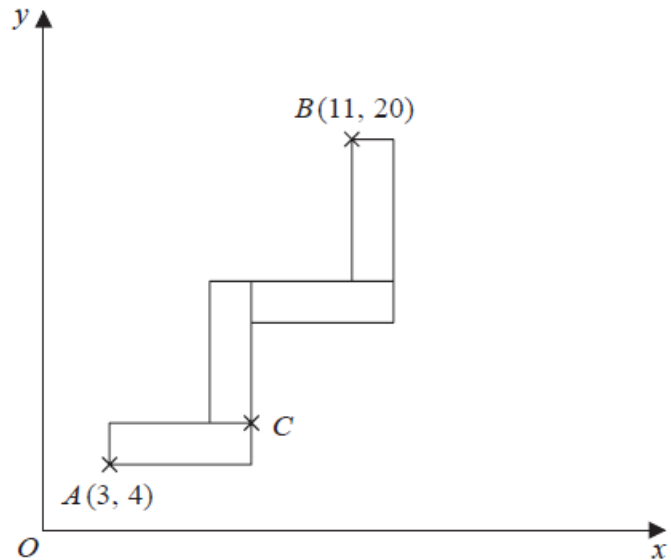
P0 No correct process to find the length of CB . Uses a scale factor of 1.5 instead of 2.5 and does not seem to realise this gives the length of CE , not CB

A0 Incorrect answer

Exemplar Question 3

Higher tier Paper 3

- 13 A pattern is made from four identical rectangles.
The sides of the rectangles are parallel to the axes.



Point A has coordinates $(3, 4)$
Point B has coordinates $(11, 20)$
Point C is marked on the diagram.

Work out the coordinates of C .
You must show all your working.

(..... , ...)

(Total for Question 13 is 5 marks)

Mean Score: 0.46

Examiner Comments

This question assesses the derivation and solution of simultaneous equations in the context of a problem involving coordinates. As such it requires students to make and use connections between different parts of mathematics. Some students find this challenging, and this was demonstrated by the relatively small proportion of fully correct answers seen. There were many unsuccessful attempts where students started by calculating the gradient of the line joining point A to point B . Students were required to show all of their working, that is, the formulation of their equations, their method for solving the equations and how they used their solutions to get the coordinates of point C .



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
13	(9, 8)	P1	for setting up an equation for one dimension (width) of the pattern eg $2b - a = 8$ oe or $2x + y = 8$ oe	a and b are the width and length x is the difference between the length and width, y is the width of the rectangle Both values correct implies this mark
		P1	for setting up an equation for the other dimension (height) of the pattern eg $2b + a = 16$ oe or $2x + 3y = 16$ oe	
		P1	(dep P2) for a full process to solve for both variables eg $4b = 24$, $b = 6$ and $12 - a = 8$, $a = 4$ or $8 = 2y$, $y = 4$ and $8 = 2x + y$, $x = 2$	
		P1	(dep P3) for a full process to find one of the coordinates of C eg $3 + 6 (= 9)$ or $4 + 4 (= 8)$ or $3 + 2 + 4 (= 9)$ or $4 + 4 (= 8)$	
		A1	cao	Award 0 marks for a correct answer with no supportive working.

Examiner Comments

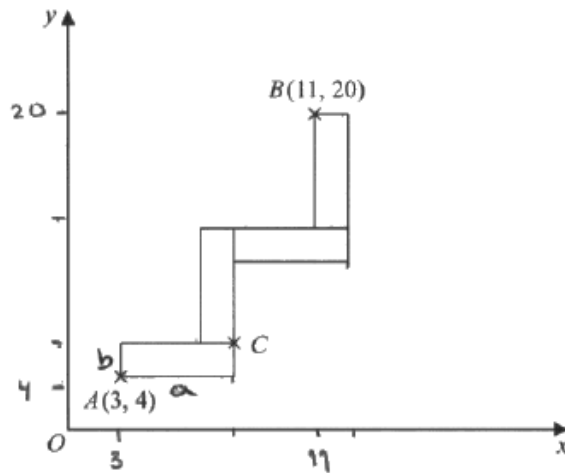
The first 2 marks are for assigning variables appropriately to lengths in the diagram and using this to formulate equations connecting these variables to the differences in the coordinates of points A and B . It is expected that the variables assigned will be for the length and width of each rectangle but this does not have to be the case. The equations may be in unsimplified form at this stage, eg $a + a - b = 8$

The third process mark is dependent on the correct formulation of two equations and is awarded for a method to solve the equations. Correct values found for the assigned variables will be sufficient evidence for the award of this mark. Similarly the fourth process mark is dependent on the first three having been awarded and if they have been given, this mark can be given on sight of one correct coordinate for point C . The final accuracy mark is for both coordinates correct. Students are not given marks if their answer (9, 8) but is not supported by working.



Student Response A

A pattern is made from four identical rectangles.
The sides of the rectangles are parallel to the axes.



Point A has coordinates (3, 4)
Point B has coordinates (11, 20)
Point C is marked on the diagram.

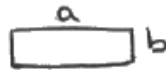
Work out the coordinates of C.
You must show all your working.

$$20 - 4 = 16$$

$$11 - 3 = 8$$

$$2a + b = 16$$

$$a + a - b = 8 \quad \text{or} \quad 2a - b = 8$$



$$\begin{array}{r} 2a + b = 16 \\ + \quad 2a - b = 8 \\ \hline 4a = 24 \\ a = 6 \end{array}$$

$$\begin{array}{r} 2a - b = 8 \quad \text{or} \\ \text{or} \quad 2 \times 6 - b = 8 \quad \text{or} \\ \text{or} \quad 12 - b = 8 \quad \text{or} \\ \text{or} \quad b = 4 \end{array}$$

$$x_c = 3 + 6 = 9$$

$$y_c = 4 + 4 = 8$$

(9 , 8)

5/5

Examiner Comments

P1 This is given for writing down one equation, for the width, $2a - b = 8$

P1 A second equation for the height is written down, $2a + b = 16$

P1 A full process is seen to solve the equations to give $a = 6$, $b = 4$

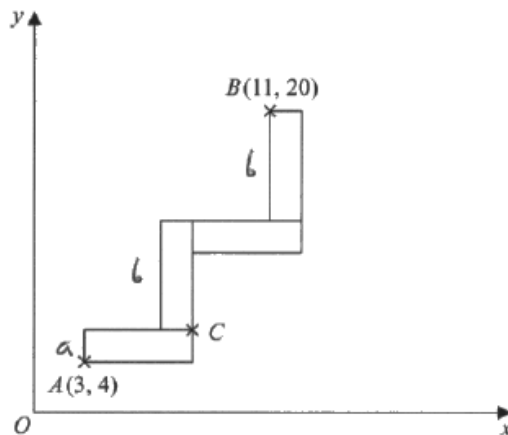
P1 A process to find one of the coordinates of C leads to the award of one mark.

A1 A fully correct answer with supportive working is seen



Student Response B

A pattern is made from four identical rectangles.
The sides of the rectangles are parallel to the axes.



Point A has coordinates $(3, 4)$

Point B has coordinates $(11, 20)$

Point C is marked on the diagram.

Work out the coordinates of C .

You must show all your working.

$$a + b + b = 16$$

$$b + b - a = 8$$

$$16 - 8 = 8$$

$$a = 8$$

$$b \div 2 = 4$$

$$3 + 4 = 7$$

$$4 + 8 = 12$$

$$(7, 12)$$

2/5

Examiner Comments

P1 Sets up an equation for the height, $a + b + b = 16$

P1 Sets up an equation for the width, $b + b - a = 8$

P0 The process for solving the equations is incorrect, $a \neq 8$ and there is no evidence of a correct process to eliminate b

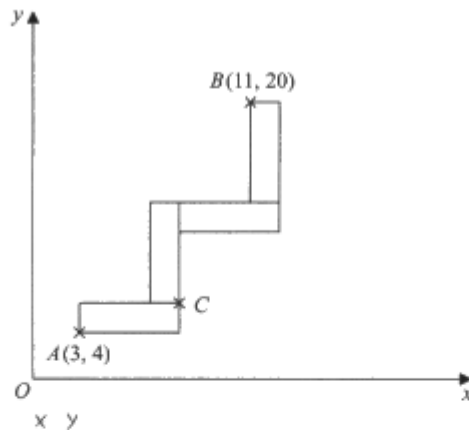
P0 This mark is dependent on P3 and cannot be awarded

A0 No fully correct answer.



Student Response C

A pattern is made from four identical rectangles.
The sides of the rectangles are parallel to the axes.



Point A has coordinates (3, 4).
Point B has coordinates (11, 20).
Point C is marked on the diagram.

Work out the coordinates of C.
You must show all your working.

$$\frac{\Delta y}{\Delta x} \rightarrow \frac{16}{8} = 2 \quad \boxed{m=2}$$

$$y = mx + c$$

$$x = 2x + c$$

$$20 = 2(11) + c$$

$$20 = 22 + c$$

$$-2 \quad \boxed{-2 = c}$$

$$x = 2x - 2$$

coordinates of C

$$\boxed{1}$$

$$(2, -2)$$

0/5

Examiner Comments

P0 P0 P0 P0 A0

This student finds the width and height of the pattern but this is not enough to gain any marks.

They then make a commonly seen error – that of calculating the gradient of the line joining point A to B, then the equation of the line. No marks can be awarded as the processes are not appropriate.



Exemplar Question 4

Higher tier Paper 3

- 14 Olivia and Jessica have in total half as many sweets as Fran and Gary have in total.

Fran and Gary share their sweets in the ratio 2 : 3

Olivia and Jessica share their sweets in the ratio 9 : 1

Fran got w sweets.

Gary got x sweets.

Olivia got y sweets.

Jessica got z sweets.

Find, in its simplest form, $w : x : y : z$

.....
(Total for Question 14 is 4 marks)

Mean Score: 1.17

Examiner Comments

Ratio is the key concept being tested in this question which involved several aspects of this topic including expressing the multiplicative relationship, “half as many as” as a ratio or fraction, combining ratios and reduction to simplest form.

Most students made some progress with the question and gained a mark for finding the sum of the parts of each of the given ratios, 2 : 3 (=5) and 9 : 1 (= 10). However, few students could complete the question successfully. It was surprising that not more students assigned a given total number of sweets to one of the pairs, for example 100 sweets to be shared between Fran and Gary. Students could then have worked with this to find the number of sweets each student gets before writing them as a ratio in its simplest form.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
14	8 : 12 : 9 : 1	P1	for $2 + 3 (= 5)$ and $9 + 1 (= 10)$ OR for assigning a total number of sweets for F + G and O + J eg $F + G = 100$, $O + J = 50$	May be in algebraic form, eg $2a + 3a (= 5a)$ and $9a + 1a (= 10a)$
		P1	for finding correct multiplier for relationship between totals for F + G and O + J eg $\times 4$ to get from 5, 10 to 20, 10 OR for working out the number of sweets from their totals for F, G eg 40, 60 or for O, J, eg 45, 5	May be in algebraic form, eg $F + G = 5a$, $O + J = 2.5a$
		P1	for $2 \times 4 (= 8)$ and $3 \times 4 (= 12)$ OR for ratio in unsimplified form, eg 40 : 60 : 45 : 5	
		A1	cao	

Examiner Comments

Students need to have identified a strategy to take into account all the different aspects of the question. For students who work entirely with the ratios, the first mark is for realising that they will need to consider the total amount of sweets each pair has in total and as a consequence, add the 2 and 3 and the 9 and 1. 5 and 10 seen is sufficient evidence to award this mark. The second mark for students following this route is for identifying that if Fran and Gary are to have twice as many sweets in total compared to Olivia and Jessica, the 5 : 10 ratio will need to change to 20 : 10 or equivalent, hence a $\times 4$ multiplier will need to be used. The third mark is for using this to get $2 \times 4 : 3 \times 4$ from 2 : 3. The final mark is for completing this calculation and combining the ratios to get a final answer of 8 : 12 : 9 : 1

For students who begin by assigning a certain number of sweets to one of the pairs, the first mark is for doing this and working out from this the total number of sweets for the other pair for example 100 sweets for Fran and Gary and 50 sweets for Olivia and Jessica. Here the second mark is for working out, using the ratios 2 : 3 and 9 : 1, the number of sweets for each person, eg 40, 60, 45, 5. The third mark is for writing this in ratio form and the final mark for an answer written in simplest form.

It should be noted that ratios not written in the order required by the question were not accepted unless labelled to associate them with Fran, Gary, Olivia and Jessica.



Student Response A

Olivia and Jessica have in total half as many sweets as Fran and Gary have in total.

Fran and Gary share their sweets in the ratio 2:3

Olivia and Jessica share their sweets in the ratio 9:1

Fran got w sweets.

Gary got x sweets.

Olivia got y sweets.

Jessica got z sweets.

Find, in its simplest form, $w:x:y:z$

$$\begin{array}{l} w+x \\ \hline \end{array} = a \quad \begin{array}{l} y+z \\ \hline \end{array} = b \quad a=2b \quad a:b=2:1$$

$$w:x=2:3 \quad y:z=9:1$$

$$w+x=5 \quad y+z=10$$

$$a=5 \quad b=10 \quad a=2b \text{ so } a=20$$

$$w:x=8:12 \quad y:z=9:1 \quad 8:12:9:1 = w:x:y:z$$

$$\underline{8:12:9:1}$$

4/4

Examiner Comments

P1 Totalling the ratio for each pair to get 5 and 10 earns this mark.

P1 Works out the correct multiplier between the total numbers of sweets using $a = 2b$ so $a = 20$ thereby moving from 5, 10 to 20, 10

P1 Uses the multiplier, $\times 4$ to get $w : x = 8 : 12$

A1 Combines the ratios to get a correct final answer (in simplest form)



Student Response B

Olivia and Jessica have in total half as many sweets as Fran and Gary have in total.

Fran and Gary share their sweets in the ratio 2:3

Olivia and Jessica share their sweets in the ratio 9:1

Fran got w sweets.

Gary got x sweets.

Olivia got y sweets.

Jessica got z sweets.

Find, in its simplest form, $w:x:y:z$

Fran : Gary

2 : 3

~~4~~ $\frac{2}{5}$ ~~6~~ $\frac{3}{5}$

Olivia : Jessica

9 : 1

$\frac{9}{10}$ $\frac{1}{10}$

$$\frac{2}{5} \xrightarrow{\times 2} \frac{4}{10}$$

$$\frac{3}{5} \xrightarrow{\times 2} \frac{6}{10}$$

$$\frac{4}{10} : \frac{6}{10} : \frac{9}{10} : \frac{1}{10}$$

$$\underbrace{4 : 6}_{10} : \underbrace{4.5 : 0.5}_5$$

$$4 : 6 : 4.5 : 0.5$$

3/4

Examiner Comments

P1 5 and 10 are given as part of the fractions

P1 Given for the process of finding four numbers such that the total for Oliver and Jessica is half of the total for Fran and Gary, that is 4, 6 and 4.5, 0.5

P1 Correct ratio is seen but it is not in simplest form, 4 : 6 : 4.5 : 0.5

A0 The ratio given is not in simplest form



Student Response C

Olivia and Jessica have in total half as many sweets as Fran and Gary have in total.

Fran and Gary share their sweets in the ratio 2:3

Olivia and Jessica share their sweets in the ratio 9:1

Fran got w sweets.

Gary got x sweets.

Olivia got y sweets.

Jessica got z sweets.

Find, in its simplest form, $w:x:y:z$

let Fran and Gary and Olivia and Jessica have $2x, 3x, 9x$ and x sweets respectively

$$2x + 3x = \cancel{9x + x} \quad 2(9x + x)$$

$$2x + 3x = 18x + 2x$$

1/4

Examiner Comments

P1 $2x + 3x$ and $9x + x$ is seen showing a start to find totals for the numbers of sweets for each pair

P0 An incorrect attempt to form an equation showing the relationship between the totals, as x has been used in both pairings

P0 No further progress is made in solving this problem

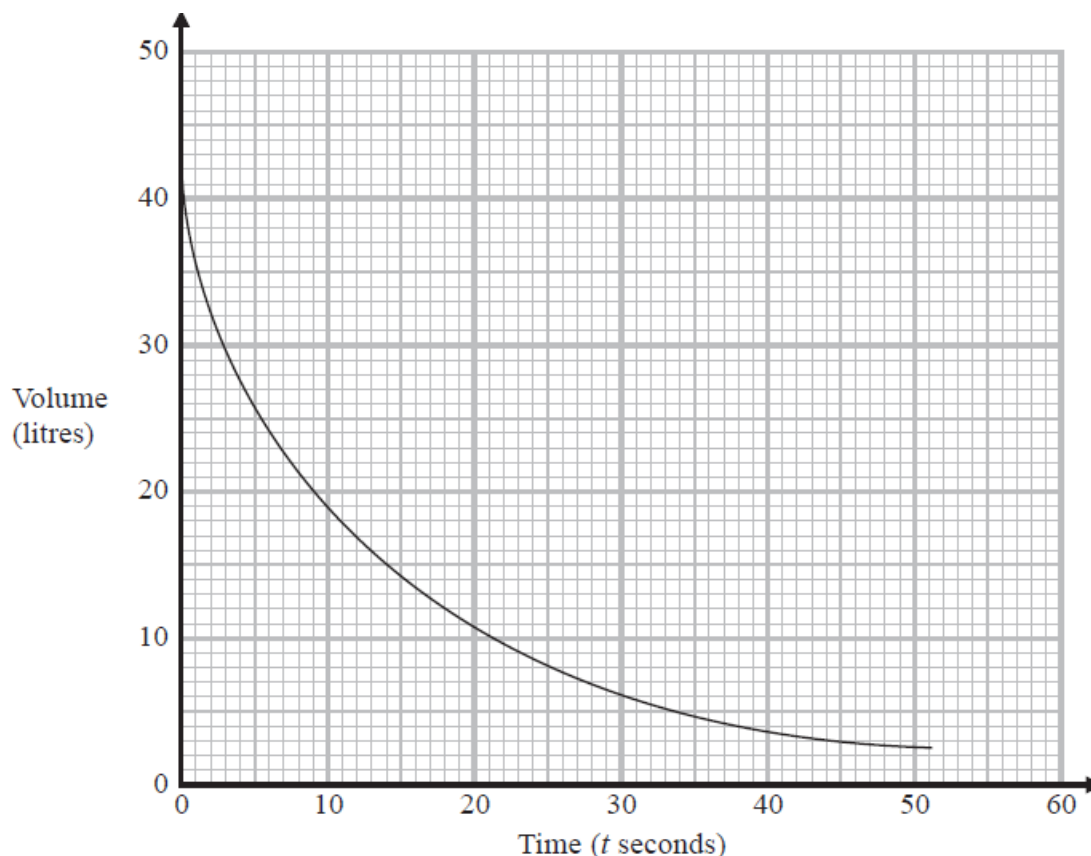
A0 No answer is given



Exemplar Question 5

Higher tier Paper 3

- 15 The graph gives the volume of water, in litres, in a container at time t seconds after the water started to flow out of the container.



Using the graph, work out an estimate for the rate at which the water is flowing out of the container when $t = 12$

You must show your working.

..... litres per second

(Total for Question 15 is 3 marks)

Mean Score: 0.43

Examiner Comments

The question is testing the appreciation that to get a rate of change at a particular point in time from a curve, a tangent needs to be drawn and its gradient calculated. Students are therefore expected to draw a tangent to the curve at the point where $t = 12$ and then calculate its gradient in order to work out an estimate for the rate at which the water is flowing at that time.

A common misconception is that to find the gradient at the point, you can divide the y coordinate by the x coordinate ($17 \div 12$). This is seen, sometimes even after a student has drawn a tangent.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
15	0.7 to 1.1	M1	for tangent to the curve drawn at $t = 12$	Working may be seen on the diagram Ignore negative signs
		M1	for method to find the gradient of their tangent, eg $28 \div 30$	
		A1	for answer in the range 0.7 to 1.1 dependent upon tangent drawn	

Examiner Comments

No marks are given for answers which do not include a tangent being drawn at the point on the curve where $t = 12$.

The first mark is given for drawing such a tangent. The second mark is for a correct method to calculate the gradient of the tangent. Evidence for this may be seen on the diagram, where for example, a triangle may be drawn and the differences in coordinates marked and divided, eg $\frac{28}{30}$.

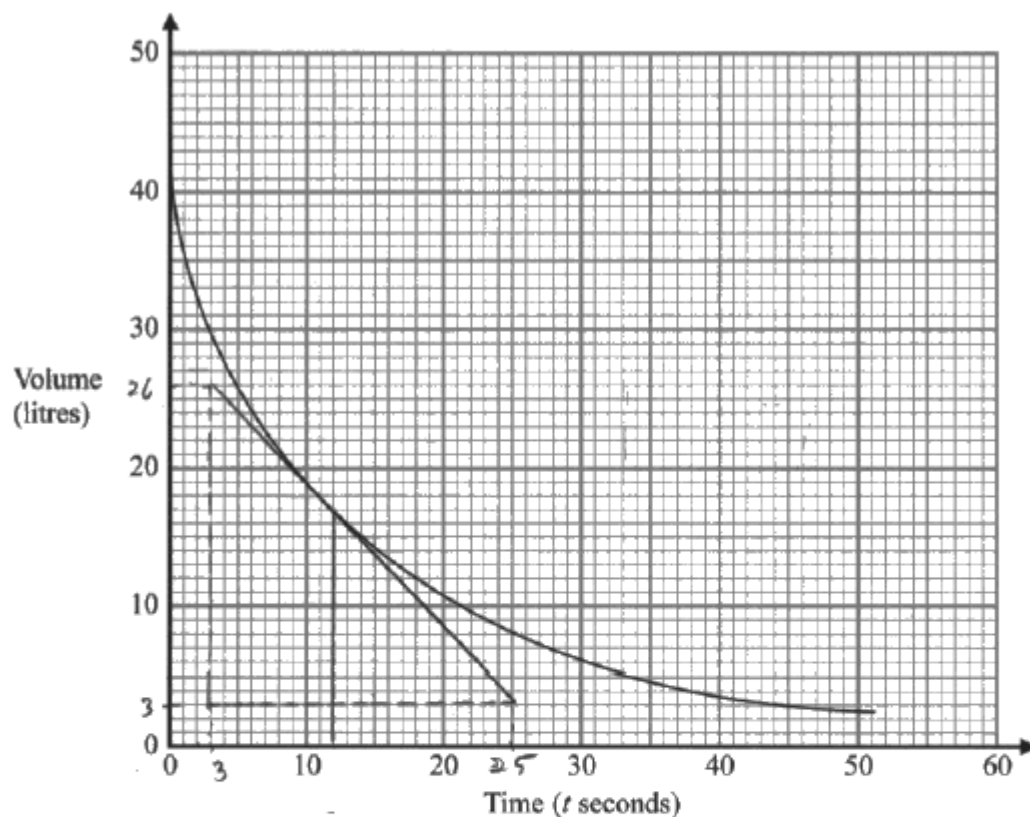
The final mark is given for a value in the range 0.7 to 1.1.

Numerical expressions used for finding the gradient where the numerator and/or denominator are negative or final answers with attached negative signs are given full credit so for example $-28 \div 30 = -0.93$ would be awarded full marks provided a suitable tangent has been drawn.



Student Response A

The graph gives the volume of water, in litres, in a container at time t seconds after the water started to flow out of the container.



Using the graph, work out an estimate for the rate at which the water is flowing out of the container when $t = 12$.

You must show your working.

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{26 - 3}{25 - 3}$$

$$= 1.045$$

$$= 1.05$$

1.05

litres per second

3/3

Examiner Comments

M1 A tangent is drawn at $t = 12$

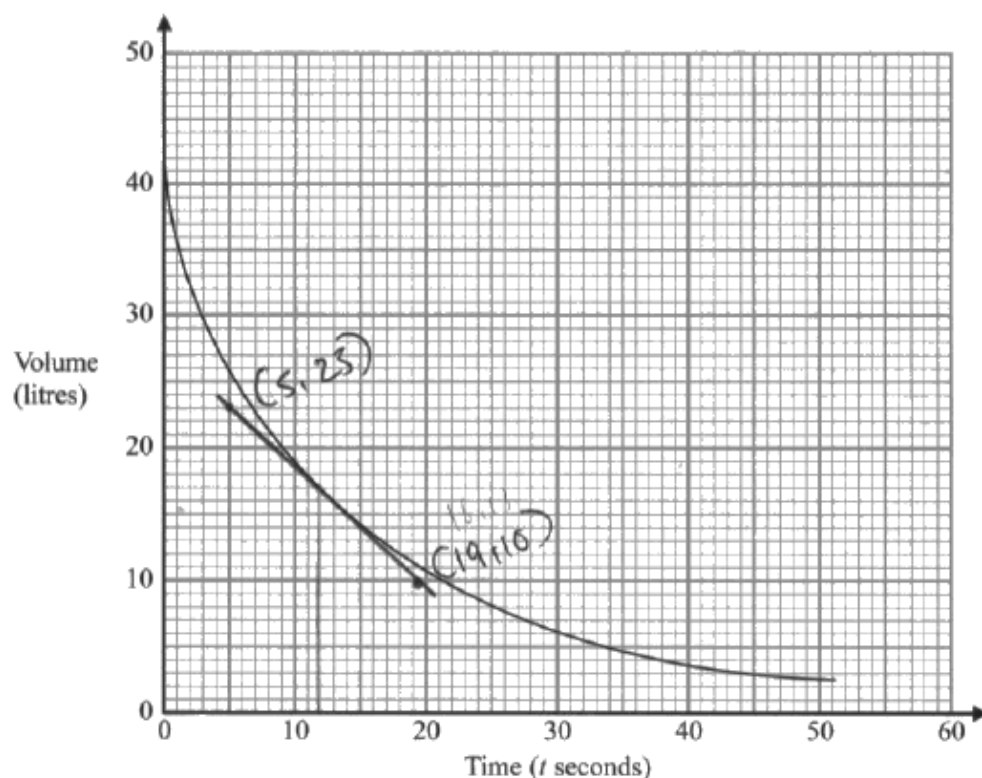
M1 A correct method to find the gradient, $\frac{26-3}{25-3}$ from the triangle drawn on their tangent is seen

A1 An answer within the range is given following a tangent having been drawn on the diagram



Student Response B

The graph gives the volume of water, in litres, in a container at time t seconds after the water started to flow out of the container.



Using the graph, work out an estimate for the rate at which the water is flowing out of the container when $t = 12$

You must show your working.

$$\frac{\Delta y}{\Delta x} \rightarrow \frac{23-10}{5-19} = \frac{13}{-4}$$

$$\frac{13}{-4} = -3.25$$

So every second, 3.25 were lost

3.25 litres per second

2/3

Examiner Comments

M1 A tangent is drawn at $t = 12$

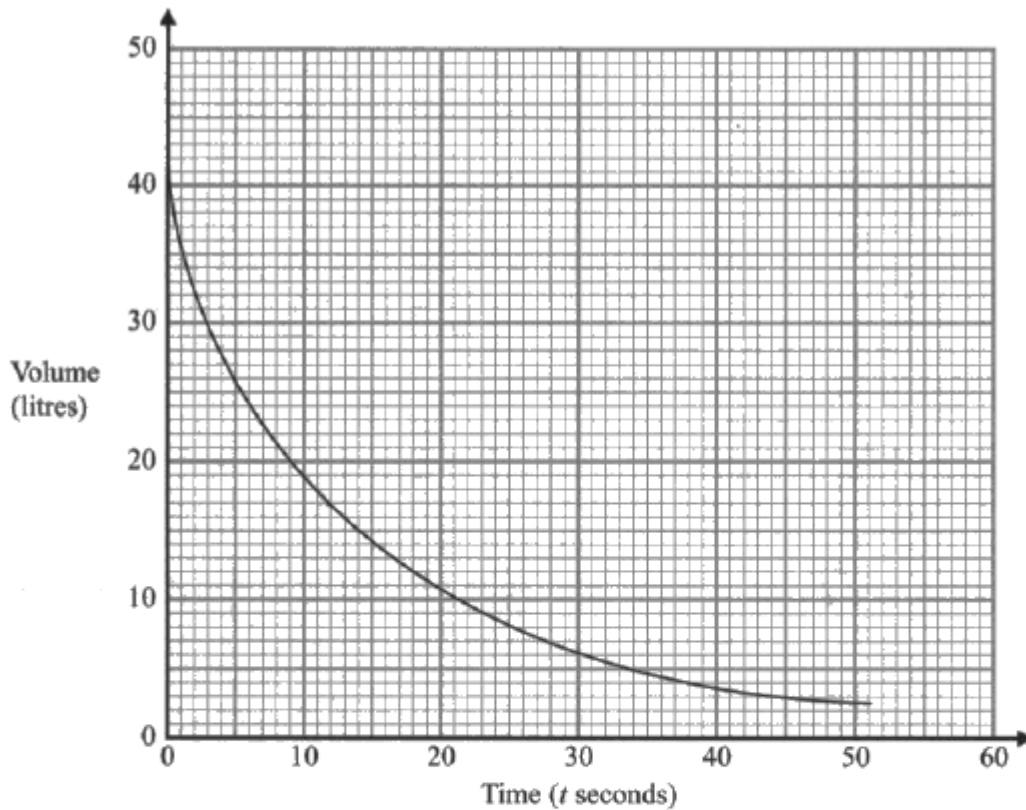
M1 A correct method is seen to find the gradient, $\frac{23-10}{5-19}$ from coordinates on their diagram. We ignore whether the numerator or denominator works out to be positive or negative.

A0 The answer does not lie within the range due to the incorrect evaluation of the denominator, $\frac{13}{-4}$. (Note that we ignore negative signs for the award of the final mark, so for example an answer of -0.93 from a tangent drawn would be awarded this mark.)



Student Response C

The graph gives the volume of water, in litres, in a container at time t seconds after the water started to flow out of the container.



Using the graph, work out an estimate for the rate at which the water is flowing out of the container when $t = 12$
You must show your working.

$$12 \text{ Seconds} = 17 \text{ Litres flowed out.}$$
$$\frac{17}{12} = 1.41\bar{6}$$

1.416 litres per second

0/3

Examiner Comments

M0 A tangent is not drawn so no marks can be awarded for this response.

M0 Here we see a common error, that of dividing the y coordinate by the x coordinate for the point at $t = 12$

A0 No marks can be given for any answer if a tangent has not been drawn



Exemplar Question 6

Higher tier Paper 3

16 The curve **C** has equation $y = x^2 + 3x - 3$

The line **L** has equation $y - 5x + 4 = 0$

Show, algebraically, that **C** and **L** have exactly one point in common.

(Total for Question 16 is 4 marks)

Mean Score: 0.59

Examiner Comments

This question demands students to follow a chain of reasoning in order to reach and state a given conclusion. Students are expected to eliminate one of the variables, probably y , to obtain an equation in x , collect the terms on one side of the equation and factorise and/or solve it to show there is only one (repeated) solution. This should lead to the conclusion that the curve and line have exactly one point in common. The most frequently seen errors involved mistakes made when students tried to manipulate the equations. Some students tried to solve the equation $x^2 + 3x - 3 = 0$. It was common for examiners to award 3 marks for students completing the algebra correctly but not the fourth communication mark because the student had not made a statement to conclude their findings.

When considering the next stage after obtaining the equation $x^2 - 2x + 1 = 0$, most students are expected to factorise the expression $x^2 - 2x + 1$ but the third mark could be awarded for students who use the quadratic formula correctly to find there is only one solution or to a student who shows that the discriminant of the equation, $b^2 - 4ac$, works out to be 0.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
16	Shown (supported)	M1	for eliminating y or x , eg $x^2 + 3x - 3 = 5x - 4$	There must be a statement in words for the award of this mark
		M1	for rearranging, collecting terms and setting to 0 eg $x^2 - 2x + 1 (= 0)$	
		M1	for factorising or solving eg $(x - 1)^2 (= 0)$	
		C1	for statement confirming only 1 point in common eg only 1 root or only 1 value of x so only 1 set of coordinates	

Examiner Comments

The first method mark is given for eliminating one variable from the 2 equations given to get one equation. The equation can be in any form at this stage but must be correct. The second method mark is for writing the equation in the form $f(x) = 0$ with all terms collected to give a quadratic equation ready for factorising or solving. Here we can accept omission of the “= 0” so, for example $x^2 - 2x + 1 = 0$ or $x^2 - 2x + 1$ is what we might expect to see for this mark. The third mark is for completing the algebra as far as is necessary to show there is only one solution. The final mark is a communication mark and is only awarded to students who provide some form of a conclusion in words confirming that their work shows that there is only one point in common.



Student Response A

The curve C has equation $y = x^2 + 3x - 3$

The line L has equation $y - 5x + 4 = 0$

Show, algebraically, that C and L have exactly one point in common.

$$y = x^2 + 3x - 3 \quad \text{①}$$

$$y = 5x - 4 \quad \text{②}$$

~~Sub ② into ①~~

Sub ② into ①.

$$x^2 + 3x - 3 = 5x - 4$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$x = 1 \quad \text{or} \quad x = 1$$

∴ C and L have exactly one point in common.

4/4

Examiner Comments

M1 y is eliminated from the two equations to give the correct equation in x , $x^2 + 3x - 3 = 5x - 4$

M1 The equation is rearranged and written in the form $ax^2 + bx + c = 0$. Note we can condone a missing “= 0” provided all terms are taken to one side and terms collected.

M1 The equation is factorised to $(x - 1)(x - 1)$

C1 A statement in words is awarded this communication mark



Student Response B

The curve C has equation $y = x^2 + 3x - 3$

The line L has equation $y - 5x + 4 = 0$

Show, algebraically, that C and L have exactly one point in common.

$$y = x^2 + 3x - 3$$

$$y - 5x + 4 = 0$$

~~$$y = 3x - 3$$~~

~~$$x^2 + 3x - 3$$~~
$$x^2 + 3x - 3 - 5x + 4 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1)$$

$$x = 1$$

$$\begin{array}{r} 1 \\ -1, 1- \end{array}$$

3/4

Examiner Comments

M1 eliminates y by substitution and sets up a correct equation in x , $x^2 + 3x - 3 - 5x + 4 = 0$. Note we can condone a missing “= 0”

M1 rearranges to get an equation of the form $x^2 - 2x + 1 (= 0)$. .

M1 factorises to $(x - 1)(x - 1)$

C0 does not provides a statement in words



Student Response C

The curve **C** has equation $y = x^2 + 3x - 3$

The line **L** has equation $y - 5x + 4 = 0$

Show, algebraically, that **C** and **L** have exactly one point in common.

$$y = x^2 + 3x - 3 \quad \text{--- (1)}$$

$$y - 5x + 4 = 0 \quad \text{--- (2)}$$

Sub (2) to (1)

$$x^2 + 3x - 3 - 5x + 4 = 0$$

$$x^2 - 2x - 1 = 0$$

∴

$$\therefore (-2)^2 - 4(1)(-1)$$

$$= 0$$

∴

∴ **C** and **L** have exactly one point in common

1/4

Examiner Comments

M1 A correct equation in x is set up, $x^2 + 3x - 3 - 5x + 4 (= 0)$

M0 The terms of the equation are collected to get an equation of the form $ax^2 + bx + c = 0$ but a mistake has been made, the equation $x^2 - 2x - 1 = 0$ is incorrect, and the mark cannot be awarded.

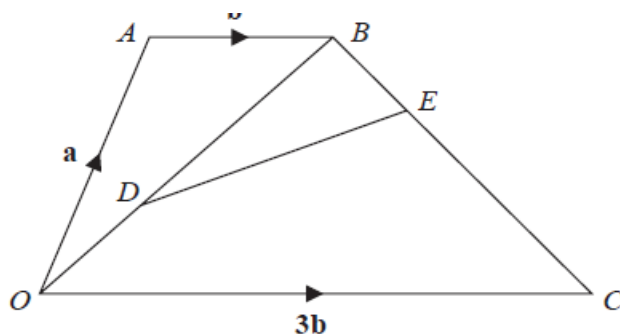
M0 The candidate tries to show that the discriminant $= 0$, a possible alternative method used by a significant number of students. However the equation is incorrect (and the discriminant is not 0) so this mark cannot be awarded.

C0 This mark cannot be awarded after incorrect working.

Exemplar Question 7

Higher tier Paper 3

18 $OABC$ is a trapezium.



$$\vec{OA} = \mathbf{a}$$

$$\vec{AB} = \mathbf{b}$$

$$\vec{OC} = 3\mathbf{b}$$

D is the point on OB such that $OD : DB = 2 : 3$

E is the point on BC such that $BE : EC = 1 : 4$

Work out the vector $\vec{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

(Total for Question 18 is 4 marks)

Mean Score: 0.51

Examiner Comments

In this question we are assessing the use and algebraic manipulation of vectors to find one vector, $\vec{DE}$ in terms of vectors $\mathbf{a}$ and $\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are defined as shown on a diagram. It is expected that students will consider how $\vec{DE}$ can be written as the sum of other vectors, for example as $\vec{DB} + \vec{BE}$. They should then find $\vec{DB}$ and $\vec{BE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and substitute into the expression $\vec{DB} + \vec{BE}$ before collecting terms to give an expression for $\vec{DE}$ in its simplest form.

Less successful attempts at answering the question often involved the incorrect use of the ratios given, for example quarters and thirds were often seen instead of fifths in expressions for $\vec{OD}$, $\vec{DB}$ and $\vec{BE}$.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18	$\frac{2}{5}\mathbf{a} + \mathbf{b}$	P1	for relationship involving D eg $\overrightarrow{OD} = \frac{2}{5}\overrightarrow{OB}$ or $\overrightarrow{DB} = \frac{3}{5}\overrightarrow{OB}$ or for relationship involving E eg $\overrightarrow{BE} = \frac{1}{5}\overrightarrow{BC}$ or $\overrightarrow{EC} = \frac{4}{5}\overrightarrow{BC}$	
		P1	for relationship involving D in terms of $\mathbf{a}$ and $\mathbf{b}$ eg $\overrightarrow{OD} = \frac{2}{5}(\mathbf{a} + \mathbf{b})$ or $\overrightarrow{DB} = \frac{3}{5}(\mathbf{a} + \mathbf{b})$ or for relationship involving E in terms of $\mathbf{a}$ and $\mathbf{b}$ eg $\overrightarrow{BE} = \frac{1}{5}(-\mathbf{b} - \mathbf{a} + 3\mathbf{b})$ oe or $\overrightarrow{EC} = \frac{4}{5}(-\mathbf{b} - \mathbf{a} + 3\mathbf{b})$ oe or $\overrightarrow{BC} = 2\mathbf{b} - \mathbf{a}$ oe or $\overrightarrow{CB} = \mathbf{a} - 2\mathbf{b}$ oe	
		P1	(dep P2) for expression for $\overrightarrow{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ eg $\overrightarrow{DE} = \frac{3}{5}(\mathbf{a} + \mathbf{b}) + \frac{1}{5}(-\mathbf{b} - \mathbf{a} + 3\mathbf{b})$	
		A1	for $\frac{2}{5}\mathbf{a} + (1)\mathbf{b}$ or $\frac{1}{5}(2\mathbf{a} + 5\mathbf{b})$	

Examiner Comments

The most obvious way to find an expression for $\overrightarrow{DE}$ is to use $\overrightarrow{DB} + \overrightarrow{BE}$ so the first process mark is likely to be awarded to a student for using one of the ratios given to express $\overrightarrow{DB}$ or $\overrightarrow{BE}$ as a fraction of $\overrightarrow{OB}$ or $\overrightarrow{BC}$ respectively. Other possibilities for a relationship involving D or E also qualify for the award of this mark. The second process mark is for writing their relationship in terms of $\mathbf{a}$ and $\mathbf{b}$. Alternatively, the second process mark can be earned by expressing $\overrightarrow{BC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. It should be noted that these expressions need not be written in simplest form.

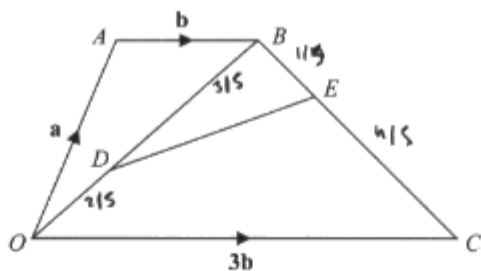
The third process mark, which can only be awarded if the first two marks have been given, is for any correct expression for $\overrightarrow{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ in unsimplified form.

The final accuracy mark is for a correct expression for $\overrightarrow{DE}$ in simplest form



Student Response A

$OABC$ is a trapezium.



$$\vec{OA} = \mathbf{a}$$

$$\vec{AB} = \mathbf{b}$$

$$\vec{OC} = 3\mathbf{b}$$

D is the point on OB such that $OD:DB = 2:3$

E is the point on BC such that $BE:EC = 1:4$

Work out the vector $\vec{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

$$\vec{DE} = \vec{DB} + \vec{BE}$$

$$\begin{aligned}\vec{BC} &= -\mathbf{b} - \mathbf{a} + 3\mathbf{b} \\ &= -\mathbf{a} + 2\mathbf{b}\end{aligned}$$

$$\begin{aligned}\vec{BE} &= \frac{1}{5}(-\mathbf{a} + 2\mathbf{b}) \\ &= -\frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}\end{aligned}$$

$$\vec{OB} = \mathbf{a} + \mathbf{b}$$

$$\begin{aligned}\vec{DB} &= \frac{3}{5}(\mathbf{a} + \mathbf{b}) \\ &= \frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}\end{aligned}$$

$$\vec{DE} = \vec{DB} + \vec{BE}$$

$$\vec{DE} = \frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$$

$$\vec{DE} = \frac{2}{5}\mathbf{a} + \frac{5}{5}\mathbf{b}$$

$$\vec{DE} = \frac{2}{5}\mathbf{a} + \mathbf{b}$$

$$\frac{2}{5}\mathbf{a} + \mathbf{b}$$

4/4

Examiner Comments

P1 P1 These 2 marks are given for $\vec{DB} = \frac{3}{5}(\mathbf{a} + \mathbf{b})$ or for $\vec{BE} = \frac{1}{5}(-\mathbf{a} + 2\mathbf{b})$. This includes the first process mark for $\vec{DB} = \frac{3}{5}\vec{OB}$ or for $\vec{BE} = \frac{1}{5}\vec{BC}$.

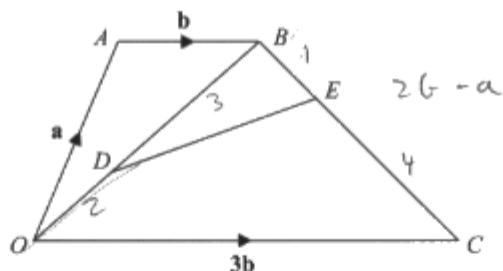
P1 A correct expression for $\vec{DE}$, $\frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ is given

A1 $\frac{2}{5}\mathbf{a} + \mathbf{b}$ is correct



Student Response B

$OABC$ is a trapezium.



$$\begin{aligned}\vec{OA} &= \mathbf{a} \\ \vec{AB} &= \mathbf{b} \\ \vec{OC} &= 3\mathbf{b}\end{aligned}$$

D is the point on OB such that $OD:DB = 2:3$

E is the point on BC such that $BE:EC = 1:4$

Work out the vector $\vec{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Give your answer in its simplest form.

$$\vec{BC} = -\mathbf{b} - \mathbf{a} + 3\mathbf{b} = \underline{\underline{2\mathbf{b} - \mathbf{a}}}$$

$$\vec{OB} = \mathbf{a} + \mathbf{b}$$

$$\vec{OD} = \frac{2}{5}(\mathbf{a} + \mathbf{b})$$

$$\vec{DE} = -\frac{2}{5}(\mathbf{a} + \mathbf{b}) + 3\mathbf{b} - \frac{4}{5}(2\mathbf{b} - \mathbf{a})$$

$$\vec{DE} = 3\mathbf{b} - \frac{6}{5}(\mathbf{a} + \mathbf{b})$$

$$\vec{DE} = 3\mathbf{b} - \frac{6}{5}\mathbf{a} - \frac{6}{5}\mathbf{b}$$

$$\vec{DE} = \frac{9}{5}\mathbf{b} - \frac{6}{5}\mathbf{a}$$

$$\vec{DE} = \frac{1}{5}(9\mathbf{b} - 6\mathbf{a})$$

$$\frac{1}{5}(9\mathbf{b} - 6\mathbf{a})$$

3/4

Examiner Comments

P1 P1 $\vec{OD} = \frac{2}{5}(\mathbf{a} + \mathbf{b})$ is awarded two marks.

P1..A correct expression for $\vec{DE}$, $\vec{DE} = -\frac{2}{5}(\mathbf{a} + \mathbf{b}) + 3\mathbf{b} - \frac{4}{5}(2\mathbf{b} - \mathbf{a})$

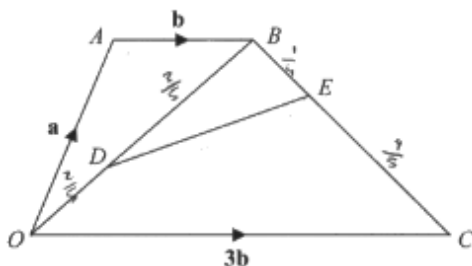
from $\vec{DE} = \vec{DO} + \vec{OC} + \vec{CE}$ gets this mark

A0..An incorrect final answer from incorrect vector algebra so this mark is not awarded



Student Response C

$OABC$ is a trapezium.



$$\begin{aligned}\vec{OA} &= \mathbf{a} \\ \vec{AB} &= \mathbf{b} \\ \vec{OC} &= 3\mathbf{b}\end{aligned}$$

D is the point on OB such that $OD:DB = 2:3$
 E is the point on BC such that $BE:EC = 1:4$

Work out the vector $\vec{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
 Give your answer in its simplest form.

$$\begin{aligned}\vec{OA} &= \mathbf{a} \\ \vec{AO} &= -\mathbf{a} \\ \vec{AB} &= \mathbf{b} \\ \vec{BA} &= -\mathbf{b} \\ [\vec{BO} &= -\mathbf{b} - \mathbf{a}] \\ \vec{OB} &= \mathbf{a} + \mathbf{b} \\ \vec{BC} &= -\mathbf{b} - \mathbf{a} + 3\mathbf{b} \\ [\vec{BD} &= -\mathbf{a} - \mathbf{b} + \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}] \\ [\vec{EO} &= -\frac{4}{5}\mathbf{a} - \frac{1}{4}\mathbf{b} - \mathbf{b} - \mathbf{a}] \\ \vec{CB} &= -3\mathbf{b} - \mathbf{a} - \mathbf{b} \\ [\vec{DO} &= -\frac{3}{5}\mathbf{b} - \mathbf{b} - \mathbf{a}]\end{aligned}$$

1/4

Examiner Comments

P0 The student starts off well giving correct expressions for $\vec{OA}$, $\vec{AO}$, $\vec{AB}$, $\vec{BA}$, $\vec{BO}$ and $\vec{OB}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$. However, these in themselves are not enough to earn the first process mark.

P1 $\vec{BC} = -\mathbf{b} - \mathbf{a} + 3\mathbf{b}$ scores this mark

P0 The other expressions given are all incorrect so no further marks can be awarded

A0 No expression for $\vec{DE}$ is given



Exemplar Question 8

Higher tier Paper 3

- 19 At the start of year n , the number of animals in a population is P_n

At the start of the following year, the number of animals in the population is P_{n+1} where

$$P_{n+1} = kP_n$$

At the start of 2017 the number of animals in the population was 4000

At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k .

.....
(Total for Question 19 is 3 marks)

Mean Score: 0.33

Examiner Comments

An understanding of the notation involved in iterative processes and some familiarity with their uses is expected in order to answer this question. The key to this question is for students to realise that the relationship $P_{n+1} = kP_n$ links one year to the next whereas the statements about the numbers of animals are for 2017 and 2019, two years apart. Higher attaining students may be able to write down the relationship $P_{n+2} = k^2P_n$ before substituting 3610 and 4000 for P_{n+2} and P_n respectively. However, most students are expected to write down two separate relationships, for example $P_{2018} = 4000k$ and $3610 = kP_{2018}$ before combining them to give $3610 = k^2 \times 4000$ and then go on to solve the equation to find the value of k . The incorrect equation $3610 = k \times 4000$ was often seen suggesting these students had either not read the question with enough care or did not have a good understanding of the processes involved.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19	0.95	P1	for initial use of the formula eg $3610 = kP_n$ or $P_{n+1} = 4000k$ or for $P_{n+2} = k^2P_n$ or for $3610 = k^2 \times 4000$	Accept n or any integer replacement for n
		P1	for a complete method to find k eg $\sqrt{\frac{3610}{4000}}$ or ± 0.95	This may be seen in steps
		A1	oe	

Examiner Comments

The first process mark is available to students who show they understand that the iterative relationship links consecutive years by writing down $3610 = kP_n$ or $P_{n+1} = 4000k$. Here n may or may not be replaced by an integer. Alternatively if a student writes down the correct relationship linking a population one year with a population two years ahead, for example $P_{n+2} = k^2P_n$

or $3610 = k^2 \times 4000$, they are given this first process mark. The second process mark is for a complete process to find the value of k . For this mark we can ignore any negative sign seen so for example, $k = \pm \sqrt{\frac{3610}{4000}}$ is awarded this second mark. The final accuracy mark is given for 0.95 or equivalent, eg $\frac{19}{20}$.



Student Response A

At the start of year n , the number of animals in a population is P_n

At the start of the following year, the number of animals in the population is P_{n+1} where

$$P_{n+1} = kP_n$$

At the start of 2017 the number of animals in the population was 4000

At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k .

$$P_{2019} =$$

$$3610 = k^2 \times 4000$$

$$k^2 = 0.9025$$

$$k = 0.95$$

$$k = 0.95$$

3/3

Examiner Comments

P1 The iterative relationship appears to be well understood and the student writes down a correct relationship of the form $P_{n+2} = k^2 P_n$ with the populations for 2019 and 2017 substituted.

P1 The statements $k^2 = 0.9025$ and $k = 0.95$ demonstrate that a complete method to find the value of k has been completed

A1 Correct answer



Student Response B

At the start of year n , the number of animals in a population is P_n

At the start of the following year, the number of animals in the population is P_{n+1} where

$$P_{n+1} = kP_n$$

At the start of 2017 the number of animals in the population was 4000

At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k .

$$4000 - 3610 = 390$$

$$P_{2017} = 4000$$

$$P_{2018} = k4000$$

$$P_{2019} = k(k4000) = k^2 4000 = 3610$$
$$\frac{3610}{k^2} = 4000$$

1/3

Examiner Comments

P1 The relationship $P_{n+1} = kP_n$ is understood and used correctly as shown by the statement

$$P_{2018} = k4000$$

P0 Though a correct equation in k is formulated, the processes to solve it are not completed.

A0 No answer given



Student Response C

At the start of year n , the number of animals in a population is P_n

At the start of the following year, the number of animals in the population is P_{n+1} where

$$P_{n+1} = kP_n$$

At the start of 2017 the number of animals in the population was 4000

At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k .

$$\begin{aligned} 1) P_{n+1} &= kP_n \\ 3610 &= 4000k \\ \frac{3610}{4000} &= k \\ k &= 0.9 \end{aligned}$$

0/3

Examiner Comments

P0 The student substitutes the populations for years 2017 and 2019 into the iterative relationship which links successive years. This was an error commonly seen.

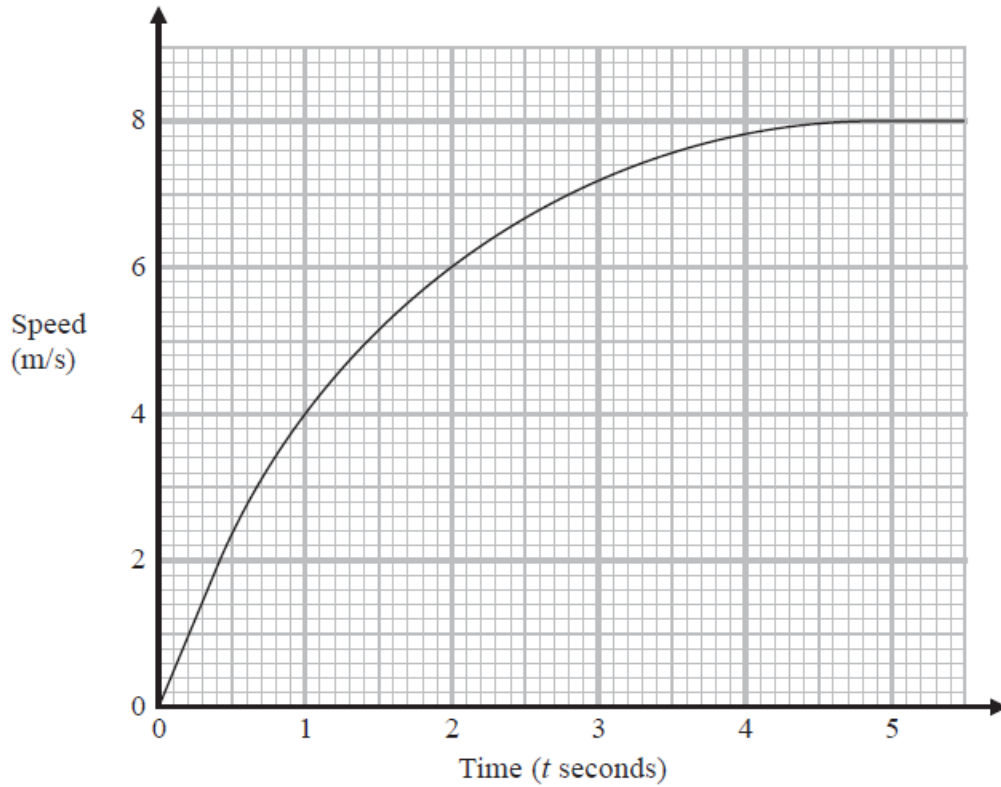
P0 The initial mistakes makes it impossible to complete a correct method to find the value of k .

A0 An incorrect answer. This is rounded from $\frac{3610}{4000} = 0.9025$ which was a frequently seen. incorrect expression for k .

Exemplar Question 9

Higher tier Paper 3

- 21 Here is a speed-time graph showing the speed, in metres per second, of an object t seconds after it started to move from rest.



- (a) Using 3 trapeziums of equal width, work out an estimate for the area under the graph between $t = 1$ and $t = 4$

.....

(3)

- (b) What does this area represent?

.....

(1)

(Total for Question 21 is 4 marks)

Mean Scores: Part (a) 0.52

Part (b) 0.35

**Examiner Comments**

In part (a) of this question students need to split the area between $t = 1$ and $t = 4$ into 3 trapeziums as stated in the question. They are expected to work out the area of each trapezium and add these together to find the total area. Students should show their working clearly. A common error seen included using incorrect or inaccurate y -values. In particular readings of 7.1 and 7.9 instead of 7.2 and 7.8 were commonly seen. Some students did not follow the instruction to use 3 trapeziums or to use trapeziums of equal width, while some students included the area between $t = 0$ and $t = 1$. In part (b) students are expected to explain that the area under the graph represents the distance in metres travelled by the object between 1 second and 4 seconds after it started to move from rest. A common misunderstanding was that some students believed the area represented the acceleration of the object.



Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
21 (a)	19.1	M1	for a method to find an estimate for the area of at least 1 trapezium under the curve eg $0.5 \times 1 \times (4 + 6)$ or $0.5 \times 1 \times (6 + 7.2)$ or $0.5 \times 1 \times (7.2 + 7.8)$	
		M1	for a complete method eg $0.5 \times 1 \times (4 + 6) + 0.5 \times 1 \times (6 + 7.2) + 0.5 \times 1 \times (7.2 + 7.8)$ or $0.5 \{(4 + 7.8) + 2(6 + 7.2)\}$	Allow a maximum of 2 errors in y values used
		A1	cao	Ignore any reference to units
(b)	Statement	C1	eg distance (travelled)	If units are given they must be correct

Examiner Comments

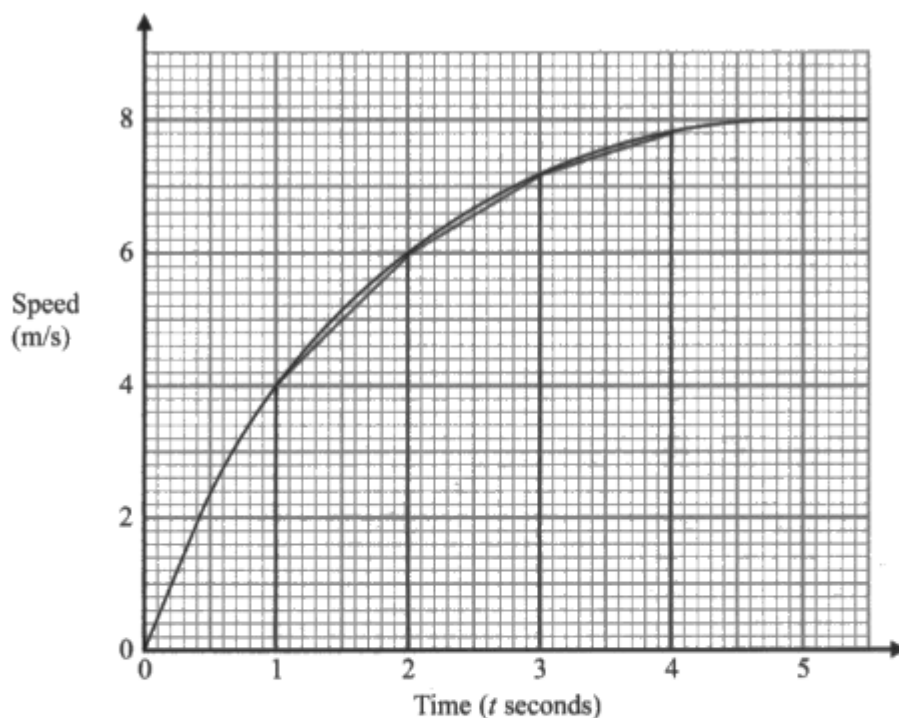
Full marks in part (a) are awarded for an answer of 19.1. If the correct answer is not given, examiners are looking to award up to 2 method marks to students who have shown clear working to find the areas of the three trapeziums of width 1 unit between $t = 1$ and $t = 4$. The first mark is given for a clear method using accurate figures to find the area of one of the trapeziums. This may be shown as, for example, $\frac{1}{2} \times (4 + 6) \times 1$. Alternatively, a student may have found the area of a triangle and a rectangle but in order to get the mark they must show the addition of the two areas. The second mark is given for a complete method to find the area required, that is the sum of the areas of the three trapeziums. For the award of this mark, up to two errors may be seen in the y-values being used.

In part (b) we need to see “distance” or equivalent, for example “how far the object travelled”. Examiners did not require reference to the time period involved or a statement of the units (metres). However, if further explanation is contradictory or if incorrect units are given, the mark for part (b) answers may be withheld.



Student Response A

Here is a speed-time graph showing the speed, in metres per second, of an object t seconds after it started to move from rest.



- (a) Using 3 trapeziums of equal width, work out an estimate for the area under the graph between $t = 1$ and $t = 4$

$$\frac{1}{2}(4 + 6) \times 1 = 5$$

$$\frac{1}{2}(6 + 7.2) \times 1 = 6.6$$

$$\frac{1}{2}(7.2 + 7.8) \times 1 = 7.5$$

$$\Rightarrow 5 + 6.6 + 7.5 = 19.1$$

(3)

3/3

Examiner Comments

Part (a)

M1..Correct expressions are seen for the area of all three trapeziums. Any one of these would lead to this first mark being awarded.

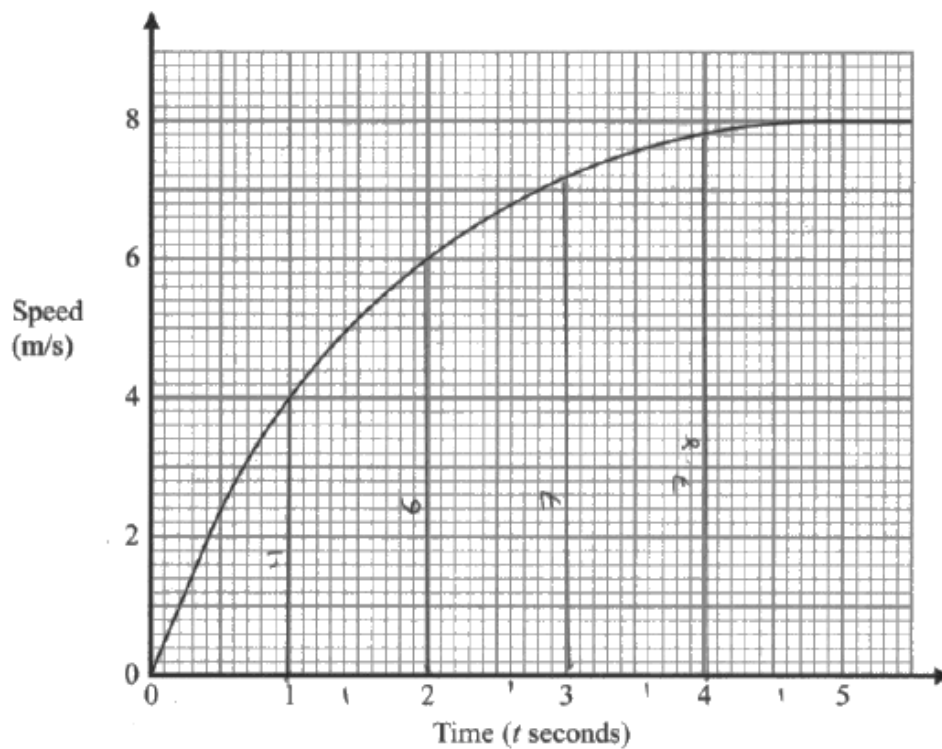
M1 Expressions for the three areas are all correct and when evaluated, the results are added. This leads to the second mark being awarded.

A1 This mark is awarded for the correct answer, 19.1



Student Response B

Here is a speed-time graph showing the speed, in metres per second, of an object t seconds after it started to move from rest.



- (a) Using 3 trapeziums of equal width, work out an estimate for the area under the graph between $t = 1$ and $t = 4$

$$\frac{1}{2}(a+b)h$$
$$\frac{1}{2}(4+6) \times 1 + \frac{1}{2}(6+7) \times 1 + \frac{1}{2}(7+7.8) \times 1 = 18.9$$

$$\begin{array}{r} 19 \\ 18.9 \\ \hline (3) \end{array}$$

2/3

Examiner Comments

Part (a)

M1 The expression $\frac{1}{2}(4+6) \times 1$ earns this first mark.

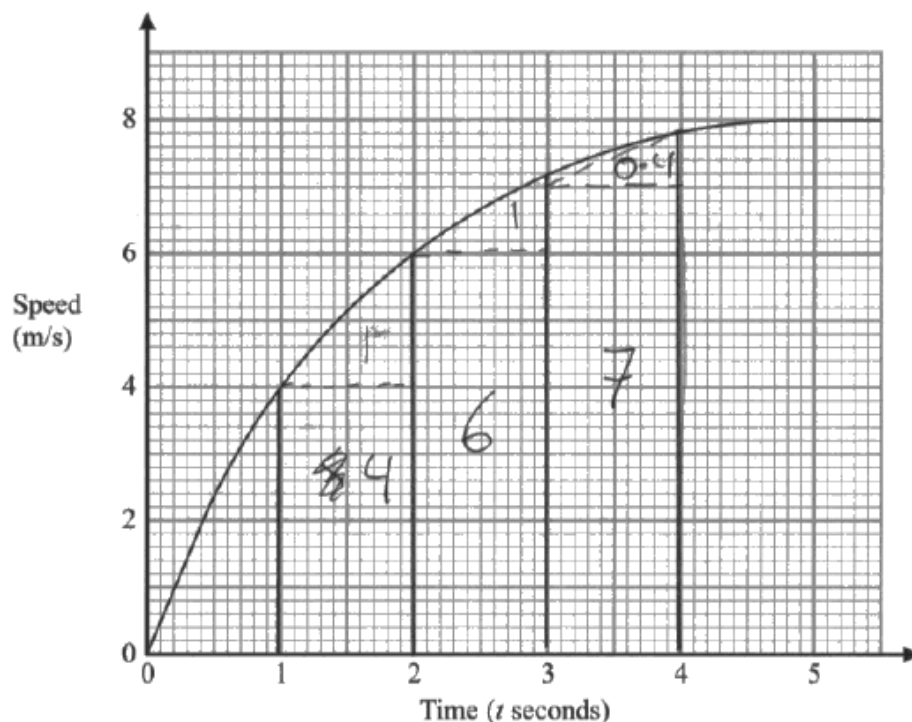
M1 A complete method showing the addition of the three areas with only one inaccurate reading is seen so this second mark is awarded.

A0 The answer is not correct.



Student Response C

Here is a speed-time graph showing the speed, in metres per second, of an object t seconds after it started to move from rest.



- (a) Using 3 trapeziums of equal width, work out an estimate for the area under the graph between $t = 1$ and $t = 4$

$$5 + 7 + 7.4 = 19.4 \text{ m}^2$$

$$19.4 \text{ m}^2$$

(3)

1/3

Examiner Comments

Part (a)

M1 The student has used a triangle and a rectangle to work out the area of each trapezium. Sight of $4 + 1$ or 5 is sufficient for the award of this mark. Note that this mark could not have been given for $6 + 1$ or 7 or for $7 + 0.4$ or 7.4 because these figures are incorrect and the student has not shown a full method.

M0 As 7 and 7.4 are incorrect and there is no method shown to illustrate where these figures come from, this mark cannot be given

A0 The answer is incorrect.



Student Response A

(b) What does this area represent?

The total ^{distance} ~~distance~~ travelled between 1 and 4 seconds

(1)

1/1

Examiner Comments

Part (b)

C1 We accept “distance” or equivalent as the minimum required for answers to this part of the question. We do not need reference to the time period 1 to 4 seconds.



Student Response B

(b) What does this area represent?

Distance in metres.

(1)

1/1

Examiner Comments

Part (b)

C1 Acceptable answer. We do not need to see “metres” but if units are given, they must be correct.



Student Response C

(b) What does this area represent?

The acceleration of the object.

(1)

0/1

Examiner Comments

Part (b)

C0 A common incorrect answer