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Level 1/Level 2

GCSE (9-1) in Mathematics (1MA1)

SUMMER 2018 EXEMPLAR STUDENT ANSWERS WITH EXAMINER COMMENTS – HIGHER

First certification 2017

ALWAYS LEARNING

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Contents

About this booklet

This booklet has been produced to support mathematics teachers delivering the new GCSE (9 – 1) Mathematics specification.

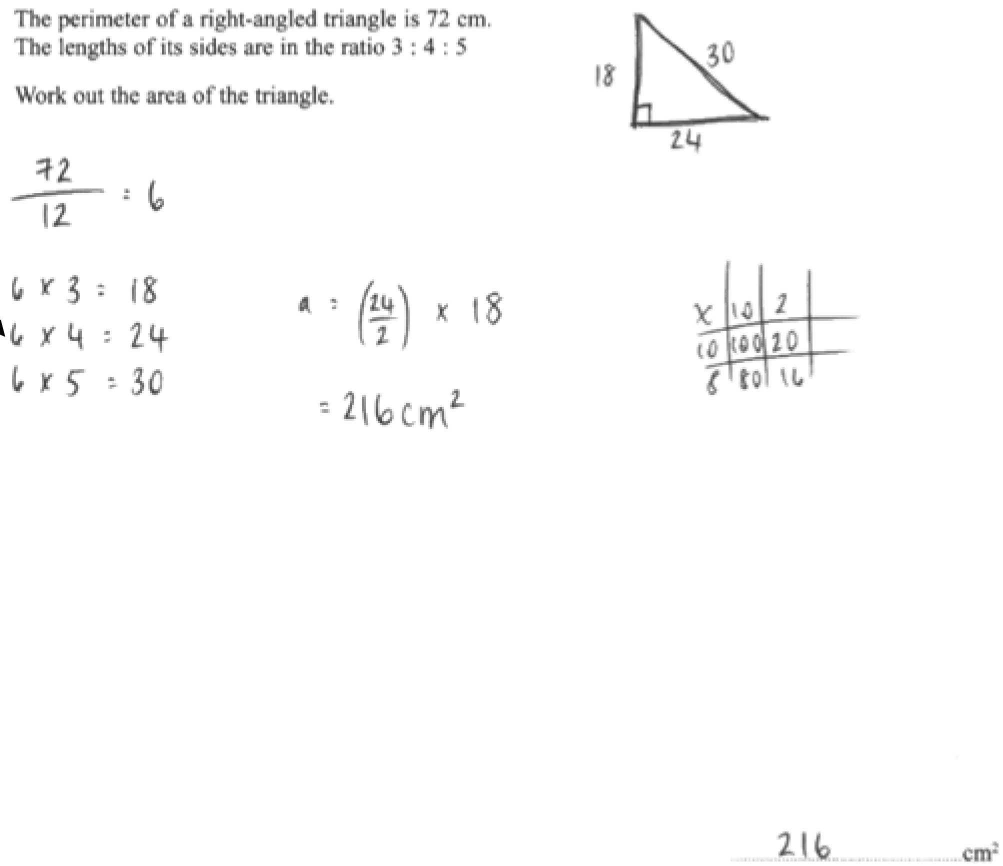
The booklet looks at a selection of questions from the Summer 2018 GCSE (9–1) Mathematics Higher tier examination. It shows real student responses to selected questions and how the examining team follow the mark schemes to demonstrate how the students would be awarded marks on these questions.

How to use this booklet

Our examining team have selected student responses to higher tier questions from the Summer 2018 examination. Following each question you will find the mark scheme for that question and then a range of student responses with accompanying examiner comments on how the mark scheme has been applied and the marks awarded, and on common errors for this sort of question.

Student response A

Student response



4/4

Examiner Comments

This is a well presented solution. The student divides 72 in the ratio 3 : 4 : 5 to find the lengths of the sides of the triangle and identifies, with the help of a diagram, that the sides with lengths 18 cm and 24 cm should be used to find the area of the triangle. The majority of students chose to multiply 18 by 24 and then divide by 2 but this student sensibly divides 24 by 2 before multiplying by 18.

Examiner commentary on the student response

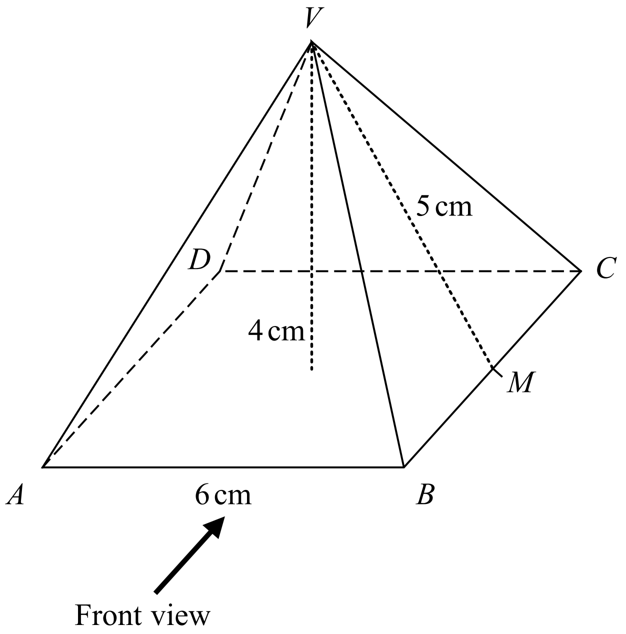
Marks awarded for the question or question parts

Paper 1H (non – calculator)

Exemplar Question 1

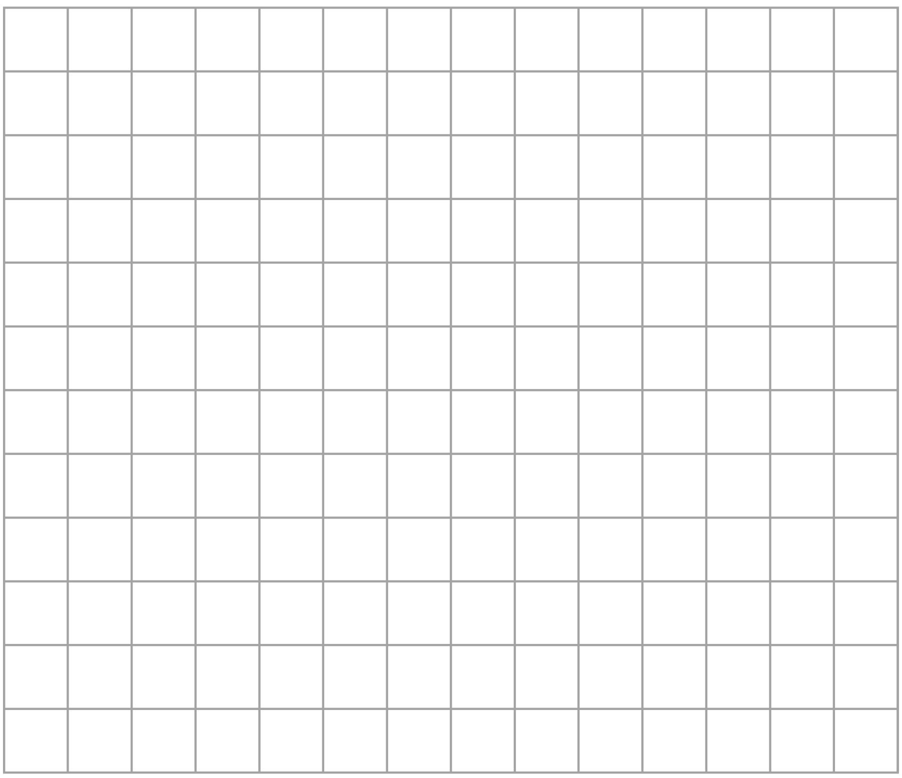
Higher tier Paper 1

5 Here is a solid square-based pyramid, *VABCD*.



The base of the pyramid is a square of side 6 cm.
The height of the pyramid is 4 cm.
M is the midpoint of *BC* and *VM* = 5 cm.

(*a*) Draw an accurate front elevation of the pyramid from the direction of the arrow.



(2)

(*b*) Work out the total surface area of the pyramid.

....

(4)

(Total for Question 5 is 6 marks)

Examiner Comments

In part (a) students are required to interpret the diagram of the square-based pyramid in order to draw an accurate front elevation. They are expected to realise that the front elevation will be in the shape of an isosceles triangle. A common mistake was to draw the triangle with an incorrect height, often 5 cm, which meant that only one mark was awarded. When the isosceles triangle was drawn as part of a 3-D shape or as part of a net then no marks were awarded.

Part (b) is a routine question that assesses the ability of students to work out the total surface area of a pyramid. The question also requires students to give the correct units with their answer. Students are expected to know that the total surface area of the pyramid is the sum of the areas of the five faces.

A common mistake was to use 4 cm, rather than 5 cm, as the height of a triangular face. Some students gave no units with their answer and some wrote cm or cm^3 instead of cm^2.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
5(a)	isosceles triangle, base 6 cm, height 4 cm	M1	for drawing an isosceles triangle **or** for drawing a triangle of base 6cm and height 4cm	Accept a freehand drawing Only a single triangle is acceptable; do **not** accept any attempted nets or 3-D diagrams
		A1	for a fully correct diagram	Condone a perpendicular drawn from base to vertex
(b)	96 cm^2	M1	for a method to find the area of a triangular face e.g. ½ × 6 × 5 (= 15)	
		M1	(dep) for finding the total surface area e.g. 4 × "15" + 6 × 6	
		A1	for a numerical answer of 96	Ignore incorrect or absent units for this mark
			SC B1 for an answer of 84 if M0 scored	[The SC is from: 4 × ½ × 6 × 4 + 6 × 6]
		B1	cm^2	Ignore incorrect or absent numerical answer for this mark

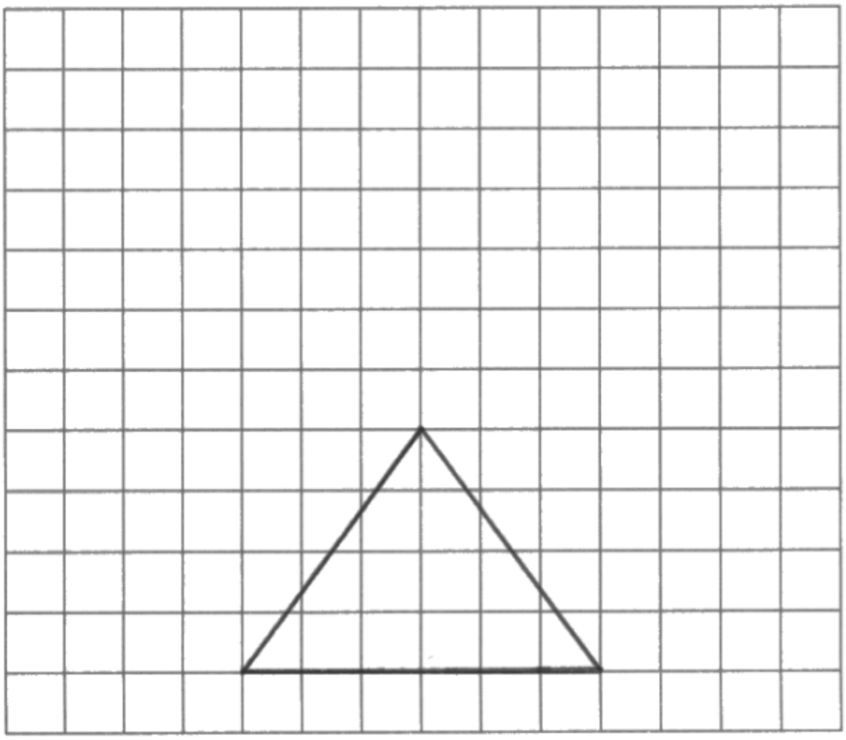
Examiner Comments

Part (a) A freehand drawing is acceptable. No marks are awarded if the triangle is drawn as part of a 3-D shape or as part of a net.

Part (b) The second method mark is dependent on the first mark and "15" has to come from a correct process to find the area of a triangular face. The special case enables students who use 4 cm rather than 5 cm as the height of a triangular face to be awarded one mark if they complete the rest of the method correctly and give an answer of 84. The final mark is for giving the correct units, cm^2. Incorrect or absent numerical answers are ignored when awarding this mark.

Student Response A

(a) Draw an accurate front elevation of the pyramid from the direction of the arrow.



(2) **2/2**

(b) Work out the total surface area of the pyramid.

6 × 6 = 36

5 × 6 = 30 ÷ 2 = 15

15 × 4 = 60

36 + 60 = 96

96 cm²

(4) **4/4**

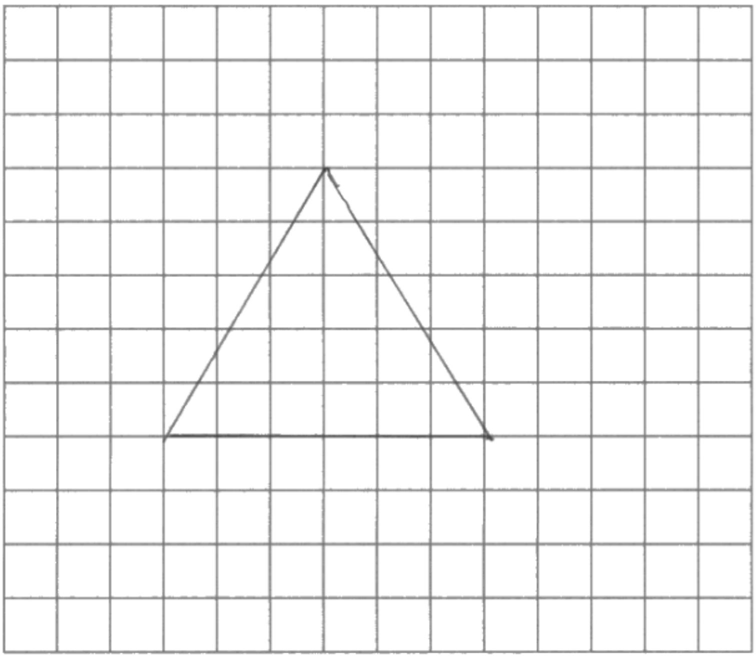
Examiner Comments

Part (a) The diagram is correct – an isosceles triangle of base 6 cm and height 4 cm.

Part (b) A correct response with each stage of working clearly shown. The correct units have been given with the total surface area.

Student Response B

(a) Draw an accurate front elevation of the pyramid from the direction of the arrow.



(2)

1/2

(b) Work out the total surface area of the pyramid.

base = 6×6 = 36cm   1 Side = 5×3 = 15
×4 = 70
4 Sides + base = 70 + 36
= 106

106
(4)

2/4

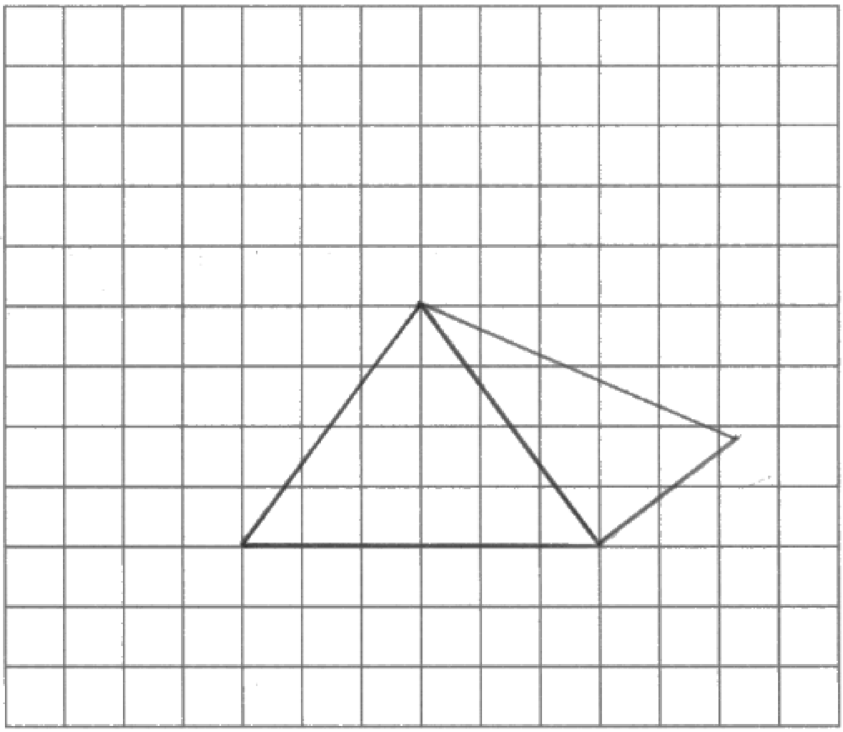
Examiner Comments

Part (a) One mark is awarded for a drawing of an isosceles triangle. The base is correct but the height is not. Drawing a triangle with a height of 5 cm was a common mistake.

Part (b) The student works out the area of a triangular face and uses a correct method to find the total surface area. An arithmetic error, $15 \times 4 = 70$, means that the accuracy mark is lost. No units have been given with the final answer.

Student Response C

(a) Draw an accurate front elevation of the pyramid from the direction of the arrow.



(2)

0/2

(b) Work out the total surface area of the pyramid.

Area of base = 6×6 = 36

Area of triangle faces = $\frac{1}{2}$bh = 12

6×4 = 24

$\frac{1}{2}$ 24 = 12

12
12
12
12
48

36
48
84

84 cm³

(4)

1/4

Examiner Comments

Part (a) An isosceles triangle with base 6 cm and height 4 cm has been drawn but this is part of a sketch of a 3-D shape. No mark can be awarded.

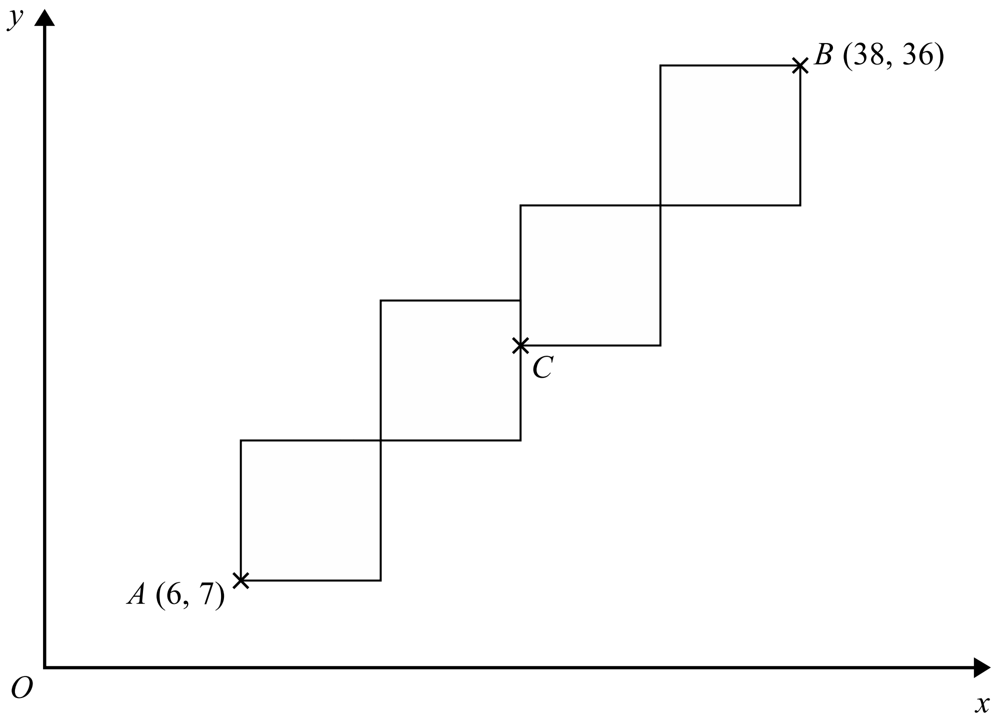
Part (b) The student has used 4 cm, not 5 cm, as the height of a triangular face in an otherwise correct method. This is covered by the special case in the mark scheme - an answer of 84 gets one mark. Incorrect units have been given with the final answer.

Exemplar Question 2

Higher tier Paper 1

6 A pattern is made from four identical squares.

The sides of the squares are parallel to the axes.



Point A has coordinates (6, 7)
Point B has coordinates (38, 36)
Point C is marked on the diagram.

Work out the coordinates of C.

(.............................. , ...)

(Total for Question 6 is 5 marks)

Examiner Comments

In this question students need to devise a suitable strategy to solve a problem that is set in a geometrical context on coordinate axes. Students are expected to identify the processes that are necessary to find the coordinates of C and to carry them out. There are several different strategies that can be used to answer this question.

A common mistake was for students to assume that the same strategy would work for both the x coordinate and the y coordinate. Finding the midpoint of AB, for example, gives the correct x coordinate but it does not help students to find the correct y coordinate.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
6	(22, 20)	P1	for process to find width or height of diagram e.g. 38 − 6 (= 32) **or** 36 – 7 (= 29)	Figures may be shown on the diagram
		P1	for process to find length of side of square e.g. "32" ÷ 4 (= 8) **or** process to find half width of diagram e.g. "32" ÷ 2 (= 16)	
		P1	for process to find x coordinate e.g. 6 + 2 × "8" (= 22) **or** 6 + "16" (= 22) **or** (6 + 38) ÷ 2 (= 22)	If (6 + 38) ÷ 2 leads to an answer other than 22, award P2 only
		P1	for process to find y coordinate e.g. 36 – 2 × "8" (= 20) **or** 36 – "16" (= 20) **or** 7 + "8" + "29" – 3 × "8" (= 20)	
		A1	cao SC: award 4 marks for (20, 22)	Award for P3 for (22, y) **or** (x, 20) **or** x = 22 or y = 20

Examiner Comments

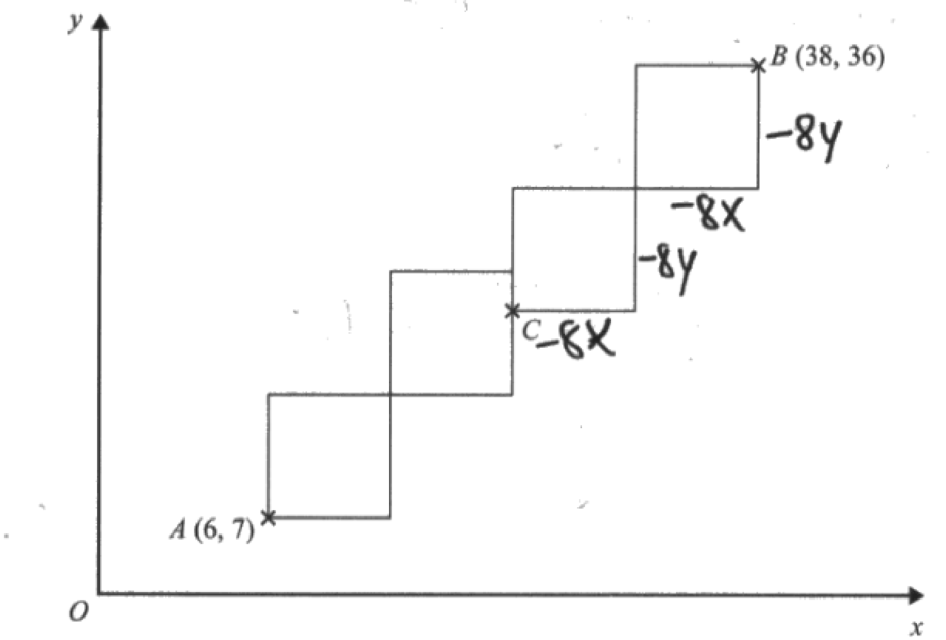
The mark scheme for this question makes extensive use of inverted commas. Any number in inverted commas must come from a correct process but the students may make arithmetic errors in their working. Answers with either the correct x coordinate or the correct y coordinate can be awarded 3 marks.

Student Response A

6

A pattern is made from four identical squares.

The sides of the squares are parallel to the axes.



Point *A* has coordinates (6, 7)
Point *B* has coordinates (38, 36)
Point *C* is marked on the diagram.

Work out the coordinates of *C*.

38 − 6 = 32 = 4 sides of square
side of square = 8

38 − 16 = 22
36 − 16 = 20

(22, 20)

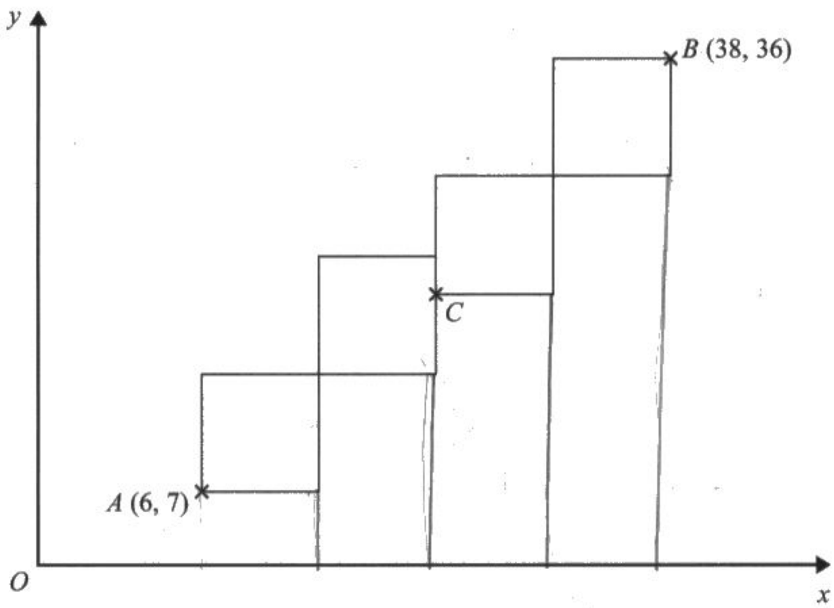
5/5

Examiner Comments

An efficient method leads to the correct coordinates of *C*.

Student Response B

6



Point A has coordinates (6, 7)
Point B has coordinates (38, 36)
Point C is marked on the diagram.

Work out the coordinates of C.

$\frac{3}{4} \times 8 = 6$

side of square = 8

$7 + 8 = 15$

$15 + 6 = 21$

$38 - 6 = 32$

$32 \div 4 = 8$

each square = 8

$6 + 8 = 14$

$14 + 8 = 22$

$x = 22$

~~36 − 7 = 29~~
~~29 ÷ 2~~
~~29 ÷ 2 = 14.5~~
~~3/4 × 14.5 = 29/2~~

(22 , 21)

3/5

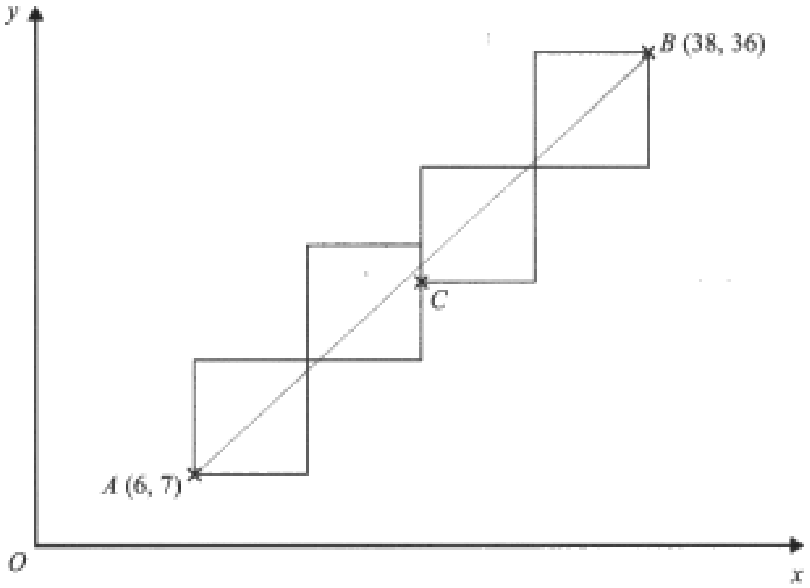
Examiner Comments

The first three marks are awarded for a correct process that leads to the x coordinate of C. To find the y coordinate of C some students worked out the vertical distance of C from the bottom of the second square, usually by considering the difference between $36 - 7 = 29$ and $3 \times 8 = 24$. This student incorrectly assumes that this vertical distance is $\frac{3}{4}$ of 8.

Student Response C

A pattern is made from four identical squares.

The sides of the squares are parallel to the axes.



Point *A* has coordinates (6, 7)
Point *B* has coordinates (38, 36)
Point *C* is marked on the diagram.

Work out the coordinates of *C*.

38 - 6 = 32 ÷ 2 = 16

36 - 7 = 29 ÷ 2 = 14.5

(16 , 14.5)

2/5

Examiner Comments

Either 38 – 6 or 36 – 7 is sufficient for the first mark and dividing 32 by 2 to find half the width of the diagram gets the second mark. The student is now in a good position to go on to find the coordinates of *C*, by working out 6 + 16 = 22 and 36 – 16 = 20 for example. Instead of using the value of 16 to find the coordinates of *C* the student divides 29 by 2 and gives (16, 14.5) as the final answer. This was a common mistake.

Exemplar Question 3

Higher tier Paper 1

8 The perimeter of a right-angled triangle is 72 cm.
The lengths of its sides are in the ratio 3 : 4 : 5

Work out the area of the triangle.

....cm^2

(Total for Question 8 is 4 marks)

Examiner Comments

This question assesses the ability of students to solve a problem by breaking it down into a series of mathematical processes. This is within the context of ratio and geometry.

To find the lengths of the sides of the triangle students need to demonstrate that they can divide a quantity in a given ratio. They need to interpret these lengths in the context of the problem and decide how to work out the area of the triangle.

Arithmetic errors, particularly in the area calculation, meant that the accuracy mark was often lost. When trying to evaluate a two stage calculation involving both multiplication and division students should try to simplify the calculation by first dividing to make the final product easier to evaluate. Some students used 18 cm and 30 cm instead of 18 cm and 24 cm in the area calculation and some forgot to divide by 2 when finding the area.

Mark Scheme

Question	Answer	Mark	Mark scheme
8	216	P1	for process to work with ratio e.g. 72 ÷ (3 + 4 + 5) (= 6) **or** 72 ÷ 12 (= 6)
		P1	for process to find length of base or height of triangle e.g. 3 × "6" (= 18) **or** 4 × "6" (= 24) **OR** process to find area scale factor e.g. "6" × "6" (= 36)
		P1	complete process to find the area of the triangle e.g. ½ × "18" × "24" **or** ½ × 3 × 4 × "6"^{2}
		A1	cao

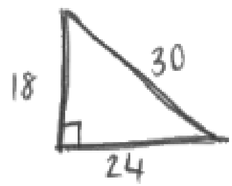
Examiner Comments

Note that the first three marks are process marks. They are not dependent on accuracy. The first two process marks are for using the perimeter of 72 cm and the ratio 3 : 4 : 5 to find the lengths of the sides of the triangle. For the second process mark "6" must come from a correct process, i.e. from dividing 72 by (3 + 4 + 5). The third mark is for a complete process to find the area of the triangle. For this mark both "18" and "24" must come from correct processes.

Student Response A

The perimeter of a right-angled triangle is 72 cm.
The lengths of its sides are in the ratio 3 : 4 : 5

Work out the area of the triangle.



$\frac{72}{12} = 6$

$6 \times 3 = 18$
$6 \times 4 = 24$
$6 \times 5 = 30$

$a = \left(\frac{24}{2}\right) \times 18$

$= 216\text{cm}^2$

x	10	2
10	100	20
8	80	16

216 cm²

4/4

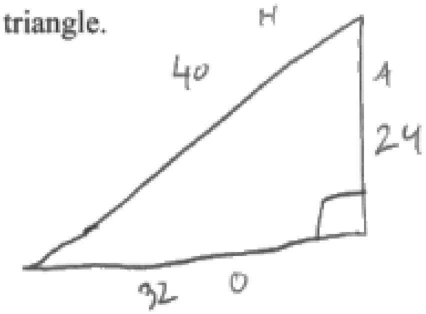
Examiner Comments

This is a well presented solution. The student divides 72 in the ratio 3 : 4 : 5 to find the lengths of the sides of the triangle and identifies, with the help of a diagram, that the sides with lengths 18 cm and 24 cm should be used to find the area of the triangle. The majority of students chose to multiply 18 by 24 and then divide by 2 but this student sensibly divides 24 by 2 before multiplying by 18.

Student Response B

The perimeter of a right-angled triangle is 72 cm.
The lengths of its sides are in the ratio 3 : 4 : 5

Work out the area of the triangle.



3 + 4 + 5 =
9
72 ÷ 9 = 8

3 × 8 = 24 cm
4 × 8 = 32
5 × 8 = 40

½ × b × h

32 ÷ 2 = 18
18 × 24 =

	10	8
20	200	160
4	40	32

200
160
40
32
372

372 cm²

3/4

Examiner Comments

This response shows a complete process to find the area of the triangle with each stage of working clearly shown. The three process marks are awarded but the solution is littered with arithmetic errors and the final answer is incorrect.

Student Response C

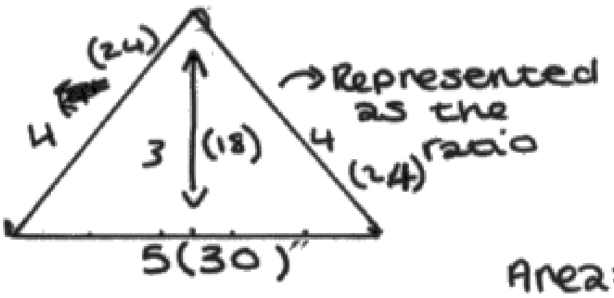
The perimeter of a right-angled triangle is 72 cm.
The lengths of its sides are in the ratio 3 : 4 : 5

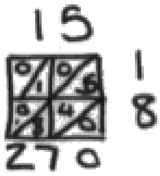
Work out the area of the triangle.

$3+4+5=12$

$72 \div 12 = 6$

18cm, 24cm, 30cm





Area $= \frac{1}{2} b \times h$

$= \frac{1}{2} 30 \times 18$

$= 15 \times 18$

$= 270$ cm²

2 24 36 48 60 72

270 cm²

2/4

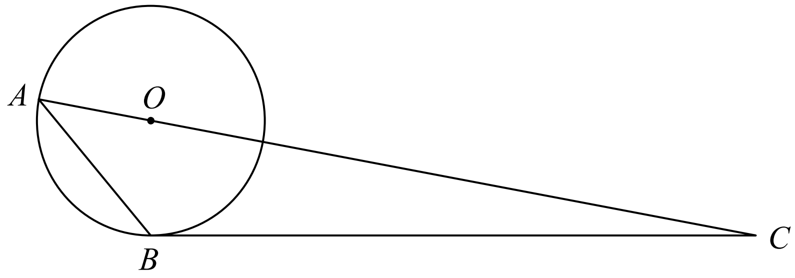
Examiner Comments

The first two marks are awarded for dividing 72 in the ratio 3 : 4 : 5. The student finds the lengths of the sides of the triangle but draws an incorrect diagram and fails to identify 18 cm and 24 cm as the base and height of the triangle. No marks are awarded for the area calculation because the correct two side lengths have not been used.

Exemplar Question 4

Higher tier Paper 1

11



A and *B* are points on a circle, centre *O*.

BC is a tangent to the circle.
AOC is a straight line.
Angle $ABO = x°$.

Find the size of angle *ACB*, in terms of *x*.
Give your answer in its simplest form.
Give reasons for each stage of your working.

(Total for Question 11 is 5 marks)

Examiner Comments

This question assesses the ability of students to use standard circle theorems and angle properties to find the size of an angle and their ability to communicate reasons. The size of one angle is given as $x°$ and students are expected to find unknown angles in terms of *x* and use these angles in an algebraic method to find angle *ACB*.

Students should set their working out in a formal manner, showing the steps that are needed to find the size of angle *ACB*. The question requires students to give reasons for each stage of their working. A very common mistake was for students to give incomplete reasons that did not use the correct terminology or to give no reasons at all.

Mark Scheme

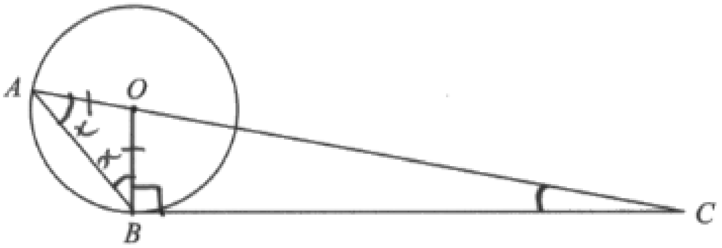
Question	Answer	Mark	Mark scheme	Additional guidance
11	$90 - 2x$	M1	for identifying an unknown angle e.g. $BAO = x$, $AOB = 180 - 2x$, $OBC = 90$, $ABC = 90 + x$	Could be shown on the diagram alone
		M1	full method to find the required angle, e.g. a method leading to $180 - x - x - 90$	Needs to be an algebraic method Accept $x + x + 90 + y = 180$ for M2
		A1	for $90 - 2x$	
		C2	(dep M2) full reasons for their method, from base angles in an <u>isosceles triangle</u> are equal <u>angles</u> in a <u>triangle</u> add up to 180° a <u>tangent</u> to a circle is perpendicular to the <u>radius</u> <u>(diameter)</u> <u>angles</u> on a straight <u>line</u> equal 180° the <u>exterior angle</u> of a triangle is <u>equal</u> to the sum of the <u>interior</u> <u>opposite angles</u>	Underlined words need to be shown; reasons need to be linked to their method; any reasons not linked do not credit.
		C1	(dep M1) for a <u>tangent</u> to a circle is perpendicular to the <u>radius</u> <u>(diameter)</u>	Apply the above criteria

Examiner Comments

The first method mark is for finding an unknown angle and this angle could be marked on the diagram. A full method is needed for the second mark. This must be an algebraic method. If $90 - 2x$ is incorrectly 'simplified' to $45 - x$ then the A mark cannot be awarded.

Reasons must contain the words that are underlined in the mark scheme and need to be linked to the method used. Because of the focus of the assessment and the level at which this question is set a correct reason linked to a circle theorem must be given before any C marks can be awarded. Usually this reason will be "a <u>tangent</u> to a circle is perpendicular to the <u>radius</u>" although some students drew a chord from *B* to the point where *OC* cuts the circle and used "<u>angles</u> in a <u>semicircle</u> are 90°".

Student Response A



A and *B* are points on a circle, centre *O*.

BC is a tangent to the circle.
AOC is a straight line.
Angle *ABO* = x°.

Find the size of angle *ACB*, in terms of x.
Give your answer in its simplest form.
Give reasons for each stage of your working.

∠CBO = 90° (tangent - radius meets at 90°)

Triangle AOB is isoceles as AO and OB are both radii of the same circle
So that ∠OAB = ∠ABO = x

As angles in a triangle sum to 180°:

∠ACB = 180 - 90 - x - x
= 90 - 2x

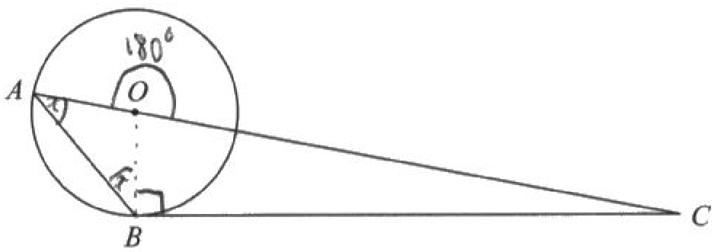
∠ACB = 90 - 2x

5/5

Examiner Comments

A complete method to find angle *ACB* leads to 90 – 2x. Full reasons, that include the words underlined in the mark scheme, are given for the method used.

Student Response B



OBC = 90° as ~~t~~angents are always at right angles to the radii of a circle.

AOC = 180° as angles on a straight line equal 180

ABC = 90 + x
BAC = x

ACB = 180 − (90 + 2x) as angles in triangles ~~have~~ add to 180°

ACB = 180 − 90 − 2x

ACB = 90 − 2x ÷2 → ACB = 45 − x

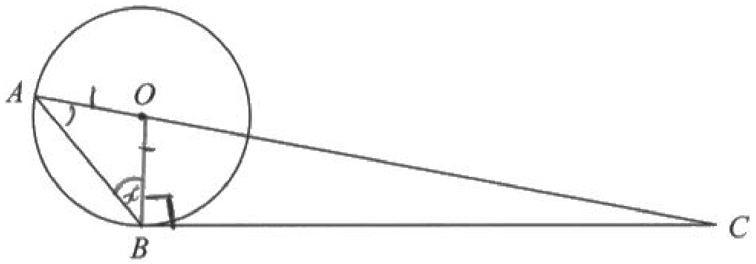
3/5

Examiner Comments

The two method marks are awarded for a complete algebraic method, 180 – (90 + 2*x*), to find angle *ACB*. Unfortunately, 90 – 2*x* has been incorrectly 'simplified' to 45 – *x* and the accuracy mark is lost. Reasons have been given for most stages of the working but there is no reason to explain why angle *BAC* = *x*. The reason for angle *OBC* = 90º gets C1.

Student Response C

11



AO = AB radii to the circle

∴ ~~BÂA = x°~~ BÂO = x°

AÔB = 180 - 2x angles in a triangle add up to 180°

OB̂C = 90° tangent to a circle = 90°

CÔB = 2x° angles on a straight line add up to 180

90° + 2x° = 180 = angles in a triangle add up to 180

180 - 90 = 2x°

90 = 2x°

45 = x°

90+45 = 135 angles in a triangle add up to 180°

180 - 135 = 45°

∴ ABO = ACB = x°

1/5

Examiner Comments

The student finds several unknown angles (*OBC* = 90, *BAO* = *x*, *AOB* = 180 – 2*x* and *COB* = 2*x*) and any one of these is sufficient for the first method mark. However, no full method to find angle *ACB* is shown. The reason that explains why angle *OBC* is a right angle contains no mention of radius (a word underlined in the mark scheme) so no C marks can be awarded. This was a very common error.

Exemplar Question 5

Higher tier Paper 1

12 Prove that the square of an odd number is always 1 more than a multiple of 4.

(Total for Question 12 is 4 marks)

Examiner Comments

This question assesses the ability of students to use algebra to construct a proof.

Students are expected to start the proof by writing down a general expression for an odd number. The most common expressions used were $2n + 1$ and, to a much lesser extent, $2n – 1$.

Proficiency at expanding and simplifying brackets is necessary if students are to present a correct proof. Some errors were made when expanding $(2n + 1)^2$. Sometimes it was expanded to $4n^2 + 1$ and occasionally the squared term was seen as $2n^2$ rather than as $4n^2$.

For those who realised that they needed to use an algebraic approach the main obstacle to a successful outcome was writing a general expression for an odd number to start the proof, with $n + 1$ a common incorrect expression. Some students used $4n + 1$ but this does not generate all the odd numbers. A common misconception was that this type of problem can be solved using a numerical approach.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
12	Statement supported by algebra	B1	writing a general expression for an odd number e.g. $2n + 1$	Could be $2n - 1$, $2n + 3$, etc
		M1	(dep) for expanding ("odd number")^2 with at least 3 out of 4 correct terms	Note that $4n^2 + 4n + 2$ or $2n^2 + 4n + 1$ in expansion of $(2n + 1)^2$ is to be regarded as 3 correct terms
		A1	for correct simplified expansion, e.g. $4n^2 + 4n + 1$	
		C1	(dep A1) for a concluding statement e.g. $4(n^2 + n) + 1$ (is one more than a multiple of 4)	

Examiner Comments

The first mark is awarded for writing a general expression for an odd number. This is needed to start the proof. The second mark is dependent on the first mark, i.e. students must square their general expression for an odd number. The final mark is dependent on the A (accuracy) mark. The proof can be concluded by factorising $4n^2 + 4n + 1$ to $4(n^2 + n) + 1$ or $4n(n + 1) + 1$ or by explaining that since both $4n^2$ and $4n$ are multiples of 4 then $4n^2 + 4n + 1$ must be 1 more than a multiple of 4. In some responses $4n + 1$ (1more than a multiple of 4) was seen on the RHS of an equation, e.g. $4n^2 + 4n + 1 = 4n + 1$. These responses were awarded at most 3 marks.

Student Response A

12 Prove that the square of an odd number is always 1 more than a multiple of 4

odd number

$$(2n+1)^2 = (2n+1)(2n+1)$$
$$= 4n^2 + 2n + 2n + 1$$
$$= 4n^2 + 4n + 1$$
$$= 4(n^2+n) + 1$$

This is a multiple of 4 — this is plus 1

odd number squared is multiple of 4, + 1

4/4

Examiner Comments

This is a fully correct proof. Factorising $4n^2 + 4n + 1$ to give $4(n^2 + n) + 1$ is sufficient for the final mark but the student also explains that $4(n^2 + n)$ is a multiple of 4 and identifies the "+1".

Student Response B

12 Prove that the square of an odd number is always 1 more than a multiple of 4

odd number = $2n + 1$

square of an odd number = $(2n+1)^2$

$= (2n+1)(2n+1)$

$= 4n^2 + 2n + 2n + 2$

$= 4n^2 + 4n + 2$

~~$4(n^2+n)+2$~~ → $= 2n^2 + 2n + 1$

even numbers multiplied by two are multiples of 4

2/4

Examiner Comments

The student writes down a general expression for an odd number and squares this expression with 3 out of 4 terms correct so the first two marks can be awarded. The expanding and simplifying does not result in $4n^2 + 4n + 1$ so neither the accuracy mark nor the final mark, which is dependent on the accuracy mark, can be awarded

Student Response C

12 Prove that the square of an odd number is always 1 more than a multiple of 4

$(2n + 1)^2 \rightarrow 2n^2 + 4n + 2$

$= 2(n^2 + 2n + 1)$

?

$(2n - 1)^2 \rightarrow 2n^2 - 4n + 2 = 2(n^2 - 2n + 1)$

$3^2 = 9$

$5^2 = 25$

1 more than a multiple of 4

1/4

Examiner Comments

The student makes a good start and gives two general expressions for an odd number. Either $2n+1$ or $2n-1$ is sufficient for the first mark. Both expressions are squared but in each expansion only 2 out of 4 terms are correct which means that no further marks can be awarded. Note: that $4n$ in the expansion $2n^2 + 4n + 2$ is regarded as two correct terms.

Exemplar Question 6

Higher tier Paper 1

14 y is inversely proportional to d^2
When $d = 10$, $y = 4$

d is directly proportional to x^2
When $x = 2$, $d = 24$

Find a formula for y in terms of x.
Give your answer in its simplest form.

....

(Total for Question 14 is 5 marks)

Examiner Comments

This question assesses the ability of students to construct and use equations that describe direct and inverse proportion. The first part of the solution is routine. Students are expected to communicate information accurately by setting up the correct proportional relationships and to then use correct processes to find the constants of proportionality. Students will be less familiar with the second part of the solution. Here they have to decide how to use two proportional relationships to find a formula for y in terms of x.

A common mistake was to use direct proportion instead of inverse proportion and vice versa. Some students did not write down an equation involving k. The main challenge for students was being able to use $y = 400/d^2$ and $d = 6x^2$ to find a formula for y in terms of x and to then give their answer in its simplest form.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
14	$y = \frac{100}{9x^4}$	P1	for setting up a correct proportional relationship, e.g. $d \propto x^2$ **or** $d = kx^2$	Condone the use of 'α' instead of '=' for the four P marks
		P1	for setting up a second proportional relationship, e.g. $y \propto \frac{1}{d^2}$ **or** $y = \frac{K}{d^2}$	
		P1	(dep P1) for a process to find one of the constants of proportionality e.g. $24 = k \times 2^2$ ($k = 6$) **or** $4 = K \div 100$ ($K = 400$)	
		P1	full process to find y in terms of x e.g. $y = \frac{"400"}{("6"x^2)^2}$ oe	Both constants must come from a correct process
		A1	$y = \frac{100}{9x^4}$ oe	Expression must have been simplified, but could be given other equivalent ways e.g. $y = 11.111...\, x^{-4}$

Examiner Comments

The use of 'α' instead of '=' is condoned for the first four P (process) marks. For example $d \propto kx^2$ would get the first mark. The third P mark is dependent on P1. The student must have set up at least one correct proportional relationship before this mark can be awarded for a process to find one of the constants of proportionality. The question requires the answer to be given in its simplest form so students who give the final answer as $y = 400/36x^4$ are not awarded the A mark.

Student Response A

14 y is inversely proportional to d^2
When $d = 10$, $y = 4$

d is directly proportional to x^2
When $x = 2$, $d = 24$

Find a formula for y in terms of x.
Give your answer in its simplest form.

$y = \frac{k}{d^2}$

$d = kx^2$

$4 = \frac{k}{100}$

$24 = k2^2$

$k = 6$

$400 = k$

$y = \frac{400}{d^2}$

$d = 6x^2$

$d^2 = (6x^2)^2$

$36x^4$

$y = \frac{400}{36x^4}$

$\frac{200}{18x^4}$ $\frac{100}{9x^4}$

$y = \frac{100}{9x^4}$

(Total for Question 14 is 5 marks)

5/5

Examiner Comments

This is a complete and fully correct solution leading to the award of 5 marks.

Student Response B

14 y is inversely proportional to d^2
When $d = 10$, $y = 4$

d is directly proportional to x^2
When $x = 2$, $d = 24$

Find a formula for y in terms of x.
Give your answer in its simplest form.

$\frac{24}{K \mid 4}$

$y = \frac{K}{d^2}$ ~~$d = K \times x^2$~~

$4 = \frac{K}{10^2} = 4 = \frac{K}{100}$

$K = 4 \times 100$
$K = 400$

$y = \frac{400}{d^2}$

$d = K \times x^2$
$24 = K \times 2^2$
$24 = K \times 4$
$K = \frac{24}{4}$
$K = 6$

$d = 6 \times x^2$

$y = \frac{400}{(6 \times x^2)}$

$y = \frac{400}{(6 \times x^2)}$

3/5

Examiner Comments

Two correct proportional relationships are set up and there are correct processes to find both constants of proportionality. The first three process marks are awarded. In order to find y in terms of x the student attempts to substitute $d = 6x^2$ into $y = 400/d^2$ but the denominator is not squared. Had the denominator been squared then the fourth process mark would have been awarded.

Student Response C

14 y is inversely proportional to d^2
When $d = 10$, $y = 4$

d is directly proportional to x^2
When $x = 2$, $d = 24$

Find a formula for y in terms of x.
Give your answer in its simplest form.

$y \propto a^2$

$y \propto d^2$

$4 \propto 10^2$

$4 \propto 100$

$d \propto x^2$

$24 \propto 2^2$

$24 \propto 4$

$\frac{24}{2}$

1/5

Examiner Comments

One mark is awarded for setting up a correct proportional relationship, $d \propto x^2$. Instead of forming the equation $d = kx^2$ and using it to find the value of k the student simply substitutes the values $d = 24$ and $x = 2$ into $d \propto x^2$. For the relationship between y and d^2 the student has used direct proportion instead of inverse proportion. This was a common mistake.

Exemplar Question 7

Higher tier Paper 1

16 There are only red counters, blue counters and purple counters in a bag.
The ratio of the number of red counters to the number of blue counters is 3 : 17

Sam takes at random a counter from the bag.
The probability that the counter is purple is 0.2

Work out the probability that Sam takes a red counter.

....

(Total for Question 16 is 3 marks)

Examiner Comments

This question assesses the ability of students to solve a problem through a series of processes. The problem is set in the context of probability but requires knowledge of ratio. Students need to apply the property that the probabilities of an exhaustive set of outcomes sum to one. A correct start did not necessarily lead to a full process to solve the problem but once students had identified a strategy to use they were often able to work out the probability that Sam takes a red counter. The difficulty for many students was finding a suitable strategy. Division of 0.8 by 20 or of 20 by 0.8 often lead to arithmetic errors.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
16	0.12	P1	for process to start e.g. (1 – 0.2) ÷ (3 + 17) (= 0.04) **or** (3 + 17) ÷ (1 – 0.2) oe (= 25) **or** (100 – 20) ÷ (3 + 17) (= 4) **or** 3 × 4 (= 12) and 17× 4 (= 68)	Just 1 – 0.2 = 0.8 is **not** sufficient for P1 May be seen in a ratio
		P1	full process to find the required probability e.g. 3 × "0.04" **or** $\frac{3}{20}$ × (1 – 0.2) oe **or** 3 ÷ "25" **or** 3 × "4" ÷ 100	
		A1	oe	0.12 using incorrect probability notation gets P2

Examiner Comments

There are several different ways in which students might attempt to solve the problem. The first mark is awarded for a process to start. Just 1 – 0.2 = 0.8 is not sufficient for this mark but using 0.8 in a correct process, e.g. 0.8 ÷ 20 or 20 ÷ 0.8, gains P1. A full process to find the required probability is required for the second mark. The A mark is awarded for 0.12 or an equivalent probability. This probability must be given as a decimal, fraction or percentage.

Student Response A

16 There are only red counters, blue counters and purple counters in a bag.
The ratio of the number of red counters to the number of blue counters is 3 : 17

Sam takes at random a counter from the bag.
The probability that the counter is purple is 0.2

Work out the probability that Sam takes a red counter.

$1 - 0.2 = 0.8$

$\frac{0.8}{20} = 0.04$

$0.04 \times 3 = 0.12$

0.12

3/3

Examiner Comments

This is a fully correct response illustrating a popular method that was used by many students.

Student Response B

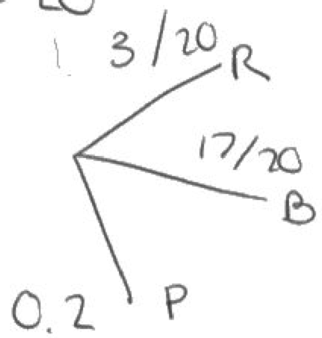
16 There are only red counters, blue counters and purple counters in a bag.
The ratio of the number of red counters to the number of blue counters is 3 : 17

Sam takes at random a counter from the bag.
The probability that the counter is purple is 0.2

Work out the probability that Sam takes a red counter.

R : B : P
3 : 17 : ? = 20

$1 - 0.2 = 0.8$



$\frac{3}{20} \times 0.8 = \frac{2.4}{20}$

0.48

2/3

Examiner Comments

Successful solutions often started with students working out that the probability of taking a red counter or a blue counter is 0.8 and then finding $\frac{3}{20}$ of 0.8. In this response the two process marks are awarded for a full process to find the required probability, $\frac{3}{20} \times 0.8$. The student is not able to divide 2.4 by 20 correctly and loses the accuracy mark.

Student Response C

16 There are only red counters, blue counters and purple counters in a bag.
The ratio of the number of red counters to the number of blue counters is 3 : 17

Sam takes at random a counter from the bag.
The probability that the counter is purple is 0.2

Work out the probability that Sam takes a red counter.

r : b : p
r : b : 0.2
3 : 17 :

3+17=20
+ 4
= 24

$\frac{20}{5} = 4$

0.2=4

$\frac{3}{24} = 8 \rightarrow 0.8$
$\rightarrow 0.08$

....

0/3

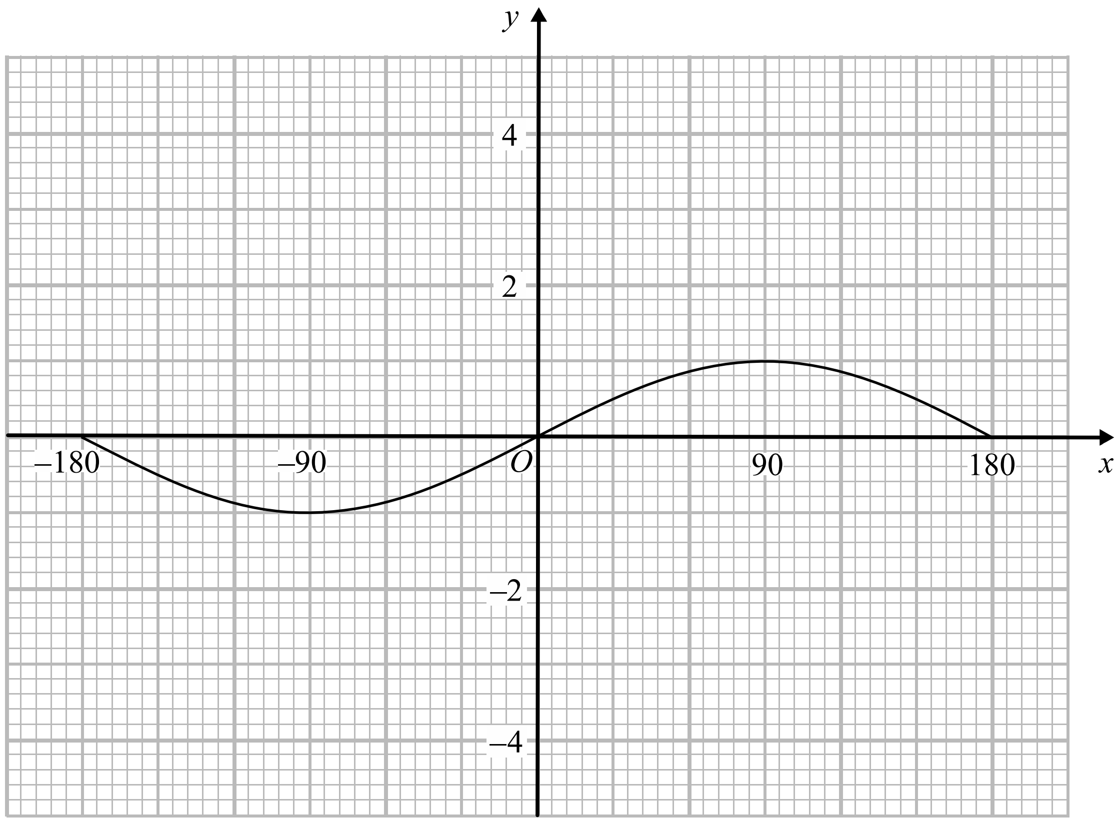
Examiner Comments

Some students worked out that if 20 red counters and blue counters represent 80% of the total number of counters then the total number of counters must be 25, often using an argument such as “80% = 20, 20% = 5”. A common error, illustrated by this response, was for students to work out the total number of counters as 24, e.g. “80% = 20, 20% of 20 = 4, 20 + 4 = 24”.

Exemplar Question 8

Higher tier Paper 1

18 Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

(Total for Question 18 is 2 marks)

Examiner Comments

This is a straightforward question in which students are expected to sketch a translation of a given function. Marks were lost when sketches were hastily drawn and did not pass through the required points. Students would be well advised to look for those points where the graph passes through integer coordinates and transform these points carefully.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18	Graph drawn	C2	for graph translated by −2 in the *y* direction	Key points: (–180, –2),(–90, –3), (0, –2), (90, –1), (180, –2)
		C1	for a graph translated in the *y* direction **OR** for a correct graph through four of the five key points	

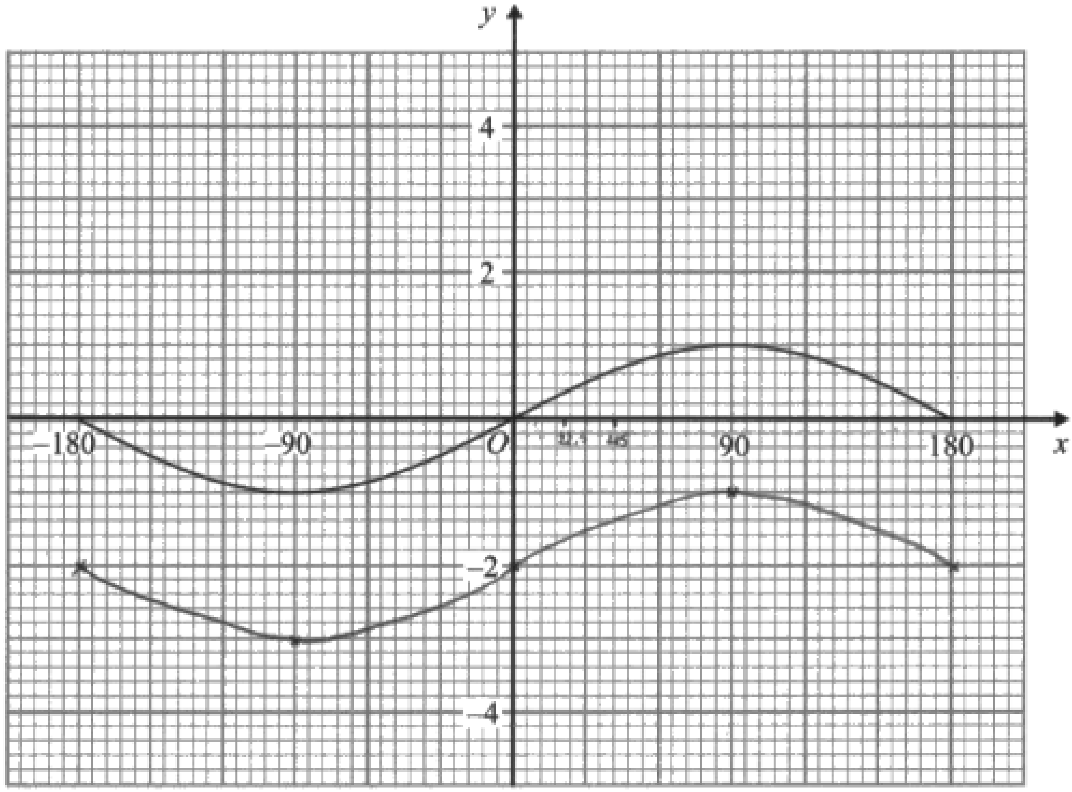
Examiner Comments

Graphs translated by −2 in the *y* direction are expected to pass though the five key points (–180, –2), (–90, –3), (0, –2), (90, –1) and (180, –2).

If the graph drawn is not fully correct then one mark can be awarded for a correct graph that passes through four of the five key points or for a translation of the graph in the *y* direction, e.g. a translation of +2 or a translation of two 2 mm squares.

Student Response A

18 Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

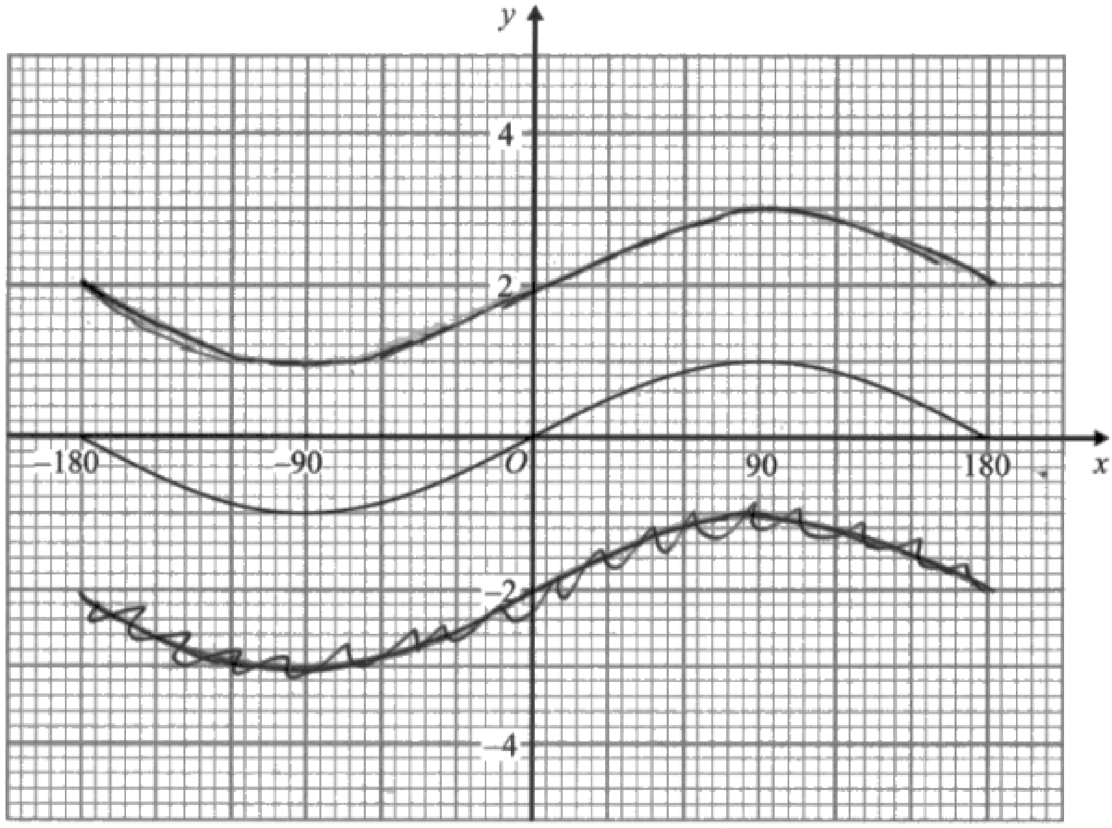
2/2

Examiner Comments

The graph has been translated by −2 in the *y* direction. The student has found the five key points where the graph passes through integer coordinates and has translated each of these points correctly.

Student Response B

18 Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



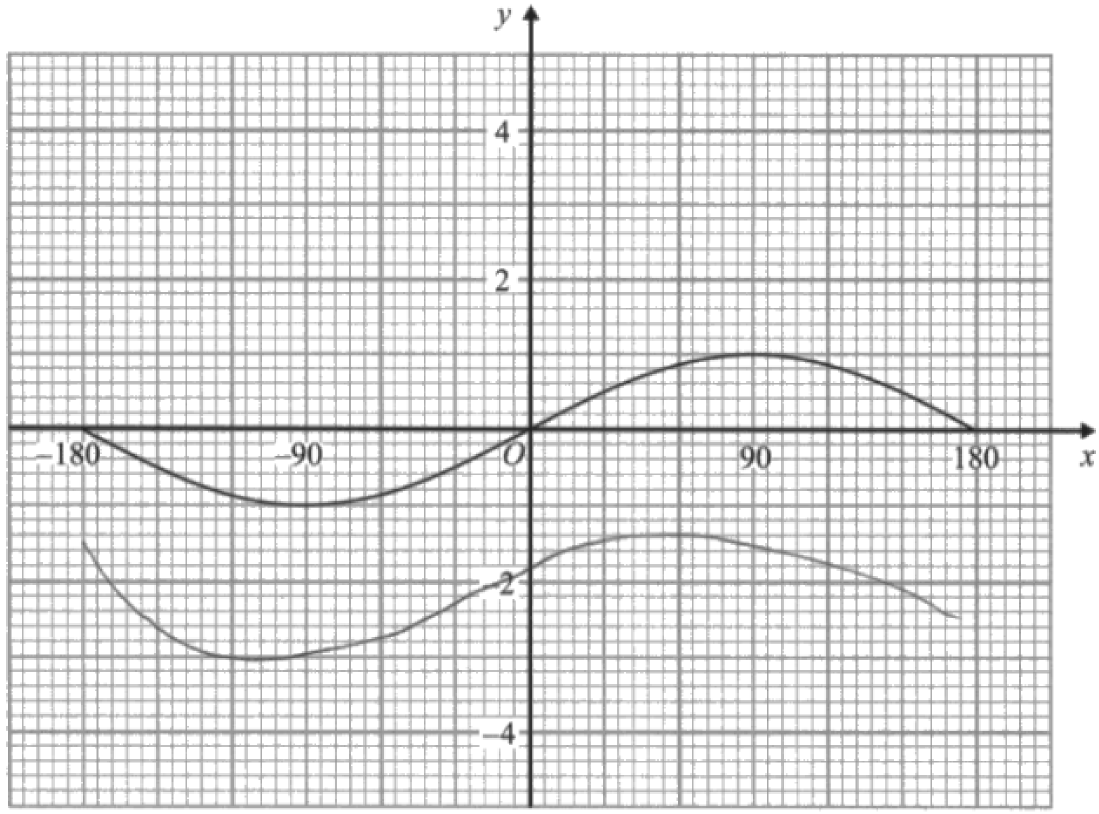
On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

1/2

Examiner Comments
The graph has been translated in the *y* direction but by +2 rather than by −2. Unfortunately, the correct answer has been crossed out and replaced.

Student Response C

18 Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

0/2

Examiner Comments

The student may well have realised that a translation of −2 in the y direction was required but in this hastily drawn sketch the points (–180, –2), (–90, –3), (0, –2), and (90, –1) have been translated by different amounts and the graph does not extend as far as x = 180

Exemplar Question 9

Higher tier Paper 1

19 The point P has coordinates (3, 4)
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $3x + 2y = 7$

Find an expression for b in terms of a.

....

(Total for Question 19 is 5 marks)

Examiner Comments

This question assesses the ability of students to translate a problem in a mathematical context into a series of processes in order to solve it. This problem is within the specific context of coordinate geometry. Familiarity with $y = mx + c$ and knowledge of how to find the gradient of a perpendicular line are necessary to make progress through the problem. Many students recognised that the first step was to rearrange the equation of the straight line to make y the subject. Students who used $y = mx + c$ did not always remember to substitute $x = 3$ and $y = 4$ to find the value of c. Some students found the gradient of the line through P and Q in terms of a and b and equated this gradient to $\frac{2}{3}$. The main difficulty faced by these students was in rearranging $\frac{b-4}{a-3} = \frac{2}{3}$ to get an expression for b in terms of a.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19	$b = \frac{2}{3}a + 2$	P1	for process to rearrange the equation to give y in terms of x e.g. $y = \frac{7-3x}{2}$ **or** $y = -\frac{3}{2}x + \left(\frac{7}{2}\right)$ **or** $m = -\frac{3}{2}$	
		P1	for using their gradient in $mn = -1$	
		P1	for showing a process to find the gradient of PQ e.g. $\frac{b-4}{a-3}$ **OR** for substituting $x = 3$ and $y = 4$ in $y =$ "$\frac{2}{3}$"$x + c$	
		P1	(dep P3) for forming an equation in a and b e.g. $\frac{b-4}{a-3} =$ "$\frac{2}{3}$" **or** $b =$ "$\frac{2}{3}$"$a +$ "2" **OR** correct equation in terms of x and y e.g. $y = \frac{2}{3}x + 2$	$y - 4 = \frac{2}{3}(x - 3)$ gets P4
		A1	for $b = \frac{2}{3}a + 2$ oe	Accept 0.66 or 0.67 oe for 2/3

Examiner Comments

The first mark is awarded for a process to rearrange the equation to give y in terms of x. An incorrect constant term is condoned as the focus is on the x term. The second mark is for using $mn = -1$. If students show understanding of $mn = -1$ by finding the negative reciprocal of what they state is the gradient of $3x + 2y = 7$ then this mark can be awarded. It is not dependent on the first mark. The fourth P mark is dependent on the previous three marks having been awarded. The final answer is awarded for $b = \frac{2}{3}a + 2$ or an equivalent expression.

Student Response A

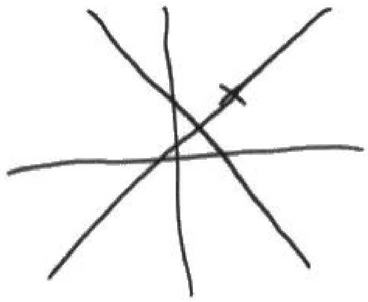
19 The point P has coordinates (3, 4)
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $3x + 2y = 7$

Find an expression for b in terms of a.

$y = \frac{7}{2} - \frac{3x}{2}$

$y = -\frac{3}{2}x + \frac{7}{2}$



$\frac{-1}{-3/2} = \frac{2}{3} =$ gradient of PQ

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{2}{3}(x - 3)$

$y = \frac{2}{3}x + 2$

$b = \frac{2}{3}a + 2$

5/5

Examiner Comments

This is a fully correct solution using an efficient method. The equation $3x + 2y = 7$ has been rearranged correctly to make y the subject and the gradient of PQ obtained by using $-1/m$. The most common approach to finding an equation for the line through P and Q was to use $y = mx + c$ or, as in this response, the equivalent form $y - y_1 = m(x - x_1)$

Student Response B

19 The point P has coordinates (3, 4)
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $3x + 2y = 7$

Find an expression for b in terms of a.

~~gradient~~

$3x + 2y = 7$

$-2y = 3x - 7$

$2y = -3x + 7$

$y = -\frac{3}{2}x + 3.5$

perpendicular gradient $= \frac{2}{3}$

$\frac{b-4}{a-3} = \frac{2}{3}$

$a - 3 = 3$

$a = 6$

$b - 4 = 2$

$b = 6$

$b = a$

4/5

Examiner Comments

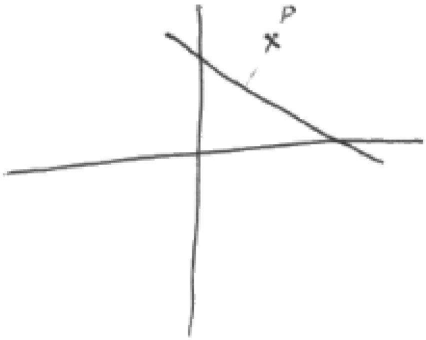
The student rearranges the equation $3x + 2y = 7$ to make y the subject and obtains the gradient of PQ by finding the negative reciprocal of the gradient from the rearranged equation. Instead of using $y = mx + c$ this student has found the gradient of the line through P and Q in terms of a and b and has equated this gradient to 2/3. The four process marks are awarded. The final answer is incorrect as the equation has not been rearranged to give b in terms of a.

Student Response C

19 The point P has coordinates (3, 4)
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $3x + 2y = 7$

Find an expression for b in terms of a.



$3x + 2y = 7$

$2y = -3x + 7$

$y = -\frac{3}{2}x + 7$

$PQ = \frac{2}{3}x - 7$

$b = \frac{2}{3}a - 7$

$y = mx + c$

$b = \frac{2}{3}a + c$

$b = \frac{2}{3}a + c$

~~$b = \frac{2}{3}a - 7$~~

2/5

Examiner Comments

The rearrangement of $3x + 2y = 7$ is not fully correct but $y = -\frac{3}{2}x$ is sufficient for the first P mark to be awarded and finding the gradient of PQ as $\frac{2}{3}$ gains the second P mark. The student uses $y = mx + c$ to find an equation for the line through P and Q but assumes that $c = -7$ instead of using $x = 3$ and $y = 4$ to find the value of c. No further marks can be awarded.

Exemplar Question 10

Higher tier Paper 1

20 n is an integer such that $3n + 2 \leqslant 14$ and $\dfrac{6n}{n^2 + 5} > 1$

Find all the possible values of n.

....

(Total for Question 20 is 5 marks)

Examiner Comments

This question assesses the ability of students to use standard techniques to solve a linear inequality in one variable and a quadratic inequality in one variable. Students should be able to rearrange the second inequality into the form $n^2 - 6n + 5$ (< 0) and to factorise the quadratic expression. Many attempts at rearranging the second inequality did not lead to $n^2 - 6n + 5 < 0$. When rearrangement did lead to $n^2 - 6n + 5$ this quadratic was usually factorised correctly but the inequalities that followed were often incorrect. A common mistake was to write $n > 1$ and $n > 5$ or $n < 1$ and $n < 5$. A quick sketch helped some students to give the correct inequalities.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
20	2, 3, 4	M1	for method to solve $3n + 2 \leqslant 14$ e.g. $n \leqslant (14 - 2) \div 3$ oe	This could be shown within an equation rather than an inequality at this stage
		M1	for complete method to rearrange $\frac{6n}{n^2 + 5} > 1$ to the form $an^2 + bn + c\ (< 0)$	For the 2nd and 3rd M marks condone no '< 0' and condone use of incorrect inequality signs or '='
		M1	for method to begin to solve $n^2 - 6n + 5\ (< 0)$ e.g. $(n \pm 5)(n \pm 1)\ (< 0)$	Accept $\frac{--6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 5}}{2 \times 1}$ (condone one sign error)
		M1	(dep on previous M2) for $n > 1$ and $n \leqslant 4$ **or** $1 < n < 5$	Must come from correct working Could be shown on a number line
		A1	(dep M4) cao	
			Alternative method	
		M1	for method to solve $3n + 2 \leqslant 14$ e.g. $n \leqslant (14 - 2) \div 3$ oe **OR** for $3 \times 4 + 2 = 14$	This could be shown within an equation rather than an inequality at this stage
		M3	for trials with 1, 2, 3 and 4 in the quadratic inequality, correctly evaluated	The values from the trials may be given as improper fractions e.g. $\frac{24}{21}, \frac{18}{14}, \frac{12}{9}, \frac{6}{6}$
		M2	for trials with three of 1, 2, 3 and 4, correctly evaluated)	
		M1	for trials with two of 1, 2, 3 and 4, correctly evaluated)	
		A1	(dep M4) cao	

Examiner Comments

There are two discrete mark schemes for this question to cater for both the algebraic and non-algebraic approaches. The first M mark in both schemes is awarded for a method to solve the linear inequality. In the first scheme the fourth M mark is dependent upon the previous two M marks. If any correct inequalities are given then this mark cannot be awarded. In the second scheme the trials of 1, 2, 3 and 4 in the second inequality must be fully and correctly evaluated for the method marks to be awarded. In both schemes, the A mark is dependent upon the award of the previous four method marks.

Student Response A

20 n is an integer such that $3n + 2 \leqslant 14$ and $\dfrac{6n}{n^2 + 5} > 1$

Find all the possible values of n.

$3n + 2 \leqslant 14$

$3n \leqslant 12$

$n \leqslant 4$

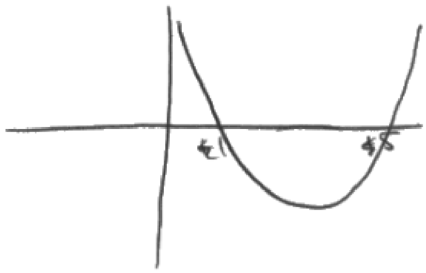
$\dfrac{6n}{n^2+5} > 1$

$6n > n^2 + 5$

$0 > n^2 - 6n + 5$

$(n-5)(n-1)$

$+5 \quad +1$



$\therefore 1 < n < 5$

so if $n \leqslant 4$ and $1 < n < 5$

possibilities for n

$= 4, 3, 2$

5/5

Examiner Comments

This is a fully correct solution in which each stage of working is set out clearly. Having factorised $n^2 - 6n + 5$ and found the critical values $n = 1$ and $n = 5$ the student draws a sketch before writing down $1 < n < 5$. The three possible values of n are listed.

Student Response B

20 n is an integer such that $3n + 2 \leqslant 14$ and $\dfrac{6n}{n^2 + 5} > 1$

Find all the possible values of n.

$3n + 2 \leqslant 14$

$3n \leqslant 12$

$n \leqslant 4$

$\dfrac{6n}{n^2+5} > 1$

$6n > n^2 + 5$

$0 > n^2 - 6n + 5$

$n^2 - 5n - n + 5$

$n(n-5) - 1(n-5)$

$0 > (n-1)(n-5)$

$n = 1 \quad n = 5$

$n > 1 \quad n > 5$

$n \leq 4 \quad n > 1 \quad n > 5$.

$1 < n \leqslant 4$

$1 < n > 5$

$2, 3, 4$

$1 < n > 5$

3/5

Examiner Comments

The first mark is awarded for a correct method to solve the linear inequality. Two further method marks are awarded for rearranging the second inequality to $0 > n^2 - 6n + 5$ and for correctly factorising $n^2 - 6n + 5$. The student finds the critical values $n = 1$ and $n = 5$ and uses these to write down two inequalities. Although $n > 1$ is correct, $n > 5$ is not and no further marks can be awarded.

Student Response C

$n - n^2 > \frac{5}{6}$

$\frac{6(4)}{4^2 + 5} > 1$

$\frac{24}{16 + 5} > 1$

$\frac{24}{21} > 1$

$1\frac{3}{21}$

$1\frac{1}{7} > 1$

$n^2 + 6n + 5 < 0$

$n \leqslant 4$

1/5

Examiner Comments

The method to solve $3n + 2 \leqslant 14$ leads to $n \leqslant 4$ and gets the first mark. The student has made two attempts to rearrange $\frac{6n}{n^2 + 5} > 1$. The first attempt does not get to the required form $an^2 + bn + c$ (< 0). The second attempt has a correct first step but the method then goes wrong and results in $0 > n^2 + 6n + 5$ rather than $0 > n^2 - 6n + 5$. The second M mark cannot be awarded.

Note that the student substitutes $n = 4$ in the second inequality and this trial is correctly evaluated to $\frac{24}{21}$. Using the alternative method on the mark scheme a minimum of two such trials are needed before any marks are awarded.

Paper 2H (calculator)

Exemplar Question 1

Higher tier Paper 2

4 Raya buys a van for £8500 plus VAT at 20%

Raya pays a deposit for the van.
She then pays the rest of the cost in 12 equal payments of £531.25 each month.

Find the ratio of the deposit Raya pays to the total of the 12 equal payments.
Give your answer in its simplest form.

....

(Total for Question 4 is 5 marks)

Examiner Comments

This is the first problem solving question on the paper and one that was well answered. Students are expected to be able to carry out a percentage increase. Find the cost of the payments and subtract this to find the value of the deposit. They are then required to state the correct ratio and simplify it. As with all such questions showing clear working is essential as is completing the steps in a correct order. A number of students added the VAT at the wrong time or stated the wrong values in their ratio.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
4	3 : 5	P1	for process to find 20% **or** 120% of the cost, e.g. 8500 × 0.2 (= 1700) oe **or** 8500 × 1.2 (= 10 200) oe	When partitioning all figures quoted must be correct or a full method shown e.g. 10% = 8500 ÷ 10 (=850) and 20% = "850" + "850" (=1700)
		P1	for process to find total cost of payments, e.g. 12 × 531.25 (= 6375)	
		P1	for complete process to find value of deposit, e.g. "10 200" – "6375" (= 3825) **or** 8500 – "6375" (=2125) and "2125" + "1700" (=3825) **or** the deposit as a proportion of the total cost, e.g. $1-\frac{"6375"}{"10200"}$ $(=\frac{3}{8})$	May be seen as a fraction of the total e.g. $\frac{3825}{10200}$ $(=\frac{3}{8})$
		P1	for finding a correct un-simplified ratio, e.g. "3825" : "6375" oe or 5:3 or $1.\dot{6}$: 1 or $\frac{5}{3}$: 1	Figures at this stage must be expressed as part of a ratio eg 51:85, $\frac{3}{8}:\frac{5}{8}$
		A1	Accept 1: $1.\dot{6}$, 1: $\frac{5}{3}$	Ignore consistent units

Examiner Comments

The first mark is for a correct process to find 20% or 120%. It is worth noting here, like with most questions, that those values need not be correct as long as the correct process or method is given. The second mark is for finding the total cost of the payments. Most students scored this mark, apart from the few who divided rather than multiplied. The third mark, requires the award of the first two. We can see this because of the use of inverted commas around the values. This indicates that although the values used don't have to be correct, the process to find them does. The final 2 marks come from stating and simplifying the ratio. Note we accepted a ratio in the form 1:*n* as a fully simplified ratio. There were a significant number of students who lost the final mark as they were unable to fully simplify the ratio.

Student Response A

4 Raya buys a van for £8500 plus VAT at 20%

Raya pays a deposit for the van.
She then pays the rest of the cost in 12 equal payments of £531.25 each month.

Find the ratio of the deposit Raya pays to the total of the 12 equal payments.
Give your answer in its simplest form.

£8500 + VAT = 8500 + (8500 × 0.2)
= 8500 + 1700
= 10200

12 × 531.25 = 6375

10200 − 6375 = 3825

deposit = £3825

total of 12 payments = £6375

85
17 5

3825 : 6375
765 : 1275
153 : 255
51 : 85
3 : 5

3 : 5

(Total for Question 4 is 5 marks)

5/5

Examiner Comments

He we can see a student who has followed the method as set out in the mark scheme perfectly. Each process is clearly seen and the addition of the annotations aid the examiner in awarding the marks. This student has taken a number of steps to simplify the ratio, whilst another may do it in a single step.

Student Response B

4 Raya buys a van for £8500 plus VAT at 20%

Raya pays a deposit for the van.
She then pays the rest of the cost in 12 equal payments of £531.25 each month.

Find the ratio of the deposit Raya pays to the total of the 12 equal payments.
Give your answer in its simplest form.

total cost

£8500 + 1700 = £10200

÷10 £8500 - 100% ÷10
£850 - 10%
×2 £1700 - 20% ×2

£531.25 × 12 = £6375

£10200 − 6375 = £3825

Deposit : Total
3825 : 6375 ÷5
765 : 1275 ÷5
153 : 255 ÷3
51 : 85

51 : 85

(Total for Question 4 is 5 marks)

4/5

Examiner Comments

This student has used a different approach to find the total cost inclusive of VAT. They have again though, clearly stated their method to find 20% showing the "÷10" and "×2". Unfortunately, this student, like many others, has failed to fully simplify the ratio, stopping at 51:85 this means that the final accuracy mark cannot be awarded.

Student Response C

4 Raya buys a van for £8500 plus VAT at 20%

Raya pays a deposit for the van.
She then pays the rest of the cost in 12 equal payments of £531.25 each month.

Find the ratio of the deposit Raya pays to the total of the 12 equal payments.
Give your answer in its simplest form.

20% of 8500 = 1700

8500 + 1700 = £10200

531.25 × 12 = £6375

÷5 10200 : 6375 ÷5
÷5 2040 : 1275 ÷5
÷3 408 : 255 ÷3
÷17 136 : 85 ÷17
8 : 5

8 : 5

(Total for Question 4 is 5 marks)

2/5

Examiner Comments

This student has started well, but then gone wrong. Their first two steps are fine and gain the first two process marks. It is worth noting here that although they haven't shown the process to find 20%, because the value gained is correct the mark is awarded. Unfortunately, once gaining the total cost and the cost of the 12 payments this student fails to find the value of the deposit and states the incorrect values in the ratio, so no further marks can be awarded.

Exemplar Question 2

Higher tier Paper 2

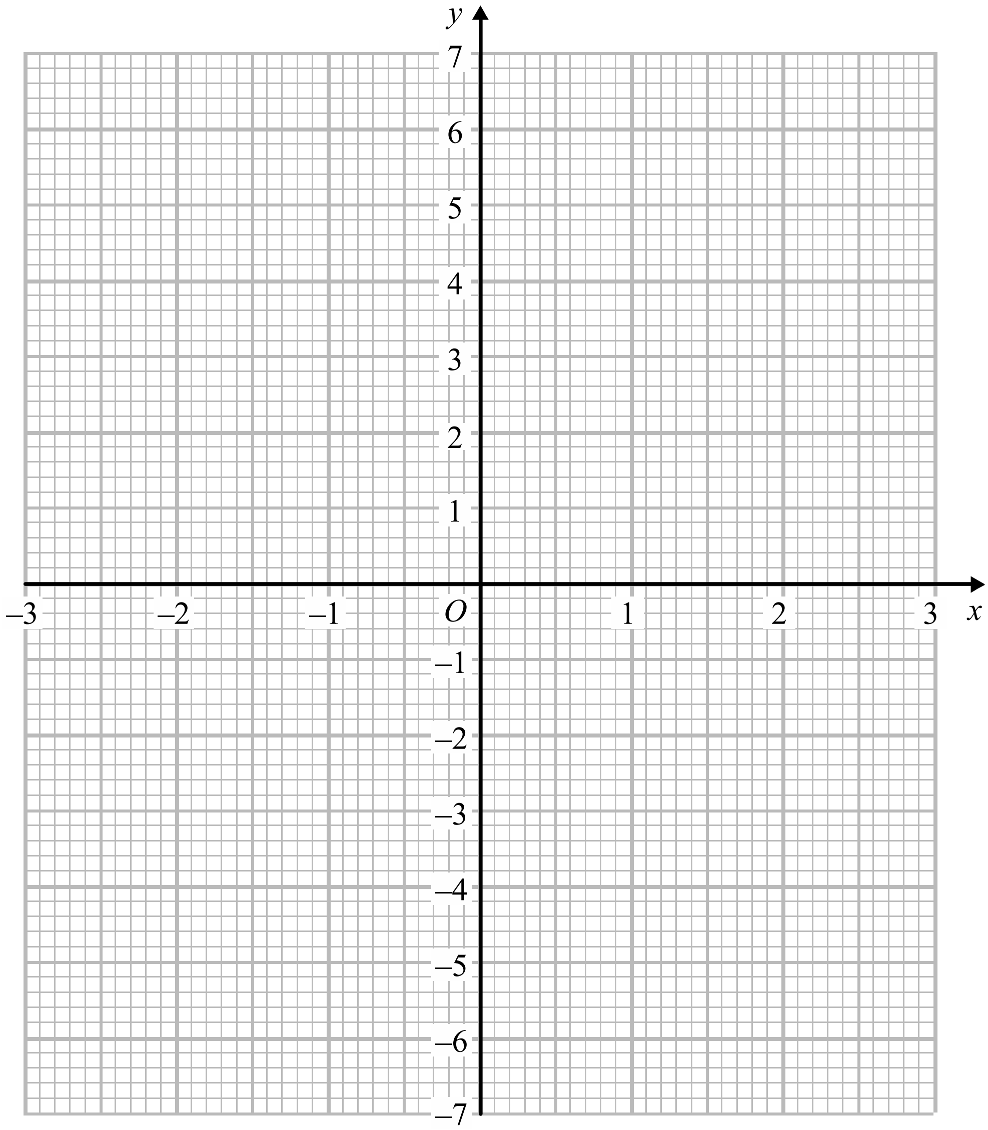
5 (*a*) Complete the table of values for $y = x^2 - x - 6$

x	–3	–2	–1	0	1	2	3
y	6			– 6			

(2)

(*b*) On the grid, draw the graph of $y = x^2 - x - 6$ for values of x from –3 to 3

(2)



(*c*) Use your graph to find estimates of the solutions to the equation $x^2 - x - 6 = -2$

....

(2)

(Total for Question 5 is 6 marks)

Examiner Comments

This question should be familiar to students. It tests three clear skills, substitution into a quadratic, drawing the graph of a quadratic, and solving a quadratic equation graphically. It was worrying to see so many students making mistakes when substituting negative values into the quadratic function, often resulting in values that would not fit on the axes. Typically though, in part (a) students scored at least 1 mark.

In part (b) students were required to plot the values generated in part (a) and then join them with a smooth curve. Many students used line segments and were unable to gain full marks.

Part (c) required students to use their graph to solve a quadratic equation. For many this meant drawing in the line $y = -2$ (or part of it) and reading off the intersections. Some however, attempted to solve the equation using the quadratic formula. This was typically unsuccessful.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
5(a)	0, –4, –6, –4, 0	B2	fully correct figures	
		B1	at least 2 correct figures	
(b)	Graph	M1	(dep B1) for at least 5 points correctly plotted ft from (a)	
		A1	fully correct graph	Must be a curve
(c)	2.6 and –1.6	M1	for $y = -2$ drawn or intersections with $y = -2$ or $y = x^2 - x - 4$ drawn or 1 correct value	If answers stated as coordinates, award M1 for both coordinates and M0 for one coordinate
		A1	ft a quadratic graph or for answers in the range 2.5 to 2.7 and –1.5 to –1.7	

Examiner Comments

In part (a) all values needed to be correct in the table to score B2. If all were not correct then B1 was awarded for any two correct values.

The method mark in part (b) was dependent on scoring at least 1 mark in part (a) and carries a follow through. This means that provided at least two values were correct in the table in (a) then a student could gain 1 mark for plotting 5 points from their table correctly. In order to gain the accuracy mark, the graph had to be fully correct. This mark was often lost when students used line segments, often giving the graph a "flat bottom".

In part (c), those who found both values correctly gained both marks. Where this wasn't the case, one mark could be awarded for either finding one correct value or for plotting $y = -2$, or for circling the points of intersection between the curve and $y = -2$, even if the second graph hadn't been drawn. To apply the follow through in this part the graph drawn had to be a quadratic graph.

Student Response A

5 (a) Complete the table of values for $y = x^2 - x - 6$

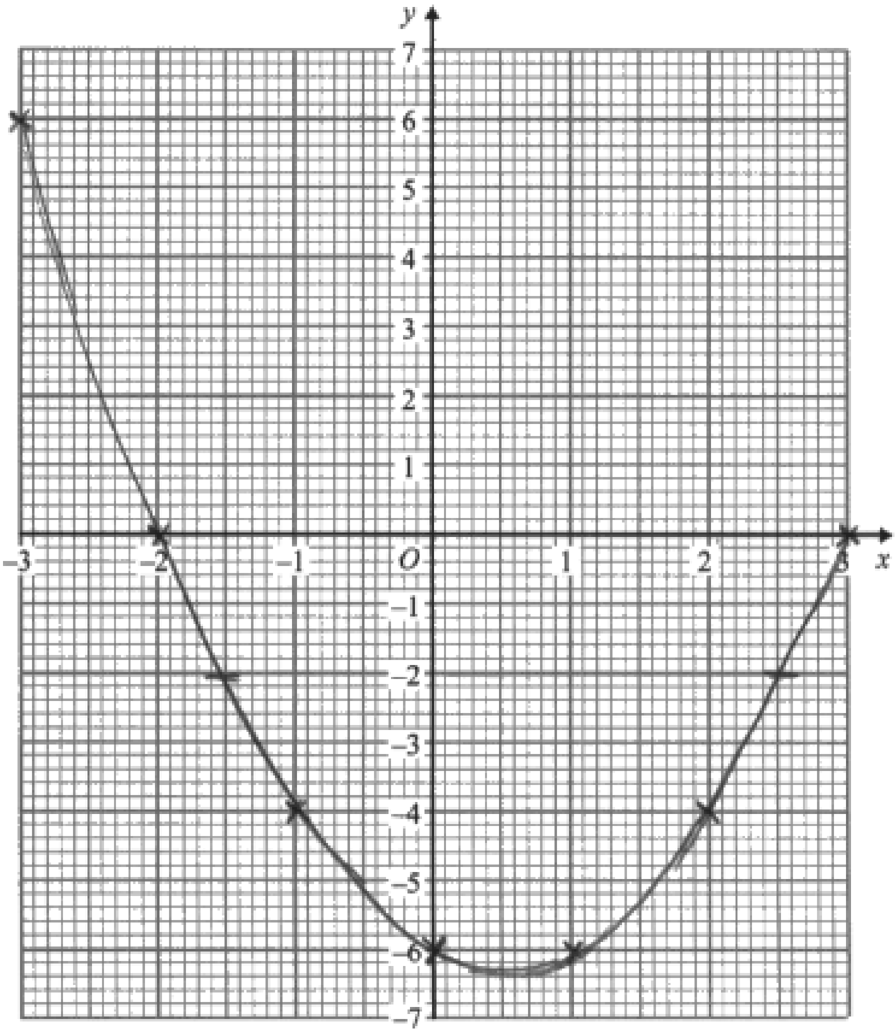
x	-3	-2	-1	0	1	2	3
y	6	0	−4	−6	−6	−4	0

(2)

2/2

(b) On the grid, draw the graph of $y = x^2 - x - 6$ for values of x from -3 to 3

(2)



2/2

(c) Use your graph to find estimates of the solutions to the equation $x^2 - x - 6 = -2$

-1.5, 2.5
(2)

(Total for Question 5 is 6 marks)

2/2

Examiner Comments

Here we have a perfect response. All values are correct in the table, followed by a fully correct graph. In part (c) both correct values are stated and, although no method is seen, they are correct and so the marks are awarded. In is worth noting that in a few places on the graph we can see slight "feathering" where the student has gone over their graph more than once. This was not penalised in the marking of this item.

Student Response B

5 (a) Complete the table of values for $y = x^2 - x - 6$

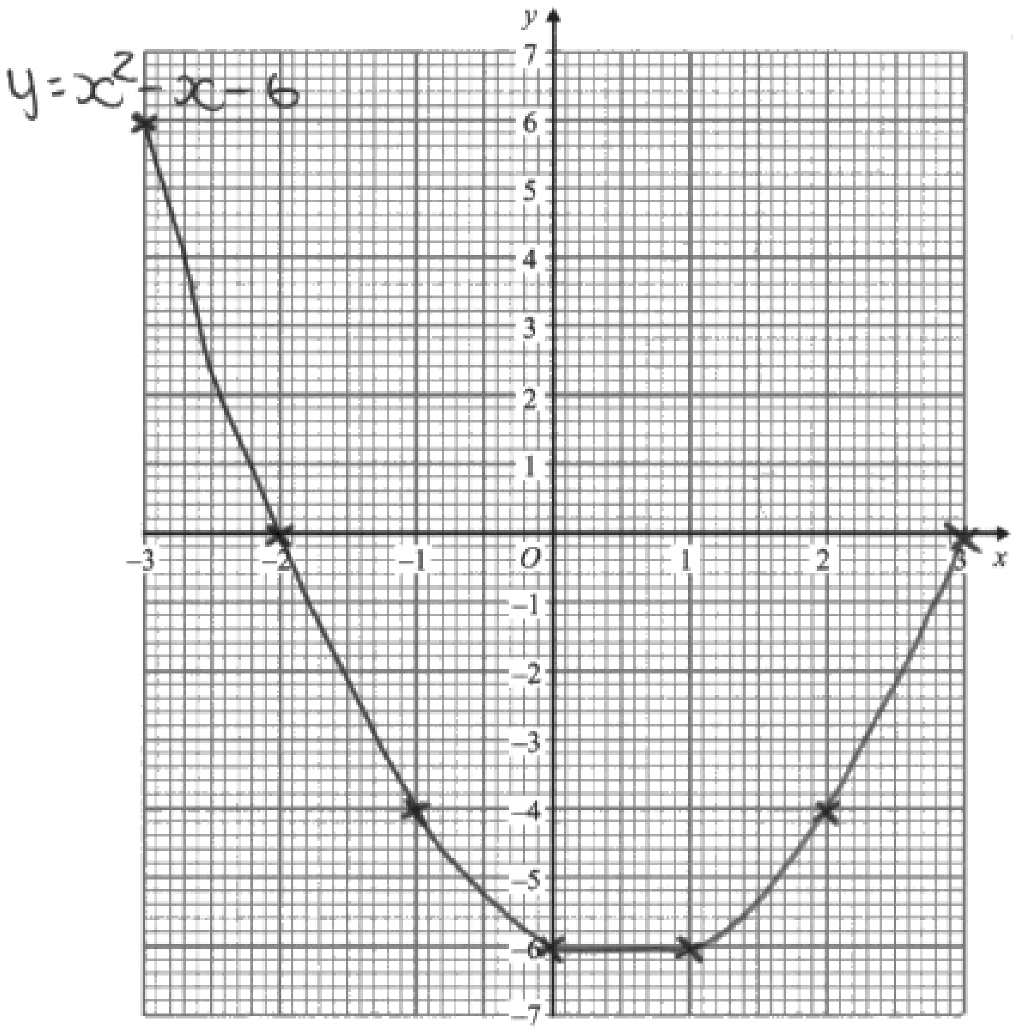
x	−3	−2	−1	0	1	2	3
y	6	0	−4	−6	−6	−4	0

(2)

2/2

(b) On the grid, draw the graph of $y = x^2 - x - 6$ for values of x from −3 to 3

(2)



1/2

(c) Use your graph to find estimates of the solutions to the equation $x^2 - x - 6 = -2$

$-3x - 3 = 9 \quad 9 + 3 - 6 = -2$

....

(2)

(Total for Question 5 is 6 marks)

0/2

Examiner Comments

This student has started well and has a fully correct table in (a). They have then plotted all their points correctly and joined the points. However, when joining the points at $x = 0$ and $x = 1$ they have used a line segment and not a curve giving the graph a “flat bottom” and thus lose the accuracy mark. In part (c) it is clear the student doesn’t know how to use the graph to solve the equation.

Student Response C

5 (a) Complete the table of values for $y = x^2 - x - 6$

x	−3	−2	−1	0	1	2	3
y	6	−2	4	−6	−6	−4	0

$y = x^2 - x - 6$ $\quad -2^2 = -4$

$-3^2 - 3 + 6$ $\quad -4 - 4 = -8 + 6$

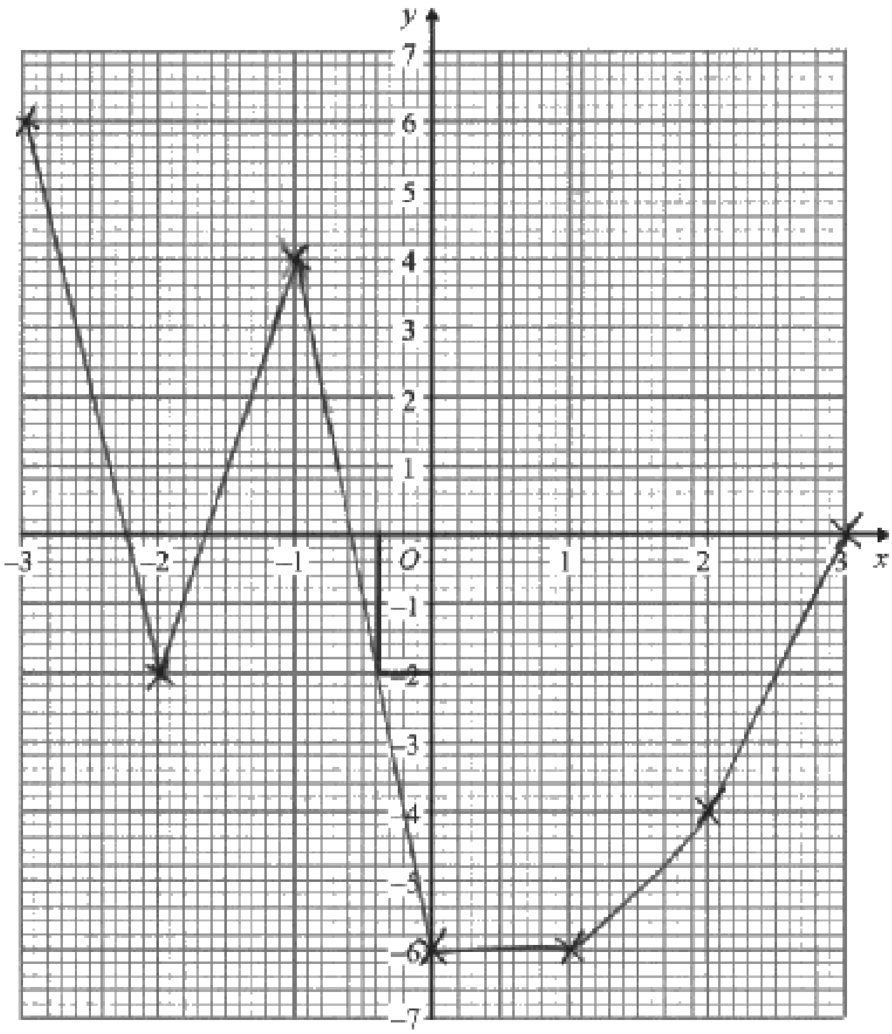
$-1^2 = -1$

$-1 - 1 = -2 + 6 = 4$ **(2)**

1/2

(b) On the grid, draw the graph of $y = x^2 - x - 6$ for values of x from −3 to 3

(2)



1/2

(c) Use your graph to find estimates of the solutions to the equation $x^2 - x - 6 = -2$

$x^2 - x - 6 = -2$

$y = -2$

$x = 0.4$
$y = -2$

$x = -0.4$

$x = -0.4$

(2)

1/2

Examiner Comments

Here we have the common sight of a student being able to correctly calculate y when x is positive, but not when it is negative. The two correct values in the table meant that B1 was scored.

In part (b) the follow through can be applied as one mark has been scored in part (a) and the method mark is awarded for at least 5 correctly plotted points. Their graph is clearly not fully correct so the accuracy mark cannot be awarded.

In part (c) the method mark can be awarded for the part of $y = -2$ which is drawn; the follow through cannot be applied for the accuracy mark as the graph is not quadratic.

Exemplar Question 3

Higher tier Paper 2

6 A force of 70 newtons acts on an area of 20 cm^2

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The force is increased by 10 newtons.
The area is increased by 10 cm^2

Helen says,

"The pressure decreases by less than 20%"

Is Helen correct?
You must show how you get your answer.

(Total for Question 6 is 3 marks)

Examiner Comments

There were many ways to approach this question. Typically, students found the original and new pressure and then did some calculation to see if Helen was correct. This could be to find the original pressure as a percentage of the new pressure, or vice versa. Some decreased the original pressure by 20% and compared this to the new pressure. Whichever way students approached the problem, a conclusion was required to score full marks.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
6	No (support-ed)	P1	For a process to calculate the initial or new pressure, e.g. (70 +10) ÷ (20+10) (=2.6 to 2.7) **or** 80 ÷ 30 (=2.6 to 2.7) **or** 70 ÷ 20 (=3.5)	Accept any value in the range 2.6 to 2.7 if unsupported by working
		P1	For a complete process to make a comparison e.g. 0.8 × "3.5" (=2.8) **or** $\frac{"3.5" - "2.6"}{"3.5"} \times 100$ (=22 to 26) **or** "3.5" × 0.2 (=0.7) and 80 ÷ 30 (=2.6 to 2.7) **or** $\frac{"2.6"}{"3.5"}$ (× 100) (= 0.74 to 0.78 **or** 74 to 78)	
		A1	for a correct conclusion supported by accurate figures e.g. 2.8 and 2.6(6…) **or** decrease is 24% (or 22% to 26%) **or** 0.7 and 2.6 to 2.7 and 3.5 **or** 0.7 and 0.9 **or** 0.76 (or 0.74 to 0.78) **or** 76% (or 74% to 78%)	Allow truncation or rounding of figures

Examiner Comments

The first process in the question was to find the original pressure or new pressure. Students typically found both.

The second mark was for a complete process that could lead to a correct conclusion. Many decreased the original amount by 20% so they could compare their found value to the new pressure. It is worth noting that for all these calculations a wide range for the final values was allowed. This was due to the fact that early rounding made no difference to a student's ability to answer the question, so they were not penalised.

The final mark required a conclusion that was supported by accurate figures. Again, the ranges for these figures were quite large.

Student Response A

6 A force of 70 newtons acts on an area of 20 cm²

The force is increased by 10 newtons.
The area is increased by 10 cm²

Helen says,
"The pressure decreases by less than 20%"

Is Helen correct?
You must show how you get your answer.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Initial pressure $= \frac{\text{force}}{\text{area}}$
$= \frac{70}{20}$
$= 3.5\text{pa}$

After pressure $= \frac{\text{force}}{\text{area}}$
$= \frac{80}{30}$
$= 2.\dot{6}$

$\frac{3.5 - 2.\dot{6}}{3.5} \times 100$

$= 23.8\%$

She is incorrect, it ~~do~~ decrease by more than 20%.

(Total for Question 6 is 3 marks)

3/3

Examiner Comments

Here we can see a perfect response. This student has calculated the initial and new pressure. Either of these is sufficient for the first P1. To make their comparison, they have calculated the percentage change correctly (23.8%). It is worth noting that had this student rounded $2.\dot{6}$ to either 2.6 or 2.7 they could still have gone on and been awarded full marks. There is then a correct conclusion stated, which along with the value (23.8%) gains the final mark.

Student Response B

6 A force of 70 newtons acts on an area of 20 cm^2

The force is increased by 10 newtons.
The area is increased by 10 cm^2

pressure = $\frac{\text{force}}{\text{area}}$

Helen says,

"The pressure decreases by less than 20%"

Is Helen correct?
You must show how you get your answer.

$P = \frac{70}{20} =$

$p = 3.5$.

$\frac{80}{30} = 2.\dot{6}$

~~0.9~~ 0.9 $3.5 - 2.6 = 0.9$.

~~20~~

$3.5 \div 10 \times 2 = \underline{0.7}$

Yes Helen is correct.

(Total for Question 6 is 3 marks)

2/3

Examiner Comments

This student starts by finding both the original and new pressure. Their second step was to find out the decrease in pressure from the original to new (0.9) and along with that they have found 20% of the original amount to compare (0.7). Unfortunately, this student has then drawn the incorrect conclusion from these figures, so they fail to gain the last mark.

Student Response C

6 A force of 70 newtons acts on an area of 20 cm²

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The force is increased by 10 newtons.
The area is increased by 10 cm²

Helen says,

"The pressure decreases by less than 20%"

Is Helen correct?
You must show how you get your answer.

$$P = \frac{70}{20} = 3.5$$

$$P = \frac{80}{30} = 2.6$$

$$2.6 \times 1.2 = 3.12$$

Helen is not correct.

(Total for Question 6 is 3 marks)

1/3

Examiner Comments

Like most students this student has calculated the original and new pressure. As in both previous example, either of these would score the first P1. The new pressure is then increased by 20%, rather than original pressure being decreased by 20%. The second method mark cannot therefore be awarded and this, in turn, means that no correct comparison can be made.

Exemplar Question 4

Higher tier Paper 2

8 60 people were asked if they prefer to go on holiday in Britain or in Spain or in Italy.

38 of the people were male.
11 of the 32 people who said Britain were female.
8 males said Italy.
12 people said Spain.

One of the females is chosen at random.

What is the probability that this female said Spain?

....

(Total for Question 8 is 4 marks)

Examiner Comments

This question is assessing the ability of students to interpret and make sense of given data. Generally, students did well in answering this question with a large proportion able to gain the first 3 marks. Often losing the last mark for using incorrect values in the probability. The best responses came when students used a structured method such as a frequency tree or two-way table. Many of the student who attempted to sort the data without such structure, often got confused or lost in the figures.

Mark Scheme

<table>
<tr><th>Question</th><th>Answer</th><th>Mark</th><th>Mark scheme</th><th>Additional guidance</th></tr>
<tr><td rowspan="4">8</td><td rowspan="4">$\frac{3}{22}$</td><td>P1</td><td>for a process to find a first value
e.g. male/Britain
= 32 – 11 (=21)
or Italy/total
= 60 – (32+12) (=16)
or female/total
= 60 – 38 (=22)</td><td><table><tr><th></th><th>Br</th><th>Sp</th><th>It</th><th>Tot</th></tr><tr><td>M</td><td>21</td><td>9</td><td>8</td><td>38</td></tr><tr><td>F</td><td>11</td><td>3</td><td>8</td><td>22</td></tr><tr><td>Tot</td><td>32</td><td>12</td><td>16</td><td>60</td></tr></table></td></tr>
<tr><td>P1</td><td>for process to find a secondary value,
e.g. male/Spain
= 38 – (“21” + 8) (=9)
or female/Italy
= “16” – 8 (=8)</td><td>May be seen in a frequency tree
Values attributed to a category or from method seen</td></tr>
<tr><td>P1</td><td>complete process to find female/Spain,
e.g. 12 – “9”
or “22” – (11 + “8”) (=3)</td><td></td></tr>
<tr><td>A1</td><td>oe accept 0.136 to 0.14
SC B3 for $\frac{3}{60}$</td><td></td></tr>
</table>

Examiner Comments

Here we have three marks for processes leading to the correct value for female/Spain.

The first mark, for finding a first value, was accessed by most students. Typically, this was female/total (22) but any of the values that could be calculated from the given figures was acceptable.

To gain the second mark, students needed to find a secondary value. This is a value that requires the use of a value already found by the student. Many of the students who used an unstructured method started to lose their way here, often down to mis-labelling values. Values had to be clearly attributed to a category to gain credit.

The third value is for correct processes to lead to the value for female/Spain. It is worth noting that in all of these marks, as long as method was seen and was right, the values need not be correct.

The final mark was for using the correct information and representing it is a fraction (or decimal or percentage). Many students lost this final mark as they chose the denominator to be the overall total rather than the female total.

Student Response A

8 60 people were asked if they prefer to go on holiday in Britain or in Spain or in Italy.

38 of the people were male.
11 of the 32 people who said Britain were female.
8 males said Italy.
12 people said Spain.

One of the females is chosen at random.

What is the probability that this female said Spain?

	B	S	I	total
M	21	9	8	38
F	11	3	8	22
Total	32	12	16	60

32 + 12 = 44
60 − 44 = 16
16 − 8 = 8
32 − 11 = 21
21 + 8 = 29
38 − 29 = 9
12 − 9 =

11 + 3 + 8 = 22

Total female = 22
Spain female = 3

$\frac{3}{22} = 0.1\dot{3}\dot{6}$

$\frac{3}{22}$

(Total for Question 8 is 4 marks)

4/4

Examiner Comments

This is a perfect response. The student has used the structure of a two-way table. All values in the table are correct and the correct values are used for the probability.

Student Response B

8 60 people were asked if they prefer to go on holiday in Britain or in Spain or in Italy.

38 of the people were male.
11 of the 32 people who said Britain were female.
8 males said Italy.
12 people said Spain.

One of the females is chosen at random.

What is the probability that this female said Spain?

Total=60

	B	S	I	Total
male	21	9	8	38
female	11	3	8	22
total	32	12	16	60

$\frac{3}{60}$

(Total for Question 8 is 4 marks)

3/4

Examiner Comments

Again, this student has used a two-way table to structure their response. This student has not shown any working, but since all values in the table are correct all three process marks are awarded. Unfortunately, this student, like many others, has chosen the incorrect denominator, for their fraction. It was very common to see 60 used instead of the correct value of 22.

Student Response C

8 60 people were asked if they prefer to go on holiday in Britain or in Spain or in Italy.

38 of the people were male.
11 of the 32 people who said Britain were female.
8 males said Italy.
12 people said Spain.

One of the females is chosen at random.

What is the probability that this female said Spain?

60 − 38 = 22 females.

22 − 11 = 11

11 females said britain → 11 left

No females said Italy

so 11 females said spain

11/60

(Total for Question 8 is 4 marks)

1/4

Examiner Comments

This student has used no structure to their response, which will have possibly contributed to their answer going wrong. They have started correctly by finding the total for females (60 – 38 = 22). Their next step is correct but not sufficient for the second mark. They have found how many females chose Italy and Spain combined. However, they have then made an incorrect assumption that none chose Italy and so all 11 must have chosen Spain. We cannot then give credit for the probability as it comes from incorrect values (and they have also chosen the wrong denominator).

Exemplar Question 5

Higher tier Paper 2

9 Jean invests £12 000 in an account paying compound interest for 2 years.

In the first year the rate of interest is x%
At the end of the first year the value of Jean's investment is £12 336

In the second year the rate of interest is $\frac{x}{2}$%

What is the value of Jean's investment at the end of 2 years?

£..

(Total for Question 9 is 4 marks)

Examiner Comments

This question assesses students' understanding of compound interest. The student has to use the two given values to calculate the interest rate for year one.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
9	12508.7(0)	P1	for start of process to find interest rate for year 1 e.g. 12336 ÷ 12000 (=1.028) **or** (12336 – 12000) ÷ 12000 (=0.028) **or** forms a suitable equation, e.g. $12000 \times (1 + \frac{x}{100}) = 12336$	
		P1	for complete process to find the interest rate for year 1 e.g. (“1.028” – 1) × 100 (=2.8) **or** “0.028” × 100 (=2.8) **or** correct process to solve correct equation e.g. (12336 – 12000) ÷ 120 (=2.8)	Rate of interest = 2.8, or $x = 2.8$ implies P2
		P1	for complete process to find the value at the end of 2 years e.g. (“2.8” ÷ 2 + 100) ÷ 100 × 12336	
		A1	accept 12508.7 to 12508.71 **or** 12509	12509 must come from correct working

Examiner Comments

Although this is a problem solving question, it has three distinct steps that need to be carried out in order. In essence each mark was dependent on the previous one, as shown by the use of inverted commas within the scheme.

The first mark is for a start to a process to find the interest rate for year one. Many students were able to gain this mark using one of the examples given in the mark scheme.

The second mark is effectively for extracting the rate from the multiplier, and it was this that many students struggled with. Many simply halved the multiplier rather than finding the rate first.

The third mark was for a complete process to find the value after two years. For the accuracy mark, students were not penalised for incorrect money notation.

Student Response A

9 Jean invests £12 000 in an account paying compound interest for 2 years.

In the first year the rate of interest is x%
At the end of the first year the value of Jean's investment is £12 336

In the second year the rate of interest is $\frac{x}{2}$%

What is the value of Jean's investment at the end of 2 years?

A First year
~~orig = ne~~ new = orig × pm^n
~~12~~ 12336 = 12000 × pm^1
pm = $\frac{12336}{12000}$
= 1.028
x = (1.028 − 1) × 100
= 2.8%
Second year
$\frac{x}{2}$% = $\frac{2.8}{2}$%
= 1.4%
new = 12336 × 1.014^1
= £12508.70 ~~ft~~ (2dp)

£ 12508.70

(Total for Question 9 is 4 marks)

4/4

Examiner Comments

Here we have a perfect response, set out really well. The student has shown each step of the problem clearly, stating all working clearly. They gain the first mark for dividing 12336 by 12000 to reach 1.028 (the division is sufficient for the mark).

The second is for then the process to get the interest rate from the 1.028. Again, the stated method would gain the mark, as would the 2.8 with no method seen.

They have then correctly dealt with the remainder of the problem, by finding the rate for year two, converting to a multiplier and then finding the product of that and 12336. The final answer is correct.

Student Response B

9 Jean invests £12 000 in an account paying compound interest for 2 years.

In the first year the rate of interest is x%
At the end of the first year the value of Jean's investment is £12 336

In the second year the rate of interest is $\frac{x}{2}$%

What is the value of Jean's investment at the end of 2 years?

$x = ?$

12,336 × 1.4 = 17 270.4

12000
12,336

$\frac{12336}{12000} = 1.028$

2.8%

$\frac{2.8}{2} = 1.4$

£ 17,270.4

(Total for Question 9 is 4 marks)

2/4

Examiner Comments

This student has started the problem well. They have gained the first process mark for finding the multiplier for year one and then the second for getting as far as 2.8%. From here they find the rate for year two as 1.4, but use it incorrectly, multiplying 12336 by 1.4 rather than 1.014 and so score no further marks.

Student Response C

9 Jean invests £12 000 in an account paying compound interest for 2 years.

In the first year the rate of interest is x%
At the end of the first year the value of Jean's investment is £12 336

In the second year the rate of interest is $\frac{x}{2}$%

What is the value of Jean's investment at the end of 2 years?

12336 ÷ 12000 = 1.028 ✓ Multiplier

$12336 \times \frac{1.028}{2} = 6340.704$

↓ Rounded into Currency

£6340.70

£ 6340.70

(Total for Question 9 is 4 marks)

1/4

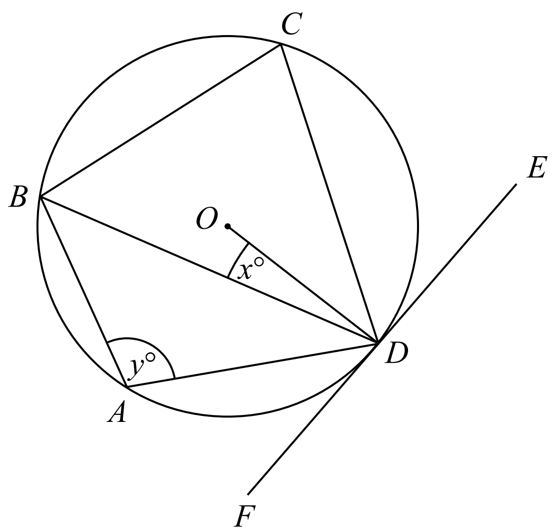
Examiner Comments

Here we have a fairly common response. The student starts well with a process leading to 1.028 and gaining the first process mark. However, rather than extract the 2.8% the student halves the multiplier and uses that. This is obviously wrong and the response scores nothing after the first mark.

Exemplar Question 6

Higher tier Paper 2

13



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
FDE is a tangent to the circle.

(*a*) Show that $y - x = 90$
You must give a reason for each stage of your working.

(3)

Dylan was asked to give some possible values for x and y.

He said,

"y could be 200 and x could be 110, because $200 - 110 = 90$"

(*b*) Is Dylan correct?
You must give a reason for your answer.

....

....

(1)

(Total for Question 13 is 4 marks)

Examiner Comments

Part (a) of this question is looking for students to put together a chain of geometric reasoning, including the use of circle theorems to show a given result. Part (b) is assessing the ability of students to use geometric reasoning to agree or disagree with a statement. In part (a) students used two or three different chains of reasoning to show the result. All methods required the use of at least one circle theorem and a number of other geometric properties. Many students were able to gain one mark for a first correct angle, but after that many struggled. The use of algebra rather than specific numbers in the question proved demanding for many students. Many students were able to access part (b) often by using angles in a triangle to state that Dylan was incorrect.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
13(a)	Shown	M1	for finding one missing angle e.g. $BDE = y$ or $ODE = 90$ or $ODF = 90$ or $DBO = x$ or $BCD = 180 - y$ or (reflex) $BOD = 2y$	Could be shown on the diagram or in working
		A1	for a complete correct method leading to $y - x = 90$	
		C1	(dep on A1) for all correct circle theorems given appropriate for their working e.g. The tangent to a circle is perpendicular (90°) to the radius (diameter) Alternate segment theorem OR Angle at the center is twice the angle at the circumference Opposite angles in a cyclic quadrilateral sum to 180°	
(b)	Explana-tion	C1	for explanation e.g. No as y must be less than 180 as it is an angle in a triangle	

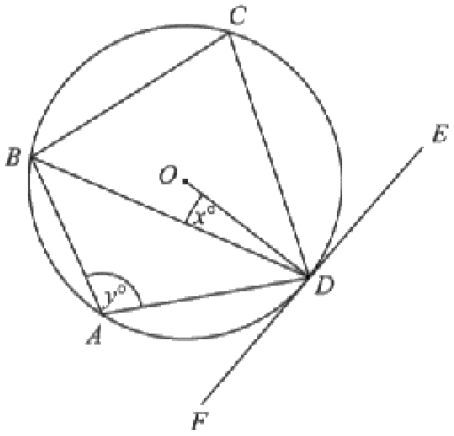
Examiner Comments

Part (a) of this question offered a first mark that was accessible to many. This was often gained for stating ODE = 90 or ODF = 90, commonly seen stated on the diagram. Others gained the mark for giving $BCD = 180 - y$

To gain the second mark the full chain of reasoning needed to be given and correct. The most efficient method was to state $BDE = y$ (using the alternate segment theorem) and $ODE = 90$ (using a tangent to a radius is 90) and therefore $x = y - 90$ which easily leads to the given result. Others used the angle at the centre is double the angle at the circumference, along with the isosceles triangle BOD. The final mark was a single communication mark for correctly stating all used circle theorems.

Student Response A

13



$180 - y = 90 - x$
$180 - 90 - y = -x$
$90 = y - x$

A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
FDE is a tangent to the circle.

(a) Show that $y - x = 90$
You must give a reason for each stage of your working.

$\angle BCD = 180° - y°$ (because opposite angles in a cyclic quadrilateral must add up to 180°)

$\angle OBD = x°$ (because OB = OD as they are both radii so $\triangle OBD$ is iscoseles)

$\angle BOD = 180 - 2x$ (because angles in a triangle must add up to 180°)

$\angle BCD = 90 - x$ (because ~~the~~ the angle at the centre from two points on the circumference is twice the angle at the circumference from those two points, in the major chord)

$\therefore 180 - y = 90 - x$
$180 - 90 = y - x$
$90 = y - x$

(3)

3/3

Dylan was asked to give some possible values for *x* and *y*.

He said,
"*y* could be 200 and *x* could be 110, because 200 – 110 = 90"

(b) Is Dylan correct?
You must give a reason for your answer.

Dylan is not correct. y must be less than 180° as y + ∠BCD = 180 ~~and~~ any ∠BCD cannot be negative. x must be less than 90° as ∠ODF = 90°, and ∠ODB must be smaller than this.

(1)

(Total for Question 13 is 4 marks)

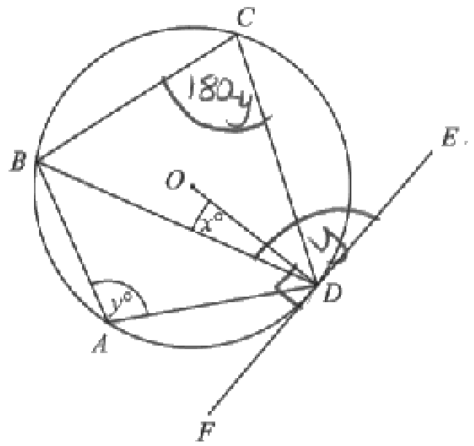
1/1

Examiner Comments

Here we have a clear and excellent response. The students gains the first mark for *BCD* = 180 – *y* stated in the first line. They then go on to complete a full chain of reasoning. In this response they find a second expression for angle *BCD* using the isosceles triangle *BOD* and angles at the centre. They then equate the two expressions and then rearrange to give the desired result. The final mark is awarded as both circle theorems used are stated correctly. In part (b) they gain the mark as using opposite angles in a cyclic quadrilateral.

Student Response B

13



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
FDE is a tangent to the circle.

(a) Show that $y - x = 90$
You must give a reason for each stage of your working.

∠BCD = 180-y → opposite angles in cyclic quadrilateral add to 180°

∠BDE = y → angles at tangent are equal to opposite angles in triangle

∠ODE = 90° → radius meets tangent at 90°
∠BDO = x

∠BDE = ∠BDO + ∠ODE
y = x + 90
y − x = 90

(3)

2/3

Dylan was asked to give some possible values for *x* and *y*.

He said,
"*y* could be 200 and *x* could be 110, because 200 – 110 = 90"

(b) Is Dylan correct?
You must give a reason for your answer.

y is in a triangle and angles in a triangle add to 180° so y can not be more than 180°. Dylan is incorrect

(1)

(Total for Question 13 is 4 marks)

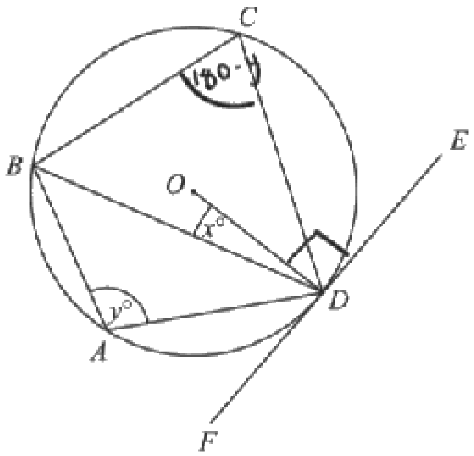
1/1

Examiner Comments

In part (a) the student gains the first mark for either $BCD = 180 - y$ or for $BDE = y$. The second mark is awarded as they have a fully correct method linking the angles *BDE, BDO* and *ODE*. The communication mark is not awarded however as the reason given for $BDE = y$ is incorrect; it should be the alternate segment theorem. The response given here for part (b) is correct and was very commonly seen.

Student Response C

13



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.
FDE is a tangent to the circle.

(a) Show that $y - x = 90$
You must give a reason for each stage of your working.

<ODE is equal to 90°, as a radius meets ~~the edge~~ a tangent at 90°.
< BCD is 180-y as opposing angles in a cyclic quadrilateral sum to 180°.

1/3

Dylan was asked to give some possible values for *x* and *y*.

He said,
"*y* could be 200 and *x* could be 110, because 200 – 110 = 90"

(b) Is Dylan correct?
You must give a reason for your answer.

x is acute, and Dylan provided an obtuse angle, so no.

(1)

1/1

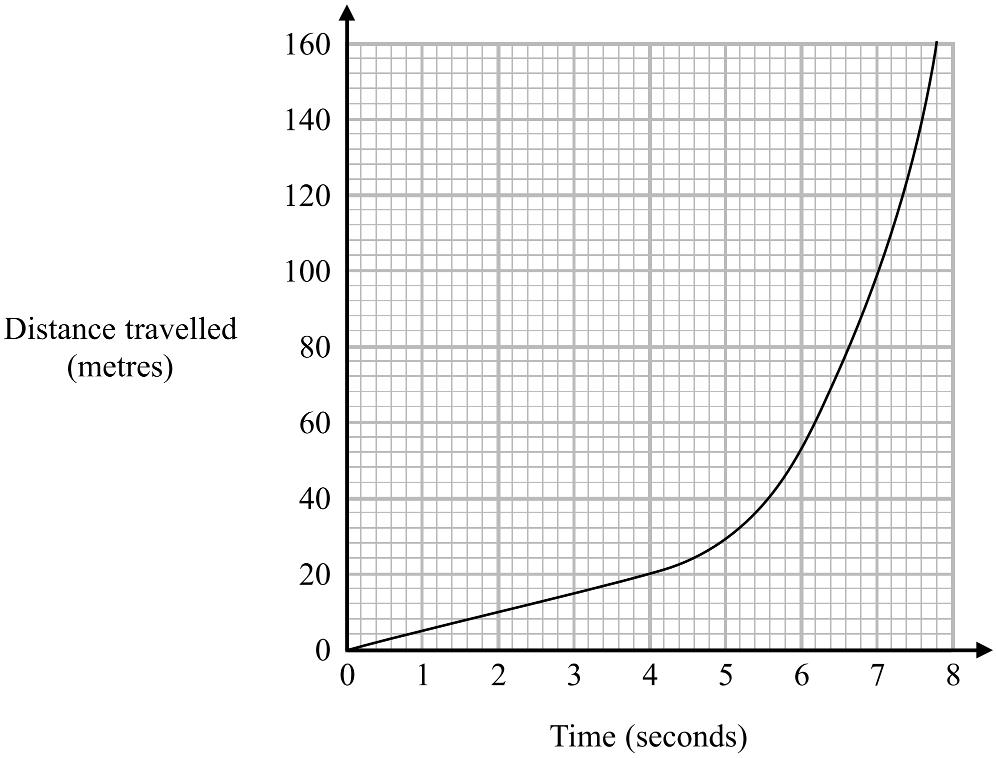
Examiner Comments

For part (a) M1 can be awarded for either *ODE* = 90 or for *BCD* = 180 – *y*. Both of these are stated in the body of the working and correctly labelled on the diagram. Again, either is acceptable. This student then doesn't know how to carry the method through and stops, and therefore scores no further marks. The response in part (b) was again commonly seen and was the minimum accepted for the mark.

Exemplar Question 7

Higher tier Paper 2

14 The distance-time graph shows information about part of a car journey.



Use the graph to estimate the speed of the car at time 5 seconds.

.... m/s

(**Total for Question 14 is 3 marks**)

Examiner Comments

This was assessing content new to GCSE Mathematics. The question required the student to understand that speed is the instantaneous rate of change on a distance-time graph, and that to calculate this required a tangent to be drawn and its gradient calculated.

Many students simply read off the distance at time = 5 and then calculated distance divided by time, and so scored no marks. A number of other students confused this with the other new topic, finding distance by calculating the area under a velocity-time graph, and again scored zero.

Mark Scheme

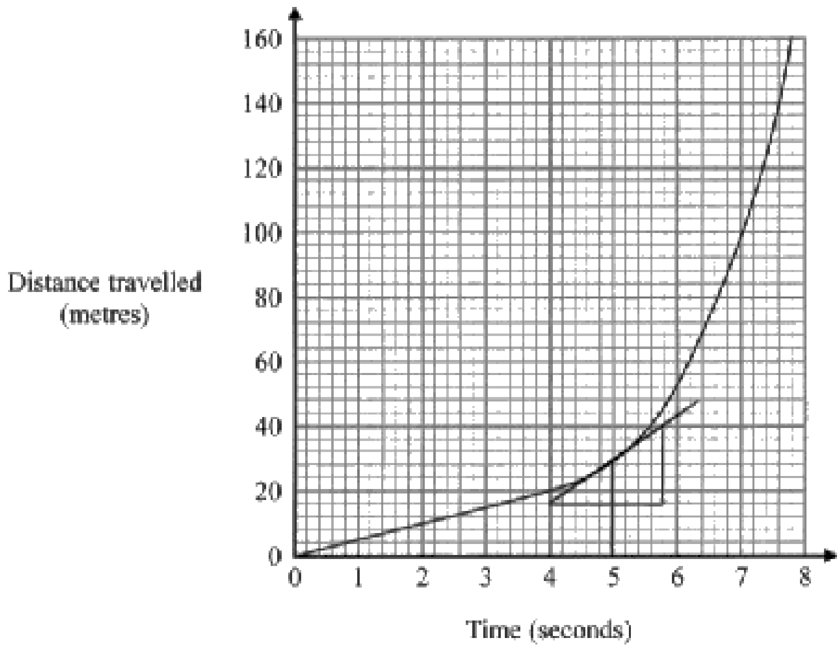
Question	Answer	Mark	Mark scheme	Additional guidance
14	11 – 19	P1	for drawing a tangent to the curve at time = 5	
		P1	for process to find the gradient, e.g. 70 ÷ 5	Using their drawn tangent, e.g. change in y ÷ change in x
		A1	(dep on 1st P1) for answer in the range 11 - 19 m/s	Must come from gradient of a tangent.

Examiner Comments

Failure to draw a tangent to the curve resulted in zero marks being scored. If the student drew a tangent at the wrong time, for example time = 6, then they couldn't access the first mark, but they could access the second mark. The award of the final mark was dependent on a tangent being drawn at the correct point on the graph.

Student Response A

14 The distance-time graph shows information about part of a car journey.



Use the graph to estimate the speed of the car at time 5 seconds.

$$\frac{24}{1.8} = 13.\dot{3}$$

..........$13.\dot{3}$.......... m/s

(Total for Question 14 is 3 marks)

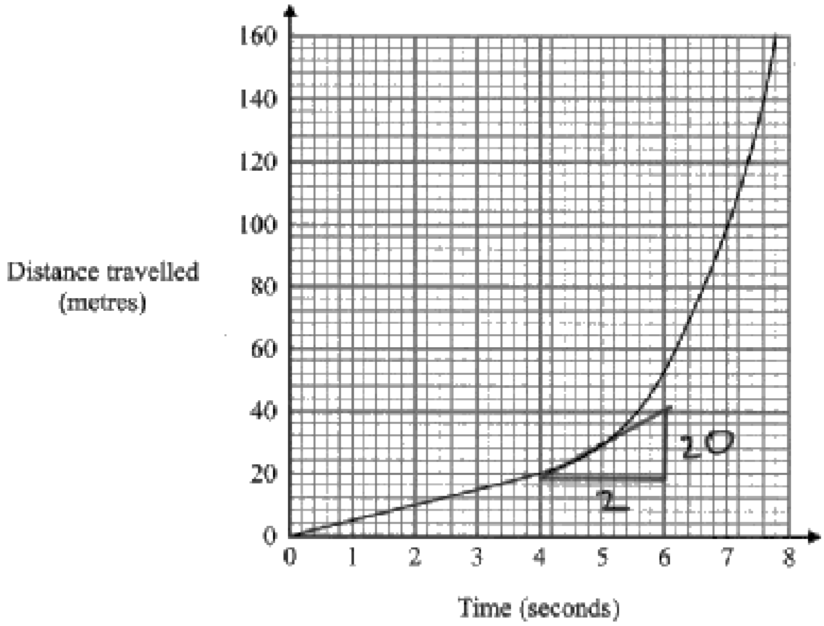
3/3

Examiner Comments

A tangent has been drawn at time = 5, so the first process mark can be awarded. The student has then drawn a right triangle and correctly read off the height and base and used these in a calculation to find the gradient of the tangent, so scores the second mark. The answer given is within range so the accuracy mark is awarded.

Student Response B

14 The distance-time graph shows information about part of a car journey.



Use the graph to estimate the speed of the car at time 5 seconds.

$\frac{20}{2} = 10$

$= 10\text{ m/s}$

10 m/s

(Total for Question 14 is 3 marks)

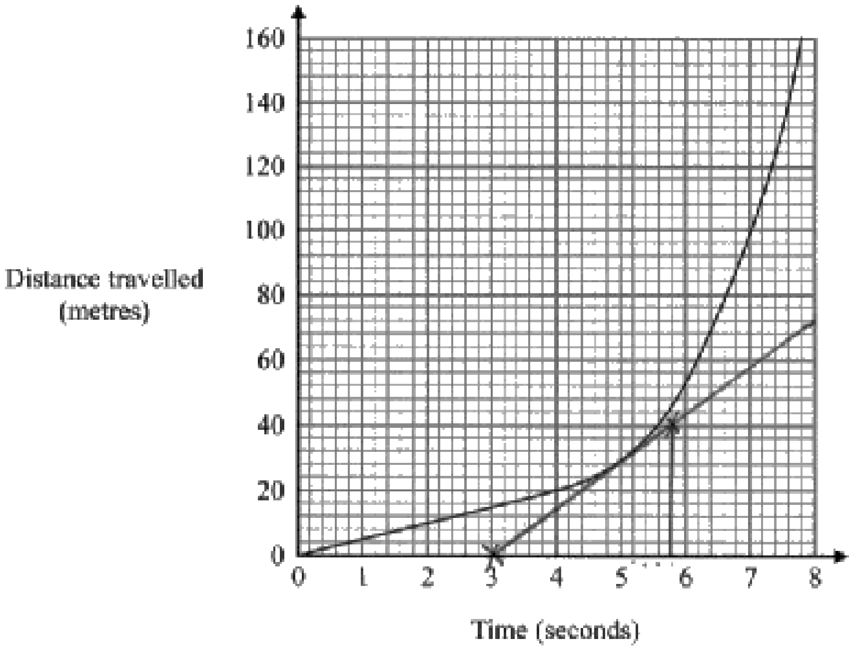
2/3

Examiner Comments

This student has clearly attempted to draw a tangent at time = 5, and so the first mark is awarded. They have drawn a right triangle and correctly used it to calculate the gradient of their tangent and so gain the second mark. Unfortunately, in the case the tangent isn't sufficiently accurate and leads to an answer outside the range given in the mark scheme which means the accuracy mark cannot be awarded.

Student Response C

14 The distance-time graph shows information about part of a car journey.



Use the graph to estimate the speed of the car at time 5 seconds.

(0 , 3)   (5.8 , 40)

$\frac{40-3}{5.8-0} = \frac{37}{5.8} = 6.379310345$ m/s

.......6.38....... m/s

(Total for Question 14 is 3 marks)

1/3

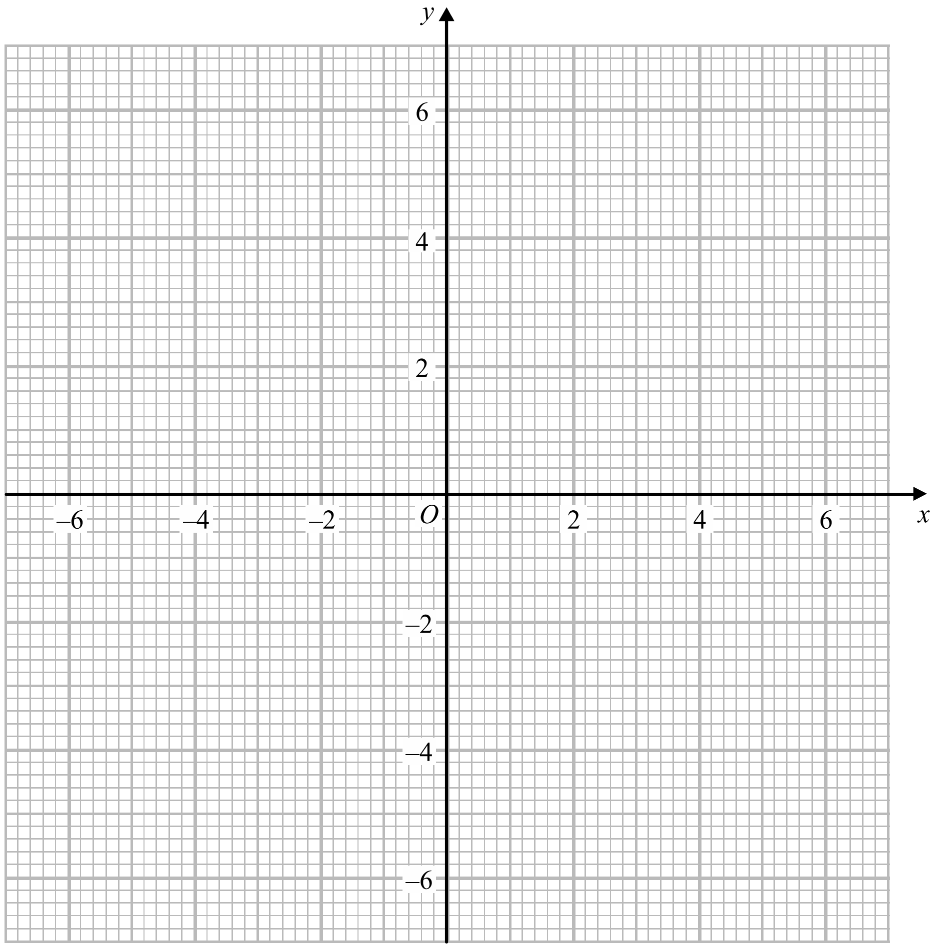
Examiner Comments

The student draws the tangent at the point time = 5 and so scores the first mark. The student then choses points on the tangent, but in writing down these coordinates makes a mistake, (0, 3) should be (3, 0). This means their calculation for the gradient is then wrong and no further marks can be awarded.

Exemplar Question 8

Higher tier Paper 2

16 (*a*) On the grid, draw the graph of $x^2 + y^2 = 12.25$



(2)

(*b*) Hence find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 12.25$$
$$2x + y = 1$$

....

(3)

(Total for Question 16 is 5 marks)

Examiner Comments

This is assessing two main skills. In part (a) understanding of the equation of a circle, and in part (b) the ability to solve simultaneous equations graphically.

Despite there being almost no working space in part (b) and the guidance in the question, a significant number of students attempted to solve the equations algebraically. Students were able to gain credit for this method. However, a number of students who chose an algebraic route failed to get beyond the first mark.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
16(a)	Correct graph	B2	for a circle radius 3.5, center (0, 0)	Circle could be drawn freehand as long as it approximates to a circle
		(B1	for a circle center (0, 0) of a different radius, or for a circle drawn of radius 3.5 center not (0, 0) or incomplete correct circle)	
(b)	$x = 2.0$, $y = -2.9$ $x = -1.2$, $y = 3.3$	M1	for $2x + y = 1$ drawn, or for correctly eliminating one variable, e.g. $x^2 + 1 - 4x + 4x^2 = 12.25$ or $x^2 + (1 - 2x)^2 = 12.25$	
		A1	for the pair of x values, or the correct pair of y values, or one correct pair of x/y values ft from (a) (dep on B1)	$2x + y = 1$ does not have to be shown Use professional judgment
		A1	for both correct pair of x/y values, unambiguously matched ft from (a) (dep on B1)	Accept values given as coordinates. Check graph for answers

Examiner Comment:

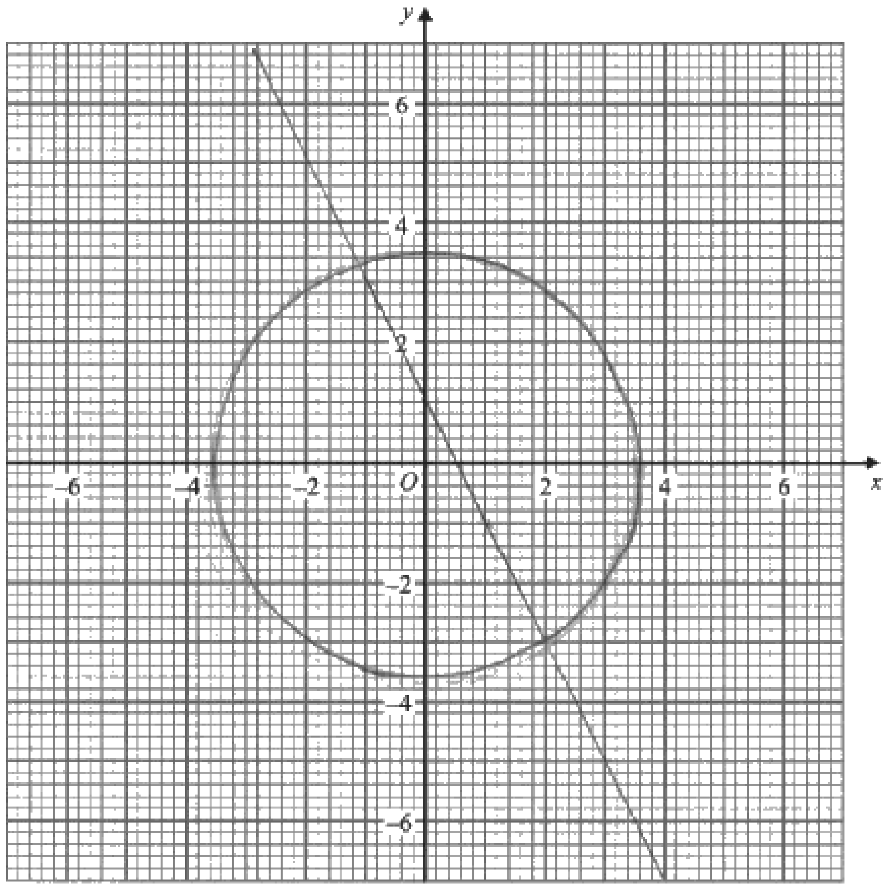
Part (a) offered two marks to those who drew the correct circle. However, there were a number of students who recognised that this was the equation for a circle, but were unsure of the radius. Providing the circle drawn had a centre (0, 0) they were able to gain one mark. Circles drawn “free hand” in part (a) were accepted as long as the intention was clear.

In part (b) the first mark was for drawing the correct graph of $2x + y = 1$. There were the two accuracy marks for the correct solutions. To get both marks the x and y vales needed to be correctly and unambiguously matched.

Provided 1 mark was scored in part (a) there was a follow through in part (b) that covered all marks. This meant a student who drew a circle with an incorrect radius in part (a) could go on and get all three marks in part (b).

Student Response A

16 (a) On the grid, draw the graph of $x^2 + y^2 = 12.25$



(2)

2/2

(b) Hence find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 12.25$$
$$2x + y = 1$$

$y = -2x + 1$

(-1.1, 3.3) (2, -3)

(3)

3/3

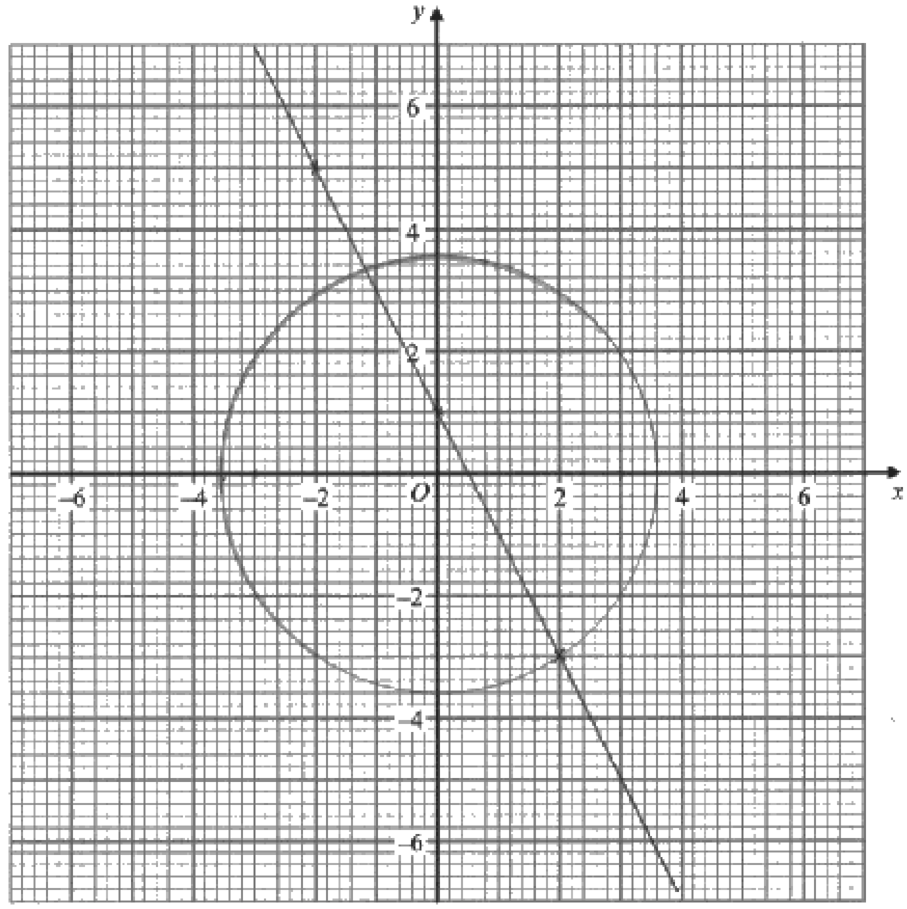
Examiner Comments

The circle drawn on the grid has centre (0, 0) and radius of 3.5 so both marks are awarded.

In part (b) the correct graph has been drawn on the grid so the first method mark can be awarded. The student has then stated both solutions as coordinates. This was taken as unambiguously matching the values and both accuracy marks awarded.

Student Response B

16 (a) On the grid, draw the graph of $x^2 + y^2 = 12.25$



$x^2+y^2=r^2$ $\sqrt{12.25}=\frac{7}{2}=3.5$

(2)

2/2

(b) Hence find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 12.25$$
$$2x + y = 1$$

$y=-2x+1$

y	-8	-2	0	2	4
x	4	5	1	-3	-15

$x=-1.2$ $y=3.3$

$x=2$ $y=3$ .... $x=-1.2$ $y=3.3$, $x=2$ $y=3$ ....

(3)

2/3

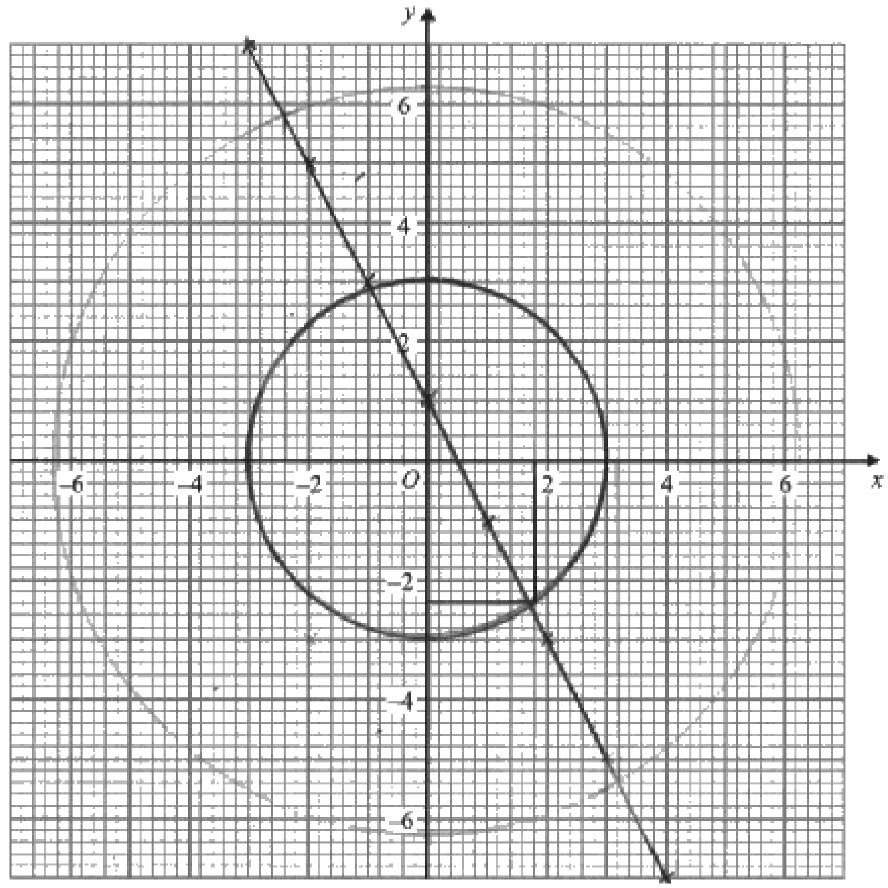
Examiner Comments

In part (a) the correct circle has been drawn and so both marks are awarded.

In part (b), it looks, at first glance, like the student has a fully correct response. The correct second graph has been drawn and the first solution is correct. However, the second solution is slightly wrong. It should be $x = 2$, $y = -3$, but the student has stated $y = 3$. This means the award of just one accuracy mark.

Student Response C

16 (a) On the grid, draw the graph of $x^2 + y^2 = 12.25$



(2)

1/2

(b) Hence find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 12.25$$
$$2x + y = 1$$

3.5

x = 1 y = 3
x = -2.4 y = -1.8

y = 3 and x = 1

(3)

1/3

Examiner Comments

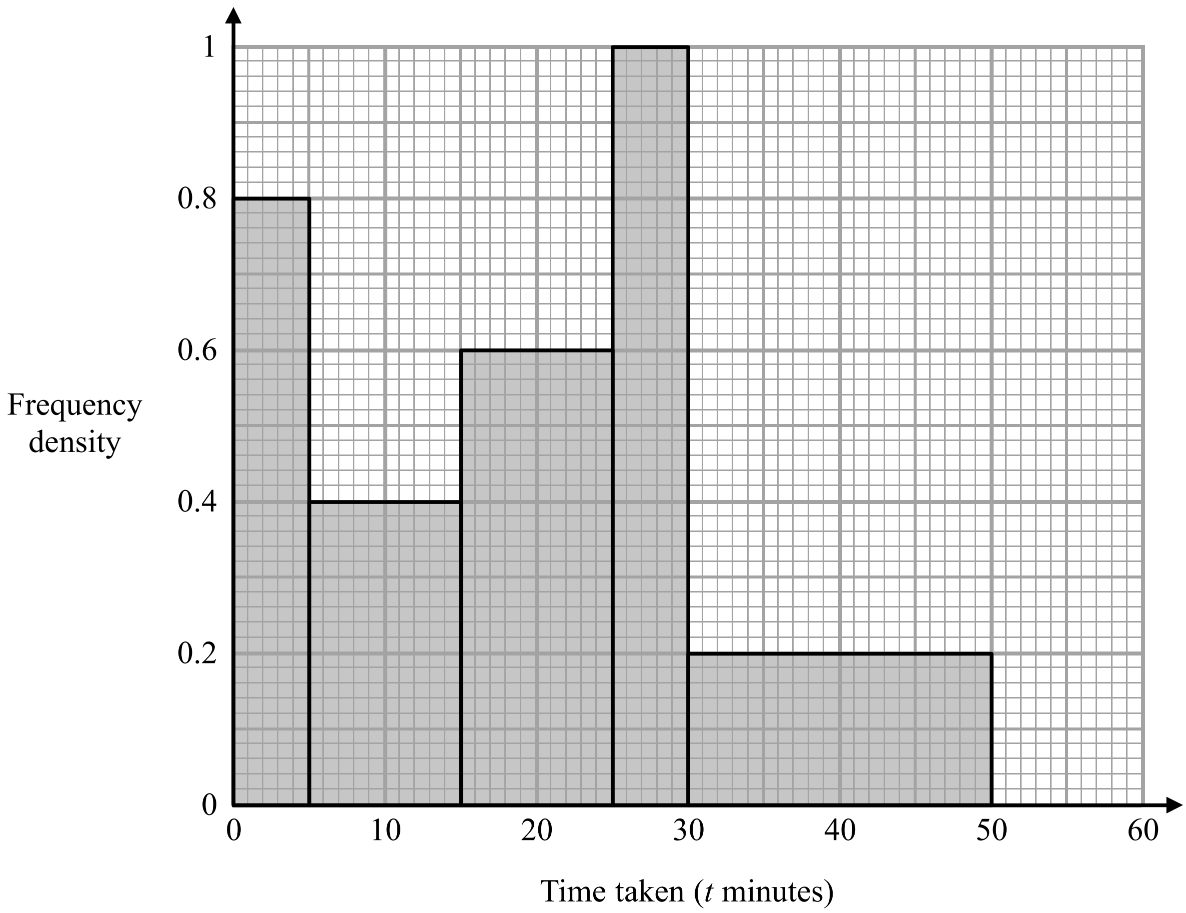
Unfortunately, despite stating the correct radius in the main body of working, this circle has a radius of 3. However, the circle does have centre (0, 0) so 1 mark can be awarded.

In part (b) the correct second graph has been drawn so the first mark can be awarded. As one mark was awarded in part (a), the follow through can be applied. However, the values given do not follow from the circle drawn in part (a) so no further marks can be awarded.

Exemplar Question 9

Higher tier Paper 2

17 The histogram shows information about the times taken by some students to finish a puzzle.



(*a*) Complete the frequency table for this information.

Time taken (*t* minutes)	**Frequency**
$0 < t \leqslant 5$	4
$5 < t \leqslant 15$	
$15 < t \leqslant 25$	
$25 < t \leqslant 30$	
$30 < t \leqslant 50$	

(2)

(*b*) Find an estimate for the lower quartile of the times taken to finish the puzzle.

.... minutes
(2)

(Total for Question 17 is 4 marks)

Examiner Comments

Part (a) requires students to use a histogram to complete a frequency table. For those students who understand histograms with unequal class intervals this is a straight forward question, assessing the relationship between area and frequency.

Part (b) required an understanding of the lower quartile. Many students tried to find ¼ of 50 rather than finding $\frac{23+1}{4}$ or $\frac{23}{4}$.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
17(a)	4, 6, 5, 4	M1	for a correct method to find at least 2 frequencies from bars of different widths, e.g. 10 × 0.4 (= 4), 10 × 0.6 (= 6), 5 × 1 (= 5), 20 × 0.2 (= 4)	
		A1	cao	
(b)	10	M1	for $\frac{23+1}{4}$ (= 6) **or** $\frac{23}{4}$ (= 5.75) could ft from their table in (a)	
		A1	for 10 or 9.375	Be aware of 10 coming from incorrect working ft does not apply to the A1

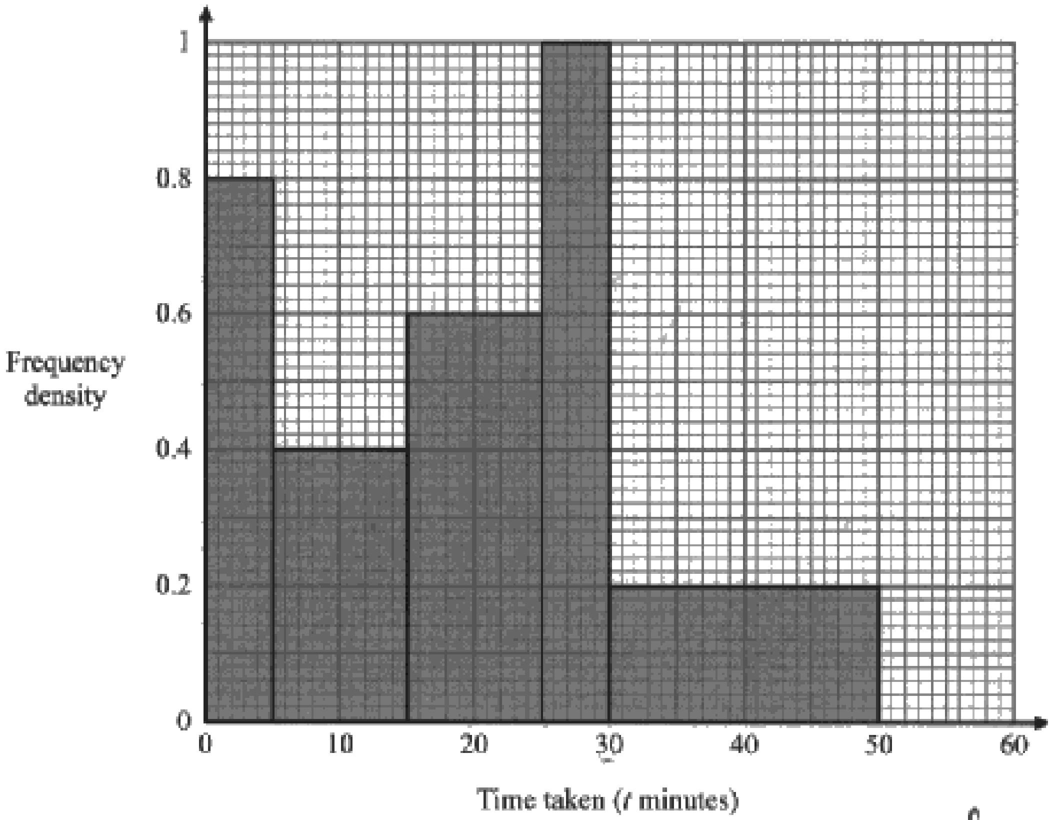
Examiner Comments

In part (a) the method mark was available to those who could show a method to find two frequencies (must be bars of different widths). A fully correct frequency table gained both marks.

In part (b) the first mark was for an attempt to find $\frac{n+1}{4}$ or $\frac{n}{4}$ where the value of n could follow through from their table in part (a) for the first mark only. The second mark was then for a correct value, and the only 2 acceptable values were 10 (from using 23 +1) or 9.375 (from using 23).

Student Response A

17 The histogram shows information about the times taken by some students to finish a puzzle.



(a) Complete the frequency table for this information.

$fd = \frac{f}{cw}$

Time taken (*t* minutes)	**Frequency**
$0 < t \leqslant 5$	4
$5 < t \leqslant 15$	4
$15 < t \leqslant 25$	6
$25 < t \leqslant 30$	5
$30 < t \leqslant 50$	4

(2)

2/2

(b) Find an estimate for the lower quartile of the times taken to finish the puzzle.

23 in total

LQ = 6

23+1 = 24

$\frac{24}{4} = 6$

$\frac{1}{2}$ way in

$5 < t \leqslant 15$

so

10 .... minutes

(2)

(Total for Question 17 is 4 marks)

2/2

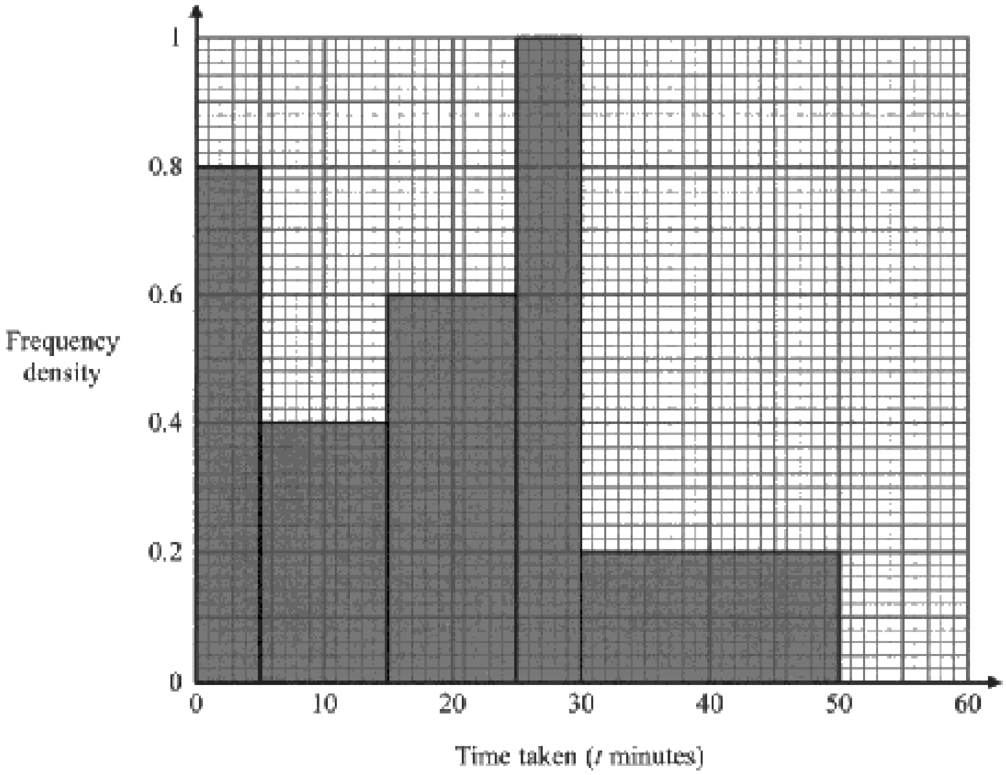
Examiner Comments

A fully correct table in part (a) gains both marks. It was very common to see no working at all in part (a). In that case two correct values would imply the award of the method mark providing the bars with of a different width.

Part (b) shows the student calculate the lower quartile as the 6th value. They then find that this is half way through the 5 to 15 bar and get the correct answer of 10.

Student Response B

17 The histogram shows information about the times taken by some students to finish a puzzle.



(a) Complete the frequency table for this information.

Time taken (t minutes)	Frequency	
$0 < t \leqslant 5$	4	
$5 < t \leqslant 15$	4	0.4 ~~4~~ x 10
$15 < t \leqslant 25$	6	0.6x10
$25 < t \leqslant 30$	5	1 x 5
$30 < t \leqslant 50$	4	0.2x20

(2)

Frequency = density x CW

2/2

(b) Find an estimate for the lower quartile of the times taken to finish the puzzle.

Total frequency = 4 + 4 + 6 + 5 + 4 = 23 students

$\frac{23+1}{4} = 6^{th}$ number

6^{th} lies in $5 < t \leq 5$

$5 < t \leq 15$

$5 < t \leq 15$ minutes

(2)

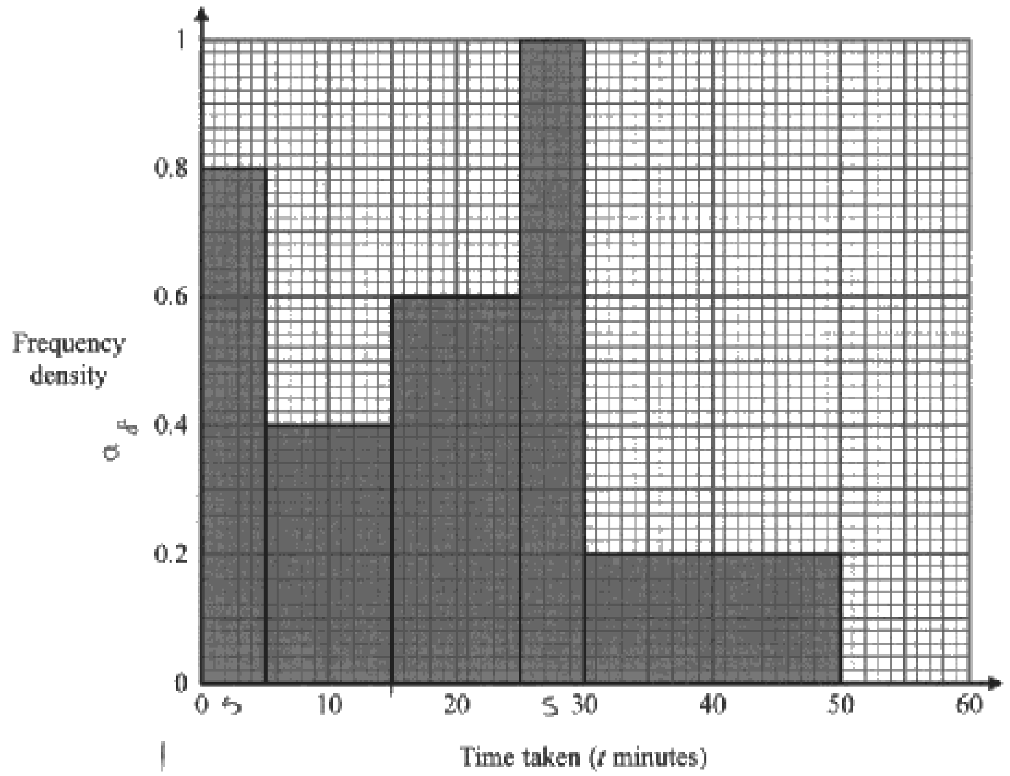
(Total for Question 17 is 4 marks)

1/2

Examiner Comments
The table is fully correct in part (a).
In part (b) the student has started correctly and found the value 6 from correct working. They know which interval the 6^{th} is in, but are unable to finish the question and so fail to gain the last mark.

Student Response C

17 The histogram shows information about the times taken by some students to finish a puzzle.



(a) Complete the frequency table for this information.

Time taken (t minutes)	Frequency
$0 < t \leqslant 5$	4
$5 < t \leqslant 15$	4
$15 < t \leqslant 25$	12
$25 < t \leqslant 30$	5
$30 < t \leqslant 50$	4

10 × 0.4
20 × 0.6
5 × 1
20 × 0.2

(2)

1/2

(b) Find an estimate for the lower quartile of the times taken to finish the puzzle.

$50 \div 4 = 12.5$

$12.5 + 12.5 = 25$

25 minutes

(2)

(Total for Question 17 is 4 marks)

0/2

Examiner Comments

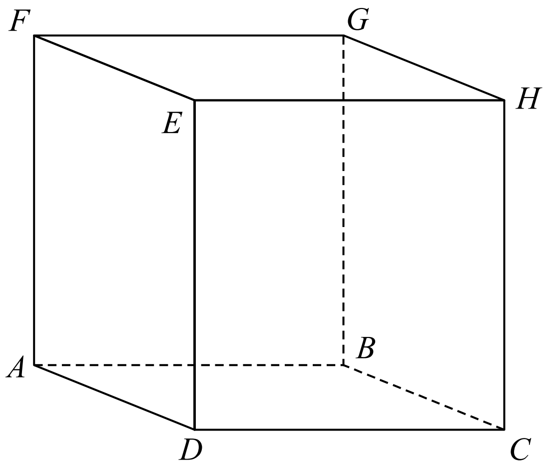
The students here has got one class width wrong (3rd bar) and used 20 instead of 10. This means their table is not fully correct. However, there are still three correct bars with different widths so the method mark can be awarded. It is worth noting that the working alone for these three bars would have still gained the method mark.

In part (b) the students uses the upper value for time (50) rather than the total frequency, and so scores no marks.

Exemplar Question 10

Higher tier Paper 2

18 *ABCDEFGH* is a cuboid.



$AB = 7.3$ cm
$CH = 8.1$ cm
Angle $BCA = 48°$

Find the size of the angle between AH and the plane $ABCD$.
Give your answer correct to 1 decimal place.

.... °

(Total for Question 18 is 4 marks)

Examiner Comments

This question assesses the use of trigonometry (and possibly Pythagoras's theorem) in three dimensions. It can be completed using the standard trigonometric ratios, or using the Sine Rule.

Many students struggled to visualise the correct angle to find, and as a result they failed to gain any marks. Others made incorrect assumptions, such as the base of the cuboid being a square.

Some students used methods that whilst being correct were not efficient, and included finding values that were not necessary.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18	39.5	P1	for a start to a process e.g., for a correct trigonometric statement, e.g. $\sin 48 = \frac{7.3}{AC}$ or $\cos 42 = \frac{7.3}{AC}$ or $\frac{AC}{\sin 90°} = \frac{7.3}{\sin 48°}$ OR angle *CAH* unambiguously identified on a diagram	Must include correct values
		P1	for a complete correct process to find *AC*, e.g. $(AC =)\ \frac{7.3}{\sin 48°}$ (=9.8..) or $(AC=)\ \frac{7.3}{\cos 42°}$ (=9.8..) or $(AC =)\ \sin 90° \times \frac{7.3}{\sin 48°}$ (=9.8..)	
		P1	for a correct statement using angle *CAH*, e.g. $\tan(CAH) = \frac{8.1}{"9.8..."}$ OR $\sqrt{8.1^2 + "9.8"^2}$ (=12.7…) and $\frac{\sin CAH}{8.1} = \frac{\sin 90}{"12.7"}$	
		A1	for answer in the range 39.5 – 39.51	If an answer is given in the range but then incorrectly rounded award full marks.

Examiner Comments

Due to the use of inverted commas around values in the mark scheme, each mark effectively becomes dependent upon the previous one.

The first mark was for any correct trigonometric statement, this was typically one of the three given as examples in the scheme. Many students who failed to score any marks did so because they either tried to use Pythagoras's Theorem as a first step and wrongly assumed *BC* was 7.3, or they labelled their diagrams incorrectly.

This first mark could also be awarded to the student who completed no correct working, but was able to identify correctly, and unambiguously the angle *CAH* on the diagram as the angle to be found.

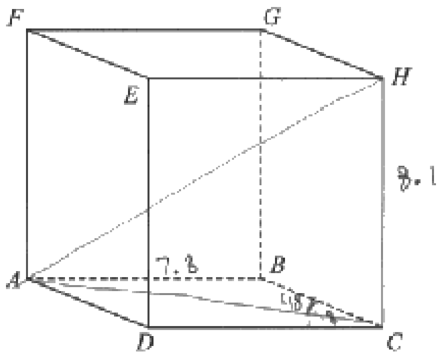
The second mark is for a complete correct process to find *AC*.

The third process mark was for using angle *CAH* in a correct trigonometric statement.

The final mark was for an answer in the given range. Premature rounding of values frequently led to an answer outside the given range and thus the loss of the final mark.

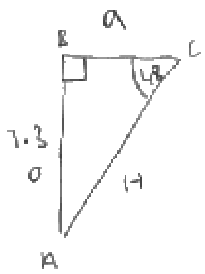
Student Response A

18 *ABCDEFGH* is a cuboid.



$AB = 7.3$ cm
$CH = 8.1$ cm
Angle $BCA = 48°$

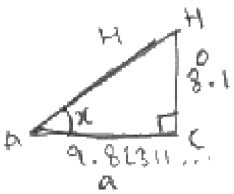
Find the size of the angle between *AH* and the plane *ABCD*.
Give your answer correct to 1 decimal place.



SOH CAH TOA

$\sin 48 = \frac{7.3}{H}$

$H = \frac{7.3}{\sin 48}$ $H = 9.823118926$ → AC



SOH CAH TOA

$\tan x = \frac{8.1}{9.823...}$

$x = \tan^{-1}\left(\frac{8.1}{9.823...}\right)$

$x = 39.50849231$

$= 39.5$

39.5 °

(Total for Question 18 is 4 marks)

4/4

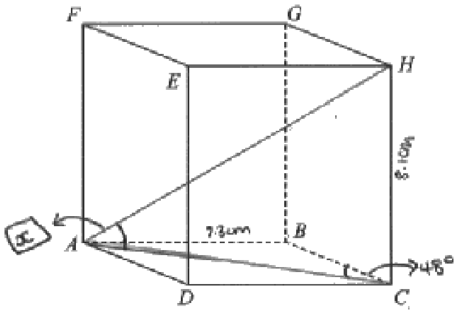
Examiner Comments

A perfect response. The student has split the question into 2 steps of standard trigonometry. The first to find *AC* and the second to find angle *HAC*.

Setting work out like this should be encouraged for benefit of the student. Here every step is clear and each stage of working is shown. The drawing of separate triangles proved beneficial to students.

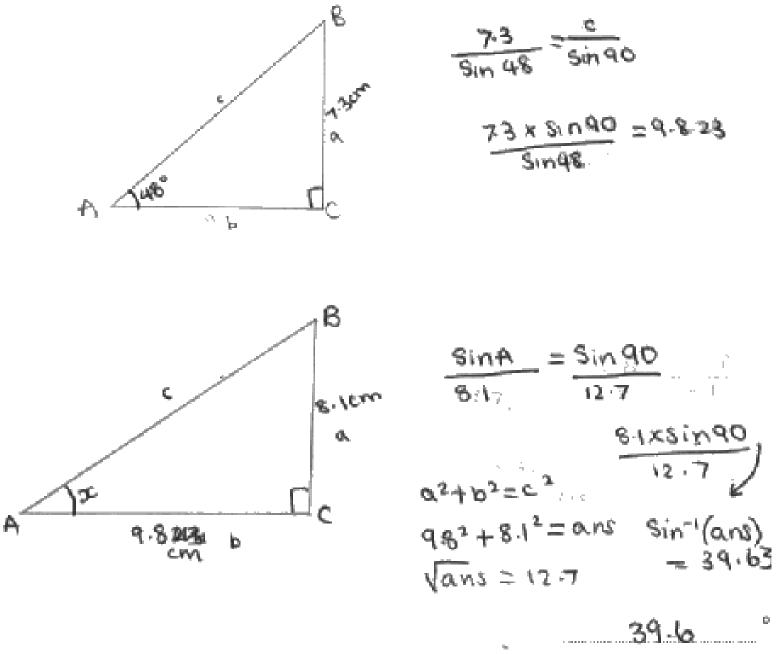
Student Response B

18 *ABCDEFGH* is a cuboid.



AB = 7.3 cm
CH = 8.1 cm
Angle *BCA* = 48°

Find the size of the angle between *AH* and the plane *ABCD*.
Give your answer correct to 1 decimal place.



(Total for Question 18 is 4 marks)

3/4

Examiner Comments

This student has used a different method to the previous one. Instead of using standard trigonometry they have used the Sine Rule, which is perfectly acceptable.

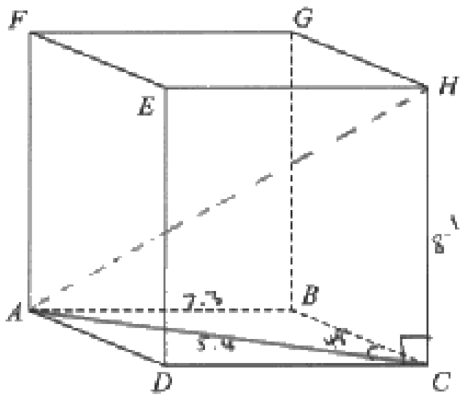
They have correctly found *AC* so are awarded the first two marks.

Instead of jumping straight to find *CAH*, because they want to use the Sine Rule again, they need to find *AH*, (labelled *AB* on their diagram). This has been done correctly and they then use this value correctly in a statement with *CAH*, which gains the third mark.

The issue with this response is the premature rounding of values; this has led to an answer outside the given range which means the accuracy mark cannot be awarded.

Student Response C

18 *ABCDEFGH* is a cuboid.



$AB = 7.3$ cm
$CH = 8.1$ cm
Angle $BCA = 48°$

Find the size of the angle between *AH* and the plane *ABCD*.
Give your answer correct to 1 decimal place.

SOHCAHTOA

$\sin 48 = \frac{\text{Opp}}{\text{Hyp}}$

$\sin 48 = \frac{7.3}{\text{Hyp}}$

$\sin 48 \times 7.3 = 5.4$

1/4

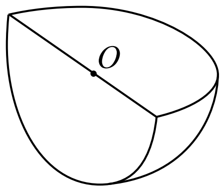
Examiner Comments

This student has started the problem well. They have correctly labelled the diagram with the given values and drawn in the line *AH*. They have then written a correct trigonometric statement to find *AC* ($\sin 48 = \frac{7.3}{hyp}$) and so gain the first mark. At this point the student cannot rearrange the expression to find "*hyp*" and so gains no further marks.

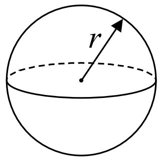
Exemplar Question 11

Higher tier Paper 2

19 Shape **S** is one quarter of a solid sphere, centre O.



Shape **S**



The volume of **S** is $576\,\pi$ cm^3

Find the surface area of **S**.
Give your answer correct to 3 significant figures.
You must show your working.

.... cm^2

(Total for Question 19 is 5 marks)

Examiner Comments

This question assesses the use of formulae for both the volume and surface area of a sphere in a problem. The student had to use the given volume of the shape to find the radius, and then use the radius to find the surface area. It was made more challenging by it being an unfamiliar shape, and also with the volume given in terms of π. Many student s failed to use the ¼ element to the problem and then struggled to access the first two marks. Others dropped the π in their working and had the same problem.

The second part of the problem however could be accessed even if the first two marks were not awarded, and many more scored the third and in some cases the fourth mark.

It is worth remembering that the formulae for the volume and surface area of a sphere will always be given and so need not be memorised. (However, formulae for the circumference and area of a circle do need to be learnt).

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19	905	P1	for correct use of formula for the volume of a sphere, e.g. $\frac{1}{4} \times \frac{4}{3} \times \pi \times r^3$ (= 576π or 1809…) OR 576π × 4 or 2304π or 7238…(= $\frac{4}{3} \times \pi \times r^3$)	We do not need to see what is in the brackets to award this mark. The contents of the bracket alone would score P0
		P1	for a complete correct process to find *r*, e.g. $r = \sqrt[3]{\frac{576 \times 4 \times 3}{4}}$ or $r = 12$	Could be shown in several stages $\sqrt[3]{\frac{576 \times 4 \times 3}{4}} = \sqrt[3]{1728}$
		P1	for a process to find the curved surface area e.g. $\frac{4 \times \pi \times [\text{radius}]^2}{4}$ (= 144π or 452…) OR the surface area of both flat surfaces e.g. (2 × $\frac{\pi \times [\text{radius}]^2}{2}$) OR complete expression for the total surface area e.g. $\frac{4\pi r^2}{4} + \frac{\pi r^2}{2} \times 2$ oe	Radius used must be clearly identified as their radius of the solid
		P1	for process to find the complete surface area e.g. $\frac{4 \times \pi \times [\text{radius}]^2}{4}$ + (2 × $\frac{\pi \times [\text{radius}]^2}{2}$)	
		A1	answer in the range 904.7 – 905 or 288π (SCB2 for an answer in the range 358.1 – 359.2)	If an answer is given in the range but then incorrectly rounded, award full marks.

Examiner Comments

This mark scheme is effectively split into two parts. One dealing with volume and credited in the first two process marks, and the other dealing with surface area and credited in the next two process marks. To gain the first process mark, the student had to link either the formula for a circle with $\frac{1}{4}$ or the given volume with ×4. Many student s struggled to do this correctly.

The second mark required all the correct processes from then on to find the value of r. This level of algebraic manipulation proved too difficult for many.

The second part of the mark schemes could be accessed by student s who failed to gain the first two process marks, provided they stated their radius. If they did this and went on to use the value of the radius correctly in a surface area calculation or calculations they could score one or two marks. One mark was awarded when a partial surface area was found (either the curved surface or both flat surfaces) or two marks for the complete surface area for their radius.

Student Response A

19 Shape **S** is one quarter of a solid sphere, centre O.

Shape **S**

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

The volume of **S** is 576π cm³

Find the surface area of **S**.
Give your answer correct to 3 significant figures.
You must show your working.

$\frac{1}{4} \times \frac{4}{3}\pi r^3 = \frac{4}{12}\pi r^3$

$\frac{1}{3}\pi r^3 = 576\pi$

$r^3 = 576 \times 3$

$r = \sqrt[3]{1728}$

$r = 12$

$\frac{1}{4} \times 4\pi r^2 = \pi r^2$

Sides $= \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 = \pi r^2$

total $= \pi r^2 + \pi r^2 = 2\pi r^2$

$2\pi \times 12^2 = 904.7$

905

........905........ cm²

(Total for Question 19 is 5 marks)

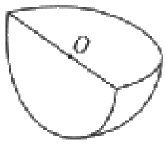
5/5

Examiner Comments

An excellent response. The student has started by multiplying the formula for volume by $\frac{1}{4}$ to get a formula for the volume of **S**. They have then equated this to 576π and followed all correct process to find r and thus score the first 2 marks. The student has then found an expression for the both the curved surface area and the two flat faces and combined them to give an expression for the surface area of **S**, and thus score the 3rd process mark. This student has then correctly substituted their value of r into this expression to gain the 4th mark. Their answer is correct and in range so the accuracy mark is awarded.

Student Response B

19 Shape **S** is one quarter of a solid sphere, centre O.



Shape **S**

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

The volume of **S** is 576π cm^3

Find the surface area of **S**.
Give your answer correct to 3 significant figures.
You must show your working.

$$\frac{\frac{4}{3}\pi r^3}{4} = 576\pi$$

$$\frac{4}{3}r^3 = 2304$$

$$\frac{\frac{4}{3}r^3}{4} = 576$$

$$2304 \div \frac{4}{3} = 1728$$

$$\sqrt[3]{1728} = 12$$

$$r = 12$$

3 s.f.

$$\frac{4 \times \pi \times 12^2}{4} = 144\pi = 452.39$$

452 cm^2

(Total for Question 19 is 5 marks)

3/5

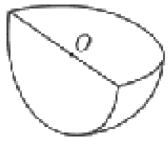
Examiner Comments

This response was typical of many student s, and shows where one error or omission can cost two marks. The student has started with a correct expression for the volume of **S** (seen here in an equation). They have followed all correct processes and arrived at $r = 12$ and so gained the first two marks.

In the second part of the problem the student has gained the 3rd mark as they have correctly calculated the curved surface area for **S**. However, this student, and many others, failed to then find the areas of the two flat faces and so cannot access any further marks.

Student Response C

19 Shape **S** is one quarter of a solid sphere, centre O.



Shape **S**

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

The volume of **S** is 576π cm^3

Find the surface area of **S**.
Give your answer correct to 3 significant figures.
You must show your working.

$\frac{4}{3}\pi \times r^3 / 4 = $ ~~476~~ 576π

$\frac{4}{3}\pi r^3 = 2304\pi$

$\pi r^3 = 1728\pi$

$r^3 = 1728$

$r = 12$

~~$4 \times \pi \times r$~~

$4 \times \pi \times r^2 = 576\pi$

$\pi \times 24 = 24\pi$

$24\pi + 576\pi = 600\pi = $ ~~1880.49~~ 1884.955592

1880 cm^2

(Total for Question 19 is 5 marks)

2/5

Examiner Comments

Again, this student makes a good start to the problem. They start by forming a correct equation for **S** and solve it successfully to find r and in doing so gain the first two marks.

However, when finding the surface area they make two separate errors. For the curved surface area they fail to divide by 4 (thus finding the surface area of a full sphere) so cannot gain credit for that. When dealing with the flat faces they use the formula for circumference rather than area, so again are unable to be credited. No further marks are awarded.

Exemplar Question 12

Higher tier Paper 2

21 Jackson is trying to find the density, in g/cm^3, of a block of wood.
The block of wood is in the shape of a cuboid.

He measures

the length as 13.2 cm, correct to the nearest mm
the width as 16.0 cm, correct to the nearest mm
the height as 21.7 cm, correct to the nearest mm

He measures the mass as 1970 g, correct to the nearest 5 g.

By considering bounds, work out the density of the wood.
Give your answer to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

(**Total for Question 21 is 5 marks**)

Examiner Comments

This question assesses the understanding of bounds of accuracy. The question gives measurements to differing degrees of accuracy and students have to find bounds. The next step is to apply these bounds to a compound measure, and thus into a formula involving division, which causes the next issue. Finally the bounds have to be used to find an answer to a suitable degree of accuracy. In this case this is the highest degree of accuracy to which the upper and lower bound for density agree.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
21	0.43	B1	for one correct bound for mass or length e.g. 1967.5 or 1972.5 or 13.15 or 15.95 or 21.65 or 13.25 or 16.05 or 21.75	Can work in any units
		P1	for a correct process to find a bound for the volume, e.g. 13.15 × 15.95 × 21.65 (= 454(0.925125)) **or** 13.25 × 16.05 × 21.75 (= 462(5.409375))	Accept volumes truncated or rounded to at least 3 sig fig
		P1	for a correct process to find a bound for density, e.g. [mass LB] ÷ "462(5.409375)" (= 0.425(367755)) where 1965 ⩽ mass LB < 1970 **or** [mass UB] ÷ "454(0.925125)" (= 0.434(3828506)) where 1970 < mass UB ⩽ 1975	Accept densities truncated or rounded to at least 3 sig fig
		A1	for both correct bounds, 0.425(367755) and 0.434(3828506)	Accept bounds truncated or rounded to at least 3 sig fig At this point correct units must be used
		C1	(dep on A1) for a correct statement on degree of accuracy e.g. UB and LB both round to 0.43 to 2 decimal places or 2 significant figures	Must be 0.43 not 0.4

Examiner Comments

The first mark was for generating any one correct bound of either mass or length. The first process mark is then for realising that volume is needed to find density, and for calculating one correct bound for volume. Since either is acceptable it meant that the student who got one length bound incorrect, could still score this mark

The third mark was for a process to find either the upper or lower bound for density. As can be seen from the inverted commas, the volume used must have come from a correct process (effectively making it dependent on the previous mark) but the mass could be incorrect provided it still sat in the range given in the mark scheme. This meant that those who struggled to find the bounds for mass could still potentially access three marks. The next mark is the accuracy mark for the question and was for both correct bounds found. Notice these could have been rounded or truncated to 3 (or more) significant figures.

The final mark was dependent on the award of the accuracy mark (so both correct bounds had to be found) and was for a correct statement about accuracy.

Student Response A

21 Jackson is trying to find the density, in g/cm³, of a block of wood.
The block of wood is in the shape of a cuboid.

He measures 13.15 < 13.2 < 13.25
the length as 13.2 cm, correct to the nearest mm
the width as 16.0 cm, correct to the nearest mm 15.95 < 16 < 16.05
the height as 21.7 cm, correct to the nearest mm 21.65 < 21.7 < 21.75

He measures the mass as 1970 g, correct to the nearest 5 g.
1967.5 < 1970 < 1972.5

By considering bounds, work out the density of the wood.
Give your answer to a suitable degree of accuracy.

low / high = low

You must show all your working and give a reason for your final answer.

high / low = high

13.15 × 15.95 × 21.65 = 4540.925125
↳ lower bound volume

1972.5 / 4540.925125 = 0.434382850 6 → high

1967.5 / (13.25 × 16.05 × 21.75) = 0.4253677546 → low

0.43 g/cm³ as they both agree to 2 significant figures.

(Total for Question 21 is 5 marks)

5/5

Examiner Comments

This student has started by correctly stating all bounds for length and mass at the top of the page. Any one of these would be worth the first mark. Next the student has found the lower bound for volume to score the first process mark. The denominator of the second fraction (finding the upper bound for volume) is also sufficient for the award of the first process mark, we do not need to see it evaluated. Either of the density calculations is sufficient for the award of the second process mark. Both correct bounds are stated and correct and so the accuracy mark is awarded. The student has then completed their response by finding the greatest degree of accuracy to which these both agree and stated it, thus gaining the final mark.

Student Response B

21 Jackson is trying to find the density, in g/cm³, of a block of wood.
The block of wood is in the shape of a cuboid.

He measures
the length as 13.2 cm, correct to the nearest mm
the width as 16.0 cm, correct to the nearest mm
the height as 21.7 cm, correct to the nearest mm

He measures the mass as 1970 g, correct to the nearest 5 g.

By considering bounds, work out the density of the wood.
Give your answer to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

$13.15 \leq 13.2 < 13.25$

$15.95 \leq 16.0 < 16.05$

$21.65 \leq 21.7 < 21.75$

upper bound for volume =
$13.25 \times 16.05 \times 21.75 = 4625.409375 \text{ cm}^3$ 41

lower bound for volume = 4540.925125 93
$13.15 \times 15.95 \times 21.65$

$1967.5 \leq 1970 < 1972.5$

upper bound for Density =
$\frac{1972.5}{4540.93} = 0.434 \text{ g/cm}^3$

lower bound = $\frac{1967.5}{4625.41} = 0.425 \text{ g/cm}^3$

$0.425 \leq D \leq 0.434$

Density = 0.4295 g/cm³
as this is the midpoint between the upper & lower bounds of the density, according to the measurements of the volume & mass.

(Total for Question 21 is 5 marks)

4/5

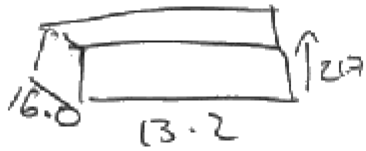
Examiner Comments

Much like the previous student, this student has correctly stated all the bounds for length and mass (down the left of the page). Again, any one of these would score the first mark. This student has then found both correct bounds for volume to score the first process mark, and then both bounds for density to score the second process mark. Both bounds are stated correctly to 3 significant figures which is sufficient to gain the accuracy mark. Unfortunately though, the student has then found the mid-point of the two bounds rather than looking at rounding and so cannot be awarded the communication mark.

Student Response C

21 Jackson is trying to find the density, in g/cm³, of a block of wood.
The block of wood is in the shape of a cuboid.

He measures
the length as 13.2 cm, correct to the nearest mm
the width as 16.0 cm, correct to the nearest mm
the height as 21.7 cm, correct to the nearest mm



He measures the mass as 1970 g, correct to the nearest 5 g.

By considering bounds, work out the density of the wood.
Give your answer to a suitable degree of accuracy.

You must show all your working and give a reason for your final answer.

work out lowest & highest

13.15 × 15.95 × 21.65 = ~~4540.925125cm³~~ 4540.925125cm³

1965/4540.925125 = 0.43g/cm³

~~4.33 × 10⁻⁷ g/cm³ ≤ x~~

13.25 × 16.05 × 21.75 = 4625.409375

1975/4625.409375 = 0.426989205g/cm³

(Total for Question 21 is 5 marks)

2/5

Examiner Comments

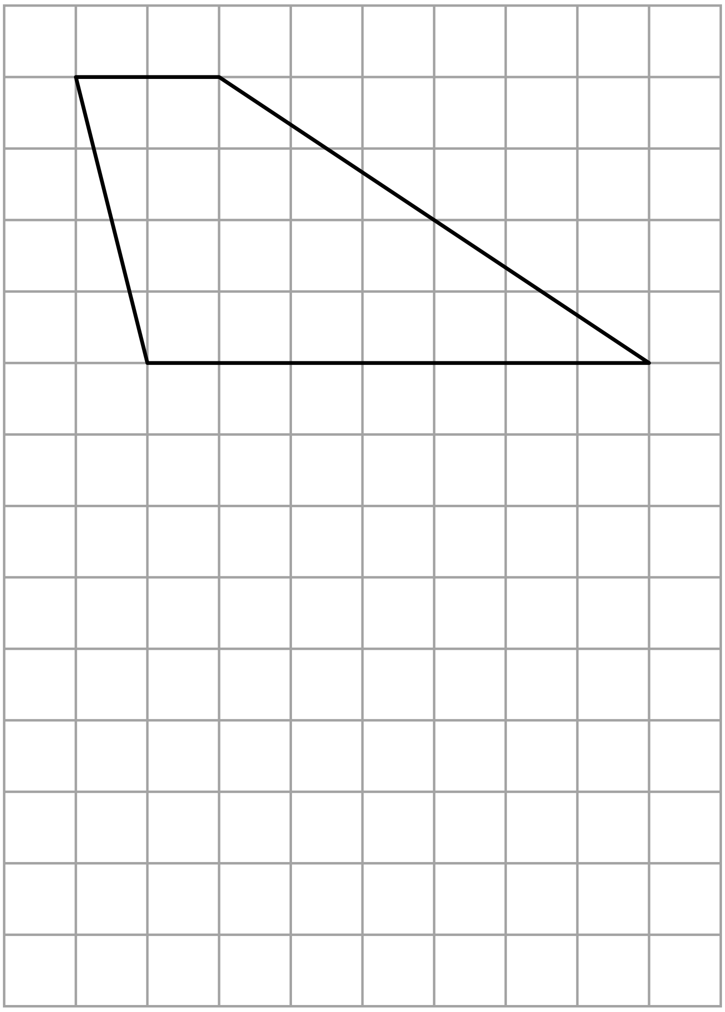
This response starts well. They have found both correct bounds for volume, stating their calculations, and thus score the first two marks. Unfortunately, when calculating density they have made an error. Rather than finding UB/LB and LB/UB this student has found UB/UB and LB/LB and thus cannot be awarded any subsequent marks.

Paper 3H (calculator)

Exemplar Question 1

Higher tier Paper 3

3 Here is a trapezium drawn on a centimetre grid.



On the grid, draw a triangle equal in area to this trapezium.

(Total for Question 3 is 2 marks)

Examiner Comments

This question assesses whether students know and can use the formulae for the area of a triangle and the area of a trapezium. It is expected that students will use the formula for the area of a trapezium to work out the area of the given shape and record their working. They are then expected to work out the dimensions of a triangle with the same area and draw it on the grid provided. Students who do not know the formula for the area of a trapezium might alternatively split the shape into two (or more) triangles. Weaker students might count squares. These students are less likely to obtain a correct value for the area.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
3	Triangle of area 18	M1	for a complete method to find area of trapezium e.g. $\frac{1}{2}(2 + 7) \times 4$ (= 18) OR for a triangle drawn of area 36 OR for a triangle that would give an area ft their area of trapezium	The value for the area of the trapezium must be clear for the ft to be checked.
		A1	for a triangle drawn of area 18 e.g. base = 6, height = 6 **or** base = 9, height = 4	Accept use of dimensions that are not whole numbers as long as the intention is clear

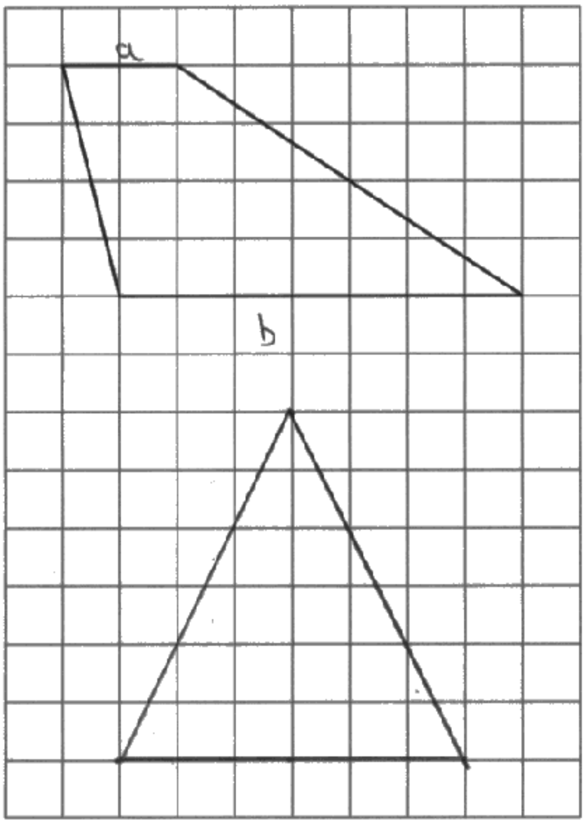
Examiner Comments

Students are credited the first mark for recording a correct method for finding the area of the trapezium. The correct value, 18, is enough evidence of this as long as it is clearly linked to the trapezium. The method mark can also be awarded where it was not possible to award the mark for a correct method to work out the area of the trapezium but the student has gone on to draw a triangle for their stated area. For example, this could be awarded where a student has written "area of trapezium = 16" without working then draws a triangle with area 16 cm^2.

The second mark can only be awarded for a triangle with area 18 cm^2.

Student Response A

3 Here is a trapezium drawn on a centimetre grid.



On the grid, draw a triangle equal in area to this trapezium.

$\frac{1}{2}(2+7)\,4 = 18\text{cm}^2$

$\frac{1}{2}(6\times6) = 18\text{cm}^2$

(Total for Question 3 is 2 marks)

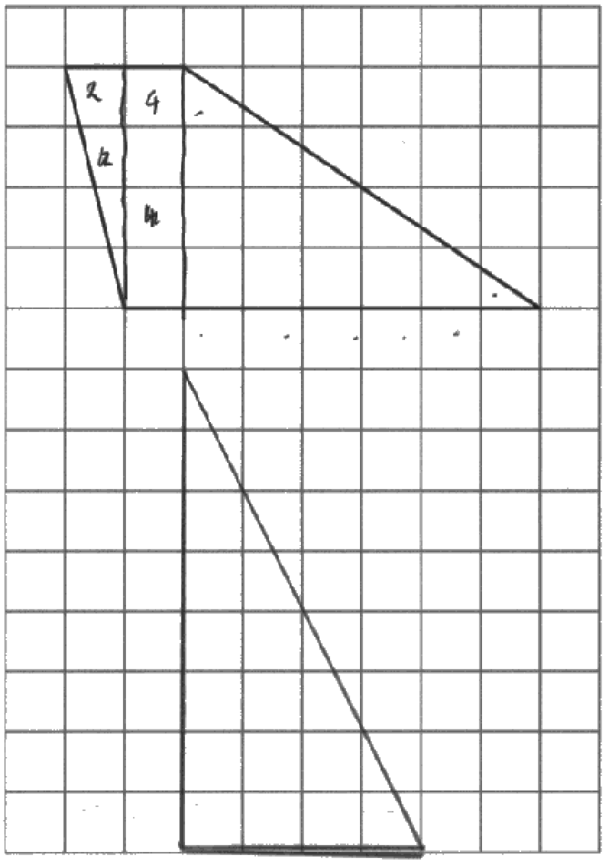
2/2

Examiner Comments

This is a perfect solution. The correct area is clearly shown and a triangle with base length 6 cm and vertical height 6 cm is drawn. It is likely the student has multiplied 18 by 2 and then used 36 = 6 × 6 before drawing their triangle.

Student Response B

3 Here is a trapezium drawn on a centimetre grid.



On the grid, draw a triangle equal in area to this trapezium.

6x4= 24
24 ÷ 2=12

12+2+4 = 18
$\frac{1}{2}$ b×h = 18
×2 ×2
b×h = 32
4 8

(Total for Question 3 is 2 marks)

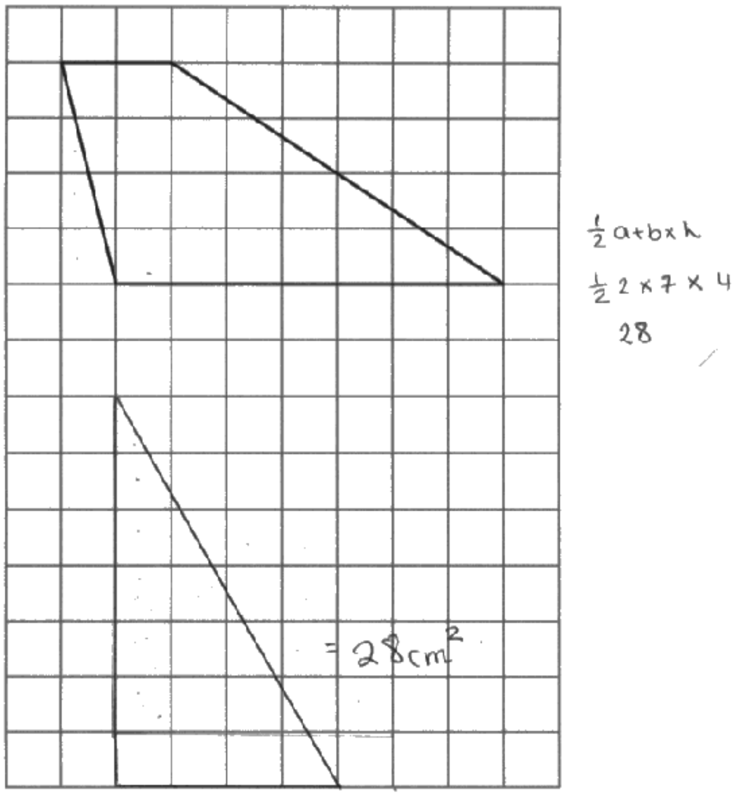
1/2

Examiner Comments

This student has a correct answer for the area of the trapezium (18) but then makes an error when multiplying this by 2 to get 32. This should be avoidable in a paper where a calculator may be used. The accuracy mark cannot be given for an incorrect triangle.

Student Response C

3 Here is a trapezium drawn on a centimetre grid.



On the grid, draw a triangle equal in area to this trapezium.

$\frac{1}{2}(a+b) \times h$

$\frac{1}{2}(2 \times 7) \times 4$

$= 28\text{ cm}^2$

(Total for Question 3 is 2 marks)

0/2

Examiner Comments

A correct formula has been written down but then it has been used incorrectly to find the area of the trapezium. If the student had then drawn a triangle of area 28 cm², they could have been awarded one mark. Unfortunately, they are mistaken in thinking the triangle they have drawn has area 28 cm² so no marks can be awarded.

Exemplar Question 2

Higher tier Paper 3

6 There are some counters in a bag.
The counters are red or white or blue or yellow.

Bob is going to take at random a counter from the bag.

The table shows each of the probabilities that the counter will be blue or will be yellow.

Colour	red	white	blue	yellow
Probability			0.45	0.25

There are 18 blue counters in the bag.

The probability that the counter Bob takes will be red is twice the probability that the counter will be white.

(*a*) Work out the number of red counters in the bag.

....
(4)

A marble is going to be taken at random from a box of marbles.
The probability that the marble will be silver is 0.5.

There must be an even number of marbles in the box.

(*b*) Explain why.

....

....
(1)

(Total for Question 6 is 5 marks)

Examiner Comments

Part (a) of this question tests a student's knowledge that the probabilities of an exhaustive set of outcomes sum to one, together with aspects of ratio, for example the ability to interpret a multiplicative relationship as a ratio. Students need to find that the probability that a counter taken will be either red or white is 0.3 then divide this in the ratio 2 : 1 to find that the probability of taking a red counter is 0.2. To find the number of red counters in the bag they will need to calculate the total number of counters in the bag or use the probability that a blue counter is taken (0.45) together with the stated number of blue counters in the bag (18). As there are 3 process marks available, students should write down each stage of their working clearly in case they make errors in working out their calculations.

Part (b) of the question requires the student to assess the validity of an argument and to make a clear and complete statement using the fact that 0.5 multiplied by an odd number will not give an integer. Students need to avoid vague (and incorrect) statements such as "you cannot find half of an odd number".

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
6(a)	8	P1	for process to find sum of unknown probabilities, e.g. 1 – 0.45 – 0.25 (= 0.3) **or** to find the total number of counters in the bag, e.g. $\frac{18}{0.45}$ (= 40) OR to find the number of yellow counters, e.g. $\frac{0.25}{0.45} \times 18$ (= 10)	Award mark for any two probabilities given that sum to 0.3 e.g. given in the table.
		P1	for process to find P(red) = 0.2 oe or P(white) = 0.1 oe OR for process to find the total number of red and white counters, eg "40" – 18 – "10" (=12) **or** for process to derive an equation in x, e.g. $2x + x = 1 - 0.45 - 0.25$ **or** $2x + x =$ "0.3" **or** $x = 0.1$	Award P2 for P(red) or P(white) (could be shown in table) Equations could be given as written statements or working but must be fully equivalent.
		P1	for a complete process to find the number of red counters, e.g. $\frac{2 \times 0.1}{0.45} \times 18$ **or** $\frac{2}{3} \times$ "12" **or** 0.2 × "40" **or** $\frac{0.2}{0.025}$	
		A1	cao	
(b)	Explana-tion	C1	for explanation e.g. 0.5 multiplied by an odd number will never be a whole number, for half of a number to be an integer that number must be even, you can't have half a marble	

Examiner Comments

Part (a) Students often put the probabilities found into the table so this should be given due credit. Give the first mark if the two probabilities of taking a red counter and taking a white counter add to give 0.3 – this may be seen in the table. Award the second mark if the two probabilities (0.2, 0.1) are correct. The third mark can only be awarded for a complete process to find the number of red counters.

Part (b) Examples of acceptable statements in part (b) include

"0.5 multiplied by an odd number would not give a whole number"

"only even numbers when divided by 2 will give a whole number"

Students are expected to refer to whole numbers (or integers) in their explanations.

Student Response A

6 There are some counters in a bag.
The counters are red or white or blue or yellow.

Bob is going to take at random a counter from the bag.

The table shows each of the probabilities that the counter will be blue or will be yellow.

Colour	red	white	blue	yellow
Probability	0·2	0·1	0.45	0.25

There are 18 blue counters in the bag.

The probability that the counter Bob takes will be red is twice the probability that the counter will be white.

(a) Work out the number of red counters in the bag.

$0{\cdot}45 + 0{\cdot}25 = 0{\cdot}7$

P (Red) = 2 × P(White)

Red = $2x$
White = x

P(Red) = 0·2
P(White) = 0·1

$3x = 0{\cdot}3$
$x = 0{\cdot}1$

$\frac{1}{0{\cdot}45} \times 18 = 40$ = total # of counters

$0{\cdot}2 \times 40 = 8$

8 red counters
(4)

4/4

A marble is going to be taken at random from a box of marbles.
The probability that the marble will be silver is 0.5

There must be an even number of marbles in the box.

(b) Explain why.

an odd number cant divide by 2 and leave an interger
(1)

(Total for Question 6 is 5 marks)

1/1

Examiner Comments

A perfect though not very common approach to part (a) which uses the number of blue counters and the probability of taking a blue counter to work out the total number of counters.

The mark is scored in part (b) as the statement clearly implies there must be an even number of counters because an odd number divided by 2 will not give an integer.

Student Response B

6 There are some counters in a bag.
The counters are red or white or blue or yellow.

Bob is going to take at random a counter from the bag.

The table shows each of the probabilities that the counter will be blue or will be yellow.

Colour	red	white	blue	yellow
Probability	0.2	0.1	0.45	0.25

There are 18 blue counters in the bag.

The probability that the counter Bob takes will be red is twice the probability that the counter will be white.

(a) Work out the number of red counters in the bag.

0.45
+0.25
0.70

1 - 0.7 = 0.3

0.2 : 0.1 : 0.45 : 0.25

20 : 10 : 45 : 25

4 : 2 : 9 : 5

18 ÷ 9 = 2   1 = 2

4 × 2 = 8
2 × 2 = 4
5 × 2 = 10
9 × 2 = 18
40
2

40

(4)

3/4

A marble is going to be taken at random from a box of marbles.
The probability that the marble will be silver is 0.5

There must be an even number of marbles in the box.

(b) Explain why.

Because the number of silver marbles is half of the total.

(1)

(Total for Question 6 is 5 marks)

0/1

Examiner Comments

In part (a) the student works out the number of counters of each colour by using a ratio approach. The working is fully correct. However, the number of red counters is not clearly indicated and so the final mark cannot be given. Instead the student has written the total number of counters on the answer line.

The student has not said enough in part (b) to score the mark. If they had continued their explanation by stating that when you multiply any (whole) number, odd or even, by 2 you get an even number the mark could have been awarded.

Student Response C

6 There are some counters in a bag.
The counters are red or white or blue or yellow.

Bob is going to take at random a counter from the bag.

The table shows each of the probabilities that the counter will be blue or will be yellow.

Colour	red	white	blue	yellow
Probability			0.45	0.25

18

There are 18 blue counters in the bag.

The probability that the counter Bob takes will be red is twice the probability that the counter will be white.

(a) Work out the number of red counters in the bag.

$0.45 + 0.25 = 0.7$

$r = 2w$ $\qquad 18 \times \frac{7}{15} = 8.4$

$2w + w = 0.7$

$3w = 0.7$

$\frac{0.7}{3} = 0.2\dot{3} \times 2 = r = \frac{7}{15}$

8.4

(4)

0/4

A marble is going to be taken at random from a box of marbles.
The probability that the marble will be silver is 0.5

There must be an even number of marbles in the box.

(b) Explain why.

Because the probability of it being silver is half the chance (50, 50)

(1)

0/1

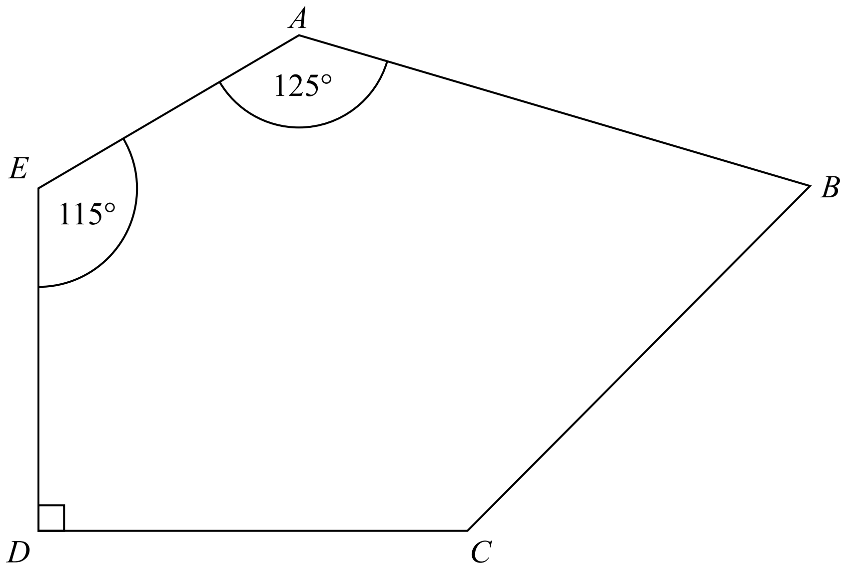
Examiner Comments

In part (a) the student has added the two probabilities given in the table but has divided this in the ratio 2 : 1 instead of subtracting from 1 first to get the total probability of taking a red or white counter. No marks can be awarded.

Exemplar Question 3

Foundation tier/Higher tier Paper 3

8 *ABCDE* is a pentagon.



Angle *BCD* = 2 × angle *ABC*

Work out the size of angle *BCD*.
You must show all your working.

....°

(Total for Question 8 is 5 marks)

Examiner Comments

This question assesses the ability of students to make and use connections between different parts of mathematics. The areas that are linked in this question are the sum of angles in a polygon and expressing a multiplicative relationship between two quantities as a ratio or fraction. Students may also choose to solve the problem by formulating and solving an equation.

Students should show all their working and in particular whether they are considering the sum of the interior angles of the pentagon or the sum of the exterior angles of a polygon.

A common misconception seen in the responses from less able students was that the sum of the interior angles of a pentagon is 360°. Some students made false assumptions such as *AB* = *CB*, angle *BAC* = angle *BCA* or angle *BAC* = angle *CAE*

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
8	140	P1	for complete process to find sum of the interior angles of a pentagon e.g. (5 – 2) × 180 **or** exterior 360 ÷ 5 = 72, interior 180 – 72 = 108, 108 × 5 **or** for complete process to find sum of the exterior angles of the pentagon e.g. (180 – x) + (180 – $2x$) + (180 – 125) + (180 – 115) + (180 – 90)	Must be a complete process that could lead to a figure of 540 if that process is evaluated incorrectly
		A1	for sum of interior angles is 540 **or** for sum of exterior angles is 360	360 must be identified as the sum of the exterior angles
		P1	for start to process to find angle *ABC* e.g. [angles in a pentagon] – 115 – 125 – 90 (= 210) **or** 115 + 125 + 90 + x + $2x$ = [angles in a pentagon] **or** (180 – x) + (180 – $2x$) + (180 – 125) + (180 – 115) + (180 – 90) = 360	Award provided [angles in a pentagon] is greater than 400 Algebraic route needs to show both sides of the equation. LHS of equation may be simplified
		P1	for process to find angle *ABC* e.g. "210" ÷ 3 (= 70), "210" divided in the ratio 2 : 1 or for process to find angle *BCD* e.g. $\frac{2}{3}$ × "210" or for $3x$ = "210" **or** $-3x$ = –"210"	Award if 70 is given for either *ABC* or *BCD* on the diagram
		A1	cao	Award marks for 140 on the diagram with working and not contradicted by the answer line. Award 0 marks for 140 without working.

Examiner Comments

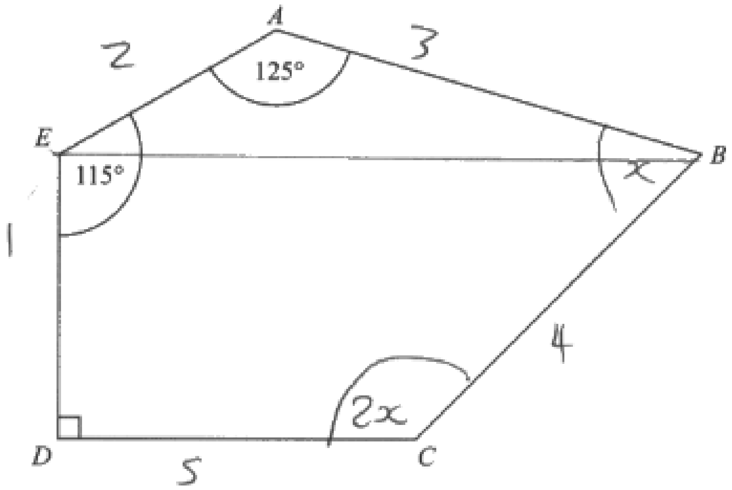
The most common correct approach to this question is by using the sum of the interior angles. A student clearly showing this is 540° scores the first 2 marks.

The third and fourth marks are awarded for using the sum of the interior angles and the known angles to find the sum of angles *ABC* and *BCD* and then divide this in the ratio 1 : 2. Student s can be awarded these marks if they clearly indicate what value they are using for their interior angle sum and this value is greater than 400. These marks are not dependent on the first two marks. This is indicated by the use of square brackets in the mark scheme.

The final mark is for a correct answer.

Student Response A

8 *ABCDE* is a pentagon.



Angle *BCD* = 2 × angle *ABC*

Work out the size of angle *BCD*.
You must show all your working.

5 sides

$(n-2) \times 180$

$(5-2) \times 180 = 540$

$540 - 115 - 125 - 90 = 210$

$BCD + ABC = x + 2x = 3x$

$3x = 210$

$x = 70$

$x = 140$

140 °

(Total for Question 8 is 5 marks)

5/5

Examiner Comments

A perfect and well-presented response. The student uses a simple algebraic approach to divide 210 in the ratio 1:2. The diagram is annotated to support their working.

Student Response B

BCD ABC
2 : 1

$n - 2 \times 180$

$5 - 2 \times 180 = 540$

$125 + 115 + 90 = 330$

$540 - 330 = 220$

$220 \div 3 = 73.\dot{3}$

$73.3 \times 2 = 146.\dot{6}$

146.6 °

(Total for Question 8 is 5 marks)

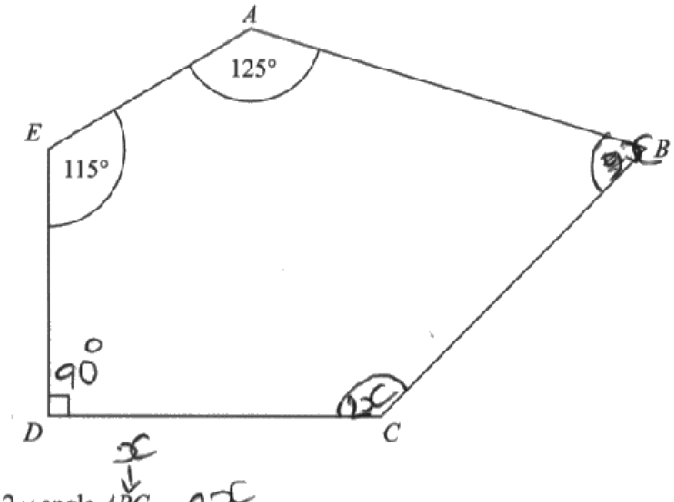
4/5

Examiner Comments

This student shows fully correct processes but makes one arithmetic error. Their written working is clear and the first 4 marks are awarded. Note that though the student has omitted brackets in their working to find the sum of the interior angles, their intent is clear and this is confirmed by the correct value, 540.

Student Response C

8 *ABCDE* is a pentagon.



Angle *BCD* = 2 × angle *ABC*

Work out the size of angle *BCD*.
You must show all your working.

$115° + 125° + 90° + 2x + x = 360°$

$330 + 3x = 360°$

$-330 \quad -330$

$3x = 30$

$\div 3$

$x = 10$

$BCD \rightarrow 2x \quad 10 \times 2 = 20$

.......................20°

(Total for Question 8 is 5 marks)

0/5

Examiner Comments

The student makes a good start by formulating an expression for the sum of the interior angles. If this expression had been equated to a total angle greater than 400°, the student could have been awarded the third and fourth marks. Unfortunately, the student uses "360" for the sum of the interior angles.

Exemplar Question 4

Higher tier Paper 3

11 In 2003, Jerry bought a house.

In 2007, Jerry sold the house to Mia.
He made a profit of 20%

In 2012, Mia sold the house for £162 000.
She made a loss of 10%

Work out how much Jerry paid for the house in 2003.

£..

(Total for Question 11 is 3 marks)

Examiner Comments

This question assesses the ability of a student to translate a problem in a real context into a series f mathematical processes. The student is required to find the original cost of a house prior to it being sold twice over a period of time. This type of problem is often referred to as a reverse percentage problem.

Students are expected to be able to relate a profit of 20% to the multiplier 1.20 and a loss of 10% to the multiplier 0.9 then solve the problem by dividing 162,000 successively by 0.9 and 1.20

Students often mistakenly believe that to find the price paid for the house in 2007 before a 10% loss is made can be done by adding 10% (equivalent to multiplying by 1.10) to the value of the house when it was sold in 2012. Similarly, they believe that to find the price the house was bought for in 2003 they can subtract 20% (equivalent to multiplying by 0.80) from the price it was bought for in 2007.

Mark scheme

Question	Answer	Mark	Mark scheme	Additional guidance
11	150 000	P1	for process to find cost in 2007, e.g. 162 000 ÷ 0.9 (= 180 000) oe	
		P1	for process to find cost in 2003, e.g. (cost in 2007) ÷ 1.2 (= 150 000) oe	Award 2 marks for 162 000 ÷ 1.08 oe
		A1	cao	

Examiner Comments

Marks are awarded in this question for a fully correct process at each stage. The first stage, working from the value of the house in 2012 to the value of the house in 2007 can only be awarded for 162,000 ÷ 0.9 or equivalent. The second mark is not dependent on the award of the first mark and is for using a correct process (÷1.2 or equivalent) with their 2007 value. This is indicated by the use of square brackets on the mark scheme. For example, the response 16200 × 1.10 = 178,200 followed by 178,200 ÷ 1.2 = 148,500 scores P0, P1, A0.

Student Response A

11 In 2003, Jerry bought a house.

In 2007, Jerry sold the house to Mia.
He made a profit of 20%

In 2012, Mia sold the house for £162 000
She made a loss of 10%

Work out how much Jerry paid for the house in 2003

162,000 ÷ 0.9 = 180,000

180,000 ÷ 1.2 = 150,000

£150,000

£ 150,000

(Total for Question 11 is 3 marks)

3/3

Examiner Comments

This response shows a fully correct, concise and clear sequence of processes leading to a correct answer by using the multipliers 0.9 and 1.2

Student Response B

11 In 2003, Jerry bought a house.

In 2007, Jerry sold the house to Mia.
He made a profit of 20%

In 2012, Mia sold the house for £162 000
She made a loss of 10%

Work out how much Jerry paid for the house in 2003

162000 = 90%

$\frac{162000}{90} \times 100 = 18000$

18000 = 120%

$\frac{18000}{120} \times 100$

£ 15000

(Total for Question 11 is 3 marks)

2/3

Examiner Comments

Note that "÷ 0.9" may be shown as "÷ 90" and "× 100" and "÷ 1.2" may be shown as "÷ 120" and "× 100". This response uses this approach and the only error is an incorrect evaluation of the first stage calculation. This error, probably due to a lack of care or checking when using a calculator, means that the accuracy mark is not awarded.

Student Response C

11 In 2003, Jerry bought a house.

In 2007, Jerry sold the house to Mia.
He made a profit of 20%

In 2012, Mia sold the house for £162 000
She made a loss of 10%

Work out how much Jerry paid for the house in 2003

~~for~~ 10% of 162,000 = 16200

162000 + 16200 = 178200

Jerry sold the house to Mia for 178200

20% of 178,200 = 35640

178,200 - 35640 = 142560

£ 142560

(Total for Question 11 is 3 marks)

0/3

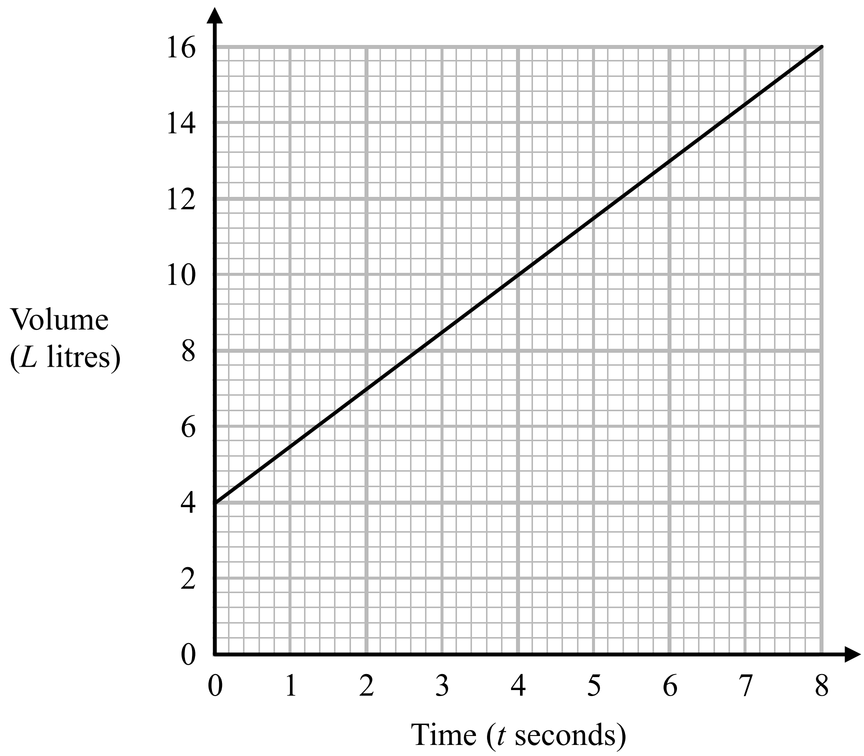
Examiner Comments

This response illustrates a classic error seen in this type of problem. The student mistakenly thinks that to reverse a percentage loss of 10%, they can add 10% and to reverse a percentage gain of 20% they can subtract 20%. This crucial misunderstanding means the student cannot be awarded any marks.

Exemplar Question 5

Higher tier Paper 3

12 The graph shows the volume of liquid (L litres) in a container at time t seconds.



(*a*) Find the gradient of the graph.

....

(2)

(*b*) Explain what this gradient represents.

....

....

(1)

The graph intersects the volume axis at $L = 4$

(*c*) Explain what this intercept represents.

....

....

(1)

(Total for Question 12 is 4 marks)

Examiner Comments

This question assesses the ability of students to calculate the gradient of a straight line in a real life context and to interpret it as a rate of change. It also requires the student to explain what the point where the straight line intersects the y-axis means in the given context.

Students should show their method for finding the gradient, for example by drawing a triangle on the line and marking in the values for the “increase in y” and the “increase in x” before showing the division of one by the other to find the gradient.

A common error is for students to ignore the scales used on the axes and find the gradient by counting squares.

Some students might find it difficult to explain what the value calculated in part (a) represents and/or not be able to interpret the gradient and intercept in the context given.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
12(a)	1.5	M1	for method to find the gradient of the line, e.g. $\frac{12}{8}$	Must see use of scales.
		A1	for 1.5 oe	
(b)	Explanation	C1	Explanation relating to rate of change of volume with time, e.g. rate at which the container fills or change in number of litres per second or number of litres added per second	Ignore any quantities given. Award the mark for an explanation involving rate.
(c)	Explanation	C1	Explanation relating to volume (amount) of liquid in the container at the start e.g. number of litres in the container when $t = 0$, amount of liquid in the container to start with	

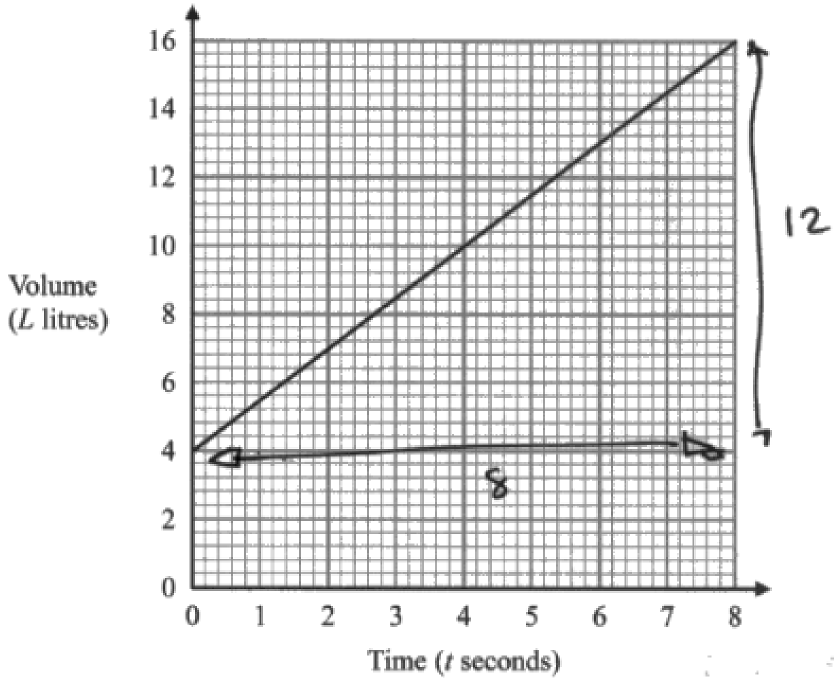
Examiner Comments

In part (a) student s need to show they have a fully correct method to find the gradient using the scales on each axis and not merely counting squares so for example 12 ÷ 8 scores the method mark but 6 ÷ 8 will not. The accuracy mark is given for a correct answer only. Some student s draw a triangle onto the line but use one which is too small or one where it is difficult to read off exact values. These student s sometimes lose the accuracy mark because they do not get 1.5 or equivalent.

For the explanations in parts (b) and (c) answers must be given using the context of the problem. In part (b) the concept of a rate must be communicated and in part (c) the concept of volume “at the start” is expected.

Student Response A

12 The graph shows the volume of liquid (L litres) in a container at time t seconds.



(a) Find the gradient of the graph.

$\frac{\Delta y}{\Delta x}$ $\frac{12}{8} = 1.5$

1.5

(2)

2/2

(b) Explain what this gradient represents.

How many litres is added every second at a Constant rate of 1.5 litres per second.

(1)

1/1

The graph intersects the volume axis at $L = 4$

(c) Explain what this intercept represents.

This intercept represents the volume already inside the container at the start. ~~before more was added.~~

(1)

(Total for Question 12 is 4 marks)

1/1

Examiner Comments

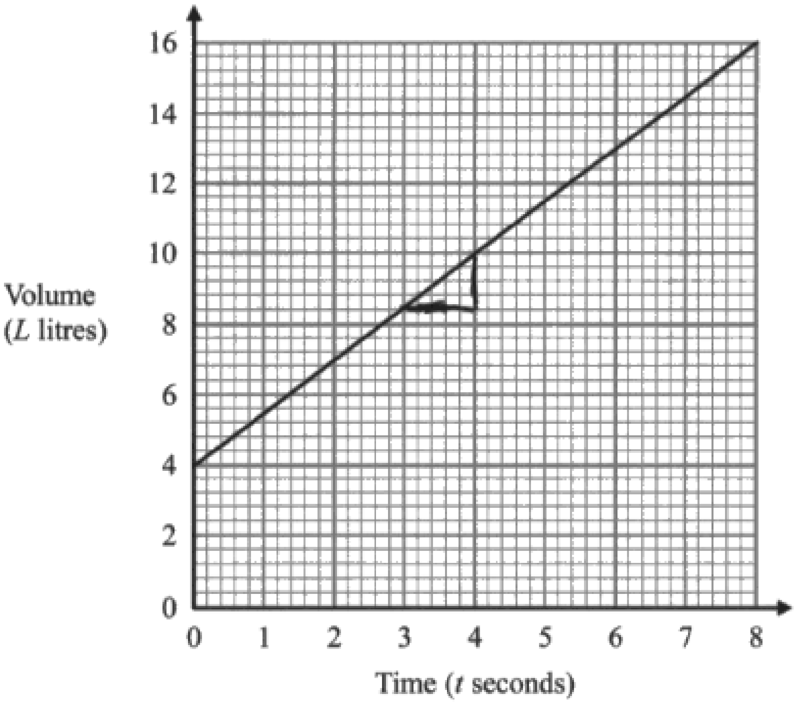
A correct answer to part (a) is supported by clear annotation of the diagram together with clear working.

"how many litres is added every second" is enough for the award of the mark in part (b) as the student communicates the idea of a rate of filling the container. The second sentence gives further evidence that the student has a clear understanding of what the gradient represents.

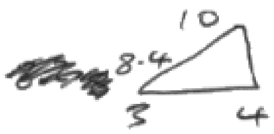
The statement given in (c) is also clear and in context so is awarded the mark.

Student Response B

12 The graph shows the volume of liquid (L litres) in a container at time t seconds.



(a) Find the gradient of the graph.



10 − 8.4 / 4 − 3 = 1.6

1.6

(2)

1/2

(b) Explain what this gradient represents.

Litres per second

(1)

0/1

The graph intersects the volume axis at $L = 4$

(c) Explain what this intercept represents.

Starting volume

(1)

1/1

Examiner Comments

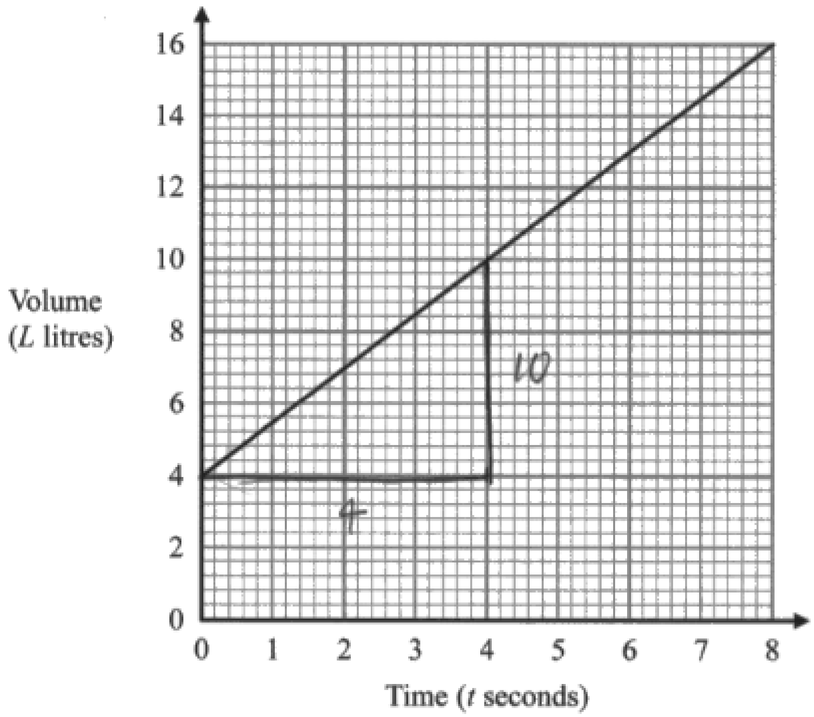
This response to part (a) reflects a clear understanding of how to find the gradient but a poor choice of triangle which leads to an inaccurate value. 1 of the 2 marks is scored.

“litres per second” is a unit of measurement and although it reflects a rate there is no reference to what is happening so the mark in part (b) cannot be awarded. The mark could have been given if the student had written something along the lines of “how the container is being filled in litres per second”.

In part (c) “starting volume” is just about enough to award the mark.

Student Response C

12 The graph shows the volume of liquid (L litres) in a container at time t seconds.



(a) Find the gradient of the graph.

$\frac{10}{4}$ = ~~0.25~~ 2.5

2.5
~~0.25~~

(2)

0/2

(b) Explain what this gradient represents.

The steepness of the graph is 1.5

(1)

0/1

The graph intersects the volume axis at $L = 4$

(c) Explain what this intercept represents.

This represents that the line passes through 4 on the Volume axis.

(1)

0/1

Examiner Comments

This student starts well by drawing a triangle on their diagram but does not find the difference in y values so cannot score any marks in part (a).

For part (b) the student s gives a general interpretation of gradient but does not relate it to rate of change of volume. 0 marks scored.

In part (c) the student rephrases the information given and does not relate it sufficiently to the context of the problem so cannot score the mark.

Exemplar Question 6

Higher tier Paper 3

14 There are 16 hockey teams in a league.
Each team played two matches against each of the other teams.

Work out the total number of matches played.

....

(Total for Question 14 is 2 marks)

Examiner Comments

This question assesses the ability of a student to use the product rule for counting in the context of hockey teams playing each other in a league.

Students should show the calculation they carry out in order to get their answer or alternatively their strategy. For example their systematic listing of pairings considered.

Students had to understand that they should only use pairings of two different teams and not include pairings where they include the same team twice.

Most students found it difficult to understand how many pairings are needed so that each team plays **two** matches against each of the other teams.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
14	240	M1	for start to method to find total number of matches, e.g. 16×15 **or** $16^2 - 16$ **or** $16 \times 15 \times 2$ (= 480) **or** $\frac{16 \times 15}{2}$ (= 120)	Credit complete listing strategies
		A1	cao	

Examiner Comments

The student needs to at least show the use of 16 multiplied by 15 (or equivalent) to score the first mark. Final answers of 480 or 120 can be taken as sufficient evidence of this and score 1 mark. To award the mark for a correct strategy communicated by for example listing pairs in a table, a complete method must be seen.

Student Response A

14 There are 16 hockey teams in a league.
Each team played two matches against each of the other teams.

Work out the total number of matches played.

$$\frac{16 \times 15}{2} = 120$$

$$120 \times 2 = 240$$

240

(Total for Question 14 is 2 marks)

2/2

Examiner Comments

This fully correct response indicates that the student has calculated how many matches are played if each team plays every other team exactly once then doubles it to get their final answer.

Student Response B

14 There are 16 hockey teams in a league.
Each team played two matches against each of the other teams.

Work out the total number of matches played.

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

1-2 2-3
1-3 2-4
1-4 2-5
1-5 2-6
1-6 -2
1-7
1-8
1-9
1-10
1-11
1-12
1-13
1-14
1-15
1-16

15 + 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 120 matches played

120

(Total for Question 14 is 2 marks)

1/2

Examiner Comments

A listing strategy is used and the answer 120 is awarded 1 mark. Without 120 seen, the sum of the first 15 positive integers is sufficient to show a correct method to find the number of matches if each team plays each other team once and so allow the award of the method mark.

Student Response C

14 There are 16 hockey teams in a league.
Each team played two matches against each of the other teams.

Work out the total number of matches played.

16 teams $16 \times 16 = 256$

$256 \times 2 = 512$

..................512..................

(Total for Question 14 is 2 marks)

0/2

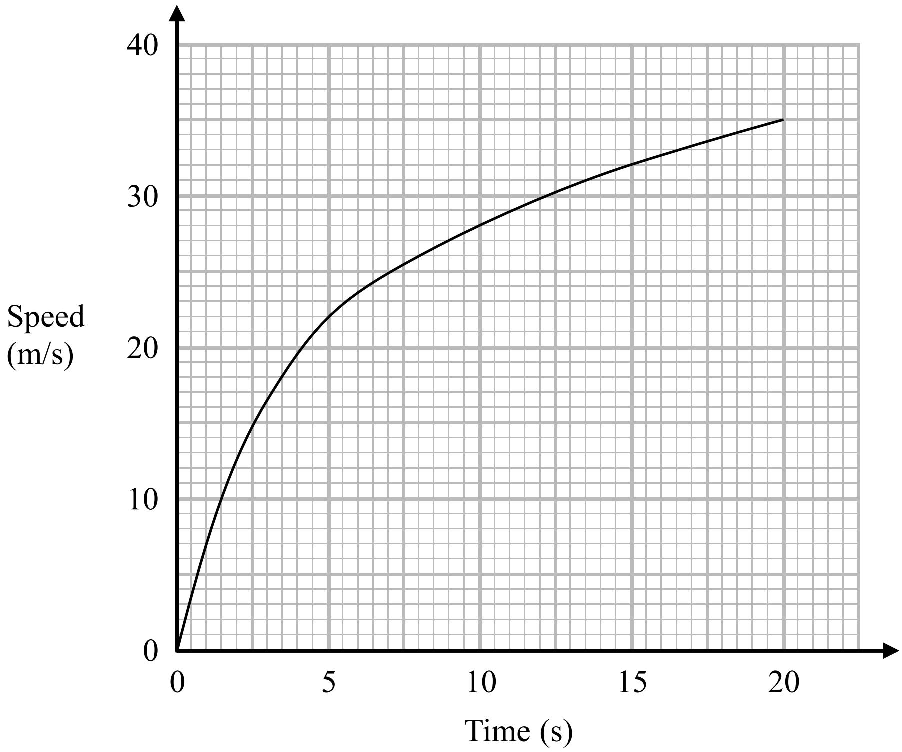
Examiner Comments

This student has some idea of what to do but fails to realise that the 16 × 16 would include pairings of each team with itself so no marks can be given.

Exemplar Question 7

Higher tier Paper 3

15 The graph shows the speed of a car, in metres per second, during the first 20 seconds of a journey.



(*a*) Work out an estimate for the distance the car travelled in the first 20 seconds.
Use 4 strips of equal width.

.... metres
(3)

(*b*) Is your answer to part (*a*) an underestimate or an overestimate of the actual distance the car travelled in the first 20 seconds?
Give a reason for your answer.

....

....
(1)

(Total for Question 15 is 4 marks)

Examiner Comments

Part (a) of this question assesses the ability of students to relate the distance travelled to the area under the graph and be able to find an estimate for this area. Students are told to use 4 strips of equal widths; trapezia are not explicitly specified.

Part (b) of this question assesses the ability of students to evaluate the result they obtained in the first part of the question and explain why their estimate of the area under the graph is an underestimate or an overestimate.

It is expected that students will make a comparison between their estimate and the actual area under the graph by considering the method they used in part (a) of the question.

Students need to show or explain clearly how they got their answer to part (a) and relate this to their decision about whether their answer is an underestimate or overestimate. The use of the diagram together with a linked explanation provides one way of doing this.

Some students tried to use an argument which suggested that the rounding of values taken from the graph was justification for their decision when, in fact, they had used exact values in their calculations.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
15(a)	488 to 507	M1	for method to find area of one strip using trapezia, e.g. $\frac{1}{2} \times 5 \times 22$ (= 55) **or** $\frac{1}{2} \times 5 \times (22 + 28)$ (= 125) **or** $\frac{1}{2} \times 5 \times (28 + 32)$ (= 150) **or** $\frac{1}{2} \times 5 \times (32 + 35)$ (= 167.5) **OR** for a method to find an estimate for the area using rectangles e.g. 5 × 22 or 5 × 28 **or** 5 × 32 or 5 × 35	May use area of triangle + area of rectangle for the second, third and fourth strips – lengths must be correct. May use triangle for first strip, $\frac{1}{2} \times 5 \times 22$
		M1	for complete and correct method to find the area using four strips, e.g. $\frac{1}{2} \times 5 \times 22 + \frac{1}{2} \times 5 \times (22 + 28) + \frac{1}{2} \times 5 \times (28 + 32) + \frac{1}{2} \times 5 \times (32 + 35)$ **or** 5 × 22 + 5 × 28 + 5 × 32 + 5 × 35	May use triangle for first strip, $\frac{1}{2} \times 5 \times 22$
		A1	for answer in the range 488 to 507 (SC B1 for using area under the curve)	
(b)	Underes-timate (support-ed)	C1	(dep M1) for underestimate since parts not included below the graph OR ft their method	

Examiner Comments

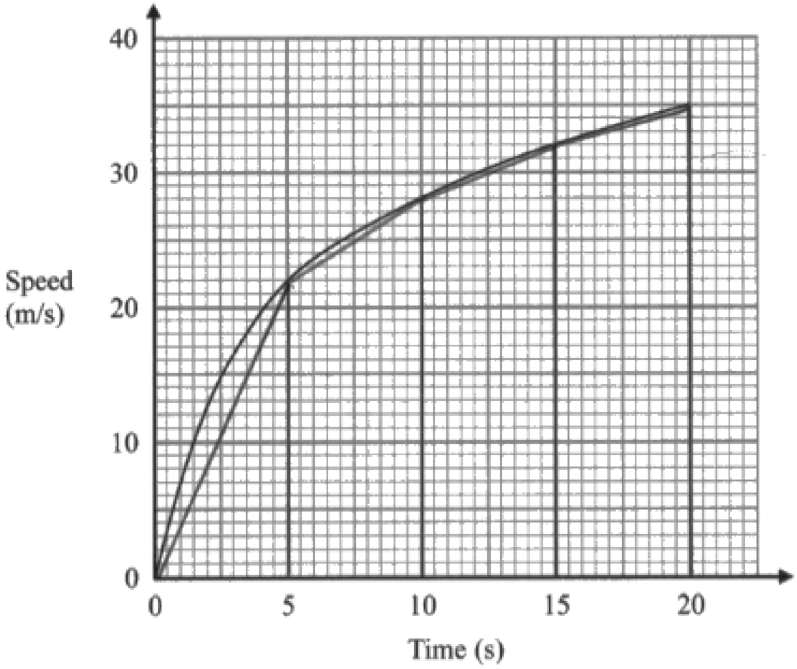
In part (a) students who gave an answer in the range given on the mark scheme were awarded full marks. The use of rectangles for the four 'strips' was allowed but this frequently led to an answer outside the range given in the mark scheme.

Note that at least 1 mark must have been given in part (a) before examiners can consider whether the answer to part (b) is worthy of the mark available.

The answer to part (b) must be clearly linked and reach a correct decision of "underestimate" or "overestimate" (or "cannot tell") based on the method used in part (a). This must be supported by a valid reason.

Student Response A

15 The graph shows the speed of a car, in metres per second, during the first 20 seconds of a journey.



(a) Work out an estimate for the distance the car travelled in the first 20 seconds.
Use 4 strips of equal width.

$\left(\frac{5\times22}{2}\right)+\left(5\times\frac{22+28}{2}\right)+\left(5\times\frac{28+32}{2}\right)+\left(5\times\frac{32+35}{2}\right)$

$\frac{1}{2}bh$ = area of △

$\frac{1}{2}(a+b)h$ = area of ⏢

$55 + 125 + 150 + 167.5 =$
497.5

497.5 metres
(3)

3/3

(b) Is your answer to part (a) an underestimate or an overestimate of the actual distance the car travelled in the first 20 seconds?
Give a reason for your answer.

under estimate as the curve goes over the triangle an trapeziums

(1)

(Total for Question 15 is 4 marks)

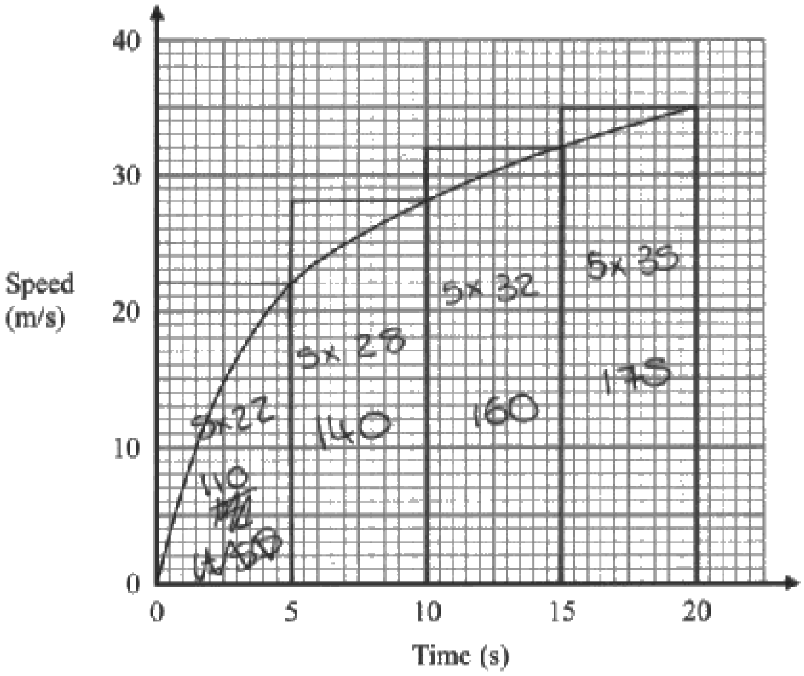
1/1

Examiner Comments

The student scores full marks in part (a) and so qualifies for examiners to assess part (b). The reason given suggests that part of the actual area under the curve has been ignored and this is evidenced by the diagram. 1 mark scored.

Student Response B

15 The graph shows the speed of a car, in metres per second, during the first 20 seconds of a journey.



(a) Work out an estimate for the distance the car travelled in the first 20 seconds.
Use 4 strips of equal width.

110 + 140 + 160 + 175 = 585

585 metres
(3)

2/3

(b) Is your answer to part (a) an underestimate or an overestimate of the actual distance the car travelled in the first 20 seconds?
Give a reason for your answer.

My answer is an overestimate because I gained my results through calculating above the curved line of best fit instead of going below it.

(1)

1/1

Examiner Comments

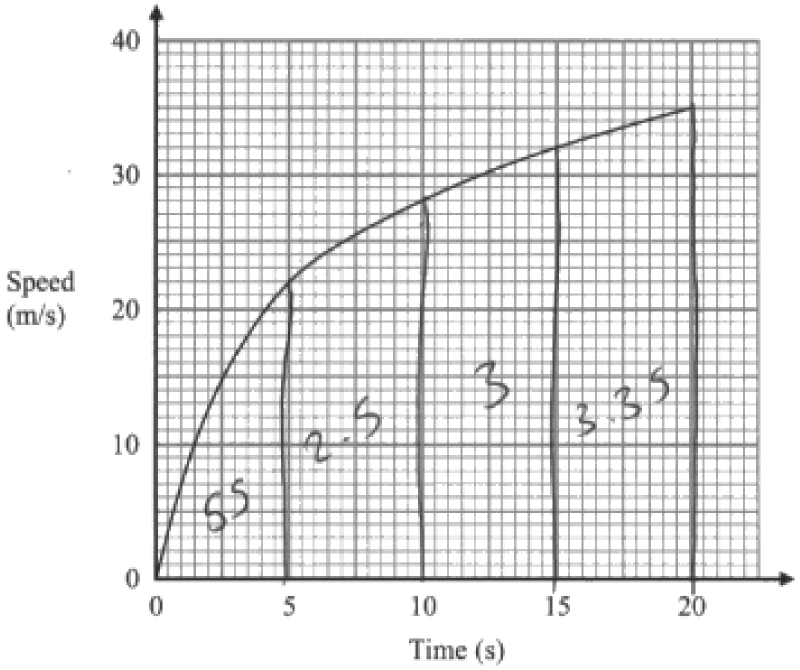
The student scores 2 marks in part (a) so part (b) is assessed.

In part (a) the student uses rectangles rather than trapezia; this allows the award of the method marks but not the accuracy mark.

Considering the diagram and working for part (a), we can see that rectangles are used with heights at the maximum y value for each strip. The student's explanation, though a little crude, describes the fact that the area of each rectangle overestimate the area of each strip. This is enough to score the mark.

Student Response C

15 The graph shows the speed of a car, in metres per second, during the first 20 seconds of a journey.



(a) Work out an estimate for the distance the car travelled in the first 20 seconds.
Use 4 strips of equal width.

$2.5 \times 22 = 55$

$\frac{1}{2} b \times h$

$\frac{1}{2}(a+b)h$

$\left(\frac{22+28}{10}\right)$

$\left(\frac{28+32}{10}\right) \div 2 = 3$

$\left(\frac{32+35}{10}\right) \div 2 = 3.35$

$55 + 2.5 + 3 + 3.35 = 63.85$

63.85 metres

(3)

1/3

(b) Is your answer to part (a) an underestimate or an overestimate of the actual distance the car travelled in the first 20 seconds?
Give a reason for your answer.

An overestimate because the answers I got were rounded up.

(1)

0/1

Examiner Comments

The student scores 1 mark in part (a) so the examiner assesses part (b).

In part (a) the area given for the first strip is correct but then an incorrect formula is used for the area of all other strips so only the first method mark can be awarded.

The diagram and working seem to indicate that the areas used are a triangle and three trapezia so the estimate would be an underestimate of the actual area. As the student has written "overestimate" they cannot be given the mark in part (b).

Note that even if they had written "underestimate", their reason is not acceptable as "rounding" is not the reason the area is not exact. The student has failed to appreciate that some of the area under the curve has not been included.

Exemplar Question 8

Higher tier Paper 3

16 The nth term of a sequence is given by $an^2 + bn$ where a and b are integers.

The 2nd term of the sequence is –2.
The 4th term of the sequence is 12.

(*a*) Find the 6th term of the sequence.

....
(4)

Here are the first five terms of a different quadratic sequence.

0 2 6 12 20

(*b*) Find an expression, in terms of n, for the nth term of this sequence.

....
(2)

(Total for Question 16 is 6 marks)

Examiner Comments

This question assesses a student's understanding of several topics within algebra using a context involving quadratic sequences. Part (a) tests the formulation, solution and interpretation of two linear simultaneous equations and part (b) tests finding the nth term of a quadratic sequence.

Part (a) tests the translation of a problem into a series of processes whereas part (b) tests a routine procedure.

Many students do not identify the need to use the expression for the nth term together with the given terms in part (a) to find the values of a and b and then, in turn, use the expression to find the 6^{th} term. Instead, students often found the difference between the 2^{nd} and 4^{th} term (14) and added this to 12 to find the 6^{th} term. These students wrote 26 as their answer.

In part (b) some students mistakenly use the second differences of 2 to write down $2n^2$ as part of their expression for the nth term.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
16(a)	42	P1	for process to find an equation in a and b, e.g. $a \times 2^2 + b \times 2 = –2$ $(4a + 2b = –2)$ **or** $a \times 4^2 + b \times 4 = 12$ $(16a + 4b = 12)$	
		P1	for process to find a pair of simultaneous equations and eliminate one unknown, e.g. $16a + 8b = –8$ and $16a + 4b = 12$ and subtraction **or** $16a + 4b = 12$ and $8a + 4b = –4$ and subtraction	Allow one arithmetic error in elimination, eg $16a + 8b = –8$ and $16a + 4b = 12$ leading to $4b = 20$ but no subtraction sign seen
		A1	for $a = 2$ and $b = –5$	
		A1	cao	
(b)	$n^2 – n$	M1	for correct method, e.g. n^2 seen as a term	
		A1	for $n^2 – n$ oe	

Examiner Comments

In part (a) the first mark required students to make substitutions to get at least one equation. The equation does not need to be simplified for the award of this mark.

The second mark was awarded to students who had two correct equations and eliminated one variable using a correct method. If the method shown was convincing the mark was awarded even if the student had made an arithmetic mistake.

The third and fourth marks were for getting correct values for a and b then use the correct expression for the nth term to find the 6^{th} term of the sequence.

In part (b) sight of the term n^2 in the student's answer is enough to give the method mark. This mark is not awarded for a term in n^2 which does not have a coefficient of 1. Any expression equivalent to $n^2 - n$ earns the accuracy mark.

Student Response A

16 The nth term of a sequence is given by $an^2 + bn$ where a and b are integers.

The 2nd term of the sequence is –2
The 4th term of the sequence is 12

(a) Find the 6th term of the sequence.

$a(+2)^2 + b(+2) = -2$
$4a + 2b = -2$
$16a + 4b = 12$
$8a + 4b = -4$
$8a = 16$
$a = 2$

$8 + 2b = -2$
$2b = -10$
$b = -5$

$2n^2 - 5n$
$2(6)^2 - 5(6)$
$= 72 - 30$
$= 42$

42
(4)

4/4

(b) Find an expression, in terms of n, for the nth term of this sequence.

0, 2, 6, 12, 20
2 4 6 8
2 2 2

0, 2, 6, 12, 20
n^2 1 4 9 16
-1 -2 -3 -4
-1 -1 -1

-1, -2, -3, -4

$n^2 - n$
(2)

(Total for Question 16 is 6 marks)

2/2

Examiner Comments

This student follows the expected route to get an expression for the nth term in part (a) then uses a substitution of $n = 6$ to find the correct answer, 42. The method of differences is clearly shown in part (b) leading to a correct answer.

Student Response B

16 The nth term of a sequence is given by $an^2 + bn$ where a and b are integers.

The 2nd term of the sequence is –2
The 4th term of the sequence is 12

(a) Find the 6th term of the sequence.

$a(2^2) + b(2) = -2 = (4a + 2b = -2)^{4\times}$

$a(4^2) + b(4) = 12 = 16a + 4b = 12$

$16a + 8b = -8$

$16a + 4b = 12$

$4b = -20$

$b = -5$

$4a + -10 = -2$

$4a = 8$

$a = 2$

$2(6^2) + -5(6) = ?$

$72 + -30$ -

$72 - 30 = 40$

40
(4)

3/4

Here are the first five terms of a different quadratic sequence.

0 2 6 12 20

(b) Find an expression, in terms of n, for the nth term of this sequence.

2 4 6

2

n^2

0 2 6 12

$n^2 = 1, 4, 9, 16$

$1, 2, 3, 4$

$1n + 1$

$n^2 + n + 1$
(2)

(Total for Question 16 is 6 marks)

1/2

Examiner Comments

A correct method in part (a) leads to a correct expression being obtained for the nth term. The correct substitution is then used but an unnecessary arithmetic error in the last stage of the working means the answer given is incorrect so the final accuracy mark cannot be given.

In part (b) correct second differences of 2 are obtained and the student deduces that one term in the required expression will be n^2. The student compares values of n^2 with terms of the sequence but wrongly concludes that they need to add "$n + 1$" to n^2 in order to get a complete expression for the nth term of the sequence.

Student Response C

16 The nth term of a sequence is given by $an^2 + bn$ where a and b are integers.

The 2nd term of the sequence is –2
The 4th term of the sequence is 12

(a) Find the 6th term of the sequence.

$an^2 + bn = -2$

$a2^2 + b2 = -2$

$4a^2 + 2b = -2$

$a12^2 + b12 = 12$

$(a \times 12)^2 + (b \times 12) = 12$

$12a^2 + 12b = 12$

$2a^2 + 2b = -2.$

$2a^2 + 2(1-a^2) = -2$

$2a^2 + 2 - 2a^2 = -2$

$12a^2 + 12b = 12$

$12 - 12a^2 = 12b$

$1 - a^2 = b$

....
(4)

1/4

Here are the first five terms of a different quadratic sequence.

0 2 6 12 20

(b) Find an expression, in terms of n, for the nth term of this sequence.

2 4 6 8

2 2 2

$2n - 2$

$2n - 2.$
(2)

(Total for Question 16 is 6 marks)

0/2

Examiner Comments

The second line in the working for part (a) shows a correct substitution so the first mark is given. However, the next line shows an error and the second equation written down is incorrect so no further marks can be scored.

In part (b) second differences are found but they are incorrectly used so the method mark cannot be awarded.

Exemplar Question 9

Higher tier Paper 3

18 (*a*) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2.

(2)

(*b*) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

(1)

(*c*) Starting with $x_0 = 2$,

use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

....

(3)

(Total for Question 18 is 6 marks)

Examiner Comments

This question assesses the method to find approximate solutions to equations numerically using iteration. In part (a) students are expected to construct a chain of reasoning to show an equation has a solution between 1 and 2. This requires the student to substitute two values of x and explain how their answers lead to the stated result. A common error was for students not to complete the argument but to stop after the substitution.

The ability of students to rearrange an equation into a given form is tested in part (b). Most students showed the critical intermediate step and so obtained the mark for this part of the question.

Part (c) assessed whether students can use the iteration to find an estimate for the solution of the equation. A common omission was for students not to write down the intermediate values of x_1 and x_2 .

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
18(a)	Correct stateme-nt	C1	for substituting both 1 and 2 into $x^3 + x$ or into $x^3 + x - 7$	All arithmetic shown must be correct. Ignore any additional trials shown.
		C1	for values 2 and 10 plus explanation that these are above and below 7, or for values −5 and 3 plus explanation that there is a change of sign, thus implying a solution lies between 1 and 2	
(b)	Correct rearrang-ement	C1	for correct algebraic rearrangement	
(c)	1.74	M1	for substitution of 2 into the formula e.g. $\sqrt[3]{7-2}$ (= 1.70997…)	x_1=1.70997… x_2=1.74241… x_3=1.73884… Accept an accuracy of 2 dp or more rounded or truncated for values of x_1 and x_2
		M1	for a substitution of x_1 to give x_2 (= 1.74241...)	
		A1	for answer in the range 1.738 to 1.74	Award the marks for 1.7 on the answer line provided correct iterations are shown in the working space.

Examiner Comments

Students who substitute both 1 and 2 into either $x^3 + x$ or $x^3 + x - 7$ are given the first mark in part (a). They do not need to evaluate the expression correctly for the award of the first mark. The second mark is for comparing correct values of the expression with 7 or 0 respectively leading to a correct conclusion. A number of student s did not do this second stage convincingly.

Care needs to be taken to check student s write a correct intermediate step in part (b), for example $x^3 = 7 - x$, before the mark is awarded.

In part (c), correct values seen lead to the award of the marks but if correct values are not seen, correct substitutions can lead to the award of the first two marks.

Student Response A

18 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

$1^3 + 1 = 2$

$2^3 + 2 = 10$

7 is inbetween 2 and 10
So ~~most be~~ x must be
between 1 and 2

(2)

2/2

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$x^3 + x = 7$

$x^3 = 7 - x$

$x = \sqrt[3]{7 - x}$

(1)

1/1

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$x_0 = \sqrt[3]{7 - 2} = 1.709975947$

$x_1 = \sqrt[3]{7 - \text{ans}} = 1.742418802$

$x_2 = 1.738849506$

1.74

1.74

(3)

(Total for Question 18 is 6 marks)

3/3

Examiner Comments

A clearly written and accurate answer with a well explained conclusion in part (a) is awarded full marks. The rearrangement in part (b) is convincing and the values in part (c) are correct.

Note: that an accuracy of 2 decimal places or better is accepted in part (c).

Student Response B

18 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

$1^3 + 1 = 2 \quad x = -5$

$2^3 + 2 = 10 \quad x = -3$

(2)

1/2

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$x^3 + x = 7$
$-x \quad -x$

$x^3 = 7 - x$

$x = \sqrt[3]{7 - x}$

(1)

1/1

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$x_0 = 2 \quad x_1 = 1.709975947$

$x_2 = 1.742418802 \quad x_3 = 1.738849506$

$1.738849506^3 + 1.738849506 = 6.996430707$

(can be rounded to 7

6.996430707

(3)

2/3

Examiner Comments

A correct substitution of both 1 and 2 into a correct expression is made in part (a) but no comparison of the values obtained with 7 is made and so the second mark is not given.

A correct re-arrangement in part (b) scores the mark available in this part of the question.

The student makes correct evaluations of x_1, x_2 and x_3 but their final answer (6.99...) is not an estimate for a solution so the third mark cannot be awarded.

Student Response C

18 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

$1^3 + 1 = 2$
$2^3 + 2 = 10$
$1.2^3 + 1.2 = 2.928$
$1.6^3 + 1.6 = 5.696$
$1.8^3 + 1.8 = 7.632$
$1.7^3 + 1.7 = 6.613$

$1.75^3 + 1.75 = 7.109375$
$1.73^3 + 1.73 = 6.907717$
$1.74^3 + 1.74 = 7.008024$

$x = 1.74.$

(2)

1/2

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$x^3 + x = 7$
$x^3 = 7 - x$
$x = \sqrt[3]{7 - x}$

(1)

1/1

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$x_0 = 2$
$x_0 = 1.709975947$
$x_1 = 1.587401052$
$x_2 = 1.44224957$
$x_3 = 1.25992105$

1.25992105

(3)

1/3

Examiner Comments

A correct substitution of both 1 and 2 into a correct expression is made in part (a) and scores the first mark but then the student uses a trial and improvement strategy to find an estimate for a solution. The student does not link this to the conclusion required so cannot score the second mark.

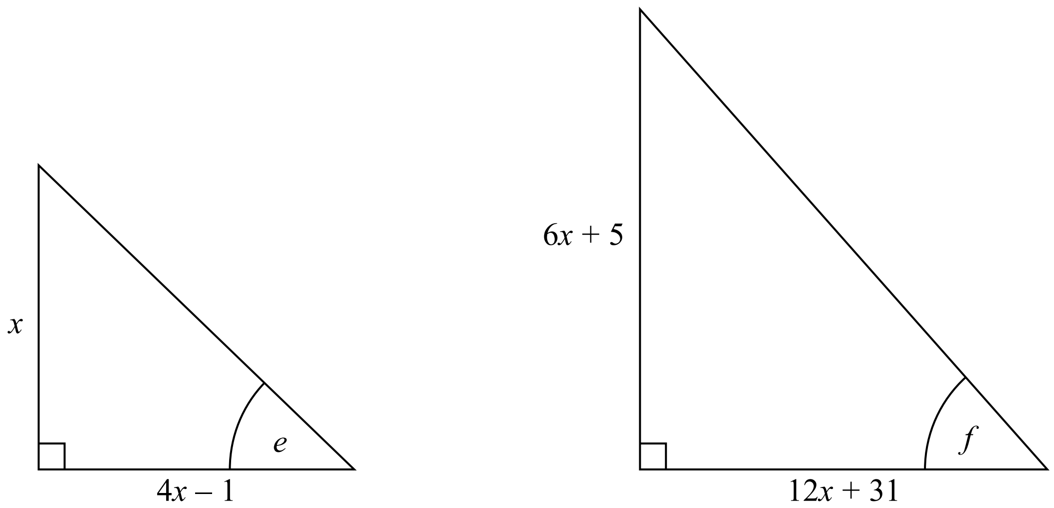
A correct re-arrangement is shown in part (b).

The student makes a correct evaluation for x_1 to score the first mark. However, x_2 and x_3 are not correct and there is no substitution shown so the second mark cannot be awarded. The value on the answer line is incorrect so the third mark is not given. Note that the use of an incorrect 'label' (x_0 rather than x_1) has been condoned by the examiner.

Exemplar Question 10

Higher tier Paper 3

19 Here are two right-angled triangles.



Given that

$$\tan e = \tan f$$

find the value of x.

You must show all your working.

....

(Total for Question 19 is 5 marks)

Examiner Comments

This question assesses the ability of students to solve a mathematical problem by splitting it into a series of processes then interpret the results in the context of the given problem.

The question is based on the trigonometry of a right-angled triangle but also involves the formulation and solution of a quadratic equation.

Many students formulated an initial equation but could not deal with the algebraic fractions with accuracy. Some students who successfully solved the quadratic equation failed to rule out the negative solution and gave two values on the answer line despite the question asking "find the value of x".

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
19	$\frac{5}{3}$	P1	for process to derive an equation in x, e.g. $\frac{x}{4x-1} = \frac{6x+5}{12x+31}$	
		P1	for complete process to remove fractions, e.g. $x(12x + 31) = (6x + 5)(4x - 1)$	Must be correct use of brackets
		P1	for process to reduce to a quadratic equation, e.g. $12x^2 - 17x - 5 = 0$	Award for correct LHS only.
		P1	for process to solve the quadratic equation by factorisation or use of quadratic formula, e.g. $(4x + 1)(3x - 5) = 0$	Award for correct LHS only. Accept substitution into the formula; $\frac{--17 \pm \sqrt{(-17)^2 - (4 \times 12 \times -5)}}{2 \times 12}$
		A1	for $\frac{5}{3}$ oe	Accept answers in the range 1.66 to 1.67 as equivalent

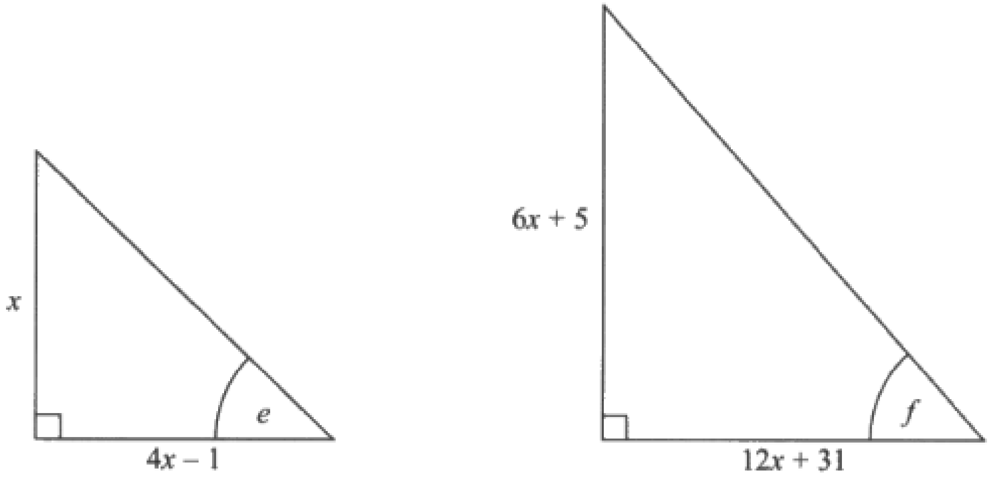
Examiner Comments

The first process mark is for forming a correct equation. This may be done in a number of ways, for example by equating $\frac{x}{4x-1}$ and $\frac{6x+5}{12x+31}$.Note that some student s write $\tan^{-1}\frac{x}{4x-1} = \tan^{-1}\frac{6x+5}{12x+31}$. This is acceptable for the award of the first mark. The second mark is given for dealing with the fractions and can be awarded without the expansion of the brackets. The third mark is for reducing the equation to a correct quadratic equation in a form ready to solve, ie with all terms on one side (though the absence of "= 0" does not prevent the award of the mark). The fourth mark is for a correct factorisation or for correct substitution into the formula. In the case of using the formula, examiners accepted $(-17)^2$ or -17^2 or 17^2 as the first term in the discriminant.

For full marks student s must give only the positive, correct solution $\frac{5}{3}$ as their final answer.

Student Response A

19 Here are two right-angled triangles.



Given that

$$\tan e = \tan f$$

find the value of x.

You must show all your working.

$\tan e = \dfrac{x}{4x-1}$

$\tan f = \dfrac{6x+5}{12x+31}$

$12x^2 + 31x = (6x+5)(4x-1)$

$12x^2 + 31x = 24x^2 - 6x + 20x - 5$

$0 = 12x^2 - 17x - 5$

-60
$1 \quad 60$
$2 \quad 30$
$3 \quad 20$

$12x^2 + 3x - 20x - 5 = 0$

$3x(4x+1) - 5(4x+1) = 0$

$(3x-5)(4x+1) = 0$

$3x = 5 \qquad 4x = -1$

$x = \frac{5}{3} \qquad x = \frac{-1}{4}$ — cannot have negative side length

$\frac{5}{3}$

(Total for Question 19 is 5 marks)

5/5

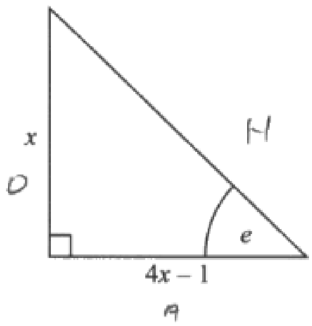
Examiner Comments

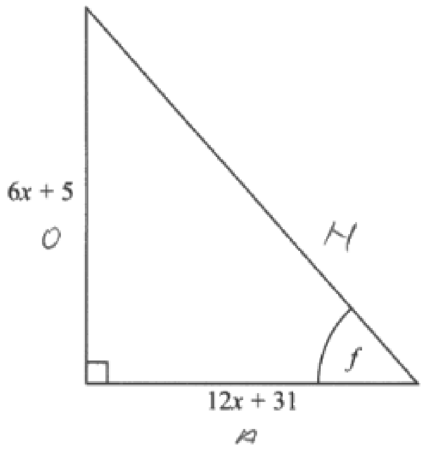
A well set out, concise and fully correct solution which uses factorisation to solve the equation and clearly indicates why the student rules out the negative value for x.

Student Response B

19 Here are two right-angled triangles.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$





Given that

$\tan e = \tan f$

find the value of x.

You must show all your working.

$$\tan^{-1}\left(\frac{x}{4x-1}\right) = \tan^{-1}\left(\frac{6x+5}{12x+31}\right)$$

$$\frac{x}{4x-1} = \frac{6x+5}{12x+31}$$

$$\frac{x(12x+31)}{4x-1} = 6x+5$$

$$x(12x+31) = 6x+5(4x-1)$$

$$12x^2 + 31x = 24x^2 - 6x + 20x - 5$$

$$= 24x^2 + 14x - 5$$

$$31x = 12x^2 + 14x - 5$$

$$17x = 12x^2 - 5$$

$$-12x^2 + 17x + 5 = 0$$

$$\frac{-17 \pm \sqrt{17^2 - 4 \times -12 \times 5}}{2 \times -12} = \begin{matrix} -0.25 \\ 1.6\dot{} \end{matrix}$$

$x = -0.25$ or 1.6

(Total for Question 19 is 5 marks)

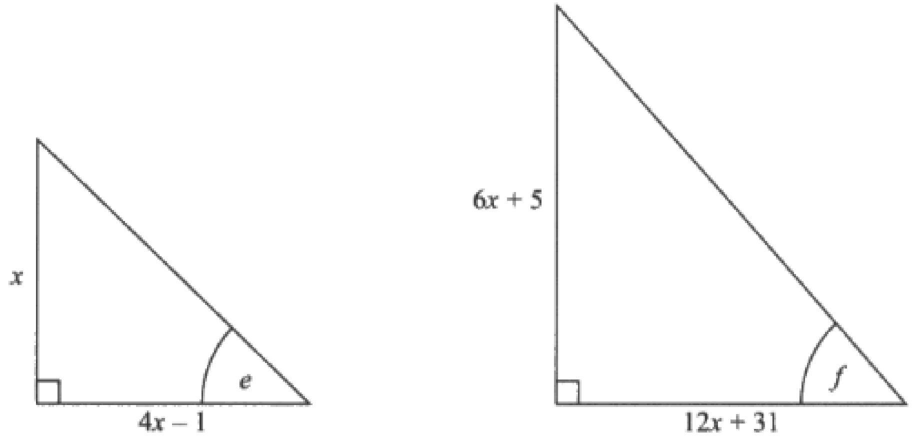
4/5

Examiner Comments

A correct equation is written down and the fractions are dealt with. Despite the absence of brackets in line 4 of the solution, correct expansions of the products in the next line convince us the student has expanded $(6x + 5)(4x - 1)$. A correct quadratic equation is found with terms collected on one side of the equals sign and the equation is solved using the formula to obtain two correct values. However, the student fails to realise that -0.25 cannot be a value of x as its use would not give positive values for all the lengths of sides of the triangles. The fifth mark is not given.

Student Response C

19 Here are two right-angled triangles.



Given that

$$\tan e = \tan f$$

find the value of x.

SOH CAH TOA

You must show all your working.

$\text{Tan}(f) = \frac{6x+5}{12x+31}$ $\quad$ $\tan = \frac{O}{A}$

$\text{Tan}(e) = \frac{6x+5}{12x+31}$

$\text{Tan}(e) = \frac{x}{4x-1}$

$\times\ 4x-1$ $\quad$ $\times\ 4x-1$

$4x-1 \times \text{Tan}(e) = x$

....

(Total for Question 19 is 5 marks)

0/5

Examiner Comments

The student writes down correct expressions for tan *e* and tan *f* and begins the process of formulating an equation but unfortunately does not get as far as writing down a complete equation in *x* and cannot make any further progress.

Exemplar Question 11

Higher tier Paper 3

20 50 people were asked if they speak French or German or Spanish.

Of these people,
- 31 speak French
- 2 speak French, German and Spanish
- 4 speak French and Spanish but not German
- 7 speak German and Spanish
- 8 do not speak any of the languages
- all 10 people who speak German speak at least one other language

Two of the 50 people are chosen at random.

Work out the probability that they both only speak Spanish.

....

(Total for Question 20 is 5 marks)

Examiner Comments

It was expected that this question would be solved by the use of a Venn diagram to find the number of people who only speak Spanish then use this to work out the required probability.

Many students interpreted the statement "7 speak German and Spanish" as "7 speak German and Spanish but not French" which led them to placing incorrect values in their diagram. Some students thought it unnecessary to put a number in the diagram to represent the number of people who speak only German (0) so left a blank space. In working out the probability, a good proportion of students failed to appreciate the non-replacement nature of the probability calculation.

Mark Scheme

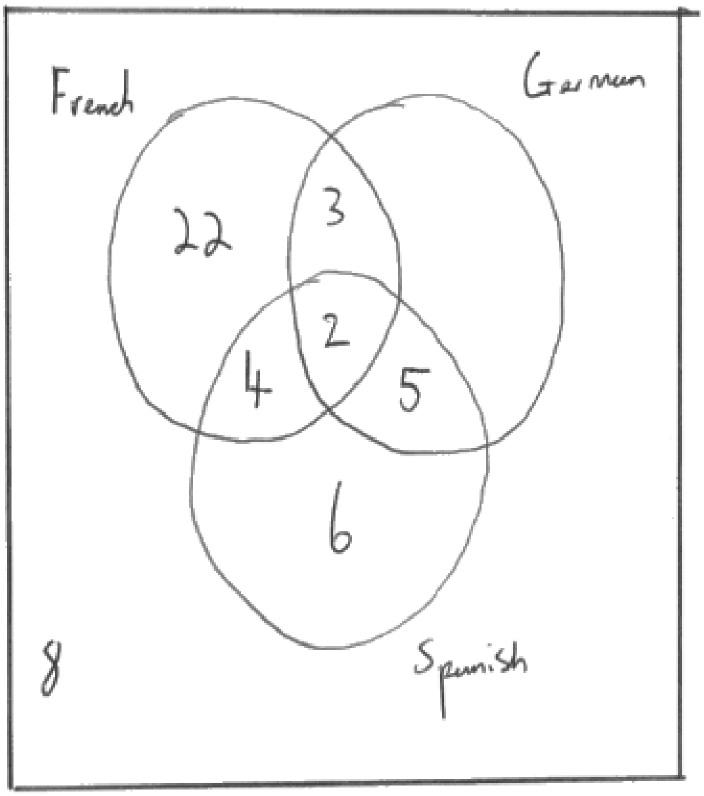
Question	Answer	Mark	Mark scheme	Additional guidance
20	$\frac{6}{490}$	P1	for start to process information, e.g. draws Venn diagram and shows at least 1 unknown amount, e.g. 5 speak German and Spanish but not French	See Venn Diagram at end of mark scheme – rectangle not needed
		P1	for process to find at least 3 unknown amounts from, e.g. 5 speak German and Spanish but not French 3 speak French and German but not Spanish 22 speak French but not German or Spanish 0 speak German but not French or Spanish	
		P1	for complete process to find number of people who speak only Spanish (= 6)	Award first 3 marks to students who show this on the Venn diagram or in a statement.
		P1	For $\frac{[\text{number speaking Spanish only}]}{50} \times \frac{[\text{number speaking Spanish only}]-1}{49}$, e.g. $\frac{6}{50} \times \frac{5}{49}$	Award this mark for use of their number of students who speak Spanish. Must be a clear link, eg from Venn diagram
		A1	for $\frac{6}{490}$ oe	See note 8 in general marking guidance but 0.01 or 1% must be from seen correct working.

Examiner Comments

It is rare to see an approach to solving this question without the use of a Venn diagram scoring any marks. An indicator for the award of the first three marks is the correct value, 6, in the region of the Venn diagram representing the number of people who only speak Spanish.

Student s who have an incorrect value in this region can still be awarded the fourth P mark if they use their value together with the concept of non-replacement to write down a correct process for calculating the probability that two of the 50 people only speak Spanish.

Student Response A



50 − 2 = 48
48 − 4 = 44
44 − 5 = 39
39 − 8 = 31
31 − 3 = 28

31 − 4 = 22
28 − 22 = 6

Person 1 = $\frac{6}{50}$
Person 2 = $\frac{5}{49}$

$\frac{6}{50} \times \frac{5}{49} = \frac{3}{245}$

$\frac{3}{245}$

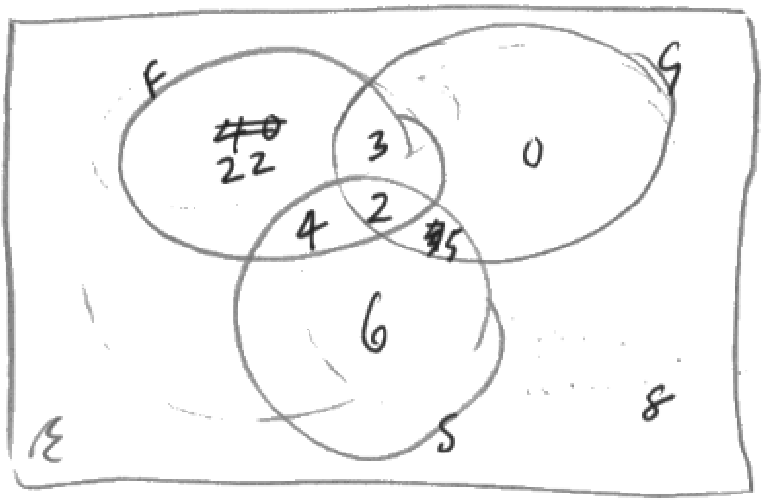
(Total for Question 20 is 5 marks)

5/5

Examiner Comments

A complete and correct Venn diagram with the exception that the student has not placed a 0 in the region for "speaks only German". A correct probability is given on the answer line so full marks are scored. Note that in cases where the probability is given as a fraction, it does not need to be fully simplified to score all five marks.

Student Response B



$4+2+3=9$

$31-9=$ ~~40~~ 22

$4+2+3=9$

$31-9=$ ~~40~~ 22

~~40~~ $22 + 3+2+4+5+8 =$ ~~6~~ 44

$50-44=6$

$\frac{6}{50} \times \frac{6}{50} = \frac{9}{625}$

$\frac{9}{625}$

(Total for Question 20 is 5 marks)

3/5

Examiner Comments

This student draws a complete and correct Venn diagram so scores the first 3 marks. The probability calculation does not take into account the fact that when the first person who speaks only Spanish is chosen there will only be 49 people left to choose from of which 5 speak only Spanish. Therefore the two final marks cannot be awarded.

Student Response C

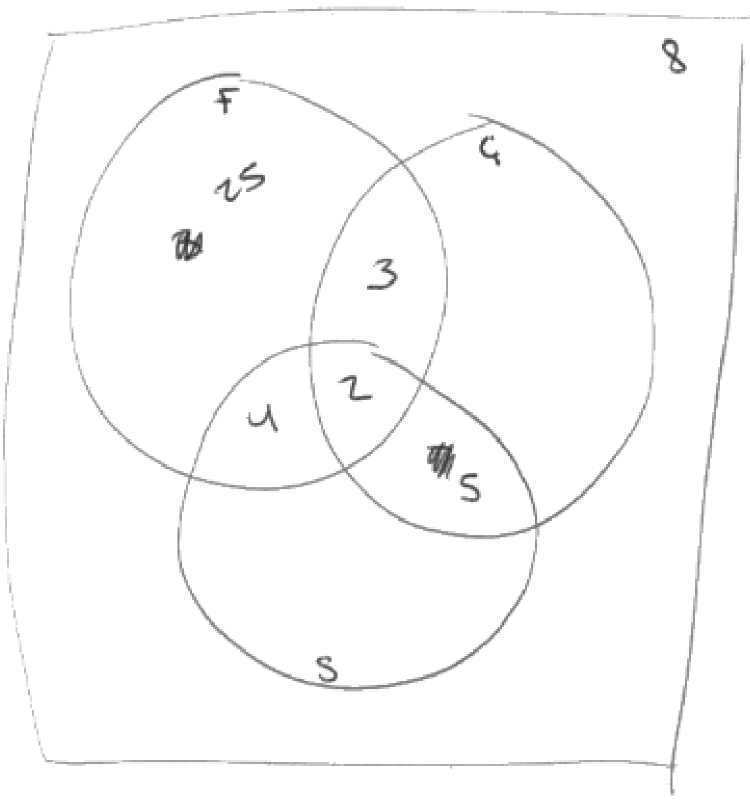
20 50 people were asked if they speak French or German or Spanish.

Of these people,
31 speak French
2 speak French, German and Spanish
4 speak French and Spanish but not German
7 speak German and Spanish
8 do not speak any of the languages
all 10 people who speak German speak at least one other language

Two of the 50 people are chosen at random.

Work out the probability that they both only speak Spanish.

31 - 2 - 4 = 25



.... 2/11 ....

(Total for Question 20 is 5 marks)

1/5

Examiner Comments

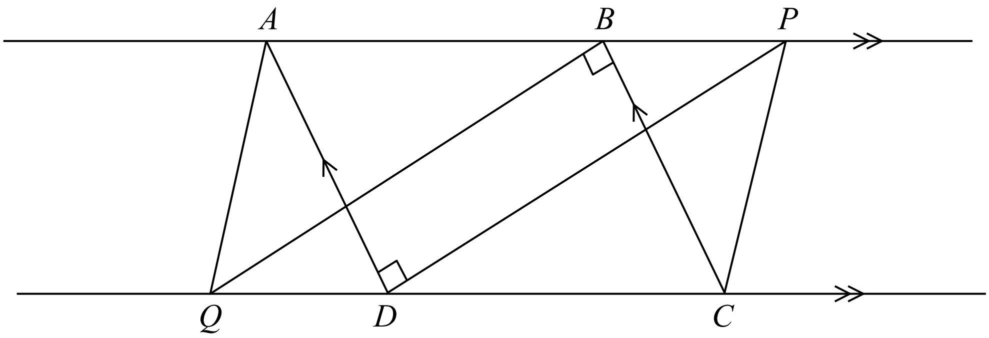
A Venn diagram is drawn and the information given in the question is placed in the appropriate regions. Two of the other regions have correct entries so 1 mark can be awarded.

The probability on the answer line is incorrect and there is no written evidence to show how it was obtained so neither of the final 2 marks can be given.

Exemplar Question 12

Higher tier Paper 3

21



ABCD is a parallelogram.
ABP and *QDC* are straight lines.
Angle *ADP* = angle *CBQ* = 90°

(*a*) Prove that triangle *ADP* is congruent to triangle *CBQ*.

(3)

(*b*) Explain why *AQ* is parallel to *PC*.

(2)

(Total for Question 21 is 5 marks)

Examiner Comments

This question assessed the ability of a student to present a proof of the congruency of two triangles and then to make deductions in order to draw a conclusion from mathematical information.

A complete and well - reasoned proof that two triangles are congruent was required to gain full credit in part (a) of the question. Students' statements to support the pairing up of sides and angles were usually incorrect or incomplete. This restricted the number of marks that could be awarded.

Few students used the congruency of the triangles *ADP* and *CBQ* to start their explanation in part (b). Instead they often stated that *APCQ* was a parallelogram without any or with insufficient justification.

Mark Scheme

Question	Answer	Mark	Mark scheme	Additional guidance
21(a)	Proof	C1	for starting the proof, identifying a pair of relevant equal sides or angles with reasons from *AD* = *BC* (opposite sides of a parallelogram are equal) angle *PAD* = angle *QCB* (opposite angles of a parallelogram are equal) angle *ADP* = angle *CBQ* (given or both 90°)	
		C1	(dep C1) for complete identification of all three equal aspects with reasons	
		C1	(dep C2) for conclusion of congruency proof	Congruency conclusion must include a reference to ASA
(b)	Explana-tion	C1	for identifying a pair of equal sides or angles in *APCQ*, with reason, e.g. *AP* = *QC* since triangle *ADP* is congruent to triangle *CBQ*	
		C1	(dep C1) for reasoning that *APCQ* is a parallelogram so opposite sides of a parallelogram are parallel	

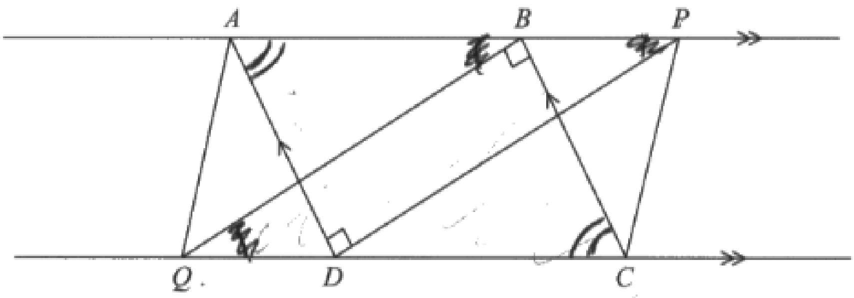
Examiner Comments

In part (a) both the identification of a pair of relevant equal sides or angles and an associated reason is needed for the award of any marks. Reasons must be complete and unambiguous.

In part (b) the use of the congruency of triangles *ADP* and *CBQ* to state that *AP* is equal to *QC*.is needed as a first step in explain that *APCQ* is a parallelogram so leading to the required result.

Student Response A

21



ABCD is a parallelogram.
ABP and *QDC* are straight lines.
Angle *ADP* = angle *CBQ* = 90°

(a) Prove that triangle *ADP* is congruent to triangle *CBQ*.

∠ADP = ∠CBQ as given

~~∠BQC = ∠QBA as alternate angles are equal~~

~~∴ ∠BQC = ∠DPA as corresponding angles are equal.~~

∠DCB = ∠DAP as opposite angles in a parallelogram are equal.

~~∴ congruent by~~

BC = AD as opposite lengths in a parallelogram are equal.

∴ congruent by ASA.

(3)

3/3

(b) Explain why *AQ* is parallel to *PC*.

As Δ ADP is congruent to ΔCBQ, AP must equal QC.

∴ AQ must be parallel to PC as the opposite lengths are equal, making it a parallelogram.

2/2

Examiner Comments

A model solution for the congruency proof gains all 3 marks in part (a).

The reasoning in part (b) is a little less clear but it does include the use of the congruency proven in part (a), together with the property that opposite sides of a parallelogram are equal, to show a parallelogram is formed. Examiners judged that this was just about enough to score both marks in part (b).

Student Response B

(a) Prove that triangle *ADP* is congruent to triangle *CBQ*.

Opposite angles in a parrallelogram are the same so angle DABP is equal to ~~B~~CB

AD = BC They are parrallel to each other and finish and start ~~at the~~ same on ~~two~~ the same two lines which are parralel

Using ~~thas~~ the knowledge Angles QBC = ADP and BAP = ~~B~~CB and the line AD ~~is the same distance~~ = BC proves it is congruent as they must have the same angles and lengths if it has the same 2 angles and shares a length

(3)

1/3

(b) Explain why *AQ* is parallel to *PC*.

Triangle ADP is ~~a~~ congruent to QBC and the lines AP and QC are parrallel and the same lengths. Therefore by joining AQ and ~~P~~PC you are forming a parrallelogram. This proves that AQ is parralled to PC as opposite sides of ~~the~~ a parrallelogram are parralel.

(2)

(Total for Question 21 is 5 marks)

2/3

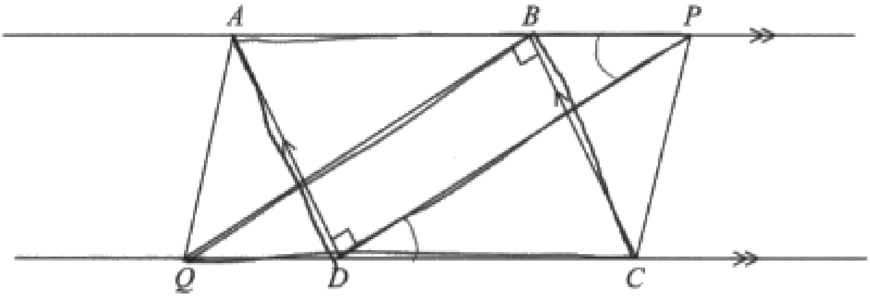
Examiner Comments

The student's first statement in part (a) qualifies for the award of 1 mark but reasons for the other pairings are insufficient for the award of any further marks.

The argument in part (b) is clear and completes and scores 2 marks.

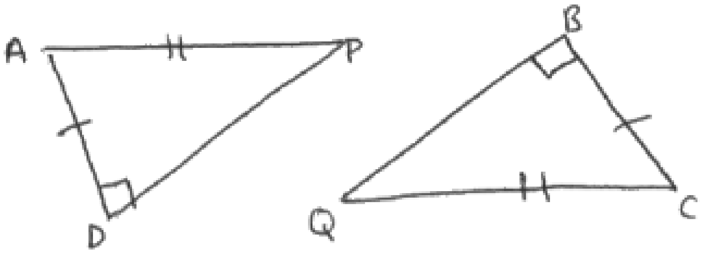
Student Response C

21



ABCD is a parallelogram.
ABP and *QDC* are straight lines.
Angle *ADP* = angle *CBQ* = 90°

(a) Prove that triangle *ADP* is congruent to triangle *CBQ*.



• ADP = QBC = 90° (given to us)
• AP = QC as parallelogram.
• DA = BC as parralell and in lines AP and QC.

Δ are congruent (RHS)

(3)

1/3

(b) Explain why *AQ* is parallel to *PC*.

ABCD is a parallelogram

0/2

Examiner Comments

The statement that angles *ADP* and *QBC* are equal (given) is enough for the award of 1 mark. Other reasons are incomplete so 1 mark in total is given for the response to part (a).

The statement written in part (b) is insufficient for the award of any marks.