edexcel

Pearson Edexcel Level 1/Level 2 GCSE (9–1) in Mathematics (1MA1)

Modified One-year Scheme of Work

For first teaching from September 2015

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Introduction

This scheme of work is specifically designed for a post-16 one-year revision course based on the assumption that candidates have already completed a full GCSE Mathematics course of study and are resitting the qualification. **It is not suitable as a teaching programme for those who have not completed the full GCSE Mathematics course of study.** After extensive research across the post-16 sector, experienced practitioners identified these key skills and themes to ensure that post-16 students mastered the core elements of the foundation tier content. The scheme of work draws upon the Two-year Scheme of Work published on the Edexcel mathematics website.

It can be used directly as a scheme of work for the Edexcel GCSE Mathematics (9–1) specification (1MA1).

This scheme of work has been written for a post-16 setting, where students are resitting their GCSE after not achieving the required grade. It therefore focuses on the middle grades – the 'underlined' objectives as specified by the DfE, for example 'calculate with roots, and with integer indices'. These objectives appear on both Higher and Foundation specifications.

The scheme of work is broken up into weekly units, so that there is greater flexibility for moving topics around to meet planning needs. Teachers should use this scheme of work for the basis of their planning, differentiating where necessary.

Units contain:

- reference to the specification
- suggested objectives for students at the end of the weekly unit
- possible success criteria for students at the end of the weekly unit
- opportunities for reasoning/problem-solving
- common misconceptions
- suggested career areas
- keywords.

Objectives have been separated into weekly units, each of approximately 3 hours in length, but we appreciate that the amount of teaching time for GCSE Mathematics differs greatly across the post-16 sector. Further information on teaching time for the GCSE Mathematics specification (1MA1) can be found on page 20 of our *Getting Started* document on the Edexcel mathematics website (http://qualifications.pearson.com/en/home.html).

Our free support for the GCSE Mathematics specification (1MA1) can be found on the Edexcel mathematics website (http://qualifications.pearson.com/en/home.html) and on the Emporium (www.edexcelmaths.com).

GCSE Mathematics (1MA1)

Foundation Tier

Modified Scheme of Work

Weekly Unit (Foundation)	Title
1	Groundwork: Number
2	Groundwork: Algebra
3	Groundwork: Geometry
4	Groundwork: Statistics
5	Percentages
6	Algebraic manipulation
7	Angle properties in shapes
8	Accuracy
9	Circles
10	Probability
11	Sequences
12	Quadratics
13	Ratio and compound measures
14	Proportion
15	Statistical graphs and measures
16	Transformations of shapes and vectors
17	Bivariate data
18	Volume and surface area

Unit 1. Groundwork: Number
(N1, N2, N3, N4, N5, N6, N7, N10, N12, N14, N15, R9, A4)

This unit focuses on the number skills required throughout the one-year GCSE course. Opportunity should be taken to assess prior knowledge and adapt as required. These skills should be kept on a 'rolling boil' throughout the course.

OBJECTIVES

By the end of the unit, students should be able to:

- Use, order and compare positive and negative numbers (integers), decimals, fractions and percentages; use the symbols <, > and understand the ≠ symbol;
- Add, subtract, multiply and divide positive and negative numbers (integers), decimals (including money) and fractions; multiply or divide any number by powers of 10;
- Recall all multiplication facts to 10 × 10, and use them to derive quickly the corresponding division facts;
- Use brackets and the hierarchy of operations (including positive integer powers);
- Round numbers to a given power of 10, to the nearest integer or to a given number of decimal places or significant figures;
- Express a given number as a percentage of another number;
- Convert between fractions, decimals and percentages;
- Check answers by rounding and using inverse operations;
- Use one calculation to find the answer to another;
- Use index notation for powers of 10, including negative powers;
- Use the laws of indices to multiply and divide numbers written in index notation;
- Find the prime factor decomposition of positive integers and write as a product using index notation; understand that the prime factor decomposition of a positive integer is unique;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors: include finding LCM and HCF given the prime factorisation of two numbers;
- Solve simple problems using HCF, LCM and prime numbers.

POSSIBLE SUCCESS CRITERIA

Use mental methods for × and ÷, e.g. 5 × 0.6, 1.8 ÷ 3.
Solve a problem involving division by a decimal (up to 2 decimal places).
Given 2.6 × 15.8 = 41.08, what is 26 × 0.158? What is 4108 ÷ 26?
Given five digits, what are the largest or smallest answers when subtracting a two-digit number from a three-digit number?
Use inverse operations to justify answers, e.g. 9 × 23 = 207 so 207 ÷ 9 = 23.
Check answers by rounding to nearest 10, 100, or 1000 as appropriate, e.g. 29 × 31 ≈ 30 × 30.
Express a given number as a fraction or percentage of another, including where the fraction > 1.
Simplify $\frac{120}{100}$.
Calculate: $\frac{3}{5}$ × 15, 20 × $\frac{3}{4}$, $\frac{1}{2}$ of 36 m, $\frac{1}{4}$ of £20, $\frac{1}{2}$ × $\frac{6}{7}$, $\frac{3}{5}$ ÷ 3.
Write terminating decimals (up to 3 d.p.) as fractions.
Convince me that 8 is not prime.
What is the value of 2^3? Evaluate $2^3 \times 2^5$.
Understand the meaning of 'prime factor'.
Understand that every number can be written as a unique product of its prime factors.
Recall prime numbers up to 100.
Write a number as a product of its prime factors.
Use a Venn diagram to sort information.

Foundation tier

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Missing digits in calculations involving the four operations.
Questions such as: 'Phil states 3.44 × 10 = 34.4 and Chris states 3.44 × 10 = 34.40. Who is correct?'
Show me another number with 4, 5, 6, 7 digits that includes a 6 with the same value as the '6' in the number 36 754.
Problems involving shopping for multiple items, such as: 'Noah purchases a magazine costing £2.10, a newspaper costing 82p and two bars of chocolate. He pays with a £10 note and gets £5.40 change. Work out the cost of one bar of chocolate.'
When estimating, students should be able to justify whether the answer will be an overestimate or underestimate.
Questions that involve rates of overtime pay including simple calculations involving fractional (>1, e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.
Working out the number of people/things where the number of people/things in different categories is given as a fraction, decimal or percentage.
Students should be able to provide convincing counter-arguments to statements concerning properties of stated numbers, e.g. 'Sharon says 108 is a prime number. Is she correct?'
Questions that require multiple layers of operations such as: 'Aisha writes down one multiple of 9 and two different factors of 40. She then adds together her three numbers. Her answer is greater than 20 but less than 30. Find three numbers that Aisha could have written down.'

COMMON MISCONCEPTIONS

- 150 – 210 = 60.
- Confusion of significant figure and decimal place rounding.
- 35 877 = 36 to two significant figures.
- The larger the denominator, the larger the fraction.
- Incorrect links between fractions and decimals, such as $\frac{1}{5}$ = 0.15, 5% = 0.5, 4% = 0.4, etc.
- It is not possible to have a percentage greater than 100%.
- 1 is a prime number.

Stress the importance of knowing the multiplication tables to aid fluency.
Particular emphasis should be put on the definition of 'product' as multiplication, as many students get confused and think it relates to addition.

KEYWORDS

Integer, number, digit, negative, positive, decimal, operation, addition, subtraction, multiplication, division, remainder, estimate, power, root, factor, multiple, prime, square, cube, even, odd, inverse, round, fraction, mixed, improper, recurring, terminating, percentage, Venn diagram

Unit 2. Groundwork: Algebra
(A1, A2, A3, A4, A6, A8, A21)

This unit focuses on the algebra skills required throughout the one-year GCSE course. Opportunity should be taken to assess prior knowledge and adapt as required. These skills should be kept on a 'rolling boil' throughout the course.

OBJECTIVES

By the end of the unit, students should be able to:
- Select an expression/equation/formula/identity from a list;
- Manipulate and simplify algebraic expressions by collecting like terms;
- Multiply together two simple algebraic expressions, e.g. $2a \times 3b$;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$;
- Use index notation and the laws of indices when multiplying or dividing algebraic terms;
- Understand the ≠ symbol and introduce the identity ≡ sign;
- Substitute numbers into algebraic expressions;
- Write expressions to solve problems representing a situation;
- Substitute numbers into a (word) formula;
- Plot coordinates in all four quadrants, and read graph scales.

POSSIBLE SUCCESS CRITERIA

Simplify $4p - 2q + 3p + 5q$.
Simplify $z^4 \times z^3$, $y^3 \div y^2$.
Simplify $x^{-4} \times x^2$, $w^2 \div w^{-1}$.
Evaluate the expressions for different values of x: $3x^2 + 4$ or $2x^3$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming expressions and equations using area and perimeter of 2D shapes.

COMMON MISCONCEPTIONS

- When working with variables a, b and c, it is always true that $a = 1$, $b = 2$, $c = 3$.
- If $a = 2$, $3a = 32$.
- $(-2)^2 = -4$
- Misunderstanding the rules for working with negative numbers.

Any poor number skills involving negatives and times tables will become evident.
Stress the convention of not writing a coefficient with a value of 1, i.e. x instead of $1x$, as this may cause confusion.

KEYWORDS

Expression, identity, equation, formula, substitute, term, like terms, index, power, collect, simplify, quadrant

Unit 3. Groundwork: Geometry
(G1, G3, G4, G6, G11, G12, G14, G16, G17)

This unit focuses on the geometry skills required throughout the one-year GCSE course. Opportunity should be taken to assess prior knowledge and adapt as required. These skills should be kept on a 'rolling boil' throughout the course.

OBJECTIVES

By the end of the unit, students should be able to:

- Estimate sizes of angles and measure angles using a protractor;
- Use geometric language and notation appropriately;
- Identify a line perpendicular to a given line on a diagram and use their properties;
- Identify parallel lines on a diagram and use their properties;
- Find missing angles using properties of corresponding and alternate angles;
- Classify quadrilaterals by their geometric properties and name all quadrilaterals that have a specific property;
- Given some information about a shape on coordinate axes, complete the shape;
- Understand and use the angle properties of quadrilaterals;
- Use the fact that the angle sum of a quadrilateral is 360°;
- Recall and use properties of angles at a point, angles at a point on a straight line, right angles and vertically opposite angles;
- Distinguish between scalene, equilateral, isosceles and right-angled triangles;
- Derive and use the sum of angles in a triangle;
- Understand and use the angle properties of triangles; use the symmetry property of isosceles triangles to show that base angles are equal; use the side/angle properties of isosceles and equilateral triangles;
- Give reasons for angle calculations and show step-by-step deduction when solving problems;
- Describe the properties of common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone;
- Find the perimeter of:
  - rectangles and triangles;
  - parallelograms and trapezia;
  - compound shapes;
- Recall and use the formulae for the area of a triangle and a rectangle;
- Find the area of a parallelogram;
- Find the area of a trapezium and recall the formula;
- Sketch nets of cuboids and prisms.

POSSIBLE SUCCESS CRITERIA

Name all the quadrilaterals that have a specific property.
Use geometric reasoning to answer problems, giving detailed reasons.
Find the size of missing angles at a point or at a point on a straight line.
Convince me that a rhombus is a parallelogram.
Find the area/perimeter of a given shape, stating the correct units.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step 'angle-chasing' style problems that involve justifying how students have found a specific angle.
Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.
What is the same, and what is different between families of polygons?

Foundation tier

Problems involving straightforward and compound shapes in a real-life context should be explored to reinforce the concept of area, for example the floor plan of a garden linked to the purchase of grass seed.

COMMON MISCONCEPTIONS

- Perpendicular lines have to be horizontal/vertical.
- All triangles have rotational symmetry of order 3.
- All trapezia are isosceles.
- A square is only square if 'horizontal', or a 'non-horizontal' square is called a diamond.
- Incorrectly identifying the 'base angles' (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.
- Confusing perimeter and area.

Shapes involving missing lengths of sides often result in incorrect answers.

KEYWORDS

Angle, acute, obtuse, reflex, polygon, regular, irregular, interior, exterior, proof, tessellation, rotational symmetry, order, clockwise, anticlockwise, parallel, perpendicular, corresponding, alternate, co-interior, vertically opposite, two-dimensional, three-dimensional, vertex, edge, face, side, triangle, equilateral, isosceles, scalene, quadrilateral, square, rectangle, parallelogram, rhombus, trapezium, kite, pentagon, hexagon, heptagon, octagon, decagon, prism, cube, cuboid, cylinder, pyramid, sphere, cone, measure, line, intersecting, area, perimeter, formula, length, width, compound

Unit 4. Groundwork: Statistics
(S2, S5)

This unit focuses on the statistical skills required throughout the one-year GCSE course. Opportunity should be taken to assess prior knowledge and adapt as required. These skills should be kept on a 'rolling boil' throughout the course.

OBJECTIVES

By the end of the unit, students should be able to:

- Design and use data-collection sheets for grouped, discrete and continuous data, use inequalities for grouped data, using ≤ and ≥ signs;
- Design, complete and use two-way tables for discrete and grouped data;
- Calculate the total frequency from a frequency table;
- Find greatest and least values, and the mode, from a frequency table;
- Identify the modal class from a grouped frequency table;
- Produce and interpret:
  - pictograms;
  - composite bar charts;
  - dual/comparative bar charts for categorical and ungrouped discrete data;
  - bar-line charts;
  - vertical line charts;
  - line graphs;
  - stem and leaf diagrams (including back-to-back);
  - pie charts;
- Calculate total population from a bar chart or table;
- Find greatest and least values, and the mode, from a bar chart or table;
- Recognise simple patterns, characteristics and relationships in bar charts and line graphs;
- From a pie chart:
  - find the mode;
  - find the total frequency;
- Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts;
- Interpret and discuss any data.

POSSIBLE SUCCESS CRITERIA

Construct a frequency table for a continuous data set, deciding on appropriate intervals using inequalities.
Plan a journey using timetables.
Decide the most appropriate chart or table given a data set.
State the mode, smallest value or largest value from a stem and leaf diagram.

From a simple pie chart, identify the frequency represented by $\frac{1}{4}$ and $\frac{1}{2}$ sections.

From a simple pie chart, identify the mode.
Find the angle for one item in a pie chart.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Misleading graphs, charts or tables can provide an opportunity for students to critically evaluate the way information is presented.
Students should be able to decide what the scales on any axis should be to be able to present information.

Foundation tier

From inspection of a pie chart, students should be able to identify the fraction of the total represented and know when that total can be calculated and compared with another pie chart.

COMMON MISCONCEPTIONS

- Misunderstanding the link between what the data in a frequency table represents, for example stating the frequency rather than the interval when asked for the modal group.
- Same size sectors on pie charts for different-sized data sets represent the same number rather than the same proportion.

SUGGESTED CAREER PATHWAYS

KEYWORDS

Mode, modal class, range, data, discrete, continuous, qualitative, quantitative, population, frequency, table, grouped, class interval, sort, pictogram, bar chart, composite bar chart, bar-line chart, vertical line chart, line graph, stem and leaf diagram, pie chart, estimate

Unit 5. Percentages
(N12, N13, R9, R16)

OBJECTIVES

By the end of the unit, students should be able to:

- Express a given number as a percentage of another number;
- Find a percentage of a quantity without a calculator: 50%, 25% and multiples of 10% and 5%;
- Find a percentage of a quantity or measurement;
- Calculate amount of increase/decrease;
- Use percentages to solve problems;
- Use percentages in real-life situations, including percentages greater than 100%:
  - price after VAT (not price before VAT);
  - value of profit or loss;
  - simple interest;
  - compound interest;
  - income tax calculations;
- Find a percentage of a quantity, including using a multiplier;
- Use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Make calculations involving repeated percentage change, not using the formula;
- Find the original amount given the final amount after a percentage increase or decrease;
- Set up, solve and interpret the answers in growth and decay problems;
- Understand the multiplicative nature of percentages as operators.

POSSIBLE SUCCESS CRITERIA

Express a given number as a percentage of another, including where the percentage > 100%.
What is 10%, 15%, 17.5% of £30?
What is the multiplier for the value after a 25% increase/decrease?
How much interest is earned over 5 years on £500 invested at 2.3% simple interest?
A car costs £8,000 and depreciates at 20% each year. How much is it worth after 3 years?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Sale prices offer an ideal opportunity for solving problems, allowing students the opportunity to investigate the most effective way to work out the 'sale' price.
Problems that involve consecutive reductions such as: 'Sale prices are 10% off the previous day's price. If a jacket is £90 on Monday, what is the price on Wednesday?'

COMMON MISCONCEPTIONS

- It is not possible to have a percentage greater than 100%.
- Compound interest and simple interest are the same method of calculating interest.
- Incomplete methods when using multipliers, e.g. reduce £80 by 15% = 80 × 0.15.

KEYWORDS

Percentage, increase, decrease, multiplier, VAT, income, tax, profit, loss, repeated, interest, simple, compound, inverse, growth, decay

Unit 6. Algebraic manipulation
(A1, A3, A4)

OBJECTIVES

By the end of the unit, students should be able to:
- Multiply a single number term over a bracket;
- Multiply together two algebraic expressions with brackets;
- Square a linear expression, e.g. $(x + 1)^2$;
- Rearrange simple expressions;
- Factorise algebraic expressions by taking out common factors.

POSSIBLE SUCCESS CRITERIA

Expand and simplify $3(t - 1)$.

Expand $(x + 2)(x + 6)$.

Understand $6x + 4 \neq 3(x + 2)$.

Argue mathematically that $2(x + 5) = 2x + 10$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming expressions and equations using area and perimeter of 2D shapes.

COMMON MISCONCEPTIONS

- $2x - 2 = 0$.
- $3(x + 4) = 3x + 4$.

KEYWORDS

Expression, identity, equation, formula, term, like terms, index, power, collect, expand, bracket, factor, factorise, linear, simplify

Unit 7. Angle properties in shapes
(G1, G3, G5, G6, G19)

OBJECTIVES

By the end of the unit, students should be able to:
- Understand 'regular' and 'irregular' as applied to polygons;
- Use the sum of the interior angles of an n-sided polygon;
- Use the sum of the exterior angles of any polygon is 360°;
- Use the sum of the interior angle and the exterior angle is 180°;
- Identify shapes which are congruent (by eye);
- Explain why some polygons fit together and others do not;
- Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS);
- Solve angle problems involving congruence;
- Find missing angles and lengths in similar shapes.

POSSIBLE SUCCESS CRITERIA

Deduce and use the angle sum in any polygon.
Derive the angle properties of regular polygons.
Given the size of its exterior angle, how many sides does the polygon have?
Use *AB* notation for describing lengths and $\angle ABC$ notation for describing angles.
Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not equal in size.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems whereby students have to justify the number of sides that a regular polygon has given an interior or exterior angle.

COMMON MISCONCEPTIONS

- All polygons are regular.
- All triangles have rotational symmetry of order 3.
- When a shape is enlarged, the angles also get bigger.

KEYWORDS

Angle, acute, obtuse, reflex, polygon, regular, irregular, interior, exterior, proof, tessellation, parallel, perpendicular, vertex, side, triangle, equilateral, isosceles, scalene, quadrilateral, pentagon, hexagon, heptagon, octagon, decagon, congruent, similar, corresponding

Unit 8. Accuracy
(N14, N15, N16)

OBJECTIVES

By the end of the unit, students should be able to:
- Estimate answers and check calculations using approximation and estimation;
- Round answers to a given degree of accuracy;
- Use inequality notation to specify simple error intervals due to truncation or rounding.

POSSIBLE SUCCESS CRITERIA

Recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction.
Round 16 000 people to the nearest 1000.
Round 1100 g to 1 significant figure.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This unit provides many opportunities for students to evaluate their answers and provide counter-arguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

COMMON MISCONCEPTIONS

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

KEYWORDS

Round, truncate, accuracy, significant figures, decimal places, estimate, error interval

Unit 9. Circles
(G9, G16, G17, G18, N8)

OBJECTIVES

By the end of the unit, students should be able to:

- Recall the definition of a circle and identify, name and draw parts of a circle, including tangent, chord and segment;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle; circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2;
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Give an answer to a question involving the circumference or area of a circle in terms of π;
- Find the radius or diameter, given the area or circumference of a circle;
- Find the perimeters and areas of semicircles and quarter-circles;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles;
- Calculate arc lengths, angles and areas of sectors of circles.

POSSIBLE SUCCESS CRITERIA

Recall terms related to a circle.
Find the circumference and area of a circle with radius 3 cm.
Understand that answers in terms of π are more accurate.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions involving calculating the radius/diameter given the area/circumference could be explored, including questions that require evaluation of statements, such as: 'Huan states "diameter = 2 × radius" and Kim states "radius = 2 × diameter". Who is correct?'

COMMON MISCONCEPTIONS

- Radius = 2 × diameter.

Recollection of which formula to use for area and circumference of circles is often poor.

KEYWORDS

Area, perimeter, formula, length, measurement, circle, tangent, chord, arc, sector, circumference, radius, diameter, segment, pi (π), cylinder, surface area, volume

Unit 10. Probability
(P1, P2, P3, P4, P5, P7, A17)

OBJECTIVES

By the end of the unit, students should be able to:

- Distinguish between events which are impossible, unlikely, even chance, likely and certain to occur;
- Mark events and/or probabilities on a probability scale of 0 to 1;
- Write probabilities in words or fractions, decimals and percentages;
- Find the probability of an event happening using theoretical probability;
- Use theoretical models to include outcomes using dice, spinners, coins;
- List all outcomes for single events systematically;
- Work out probabilities from frequency tables and frequency trees;
- Record outcomes of probability experiments in tables;
- Add simple probabilities;
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all outcomes is 1;
- Use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring;
- Find a missing probability from a list or table, including algebraic terms;
- Find the probability of an event happening using relative frequency;
- Estimate the number of times an event will occur, given the probability and the number of trials, for both experimental and theoretical probabilities;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes.

POSSIBLE SUCCESS CRITERIA

Mark events on a probability scale and use the language of probability.
If the probability of the possible outcomes are x, $2x$, $4x$, $3x$, calculate x.
Calculate the probability of an event from a frequency table.
Decide if a coin, spinner or game is fair.
Understand the use of the 0–1 scale to measure probability.
List all the outcomes for an experiment.
Know and apply the fact that the sum of probabilities for all outcomes is 1.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Lotteries provide a real-life link to probability. Work out the probabilities of winning on different lotteries.
Students should be given the opportunity to justify the probability of events happening or not happening.

COMMON MISCONCEPTIONS

- If a 6 has been rolled on a fair dice, the next roll is less likely to be a 6

KEYWORDS

Probability, event, outcome, impossible, unlikely, even chance, likely, certain, probability scale, theoretical, experimental, mutually exclusive, relative frequency, frequency tree, fairness

Unit 11. Sequences
(A23, A24, A25)

OBJECTIVES

By the end of the unit, students should be able to:

- Recognise sequences of odd and even numbers, and other sequences including Fibonacci sequences;
- Write the term-to-term definition of a sequence in words;
- Find a specific term in a sequence using position-to-term or term-to-term rules;
- Generate arithmetic sequences of numbers, triangular numbers, square and cube integers and sequences derived from diagrams;
- Recognise such sequences from diagrams and draw the next term in a pattern sequence;
- Find the next term in a sequence, including negative values;
- Find the nth term:
  - for a pattern sequence;
  - of a linear sequence;
- Use the nth term of an arithmetic sequence to:
  - generate terms;
  - decide if a given number is a term in the sequence;
  - find the first term greater/less than a certain number;
- Continue a geometric progression and find the term-to-term rule, including negative, fraction and decimal terms;
- Continue a quadratic sequence and use the nth term to generate terms;
- Distinguish between arithmetic and geometric sequences.

POSSIBLE SUCCESS CRITERIA

Given a sequence, which is the first term greater than 50?
What is the amount of money after x months of saving the same amount or the height of a tree that grows 6 m per year?
What are the next terms in the following sequences?
1, 3, 9, … 100, 50, 25, … 2, 4, 8, 16, …
Write down an expression for the nth term of the arithmetic sequence 2, 5, 8, 11, …
Is 67 a term in the sequence 4, 7, 10, 13, …?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluating statements about whether or not specific numbers or patterns are in a sequence and justifying the reasons.

COMMON MISCONCEPTIONS

- Finding the difference between the first two terms of a sequence and assuming that the sequence is a linear sequence with this common difference.

KEYWORDS

Arithmetic, geometric, sequence, progression, nth term, common difference, derive, linear, quadratic, triangular, cube, square, Fibonacci, odd, even

Unit 12. Quadratics
(A4)

OBJECTIVES

By the end of the unit, students should be able to:

- Multiply together two algebraic expressions with brackets;

POSSIBLE SUCCESS CRITERIA

Expand $(x + 2)(x + 6)$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

COMMON MISCONCEPTIONS

- Collecting x terms with x^2 terms.

KEYWORDS

Expression, quadratic, linear, function, solve, root, simplify, expand, factorise, factor, coefficient, bracket

Unit 13. Ratio and compound measures
(R1, R3, R4, R5, R6, R8, R11, R12, N11, N13)

OBJECTIVES

By the end of the unit, students should be able to:

- Write ratios in their simplest form;
- Write/interpret a ratio to describe a situation;
- Share a quantity in a given ratio, including three-part ratios;
- Solve a ratio problem in context;
- Compare ratios;
- Write ratios in the form $1 : m$ or $m : 1$;
- Write a ratio as a fraction or as a linear function;
- Write lengths, areas and volumes of two shapes as ratios in their simplest form;
- Express a multiplicative relationship between two quantities as a ratio or a fraction;
- Understand and use compound measures:
  - speed:
    - read values in km/h and mph from a speedometer;
    - calculate average speed, distance, time – in miles per hour as well as metric measures;
- Convert between:
  - metric speed measures;
- Change $\frac{d}{t}$ in m/s to a formula in km/h, i.e. $\frac{d}{t} \times \frac{60 \times 60}{1000}$ (with support).

POSSIBLE SUCCESS CRITERIA

Write a ratio to describe a situation such as 1 blue for every 2 red, or 3 adults for every 10 children.
Recognise that two paints mixed red to yellow $5 : 4$ and $20 : 16$ are the same colour.
Express the statement 'There are twice as many girls as boys' as the ratio $2 : 1$ or the linear function $y = 2x$, where x is the number of boys and y is the number of girls.
Know that measurements using real numbers depend upon the choice of unit, with speedometers and rates of change.
Change g/cm^3 to kg/m^3, kg/m^3 to g/cm^3, N/m^2 to N/cm^2, m/s to km/h.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving sharing in a ratio that include percentages rather than specific numbers, such as: 'In a youth club the ratio of the number of boys to the number of girls is $3 : 2$. 30% of the boys are under the age of 14 and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?'
Speed–distance type problems that involve students justifying their reasons why one vehicle is faster than another.

COMMON MISCONCEPTIONS

- When using a ratio to find one quantity when the other is known, 'sharing' the known amount.

Students find three-part ratios difficult.

KEYWORDS

Ratio, share, parts, fraction, function, compound measure, density, mass, volume, speed, distance, time, pressure, force, area, velocity, acceleration

Unit 14. Proportion
(R6, R10, R11, R13, R14)

OBJECTIVES

By the end of the unit, students should be able to:
- Solve proportion problems using the unitary method;
- Work out which product offers best value and consider rates of pay;

POSSIBLE SUCCESS CRITERIA

Solve word problems involving direct and inverse proportion.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Calculations involving value for money are a good reasoning opportunity that utilise different skills.
Working out best value of items using different currencies given an exchange rate.

COMMON MISCONCEPTIONS

- Any straight-line graph shows direct proportion.

KEYWORDS

Ratio, fraction, proportion, best value, unitary, proportional change, direct proportion, inverse proportion, constant of proportionality

Unit 15. Statistical graphs and measures
(S2, S4)

OBJECTIVES

By the end of the unit, students should be able to:

- Find the median, mean, mode and range of a data set;
- Recognise the advantages and disadvantages between measures of average;
- Construct and interpret stem and leaf diagrams (including back-to-back diagrams):
  - find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
- Calculate the mean, mode, median and range from a frequency table (discrete data);
- Construct and interpret grouped frequency tables for continuous data:
  - for grouped data, find the interval which contains the median and the modal class;
  - estimate the mean with grouped data.

POSSIBLE SUCCESS CRITERIA

Extract the averages from a stem and leaf diagram.
Estimate the mean from a grouped frequency table.
Compare two data sets and justify their comparisons based on measures extracted from their diagrams where appropriate in terms of the context of the data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide reasons for choosing to use a specific average to support a point of view.
Students should be able to provide a correct solution as a counter-argument to statements involving the 'averages', e.g. 'Susan states that the median is 15. She is wrong. Explain why.'

COMMON MISCONCEPTIONS

- Misunderstanding the link between what the data in a frequency table represents, for example stating the frequency rather than the interval when asked for the modal group.

KEYWORDS

Mean, median, mode, range, average, data, discrete, continuous, grouped, stem and leaf diagram, frequency, table, sort, estimate

Unit 16. Transformations of shapes
(G7, G19, G24, G25, R2)

OBJECTIVES

By the end of the unit, students should be able to:

- Find the centre of rotation, angle and direction of rotation and describe rotations fully using the angle, direction of turn and centre;
- Rotate and draw the position of a shape after rotation about the origin or any other point, including rotations on a coordinate grid;
- Understand that translations are specified by a distance and direction using a vector;
- Translate a given shape by a vector;
- Use column vectors to describe and transform 2D shapes using single translations on a coordinate grid;
- Identify two column vectors that are parallel;
- Calculate using column vectors, and represent graphically, the sum of two vectors, the difference of two vectors and a scalar multiple of a vector;
- Understand that distances and angles are preserved under rotations, reflections and translations, so that any figure is congruent under any of these transformations;
- Understand that reflections are specified by a mirror line;
- Identify the equation of a line of symmetry;
- Transform 2D shapes using single reflections with vertical, horizontal and diagonal mirror lines, including reflections on a coordinate grid;
- Describe reflections on a coordinate grid;
- Enlarge a given shape using (0, 0) as the centre of enlargement, and enlarge shapes with a centre other than (0, 0);
- Find the centre of enlargement by drawing;
- Describe and transform 2D shapes using enlargements by:
  - a positive integer scale factor;
  - a fractional scale factor;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides, simple integer scale factors or simple fractions;
- Understand that similar shapes are enlargements of each other and angles are preserved;
- Transform 2D shapes by a combination of two transformations.

POSSIBLE SUCCESS CRITERIA

Understand that translations are specified by a distance and direction (using a vector).
Describe and transform a given shape by a reflection, a rotation or a translation.
Convince me the scale factor is 2.5.
Know that if one vector is a multiple of the other, they are parallel.
Add and subtract vectors using column vectors.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students can explore the effect of reflecting in two parallel mirror lines and combining transformations.
Investigations involving vectors around 2D shapes such as a square can be extended to include considering the area enclosed in the same shapes.

COMMON MISCONCEPTIONS

- Confusing the directions in a column vector.

Foundation tier

Emphasise that the units of movement are those on the axes, and that care needs to be taken to check the scale.
Correct language must be used: students often use 'turn' rather than 'rotate'.
Stress that two vectors in different locations in the plane can be parallel and equal.

KEYWORDS

Transformation, rotation, reflection, enlargement, translation, single, combination, mirror line, centre of rotation, clockwise, anticlockwise, centre of enlargement, scale factor, column vector, similar, congruent, angle, direction, coordinate, describe, scalar, multiple, parallel

Unit 17. Bivariate data
(S6)

OBJECTIVES

By the end of the unit, students should be able to:
- Draw scatter graphs;
- Interpret points on a scatter graph;
- Identify outliers and ignore them on scatter graphs;
- Draw the line of best fit on a scatter graph by eye, and understand what it represents;
- Use the line of best fit to make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing;
- Distinguish between positive, negative and no correlation;
- Interpret scatter graphs in terms of the relationship between two variables;
- Interpret correlation in terms of the problem;
- Understand that correlation does not imply causality;
- State how reliable their predictions are, i.e. not reliable if extrapolated.

POSSIBLE SUCCESS CRITERIA

Justify an estimate they have made using a line of best fit.
Identify outliers and explain why they may occur.
Given two sets of data in a table, model the relationship and make predictions.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many real-life situations that give rise to two variables provide opportunities for students to extrapolate and interpret the resulting relationship (if any) between the variables.

COMMON MISCONCEPTIONS

- Interpreting scales of different measurements and confusion between x and y axes when plotting points.

Emphasise that lines of best fit need to be drawn when predicting values, rather than obtaining answers by sight.

KEYWORDS

Scatter graph, variable, outlier, line of best fit, extrapolate, interpolate, correlation, positive, negative

Unit 18. Volume and surface area
(G14, G16, G17, N14)

OBJECTIVES

By the end of the unit, students should be able to:
- Recall and use the formula for the volume of a cuboid;
- Find the surface area of a prism, including a triangular prism, cube and cuboid;
- Calculate volumes of right prisms and shapes made from cubes and cuboids;
- Estimate volumes, etc. by rounding measurements to 1 significant figure;
- Convert between measures of area and volume;
- Find the surface area and volume of a cylinder;
- Find the surface area and volume of spheres, pyramids, cones and composite solids.

POSSIBLE SUCCESS CRITERIA

Justify whether a certain number of small boxes fit inside a larger box.
Calculate the volume of a triangular prism with correct units.
Understand that answers in terms of π are more accurate.

COMMON MISCONCEPTIONS

- Confusing volume with surface area.
- 1 m = 100 cm, so 1 m^2 = 100 cm^2 and 1 m^3 = 100 cm^3.

KEYWORDS

Volume, surface area, formula, cube, cuboid, prism, cylinder, sphere, cone, hemisphere, radius, diameter, pi (π), accuracy, compound shape