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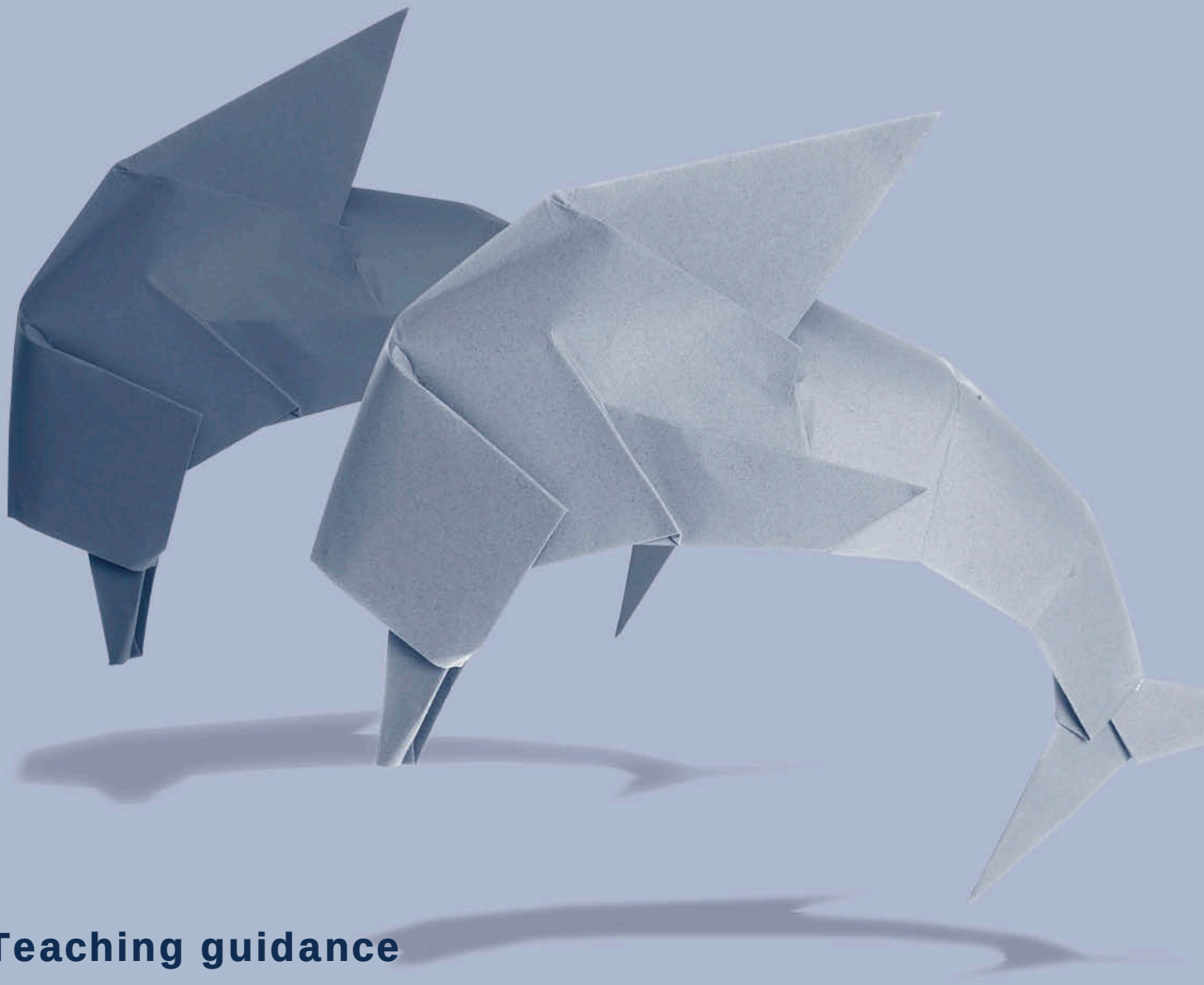
GCSE (9–1) Mathematics



Teaching guidance

Pearson Edexcel Level 1/Level 2 GCSE (9–1) in Mathematics (1MA1)
First teaching from September 2015
First certification from June 2017 Issue 4

ALWAYS LEARNING PEARSON

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About this booklet

This teaching guidance booklet has been produced to support mathematics teachers delivering the new GCSE (9–1) in Mathematics (1MA1) specification (first assessment summer 2017).

How to use this booklet

This booklet provides information on each specification point in turn, drawing together both new information and information already supplied in other documents.

For each specification point, the booklet provides the following information, where relevant:

- **content guidance** – commentary from the senior examiner team on how the specification will be assessed;
- **new to GCSE (9–1) Maths**, for any content nor previously assessed in 1MA0 or 2MB01;
- **new to Foundation tier**, for any content previously assessed at Higher tier in 1MA0 or 2MB01;
- **mapping to 1MA0**, listing the content descriptors;
- **learning objectives** at both tiers, with two-year scheme of work unit references;
- **selected questions** from the New Sample Assessment Materials, along with a few from the original SAMs; questions from the specimen papers will be added in due course;
- **Exam Wizard topics**, where you will find past paper questions from 1MA0, 2MB01 and 1MA1.

Topics in 1MA0 not included in 1MA1

Some topics have moved down to Key Stage 3, and so are not referenced in this booklet. A full list can be found in the **Getting Started** guide.

The 1MA0 specification points explicitly about calculator display (N g, N v and SP u) are now implicit in 1MA1 only.

Free support

Our free support for the GCSE (9–1) Mathematics specification (1MA1) can be found on the Edexcel mathematics website (http://qualifications.pearson.com/en/home.html) and on the Emporium (www.edexcel.com).

Issue 2 (December 2015)

This booklet has been updated to include sample questions from the two sets of specimen papers released in September 2015. These papers can be downloaded from the Emporium or from the Edexcel website here.

Issue 3 (December 2016)

This booklet has been updated to include the FAQs, Geometrical statements and Command words that were added to the Content Guidance Issue 3.

Issue 4 (December 2017)

This booklet has been updated to include questions from the three sets of mock papers issued in 2016 and 2017.

1. Number

Structure and calculation

N1	order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

Mapping to 1MA0 specification content descriptors

Nb (F/H) Order rational numbers
Nh (F) Understand equivalent fractions, simplifying a fraction by cancelling all common factors
Nl (F) Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions

Learning objectives

Foundation tier (Units 1a, 1b, 2a, 4a, 5a, 20)

- Use, compare and order positive and negative numbers (integers), decimals, fractions and percentages; use the symbols <, > and understand the ≠ symbol;
- Use decimal notation and place value and identify the value of digits in a decimal or whole number;
- Use diagrams to find equivalent fractions or compare fractions;
- Use notation and symbols correctly; understand the ≠ symbol and introduce the identity ≡ sign; know the difference between an equation and an identity;
- Use the correct notation to show inclusive and exclusive inequalities.

Higher tier (Units 2a, 4a, 9b)

- Find equivalent fractions and compare the size of fractions;
- Use algebraic notation and symbols correctly;
- Understand the ≠ symbol (not equal), e.g. $6x + 4 \neq 3\,(x + 2)$, and introduce the identity ≡ sign;
- Use the correct notation to show inclusive and exclusive inequalities.

Sample questions

Write the following numbers in order of size.
Start with the smallest number.

0.61 0.1 0.16 0.106

(Total 1 mark)
New SAMs Paper 1F qu.1 (N1 – AO1)

Here are four numbers.

0.43 $\frac{3}{7}$ 43.8% $\frac{7}{16}$

Write these numbers in order of size.
Start with the smallest number.

(Total 2 marks)
New SAMs Paper 3F qu.10 (N1, N10 – AO1)

(*a*) Factorise $y^2 + 7y + 6$

(2)

(*b*) Solve $6x + 4 > x + 17$

(2)

(*c*) n is an integer with $-5 < 2n \leq 6$
Write down all the values of n

(2)

(Total 6 marks)
New SAMs Paper 3H qu.9 (N1, A4, A22 – AO1)

Exam Wizard topics

Integers
Directed numbers
Fractions
Decimals
Place value

N2	apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

Content guidance

The examiners will test non-calculator arithmetic, including long multiplication and division, on the non-calculator paper. No method will be specified; any correct method will be accepted.

If the question was, for example, 45 × 289, then full marks would be given for a correct answer of 13 005. If the answer was incorrect then the working would be looked at and partial marks, if appropriate, would be awarded for *any* correct method used to multiply the two numbers (as at present) – examiners will not be prescribing the method that candidates should use.

Mapping to 1MA0 specification content descriptors

Na (F/H) Add, subtract, multiply and divide any number
Ni (F/H) Add and subtract fractions
Nj (F) Use decimal notation and recognise that each terminating decimal is a fraction
No (F/H) Interpret fractions, decimals and percentages as operators
Nq (F/H) Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

Learning objectives

Foundation tier (Units 1a, 1b, 4a, 18a)

- Add, subtract, multiply and divide positive and negative numbers (integers) and decimals;
- Add, subtract, multiply and divide fractions with integers and fractions, including mixed numbers; write the answer as a mixed number; find fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;
- Multiply or divide any number by powers of 10, or by any number between 0 and 1;
- Use decimal notation and place value; identify the value of digits in a decimal or whole number;
- Simplify fraction multiplication by cancelling first;
- Use calculators for all calculations: positive and negative numbers, brackets, square, cube, powers and roots, and all four operations.

Higher tier (Units 1a, 1b, 4a)

- Add, subtract, multiply and divide decimals, whole numbers including any number between 0 and 1;
- Add, subtract, multiply and divide fractions, including mixed numbers and whole numbers; find a fraction of a quantity or measurement;
- Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
- Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.

Sample questions

Write down the value of the 3 in the number 4376

(Total 1 mark)
New SAMs Paper 2F qu.1 (N2 – AO1)

There are 6760 people at a rugby match.
3879 of the people are men.
1241 of the people are women.
$\frac{1}{4}$ of the children are girls.

Work out how many boys are at the rugby match.

(Total 3 marks)
New SAMs Paper 3F qu.3 (N2 – AO1/AO3)

Work out 6.34 × 5.2

(Total 3 marks)
New SAMs Paper 1F qu.21 / 1H qu.1 (N2 – AO1)

Henry is thinking of having a water meter.

These are the two ways he can pay for the water he uses.

Water Meter

A charge of £28.20 per year

plus

91.22p for every cubic metre of water used

1 cubic metre = 1000 litres

No Water Meter

A charge of £107 per year

Henry uses an average of 180 litres of water each day.

Use this information to determine whether or not Henry should have a water meter

(Total 5 marks)
New SAMs Paper 3F qu.22 / 3H qu.5 (N2, R10, R11 – AO1/AO3)

Exam Wizard topics

Integers
Four operations
Directed numbers
Fractions
Decimals
Place value

N3	recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

Mapping to 1MA0 specification content descriptors

Na (F) Add, subtract, multiply and divide any number
Ne (F/H) Use index notation for squares, cubes and powers of 10
Nh (F/H) Understand equivalent fractions, simplifying a fraction by cancelling all common factors
Nq (F/H) Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

Learning objectives

Foundation tier (Units 1a, 1b, 1c, 2a, 4a, 18a)

- Recall all multiplication facts to 10 × 10, and use them to derive quickly the corresponding division facts;
- Use one calculation to find the answer to another; check answers by rounding and using inverse operations;
- Find the reciprocal of an integer, decimal or fraction; understand and use unit fraction and 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is not defined);
- Evaluate expressions involving squares, cubes and roots: add, subtract, multiply and divide numbers in index form; cancel to simplify a calculation;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$;
- Use brackets and the hierarchy of operations, including with powers inside the brackets, or raising brackets to powers;
- Use calculators for all calculations: positive and negative numbers, brackets, square, cube, powers and roots, and all four operations.

Higher tier (Units 1a, 1b, 2a, 4a)

- Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
- Understand that the inverse operation of raising a positive number to a power n is raising the result of this operation to the power $\frac{1}{n}$;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$;
- Write a fraction in its simplest form, including using it to simplify a calculation, e.g. $50 \div 20 = \frac{50}{20} = \frac{5}{2} = 2.5$;
- Understand and use unit fractions as multiplicative inverses;
- Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets.

Sample questions

Work out $\dfrac{30+12}{5+3}$

(Total 1 mark)
Specimen Papers Set 1, Paper 3F qu.2 (N3 – AO1)

Work out the reciprocal of 0.125.

(Total 1 mark)
Specimen Papers Set 1, Paper 3F qu.3 (N3 – AO1)

Exam Wizard topics

Four operations
Reciprocals
Roots and powers
Index notation

N4	use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

Content guidance

The unique factorisation theorem will be tested by the requirement to carry out the prime factorisation of a given number.

Mapping to 1MA0 specification content descriptors

Nc (F/H) Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor (HCF), Least Common Multiple (LCM), prime number and prime factor decomposition

Learning objectives

Foundation tier (Units 1a, 1d)

- Recall all multiplication facts to 10 × 10, and use them to derive quickly the corresponding division facts;
- Recognise odd, even and prime (two digit) numbers;
- Identify factors and multiples and list all factors and multiples of a number systematically;
- Find the prime factor decomposition of positive integers and write as a product using index notation; understand that the prime factor decomposition of a positive integer is unique – whichever factor pair you start with – and that every number can be written as a product of two factors;
- Find common factors and common multiples of two numbers; find the LCM and HCF of two numbers by listing, Venn diagrams and using prime factors: include finding LCM and HCF given the prime factorisation of two numbers;
- Solve simple problems using HCF, LCM and prime numbers.

Higher tier (Unit 1c)

- Identify factors, multiples and prime numbers;
- Find the prime factor decomposition of positive integers – write as a product using index notation; understand that the prime factor decomposition of a positive integer is unique – whichever factor pair you start with – and that every number can be written as a product of two factors;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers by listing, Venn diagrams and using prime factors – include finding LCM and HCF given the prime factorisation of two numbers;
- Solve problems using HCF, LCM and prime numbers.

Sample questions

Write down all the factors of 20

(Total 2 marks)

New SAMs Paper 1F qu.4 (N4 – AO1)

Jan writes down

one multiple of 9
and two different factors of 40

Jan adds together her three numbers.
Her answer is greater than 20 but less than 30

Find three numbers that Jan could have written down.

(Total 3 marks)
New SAMs Paper 2F qu.6 (N4 – AO1/AO3)

Adam says,

"When you multiply an even number by an odd number
the answer is always an odd number."

(*a*) Write down an example to show Adam is wrong.

(1)

Betty says,

"When you multiply two prime numbers together
the answer is always an odd number."

(*b*) Betty is wrong.
Explain why.

(2)

(Total 3 marks)
New SAMs Paper 2F qu.11 (N4, N2 – AO2)

Liz buys packets of coloured buttons.

There are 8 red buttons in each packet of red buttons.
There are 6 silver buttons in each packet of silver buttons.
There are 5 gold buttons in each packet of gold buttons.

Liz buys equal numbers of red buttons, silver buttons and gold buttons.
How many packets of each colour of buttons did Liz buy?

(Total 3 marks)
New SAMs Paper 3H qu.6 (N4 – AO1/AO3)

Exam Wizard topics

Primes; factors; multiples

N5	apply systematic listing strategies, including use of the product rule for counting **(i.e. if there are *m* ways of doing one task and for each of these, there are *n* ways of doing another task, then the total number of ways the two tasks can be done is *m* × *n* ways)**

New to GCSE (9–1) Maths

Use of the product rule for counting.

Mapping to 1MA0 specification content descriptors

SPo (F/H) List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities

Learning objectives

Foundation tier (Units 1d, 13)

- List all three-digit numbers that can be made from three given integers;
- Find the LCM and HCF of two numbers by listing, Venn diagrams and using prime factors;
- List all outcomes for single events, and combined events, systematically;
- Use and draw sample space diagrams.

Higher tier (Units 1a, 10)

- Use the product rule for counting (i.e. if there are *m* ways of doing one task and for each of these, there are *n* ways of doing another task, then the total number of ways the two tasks can be done is *m* × *n* ways);
- List all outcomes for single events, and combined events, systematically;
- Use and draw sample space diagrams.

Sample questions

Sally has three tiles.
Each tile has a different number on it.
Sally puts the three tiles down to make a number.
Each number is made with all three tiles.

How many different numbers can Sally make?

(Total 2 marks)
New SAMs Paper 1F qu.11 (N5 – AO1)

There are 24 girls and 12 boys in a club.

One girl and one boy are going to be chosen to go to a meeting.

Work out the total number of ways of choosing a girl and a boy.

(Total 2 marks)
Mock Papers Set 2, Paper 3H qu.12 (N5 – AO1)

There are 14 boys and 12 girls in a class.

Work out the total number of ways that 1 boy and 1 girl can be chosen from the class.

(Total 2 marks)

Specimen Papers Set 1, Paper 3H qu.13 (N5 – AO1)

A cafe owner sells 10 different types of sandwich.

Rayheem buys a different type of sandwich on Monday, on Tuesday and on Wednesday.

In how many ways can he do this?

(Total for Question 14 is 2 marks)

Mock Papers Set 3, Paper 2H qu.14 (N5 – AO1)

Marie has 25 cards.
Each card has a different symbol on it.

Marie gives one card to Shelley and one card to Pauline.

(*a*) In how many different ways can Marie do this?

(2)

There are 12 boys and 10 girls in David's class.
David is going to pick three different students from his class and write their names in a list in order.

The order will be

boy girl boy	**or**	girl boy girl

(*b*) How many different lists can David write?

(3)

(Total 5 marks)

Specimen Papers Set 2, Paper 2H qu.12 (N5 – AO1/AO3)

There are 17 men and 26 women in a choir.
The choir is going to sing at a concert.
One of the men and one of the women are going to be chosen to make a pair to sing the first song.

(*a*) Work out the number of different pairs that can be chosen.

(2)

Two of the men are going to be chosen to make a pair to sing the second song.
Ben thinks the number of different pairs that can be chosen is 136
Mark thinks the number of different pairs that can be chosen is 272

(*b*) Who is correct, Ben or Mark?
Give a reason for your answer.

(1)

(Total 3 marks)
New SAMs Paper 3H qu.15 (N5 – AO1/AO2)

There are 95 girls and 87 boys in Year 13 at a school.

One girl is going to be chosen for the role of Head Girl.
A different girl is going to be chosen for the role of Deputy Head Girl.
One boy is going to be chosen for the role of Head Boy.
A different boy is going to be chosen for the role of Deputy Head Boy.

Work out how many different ways this can be done.

(Total 3 marks)
Mock Papers Set 1, Paper 2H qu.18 (N5 – AO1)

Exam Wizard topics

Product rule for counting
Theoretical probability
New to GCSE (9–1)

N6	use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5; **estimate powers and roots of any given positive number**

Content guidance

The accuracy that candidates will be expected to estimate a square root of a positive number will depend on the context of the question. For a straightforward AO1 question such as "estimate the square root of 85", then knowing that the answer lies between 9 and 10 and closer to 9 is all that examiners would expect.

New to GCSE (9–1) Maths

Estimating powers and roots of any given positive number.

Mapping to 1MA0 specification content descriptors

Nd (F/H) Use the terms square, positive and negative square root, cube and cube root
Ne (F/H) Use index notation for squares, cubes and powers of 10

Learning objectives

Foundation tier (Unit 1c)

- Find squares and cubes: recall integer squares up to 10 × 10 and the corresponding square roots; understand the difference between positive and negative square roots; recall the cubes of 1, 2, 3, 4, 5 and 10;
- Recognise powers of 2, 3, 4, 5;
- Use index notation for squares, cubes and powers of 10;
- Evaluate expressions involving squares, cubes and roots: add, subtract, multiply and divide numbers in index form; cancel to simplify a calculation;
- Use calculators for all calculations: positive and negative numbers, brackets, square, cube, powers and roots, and all four operations.

Higher tier (Unit 1b)

- Use index notation for integer powers of 10, including negative powers;
- Recognise powers of 2, 3, 4, 5;
- Estimate powers and roots of any given positive number by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
- Use calculators for all calculations: positive and negative numbers, brackets, four operations, x^2, $\sqrt{x}$, x^3, $\sqrt[3]{x}$, memory, x^y, $x^{\frac{1}{y}}$.

Sample questions

Here is a list of numbers

4 7 9 25 27 31 64

From the numbers in the list, write down a cube number.

(Total 1 mark)
New SAMs Paper 2F qu.3 (N6 – AO1)

Find the value of $(2.8-0.45)^2+\sqrt[3]{5.832}$

(Total 2 marks)
New SAMs Paper 2F qu.4 (N6 – AO1)

Here are two numbers.

29 37

Nadia says both of these numbers can be written as the **sum** of two square numbers.

Is Nadia correct?
You must show how you get your answer.

(Total 3 marks)
Specimen Papers Set 1, Paper 3F qu.12 (N6 – AO1/AO3)

Exam Wizard topics

Roots and powers
Index notation
New to GCSE (9–1)

N7 calculate with roots, and with integer **and fractional** indices

Content guidance

To include the laws of indices applied to numbers with integer powers (integer power could be positive, negative or zero; positive and negative fractional powers on the Higher tier only), e.g. simplify $2^7 \times 2^{-4}$, write down the value of 3^0.

New to Foundation tier

Calculating with zero and negative powers. (NB: calculating with fractional integers is still Higher tier only.)

Mapping to 1MA0 specification content descriptors

Ne (F/H) Use index notation for squares, cubes and powers of 10
Nf (F) Use index laws for multiplication and division of integer powers
Nf (H) Use index laws for multiplication and division of integer, fractional and negative powers
Nq (F/H) Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

Learning objectives

Foundation tier (Units 1c, 12, 18b)

- Use index notation for squares, cubes and powers of 10, including negative powers;
- Evaluate expressions involving squares, cubes and roots: add, subtract, multiply and divide numbers in index form; cancel to simplify a calculation;
- Use the laws of indices to multiply and divide numbers written in index notation, and to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractions and powers of a power;
- Use numbers raised to the power zero, including the zero power of 10;
- Use calculators for all calculations: positive and negative numbers, brackets, square, cube, powers and roots, and all four operations;
- Understand, recall and use Pythagoras' theorem in 2D to justify if a triangle is right-angled or not; calculate unknown sides in a right-angled triangle, including on a coordinate grid; calculate length of a line segment AB given pairs of points; leave answers in surd form.

Higher tier (Units 1b, 5b)

- Use index notation for integer powers of 10, including negative powers;
- Find the value of calculations using indices including positive, fractional and negative indices;
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power, and to solve problems;
- Recall that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers n as well as, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n; understand that the inverse operation of raising a positive number to a power n is raising the result of this operation to the power $\frac{1}{n}$;
- Use calculators for all calculations: positive and negative numbers, brackets, four operations, x^2, $\sqrt{x}$, x^3, $\sqrt[3]{x}$, memory, x^y, $x^{\frac{1}{y}}$;
- Understand, recall and use Pythagoras' theorem in 2D to justify if a triangle is right-angled or not; calculate unknown sides in a right-angled triangle, including on a coordinate grid; calculate length of a line segment AB given pairs of points; leave answers in surd form.

Sample questions

(*a*) Write down the value of $64^{\frac{1}{2}}$

(1)

(*b*) Find the value of $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

(2)

(Total 3 marks)
New SAMs Paper 1H qu.10 (N7, N8 – AO1)

(i) Find the value of $\sqrt[5]{3.2 \times 10^{11}}$

(ii) Find the value of $10^{\frac{3}{4}}$
Give your answer correct to 1 decimal place.

(Total 2 marks)
Specimen Papers Set 2, Paper 2H qu.16 (N7, N9 – AO1)

Exam Wizard topics

Roots and powers
Index notation
New to Foundation from Higher in GCSE (9–1)

N8	calculate exactly with fractions, **surds** and multiples of π; **simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators**

Content guidance

Candidates could be asked to rationalise the denominator of any fraction which may involve utilising the difference of two squares. For example, $\frac{\sqrt{2}}{\sqrt{2}+1}$.

New to Foundation tier

Calculating exactly with multiples of π. (NB: calculating exactly with surds is still Higher tier only). Calculating lengths of arcs and areas of sectors of circles, including answers in terms of π (semicircles and quarter-circles are already in Foundation tier).

Mapping to 1MA0 specification content descriptors

Na (F/H) Add, subtract, multiply and divide any number
No (F/H) Interpret fractions, decimals and percentages as operators
Nr (H) Use surds and π in exact calculations
GMz (H) Find circumferences and areas of circles

Learning objectives

Foundation tier (Units 4a, 17, 18a)

- Add, subtract, multiply and divide fractions with integers and fractions, including mixed numbers; write the answer as a mixed number; simplify calculations by cancelling first; find fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;
- Convert between mixed numbers and improper fractions;
- Understand and use unit fractions as multiplicative inverses;
- Find the reciprocal of an integer, decimal or fraction;
- Recall the fraction-to-decimal conversion and convert between fractions, decimals and percentages, including terminating and recurring decimals; convert a fraction to a decimal to make a calculation easier, e.g. $0.25 \times 8 = \frac{1}{4} \times 8$, or $\frac{3}{8} \times 10 = 0.375 \times 10$, and to compare and order fractions, decimals and integers, using inequality signs;
- Give an answer to a question involving the circumference or area of a circle in terms of π.

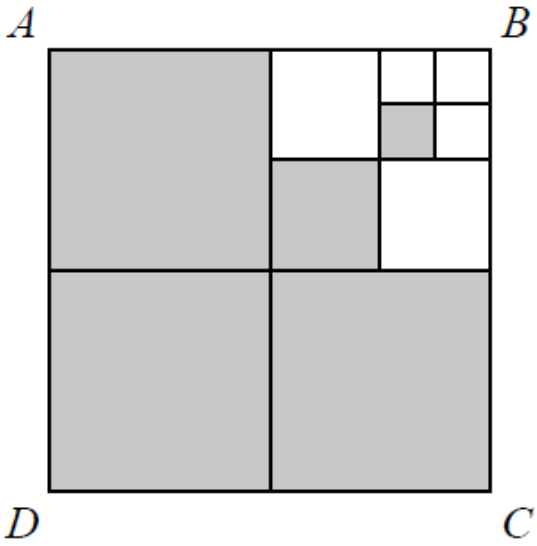
Higher tier (Units 1c, 2b, 4a, 5b, 7a, 7b, 9a, 17)

- Add, subtract, multiply and divide fractions with integers and fractions, including mixed numbers; write the answer as a mixed number; simplify calculations by cancelling first; find a fraction of a quantity or measurement, including within a context;
- Find equivalent fractions, and compare and order fractions by using a common denominator;
- Write a fraction in its simplest form, including using it to simplify a calculation, e.g. $50 \div 20 = \frac{50}{20} = \frac{5}{2} = 2.5$;
- Convert between mixed numbers and improper fractions;
- Convert between fractions, decimals and percentages, including terminating and recurring decimals, and vice versa; by writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals; convert a fraction to a decimal to make a calculation easier; compare and order fractions, decimals and integers, using inequality signs;
- Understand and use unit fractions as multiplicative inverses;
- Find the reciprocal of an integer, decimal or fraction;

- Understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents, and choose fractions, decimals or percentages appropriately for calculations;
- Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2; simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$); rationalise the denominator involving surds;
- Recognise and use simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 or a surd);
- Give an answer to the use of Pythagoras' theorem in surd form;
- Solve quadratic equations by factorisation, completing the square and using the quadratic formula;
- Give an answer to a question involving the circumference or area of a circle, or the surface area or volume of a sphere or cone in terms of π.

Sample questions

ABCD is a square.
This diagram is drawn accurately



What fraction of the square *ABCD* is shaded?

(Total 2 marks)
New SAMs Paper 2F qu.7 (N8 – AO1/AO2)

(*a*) Work out $\frac{2}{7} + \frac{1}{5}$

(2)

(*b*) Work out $1\frac{2}{3} \div \frac{3}{4}$

(2)

(Total 4 marks)
New SAMs Paper 1F qu.18 (N8 – AO1)

Show that $\dfrac{1}{1+\dfrac{1}{\sqrt{2}}}$ can be written as $2 - \sqrt{2}$

(Total 3 marks)

New SAMs Paper 1H qu.23 (N8 – AO2)

Exam Wizard topics

Four operations
Fractions
Surds
Area
Perimeter
Circles
New to Foundation from Higher in GCSE (9–1)

N9	calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

New to Foundation tier

Calculating with and interpreting standard form.

Mapping to 1MA0 specification content descriptors

Ng (H) Interpret, order and calculate numbers written in standard index form

Learning objectives

Foundation tier (Unit 18b)

- Convert large and small numbers into standard form and vice versa;
- Add, subtract, multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form.

Higher tier (Unit 1c)

- Convert large and small numbers into standard form and vice versa;
- Add, subtract, multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form.

Sample questions

Work out the value of $(9 \times 10^{-4}) \times (3 \times 10^{7})$
Give your answer in standard form.

(Total 2 marks)
New SAMs Paper 1H qu.9 (N9 – AO1)

One uranium atom has a mass of 3.95×10^{-22} grams.

(*a*) Work out an estimate for the number of uranium atoms in 1 kg of uranium.

(3)

(*b*) Is your answer to (*a*) an underestimate or an overestimate?
Give a reason for your answer.

(1)

(Total 4 marks)
New SAMs Paper 1H qu.11 (N9, N14, R1, R10 – AO1/AO3)

Write 0.000068 in standard form.

(Total 1 mark)
New SAMs Paper 3H qu.8 (N9, N7 – AO1)

Exam Wizard topics

Standard form
New to Foundation from Higher in GCSE (9–1)

Fractions, decimals and percentages

N10	work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 or $\frac{3}{8}$); **change recurring decimals into their corresponding fractions and vice versa**

Content guidance

Students may need to change a fraction into a recurring decimal in the context of a problem, e.g. order 30%, 0.35, $\frac{1}{3}$, 32%.

Mapping to 1MA0 specification content descriptors

Na (F) Add, subtract, multiply and divide any number
Nh (F/H) Understand equivalent fractions, simplifying a fraction by cancelling all common factors
Nj (F/H) Use decimal notation and recognise that each terminating decimal is a fraction
Nk (H) Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals

Learning objectives

Foundation tier (Unit 4a)

- Recognise recurring decimals and convert fractions such as $\frac{3}{7}$, $\frac{1}{3}$ and $\frac{2}{3}$ into recurring decimals.

Higher tier (Unit 4a)

- By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
- Convert a fraction to a recurring decimal and vice versa.

Sample questions

Write 0.037 as a fraction.

(Total 1 mark)
New SAMs Paper 1F qu.2 (N10 – AO1)

Prove algebraically that the recurring decimal $0.2\dot{5}$ has the value $\frac{23}{90}$

(Total 2 marks)
New SAMs Paper 2H qu.15 (N10 – AO1)

Exam Wizard topics

Fractions
Decimals

N11 identify and work with fractions in ratio problems

Mapping to 1MA0 specification content descriptors

Nt (F/H) Divide a quantity in a given ratio

Learning objectives

Foundation tier (Unit 11a)

- Share a quantity in a given ratio including three-part ratios;
- Solve a ratio problem in context: find one quantity when the other is known; in problems involving mixing, e.g. paint colours, cement and drawn conclusions;
- Write a ratio as a fraction;
- Express a multiplicative relationship between two quantities as a ratio or a fraction.

Higher tier (Units 4b, 11)

- Divide a given quantity into two or more parts in a given part : part or part : whole ratio;
- Use a ratio to find one quantity when the other is known;
- Write a ratio as a fraction and as a linear function;
- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when A : B are in the ratio 3 : 5, A is $\frac{3}{5}B$. When $4a = 7b$, then $a = \frac{7b}{4}$ or a : b is 7 : 4.

Sample questions

Living to 100 years old

1 in 3 babies born last year
are expected to live
to 100 years old

720 000 babies were born last year.

How many of these babies are expected to live to 100 years old?

(Total 2 marks)

Specimen Papers Set 2, Paper 2F qu.5 (N11 – AO1/AO3)

A and B are two companies.

The table shows some information about the sales of each company and the number of workers for each company in 2004 and in 2014

	Company A		Company B	
	Sales (£ millions)	Number of workers	Sales (£ millions)	Number of workers
2004	320	2960	48	605
2014	388	3200	57	640

(*a*) Work out the percentage increase in sales from 2004 to 2014 for Company A.

(2)

(*b*) Which company had the most sales per worker in 2014, Company A or Company B?
You must show how you get your answer.

(3)

(Total 5 marks)

New SAMs Paper 3F qu.23 (N11, R9, N2 – AO1/AO2)

Exam Wizard topics

Ratio
Proportion

N12 interpret fractions and percentages as operators

Content guidance

E.g. interpret $\frac{2}{5}$ of 40 as $\frac{2}{5} \times 40$; interpret 20% of 40 as 0.2×40.

Mapping to 1MA0 specification content descriptors

Nm (F) Use percentage
Nm (H) Use percentage, repeated proportional change
No (F/H) Interpret fractions, decimals and percentages as operators

Learning objectives

Foundation tier (Units 4a, 4b, 14)

- Multiply and divide a fraction by an integer, including finding fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;
- Find a percentage of a quantity or measurement: without a calculator (50%, 25% and multiples of 10% and 5%) and with a calculator; calculate amount of increase/decrease;
- Use percentages to solve problems, including comparisons of two quantities and tax, profit and loss problems;
- Use percentages in real-life situations, including percentages greater than 100%;
- Use a multiplier to find a percentage of a quantity or an increase or decrease by a percentage in any scenario where percentages are used; find the original amount given the final amount after a percentage increase or decrease;
- Make calculations involving repeated percentage change, not using the formula; use compound interest;
- Understand the multiplicative nature of percentages as operators.

Higher tier (Units 4a, 4b, 11)

- Find a fraction of a quantity or measurement, including within a context;
- Find a percentage of a quantity; a percentage increase or decrease including: simple interest, income tax calculations, value of profit or loss, percentage profit or loss; the new amount after a percentage increase or decrease;
- Compare two quantities using percentages, including a range of calculations and contexts such as those involving time or money;
- Find a percentage of a quantity using a multiplier and use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Represent repeated proportional change as a single decimal multiplier or as a multiplier raised to a power; use this to solve problems involving compound interest and depreciation;
- Find the original amount given the final amount after a percentage increase or decrease (reverse percentages), including VAT; use calculators for reverse percentage calculations by doing an appropriate division;
- Use percentages in real-life situations, including percentages greater than 100%;
- Describe percentage increase/decrease with fractions, e.g. 150% increase means $2\frac{1}{2}$ times as big.

Sample questions

Work out 15% of 80

(Total 2 marks)
New SAMs Paper 1F qu.8 (N12 – AO1)

Ryan and Carl each get paid a basic pay of £60 per day.

One day, Ryan also gets a bonus of 25% of his basic pay.
Carl also gets £20 in tips from customers.

Work out the difference between the total amounts of money that Ryan and Carl each get.

(Total 3 marks)
Specimen Papers Set 2, Paper 1F qu.13 (N12, N2, R9 – AO1/AO3)

Katy invests £2000 in a savings account for 3 years.

The account pays compound interest at an annual rate of

2.5% for the first year

x % for the second year

x % for the third year

There is a total amount of £2124.46 in the savings account at the end of 3 years.

(*a*) Work out the rate of interest in the second year.

(4)

Katy goes to work by train.

The cost of her weekly train ticket increases by 12.5% to £225

(*b*) Work out the cost of her weekly train ticket before this increase.

(2)

(Total 6 marks)
New SAMs Paper 2H qu.10 (N12, N1, R9 – AO1/AO3)

Exam Wizard topics

Fractions
Percentages

Measures and accuracy

N13	use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

Content guidance

Standard compound measure:
- speed (e.g. m/s, miles per hour),
- acceleration (e.g. m/s^2),
- density (e.g. g/cm^3),
- pressure (e.g. N/m^2).

New to Foundation tier

Using compound measures such as density and pressure (speed already in Foundation tier).

Mapping to 1MA0 specification content descriptors

GMo (F/H) Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GMs (F/H) Understand and use compound measures

Learning objectives

Foundation tier (Units 9a, 11a, 11b, 14)

- Draw straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bill graphs, fixed charge and cost per unit; interpret information presented in a range of linear and non-linear graphs, including graphs with negative values;
- Work out time intervals for graph scales;
- Draw distance–time graphs and velocity–time graphs; interpret gradient as the rate of change; interpret distance–time graphs and calculate: the speed of individual sections, total distance and total time;
- Solve a ratio problem in context, using a variety of measures; share a quantity in a given ratio including three-part ratios; find one quantity when the other is known; compare a scale model to a real-life object; convert between measures and currencies; in problems involving currency conversion, rates of pay, best value, mixing, e.g. paint colours, cement and drawn conclusions; scale up recipes;
- Write lengths, areas and volumes of two shapes as ratios in simplest form;
- Understand and use compound measures: density, pressure, speed including converting between metric speed measures; reading values in km/h and mph from a speedometer; calculating average speed, distance, time – in miles per hour as well as metric measures; using kinematics formulae from the formulae sheet to calculate speed, acceleration (with variables defined in the question); changing d/t in m/s to a formula in km/h, i.e. $d/t \times (60 \times 60) \div 1000$ with support.

Higher tier (Units 6a, 11)

- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bill graphs, fixed charge and cost per unit;
- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Work out which product offers best value and consider rates of pay;
- Understand and use compound measures and convert between: metric speed measures, density measures, pressure measures;
- Use kinematics formulae from the formulae sheet to calculate speed, acceleration etc. (with variables defined in the question).

Sample questions

Faiza buys

one magazine costing £2.30
one paper costing 92p
two identical bars of chocolate

Faiza pays with a £5 note.
She gets 40p change.

Work out the cost of **one** bar of chocolate.

(Total 3 marks)
New SAMs Paper 3F qu.5 (N13, N2 – AO1/AO3)

Three companies sell the same type of furniture.

The price of the furniture from Pooles of London is £1480
The price of the furniture from Jardins of Paris is €1980
The price of the furniture from Outways of New York is $2250

The exchange rates are

£1 = €1.34

£1 = $1.52

Which company sells this furniture at the lowest price?
You must show how you get your answer.

(Total 3 marks)
Specimen Papers Set 1, Paper 2F qu.21 / 2H qu.2 (N13, N2 – AO3)

Exam Wizard topics

Money calculations
Interpret and estimate measures
Conversions
Compound measures
Time calculations
New to Foundation from Higher in GCSE (9–1)

N14	estimate answers; check calculations using approximation and estimation, including answers obtained using technology

Mapping to 1MA0 specification content descriptors

Nu (F/H) Approximate to specified or appropriate degrees of accuracy including a given power of 10, number of decimal places and significant figures

Learning objectives

Foundation tier (Units 1a, 1b, 8)

- Check answers by rounding and using inverse operations;
- Estimate answers to calculations by rounding numbers to 1 significant figure;
- Estimate surface areas and volumes by rounding measurements to 1 significant figure.

Higher tier (Units 1a, 1b, 7a, 7b)

- Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals;
- Use the square, cube and power keys on a calculator and estimate powers and roots of any given positive number by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
- Estimate area, perimeter, surface area and volume by rounding measurements to 1 significant figure to check reasonableness of answers.

Sample questions

Jayne writes down the following

$$3.4 \times 5.3 = 180.2$$

Without doing the exact calculation, explain why Jayne's answer cannot be correct.

(Total 1 mark)

Specimen Papers Set 1, Paper 3F qu.8 (N14 – AO3)

Paul organised an event for a charity.

Each ticket for the event cost £19.95
Paul sold 395 tickets.

Paul paid costs of £6000
He gave all money left to the charity.

(*a*) Work out an estimate for the amount of money Paul gave to the charity.

(3)

(*b*) Is your answer to (*a*) an underestimate or an overestimate?
Give a reason for your answer.

(1)

(Total 4 marks)
New SAMs Paper 1F qu.13 (N14, R10, N2 – AO1/AO3)

One uranium atom has a mass of 3.95×10^{-22} grams.

(*a*) Work out an estimate for the number of uranium atoms in 1 kg of uranium.

(3)

(*b*) Is your answer to (*a*) an underestimate or an overestimate?
Give a reason for your answer.

(1)

(Total 4 marks)
New SAMs Paper 1H qu.11 (N14, N9, R1, R10 – AO1/AO3)

Exam Wizard topics

Approximation; estimation; rounding

N15	round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

Content guidance

E.g. $x = 2.3$ correct to 2 s.f. implies that $2.25 \leq x < 2.35$.

If a piece of wood has been measured as 7 cm to the nearest whole number how long could the piece of wood actually be?
(Answer: $6.5 \leq L < 7.5$)

Jim used his calculator to work out the value of a number x. He wrote down the first two digits of the answer on his calculator.
He wrote down 4.6.
Write down the error interval for x.
(Answer: $4.6 \leq x < 4.7$)

Or (more suitable for higher tier):

Jim truncates to 1 digit a number x. The result is 5.
Write down the error interval for x.
(Answer $5 \leq x < 6$)

New to GCSE (9–1) Maths

Using inequality notation to specify simple error intervals due to truncation or rounding.

Mapping to 1MA0 specification content descriptors

Nu (F/H) Approximate to specified or appropriate degrees of accuracy, including a given power of 10, number of decimal places and significant figures

Learning objectives

Foundation tier (Units 1a, 1b, 5a, 12, 17)

- Round numbers to a given power of 10, the nearest integer or a given number of decimal places and significant figures;
- Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question.
- Use inequality notation to specify simple error intervals due to truncation or rounding.

Higher tier (Units 1a, 7c)

- Round numbers to the nearest 10, 100, 1000, the nearest integer, to a given number of decimal places and to a given number of significant figures;
- Use inequality notation to specify an error bound.

Sample questions

Write 2148 correct to the nearest 100

(Total 1 mark)
New SAMs Paper 3F qu.1 (N15 – AO1)

The length, L cm, of a line is measured as 13 cm correct to the nearest centimetre.

Complete the following statement to show the range of possible values of L.

..... $\leq L <$ .....

(Total 2 marks)

Specimen Papers Set 1, Paper 3F qu.22 / 3H qu.2 (N15 – AO1)

Jim rounds a number, x, to one decimal place.
The result is 7.2.

Write down the error interval for x.

(Total 2 marks)

Specimen Papers Set 2, Paper 3F qu.25 / 3H qu.4 (N15 – AO1)

Lyn measures the length, x cm, of a piece of string as 3.5 cm correct to the nearest millimetre.

Write down the error interval for x.

(Total 2 marks)

Mock Papers Set 2, Paper 2F qu.20 (N15 – AO1)

A number, y, is rounded to 2 significant figures.

The result is 0.46.

Write down the error interval for y.

(Total 2 marks)

Mock Papers Set 1, Paper 1F qu.23 (N15 – AO1)

Kiera used her calculator to work out the value of a number x.
She wrote down the first two digits of the answer on her calculator.

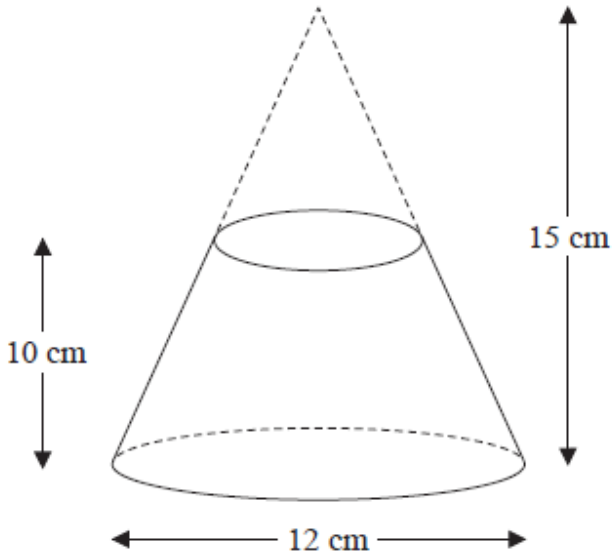
She wrote down 7.3

Write down the error interval for x.

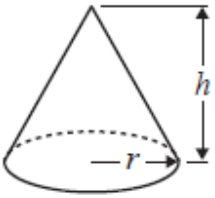
(Total 2 marks)

Mock Papers Set 3, Paper 3H qu.8 (N15 – AO1)

A frustum is made by removing a small cone from a large cone as shown in the diagram.



Volume of cone $= \frac{1}{3}\pi r^2 h$



The frustum is made from glass.
The glass has a density of 2.5 g/cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

(Total 5 marks)

New SAMs Paper 2H qu.22 (N15, G17, G19, R11 – AO1/AO3)

Exam Wizard topics

Approximation; estimation; rounding
New to GCSE (9–1)

N16 apply and interpret limits of accuracy, **including upper and lower bounds**

Content guidance

E.g. a gap between two cupboards is 0.90 m correct to the nearest centimetre: is it possible that a cupboard with a width of 90.4 cm will fit into this gap?

Mapping to 1MA0 specification content descriptors

Ns (H) Calculate upper and lower bounds
GMo (F/H) Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GMs (F/H) Understand and use compound measures

Learning objectives

Foundation tier (Unit 5a)

- Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question..

Higher tier (Unit 7c)

- Calculate the upper and lowers bounds of numbers given to varying degrees of accuracy;
- Calculate the upper and lower bounds of calculations and expressions involving the four operations, particularly when working with measurements and including perimeters, areas and volumes of 2D and 3D shapes;
- Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
- Use inequality notation to specify an error bound.

Sample questions

$m = \dfrac{\sqrt{s}}{t}$ s = 3.47 correct to 3 significant figures

t = 8.132 correct to 4 significant figures

By considering bounds, work out the value of m to a suitable degree of accuracy.
Give a reason for your answer.

(Total 5 marks)
New SAMs Paper 2H qu.18 (N16 – AO1/AO2)

Exam Wizard topics

Bounds
Interpret and estimate measures

2. Algebra

Notation, vocabulary and manipulation

A1 use and interpret algebraic manipulation, including:
- ab in place of $a \times b$
- $3y$ in place of $y + y + y$ and $3 \times y$
- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
- $\frac{a}{b}$ in place of $a \div b$
- coefficients written as fractions rather than as decimals
- brackets

Mapping to 1MA0 specification content descriptors

Aa (F/H) Distinguish the different roles played by letter symbols in algebra, using the correct notation

Learning objectives

Foundation tier (Unit 2a)

- Use notation and symbols correctly.

Higher tier (Unit 2a)

- Use algebraic notation and symbols correctly.

Sample questions

(*a*) Simplify $5f - f + 2f$

(1)

(*b*) Simplify $2 \times m \times n \times 8$

(1)

(*c*) Simplify $t^2 + t^2$

(1)

(Total 3 marks)

Specimen Papers Set 1, Paper 2F qu.4 (A1, A4 – AO1)

Prove algebraically that

$(2n + 1)^2 - (2n + 1)$ is an even number

for all positive integer values of n.

(Total 3 marks)

New SAMs Paper 2H qu.14 (A1, A4 – AO1/AO2)

Exam Wizard topics

Simplify expressions

A2	substitute numerical values into formulae and expressions, including scientific formulae

Content guidance

Numerical values could be given in any form (integer, decimal, fraction) or given in standard form.

Mapping to 1MA0 specification content descriptors

Af (F/H) Derive a formula, substitute numbers into a formula and change the subject of a formula

Learning objectives

Foundation tier (Units 2b, 5a)

- Substitute positive and negative numbers into simple algebraic expressions and expressions involving brackets and powers;
- Substitute numbers into a (word) formula;
- Substitute into a formula and solve the resulting equation.

Higher tier (Units 2a)

- Substitute positive and negative numbers into expressions such as $3x + 4$ and $2x^3$ and then into expressions involving brackets and powers;
- Substitute positive and negative numbers into a formula, solve the resulting equation, including brackets, powers or standard form;
- Use and substitute formulae from mathematics and other subjects including simple linear formulae, e.g. $l \times w$, and the kinematics formulae $v = u + at$, $v^2 - u^2 = 2as$ and $s = ut + \frac{1}{2}at^2$.

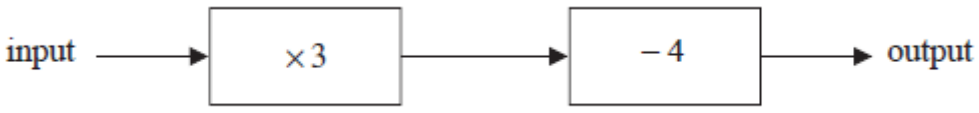
Sample questions

$f = 5x + 2y$
$x = 3$ and $y = -2$

Find the value of f.

(Total 2 marks)
New SAMs Paper 2F qu.18 (A2 – AO1)

Here is a number machine.



(*a*) Work out the **output** when the input is 4

(1)

(*b*) Work out the **input** when the output is 11

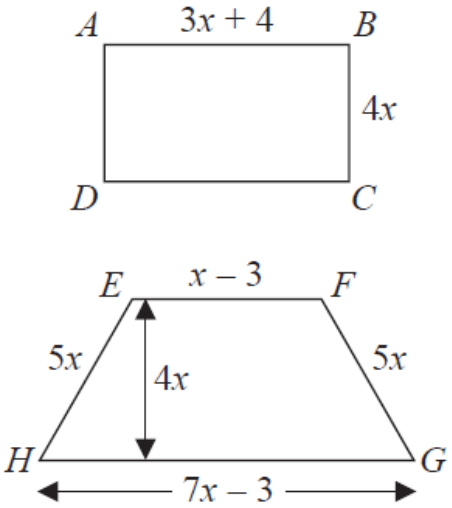
(2)

(*c*) Show that there is a value of the input for which the input and the output have the same value.

(2)

(Total 5 marks)
New SAMs Paper 3F qu.7 (A2, A7, A17 – AO1/AO2)

ABCD is a rectangle.
EFGH is a trapezium.



All measurements are in centimetres.
The perimeters of these two shapes are the same.

Work out the area of the rectangle.

(Total 5 marks)
New SAMs Paper 2H qu.9 (A2, A4, A17, A21 – AO1/AO3)

Exam Wizard topics

Substitute into expressions and formulae

A3	understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

Content guidance

Examiners do not anticipate using the identity symbol on Foundation tier papers.

Pick a word from the list of *equation*, *formula*, *identity* that would best describe
(i) $3x + 5 = 12$
(ii) $2x + 4 = 2(x + 2)$

New to Foundation tier

Recognising identities and knowing the difference between an equation and an identity.

Mapping to 1MA0 specification content descriptors

Ab (F) Distinguish in meaning between the words 'equation', 'formula' and 'expression'
Ab (H) Distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression'

Learning objectives

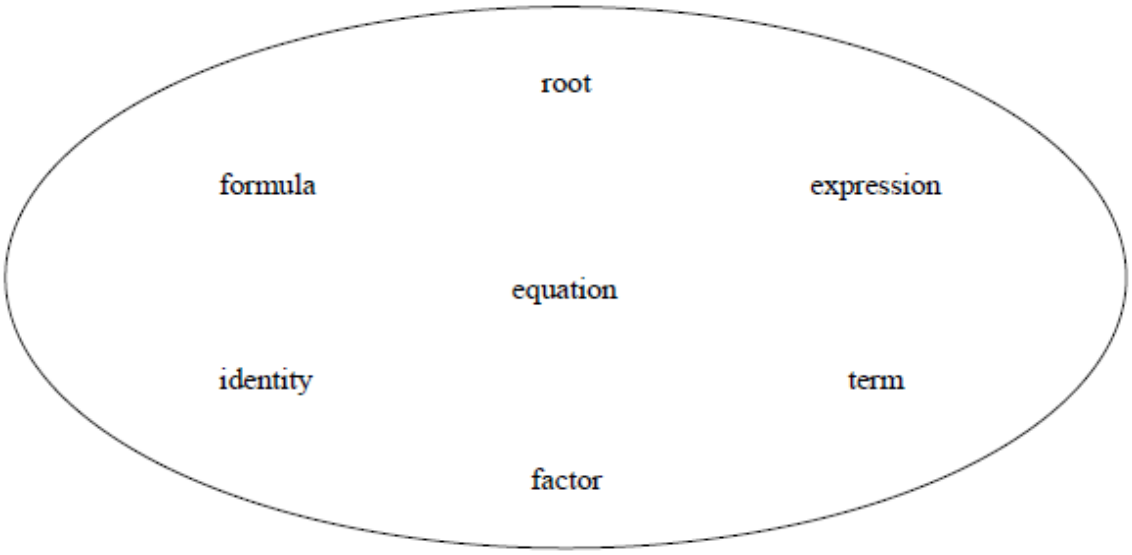
Foundation tier (Units 2a, 5a, 20)

- Select an expression/equation/formula/identity from a list;
- Know the difference between an equation and an identity, and use and understand the ≠ and identity ≡ symbols.

Higher tier (Unit 2a)

- Know the difference between a term, expression, equation, formula and identity;
- Understand the ≠ symbol (not equal), e.g. $6x + 4 \neq 3(x + 2)$, and introduce identity ≡ sign;
- Simple proofs and use of ≡ in 'show that' style questions; know the difference between an equation and an identity.

Sample questions



Choose a word from those above that makes this statement correct.

(*a*) x^2 is a ..... in $x^2 + 4y$

(1)

Choose a word from those above that makes this statement correct.

(*b*) $(y + 2)$ is a ..... of $3y + 6$

(1)

(Total 2 marks)
Original SAMs Paper 2F qu.9 (A3 – AO1)

Exam Wizard topics

Derive expressions; equations; formulae
New to Foundation from Higher in GCSE (9–1)

A4 simplify and manipulate algebraic expressions (including those involving surds **and algebraic fractions**) by:
- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two **or more** binomials
- factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **factorising quadratic expressions of the form $ax^2 + bx + c$**
- simplifying expressions involving sums, products and powers, including the laws of indices

Content guidance

Will be limited to expanded products of **three** binomials (i.e. cubics).

New to GCSE (9–1) Maths

Expanding the products of more than two binomials.

New to Foundation tier

Using zero and negative powers for index laws. (NB: fractional integers are still Higher tier only.)

Expanding the product of two linear expressions.

Factorising quadratic expressions in the form $x^2 + bx + c$. (NB: expressions in the form $ax^2 + bx + c$ are still Higher tier only.)

Simplifying and manipulating algebraic expressions that include surds.

Mapping to 1MA0 specification content descriptors

Ac (F) Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
Ac (H) Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions

Learning objectives

Foundation tier (Units 2a, 2b, 12, 16a)

- Manipulate and simplify algebraic expressions by collecting 'like' terms, including squares and cubes; by cancelling, e.g. $\frac{4x}{2} = 2x$; involving brackets, i.e. expand the brackets, then add/subtract;
- Multiply together two simple algebraic expressions, e.g. $2a \times 3b$; a single number term over a bracket; two algebraic expressions with brackets, including squaring a linear expression, e.g. $(x + 1)^2$;
- Use index notation and the index laws when multiplying or dividing algebraic terms;
- Factorise algebraic expressions by taking out common factors with single brackets; factorise quadratic expressions of the form $x^2 + bx + c$ and $x^2 – a^2$ using the difference of two squares;
- Define a 'quadratic' expression;
- Understand, recall and use Pythagoras' theorem in 2D, including leaving answers in surd form and being able to justify if a triangle is right-angled or not.

Higher tier (Units 1c, 2a, 9a, 15, 17)

- Write, manipulate and simplify algebraic expressions by collecting 'like' terms; by cancelling, e.g. $\frac{4x}{2} = 2x$;
- Multiply a single number term over a bracket; two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to $(2x + 3y)(3x – y)$;
- Know that squaring a linear expression is the same as expanding double brackets;
- Factorise algebraic expressions by taking out common factors with single brackets, including subsequently collecting like terms; factorise quadratic expressions of the form $x^2 + bx + c$ and $x^2 – a^2$ using the difference of two squares;
- Use instances of index laws: for positive integer powers including when multiplying or dividing algebraic terms; use of zero, fractional and negative powers;
- Simplify, multiply and divide algebraic fractions;
- Expand the product of more than two linear expressions.

Sample questions

(a) Simplify $8x – 3x + 2x$

(1)

(b) Simplify $4y \times 2y$

(1)

(Total 2 marks)
New SAMs Paper 3F qu.2 (A4 – AO1)

(a) Factorise $3f + 9$

(1)

(b) Factorise $x^2 – 2x – 15$

(2)

(Total 3 marks)
New SAMs Paper 2F qu.20 (A4 – AO1)

Expand and simplify $(m + 7)(m + 3)$

(Total 2 marks)
New SAMs Paper 1F qu.22 / 1H qu.2 (A4 – AO1)

Show that

$$(3x – 1)(x + 5)(4x – 3) = 12x^3 + 47x^2 – 62x + 15$$

for all values of x.

(Total 3 marks)
Specimen Papers Set 1, Paper 2H qu.13 (A4, A6 – AO2)

Expand and simplify $(x + 2)(x + 8)(x – 4)$

(Total 3 marks)
Mock Papers Set 3, Paper 3H qu.11 (A4 – AO1)

Expand and simplify $(x + 2)(2x – 3)(3x + 1)$

(Total 3 marks)
Mock Papers Set 2, Paper 3H qu.13a (A4 – AO1)

Martin expands $(2x + 1)(2x − 3)(3x + 2)$

He gets $12x^3 − 4x^2 − 17x + 6$

Explain why Martin's solution cannot be correct.

(Total 1 mark)
Mock Papers Set 1, Paper 1H qu.16b (A4 – AO3)

Show that $\frac{1}{6x^2+7x-5} \div \frac{1}{4x^2-1}$ simplifies to $\frac{ax+b}{cx+d}$ where a, b, c and d are integers.

(Total 3 marks)
New SAMs Paper 2H qu.16 (A4 – AO2)

Exam Wizard topics

Simplify expressions
Expand expressions
Factorise expressions
Algebraic fractions
New to GCSE (9–1)
New to Foundation from Higher in GCSE (9–1)

A5	understand and use standard mathematical formulae; rearrange formulae to change the subject

Content guidance

The rearrangement of formulae where the intended subject appears twice (and so needs to be taken out as a common factor) will be tested at Higher tier only.

New to Foundation tier

Changing the subject of the formula where the subject appears on both sides, or with a power of the subject.

Mapping to 1MA0 specification content descriptors

Af (F/H) Derive a formula, substitute numbers into a formula and change the subject of a formula

Learning objectives

Foundation tier (Units 2b, 5a, 8, 12, 14, 17, 20)

- Derive a simple formula, including those with squares, cubes and roots;
- Rearrange simple equations;
- Substitute into a formula and solve the resulting equation;
- Recall and use the formulae for the area of a triangle, rectangle and trapezium; the volume of a cuboid;
- Understand, recall and use Pythagoras' theorem in 2D, including leaving answers in surd form and being able to justify if a triangle is right-angled or not;
- Understand and use compound measures, including speed: use given kinematics formulae to calculate speed, acceleration (with variables defined in the question); changing d/t in m/s to a formula in km/h, i.e. d/t × (60 × 60) ÷ 1000 with support;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; the surface area and volume of a cylinder sphere, pyramid, cone and composite solids;
- Change the subject of a formula involving the use of square roots and squares.

Higher tier (Units 2a, 5b, 7a, 7b, 9a, 13b, 17)

- Use and substitute formulae from mathematics and other subjects including simple linear formulae, e.g. $l \times w$, and the kinematics formulae $v = u + at$, $v^2 - u^2 = 2as$, and $s = ut + \frac{1}{2}at^2$;
- Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram, including to find surface areas of prisms and the circumference and area of a circle, using a variety of metric measures;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures;
- Find the volume and surface area of a cylinder, pyramid, sphere and cone;
- Solve quadratic equations by using the quadratic formula;
- Know and apply area = $\frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle, and the sine and cosine rules, and use to solve 2D problems (including involving bearings);
- Understand, recall and use trigonometric relationships and Pythagoras' theorem in right-angled triangles, and use these to solve 2D and 3D problems;
- Change the subject of a simple formula, i.e. linear one-step, such as $x = 4y$; a more complex formula, including cases where the subject occurs on both sides of the formula, or where a power of the subject appears; a formula such as $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where all variables are in the denominators.

Sample questions

$q = \frac{p}{r} + s$

Make p the subject of this formula.

(Total 2 marks)
New SAMs Paper 2F qu.21 (A5 – AO1)

Make a the subject of $a + 3 = \frac{2a + 7}{r}$

(Total 3 marks)
New SAMs Paper 1H qu.17 (A5 – AO1)

Exam Wizard topics

Derive expressions; equations; formulae
Manipulate expressions and formulae
New to Foundation from Higher in GCSE (9–1)

A6	know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

Mapping to 1MA0 specification content descriptors

Ab (H) Distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression'
Ac (F) Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
Ac (H) Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions

Learning objectives

Foundation tier (Units 2b, 20)

- Know the difference between an equation and an identity, and use and understand the ≠ symbol;
- Argue mathematically to show algebraic expressions are equivalent;
- Answer 'show that' questions using consecutive integers (n, $n + 1$), squares a^2, b^2, even numbers $2n$, and odd numbers $2n + 1$.

Higher tier (Units 2a, 17)

- Simple proofs and use of ≡ in 'show that' style questions; know the difference between an equation and an identity;
- Solve 'show that' and proof questions using consecutive integers (n, $n + 1$), squares a^2, b^2, even numbers $2n$, odd numbers $2n + 1$.

Sample questions

The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

(Total 3 marks)
Specimen Papers Set 2, Paper 3H qu.17 (A6 – AO2)

Exam Wizard topics

Manipulate expressions and formulae
Algebraic proofs

A7	where appropriate, interpret simple expressions as functions with inputs and outputs; **interpret the reverse process as the 'inverse function'**; **interpret the succession of two functions as a 'composite function' (the use of formal function notation is expected)**

Content guidance

Candidates could be asked to produce the graph of a function or an inverse function. It is possible that this could then be linked into a geometrical interpretation.

Candidates will be expected to use notation $f^{-1}(x)$ for work on inverse functions and $gf(x)$ for work on composite functions.

New to GCSE (9–1) Maths

Interpreting the reverse process as the 'inverse function'.

Interpreting the succession of two functions as a 'composite function'.

Mapping to 1MA0 specification content descriptors

Af (F) Derive a formula, substitute numbers into a formula and change the subject of a formula

Learning objectives

Foundation tier (Units 5a, 5b, 9a, 9b)

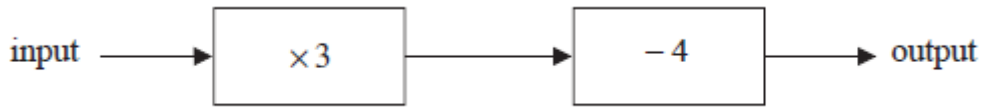
- Use function machines, including to find terms of a sequence; coordinates (i.e. given the input x, find the output y);
- Use input/output diagrams.

Higher tier (Units 2a, 17)

- Use function notation;
- Find $f(x) + g(x)$ and $f(x) – g(x)$, $2f(x)$, $f(3x)$ etc. algebraically;
- Find the inverse of a linear function; know that $f^{-1}(x)$ refers to the inverse function;
- For two functions $f(x)$ and $g(x)$, find $gf(x)$.

Sample questions

Here is a number machine.



(*a*) Work out the **output** when the input is 4

(1)

(*b*) Work out the **input** when the output is 11

(2)

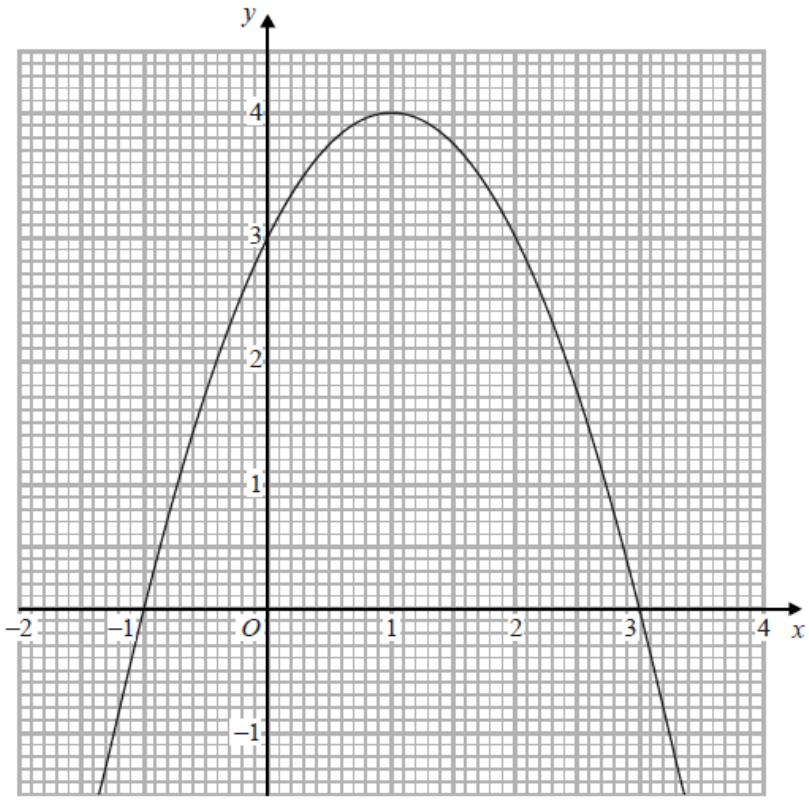
(*c*) Show that there is a value of the input for which the input and the output have the same value.

(2)

(Total 5 marks)

New SAMs Paper 3F qu.7 (A7, A17, A2 – AO1/AO2)

The graph of $y = f(x)$ is drawn on the grid.



(*a*) Write down the coordinates of the turning point of the graph.

(1)

(*b*) Write down the roots of $f(x) = 2$

(1)

(*c*) Write down the value of $f(0.5)$

(1)

(Total 3 marks)

New SAMs Paper 2H qu.7 (A7, A11 – AO2)

The functions f and g are such that

$$f(x) = 3(x - 4) \quad \text{and} \quad g(x) = \frac{x}{5} + 1$$

(*a*) Find the value of $f(10)$

(1)

(*b*) Find $g^{-1}(x)$

(2)

(*c*) Show that $ff(x) = 9x - 48$

(2)

(Total 5 marks)

Specimen Papers Set 2, Paper 2H qu.9 (A7 – AO1/AO2)

The function f is such that

$$f(x) = 4x - 1$$

(*a*) Find $f^{-1}(x)$

(2)

The function g is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that fg(2) = 12

(*b*) work out the value of *k*

(2)

(Total 4 marks)
New SAMs Paper 3H qu.10 (A7 – AO1/AO3)

$f(x) = 3x^2 - 2x - 8$

Express $f(x + 2)$ in the form $ax^2 + bx$

(Total 3 marks)
Specimen Papers Set 1, Paper 2H qu.18 (A7 – AO1/AO2)

For all values of *x*

$$f(x) = 2x - 3 \quad \text{and} \quad g(x) = x^2 + 2$$

(*a*) Find g(−4)

(1)

(*b*) Show that $gf(x) = 4x^2 - 12x + 11$

(2)

(*c*) Solve $fg(x) = gf(x)$

(4)

(Total 7 marks)
Mock Papers Set 1, Paper 3H qu.20 (A7, A4, A18 – AO1/AO2/AO3)

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f and g are functions such that

$$f(x) = 3x^2 \text{ and } g(x) = \frac{1}{x-2}$$

Find gf(4).
Give your answer as a fraction.

(Total 2 marks)
Mock Papers Set 2, Paper 2H qu.21 (A7 – AO1)

$f(x) = x^3$
$g(x) = 4x - 1$

(*a*) Find fg(2)

(2)

$h(x) = fg(x)$

(*b*) Find an expression for $h^{-1}(x)$

$h^{-1}(x) =$ .....

(3)

(Total 5 marks)

Mock Papers Set 3, Paper 3H qu.21 (A7 – AO1)

Exam Wizard topics

Substitute into expressions and formulae
Functions and function notation
New to GCSE (9–1)

Graphs

A8	work with coordinates in all four quadrants

Content guidance

To include finding the midpoint of a line joining two coordinates.

Mapping to 1MA0 specification content descriptors

Ak (F/H) Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

Learning objectives

Foundation tier (Units 6a, 9a)

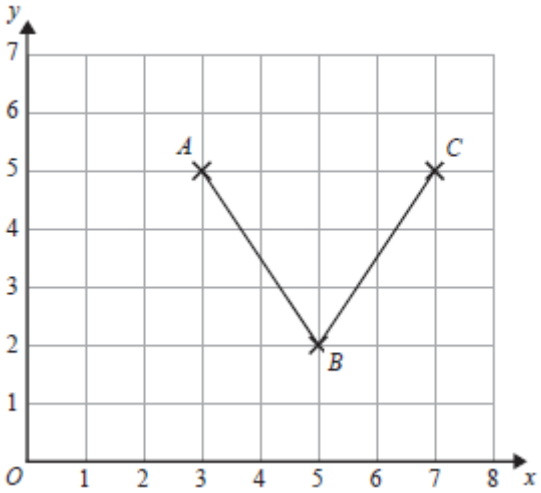
- Given some information about a shape on coordinate axes, complete the shape;
- Draw, label and scale axes, and use axes and coordinates to specify points in all four quadrants in 2D;
- Identify points with given coordinates and coordinates of a given point in all four quadrants;
- Find the coordinates of points identified by geometrical information in 2D (all four quadrants);
- Find the coordinates of the midpoint of a line segment; read values from straight-line graphs for real-life situations.

Higher tier (Unit 6a)

- Identify and plot points in all four quadrants;
- Find the coordinates of the midpoint of a line segment with given diagram and coordinates, and from coordinates only;
- Calculate the length of a line segment given the coordinates of the end points;
- Find the coordinates of points identified by geometrical information.

Sample questions

Here is a grid showing the points *A*, *B* and *C*.



(*a*) Write down the coordinates of the point *A*.

(1)

(*b*) On the grid, mark with a cross (×) the point (1, 2).
Label this point *D*.

(1)

(*c*) On the grid, mark with a cross (×) a point *E*, so that the quadrilateral *ABCE* is a kite.

(1)

(Total 3 marks)
New SAMs Paper 3F qu.4 (A8 – AO1)

Exam Wizard topics

Coordinates in 2D

A9	plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular** lines; find the equation of the line through two given points or through one point with a given gradient

New to Foundation tier

Using the gradients of parallel lines.

Finding the equation of the line through two given points or through one point with a given gradient.

Mapping to 1MA0 specification content descriptors

Al (F/H) Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
An (H) Understand the gradients of parallel lines

Learning objectives

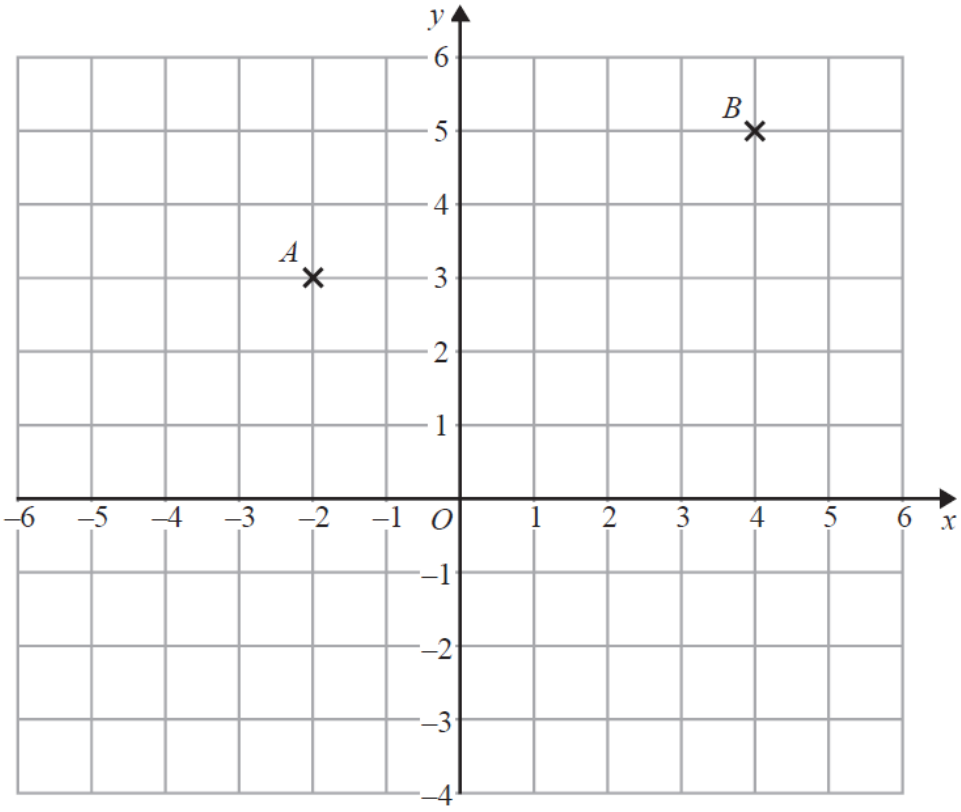
Foundation tier (Units 9a, 9b, 20)

- Plot, draw and recognise graphs of $y = a$, $x = a$; plot and draw graphs of $y = x$ and $y = –x$;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Plot and draw graphs of straight lines of the form $y = mx + c$ and $ax + by = c$ using a table of values;
- Identify and interpret gradient from an equation $y = mx + c$, including identifying parallel lines;
- Draw straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bill graphs, fixed charge and cost per unit;
- Find the equation of a straight line from a graph; through one point with a given gradient; through two given points.

Higher tier (Units 6a, 6b)

- Plot, draw and recognise graphs of $y = a$, $x = a$, $y = x$ and $y = –x$;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Plot and draw graphs of straight lines of the form $y = mx + c$ and with and without a table of values; and of the form $ax + by = c$;
- Find the equation of a straight line from a graph in the form $y = mx + c$ or $ax + by = c$; through one point with a given gradient; through two given points;
- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bill graphs, fixed charge and cost per unit;
- Explore the gradients of parallel lines and lines perpendicular to each other;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line; use the fact that when $y = mx + c$ is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of m and a line perpendicular to this line will have a gradient of $-\frac{1}{m}$.

Sample questions



(*a*) Write down the coordinates of point *B*.

(1)

(*b*) Find the coordinates of the midpoint of *AB*.

(1)

(*c*) On the grid, draw the line with equation $y = -3$

(1)

(Total 3 marks)

Specimen Papers Set 2, Paper 1F qu.5 (A9, A8 – AO1/AO2)

Here are the equations of four straight lines.

Line A $y = 2x + 4$
Line B $2y = x + 4$
Line C $2x + 2y = 4$
Line D $2x - y = 4$

Two of these lines are parallel.

Write down the two parallel lines.

(Total 1 mark)

Specimen Papers Set 1, Paper 3F qu.27 / 3H qu.7 (A9 – AO2)

$A(-2, 1)$, $B(6, 5)$, and $C(4, k)$ are the vertices of a right-angled triangle *ABC*.
Angle *ABC* is the right angle.

Find an equation of the line that passes through *A* and *C*.
Give your answer in the form $ay + bx = c$ where *a*, *b* and *c* are integers.

(Total 5 marks)
New SAMs Paper 1H qu.25 (A9 – AO1/AO3)

Exam Wizard topics

Graphs of linear equations
Gradients of straight lines
New to Foundation from Higher in GCSE (9–1)

A10	identify and interpret gradients and intercepts of linear functions graphically and algebraically

Content guidance

When sketching the graph of a linear function, intercepts with the axes should be shown.

New to Foundation tier

Finding and analysing gradients for graphs in the form $y = mx + c$.

Interpreting and analysing straight-line graphs.

Mapping to 1MA0 specification content descriptors

AI (F/H) Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
Am (H) Understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line and c is the value of the y-intercept

Learning objectives

Foundation tier (Units 9a, 9b, 20)

- Interpret distance–time graphs and calculate: the speed of individual sections, total distance and total time;
- Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs; find the gradient of a straight line from real-life graphs;
- Sketch a graph of a linear function, using the gradient and y-intercept;
- Identify and interpret gradient from equations of the form $y = mx + c$ and $ax + by = c$;
- Identify parallel lines from their equations;
- Find the equation of a straight line from a graph.

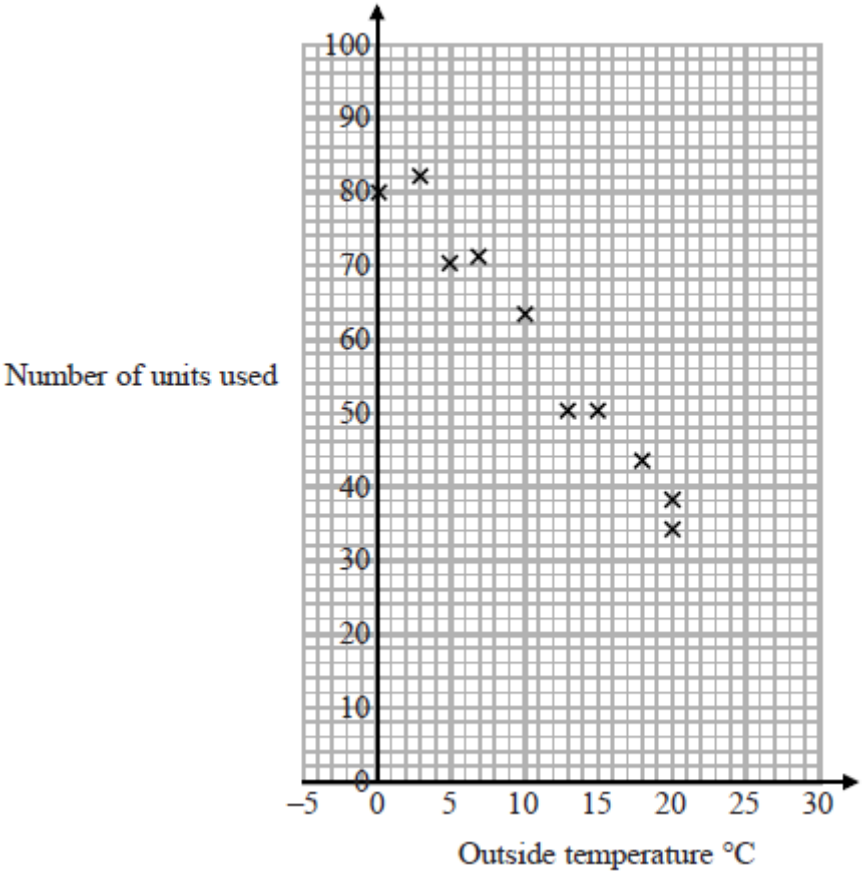
Higher tier (Units 6a, 6b)

- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bill graphs, fixed charge and cost per unit;
- Use graphs to calculate various measures (of individual sections) including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Identify and interpret the gradient of a line segment; the gradient and y-intercept of linear graphs given by equations of the form $y = mx + c$ and $ax + by = c$;
- Find the equation of a straight line from a graph in the form $y = mx + c$;
- Plot, draw and sketch graphs of straight lines of the form $y = mx + c$ with and without a table of values;
- Interpret and analyse information presented in a range of linear graphs: use gradients to interpret how one variable changes in relation to another; find approximate solutions to a linear equation from a graph; identify direct proportion from a graph; find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
- Explore the gradients of parallel lines and lines perpendicular to each other;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line.

Sample questions

In a survey, the outside temperature and the number of units of electricity used for heating were recorded for ten homes.

The scatter diagram shows this information.



Molly says,

> "On average the number of units of electricity used for heating decreases by 4 units for each °C increase in outside temperature."

(*a*) Is Molly right?
Show how you get your answer.

(3)

(*b*) You should **not** use a line of best fit to predict the number of units of electricity used for heating when the outside temperature is 30°C.

Give one reason why.

(1)

(Total 4 marks)

New SAMs Paper 3F qu.21 / 3H qu.4 (A10, S6 – AO2)

Exam Wizard topics

Graphs of linear equations
Gradients of straight lines
New to Foundation from Higher in GCSE (9–1)

A11	identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically **and turning points by completing the square**

Content guidance

The coordinates of the maximum/minimum could be determined either by completing the square or by considerations of symmetry.

No use of calculus is expected. However, if candidates use an AS/A level skill correctly then they will be awarded marks if this is used correctly; partial marks would be awarded for a partially correct answer. The only exception to this could be if a particular method is specified in the question, in which case that method should be used.

Candidates at Higher tier could be asked to complete the square for any quadratic expression of the form $ax^2 + bx + c$. The difficulty of the expression will affect the demand at which the question is set.

New to GCSE (9–1) Maths

The content in A11 is new to GCSE (9–1) Maths.

Learning objectives

Foundation tier (Units 16a, 16b)

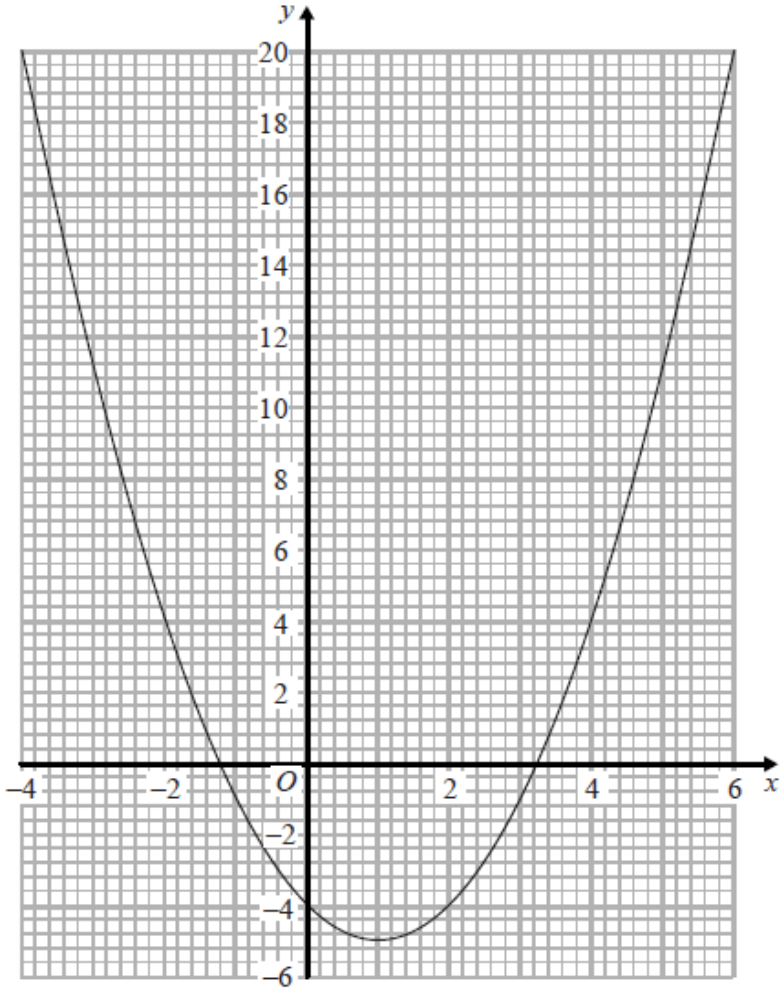
- Solve quadratic equations by factorising;
- Find the roots of a quadratic function algebraically;
- Find approximate solutions to quadratic equations using a graph;
- Identify and interpret roots, intercepts and turning points of quadratic graphs.

Higher tier (Units 6c, 9a, 15)

- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
- Set up and solve quadratic equations, including those that need rearranging, by: factorisation and completing the square; using the quadratic formula;
- Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots and *y*-intercept, turning point;
- Be able to identify from a graph if a quadratic equation has any real roots;
- Find approximate solutions to quadratic equations using a graph.

Sample questions

Here is the graph of $y = x^2 - 2x - 4$



(*a*) Write down estimates for the roots of $x^2 - 2x - 4 = 0$

(2)

(*b*) Write down the coordinates of the turning point of $y = x^2 - 2x - 4$

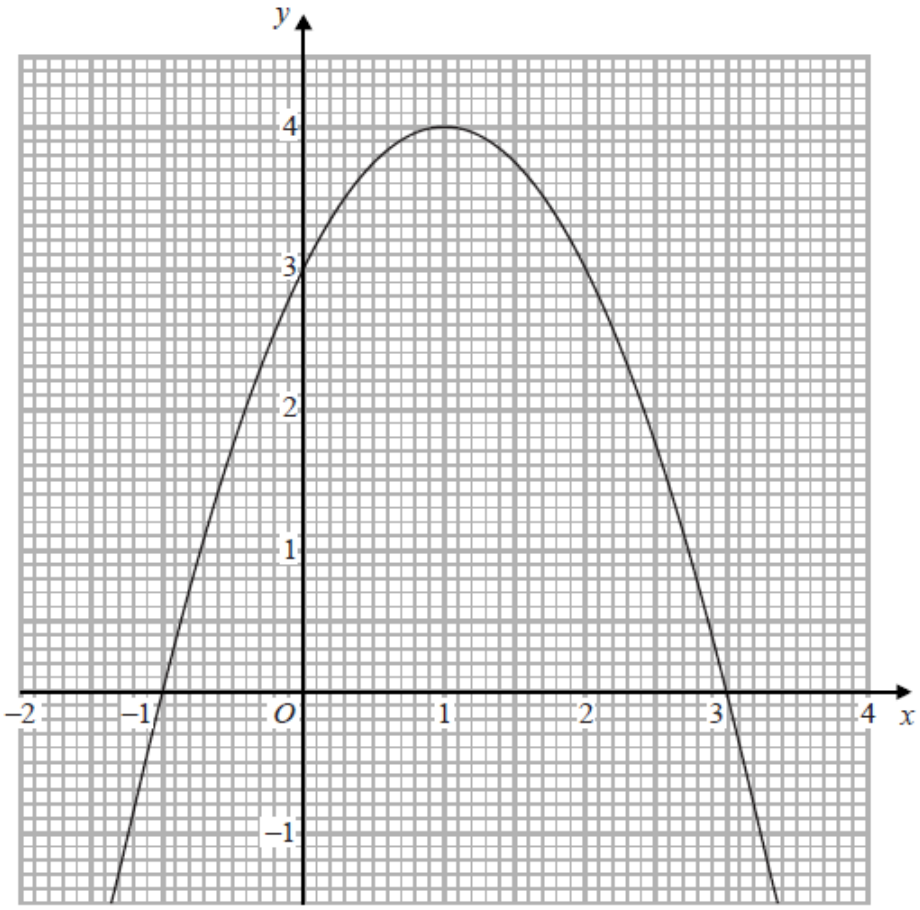
(..... , ...)

(1)

(Total 3 marks)

Mock Papers Set 2, Paper 2F qu.25 / 2H qu.4 (A11 – AO2)

The graph of $y = f(x)$ is drawn on the grid.



(*a*) Write down the coordinates of the turning point of the graph.

(1)

(*b*) Write down the roots of $f(x) = 2$

(1)

(*c*) Write down the value of $f(0.5)$

(1)

(Total 3 marks)

New SAMs Paper 2H qu.7 (A11, A7 – AO2)

(*a*) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

(3)

(*b*) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

(1)

(Total 4 marks)

Specimen Papers Set 2, Paper 3H qu.23 (A11 – AO1)

Exam Wizard topics

Roots, intercepts, turning points in quadratic graphs
New to GCSE (9–1)

A12	recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **exponential functions $y = k^x$ for positive values of k, and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size**

Content guidance

Students will be expected to be able to sketch quadratic functions showing any intercepts with axes and possibly the maximum/minimum as well. For recognition and/or sketching other functions, then the general shape of the graph should be known.

New to Foundation tier

Plotting cubic and reciprocal graphs, and recognising the shapes of quadratic and cubic graphs.

Mapping to 1MA0 specification content descriptors

Ap (H) Draw, sketch and recognise graphs of simple cubic functions, the reciprocal function $y = 1/x$ with $x \neq 0$, the function $y = k^x$ for integer values of x and simple positive values of k, the trigonometric functions $y = \sin x$ and $y = \cos x$
Ar (H) Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs
At (F/H) Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions

Learning objectives

Foundation tier (Units 9b, 16b, 20)

- Sketch a graph of a linear function, using the gradient and y-intercept;
- Find approximate solutions to a linear equation from a graph;
- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Use a quadratic graph to identify the line of symmetry; find approximate solutions; identify and interpret roots, intercepts and turning points;
- Recognise, sketch and interpret graphs of simple cubic functions and the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$.

Higher tier (Units 6b, 6c, 13a, 15, 19a)

- Identify and interpret the gradient and y-intercept of a linear graph given by equations of the form $y = mx + c$;
- Sketch a graph of a linear function, using the gradient and y-intercept (i.e. without a table of values);
- Interpret and analyse information presented in a range of linear graphs: use gradients to interpret how one variable changes in relation to another; find approximate solutions to a linear equation from a graph; identify direct proportion from a graph;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions; sketch a graph of a quadratic function by factorising or by using the formula, identifying roots and y-intercept, turning point;
- Use a quadratic graph to find approximate solutions; to identify if a quadratic equation has any real roots;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points;

- Draw graphs of simple cubic functions using tables of values; sketch graphs of simple cubic functions, given as three linear expressions; interpret graphs of simple cubic functions, including finding solutions to cubic equations;
- Recognise, draw, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$; state the value of x for which the equation is not defined;
- Recognise, sketch and interpret graphs of the trigonometric functions (in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size;
- Recognise, sketch and interpret graphs of exponential functions $y = k^x$ for positive values of k and integer values of x;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient.

Sample questions

In triangle *RPQ*,

RP = 8.7 cm
PQ = 5.2 cm
Angle *PRQ* = 32°

(*a*) Assuming that angle *PQR* is an acute angle,
calculate the area of triangle *RPQ*.
Give your answer correct to 3 significant figures.

(4)

(*b*) If you did not know that angle *PQR* is an acute angle, what effect would this have on your calculation of the area of triangle *RPQ*?

(1)

(Total 5 marks)
New SAMs Paper 2H qu.21 (A12, G23 – AO1/AO3)

Louis and Robert are investigating the growth in the population of a type of bacteria.
They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.
The population of bacteria grows exponentially at the rate of 50% per day.

(*a*) Show that the population of bacteria in flask A at the start of each day forms a geometric progression.

(2)

The population of bacteria in flask A at the start of the 10th day is *k* times the population of bacteria in flask A at the start of the 6th day.

(*b*) Find the value of *k*.

(2)

At the start of day 1 there are 1000 bacteria in flask B.
The population of bacteria in flask B grows exponentially at the rate of 30% per day.

(*c*) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

(1)

(Total 5 marks)
New SAMs Paper 3H qu.17 (A12, R16 – AO1/AO2/AO3)

Exam Wizard topics

Graphs of quadratic equations
Graphs of cubic equations
Graphs of reciprocal functions
Graphs of exponential functions
Graphs of trigonometrical functions
Line graphs of real situations
New to Foundation from Higher in GCSE (9–1)

A13 sketch translations and reflections of a given function

Content guidance

Stretches are *not* on the new specification; transformations are limited to reflections and translations.

E.g. given the graph of $y = f(x)$, sketch the graph of $y = f(x + 2)$.

Mapping to 1MA0 specification content descriptors

Av (H) Transformation of functions

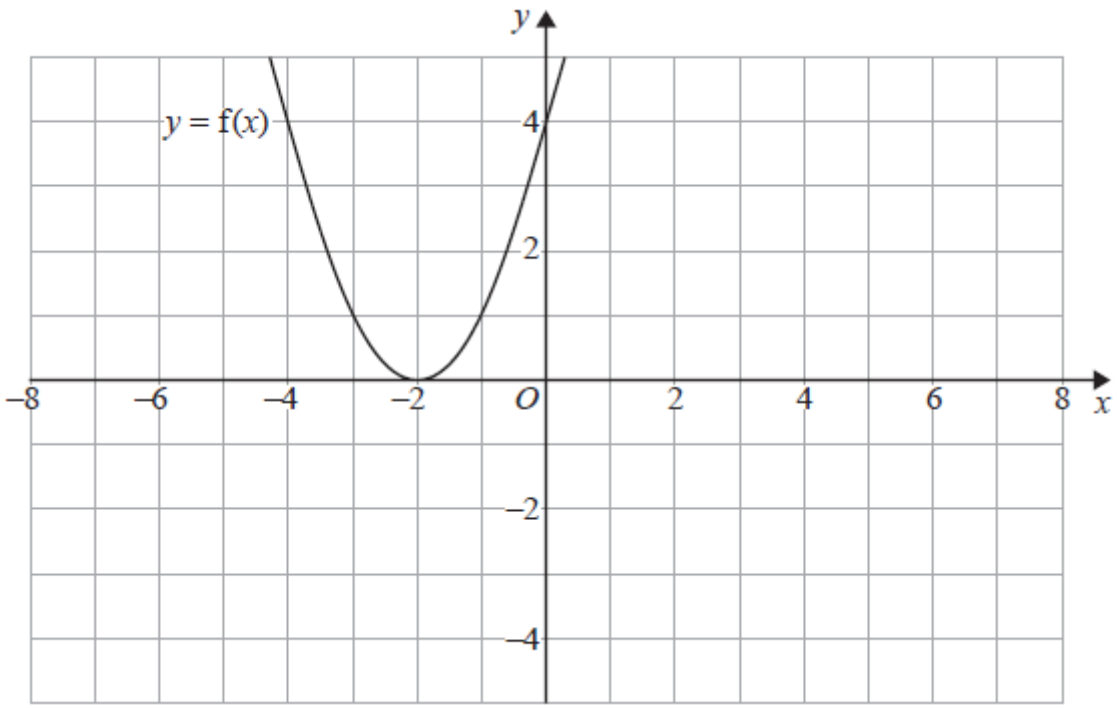
Learning objectives

Higher tier (Units 13a, 19a)

- Apply to the graph of $y = f(x)$ the transformations $y = -f(x)$, $y = f(-x)$, $y = f(x) + a$, $y = f(x + a)$ for sine, cosine and tan functions $f(x)$;
- Interpret and analyse transformations $y = -f(x)$, $y = f(-x)$, $y = f(x) + a$, $y = f(x + a)$ of graphs of linear, quadratic and cubic functions, and write the functions algebraically, e.g. write the equation of $f(x) + a$ or $f(x - a)$.

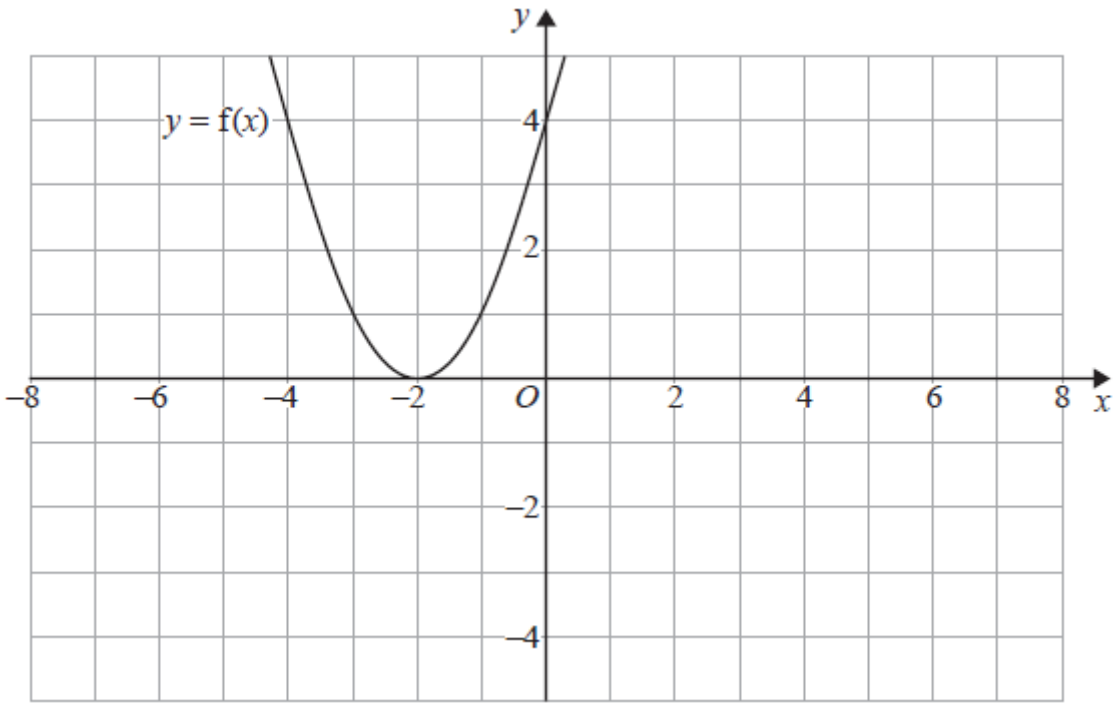
Sample questions

The graph of $y = f(x)$ is shown on both grids below.



(*a*) On the grid above, sketch the graph of $y = f(-x)$

(1)



(*b*) On this grid, sketch the graph of $y = -f(x) + 3$

(1)

(Total 2 marks)

New SAMs Paper 2H qu.19 (A13, A12 – AO2)

The graph of $y = f(x)$ is transformed to give the graph of $y = -f(x + 3)$
The point A on the graph of $y = f(x)$ is mapped to the point P on the graph of $y = -f(x + 3)$

The coordinates of point A are (9, 1)
Find the coordinates of point P.

(Total 2 marks)

Specimen Papers Set 1, Paper 3H qu.16 (A13, A12 – AO1/AO2)

Exam Wizard topics

Transformations of functions

A14	plot and interpret graphs (including reciprocal graphs **and exponential graphs**) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Content guidance

At Higher tier, to include $y = \frac{k}{x}$ and $y = ak^x$.

Candidates will be expected to answer simple kinematics problems from graphs involving speed, distance and time.

The *suvat* formulae were included on our formulae sheet, which has now been withdrawn. Knowledge of the *suvat* formulae is not part of our specification and will not form part of our assessment.

There may be questions that students could solve by using the *suvat* formulae, but no questions will be set where these formulae have to be used.

Students could be presented with one of the *suvat* equations and asked, for example, to change the subject of the formula or substitute in values to find the value of one of the variables, but no application of these formulae will be expected.

New to Foundation tier

Plotting and interpreting reciprocal graphs in real contexts.

Mapping to 1MA0 specification content descriptors

Ar (F) Construct linear functions from real-life problems and plot their corresponding graphs
Ar (H) Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs
As (F/H) Discuss, plot and interpret graphs (which may be non-linear) modelling real situations

Learning objectives

Foundation tier (Units 9a, 16b, 20)

- Draw straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills graphs, fixed charge and cost per unit; read values from straight-line graphs for real-life situations; find the gradient of a straight line from real-life graphs;
- Work out time intervals for graph scales;
- Draw distance–time graphs and velocity–time graphs; interpret distance–time graphs, and calculate: the speed of individual sections, total distance and total time;
- Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs;
- Interpret information presented in a range of linear and non-linear graphs; interpret graphs with negative values on axes;
- Interpret graphs of quadratic functions from real-life problems;
- Use graphical representations of indirect proportion to solve problems in context;
- Solve simultaneous equations representing a real-life situation, graphically and algebraically, and interpret the solution in the context of the problem.

2. Algebra

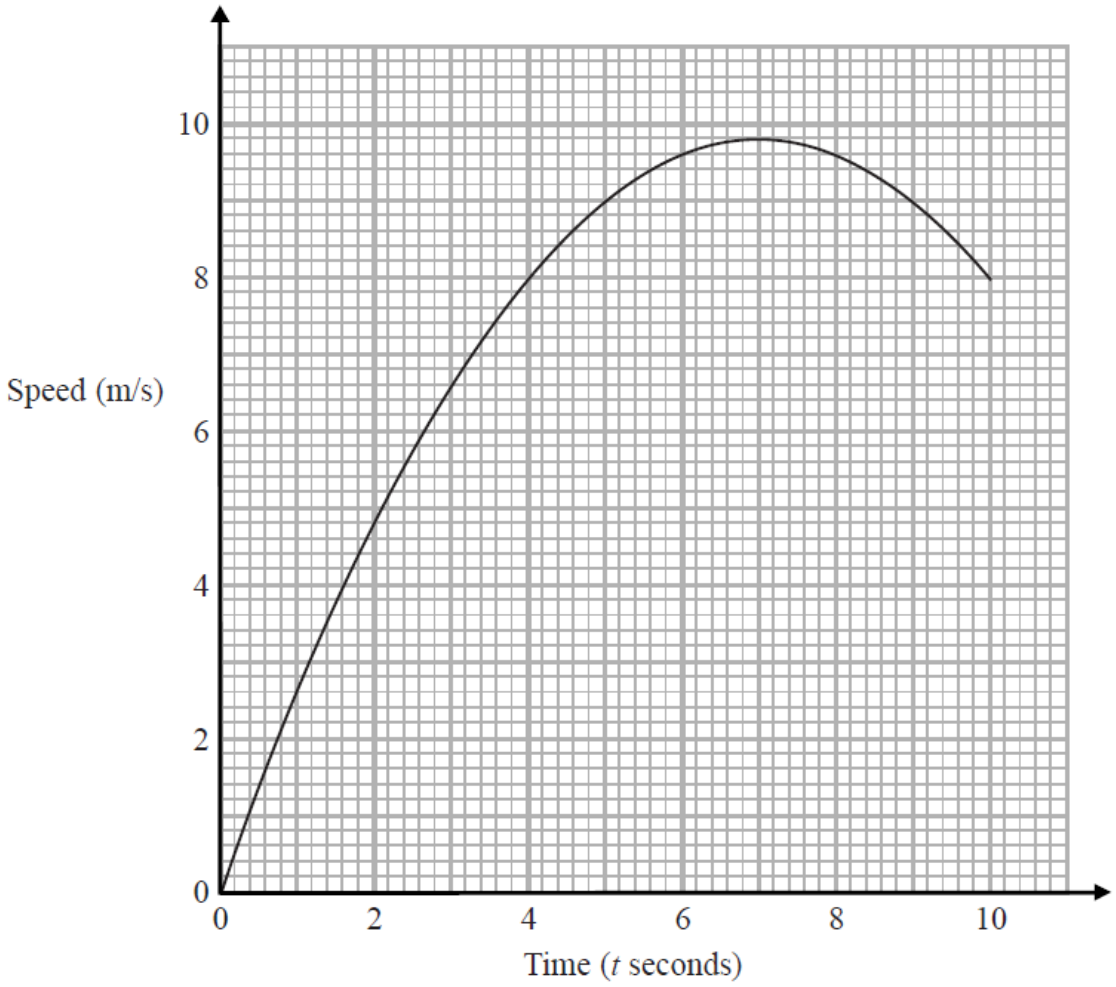
Higher tier (Units 6a, 6c, 19a)

- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bill graphs, fixed charge and cost per unit;
- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections) including: unit price (gradient), average speed, distance, time, acceleration; use enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Interpret graphs of quadratic functions from real-life problems;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
    - for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
    - for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts; the rate of change of graphs of containers filling and emptying; of unit price in price graphs;
- Interpret the area under a linear or non-linear graph in real-life contexts.

Sample questions

Karol ran in a race.

The graph shows her speed, in metres per second, *t* seconds after the start of the race.



(*a*) Write down Karol's speed 3 seconds after the start of the race.

(1)

(*b*) Write down Karol's greatest speed.

(1)

There were two times when Karol's speed was 9 m/s.

(*c*) Write down these two times.

(1)

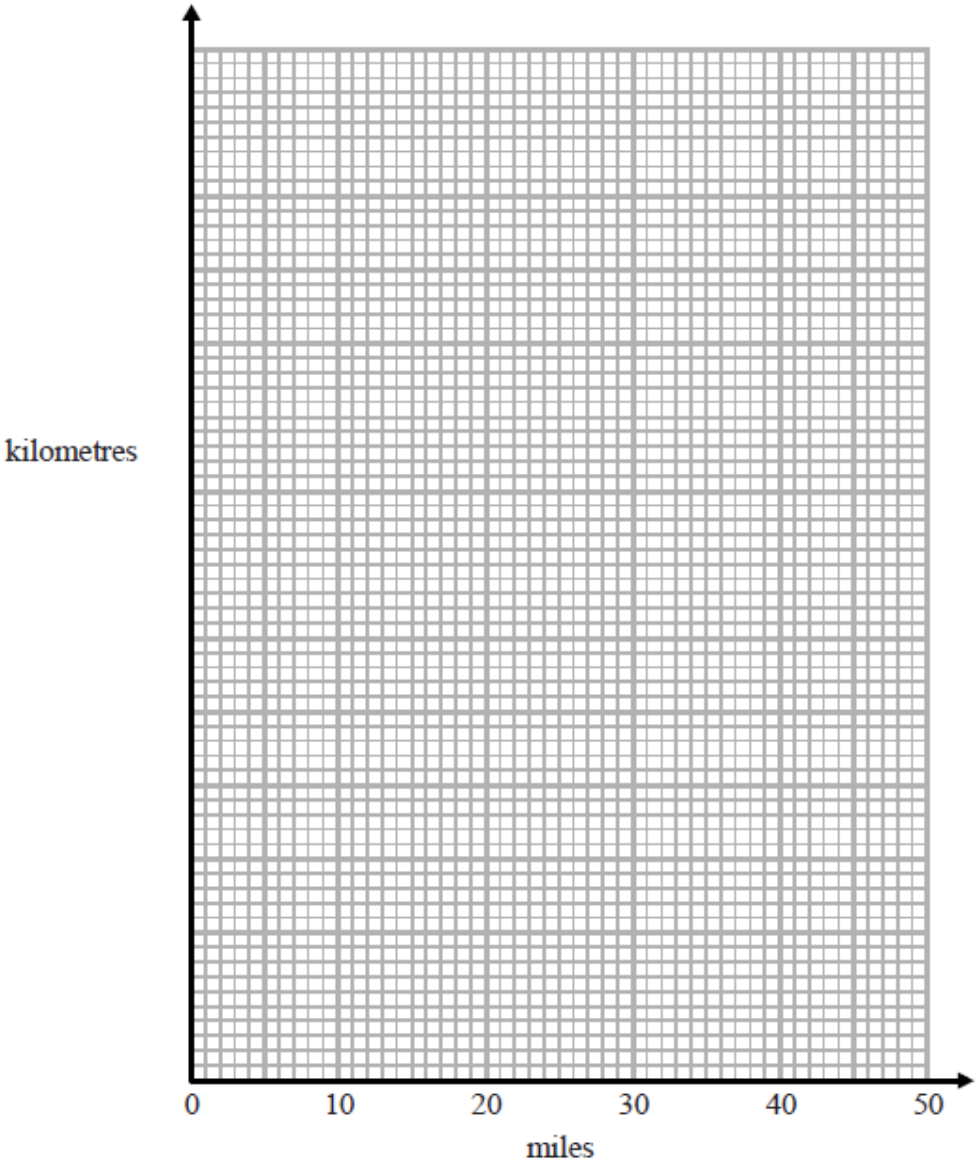
(Total 3 marks)

Specimen Papers Set 2, Paper 2F qu.10 (A14 –AO2)

You can use the information in the table to convert between kilometres and miles.

miles	0	5	20	40
kilometres	0	8	32	64

(*a*) Use this information to draw a conversion graph.



(3)

(*b*) Which is further, 20 kilometres or 15 miles?
You must show how you got your answer.

(2)

(Total 5 marks)
New SAMs Paper 2F qu.12 (A14, R1 – AO2)

Exam Wizard topics

Distance-time/travel graphs
Conversion graphs
Line graphs of real situations
New to Foundation from Higher in GCSE (9–1)

A15 calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance–time graphs, velocity–time graphs and graphs in financial contexts (this does not include calculus)

Content guidance

When estimating area under a curve, a maximum of four equal intervals will be expected.

At Higher tier, candidates will be expected to find gradients of graphs and areas under graphs and interpret these results in distance-time and velocity-time graphs.

New to GCSE (9–1) Maths

The content in A15 is new to GCSE (9–1) Maths.

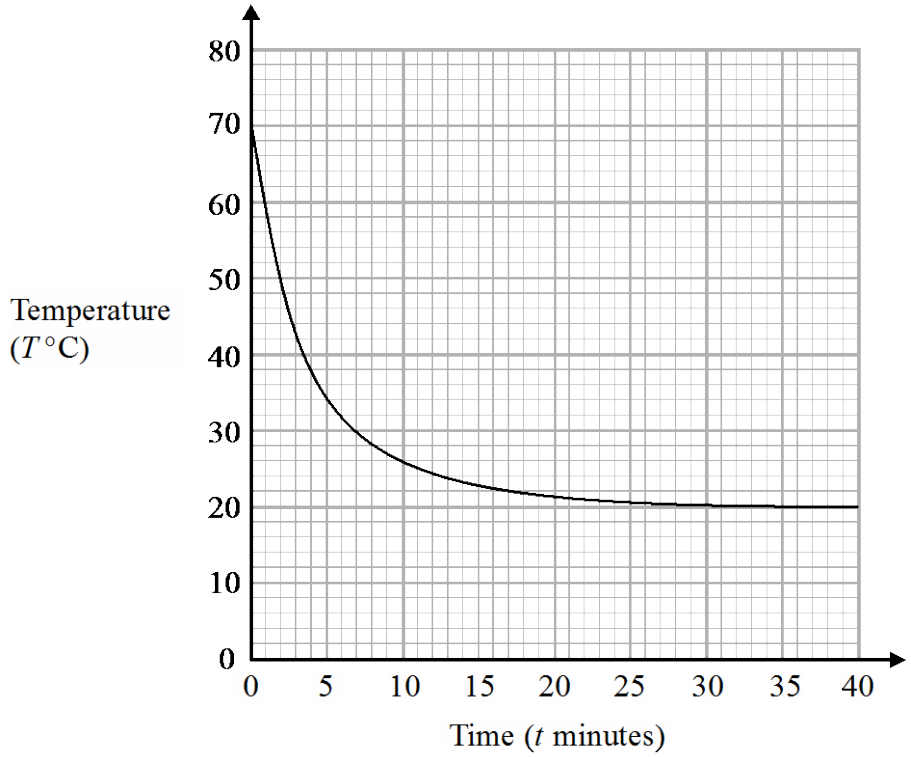
Learning objectives

Higher tier (Units 6a, 19a)

- Use graphs to calculate various measures (of individual sections) including: unit price (gradient), average speed, distance, time, acceleration; use enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Estimate area under a quadratic or other graph by dividing it into trapezia;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
  - for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
  - for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts; the rate of change of graphs of containers filling and emptying; of unit price in price graphs;
- Interpret the area under a linear or non-linear graph in real-life contexts;
- Interpret the rate of change of graphs of containers filling and emptying.

Sample questions

The graph shows the temperature, T °C, of the coffee in a cup at a time t minutes.



(*a*) Find an estimate for the gradient of the graph at time 5 minutes.

(2)

(*b*) Explain what this gradient represents.

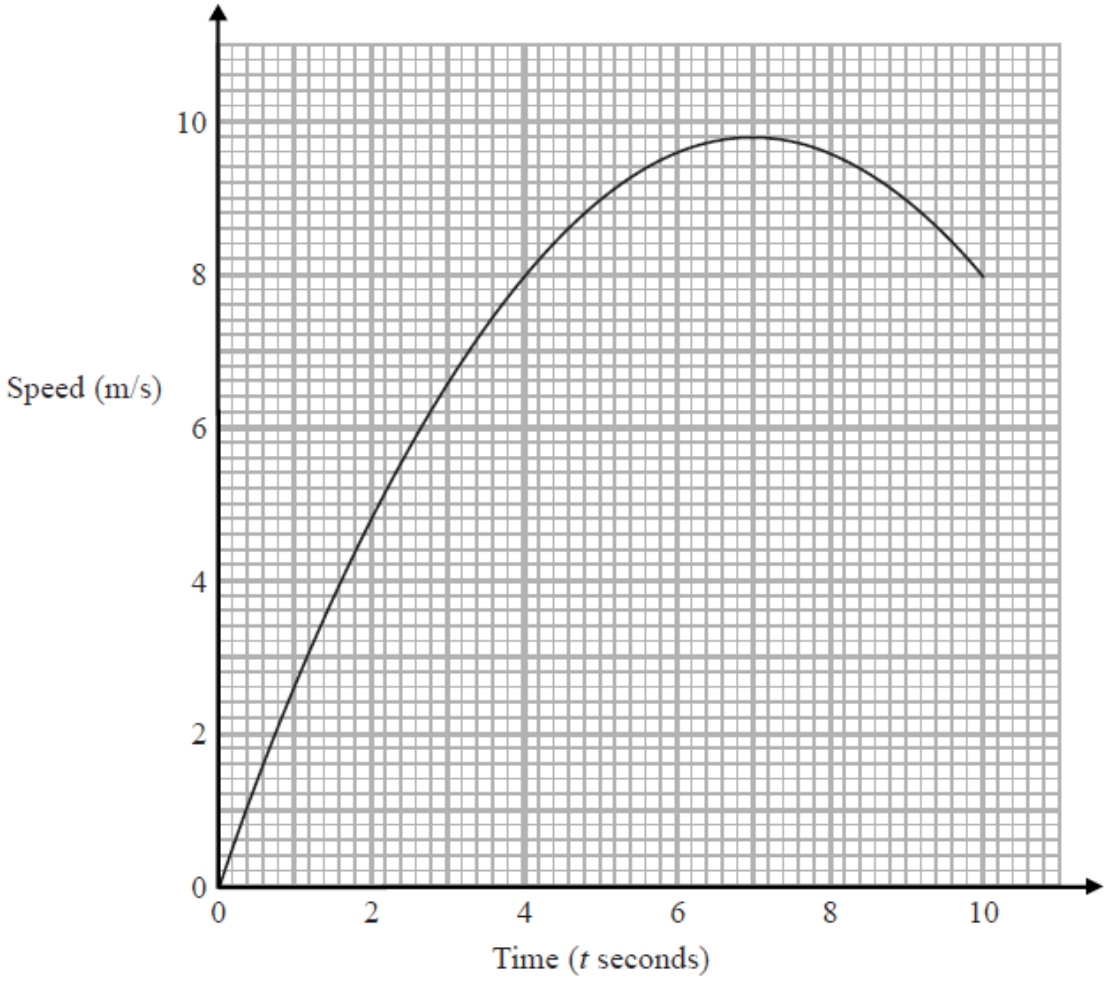
(1)

(Total 3 marks)

Mock Papers Set 3, Paper 1H qu.13 (A15, R15 – AO1/AO2)

Karol runs in a race.

The graph shows her speed, in metres per second, t seconds after the start of the race.



(a) Calculate an estimate for the gradient of the graph when $t = 4$
You must show how you get your answer.

(3)

(b) Describe fully what your answer to part (a) represents.

(2)

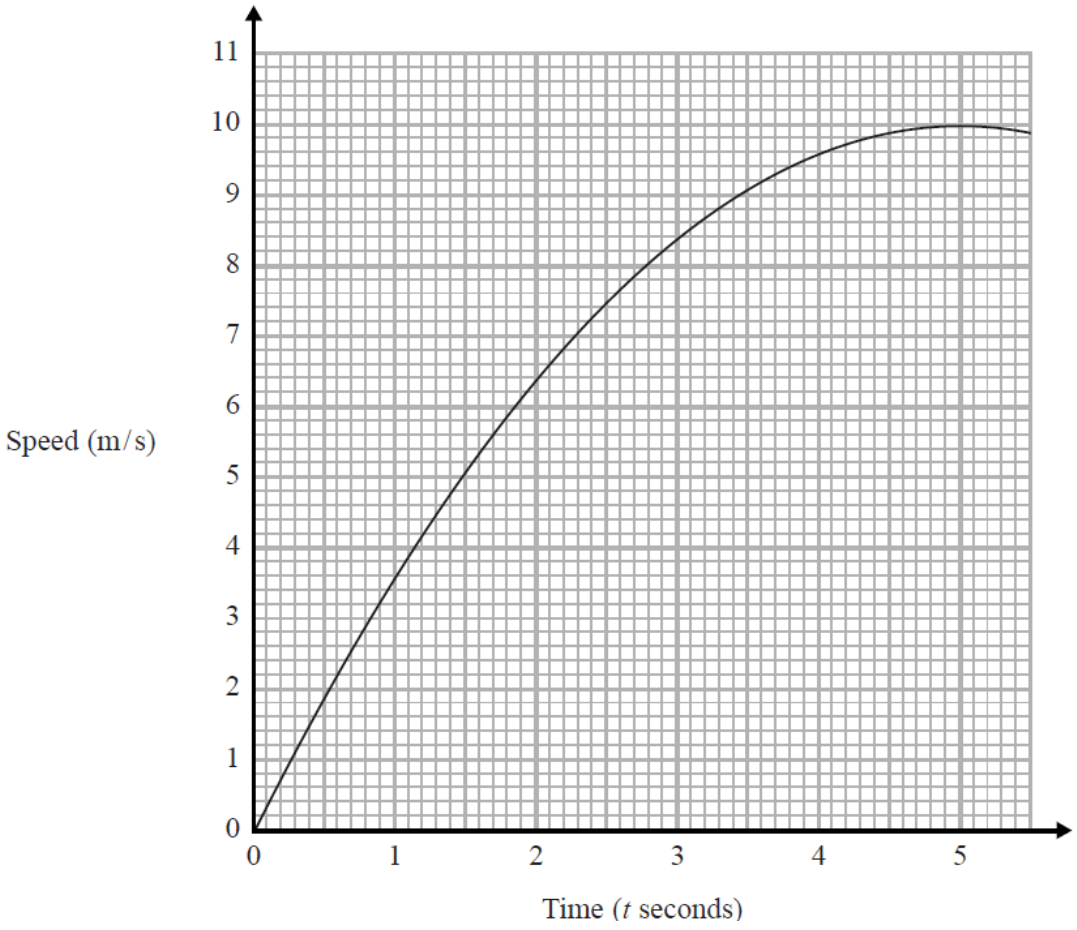
(c) Explain why your answer to part (a) is only an estimate.

(1)

(Total 6 marks)

Specimen Papers Set 2, Paper 2H qu.15 (A15 –AO1/AO2/AO3)

Here is a speed-time graph showing the speed, in metres per second, of an object t seconds after it started to move.



(a) Use 3 strips of equal width to find an estimate for the area under the graph between $t = 1$ and $t = 4$

(3)

(b) Describe fully what your answer to part (a) represents.

(2)

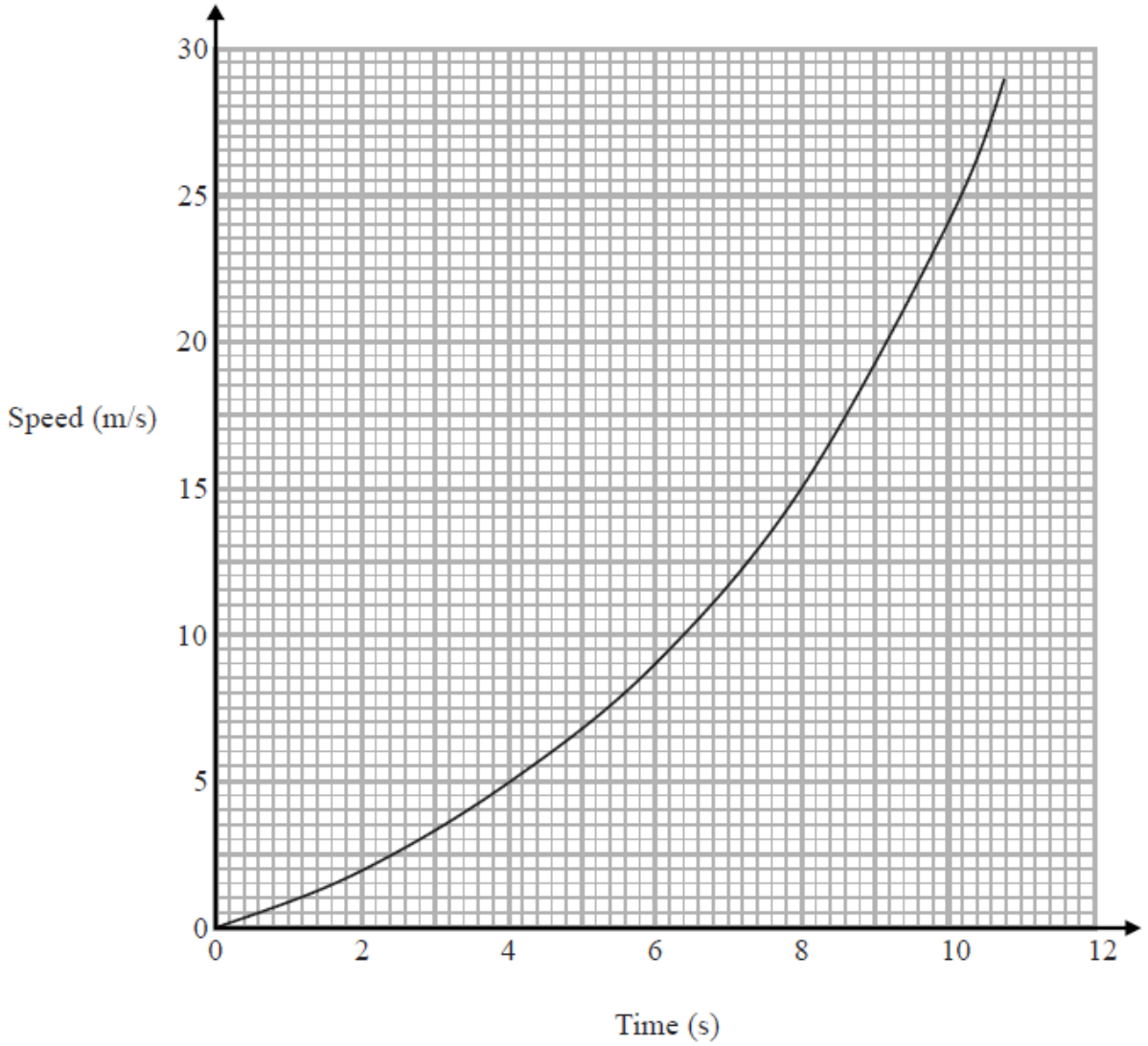
(c) Explain whether your answer in part (a) gives an underestimate or an overestimate for the area under the graph.

(1)

(Total 6 marks)

Mock Papers Set 1, Paper 2H qu.17 (A15 – AO1/AO2/AO3)

Here is a speed-time graph for a car.



(*a*) Work out an estimate for the distance the car travelled in the first 10 seconds.
Use 5 strips of equal width.

(3)

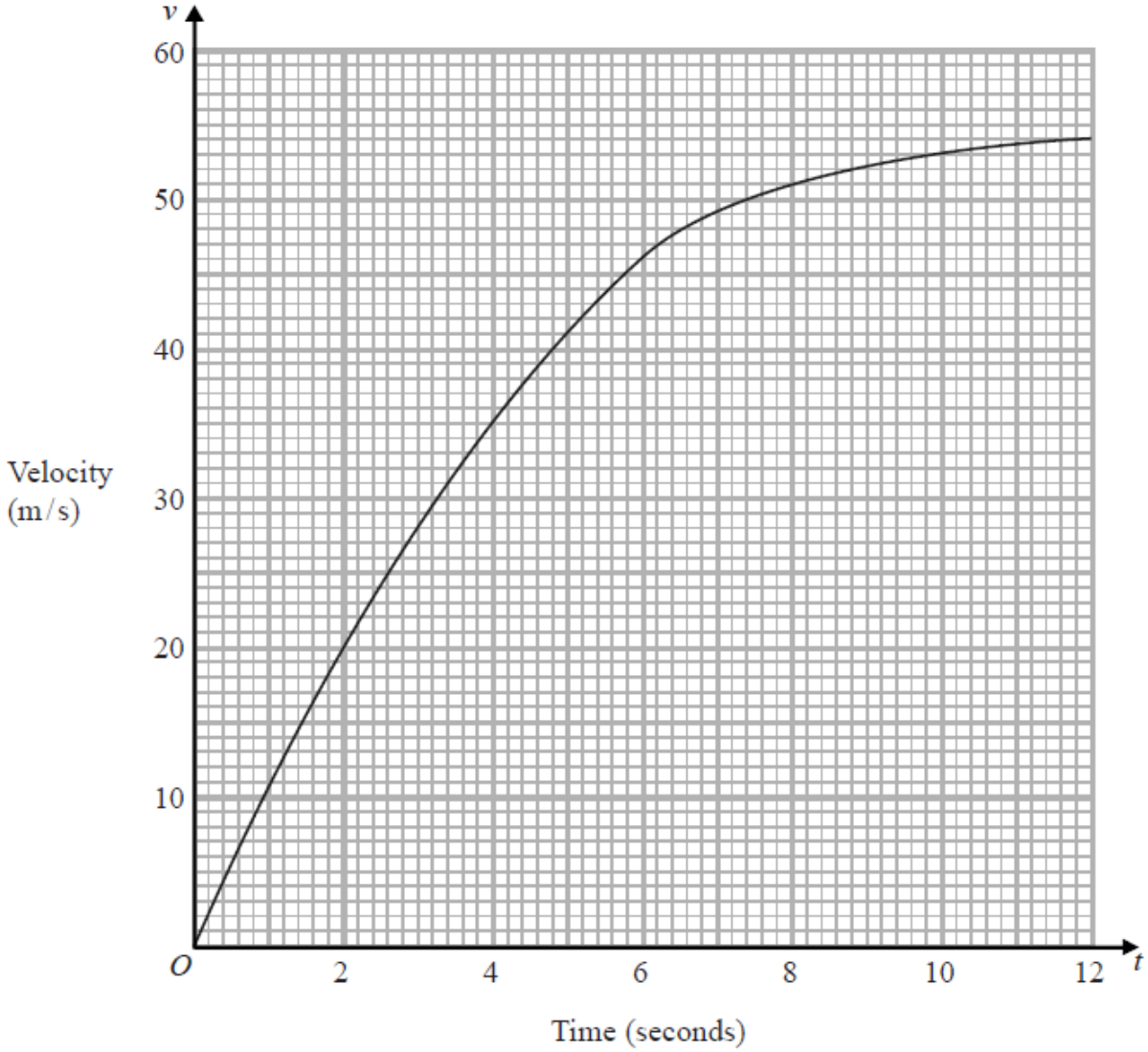
(*b*) Is your answer to (*a*) an underestimate or an overestimate of the actual distance?
Give a reason for your answer.

(1)

(Total 4 marks)

Specimen Papers Set 2, Paper 3H qu.18 (A15 –AO1/AO3)

The graph shows information about the velocity, v m/s, of a parachutist t seconds after leaving a plane.



(*a*) Work out an estimate for the acceleration of the parachutist at $t = 6$

(2)

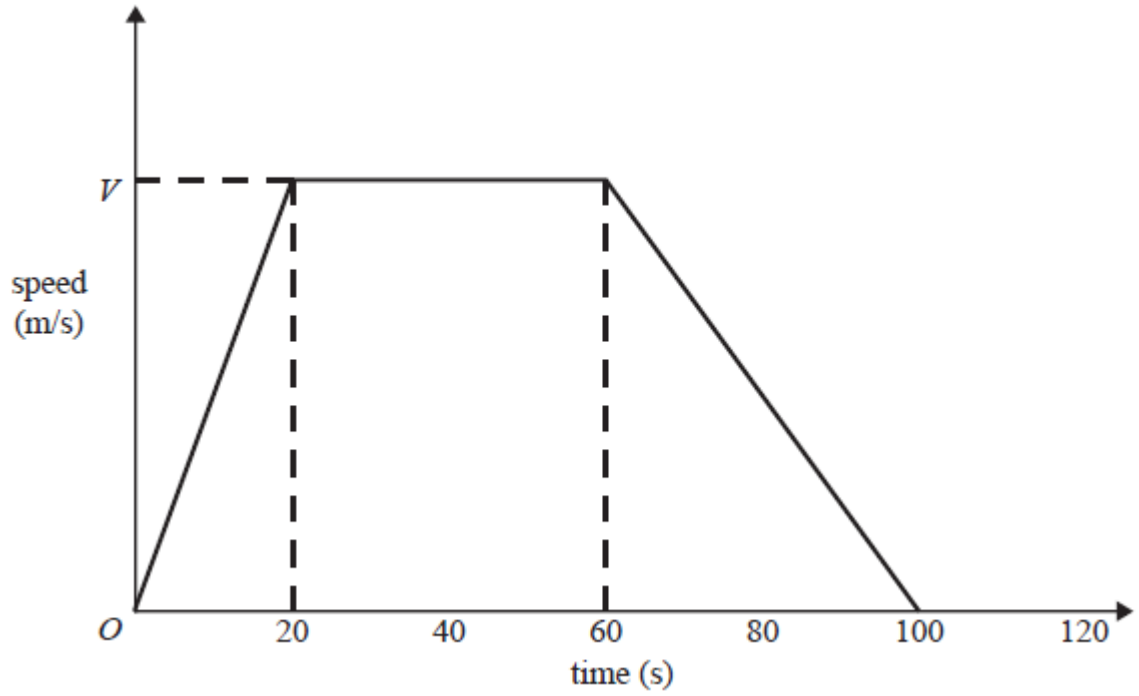
(*b*) Work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane.
Use 3 strips of equal width.

(3)

(Total 5 marks)

Specimen Papers Set 1, Paper 2H qu.20 (A15 –AO1/AO2)

Here is a speed-time graph for a car journey.
The journey took 100 seconds.



The car travelled 1.75 km in the 100 seconds.

(*a*) Work out the value of *V*.

(3)

(*b*) Describe the acceleration of the car for each part of this journey.

(2)

(Total 5 marks)
New SAMs Paper 1H qu.21 (A15 – AO1/AO2/AO3)

Exam Wizard topics

Gradients of graphs and area under graphs
New to GCSE (9–1)

A16 recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point

New to GCSE (9–1) Maths

Finding the equation of a tangent to a circle at a given point.

Mapping to 1MA0 specification content descriptors

Aq (H) Construct the graphs of simple loci

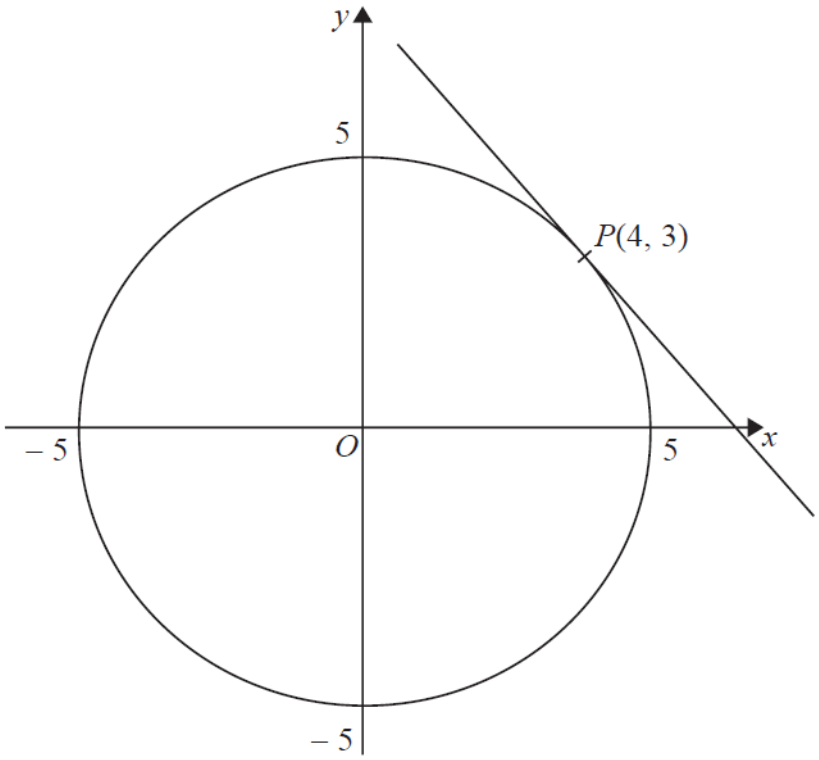
Learning objectives

Higher tier (Units 6c, 15, 16b)

- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
- Recognise and draw circles, centre the origin, equation $x^2 + y^2 = r^2$ for radius r centred at the origin of coordinates;
- Find the equation of a tangent to a circle at a given point by: finding the gradient of the radius that meets the circle at that point (circles all centre the origin); finding the gradient of the tangent perpendicular to it; using the given point.

Sample questions

Here is a circle, centre O, and the tangent to the circle at the point $P(4, 3)$ on the circle.

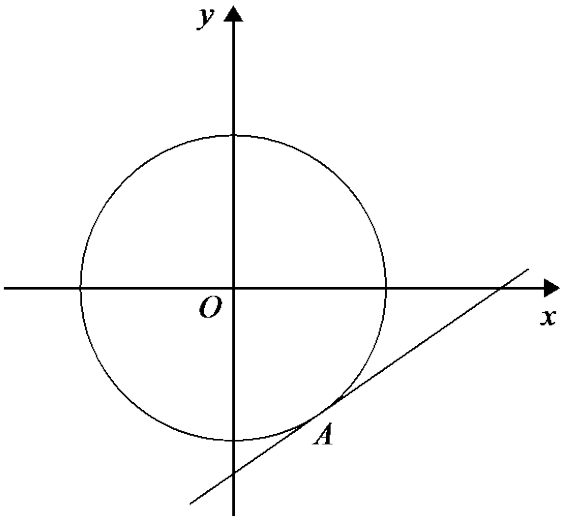


Find an equation of the tangent at the point P.

(Total 3 marks)

Specimen Papers Set 1, Paper 2H qu.23 (A16 –AO1)

The diagram shows the circle with equation $x^2 + y^2 = 261$



A tangent to the circle is drawn at point A with coordinates $(p, -15)$, where $p > 0$

Find an equation of the tangent at A.

(Total 5 marks)

Mock Papers Set 3, Paper 3H qu.22 (A16, A9, A10 – AO1/AO3)

The line l is a tangent to the circle $x^2 + y^2 = 40$ at the point A.
A is the point (2, 6).

The line l crosses the x-axis at the point P.

Work out the area of triangle OAP.

(Total 5 marks)

Specimen Papers Set 2, Paper 1H qu.22 (A16, A9, G16 –AO1/AO3)

Exam Wizard topics

Graphs of circles

Solving equations and inequalities

A17	solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph

Mapping to 1MA0 specification content descriptors

Ad (F) Set up and solve simple equations
Ad (H) Set up and solve simple equations including simultaneous equations in two unknowns

Learning objectives

Foundation tier (Units 5a, 9b)

- Solve simple equations including those: with integer coefficients, in which the unknown appears on either side or on both sides of the equation; which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution; with one unknown, with integer or fractional coefficients;
- Substitute into a formula and solve the resulting equation;
- Solve angle or perimeter problems using algebra;
- Find an approximate solution to a linear equation using a graph.

Higher tier (Units 2a, 6b)

- Solve linear equations including those: with integer coefficients, in which the unknown appears on either side or on both sides of the equation; which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution; with one unknown, with integer or fractional coefficients;
- Set up and solve linear equations to solve a problem;
- Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
- Substitute positive and negative numbers into a formula; solve the resulting equation;
- Find approximate solutions to a linear equation from a graph.

Sample questions

(*a*) Rob buys *p* packets of plain crisps and c packets of cheese crisps.

Write down an expression for the total number of packets of crisps Rob buys.

(1)

(*b*) Solve $3x - 5 = 9$

(2)

(Total 3 marks)
New SAMs Paper 2F qu.10 (A17, A21 – AO1)

Solve $4x + 5 = x + 26$

(Total 2 marks)
New SAMs Paper 1F qu.19 (A17 – AO1)

Solve $\frac{x+2}{3x}+\frac{x-2}{2x}=3$

(Total 3 marks)

Specimen Papers Set 1, Paper 1H qu.14 (A17 –AO1)

Exam Wizard topics

Derive expressions; equations; formulae
Algebraic fractions
Solve linear equations

A18	solve quadratic equations **(including those that require rearrangement)** algebraically by factorising, **by completing the square and by using the quadratic formula**; find approximate solutions using a graph

Content guidance

The solution of quadratic equations on the Foundation tier will be limited to solution by factorising only and to the type $x^2 + bx + c = 0$.

Candidates at Higher tier could be asked to complete the square for any quadratic expression of the form $ax^2 + bx + c$. The difficulty of the expression will affect the demand at which the question is set.

New to Foundation tier

Solving quadratic equations by factorisation. (NB: completing the square is still Higher tier only.)

Mapping to 1MA0 specification content descriptors

Ae (H) Solve quadratic equations
Ar (H) Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs
At (F/H) Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions

Learning objectives

Foundation tier (Units 16a, 16b)

- Solve quadratic equations by factorising;
- Find approximate solutions to quadratic equations using a graph;
- Identify and interpret roots, intercepts and turning points of quadratic graphs.

Higher tier (Units 6c, 9a, 15, 17)

- Set up and solve quadratic equations;
- Solve quadratic equations by factorisation: completing the square; using the quadratic formula;
- Solve quadratic equations that need rearranging;
- Find approximate solutions to quadratic equations using a graph;
- Solve quadratic equations arising from algebraic fraction equations.

Sample questions

Solve $x^2 - 5x + 3 = 0$

Give your solutions correct to 3 significant figures.

(Total 3 marks)
New SAMs Paper 3H qu.11 (A18 – AO1)

Steve is asked to solve the equation $5(x + 2) = 47$
Here is his working.

$$\begin{aligned} 5(x + 2) &= 47 \\ 5x + 2 &= 47 \\ 5x &= 45 \\ x &= 9 \end{aligned}$$

Steve's answer is wrong.

(*a*) What mistake did he make?

(1)

Liz is asked to solve the equation $3x^2 + 8 = 83$

Here is her working.

$$\begin{aligned} 3x^2 + 8 &= 83 \\ 3x^2 &= 75 \\ x^2 &= 25 \\ x &= 5 \end{aligned}$$

(*b*) Explain what is wrong with Liz's answer.

(1)

(Total 2 marks)
Specimen Papers Set 2, Paper 2H qu.8 (A18, A17 –AO2/AO3)

Solve $x^2 - 6x - 8 = 0$
Write your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

(Total 3 marks)
Specimen Papers Set 2, Paper 1H qu.17 (A18 –AO2)

Exam Wizard topics

Solve quadratic equations
Graphs of quadratic equations
Line graphs of real situations
New to Foundation from Higher in GCSE (9–1)

A19	solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph

Content guidance

E.g. to include, at Higher tier, solve $x^2 + y^2 = 10$ and $x + y = 4$.

New to Foundation tier

Solving linear/linear simultaneous equations. (NB: solving linear/quadratic simultaneous equations is still Higher tier only.)

Mapping to 1MA0 specification content descriptors

Ad (H) Set up and solve simple equations including simultaneous equations in two unknowns
Ao (H) Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions
Ar (H) Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs

Learning objectives

Foundation tier (Unit 20)

- Solve simultaneous equations (linear/linear) algebraically and graphically;
- Solve simultaneous equations representing a real-life situation, graphically and algebraically, and interpret the solution in the context of the problem.

Higher tier (Units 9a, 15)

- Find the exact solutions of two simultaneous equations in two unknowns;
- Use elimination or substitution to solve simultaneous equations;
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns: linear/linear, including where both need multiplying; linear/quadratic; linear/$x^2 + y^2 = r^2$;
- Set up and solve a pair of simultaneous equations in two variables for each of the above scenarios, including to represent a situation; interpret the solution in the context of the problem;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points;
- Solve simultaneous equations graphically: find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach; find graphically the intersection points of a given straight line with a circle; solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem.

Sample questions

Solve the simultaneous equations

$$4x + y = 25$$
$$x - 3y = 16$$

(Total 3 marks)
Specimen Papers Set 1, Paper 1F qu.29 (A19 –AO1)

Solve algebraically the simultaneous equations

$$x^2 + y^2 = 25$$
$$y - 2x = 5$$

(Total 5 marks)

New SAMs Paper 2H qu.20 (A19 – AO1/AO2)

Exam Wizard topics

Solve simultaneous equations
Solve quadratic equations
Graphs of quadratic equations
Line graphs of real situations
New to Foundation from Higher in GCSE (9–1)

A20 find approximate solutions to equations numerically using iteration

Content guidance

Examiners would expect to give students a rearranged equation to use in their iteration along with a starting value and ask them to carry out, say, three iterations, feeding their solution each time into $x_{n+1} = \mathrm{f}(x_n)$ to get an improved solution and so generating x_2, x_3, etc, having been given a value for x_1. They may first be given an equation and asked to show that it can be rearranged into a given form. Students will be expected to realise that the values they are generating are converging to a root of the equation.

Candidates will be given the iterative formula within the question but might also be asked to show the rearrangement of a given equation into a particular form.

Students will be required to know the rule that "where there is a sign change, there is a solution".

New to GCSE (9–1) Maths

The content in A20 is new to GCSE (9–1) Maths.

Learning objectives

Higher tier (Units 2a, 15)

- Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations;
- Use iteration with simple converging sequences.

Sample questions

(*a*) Show that the equation $x^3 + 5x - 4 = 0$ has a solution between $x = 0$ and $x = 1$

(2)

(*b*) Show that the equation $x^3 + 5x - 4 = 0$ can be arranged to give $x = \dfrac{4}{x^2 + 5}$

(2)

(*c*) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{4}{x_n^{\,2} + 5}$ twice,

to find an estimate for the solution of $x^3 + 5x - 4 = 0$

(3)

(Total 7 marks)

Mock Papers Set 1, Paper 3H qu.13 (A20, A2, A4 – AO1/AO2)

(*a*) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

(2)

(*b*) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

(1)

(*c*) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^{\ 3}}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

(3)

(Total 6 marks)

New SAMs Paper 3H qu.14 (A20, A4 – AO1/AO2)

(*a*) Show that the equation $x^3 - 3x^2 + 3 = 0$ has a solution between $x = 2$ and $x = 3$

(2)

(*b*) Show that the equation $x^3 - 3x^2 + 3 = 0$ can be rearranged to give $x = \sqrt[3]{3x^2 - 3}$

(1)

(*c*) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{3x^2 - 3}$ to find the value of x_2. Give your answer correct to 3 decimal places.

(3)

(Total 6 marks)

Mock Papers Set 3, Paper 2H qu.16 (A20, A4 – AO1/AO2)

(*a*) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

(2)

(*b*) Using

$$x_{n+1} = 3 + \frac{3}{x_n^2} \quad \text{with } x_0 = 3.2,$$

find the values of x_1, x_2 and x_3

(3)

(*c*) Explain what the values of x_1, x_2 and x_3 represent.

(1)

(Total 6 marks)

Specimen Papers Set 1, Paper 3H qu.21 (A20, A5 –AO1/AO2/AO3)

(*a*) Show that the equation $2x^3 + 4x = 3$ has a solution between 0 and 1.

(2)

(*b*) Show that $2x^3 + 4x = 3$ can be arranged to give $x = \frac{3}{4} - \frac{x^3}{2}$

(1)

(*c*) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{3}{4} - \frac{x_n^3}{2}$ three times to find an estimate for the solution to $2x^3 + 4x = 3$

(3)

(Total 6 marks)

Mock Papers Set 2, Paper 2H qu.23 (A20, A4 – AO1/AO2)

Exam Wizard topics

Iteration
New to GCSE (9–1)

A21	translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution

New to Foundation tier

Deriving and solving linear/linear simultaneous equations. (NB: solving linear/quadratic equations is still Higher tier only.)

Mapping to 1MA0 specification content descriptors

Ab (F) Distinguish in meaning between the words 'equation', 'formula' and 'expression'
Ab (H) Distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression'
Ac (F) Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
Ac (H) Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions
Ad (F) Set up and solve simple equations
Ad (H) Set up and solve simple equations including simultaneous equations in two unknowns
Af (F/H) Derive a formula, substitute numbers into a formula and change the subject of a formula

Learning objectives

Foundation tier (Units 2b, 5a, 20)

- Write and simplify expressions using squares and cubes;
- Write expressions to solve problems representing a situation;
- Derive a simple formula, including those with squares, cubes and roots;
- Write expressions and set up simple equations including forming an equation from a word problem;
- Solve angle or perimeter problems using algebra;
- Write simultaneous equations to represent a situation.

Higher tier (Units 2a, 7a, 7b, 9a, 15)

- Write and manipulate an expression by collecting like terms;
- Set up simple equations from word problems and derive simple formulae;
- Set up and solve linear equations to solve a problem;
- Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
- Form equations involving more complex shapes and solve these equations;
- Set up and solve quadratic equations;
- Set up and solve a pair of simultaneous equations in two variables, including to represent a situation; interpret the solution in the context of the problem.

Sample questions

(*a*) Rob buys *p* packets of plain crisps and c packets of cheese crisps.

Write down an expression for the total number of packets of crisps Rob buys.

(1)

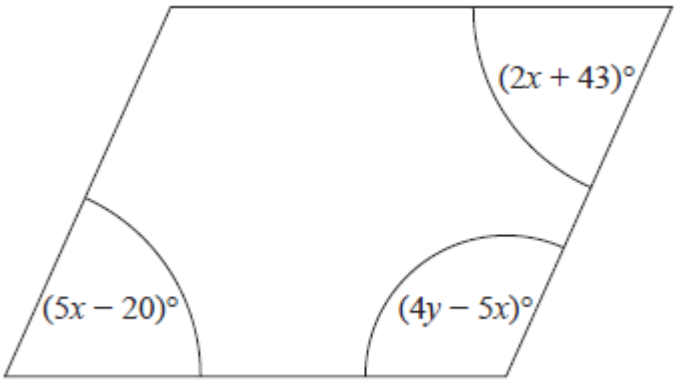
(*b*) Solve $3x - 5 = 9$

(2)

(Total 3 marks)

New SAMs Paper 2F qu.10 (A21, A17 – AO1)

Here is a parallelogram.



Work out the value of *x* and the value of *y*.

(Total 5 marks)

New SAMs Paper 1F qu.28 / 1H qu.8 (A21, G4 – AO1/AO3)

Exam Wizard topics

Derive expressions; equations; formulae
Manipulate expressions and formulae
Solve linear equations
Solve simultaneous equations
New to Foundation from Higher in GCSE (9–1)

A22	solve linear inequalities in one **or two** variable**(s), and quadratic inequalities in one variable**; represent the solution set on a number line**, using set notation and on a graph**

Content guidance

E.g. the solution of $x^2 - 1 < 0$ is $-1 < x < 1$ or $\{x : -1 < x < 1\}$.
E.g. represent the solution set to a given number of linear inequalities in two variables as a region on a graph.

Examiners will not be expecting students to use the ( ] bracket notation as part of set notation.

Representing solution sets on a graph: the examiners' wording will be along the lines of "Show, by shading, the region … label your region *R*."
This is done deliberately to allow for candidates who are taught to shade the required region and those who are taught to shade the unwanted region. Either approach is accepted by mark schemes, provided that the candidate makes their approach clear – hence the requirement to label the region.

New to GCSE (9–1) Maths

Solving quadratic inequalities in one variable and representing the solution set using set notation.

Mapping to 1MA0 specification content descriptors

Ag (F) Solve linear inequalities in one variable and represent the solution set on a number line
Ag (H) Solve linear inequalities in one or two variables, and represent the solution set on a number line or coordinate grid

Learning objectives

Foundation tier (Unit 5a)

- Show inequalities on number lines; write down whole number values that satisfy an inequality;
- Solve simple linear inequalities in one variable and represent the solution set on a number line;
- Solve two inequalities in x, such as $-3 < 2x + 1 < 7$, find the solution sets and compare them to see which value of x satisfies both; show the solution set on a number line;
- Construct inequalities to represent a set shown on a number line.

Higher tier (Units 9b, 15)

- Show inequalities on number lines; write down whole number values that satisfy an inequality;
- Solve simple linear inequalities in one variable and represent the solution set on a number line;
- Solve two linear inequalities in x, find the solution sets and compare them to see which value of x satisfies both; solve linear inequalities in two variables algebraically;
- Solve quadratic inequalities in one variable by factorising and sketching the graph to find critical values;
- Represent the solution set for inequalities using set notation, i.e. curly brackets and 'is an element of' notation; for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of $x^2 - 3x - 10 < 0$ as $\{x : -3 < x < 5\}$;
- Solve linear inequalities in two variables graphically;
- Show the solution set of several inequalities in two variables on a graph.

Sample questions

(*a*) Factorise $y^2 + 7y + 6$

(2)

(*b*) Solve $6x + 4 > x + 17$

(2)

(*c*) n is an integer with $-5 < 2n \leq 6$
Write down all the values of n

(2)

(Total 6 marks)
New SAMs Paper 3H qu.9 (A22, A4, N1 – AO1)

Solve $x^2 > 3x + 4$

(Total 3 marks)
New SAMs Paper 1H qu.19 (A22 – AO1)

Solve $2x^2 - 5x - 12 > 0$

(Total 3 marks)
Mock Papers Set 2, Paper 3H qu.19 (A22 – AO1)

Solve the inequality $x^2 > 3(x + 6)$

(Total 4 marks)
Specimen Papers Set 2, Paper 1H qu.21 (A22 –AO1)

Exam Wizard topics

Solve linear inequalities
Solve quadratic inequalities
New to GCSE (9–1)

Sequences

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

Mapping to 1MA0 specification content descriptors

Ai (F/H) Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence

Learning objectives

Foundation tier (Unit 5b)

- Use function machines to find terms of a sequence;
- Write the term-to-term definition of a sequence in words;
- Find a specific term in the sequence using position-to-term or term-to-term rules;
- Generate arithmetic sequences of numbers, triangular number, square and cube integers and sequences derived from diagrams;
- Recognise such sequences from diagrams and draw the next term in a pattern sequence;
- Find the next term in a sequence, including negative values;
- Use the *n*th term of an arithmetic sequence to generate terms; decide if a given number is a term in the sequence or find the first term over a certain number; find the first term greater/less than a certain number;
- Continue a quadratic sequence and use the *n*th term to generate terms;
- Continue a geometric progression and find the term-to-term rule, including negatives, fraction and decimal terms.

Higher tier (Unit 2b)

- Generate sequences of numbers, squared integers and sequences derived from diagrams;
- Describe in words a term-to-term sequence and identify which terms cannot be in a sequence;
- Generate specific terms in a sequence using the position-to-term rule and term-to-term rule;
- Use the *n*th term of an arithmetic sequence to generate terms; decide if a given number is a term in the sequence or find the first term over a certain number; find the first term greater/less than a certain number;
- Continue a quadratic sequence and use the *n*th term to generate terms;
- Recognise and use simple geometric progressions (r^n where n is an integer and r is a rational number > 0 or a surd);
- Continue geometric progression and find term to term rule, including negative, fraction and decimal terms.

Sample questions

Here are the first three terms of a sequence.

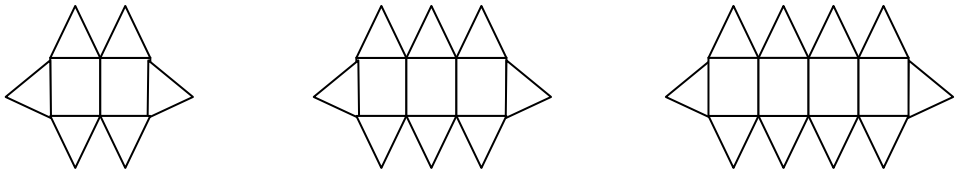
32 26 20

Find the first two terms in the sequence that are less than zero.

(Total 3 marks)

Specimen Papers Set 1, Paper 3F qu.13 (A23 –AO2)

Here are the first three patterns in a sequence.
The patterns are made from triangles and rectangles.



pattern number 1 pattern number 2 pattern number 3

(*a*) How many triangles are there in pattern number 7?

(2)

Charlie says

"There are 4 rectangles in pattern number 3 so there will be 8 rectangles in pattern number 6"

(*b*) Is Charlie right?
Give a reason for your answer.

(1)

(Total 3 marks)

New SAMs Paper 1F qu.12 (A23, A24 – AO1/AO2)

Exam Wizard topics

Linear sequences of diagrams
Linear sequences of numbers

A24	recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci-type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 **or a surd) and other sequences**

Content guidance

Other sequences to include ar^n at Higher tier.

Other sequences could include, for example,
1, 1/2, 1/3, 1/4, 1/5, 1/6, …
1, 16, 81, 256, …

New to GCSE (9–1) Maths

Recognising and using Fibonacci-type sequences, quadratic sequences and simple geometric progressions (including surds) and other sequences.

Mapping to 1MA0 specification content descriptors

Ai (F/H) Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence
Aj (F/H) Use linear expressions to describe the *n*th term of an arithmetic sequence

Learning objectives

Foundation tier (Unit 5b)

- Recognise sequences of odd and even numbers, and other sequences including Fibonacci sequences;
- Generate arithmetic sequences of numbers, triangular number, square and cube integers and sequences derived from diagrams;
- Distinguish between arithmetic and geometric sequences.

Higher tier (Unit 2b)

- Recognise simple sequences including at the most basic level odd, even, triangular, square and cube numbers and Fibonacci-type sequences;
- Distinguish between arithmetic and geometric sequences;
- Use finite/infinite and ascending/descending to describe sequences;
- Recognise and use simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 or a surd);
- Solve problems involving sequences from real-life situations.

Sample questions

Here are the first five terms of a sequence.

2 8 18 32 50

(*a*) Find the next term of this sequence.

(1)

The *n*th term of a different sequence is $3n^2 - 10$

(*b*) Work out the 5th term of this sequence.

(1)

(Total 2 marks)
Specimen Papers Set 2, Paper 1F qu.24 (A24, A23 –AO1)

Here are the first six terms of a Fibonacci sequence.

1 1 2 3 5 8

The rule to continue a Fibonacci sequence is,

the next term in the sequence is the sum of the two previous terms.

(*a*) Find the 9th term of this sequence.

(1)

The first three terms of a different Fibonacci sequence are

a b $a + b$

(*b*) Show that the 6th term of this sequence is $3a + 5b$

(2)

Given that the 3rd term is 7 and the 6th term is 29,

(*c*) find the value of a and the value of b.

(3)

(Total 6 marks)

New SAMs Paper 3F qu.20 / 3H qu.3 (A24, A19, A21 – AO1/AO2/AO3)

Here are the first four terms of a quadratic sequence.

3 8 15 24

(*a*) Find an expression, in terms of n, for the nth term of this sequence.

(3)

The nth term of a different sequence is $2^n + 5$

(*b*) Show that 36 is **not** a term of this sequence.

(1)

(Total 4 marks)

Mock Papers Set 1, Paper 2H qu.12 (A24, A25 – AO1/AO2)

Exam Wizard topics

Linear sequences of diagrams
Linear sequences of numbers
Non-linear sequences
New to GCSE (9–1)

A25 deduce expressions to calculate the *n*th term of linear **and quadratic** sequences

Content guidance

At the Higher tier, students might have to find complex *n*th terms, such as $n^2 + 3n - 5$, when given the sequence only.

Questions using sequences such as $2n^2$; $n^2 - 3$, $n^2 + 5n$, etc, could be set.

New to GCSE (9–1) Maths

Deducing expressions to calculate the *n*th term of quadratic sequences.

Mapping to 1MA0 specification content descriptors

Aj (F/H) Use linear expressions to describe the *n*th term of an arithmetic sequence

Learning objectives

Foundation tier (Unit 5b)

- Find the *n*th term for a pattern sequence; a linear sequence; of an arithmetic sequence.

Higher tier (Unit 2b)

- Find and use (to generate terms) the *n*th term of an arithmetic sequence;
- Find the *n*th term of quadratic sequences.

Sample questions

Here are the first four terms of an arithmetic sequence.

6 10 14 18

(*a*) Write an expression, in terms of *n*, for the *n*th term of this sequence.

(2)

The *n*th term of a different arithmetic sequence is $3n + 5$

(*b*) Is 108 a term of this sequence?
Show how you get your answer.

(2)

(Total 4 marks)
New SAMs Paper 2F qu.25 / 2H qu.3 (A25 – AO1/AO2)

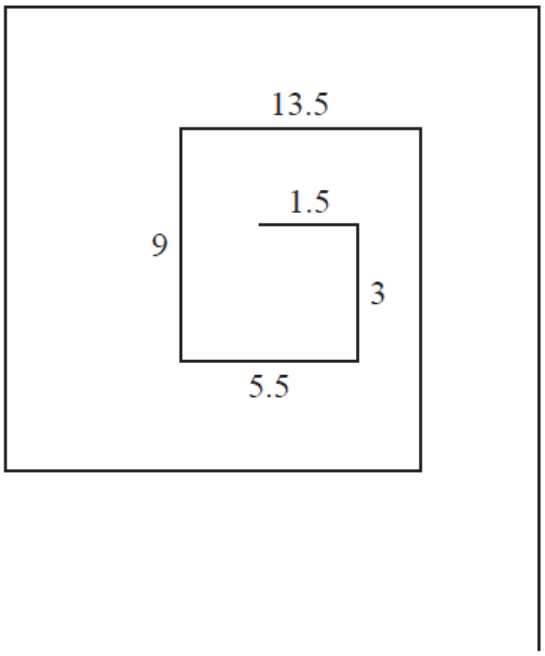
Here are the first 5 terms of a quadratic sequence.

1 3 7 13 21

Find an expression, in terms of *n*, for the *n*th term of this quadratic sequence.

(Total 3 marks)
Specimen Papers Set 1, Paper 2H qu.17 (A25 –AO1/AO2)

The diagram shows the first 10 sides of a spiral pattern.
It also gives the lengths, in cm, of the first 5 sides.



The lengths, in cm, of the sides of the spiral form a sequence.

Find an expression in terms of n for the length, in cm, of the nth side.

(Total 3 marks)

Mock Papers Set 1, Paper 1H qu.18 (A25 – AO2/AO3)

Exam Wizard topics

Linear sequences of diagrams
Linear sequences of numbers
Non-linear sequences
New to GCSE (9–1)

3. Ratio, proportion and rates of change

R1	change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

Content guidance

Any necessary conversions from metric units to imperial units will be given within the question.

E.g. A piece of wood has a mass of x kg and a volume of 0.002 m^3. Show that the density of the wood is $0.5x$ g/cm^3.

New to GCSE (9–1) Maths

Changing between related standard units and compound units in algebraic contexts.

New to Foundation tier

Changing between related compound units for density and pressure.

Mapping to 1MA0 specification content descriptors

GMp (F/H) Convert measurements from one unit to another
GMs (F/H) Understand and use compound measures

Learning objectives

Foundation tier (Units 8, 9a, 11a, 11b, 14)

- Convert between units of measure within one system, including time and metric units, to metric units of length, area, and volume and capacity, e.g. 1 ml = 1 cm^3;
- Draw distance–time graphs and velocity–time graphs;
- Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs; from distance–time graphs, calculate the speed of individual sections, total distance and total time;
- Write lengths, areas and volumes of two shapes as ratios in simplest form;
- Solve a ratio problem in context; use a ratio to convert between measures and currencies;
- Solve word problems involving direct and indirect proportion, including recipes;
- Understand and use density, pressure and speed: convert between metric speed measures; read values in km/h and mph from a speedometer; calculate average speed, distance and time; use kinematics formulae from the formulae sheet (with variables defined in the question).

Higher tier (Units 6a, 7a, 11)

- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections) including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Understand and use compound measures and convert: between metric speed measures; between density measures; and between pressure measures.
- Use a variety of metric measures to find the areas and perimeters of 2D shapes, including circles, parts of circles and compound shapes.

Sample questions

Carpet tiles are going to be used to cover a floor.

The floor is a 1200 mm by 1000 mm rectangle.
Each carpet tile is a 40 cm by 30 cm rectangle.

Exactly 10 carpets tiles can be used to cover the floor completely.

Show in a labelled sketch how this can be done.

(Total 3 marks)
New SAMs Paper 1F qu.15 (R1, G14, N2 – AO1/AO2)

Change 72 km/h into m/s.

(Total 3 marks)
New SAMs Paper 3F qu.16 (R1 – AO1)

A box exerts a force of 140 newtons on a table.
The pressure on the table is 35 newtons/m^2.

Calculate the area of the box that is in contact with the table.

$P = \frac{F}{A}$
p = pressure
F = force
A = area

(Total 3 marks)
Specimen Papers Set 1, Paper 1F qu.21 / 1H qu.2 (R1, R11, A2 – AO1)

Axel and Lethna are driving along a motorway.

They see a road sign.
The road sign shows the distance to Junction 8
It also shows the average time drivers take to get to Junction 8

To Junction 8 30 miles 26 minutes

The speed limit on the motorway is 70 mph.

Lethna says

"We will have to drive faster than the speed limit to drive 30 miles in 26 minutes."

Is Lethna right?
You must show how you get your answer.

(Total 3 marks)

New SAMs Paper 2F qu.26 / 2H qu.4 (R1, R11 – AO1/AO3)

Exam Wizard topics

Money calculations
Area
Volume
Conversions
Compound measures
New to GCSE (9–1)
New to Foundation from Higher in GCSE (9–1)

R2	use scale factors, scale diagrams and maps

Mapping to 1MA0 specification content descriptors

GMm (F/H) Use and interpret maps and scale drawings
GMu (F/H) Draw triangles and other 2D shapes using ruler and protractor

Learning objectives

Foundation tier (Units 11a, 15b)

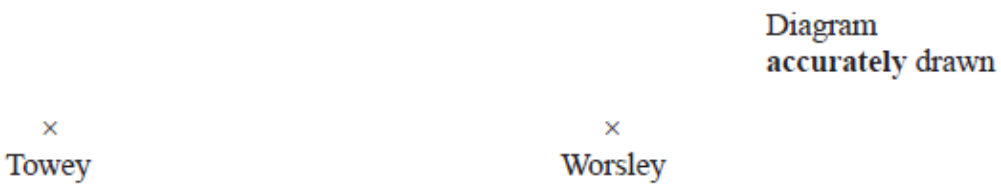
- Solve a ratio problem in context: use a ratio to compare a scale model to a real-life object;
- Use and interpret maps and scale drawings;
- Estimate lengths and give bearings between points on a scale diagram or map;
- Make an accurate scale drawing from a diagram, including for solving bearings problems.

Higher tier (Units 4b, 8b)

- Use a ratio to compare a scale model to a real-life object;
- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Estimate lengths using a scale diagram;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings;
- Use constructions to solve loci problems including with bearings.

Sample questions

The diagram below represents two towns on a map.



Scale: 1 cm represents 3 kilometres.

Work out the distance, in kilometres, between Towey and Worsley.

(Total 2 marks)
New SAMs Paper 2F qu.15 (R2 – AO1/AO2)

Exam Wizard topics

Maps and scale drawings

R3	express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

Mapping to 1MA0 specification content descriptors

No (F/H) Interpret fractions, decimals and percentages as operators

Learning objectives

Foundation tier (Units 4a, 11a)

- Write fractions to describe shaded parts of diagrams;
- Express a given number as a fraction of another, using very simple numbers, some cancelling and where the fraction is both < 1 and > 1;
- Understand that a percentage is a fraction in hundredths; express a given number as a percentage of another number;
- Write a ratio as a fraction.

Higher tier (Units 4a, 4b)

- Express a given number as a fraction of another number;
- Express a given number as a percentage of another number;
- Write a ratio as a fraction and as a linear function.

Sample questions

There are 35 pens in a box.
15 of the pens are green.
The rest of the pens are red.

(*a*) What fraction of the pens in the box are red?

(1)

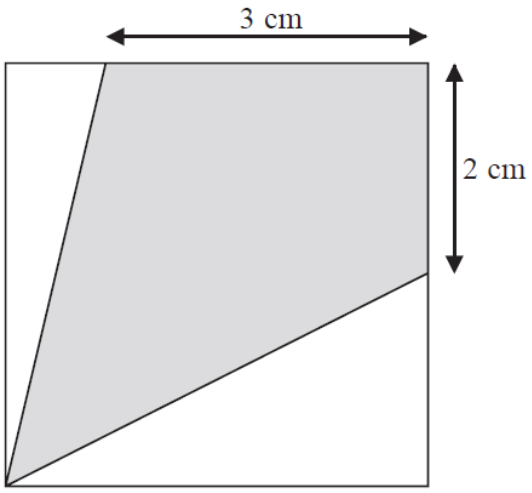
(*b*) Write down the ratio of the number of green pens to the number of red pens.
Give your ratio in its simplest form.

(2)

(Total 3 marks)

Specimen Papers Set 2, Paper 3F qu.15 (R3, R4, R5 –AO1)

The diagram shows a square with perimeter 16 cm.



Work out the proportion of the area inside the square that is shaded.

(Total 5 marks)

Specimen Papers Set 2, Paper 1F qu.20 / 1H qu.2 (R3, G16 –AO1/AO3)

Exam Wizard topics

Fractions

R4	use ratio notation, including reduction to simplest form

Mapping to 1MA0 specification content descriptors

Np (F/H) Use ratio notation, including reduction to its simplest form and its various links to fraction notation

Learning objectives

Foundation tier (Unit 11a)

- Understand and express the division of a quantity into a number of parts as a ratio;
- Write ratios in their simplest form; and in the form 1 : *m* or *m* : 1.

Higher tier (Unit 4b)

- Express the division of a quantity into a number of parts as a ratio;
- Write ratios in their simplest form, including three-part ratios; in the form 1 : *m* or *m* : 1; and to describe a situation.

Sample questions

There are 28 red pens and 84 black pens in a bag.

Write down the ratio of the number of red pens to the number of black pens.
Give your ratio in its simplest form.

(Total 2 marks)
Specimen Papers Set 2, Paper 1F qu.12 (R4 –AO1)

Frank, Mary and Seth shared some sweets in the ratio 4 : 5 : 7
Seth got 18 more sweets than Frank.

Work out the total number of sweets they shared.

(Total 3 marks)
New SAMs Paper 2F qu.23 / 2H qu.1 (R4, R5 – AO1/AO3)

Exam Wizard topics

Ratio

R5	divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)

Content guidance

To include division of a quantity into three (or more) parts.

Mapping to 1MA0 specification content descriptors

Np (F/H) Use ratio notation, including reduction to its simplest form and its various links to fraction notation
Nt (F/H) Divide a quantity in a given ratio

Learning objectives

Foundation tier (Unit 11a)

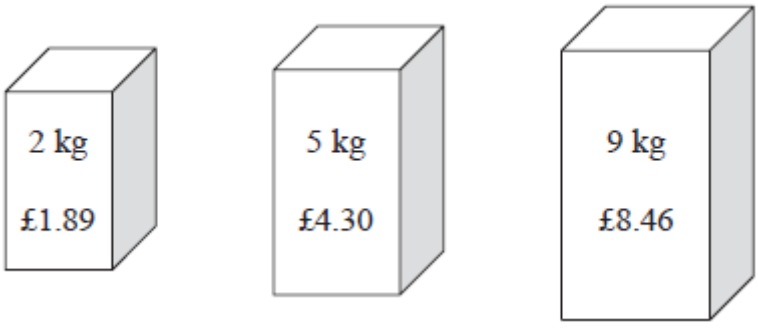
- Understand and express the division of a quantity into a number of parts as a ratio;
- Share a quantity in a given ratio including three-part ratios;
- Solve a ratio problem in context: find one quantity when the other is known; compare a scale model to a real-life object; convert between measures and currencies; in problems involving mixing, e.g. paint colours, cement and drawn conclusions;
- Compare ratios.

Higher tier (Unit 4b)

- Express the division of a quantity into a number of parts as a ratio;
- Divide a given quantity into two or more parts in a given part : part or part : whole ratio;
- Use a ratio to find one quantity when the other is known; compare a scale model to real-life object; convert between measures and currencies, e.g. £1.00 = €1.36.

Sample questions

Soap powder is sold in three sizes of box.



A 2 kg box of soap powder costs £1.89
A 5 kg box of soap powder costs £4.30
A 9 kg box of soap powder costs £8.46

Which size of box of soap powder is the best value for money?
You must show how you get your answer.

(Total 3 marks)
New SAMs Paper 2F qu.17 (R5, N2 – AO3)

Frank, Mary and Seth shared some sweets in the ratio 4 : 5 : 7
Seth got 18 more sweets than Frank.

Work out the total number of sweets they shared.

(Total 3 marks)
New SAMs Paper 2F qu.23 / 2H qu.1 (R5, R4 – AO1/AO3)

Asif is going on holiday to Turkey.
The exchange rate is £1 = 3.5601 lira.
Asif changes £550 to lira.

(*a*) Work out how many lira he should get.
Give your answer to the nearest lira.

(2)

Asif sees a pair of shoes in Turkey.
The shoes cost 210 lira.
Asif does not have a calculator.
He uses £2 = 7 lira to work out the approximate cost of the shoes in pounds.

(*b*) Use £2 = 7 lira to show that the approximate cost of the shoes is £60

(2)

(*c*) Is using £2 = 7 lira instead of using £1 = 3.5601 lira a sensible start to Asif's method to work out the cost of the shoes in pounds?
You must give a reason for your answer.

(1)

(Total 5 marks)
New SAMs Paper 3F qu.19 / 3H qu.2 (R5, R10 – AO1/AO2/AO3)

Exam Wizard topics

Ratio
Proportion

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

Content guidance

E.g. There are twice as many blue beads as red beads in a jar.
Write down the ratio of the number of blue beads to the number of red beads in the jar.

New to GCSE (9–1) Maths

The content in R6 is new to GCSE (9–1) Maths.

Learning objectives

Foundation tier (Units 10, 11a, 19a)

- Write a ratio as a fraction;
- Express a multiplicative relationship between two quantities as a ratio or a fraction;
- Describe and transform 2D shapes using enlargements by a positive integer and positive fractional scale factor;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides, simple integer scale factors or simple fractions.

Higher tier (Units 4b, 8a, 11, 12)

- Write a ratio as a fraction and as a linear function;
- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when $A : B$ are in the ratio $3 : 5$, A is $\frac{3}{5}B$. When $4a = 7b$, then $a = \frac{7b}{4}$ or $a : b$ is $7 : 4$;
- Describe and transform 2D shapes using enlargements by a positive integer, positive fractional and negative scale factor;
- Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fractional scale factors;
- Find areas after enlargement and compare with before enlargement to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
- Know and use the relationships between linear, area and volume scale factors of mathematically similar shapes and solids.

Sample questions

Mark asked each student in a different class to name their favourite colour.

For this class,
the number of students who said blue is three times the number of students who said green.

Write down the ratio of
the number of students who said blue to the number of students who said green.

(Total 1 mark)
Mock Papers Set 3, Paper 2F qu.5c (R6 – AO1)

In a box of pens, there are

three times as many red pens as green pens
and two times as many green pens as blue pens.

For the pens in the box, write down
the ratio of the number of red pens to the number of green pens to the number of blue pens.

(Total 2 marks)
New SAMs Paper 2H qu.8 (R6 – AO2)

Anna and Bill share some money in the ratio 2 : 5

Anna gets £A
Bill gets £B

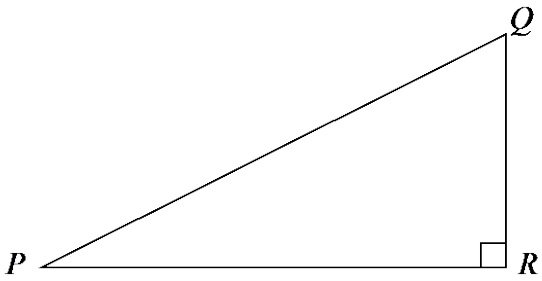
Carl and Donna share twice as much money as Anna and Bill share.
They share the money in the ratio 3 : 1

Carl gets £C
Donna gets £D

Find $A : B : C : D$
Give your answer in its simplest form.

(Total 3 marks)
Mock Papers Set 3, Paper 2H qu.11 (R6, R4 – AO1/AO3)

Here is triangle PQR.



The length of QR is 60% of the length of PR.

Find the value of sin QPR.
Give your answer correct to 3 significant figures.

(Total 3 marks)
Mock Papers Set 3, Paper 2H qu.12 (R6, G20 – AO1/AO3)

Exam Wizard topics

Fractions
Ratio
New to GCSE (9–1)

R7	understand and use proportion as equality of ratios

Mapping to 1MA0 specification content descriptors

Nt (F/H) Divide a quantity in a given ratio

Learning objectives

Foundation tier (Unit 11b)

- Understand and use proportion as equality of ratios.

Higher tier (Units 4b, 19b)

- Identify direct proportion from a table of values, by comparing ratios of values;
- Identify direct proportion from a table of values, by comparing ratios of values, for x squared and x cubed relationships.

Sample questions

There are 5 grams of fibre in every 100 grams of bread.

A loaf of bread has a weight of 400 g.
There are 10 slices of bread in a loaf.

Each slice of bread has the same weight.

Work out the weight of fibre in one slice of bread.

(Total 3 marks)
Specimen Papers Set 1, Paper 3F qu.15 (R7, R10 – AO1/AO3)

Exam Wizard topics

Ratio
Proportion

R8 relate ratios to fractions and to linear functions

Content guidance

E.g. Purple paint is made from using red paint and blue paint in the ratio 1 : 2.
Write an equation for y in terms of x to show the relationship between the amount of red paint (y) and the amount of blue paint (x).

New to GCSE (9–1) Maths

Relating ratios to linear functions.

Mapping to 1MA0 specification content descriptors

Np (F/H) Use ratio notation, including reduction to its simplest form and its various links to fraction notation

Learning objectives

Foundation tier (Unit 11a)

- Write a ratio as a fraction;
- Write a ratio as a linear function;
- Express a multiplicative relationship between two quantities as a ratio or a fraction.

Higher tier (Units 4b, 11)

- Write a ratio as a fraction and as a linear function;
- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when A : B are in the ratio 3 : 5, A is $\frac{3}{5}B$. When $4a = 7b$, then $a = \frac{7b}{4}$ or a : b is 7 : 4.

Sample questions

There are only black pens and green pens in a box.
The ratio of the number of black pens in the box to the number of green pens in the box is 2 : 5

What fraction of the pens are black?

(Total 1 mark)
New SAMs Paper 1F qu.10 (R8 – AO1)

There are only red counters, blue counters and green counters in a bag.

number of red counters : number of blue counters : number of green counters = 1 : 3 : 7

A counter is going to be taken at random from the bag.

Complete the table below to show each of the probabilities that the counter will be red or blue or green.

Colour	red	blue	green
Probability			

(Total 2 marks)
Mock Papers Set 3, Paper 3F qu.22a / 3H qu.4a (R8, P4 – AO2)

In a company, the ratio of the number of men to the number of women is 3 : 2

40% of the men are under the age of 25
10% of the women are under the age of 25

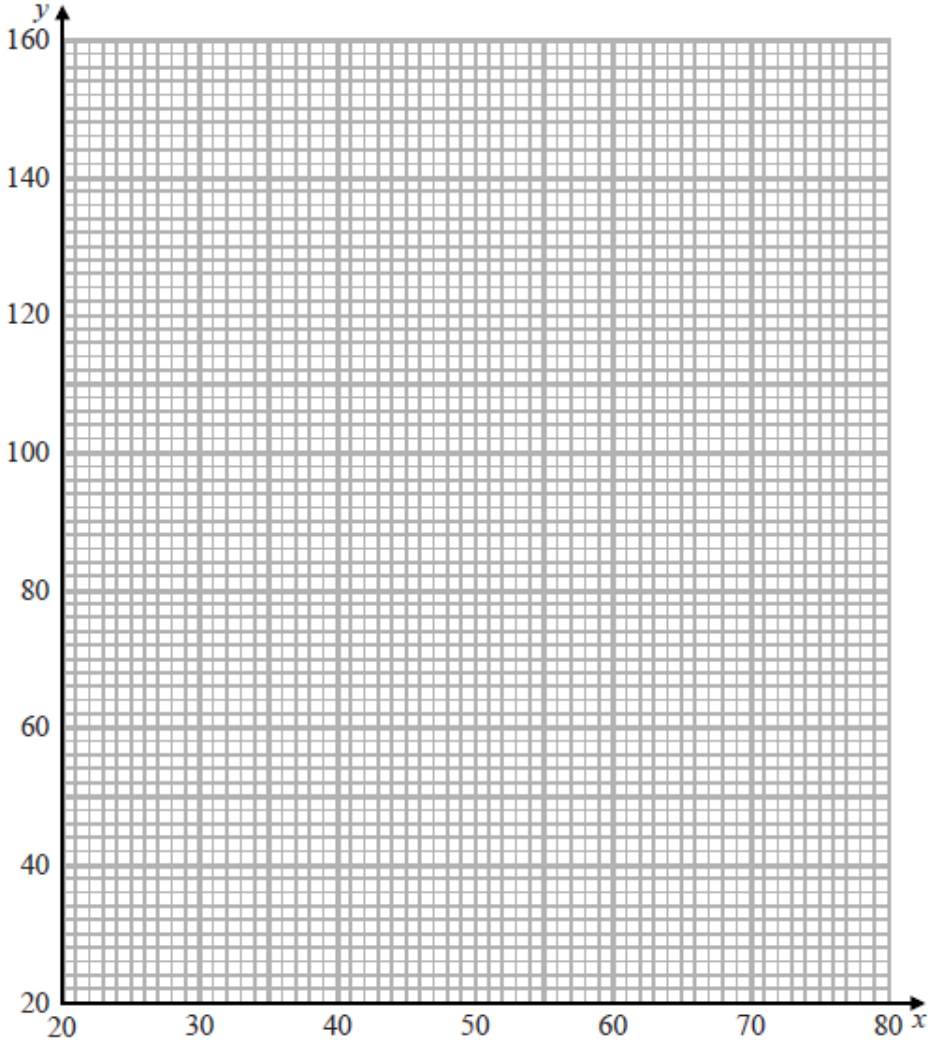
What percentage of all the people in the company are under the age of 25?

(Total 4 marks)

New SAMs Paper 1F qu.25 / 1H qu.5 (R8, R9 – AO1/AO3)

The height (x cm) and the width (y cm) of TVs are in the ratio 9 : 16

(*a*) Use this information to draw a graph to show the relationship between the height and the width of TVs.
Use values of x from 20 to 80



(2)

A TV has a width of 90 cm.

(*b*) Use your graph to work out the height of this TV.

(1)

(Total 3 marks)

Mock Papers Set 2, Paper 3F qu.21 / 3H qu.5 (R8, R14 – AO2)

$x^2 - 9y^2 = 0$ where $x > 0$ and $y > 0$

Work out the ratio $x : y$

(Total 3 marks)

Mock Papers Set 3, Paper 1H qu.17a (R8, A4 – AO1)

John has an empty box.
He puts some red counters and some blue counters into the box.

The ratio of the number of red counters to the number of blue counters is 1 : 4

Linda takes at random 2 counters from the box.

The probability that she takes 2 red counters is $\frac{6}{155}$

How many red counters did John put into the box?

(Total 4 marks)

New SAMs Paper 1H qu.24 (R8, A21, P9 – AO1/AO3)

Exam Wizard topics

Fractions
Ratio
New to GCSE (9–1)

R9	define percentage as ‘number of parts per hundred’; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics

New to Foundation tier

Finding percentage profit and loss, and using reverse percentages. (NB: finding the value of profit and loss is already in Foundation.)

Mapping to 1MA0 specification content descriptors

Nl (F/H) Understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions
Nm (F) Use percentage
Nm (H) Use percentage, repeated proportional change
No (F/H) Interpret fractions, decimals and percentages as operators
Nq (H) Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

Learning objectives

Foundation tier (Units 4a, 4b, 14)

- Understand that a percentage is a fraction in hundredths;
- Express a given number as a percentage of another number, in simple and more complex situations;
- Convert between fractions, decimals and percentages;
- Work with percentages over 100%;
- Find a percentage of a quantity or measurement (only measurements from Key Stage 3) with and without a calculator (50%, 25%, and multiples of 10% and 5%);
- Solve problems involving percentage change, including percentage increase/decrease, original value problems and comparisons of two quantities using percentages;
- Use percentages in real-life situations, including percentages greater than 100%: price after VAT (not price before VAT); value of profit or loss; simple interest; income tax calculations;
- Understand the multiplicative nature of percentages as operators; use a multiplier to find a percentage of a quantity and to increase or decrease by a percentage in any scenario where percentages are used;
- Calculate percentage profit or loss.

Higher tier (Unit 4a)

- Convert between fractions, decimals and percentages;
- Express a given number as a percentage of another number, including where the percentage is greater than 100%;
- Find a percentage of a quantity;
- Solve problems involving percentage change, including percentage increase/decrease (simple interest, income tax, value/percentage of profit or loss), original value problems and comparisons of two quantities using percentages (range of calculations and contexts such as those involving time or money);
- Find a percentage of a quantity using a multiplier; use a multiplier to increase or decrease by a percentage in any scenario where percentages are used; use calculators for reverse percentage calculations by doing an appropriate division;

- Use percentages in real-life situations, including percentages greater than 100%;
- Describe percentage increase/decrease with fractions, e.g. 150% increase means $2\frac{1}{2}$ times as big.

Sample questions

Work out 234% of 150

(Total 2 marks)
New SAMs Paper 3F qu.9 (R9 – AO1)

In a sale, normal prices are reduced by 20%.
The normal price of a coat is reduced by £15

Work out the normal price of the coat.

(Total 2 marks)
New SAMs Paper 1F qu.20 (R9 – AO1)

Katy invests £2000 in a savings account for 3 years.

The account pays compound interest at an annual rate of

2.5% for the first year

x % for the second year

x % for the third year

There is a total amount of £2124.46 in the savings account at the end of 3 years.

(*a*) Work out the rate of interest in the second year.

(4)

Katy goes to work by train.

The cost of her weekly train ticket increases by 12.5% to £225

(*b*) Work out the cost of her weekly train ticket before this increase.

(2)

(Total 6 marks)
New SAMs Paper 2H qu.10 (R9, N12, N1 – AO1/AO3)

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3. Ratio, proportion and rates of change

In a shop, all normal prices are reduced by 20% to give the sale price.

The sale price of a TV set is then reduced by 30%.

Mary says,

> " 30 + 20 = 50, so this means that the normal price of the TV set has been reduced by 50%."

Is Mary right?
You must give a reason for your answer.

(Total 2 marks)
New SAMs Paper 1H qu.15 (R9 – AO2)

Exam Wizard topics

Money calculations
Compound interest and multipliers
Fractions
Decimals
Percentages
New to Foundation from Higher in GCSE (9–1)

R10	solve problems involving direct and inverse proportion, including graphical and algebraic representations

New to Foundation tier

Solving problems involving direct and inverse proportion.

Mapping to 1MA0 specification content descriptors

Nn (H) Understand and use direct and indirect proportion
As (F/H) Discuss, plot and interpret graphs (which may be non-linear) modelling real situations
Au (H) Direct and inverse proportion

Learning objectives

Foundation tier (Units 11b, 14, 20)

- Solve word problems involving direct and inverse proportion, with a variety of measures: bets value; scaling up recipes; currency conversion; rates of pay;
- Find amounts for 3 people when amount for 1 given; solve proportion problems using the unitary method;
- Understand inverse proportion: as x increases, y decreases;
- Understand direct proportion → relationship $y = kx$;
- Interpret equations that describe direct and inverse proportion;
- Solve problems involving inverse proportion using graphs, and read values from graphs;
- Use graphical representations of inverse proportion to solve problems in context.

Higher tier (Units 4b, 6b, 11, 19b)

- Scale up recipes and convert between currencies;
- Solve proportion problems using the unitary method, including best value and rates of pay;
- Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
- Recognise and interpret graphs showing direct and inverse proportion; use a graph to find the value of k in $y = kx$; use graphs to solve inverse proportion problems by plotting and reading values;
- Set up and use equations to solve word and other problems involving direct proportion and inverse proportion, and relate algebraic solutions to graphical representation of the equations.

Sample questions

The total weight of 3 tins of beans and 4 jars of jam is 2080 g.
The total weight of 5 tins of beans is 2000 g.
Work out the weight of 1 tin of beans and the weight of 1 jar of jam.

(Total 4 marks)
New SAMs Paper 3F qu.14 (R10 – AO1/AO3)

Jane made some almond biscuits which she sold at a fete.

She had:
5 kg of flour
3 kg of butter
2.5 kg of icing sugar
320 g of almonds

Here is the list of ingredients for making 24 almond biscuits.

Ingredients for 24 almond biscuits
150 g flour 100 g butter 75 g icing sugar 10 g almonds

Jane made as many almonds biscuits as she could, using the ingredients she had.

Work out how many almond biscuits she made.

(Total 3 marks)
New SAMs Paper 2F qu.19 (R10 – AO1/AO3)

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d is inversely proportional to c

When $c = 280$, $d = 25$

Find the value of d when $c = 350$

(Total 3 marks)
New SAMs Paper 2H qu.13 (R10, R13 – AO1)

Exam Wizard topics

Proportion
Derive expressions; equations; formulae
Conversion graphs
Line graphs of real situations
Direct and inverse proportion
New to Foundation from Higher in GCSE (9–1)

R11	use compound units such as speed, rates of pay, unit pricing, density and pressure

New to Foundation tier

Using density and pressure. (NB: average speed is already in Foundation.)

Mapping to 1MA0 specification content descriptors

GMs (F/H) Understand and use compound measures

Learning objective)

Foundation tier (Units 9a, 14)

- Draw distance–time graphs and velocity–time graphs;
- Interpret distance–time graphs and calculate: the speed of individual sections, total distance and total time;
- Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs;
- Understand and use compound measures density and pressure;
- Understand and use the compound measure speed: convert between metric speed measures; read values in km/h and mph from a speedometer; calculate average speed, distance, time – in miles per hour as well as metric measures; use kinematics formulae from the formulae sheet to calculate speed, acceleration (with variables defined in the question); change d/t in m/s to a formula in km/h, i.e. $d/t \times (60 \times 60) \div 1000$ – with support.

Higher tier (Units 6a, 11, 19a)

- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections), including average speed, distance, time, acceleration;
- Use kinematics formulae from the formulae sheet to calculate speed, acceleration etc. (with variables defined in the question);
- Understand and use compound measures and: convert between metric speed, density and pressure measures;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
  - for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
  - for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord.

Sample questions

One day Sally earned £60.
She worked for 8 hours.

Work out Sally's hourly rate of pay.

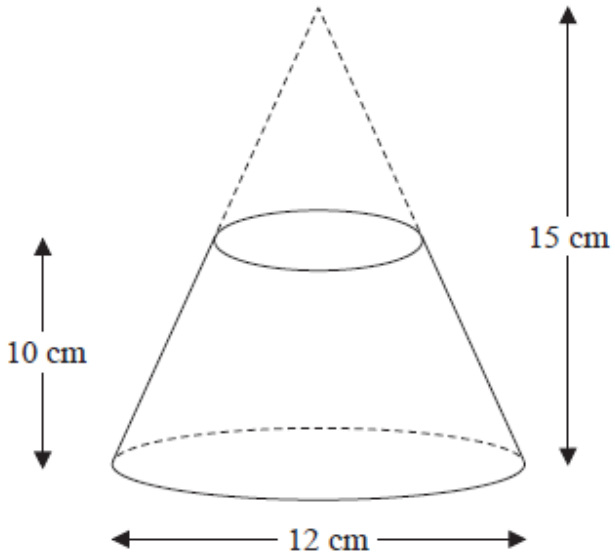
(Total 2 marks)
New SAMs Paper 1F qu.7 (R11 – AO1)

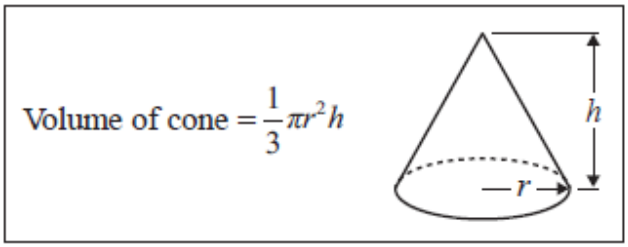
$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$

Find the pressure exerted by a force of 900 newtons on an area of 60 cm^2.
Give your answer in newtons/m^2.

(Total 2 marks)
New SAMs Paper 1H qu.12 (R11, R1 – AO1)

A frustum is made by removing a small cone from a large cone as shown in the diagram.





The frustum is made from glass.
The glass has a density of 2.5 g/cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

(Total 5 marks)
New SAMs Paper 2H qu.22 (R11, N15, G17, G19 – AO1/AO3)

Exam Wizard topics

Money calculations
Conversions
Compound measures
New to Foundation from Higher in GCSE (9–1)

R12	compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

Mapping to 1MA0 specification content descriptors

GMl (F) Describe and transform 2D shapes using single or combined rotations, reflections, translations or enlargements by a positive scale factor, and distinguish properties that are preserved under particular transformations

Learning objectives

Foundation tier (Units 10, 11a, 12, 19a)

- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides, simple integer scale factors or simple fractions;
- Write/interpret a ratio to describe a situation;
- Write lengths, areas and volumes of two shapes as ratios in simplest form;
- Solve problems to find missing lengths in similar shapes;
- Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures.

Higher tier (Units 5b, 12)

- Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;
- Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
- Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures.

Sample questions

Gary drove from London to Sheffield.
It took him 3 hours at an average speed of 80 km/h.

Lyn drove from London to Sheffield.
She took 5 hours.

Assuming that Lyn
drove along the same roads as Gary
and did not take a break,

(*a*) work out Lyn's average speed from London to Sheffield.

(3)

(*b*) If Lyn did **not** drive along the same roads as Gary, explain how this could affect your answer to part (*a*).

(1)

(Total 4 marks)

New SAMs Paper 1F qu.24 / 1H qu.4 (R12, R11 – AO1/AO3)

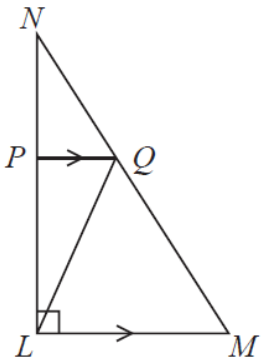
Solid **A** and solid **B** are mathematically similar.
The ratio of the surface area of solid **A** to the surface area of solid **B** is 4 : 9

The volume of solid **B** is 405 cm^3.

Show that the volume of solid **A** is 120 cm^3.

(Total 3 marks)
New SAMs Paper 1H qu.18 (R12, G19 – AO2)

LMN is a right-angled triangle.



Angle *NLM* = 90°.
PQ is parallel to *LM*.

The area of triangle *PNQ* is 8 cm^2.
The area of triangle *LPQ* is 16 cm^2.

Work out the area of triangle *LQM*.

(Total 4 marks)
Specimen Papers Set 2, Paper 1H qu.18 (R12, G19 – AO1/AO3)

Exam Wizard topics

Enlargement
Congruence and similarity
Trigonometry

R13 understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$;

construct and interpret equations that describe direct and inverse proportion

Content guidance

At Foundation tier it is appropriate to test y is directly proportional to x or $1/x$ only.

Note that *constructing* equations that describe inverse and direct proportion is in bold in the spec and so is at higher only.

New to Foundation tier

Understanding and interpreting direct and inverse proportion.

Mapping to 1MA0 specification content descriptors

Au (H) Direct and inverse proportion

Learning objectives

Foundation tier (Units 11b, 14)

- Understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$;
- Understand and interpret equations that describe direct and inverse proportion.

Higher tier (Units 11, 19b)

- Use $y = kx$ to solve direct proportion problems, including questions where students find k and then use k to find another value;
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion, and relate algebraic solutions to graphical representation of the equations.

Sample questions

d is inversely proportional to c

When $c = 280$, $d = 25$

Find the value of d when $c = 350$

(Total 3 marks)

New SAMs Paper 2H qu.13 (R13, R10 – AO1)

Exam Wizard topics

Derive expressions; equations; formulae
Direct and inverse proportion
New to Foundation from Higher in GCSE (9–1)

R14	interpret the gradient of a straight-line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

New to GCSE (9–1) Maths

Interpreting the gradient of a straight line graph as a rate of change.

Mapping to 1MA0 specification content descriptors

As (F/H) Discuss, plot and interpret graphs (which may be non-linear) modelling real situations

Learning objectives

Foundation tier (Units 9a, 11b, 20)

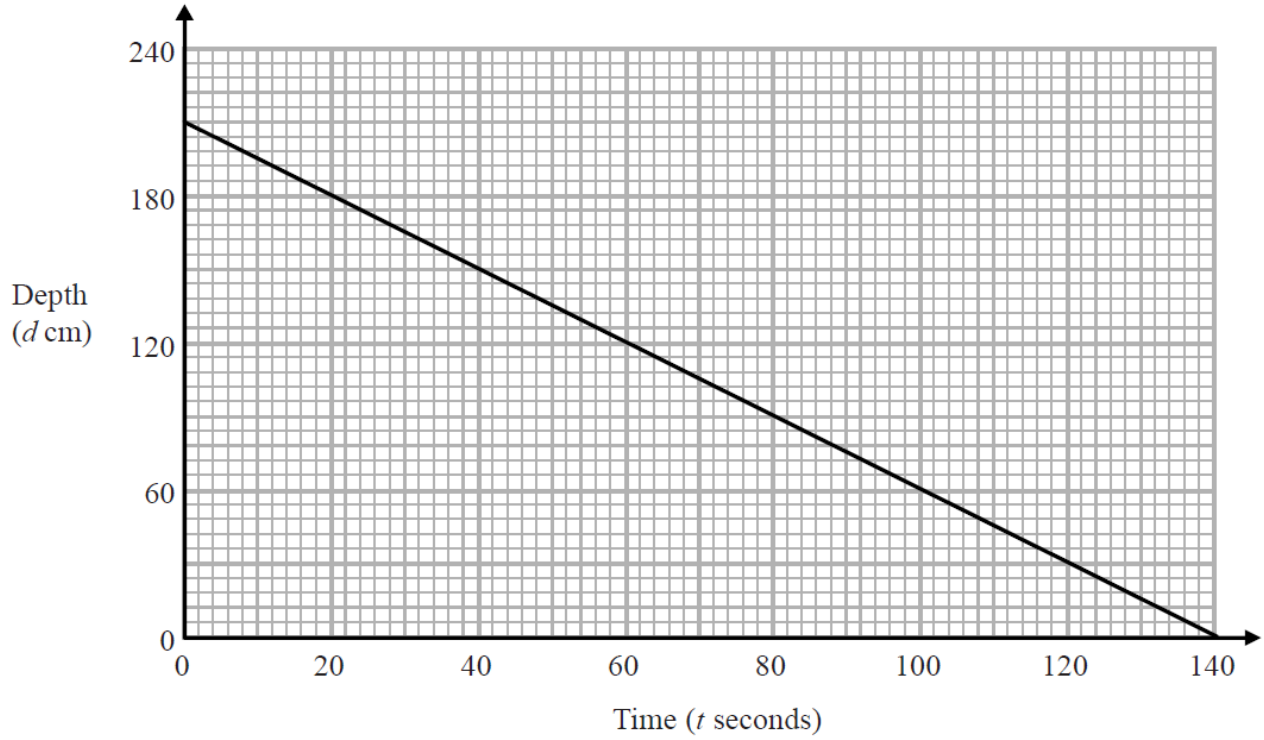
- Interpret distance–time graphs and calculate: the speed of individual sections, total distance and total time;
- Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs;
- Recognise when values are in direct proportion by reference to the graph form;
- Solve problems involving inverse proportion using graphs, and read values from graphs.

Higher tier (Units 6b, 11, 19a, 19b)

- Recognise and interpret graphs showing direct and inverse proportion; use a graph to find the value of k in $y = kx$; use graphs to solve inverse proportion problems by plotting and reading values;
- Interpret the rate of change of graphs of containers filling and emptying; of unit price in price graphs.

Sample questions

The graph shows the depth, d cm, of water in a tank after t seconds.



(a) Find the gradient of this graph.

(2)

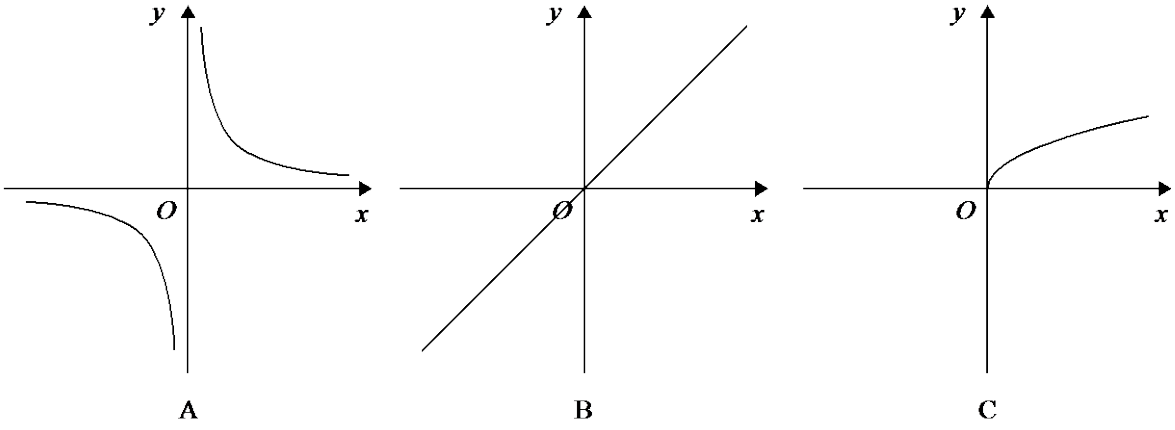
(b) Explain what this gradient represents.

(1)

(Total 3 marks)

Specimen Papers Set 1, Paper 2H qu.10 (R14 – AO1/AO2)

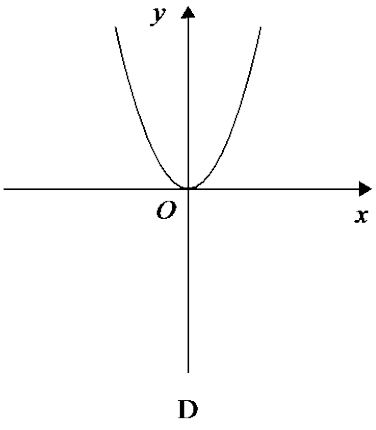
Here are five graphs.
Each graph shows either direct proportion or inverse proportion.



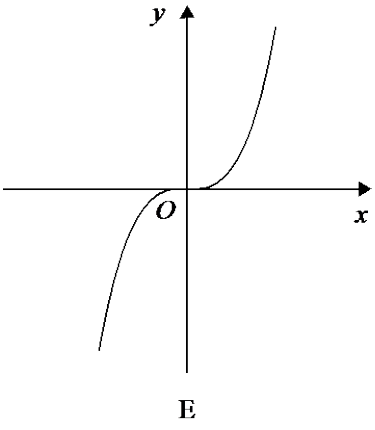
A

B

C



D



E

The table shows five equations.

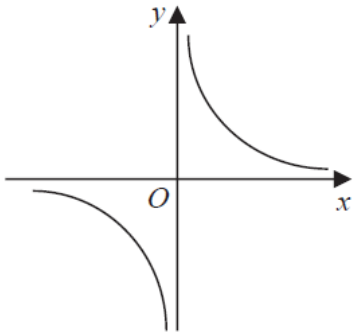
Equation	Graph
$y = kx^3$	.....
$y = k\sqrt{x}$	.....
$y = kx^2$	.....
$y = \frac{k}{x}$	.....
$y = kx$	.....

Match the letter of each graph to its equation.

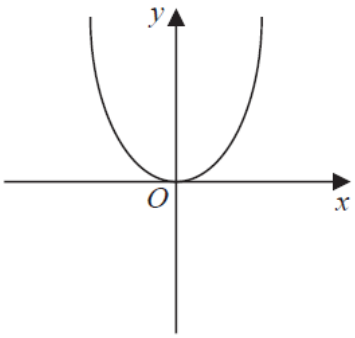
(Total 3 marks)

Mock Papers Set 3, Paper 2H qu.13 (R14 – AO2)

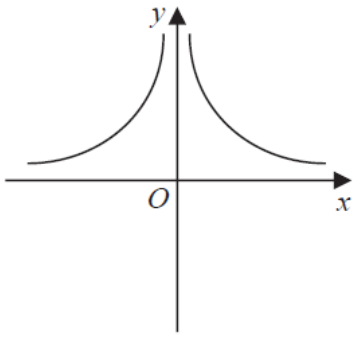
These graphs show four different proportionality relationships between y and x.



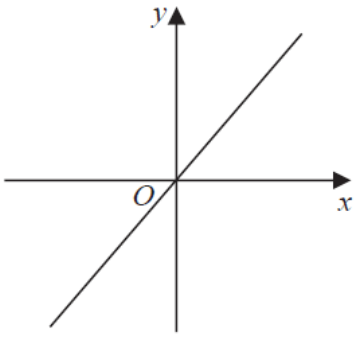
Graph A



Graph B



Graph C



Graph D

Match each graph with a statement in the table below.

Proportionality relationship	**Graph letter**
y is directly proportional to x	
y is inversely proportional to x	
y is proportional to the square of x	
y is inversely proportional to the square of x	

(Total 2 marks)

Specimen Papers Set 1, Paper 1H qu.16 (R14 – AO2)

Exam Wizard topics

Rates of change
Distance-time/travel graphs
Conversion graphs
Line graphs of real situations
Direct and inverse proportion
New to GCSE (9–1)

R15	**interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts (this does not include calculus)**

Content guidance

Unless the method of solution is specified in the question then any correct method, including calculus, is acceptable.

However, be aware that questions do not always state the equation of the curve under consideration so candidates may have to use methods other than calculus.

New to GCSE (9–1) Maths

The content in R15 is new to GCSE (9–1) Maths.

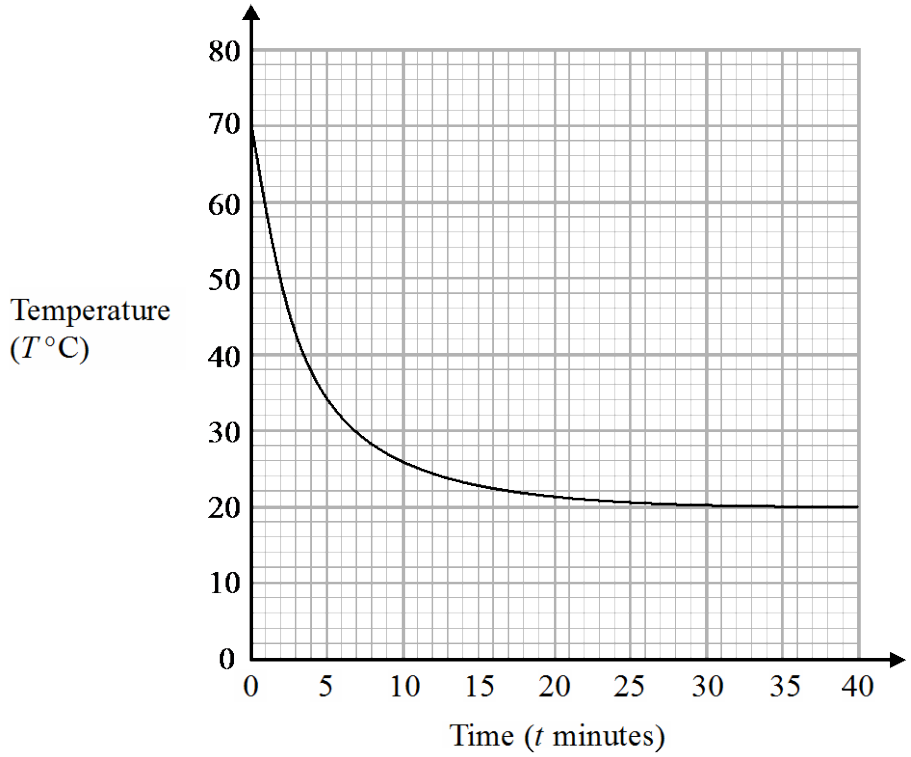
Learning objectives

Higher tier (Unit 19a)

- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
  - for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
  - for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts.

Sample questions

The graph shows the temperature, T °C, of the coffee in a cup at a time t minutes.



(*a*) Find an estimate for the gradient of the graph at time 5 minutes.

(2)

(*b*) Explain what this gradient represents.

(1)

(Total 3 marks)

Mock Papers Set 3, Paper 1H qu.13 (R15, A15 – AO1/AO2)

The quantity of heat, H calories, delivered by an electric current, I amps, acting for t seconds to heat an amount of water is given by the formula

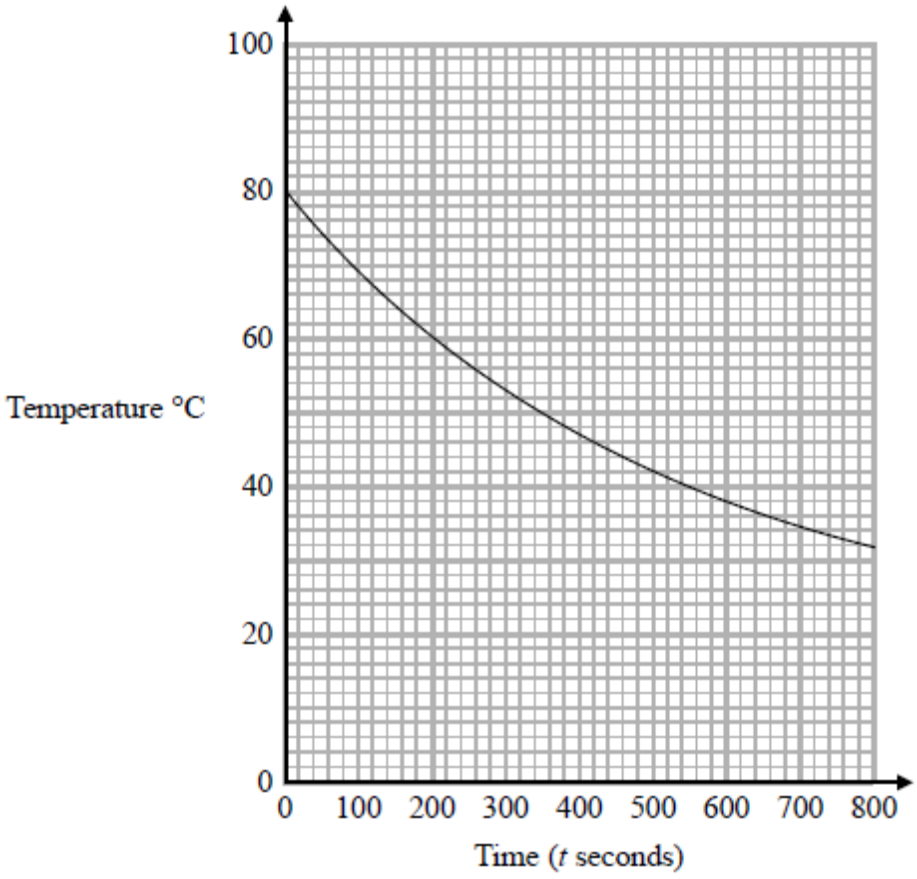
$$H = atI^2 - b$$

where a and b are constants.

(*a*) Rearrange the formula to make I the subject.

(2)

The graph gives information about the variation in the temperature, in °C, of an amount of water that is allowed to cool from 80 °C.



(*b*) (i) Work out the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$.

The instantaneous rate of decrease of the temperature of the water at time T seconds is equal to the average rate of decrease of the temperature of the water between $t = 0$ and $t = 800$.

(ii) Find an estimate for the value of T.
You must show how you got your answer.

(4)

(Total 6 marks)

Original SAMs Paper 3H qu.14 (R15, R14, A5 – AO1/AO2/AO3)

Exam Wizard topics

Rates of change
Line graphs of real situations
Gradients of graphs and area under graphs
New to GCSE (9–1)

R16 set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes**

Content guidance

General iterative processes: for example, population growth or decay.

Given $N_{t+1} = kN_t$ and $N_0 = 120$, find the value of N_3.

E.g. $P_{n+1} = kP_n$.

New to GCSE (9–1) Maths

Working with general iterative processes.

New to Foundation tier

Set up, solve and interpret the answers in compound interest and depreciation problems.

Mapping to 1MA0 specification content descriptors

Nm (H) Use percentage, repeated proportional change
No (H) Interpret fractions, decimals and percentages as operators

Learning objectives

Foundation tier (Unit 14)

- Make calculations involving repeated percentage change, not using the formula;
- Use compound interest;
- Set up, solve and interpret the answers in growth and decay problems.

Higher tier (Units 2a, 11, 15, 19a)

- Represent repeated proportional change using a multiplier raised to a power; use this to solve problems involving compound interest and depreciation;
- Use calculators to explore exponential growth and decay;
- Set up, solve and interpret the answers in growth and decay problems;
- Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations;
- Use iteration with simple converging sequences.

Sample questions

The number of slugs in a garden t days from now is p_t where

$p_0 = 100$

$p_{t+1} = 1.06p_t$

Work out the number of slugs in the garden 3 days from now.

(Total 3 marks)
Specimen Papers Set 1, Paper 2H qu.13 (R16 – AO1/AO3)

At the start of year n, the quantity of a radioactive metal is P_n
At the start of the following year, the quantity of the same metal is given by

$$P_{n+1} = 0.87P_n$$

At the start of 2016 there were 30 grams of the metal.

What will be the quantity of the metal at the start of 2019?
Give your answer to the nearest gram.

(Total 3 marks)
Mock Papers Set 3, Paper 3H qu.17 (R16 – AO1)

A virus on a computer is causing errors.
An antivirus program is run to remove these errors.

An estimate for the number of errors at the end of t hours is $10^6 \times 2^{-t}$

(*a*) Work out an estimate for the number of errors on the computer at the end of 8 hours.

(2)

(*b*) Explain whether the number of errors on this computer ever reaches zero.

(1)

(Total 3 marks)
Specimen Papers Set 1, Paper 3H qu.15 (R16 – AO1/AO2/AO3)

Louis and Robert are investigating the growth in the population of a type of bacteria.
They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.
The population of bacteria grows exponentially at the rate of 50% per day.

(*a*) Show that the population of bacteria in flask A at the start of each day forms a geometric progression.

(2)

The population of bacteria in flask A at the start of the 10th day is k times the population of bacteria in flask A at the start of the 6th day.

(*b*) Find the value of k.

(2)

At the start of day 1 there are 1000 bacteria in flask B.
The population of bacteria in flask B grows exponentially at the rate of 30% per day.

(*c*) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

(1)

(Total 5 marks)
New SAMs Paper 3H qu.17 (R16, A12 – AO1/AO2/AO3)

The number of bees in a beehive at the start of year n is P_n.
The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.

How many bees will there be in the beehive at the start of 2018?

(Total 3 marks)

Specimen Papers Set 1, Paper 2H qu.21 (R16 – AO3)

Exam Wizard topics

Money calculations
Compound interest and multipliers
Percentages
Proportion
Iteration
New to GCSE (9–1)
New to Foundation from Higher in GCSE (9–1)

4. Geometry and measures

Properties and constructions

G1	use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description

Content guidance

Symmetry does not exist as a topic within the new GCSE (9–1), so there will no questions asking students about the number of lines of symmetry or the order of rotation symmetry. However, symmetry could be used to describe a shape.

Students will have to carry out the transformations of reflection and rotation (see G7).

Mapping to 1MA0 specification content descriptors

GMa (F) Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines and opposite angles at a vertex
GMb (F) Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals
GMb (H) Understand and use the angle properties of parallel lines, triangles and quadrilaterals
GMc (F/H) Calculate and use the sums of the interior and exterior angles of polygons
GMv (F/H) Use straight edge and a pair of compasses to do constructions
GMu (F) Draw triangles and other 2D shapes using ruler and protractor

Learning objectives

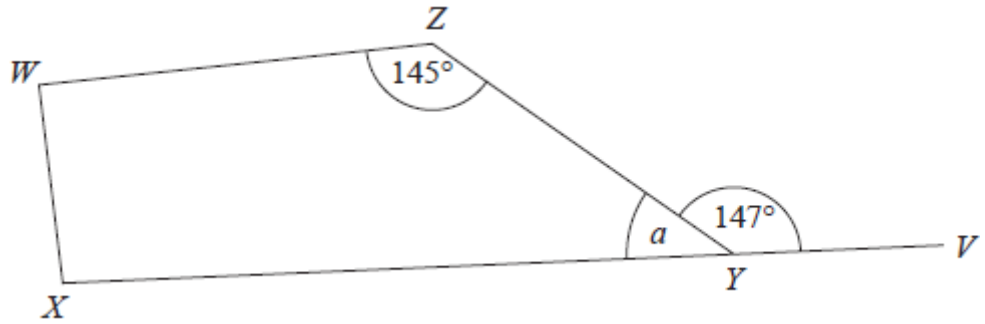
Foundation tier (Units 3b, 6a, 6b, 10, 15a)

- Use geometric language appropriately;
- Use letters to identify points, lines and angles; and use two-letter notation for a line and three-letter notation for an angle;
- Recognise and name pentagons, hexagons, heptagons, octagons and decagons; understand ‘regular’ and ‘irregular’ as applied to polygons;
- Find missing sides and angles in geometrical problems using conventional terms and notation, including when diagram needs to be drawn from written description;
- Understand that reflections are specified by a mirror line;
- Know the terms face, edge and vertex;
- Draw circles and arcs to a given radius or given the diameter;
- Measure and draw lines to the nearest mm and angles to the nearest degree;
- Draw sketches of 2D shapes and 3D solids;
- Make accurate drawings of triangles and other 2D shapes using a ruler and a protractor;
- Construct diagrams of everyday 2D situations involving rectangles, triangles, perpendicular and parallel lines.

Higher tier (Units 5a, 7a, 7b)

- Understand 'regular' and 'irregular' as applied to polygons;
- Find missing sides and angles in geometrical problems using conventional terms and notation, including when diagram needs to be drawn from written description;
- Recall the definition of a circle, and name and draw parts of a circle;
- Draw sketches of 3D solids.

Sample questions



WXYZ is a quadrilateral.
XYV is a straight line.

(*a*) (i) Find the size of the angle marked *a*.
(ii) Give a reason for your answer.

(2)

Angle *ZWX* = angle *WXY*

(*b*) Work out the size of angle *ZWX*.

(2)

(Total 4 marks)
New SAMs Paper 3F qu.13 (G1, G3 – AO1/ AO3)

Exam Wizard topics

Geometric reasoning
Properties of 2D shapes
Parallel lines
Constructions

G2	use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

Content guidance

To include the locus of points equidistant from a given point; the locus of points that are a given distance from a line.

New to Foundation tier

Know that the perpendicular distance from a point to a line is the shortest distance to the line.

Mapping to 1MA0 specification content descriptors

GMv (F/H) Use straight edge and a pair of compasses to do constructions
GMw (F/H) Construct loci

Learning objectives

Foundation tier (Unit 15b)

- Use straight edge and a pair of compasses to do standard constructions: perpendicular bisector of a given line; perpendicular from a point to a line; bisector of a given angle; angles of 90°, 45°;
- Draw and construct diagrams from given instructions including: a region bounded by a circle and an intersecting line; a given distance from a point and a given distance from a line; equal distances from two points or two line segments; regions defined by 'nearer to' or 'greater than';
- Solve 2D locus problems using constructions; including bearings; regions satisfying a combination of loci.

Higher tier (Unit 8b)

- Use the standard ruler and compass constructions: perpendicular bisector of a given line; perpendicular from a point to a line; bisector of a given angle; angles of 90°, 45°;
- Construct: a region bounded by a circle and an intersecting line; a given distance from a point and a given distance from a line; equal distances from two points or two line segments; regions defined by 'nearer to' or 'greater than';
- Solve locus problems using constructions; including bearings; regions satisfying a combination of loci, including in 3D;
- Know that the perpendicular distance from a point to a line is the shortest distance to the line.

Sample questions

The diagram shows a rectangle *ABCD*.

In the space below, use a ruler and a pair of compasses to construct a right-angled triangle equal in area to the area of the rectangle *ABCD*.

You must show all your construction lines.

The base of the triangle, which is equal in length to the side *CD*, has been drawn for you.

(Total 3 marks)

Original SAMs Paper 3F qu.17 / 3H qu.6 (G2, G16 – AO2)

Exam Wizard topics

Constructions
Loci
New to Foundation from Higher in GCSE (9–1)

G3	apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

Content guidance

To include the sum of interior angles of polygons and the exterior angles of polygons.

New to Foundation tier

Use basic angle properties in more complex problems.

Mapping to 1MA0 specification content descriptors

GMa (F/H) Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines and opposite angles at a vertex
GMb (F) Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals
GMb (H) Understand and use the angle properties of parallel lines, triangles and quadrilaterals
GMc (F/H) Calculate and use the sums of the interior and exterior angles of polygons

Learning objectives

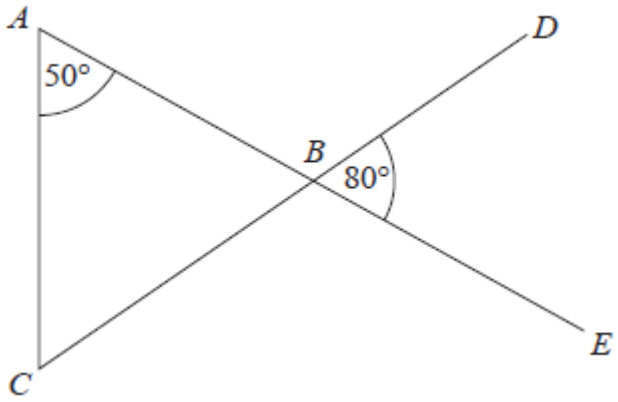
Foundation tier (Units 6a, 6b)

- Identify parallel lines on a diagram and understand and use their properties; find missing angles using properties of corresponding and alternate angles;
- Recall and use properties of angles at a point, angles at a point on a straight line, right angles and vertically opposite angles; understand and use the angle properties of intersecting lines;
- Derive and use the sum of angles in a triangle; find a missing angle in a triangle; use the symmetry property of an isosceles triangle to show that base angles are equal;
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Calculate and use the sums of the interior angles of polygons; the angles of regular polygons;
- Use the sum of angles of irregular polygons; the sum of the interior angles of an *n*-sided polygon; the sum of the exterior angles of any polygon is 360°; the sum of the interior angle and the exterior angle is 180°;
- Explain why some polygons fit together and others do not.

Higher tier (Units 5a, 8b)

- Understand the proof that the angle sum of a triangle is 180°, and derive and use the sum of angles in a triangle; use the symmetry property of an isosceles triangle to show that base angles are equal; find missing angles in a triangle using the angle sum in a triangle *and* the properties of an isosceles triangle;
- Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
- Calculate bearings and solve bearings problems;
- Calculate and use the sums of the interior angles of polygons; use the sum of angles in a triangle; use the angle sum in any polygon; to derive the properties of regular polygons;
- Use the angle sums of irregular polygons; the sum of the exterior angles of any polygon is 360°; the sum of the interior angles of an *n*-sided polygon; the sum of the interior angle and the exterior angle is 180°;
- Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon; calculate the angles of regular polygons and use these to solve problems;
- Use angle facts to demonstrate how shapes would ‘fit together’ and work out interior angles of shapes in a pattern.

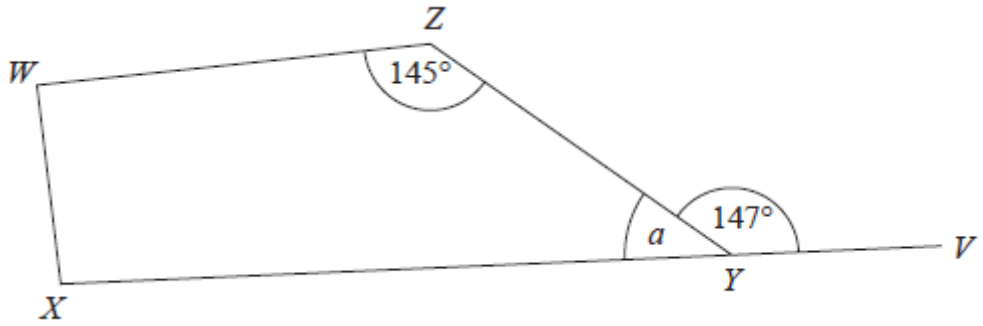
Sample questions



ABE and *CBD* are straight lines.

Show that triangle *ABC* is an isosceles triangle.
Give a reason for each stage of your working.

(Total 4 marks)
New SAMs Paper 2F qu.13 (G1, G3, G4 – AO1/AO2)



WXYZ is a quadrilateral.
XYV is a straight line.

(*a*) (i) Find the size of the angle marked *a*.
(ii) Give a reason for your answer.

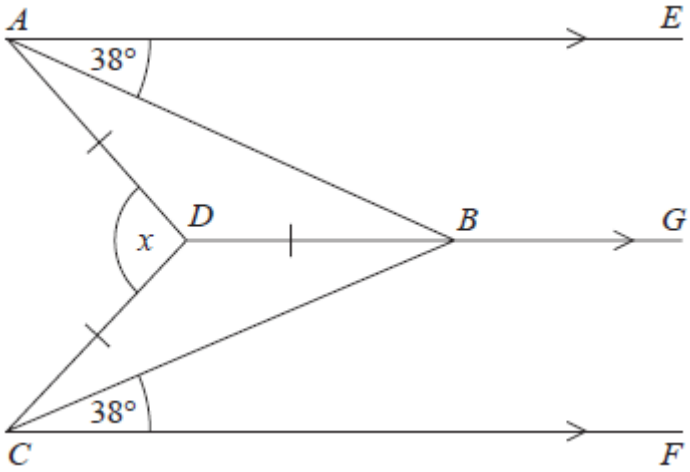
(2)

Angle *ZWX* = angle *WXY*

(*b*) Work out the size of angle *ZWX*.

(2)

(Total 4 marks)
New SAMs Paper 3F qu.13 (G3, G1 – AO1/AO3)



AE, *DBG* and *CF* are parallel.
DA = *DB* = *DC*.
Angle *EAB* = angle *BCF* = 38°

Work out the size of the angle marked *x*.
You must show your working.

(Total 3 marks)
New SAMs Paper 1F qu.23 / 1H qu.3 (G3, G6 –AO2)

Exam Wizard topics

Geometric reasoning
Properties of 2D shapes
Exterior and interior angles
Parallel lines
New to Foundation from Higher in GCSE (9–1)

G4	derive and apply the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

Mapping to 1MA0 specification content descriptors

GMb (F) Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals
GMb (H) Understand and use the angle properties of parallel lines, triangles and quadrilaterals
GMd (F/H) Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus

Learning objectives

Foundation tier (Unit 6a)

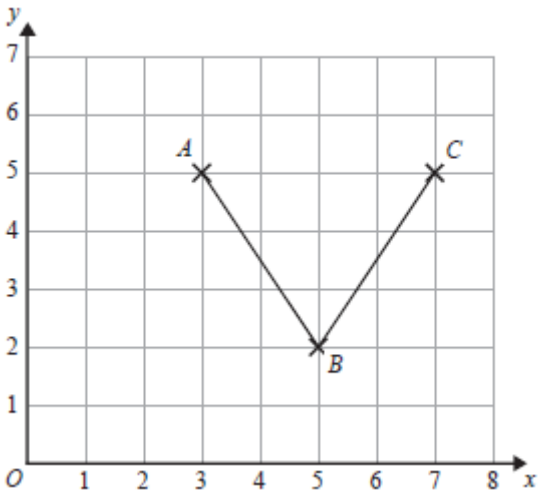
- Recall and use the properties and definitions of special types of quadrilaterals, including symmetry properties;
- Classify quadrilaterals by their geometric properties and name all quadrilaterals that have a specific property;
- Identify quadrilaterals from everyday usage;
- Given some information about a shape on coordinate axes, complete the shape;
- Understand and use the angle properties of quadrilaterals.

Higher tier (Unit 5a)

- Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles.

Sample questions

Here is a grid showing the points *A*, *B* and *C*.



(*a*) Write down the coordinates of the point *A*.

(1)

(*b*) On the grid, mark with a cross (×) the point (1, 2).
Label this point *D*.

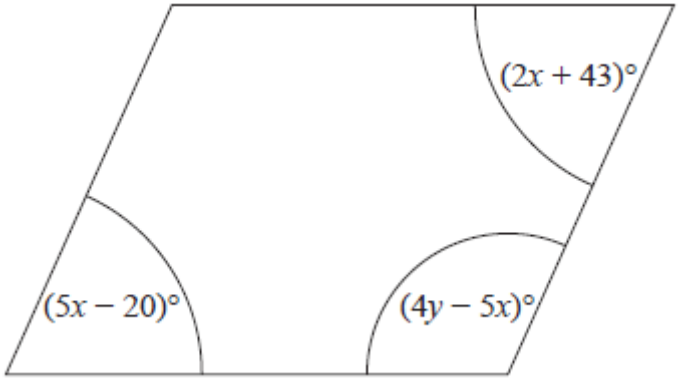
(1)

(*c*) On the grid, mark with a cross (×) a point *E*, so that the quadrilateral *ABCE* is a kite.

(1)

(Total 3 marks)
New SAMs Paper 3F qu.4 (G4, A8 – AO1)

Here is a parallelogram.



Work out the value of *x* and the value of *y*.

(Total 5 marks)
New SAMs Paper 1F qu.28 / 1H qu.8 (G4, A21 – AO1/AO3)

Exam Wizard topics

Geometric reasoning
Properties of 2D shapes
Parallel lines

G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

Content guidance

The requirement to prove two triangles are congruent is in Higher tier only.

New to Foundation tier

Understand and use congruence.

Mapping to 1MA0 specification content descriptors

GMf (F/H) Understand congruence and similarity
GMv (F/H) Use straight edge and a pair of compasses to do constructions

Learning objectives

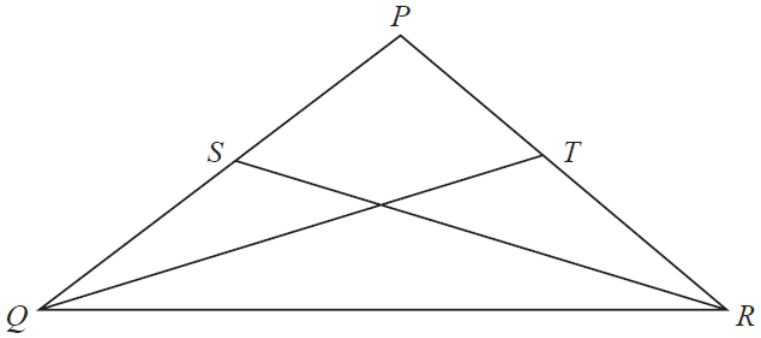
Foundation tier (Units 6b, 15b, 19a)

- Visually identify shapes which are congruent;
- Understand congruence as two shapes that are the same size and shape;
- Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not;
- Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS);
- Solve angle problems involving congruence.

Higher tier (Unit 12)

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions;
- Solve angle problems by first proving congruence.

Sample questions

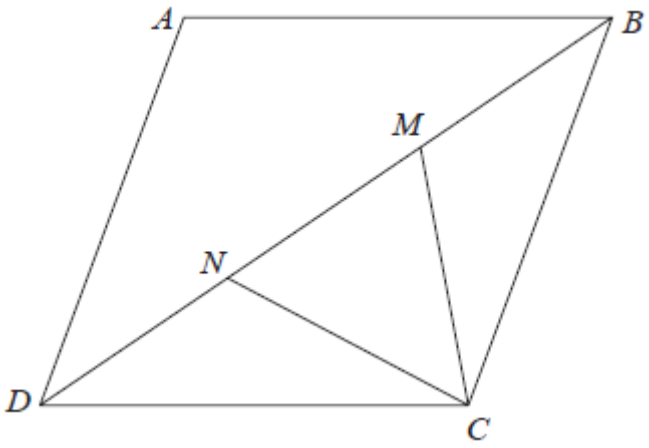


$PQ = PR$.
S is the midpoint of PQ.
T is the midpoint of PR.

Prove triangle QTR is congruent to triangle RSQ.

(Total 3 marks)
Specimen Papers Set 1, Paper 1H qu.17 (G5 – AO2)

ABCD is a rhombus.



M and *N* are points on *BD* such that $DN = MB$.
Prove that triangle *DNC* is congruent to triangle *BMC*.

(Total 3 marks)
New SAMs Paper 3H qu.13 (G5, G4 – AO2)

Exam Wizard topics

Properties of 2D shapes
Constructions
Congruence and similarity
New to Foundation from Higher in GCSE (9–1)

G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

Content guidance

At Higher tier, to include proving that two triangles are similar.

New to Foundation tier

Understand and use congruence and similarity.

Mapping to 1MA0 specification content descriptors

GMf (F/H) Understand congruence and similarity
GMg (F) Use Pythagoras' theorem in 2D
GMg (H) Use Pythagoras' theorem in 2D and 3D

Learning objectives

Foundation tier (Units 6a, 12, 19a)

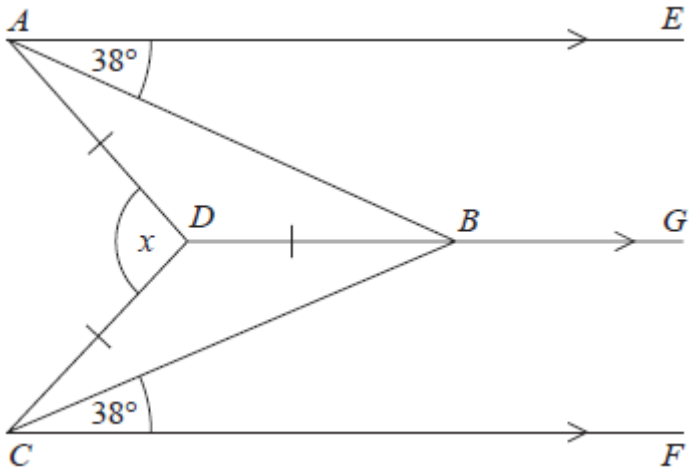
- Use geometrical language appropriately, give reasons for angle calculations and show step-by-step deduction when solving problems;
- Understand and use the angle properties of parallel lines; find missing angles using properties of corresponding and alternate angles;
- Recall and use properties of angles at a point; angles at a point on a straight line; right angles; vertically opposite angles/intersecting lines;
- Understand, derive and use the sum of angles in a triangle; find a missing angle in a triangle, using the angle sum of a triangle is 180°; use the symmetry property of an isosceles triangle to show that base angles are equal; use the side/angle properties of isosceles and equilateral triangles;
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Understand and use the angle properties of quadrilaterals; angle sum of a quadrilateral is 360°;
- Understand, recall and use Pythagoras' theorem in 2D to calculate the length of the hypotenuse and of a shorter side, including leaving answers in surd form; justify if a triangle is right-angled or not;
- Apply Pythagoras' theorem with a triangle drawn on a coordinate grid; to calculate the length of a line segment AB given pairs of points;
- Solve angle problems involving congruence;
- Understand similarity of triangles and of other plane shapes; use this to make geometric inferences; solve angle and length problems using similarity.

Higher tier (Units 5a, 5b, 12)

- Understand the proof that the angle sum of a triangle is 180°, and derive and use the sum of angles in a triangle; use symmetry property of an isosceles triangle to show that base angles are equal; find missing angles in a triangle using the angle sum in a triangle *and* the properties of an isosceles triangle;
- Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Explain why the angle sum of a quadrilateral is 360°; use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is 360°;
- Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
- Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;

- Understand, recall and use Pythagoras' theorem in 2D to calculate the length of the hypotenuse and of a shorter side, in a range of units and including decimals, and leaving answers in surd form; justify if a triangle is right-angled or not;
- Calculate the length of a line segment AB given pairs of points;
- Solve angle problems by first proving congruence;
- Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
- Find missing lengths, areas and volumes in similar 3D solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

Sample questions



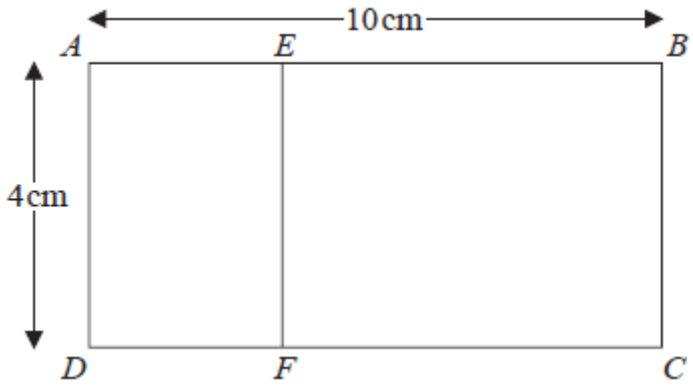
AE, *DBG* and *CF* are parallel.
DA = *DB* = *DC*.
Angle *EAB* = angle *BCF* = 38°

Work out the size of the angle marked *x*.
You must show your working.

(Total 3 marks)

New SAMs Paper 1F qu.23 / 1H qu.3 (G6, G3 – AO2)

Rectangle *ABCD* is mathematically similar to rectangle *DAEF*.



AB = 10 cm.
AD = 4 cm.

Work out the area of rectangle *DAEF*.

(Total 3 marks)
New SAMs Paper 1H qu.13 (G6, G13 – AO1/AO3)

Exam Wizard topics

Geometric reasoning
Congruence and similarity
Pythagoras
New to Foundation from Higher in GCSE (9–1)

G7	identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)

New to Foundation tier

Using fractional scale enlargements in transformations. (NB: negative scale factors are still in Higher tier only.)

Mapping to 1MA0 specification content descriptors

GMe (F/H) Recognise reflection and rotation symmetry of 2D shapes
GMl (F) Describe and transform 2D shapes using single or combined rotations, reflections, translations or enlargements by a positive scale factor, and distinguish properties that are preserved under particular transformations
GMl (H) Describe and transform 2D shapes using single or combined rotations, reflections, translations or enlargements by a positive, fractional or negative scale factor, and distinguish properties that are preserved under particular transformations

Learning objectives

Foundation tier (Unit 10)

- Understand that rotations are specified by a centre, an angle and a direction of rotation; identify correct rotations from a choice of diagrams; describe fully and transform 2D shapes after rotation about the origin or any other point, including rotations on a coordinate grid;
- Understand that translations are specified by a distance and direction using a vector; describe and transform 2D shapes on a coordinate grid using translation by a vector;
- Understand that reflections are specified by a mirror line; identify correct reflections from a choice of diagrams; identify the equation of a line of symmetry; describe and transform 2D shapes using single reflections (including those not on coordinate grids) with vertical, horizontal and diagonal mirror lines;
- Scale a shape on a grid (without a centre specified); understand that an enlargement is specified by a centre and a scale factor; find the centre of enlargement by drawing; describe and transform 2D shapes on a coordinate grid using enlargement with (0, 0) as the centre of enlargement and with a centre other than (0, 0), with a positive integer and fractional scale factor;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides: simple integer scale factors or simple fractions;
- Understand that distances and angles are preserved under rotations, translations and reflections, so that any figure is congruent under either of these transformations;
- Understand that similar shapes are enlargements of each other and angles are preserved.

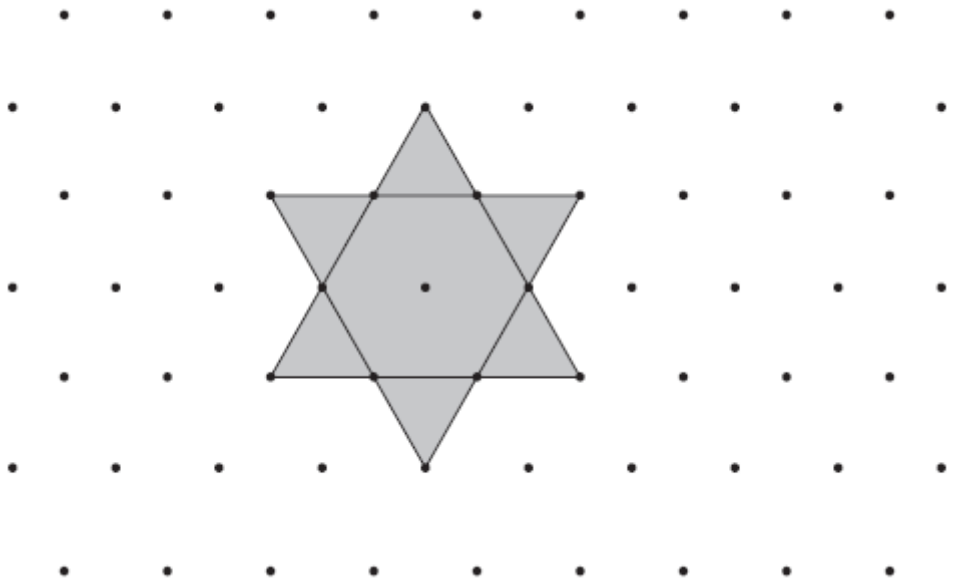
Higher tier (Unit 8a)

- Understand that rotations are specified by a centre, an angle and a direction of rotation; recognise, describe and transform 2D shapes after rotation about the origin or any other point (not necessarily on a coordinate grid);
- Understand that translations are specified by a distance and direction using a vector; recognise, describe and transform 2D shapes on a coordinate grid using translation by a vector;
- Identify the equation of a line of symmetry; recognise, describe and transform 2D shapes using single reflections with vertical, horizontal and diagonal mirror lines; include the mirror line as a simple algebraic equation, $x = a$, $y = a$, $y = x$, $y = -x$, and lines not parallel to the axes;
- Enlarge a shape on a grid (without a centre specified); know that an enlargement is specified by a centre and a scale factor; find the centre of enlargement by drawing; describe and transform 2D shapes using a given centre of enlargement with a positive integer, positive fractional and negative scale factor;
- Find areas after enlargement and compare with before enlargement to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);

- Distinguish properties that are preserved under particular transformations;
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations.

Sample questions

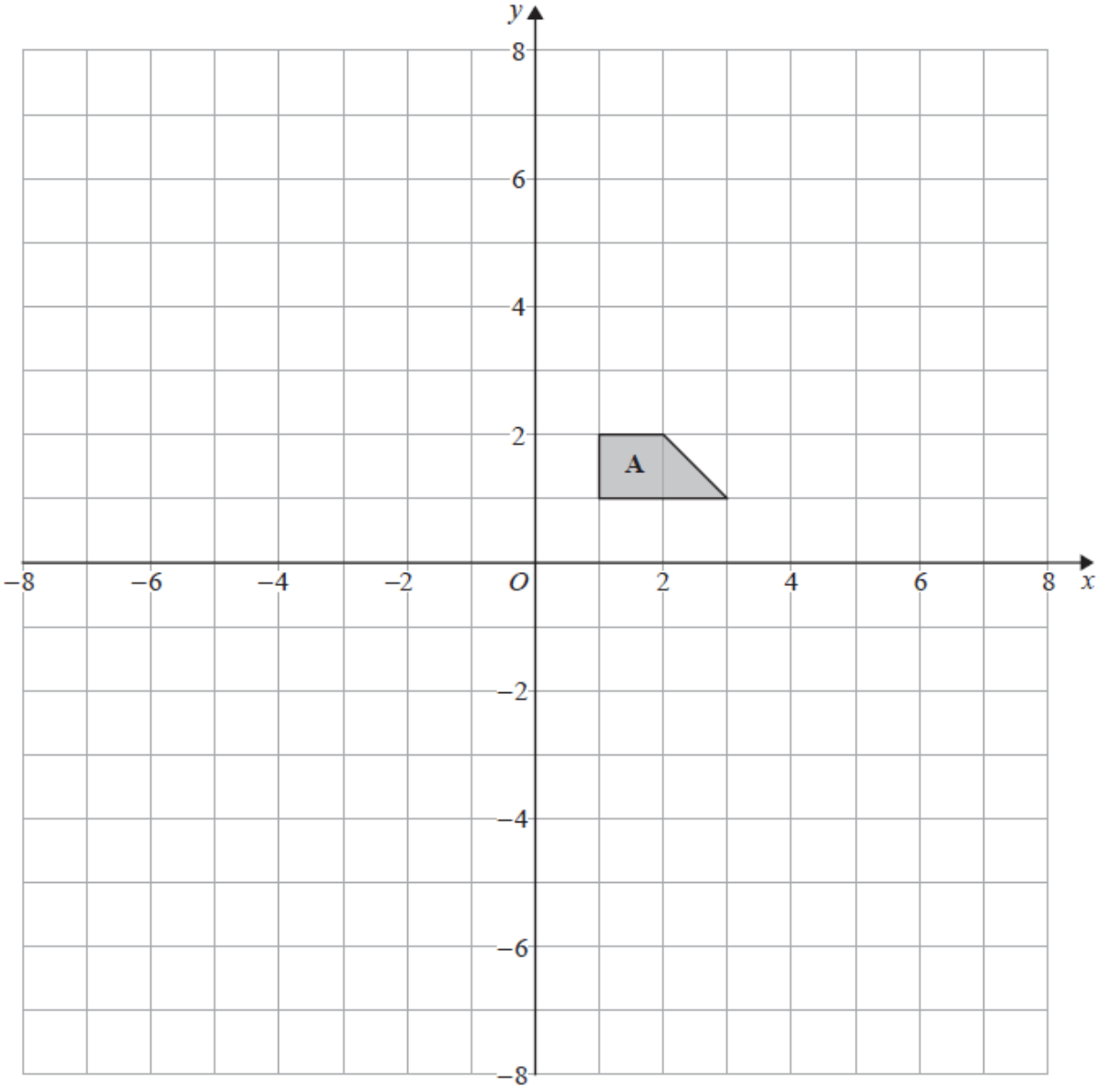
Here is a star shape.



The star shape is made from a regular hexagon and six congruent equilateral triangles.
The area of the star shape is 96 cm^2.
Work out the area of the regular hexagon.

(Total 2 marks)

New SAMs Paper 3F qu.12 (G7, G4 – AO1/AO3)



(*a*) Enlarge shape **A** by scale factor −2, centre (0, 0)
Label your image **B.**

(2)

(*b*) Describe fully the single transformation that will map shape **B** onto shape **A**.

(1)

(Total 3 marks)
New SAMs Paper 1H qu.20 (G7, G8 – AO1/AO2)

Exam Wizard topics

Properties of 2D shapes
Reflection
Rotation
Translation
Enlargement
Congruence and similarity
New to Foundation from Higher in GCSE (9–1)

G8 describe the changes and invariance achieved by combinations of rotations, reflections and translations

Content guidance

E.g. Describe the single transformation that would map A onto C given that shape A is mapped onto shape B by a reflection in the *x*-axis.
Shape B is mapped onto shape C by a reflection in the line y = –2.
Given the above mappings, would shape C be in the same orientation as shape A?
If the triangle with vertices at coordinates (0, 0) (0, 2) (2, 0) is reflected in the *y*-axis, which vertices will stay in the same position?

New to Foundation tier

NB: combinations of rotations, reflections and translations are now in Higher tier only.

Mapping to 1MA0 specification content descriptors

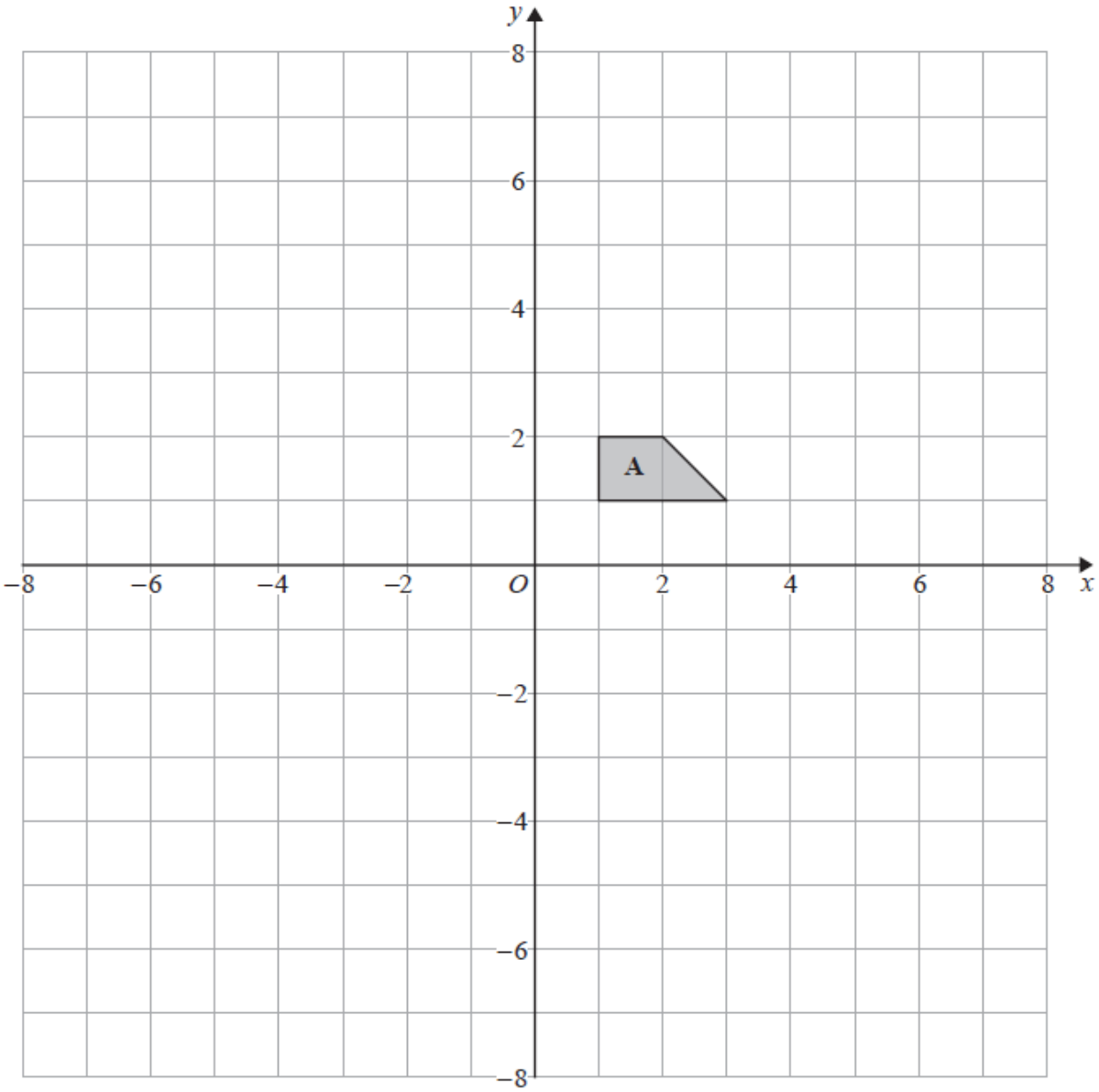
GMl (H) Describe and transform 2D shapes using single or combined rotations, reflections, translations or enlargements by a positive, fractional or negative scale factor, and distinguish properties that are preserved under particular transformations

Learning objectives

Higher tier (Unit 8a)

- Understand the effect of one translation followed by another in terms of column vectors (to introduce vectors in a concrete way);
- Describe and transform 2D shapes using combined rotations, reflections, translations or enlargements;
- Describe the changes and invariance achieved by combinations of rotations, reflections and translations.

Sample questions



(*a*) Enlarge shape **A** by scale factor −2, centre (0, 0)
Label your image **B.**

(2)

(*b*) Describe fully the single transformation that will map shape **B** onto shape **A**.

(1)

(Total 3 marks)
New SAMs Paper 1H qu.20 (G8, G7 – AO1/AO2)

Exam Wizard topics

Reflection
Rotation
Translation
Enlargement
Congruence and similarity

G9	identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

Mapping to 1MA0 specification content descriptors

GMi (F/H) Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

Learning objectives

Foundation tier (Units 15a, 17)

- Understand clockwise and anticlockwise;
- Draw circles and arcs to a given radius or given the diameter;
- Recall the definition of a circle and identify, name and draw parts of a circle including tangent, chord and segment.

Higher tier (Units 7a, 16a)

- Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment.

Sample questions

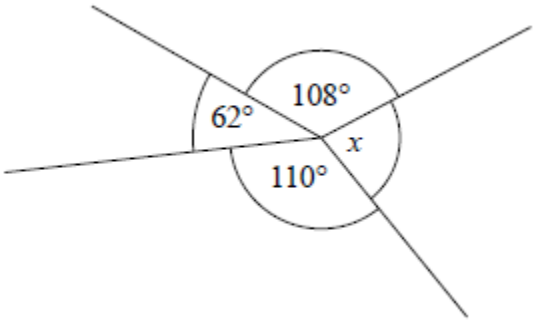
(*a*) Complete the following sentences.

(i) A cuboid has six .....

(ii) A ..... is a straight line from the centre of a circle to its circumference.

(2)

(*b*)



Explain clearly why angle *x* cannot be a right angle.

(2)

(Total 4 marks)

Original SAMs Paper 3F qu.2 (G9, G1, G3 – AO1/AO2)

Exam Wizard topics

Circles

G10 apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

Mapping to 1MA0 specification content descriptors

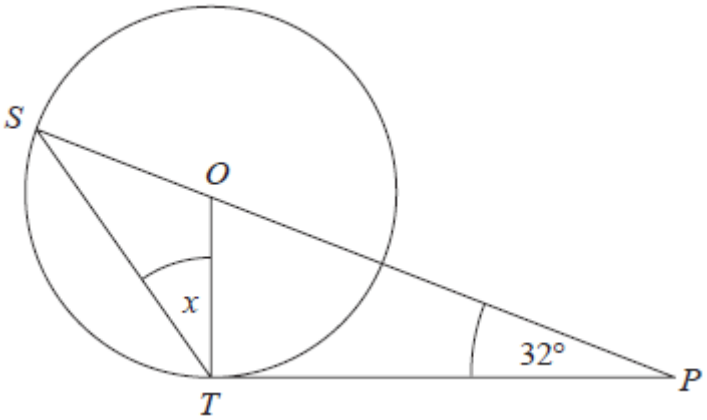
GMj (H) Understand and construct geometrical proofs using circle theorems

Learning objectives

Higher tier (Unit 16a)

- Prove and use the facts that:
  - the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
  - the angle in a semicircle is a right angle;
  - the perpendicular from the centre of a circle to a chord bisects the chord;
  - angles in the same segment are equal;
  - alternate segment theorem;
  - opposite angles of a cyclic quadrilateral sum to 180°;
- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
- Find and give reasons for missing angles on diagrams using:
  - circle theorems;
  - isosceles triangles (radius properties) in circles;
  - the fact that the angle between a tangent and radius is 90°;
  - the fact that tangents from an external point are equal in length.

Sample questions

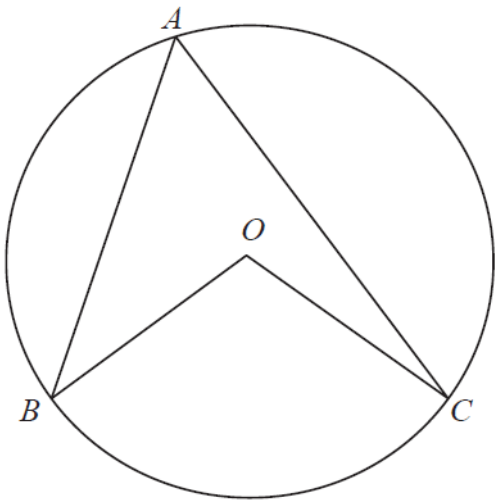


S and *T* are points on the circumference of a circle, centre *O*.
PT is a tangent to the circle.
SOP is a straight line.
Angle *OPT* = 32°

Work out the size of the angle marked *x*.
You must give a reason for each stage of your working.

(Total 4 marks)
New SAMs Paper 2H qu.11 (G10 – AO2)

A, *B* and *C* are points on the circumference of a circle centre *O*.



Prove that angle *BOC* is twice the size of angle *BAC*.

(Total 4 marks)

Specimen Papers Set 1, Paper 2H qu.24 (G10 – AO2)

Exam Wizard topics

Circle theorems

G11 solve geometrical problems on coordinate axes

Content guidance

2D coordinates only.

Mapping to 1MA0 specification content descriptors

Ak (F/H) Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

Learning objectives

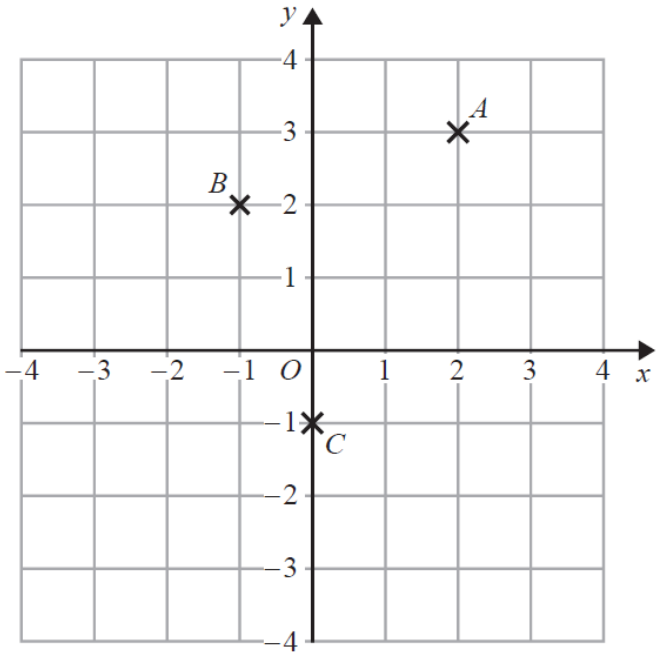
Foundation tier (Units 6a, 9a)

- Given some information about a shape on coordinate axes, complete the shape;
- Find the coordinates of points identified by geometrical information in 2D (all four quadrants).

Higher tier (Units 5a, 13b)

- Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
- Solve geometrical problems on coordinate axes.

Sample questions



(*a*) Write down the coordinates of point *C*.

(1)

ABCD is a square.

(*b*) On the grid, mark with a cross (X) the point *D* so that *ABCD* is a square.

(1)

(*c*) Write down the coordinates of the midpoint of the line segment *BC*.

(1)

(Total 3 marks)

Specimen Papers Set 1, Paper 2F qu.14 (G11, A8, G4 – AO1/AO3)

Exam Wizard topics

Coordinates in 2D

G12	identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres

Mapping to 1MA0 specification content descriptors

GMk (F/H) Use 2D representations of 3D shapes

Learning objectives

Foundation tier (Units 8, 15a)

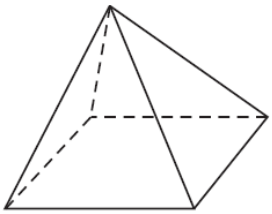
- Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone;
- Sketch nets of cuboids and prisms;
- Draw sketches of 3D solids;
- Know the terms face, edge and vertex;
- Identify and sketch planes of symmetry of 3D solids.

Higher tier (Unit 7b)

- Draw sketches of 3D solids and identify planes of symmetry of 3D solids, and sketch planes of symmetry.

Sample questions

Here is a square-based pyramid.



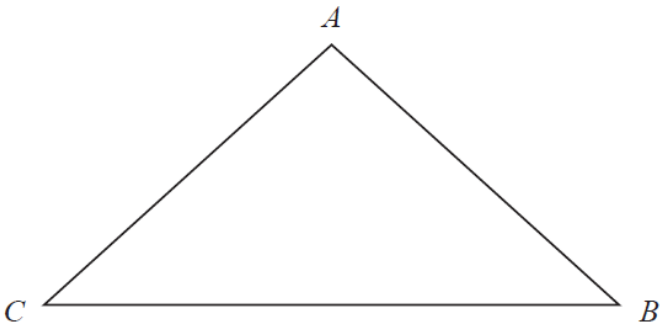
(i) How many faces does the pyramid have?

(ii) How many edges does the pyramid have?

(Total 2 marks)

Specimen Papers Set 2, Paper 1F qu.4 (G12 – AO1)

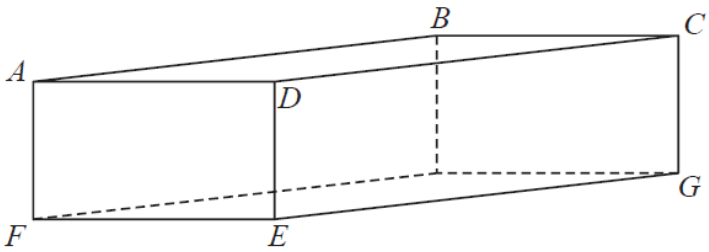
Here is a triangle *ABC*.



(*a*) Mark, with the letter *y*, the angle *CBA*.

(1)

Here is a cuboid.



Some of the vertices are labelled.

(*b*) Shade in the face *CDEG*.

(1)

(*c*) How many edges has a cuboid?

(1)

(Total 3 marks)

Specimen Papers Set 1, Paper 3F qu.14 (G12, G1 – AO1)

Exam Wizard topics

Recognise and draw 3D shapes

G13 construct and interpret plans and elevations of 3D shapes

Mapping to 1MA0 specification content descriptors

GMk (F/H) Use 2D representations of 3D shapes

Learning objectives

Foundation tier (Unit 15a)

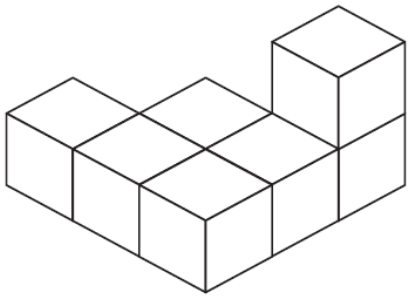
- Draw sketches of 3D solids;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid.

Higher tier (Units 7b, 8b)

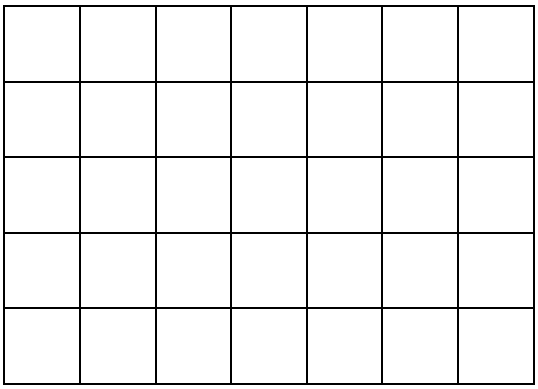
- Draw sketches of 3D solids and identify planes of symmetry of 3D solids, and sketch planes of symmetry;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid.

Sample questions

The diagram represents a solid made from seven centimetre cubes.



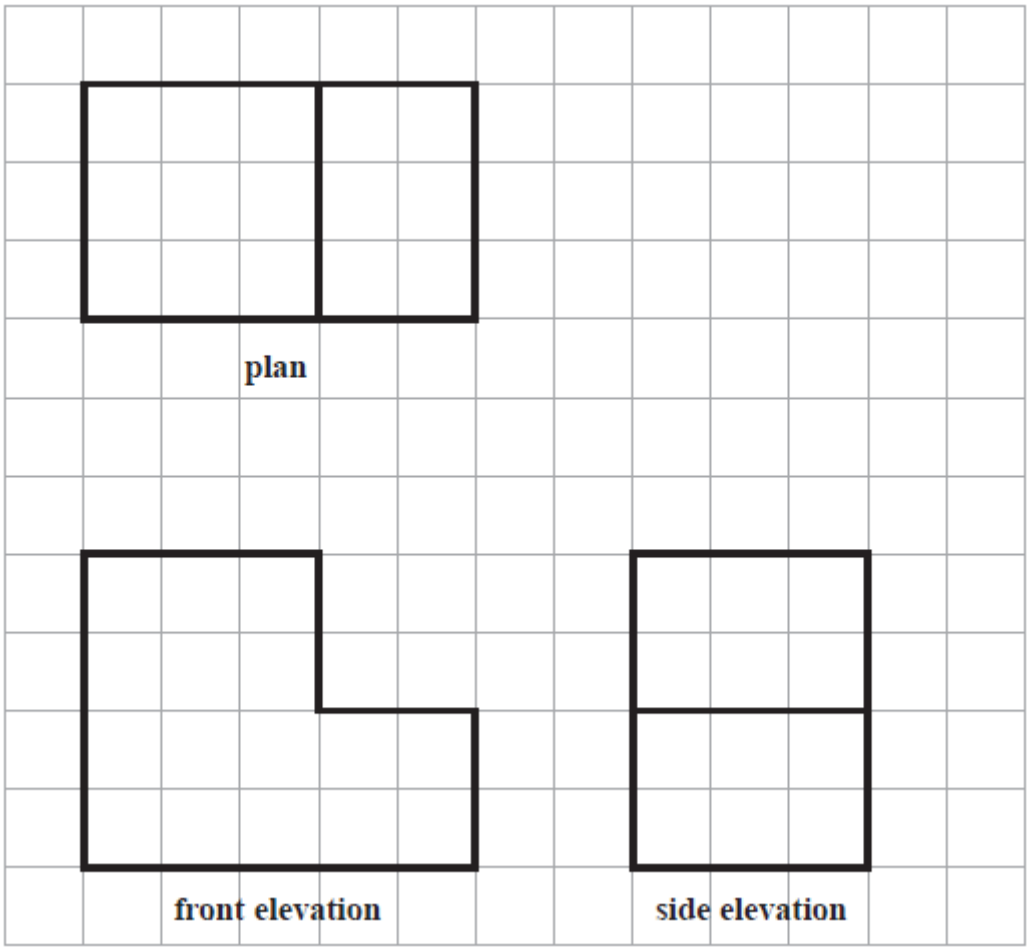
On the centimetre grid below, draw a plan of the solid.



(Total 2 marks)

Specimen Papers Set 2, Paper 3F qu.23 / 3H qu.2 (G13 – AO2)

The plan, front elevation and side elevation of a solid prism are drawn on a centimetre grid.



In the space below, draw a sketch of the solid prism.
Write the dimensions of the prism on your sketch.

(Total 2 marks)

New SAMs Paper 1F qu.26 / 1H qu.6 (G13 – AO2)

Exam Wizard topics

Plans and elevations

Mensuration and calculation

G14	use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money etc.)

New to Foundation tier

Find perimeter, area and surface area of compound shapes. (NB: compound shapes made from triangles and rectangles are already in Foundation tier.)

Mapping to 1MA0 specification content descriptors

GMx (F) Calculate perimeters and areas of shapes made from triangles and rectangles
GMx (H) Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GMo (F/H) Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GMq (F/H) Make sensible estimates of a range of measures
GMt (F/H) Measure and draw lines and angles

Learning objectives

Foundation tier (Unit 8)

- Convert between units of measure within one system, including time and metric units, to metric units of length, area and volume and capacity, e.g. 1 ml = 1 cm^3;
- Measure shapes to find perimeters and areas using a range of scales;
- Recall and use the formulae for the area of a triangle, rectangle trapezium and parallelogram; calculate areas of compound shapes made from triangles and rectangles;
- Find the surface area of a prism; using rectangles and triangles;
- Recall and use the formula for the volume of a cuboid; find the volume of a prism, including a triangular prism, cube and cuboid; calculate volumes of right prisms and shapes made from cubes and cuboids;
- Estimate surface areas and volumes etc. by rounding measurements to 1 significant figure.

Higher tier (Units 7a, 7b)

- Recall and use the formulae for areas of triangles, rectangles, trapezia, parallelograms and compound shapes (made from triangles, rectangles, trapezia and parallelograms) using a variety of metric measures;
- Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes, with and without a diagram;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures; use volume to solve problems;
- Convert between metric measures of volume and capacity, e.g. 1 ml = 1 cm^3;
- Estimate perimeter, area, surface area, perimeter and volume by rounding measurements to 1 significant figure to check reasonableness of answers.

Sample questions

Here is part of a train timetable from Swindon to London.

Swindon to London							
Swindon	06 10	06 27	06 41	06 58	07 01	07 17	07 28
Didcot	06 27	06 45	06 58	–	7 18	–	07 45
Reading	06 41	06 59	07 13	07 28	07 33	07 43	08 00
London	07 16	07 32	07 44	08 02	08 07	08 14	08 33

(*a*) How long should the 06 58 train from Swindon take to get to London?

(1)

Clare says,

"All these trains take more than one hour to get from Swindon to London."

(*b*) Is Clare correct?
You must give a reason for your answer.

(Total 2 marks)
Specimen Papers Set 2, Paper 2F qu.6 (G14, N13 – AO2)

Carpet tiles are going to be used to cover a floor.

The floor is a 1200 mm by 1000 mm rectangle.
Each carpet tile is a 40 cm by 30 cm rectangle.

Exactly 10 carpets tiles can be used to cover the floor completely.

Show in a labelled sketch how this can be done.

(Total 3 marks)
New SAMs Paper 1F qu.15 (G14, N2, R1 – AO1/AO2)

Exam Wizard topics

Geometric reasoning
Area
Perimeter
Interpret and estimate measures
Time calculations
Maps and scale drawings
Constructions
New to Foundation from Higher in GCSE (9–1)

G15	measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

Mapping to 1MA0 specification content descriptors

GMm (F/H) Use and interpret maps and scale drawings
GMr (F/H) Understand and use bearings
GMt (F/H) Measure and draw lines and angles
GMu (F/H) Draw triangles and other 2D shapes using ruler and protractor

Learning objectives

Foundation tier (Units 3b, 6a, 8, 15a, 15b)

- Measure angles using a protractor;
- Measure shapes to find perimeters and areas using a range of scales;
- Measure and draw lines (to the nearest mm) and angles (to the nearest degree);
- Use and interpret maps and scale drawings;
- Give a bearing between the points on a map or scaled plan;
- Use accurate drawing to solve bearings problems.

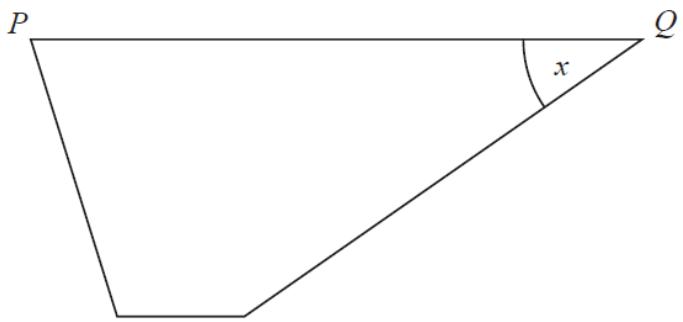
Higher tier (Unit 8b)

- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Understand, draw and measure bearings;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings.

Sample questions

Here is a trapezium.

This diagram is accurately drawn.



(*a*) Measure the length of the line *PQ*.

(1)

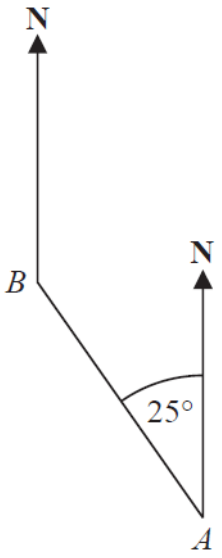
(*b*) Measure the size of the angle marked *x*.

(1)

(Total 2 marks)

Specimen Papers Set 1, Paper 3F qu.6 (G15 – AO1)

The diagram shows the positions of two churches, *A* and *B*.



Amber says,

"The bearing of church *B* from church *A* is 025°"

Amber is wrong.
Explain why.

(Total 1 mark)
Specimen Papers Set 2, Paper 3F qu.18 (G15 – AO2)

Exam Wizard topics

Bearings
Maps and scale drawings
Constructions

G16	know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)

New to Foundation tier

Find perimeter, area and surface area of compound shapes. (NB: compound shapes made from triangles and rectangles are already in Foundation tier.)
Solve mensuration problems involving more complex shapes and solids.

Mapping to 1MA0 specification content descriptors

GMx (F) Calculate perimeters and areas of shapes made from triangles and rectangles
GMx (H) Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GMaa (F/H) Calculate volumes of right prisms and shapes made from cubes and cuboids
GMbb (H) Solve mensuration problems involving more complex shapes and solids

Learning objectives

Foundation tier (Units 8, 17)

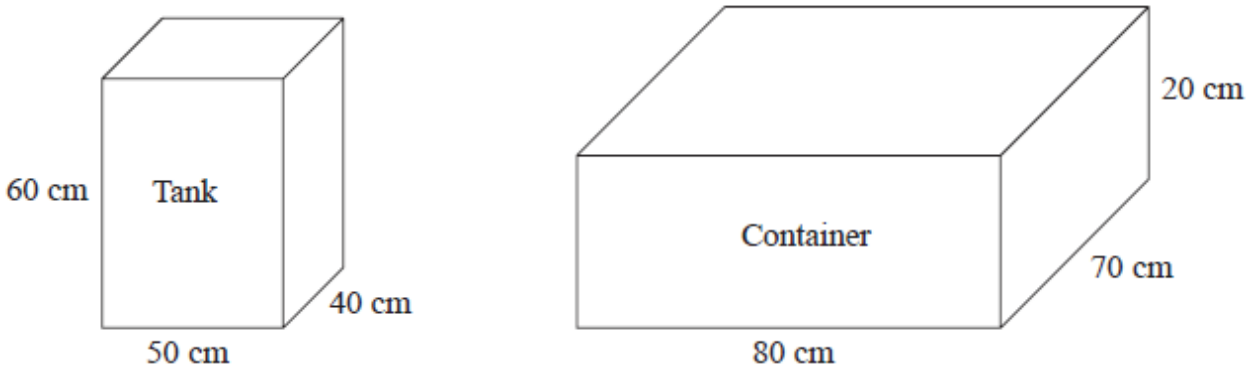
- Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram;
- Calculate areas and perimeters of compound shapes made from triangles and rectangles;
- Find surface area using rectangles and triangles, including the surface area of a prism;
- Find the volume of a prism, including a triangular prism, cube and cuboid; recall and use the formula for the volume of a cuboid;
- Calculate volumes of right prisms and shapes made from cubes and cuboids;
- Estimate surface areas and volumes by rounding measurements to 1 significant figure;
- Find the surface area and volume of a cylinder.

Higher tier (Units 7a, 7b)

- Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the area of compound shapes made from triangles, rectangles, trapezia and parallelograms using a variety of metric measures;
- Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes, with and without a diagram;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures; use volume to solve problems;
- Estimate perimeter, area, surface area and volume by rounding measurements to 1 significant figure to check reasonableness of answers;
- Find the volume and surface area of a cylinder.

Sample questions

The diagram shows a tank in the shape of a cuboid.
It also shows a container in the shape of a cuboid.



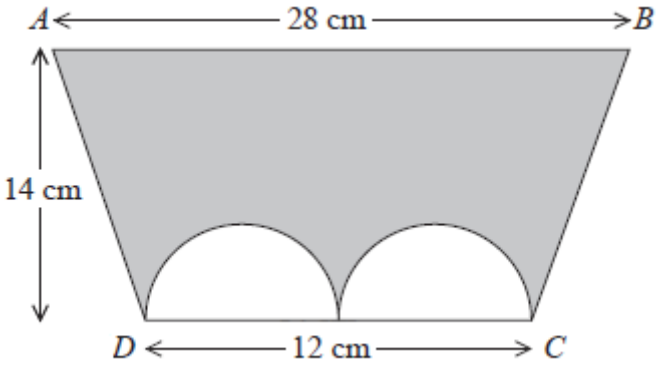
The tank is full of oil.
The container is empty

35% of the oil from the tank is spilled.
The rest of the oil from the tank is put into the container.

Work out the height of the oil in the container.
Give your answer to an appropriate degree of accuracy.

(Total 5 marks)
New SAMs Paper 2F qu.14 (G16, R9 – AO1/AO3)

The diagram shows a trapezium *ABCD* and two identical semicircles.

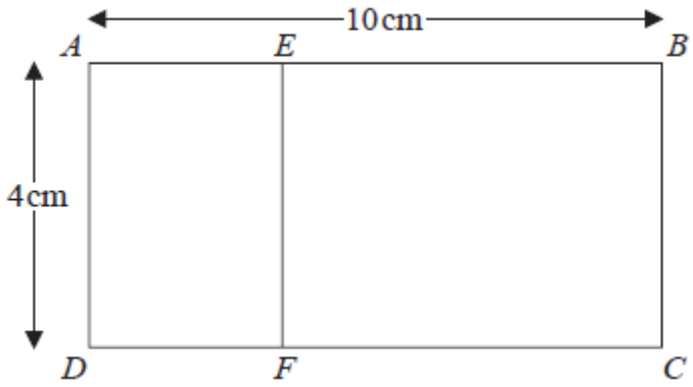


The centre of each semicircle is on *DC*.

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

(Total 4 marks)
New SAMs Paper 3F qu.18 / 3H qu.1 (G16, G17 – AO1/AO3)

Rectangle *ABCD* is mathematically similar to rectangle *DAEF*.



AB = 10 cm.
AD = 4 cm.

Work out the area of rectangle *DAEF*.

(Total 3 marks)
New SAMs Paper 1H qu.13 (G16, G6 – AO1/AO3)

Exam Wizard topics

Area
Perimeter
Volume
Circles
New to Foundation from Higher in GCSE (9–1)

G17	know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids

Content guidance

To include the surface area of cuboids and cylinders.

New to Foundation tier

Perimeter, area and surface area of compound shapes now also in Foundation (compound shapes made from triangles and rectangles already in Foundation).
Solve mensuration problems involving more complex shapes and solids.

Mapping to 1MA0 specification content descriptors

GMx (F) Calculate perimeters and areas of shapes made from triangles and rectangles
GMx (H) Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GMz (F/H) Find circumferences and areas of circles
GMbb (H) Solve mensuration problems involving more complex shapes and solids

Learning objectives

Foundation tier (Units 8, 17)

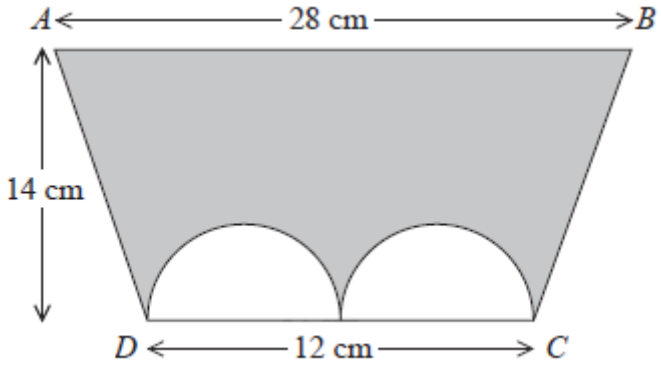
- Find the perimeters of rectangles, triangles, parallelograms, trapezia and compound shapes;
- Calculate volumes of right prisms and shapes made from cubes and cuboids;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle; circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; include giving answers in terms of π;
- Find radius or diameter, given area or perimeter of a circle;
- Find the perimeters and areas of semicircles, quarter-circles and composite shapes made from circles and parts of circles;
- Find the surface areas and volumes of cylinders, spheres, pyramids, cones and composite solids.

Higher tier (Units 7a, 7b, 12)

- Find the perimeters of rectangles, trapezia, parallelograms and compound shapes made from triangles and rectangles using a variety of metric measures;
- Estimate area and perimeter by rounding measurements to 1 significant figure to check reasonableness of answers;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle (using circumference = $2\pi r = \pi d$ and area of a circle = πr^2) using a variety of metric measures; include giving answers in terms of π;
- Find radius or diameter, given area or circumference of circles in a variety of metric measures;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);
- Find the surface areas and volumes of cylinders, spheres, pyramids, cones and compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders; recall and use the formulae for volume of pyramids, spheres and cones; include giving answers in terms of π; use volume to solve problems;
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

Sample questions

The diagram shows a trapezium *ABCD* and two identical semicircles.



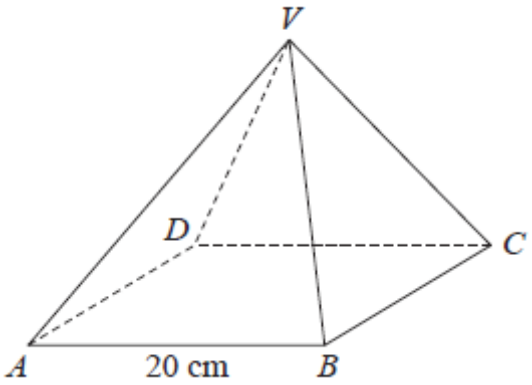
The centre of each semicircle is on *DC*.

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

(Total 4 marks)

New SAMs Paper qu.18 / 3H qu.1 (G17, G16 – AO1/AO3)

VABCD is a solid pyramid.



ABCD is a square of side 20 cm.

The angle between any sloping edge and the plane *ABCD* is 55^{o}

Calculate the surface area of the pyramid.
Give your answer correct to 2 significant figures.

(Total 5 marks)

New SAMs Paper 3H qu.16 (G17, G20 – AO1/AO3)

Exam Wizard topics

Area
Perimeter
Volume
Circles
New to Foundation from Higher in GCSE (9–1)

G18 calculate arc lengths, angles and areas of sectors of circles

New to Foundation tier

Calculate lengths of arcs and areas of sectors of circles, including answers in terms of π.
(NB: semicircles and quarter-circles are already in Foundation.)
Solve mensuration problems involving more complex shapes and solids.

Mapping to 1MA0 specification content descriptors

GMz (H) Find circumferences and areas of circles
GMbb (H) Solve mensuration problems involving more complex shapes and solids

Learning objectives

Foundation tier (Unit 17)

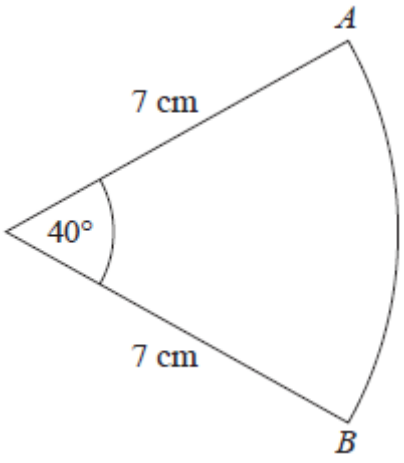
- Find the perimeters and areas of semicircles and quarter-circles;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles;
- Calculate arc lengths, angles and areas of sectors of circles.

Higher tier (Unit 7a)

- Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);
- Calculate arc lengths, angles and areas of sectors of circles.

Sample questions

The diagram shows a sector of a circle of radius 7 cm.



Work out the length of arc *AB*.
Give your answer correct to 3 significant figures.

(Total 2 marks)
New SAMs Paper 2H qu.17 (G18 – AO1)

Exam Wizard topics

Area
Perimeter
Circles
New to Foundation from Higher in GCSE (9–1)

G19 apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures

New to Foundation tier

Understand and use congruence and similarity.

Mapping to 1MA0 specification content descriptors

GMf (F/H) Understand congruence and similarity
GMn (F/H) Understand the effect of enlargement for perimeter, area and volume of shapes and solids

Learning objectives

Foundation tier (Unit 19a)

- Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS) and solve angle problems involving congruence;
- Identify shapes which are similar; including all circles or all regular polygons with equal number of sides;
- Understand similarity of triangles and of other plane shapes, use this to make geometric inferences, and solve angle problems using similarity;
- Understand the effect of enlargement on perimeter of shapes;
- Solve problems to find missing lengths in similar shapes.

Higher tier (Unit 12)

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions; solve angle problems by first proving congruence;
- Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
- Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor; use formal geometric proof for the similarity of two given triangles;
- Understand the effect of enlargement on angles, perimeter, area and volume of similar shapes and solids; know and use the relationship between linear, area and volume scale factors;
- Find missing lengths, areas and volumes in similar 3D solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

Sample questions

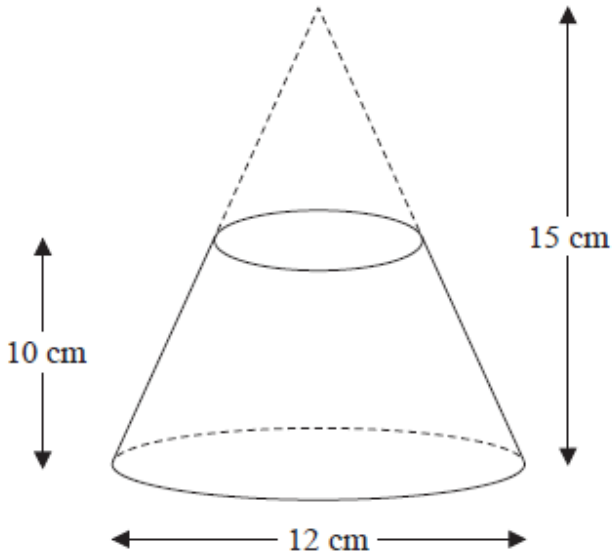
Solid **A** and solid **B** are mathematically similar.
The ratio of the surface area of solid **A** to the surface area of solid **B** is 4 : 9

The volume of solid **B** is 405 cm^3.

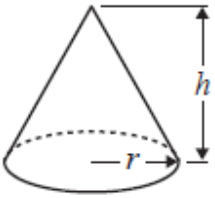
Show that the volume of solid **A** is 120 cm^3.

(Total 3 marks)
New SAMs Paper 1H qu.18 (G19, R12 – AO2)

A frustum is made by removing a small cone from a large cone as shown in the diagram.



Volume of cone $= \frac{1}{3}\pi r^2 h$



The frustum is made from glass.
The glass has a density of 2.5 g/cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

(Total 5 marks)

New SAMs Paper 2H qu.22 (G19, G17, N15, R11 – AO1/AO3)

Exam Wizard topics

Enlargement
Congruence and similarity
New to Foundation from Higher in GCSE (9–1)

G20 know the formulae for: Pythagoras' theorem $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$; apply them to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two- **and three**-dimensional figures

Content guidance

At Higher tier, to include the angle between a line and a plane.

New to Foundation tier

Use the trigonometric ratios in right-angled triangles to solve problems, and to find angles of elevation and depression. (NB: 3D problems are still in Higher tier only.)

Mapping to 1MA0 specification content descriptors

GMg (F) Use Pythagoras' theorem in 2D
GMg (H) Use Pythagoras' theorem in 2D and 3D
GMh (H) Use the trigonometric ratios and the sine and cosine rules to solve 2D and 3D problems

Learning objectives

Foundation tier (Unit 12)

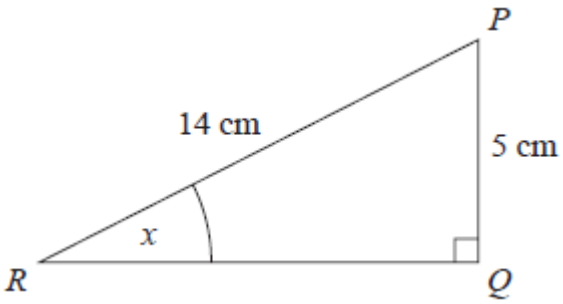
- Understand, recall and use Pythagoras' theorem in 2D, including leaving answers in surd form; justify if a triangle is right-angled or not; calculate the length of the hypotenuse and of a shorter side in a right-angled triangle, including decimal lengths and a range of units;
- Apply Pythagoras' theorem with a triangle drawn on a coordinate grid; calculate the length of a line segment AB, given pairs of points;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems; to find angles of elevation and depression.

Higher tier (Units 5b, 13b)

- Understand, recall and use Pythagoras' theorem in 2D, including leaving answers in surd form; justify if a triangle is right-angled or not; calculate the length of the hypotenuse and of a shorter side in a right-angled triangle, including decimal lengths and a range of units;
- Calculate the length of a line segment AB, given pairs of points;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems; to find angles of elevation and depression;
- Understand the language of planes and recognise the diagonals of a cuboid; use trigonometric relationships and Pythagoras' theorem to solve problems in 3D configurations, including calculating length of a diagonal of a cuboid and the angle between a line and a plane.

Sample questions

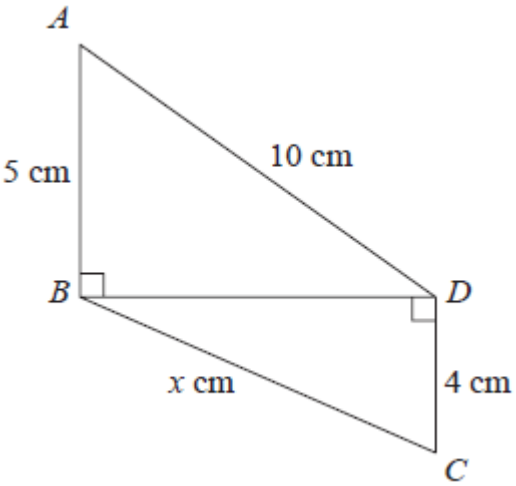
PQR is a right-angled triangle.



Work out the size of the angle marked *x*.
Give your answer correct to 1 decimal place.

(Total 2 marks)
New SAMs Paper 2F qu.24 / 2H qu.2 (G20 – AO1)

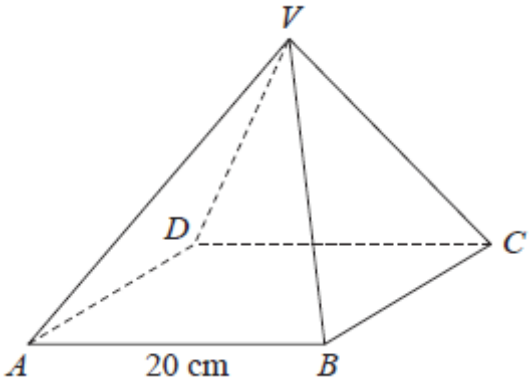
Triangles *ABD* and *BCD* are right-angled triangles.



Work out the value of *x*.
Give your answer correct to 2 decimal places.

(Total 4 marks)
New SAMs Paper 2F qu.28 / 2H qu.6 (G20 – AO1/AO3)

VABCD is a solid pyramid.



ABCD is a square of side 20 cm.

The angle between any sloping edge and the plane *ABCD* is 55^{o}

Calculate the surface area of the pyramid.
Give your answer correct to 2 significant figures.

(Total 5 marks)

New SAMs Paper 3H qu.16 (G17, G20 – AO1/AO3)

Exam Wizard topics

Pythagoras
Trigonometry
New to Foundation from Higher in GCSE (9–1)

G21	know the exact values of sin θ and cos θ for $\theta = 0°, 30°, 45°, 60°$ and $90°$; know the exact value of tan θ for $\theta = 0°, 30°, 45°$ and $60°$

New to GCSE (9–1) Maths

The content in G21 is new to GCSE (9–1) Maths.

New to Foundation tier

Know the exact values of these trigonometric ratios in right-angled triangles.

Learning objectives

Foundation tier (Unit 12)

- Know the exact values of sin θ and cos θ for $\theta = 0°, 30°, 45°, 60°$ and $90°$; know the exact value of tan θ for $\theta = 0°, 30°, 45°$ and $60°$.

Higher tier (Units 5b, 13a)

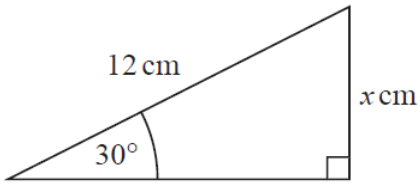
- Know the exact values of sin θ and cos θ for $\theta = 0°, 30°, 45°, 60°$ and $90°$, and of tan θ for $\theta = 0°, 30°, 45°$ and $60°$, and find them from graphs.

Sample questions

(*a*) Write down the exact value of cos30°

(1)

(*b*)

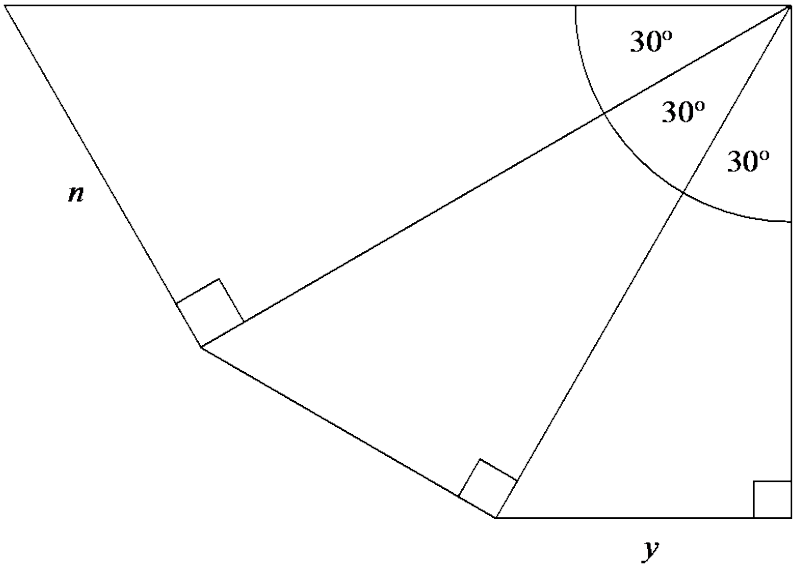


Given that sin 30° = 0.5, work out the value of x.

(2)

(Total 3 marks)

Specimen Papers Set 1, Paper 1F qu.26 / 1H qu.7 (G21, G20 – AO1)



The diagram shows three right-angled triangles.

Prove that $y = \frac{3}{4}n$

(Total 4 marks)

Mock Papers Set 3, Paper 1H qu.20 (G21, G21 – AO1/AO2)

Exam Wizard topics

Trigonometry
New to GCSE (9–1)
New to Foundation from Higher in GCSE (9–1)

G22 **know and apply the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles**

Mapping to 1MA0 specification content descriptors

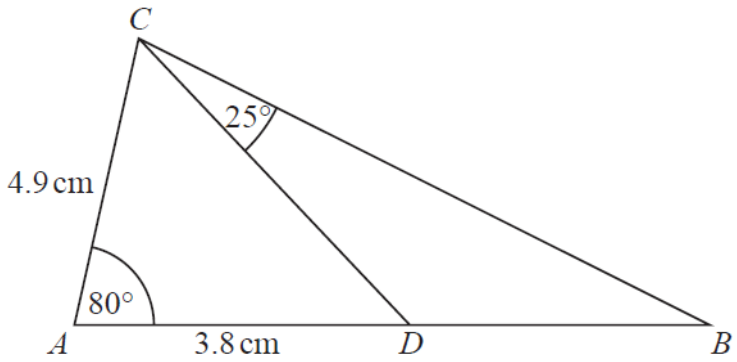
GMh (H) Use the trigonometric ratios and the sine and cosine rules to solve 2D and 3D problems

Learning objectives

Higher tier (Unit 13b)

- Know the sine and cosine rules, and use them to solve 2D problems (including involving bearings);
- Use the sine and cosine rules to solve 3D problems.

Sample questions



ABC is a triangle.
D is a point on *AB*.

Work out the area of triangle *BCD*.
Give your answer correct to 3 significant figures.

(Total 5 marks)

Specimen Papers Set 2, Paper 3H qu.21 (G22, G23 – AO1/AO3)

Exam Wizard topics

Trigonometry

G23 know and apply area = $\frac{1}{2}$ *ab* sin *C* to calculate the area, sides or angles of any triangle

Mapping to 1MA0 specification content descriptors

GMy (H) Calculate the area of a triangle using $\frac{1}{2}ab \sin C$

Learning objectives

Higher tier (Unit 13b)

- Know and apply Area = $\frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle.

Sample questions

In triangle *RPQ*,

RP = 8.7 cm
PQ = 5.2 cm
Angle *PRQ* = 32°

(*a*) Assuming that angle *PQR* is an acute angle,
calculate the area of triangle *RPQ*.
Give your answer correct to 3 significant figures.

(4)

(*b*) If you did not know that angle *PQR* is an acute angle, what effect would this have on your calculation of the area of triangle *RPQ*?

(1)

(Total 5 marks)
New SAMs Paper 2H qu.21 (G23, A12 – AO1/AO3)

Exam Wizard topics

Area
Trigonometry

Vectors

G24	describe translations as 2D vectors

Mapping to 1MA0 specification content descriptors

GMl (F) Describe and transform 2D shapes using single or combined rotations, reflections, translations or enlargements by a positive scale factor, and distinguish properties that are preserved under particular transformations
GMl (H) Describe and transform 2D shapes using single or combined rotations, reflections, translations or enlargements by a positive, fractional or negative scale factor, and distinguish properties that are preserved under particular transformations

Learning objectives

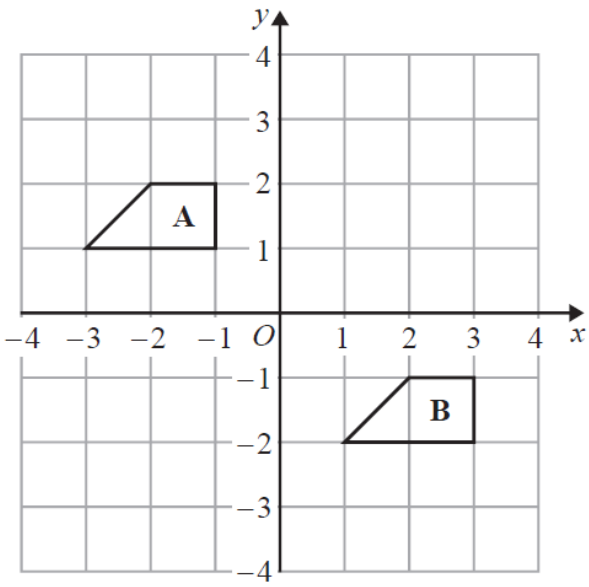
Foundation tier (Units 10, 19b)

- Understand that translations are specified by a distance and direction using a vector;
- Translate a given shape by a vector;
- Use column vectors to describe and transform 2D shapes using single translations on a coordinate grid;
- Understand and use column notation in relation to vectors.

Higher tier (Units 8a, 18)

- Recognise and describe single translations using column vectors on a coordinate grid;
- Translate a given shape by a vector;
- Understand the effect of one translation followed by another in terms of column vectors (to introduce vectors in a concrete way);
- Understand and use vector notation, including column notation.

Sample questions



Describe the single transformation that maps shape **A** onto shape **B**.

(Total 2 marks)

Specimen Papers Set 2, Paper 2F qu.22 / 2H qu.1 (G24, G7 – AO2)

Exam Wizard topics

Translation
Vectors

G25 apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors;
use vectors to construct geometric arguments and proofs

New to Foundation tier

Vector notation, sum and difference of vectors, scalar multiple and resultant of vectors.
(NB: geometric proofs and vectors still in Higher only.)

Mapping to 1MA0 specification content descriptors

GMcc (H) Use vectors to solve problems

Learning objectives

Foundation tier (Unit 19b)

- Understand and use column notation in relation to vectors;
- Be able to represent information graphically, given column vectors;
- Identify two column vectors which are parallel;
- Calculate using column vectors, and represent graphically, the sum of two vectors, the difference of two vectors and a scalar multiple of a vector.

Higher tier (Units 8a, 18)

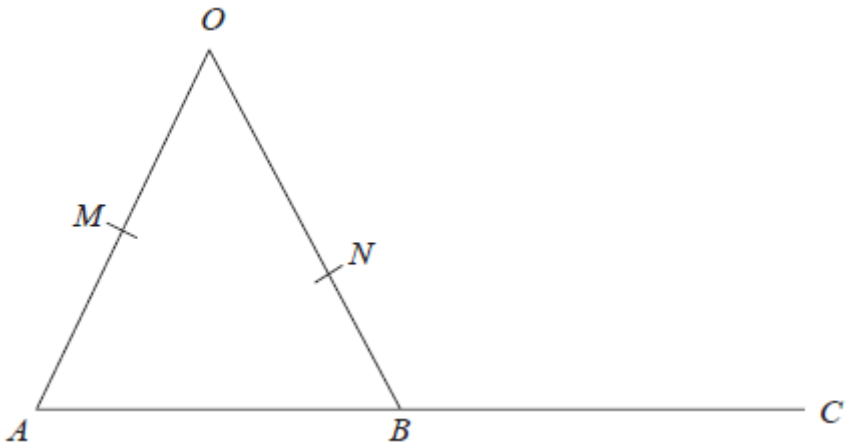
- Understand the effect of one translation followed by another in terms of column vectors (to introduce vectors in a concrete way);
- Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction;
- Understand that 2**a** is parallel to **a** and twice its length, and that **a** is parallel to –**a** in the opposite direction;
- Represent vectors, combinations of vectors and scalar multiples in the plane pictorially;
- Calculate the sum/resultant of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms);
- Find the length of a vector using Pythagoras' theorem;
- Solve geometric problems in 2D where vectors are divided in a given ratio;
- Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.

Sample questions

$\mathbf{a} = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Work out **b** – 2**a** as a column vector.

(Total 2 marks)
New SAMs Paper 1F qu.30 (G25 – AO1)



OMA, *ONB* and *ABC* are straight lines.
M is the midpoint of *OA*.
B is the midpoint of *AC*.

$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{ON} = k\mathbf{b}$ where *k* is a scalar quantity.

Given that *MNC* is a straight line, find the value of *k*.

(Total 5 marks)
New SAMs Paper 3H qu.18 (G25, A4 – AO1/AO3)

Exam Wizard topics

Vectors
New to Foundation from Higher in GCSE (9–1)

5. Probability

P1	record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

New to GCSE (9–1) Maths

Use frequency trees to record, describe and analyse the frequency of outcomes of probability experiments.

Mapping to 1MA0 specification content descriptors

SPn (F/H) Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes) or from relative frequency
SPo (F/H) List all outcomes for single events and for two successive events in a systematic way, and derive relative probabilities
SPs (F/H) Compare experimental data and theoretical probabilities

Learning objectives

Foundation tier (Unit 13)

- Work out probabilities from frequency tables, frequency trees, and two-way tables; find a missing probability from a list or table including algebraic terms;
- Record outcomes of probability experiments in tables; find the probability of an event happening using relative frequency;
- Compare experimental data and theoretical probabilities; compare relative frequencies from samples of different sizes;
- Use tree diagrams to calculate the probability of two independent events and two dependent events.

Higher tier (Unit 10)

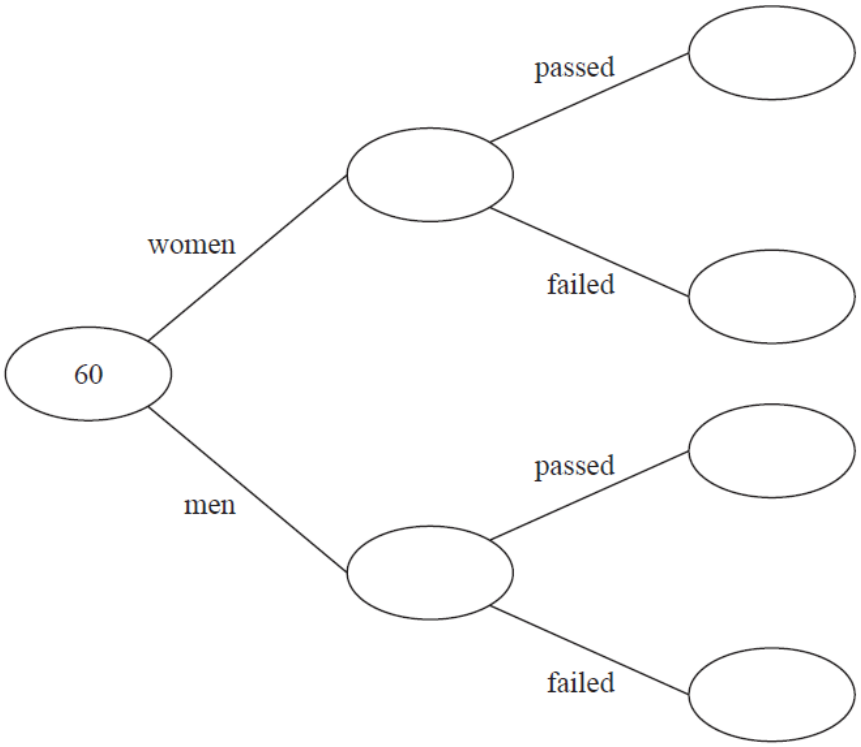
- Find a missing probability from a list or two-way table, including algebraic terms;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
- Use a two-way table or a tree diagram to calculate conditional probability;
- Compare experimental data and theoretical probabilities; compare relative frequencies from samples of different sizes.

Sample questions

60 people each took a driving test one day.

21 of these people were women.
18 of the 60 people failed their test.
27 of the men passed their test.

(*a*) Use this information to complete the frequency tree.



(3)

One of the men is chosen at random.

(*b*) Work out the probability that this man failed his test.

(2)

(Total 5 marks)

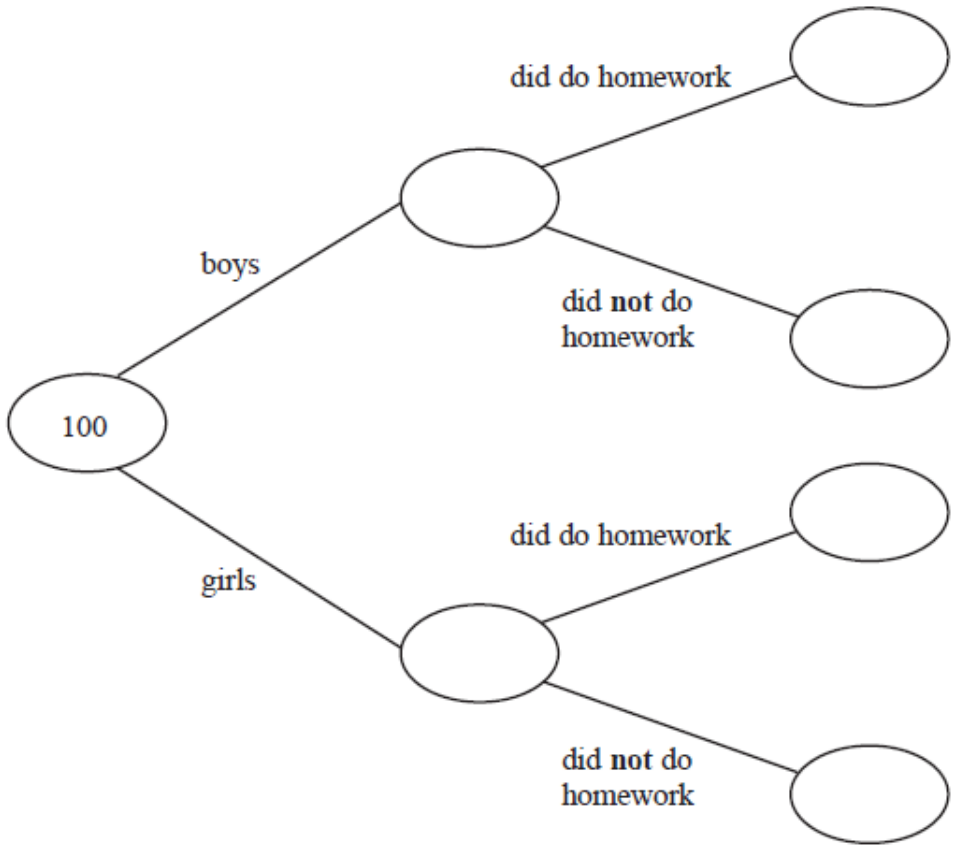
Mock Papers Set 1, Paper 1F qu.12 (P1, P3 – AO2)

100 students had some homework.

42 of these students are boys.
8 of the 100 students did **not** do their homework.
53 of the girls did do their homework.

(*a*) Use this information to complete the frequency tree.

(3)



One of the girls is chosen at random.

(*b*) Work out the probability that this girl did **not** do her homework.

(2)

(Total 5 marks)
New SAMs Paper 1F qu.17 (P1, P3 – AO1/AO2)

Exam Wizard topics

Theoretical probability
Relative frequency
Sample space diagrams
Frequency trees
New to GCSE (9–1)

P2	apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

Mapping to 1MA0 specification content descriptors

SPn (F/H) Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes) or from relative frequency

Learning objectives

Foundation tier (Unit 13)

- Find the probability of an event happening using theoretical probability; use theoretical models to include outcomes using dice, spinners, coins;
- Estimate the number of times an event will occur, given the probability and the number of trials – for both experimental and theoretical probabilities; compare experimental data and theoretical probabilities;
- Use and draw sample space diagrams;
- Find the probability of successive events, such as several throws of a single dice;
- Use tree diagrams to calculate the probability of two independent events; two dependent events.

Higher tier (Unit 10)

- Understand and use experimental and theoretical measures of probability, including relative frequency, to include outcomes using dice, spinners, coins etc.;
- Estimate the number of times an event will occur, given the probability and the number of trials;
- Find the probability of successive events, such as several throws of a single dice;
- Draw sample space diagrams and use them for adding simple probabilities;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcomes;
- Compare experimental data and theoretical probabilities.

Sample questions

In a box there are three types of chocolates.

There are 6 plain chocolates,
8 milk chocolates
and 10 white chocolates.

Ben takes at random a chocolate from the box.

(*a*) Write down the probability that Ben takes a plain chocolate.

(2)

Deon takes 2 chocolates from the box.

(*b*) Write down all the possible combinations of types of chocolates that Deon can take.

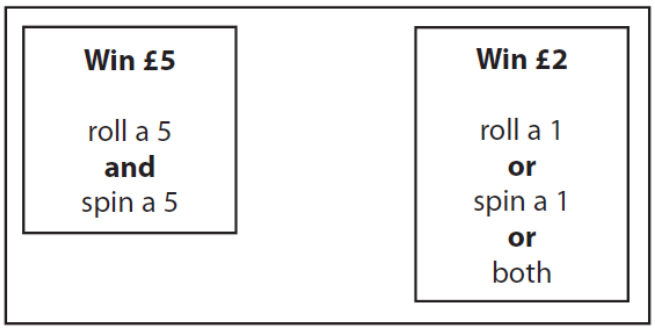
(2)

(Total 4 marks)

Specimen Papers Set 2, Paper 1F qu.7 (P2, R3, N5 – AO1/AO2)

David has designed a game.
He uses a fair 6-sided dice and a fair 5-sided spinner.
The dice is numbered 1 to 6.
The spinner is numbered 1 to 5.

Each player rolls the dice once and spins the spinner once.
A player can win £5 or win £2.



David expects 30 people will play his game.
Each person will pay David £1 to play the game.

(*a*) Work out how much profit David can expect to make.

(4)

(*b*) Give a reason why David's actual profit may be different to the profit he expects to make.

(1)

(Total 5 marks)

Specimen Papers Set 2, Paper 1F qu.21 / 1H qu.3 (P2, P5, P8 – AO1/AO3)

Exam Wizard topics

Theoretical probability
Relative frequency

P3	relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale

Mapping to 1MA0 specification content descriptors

SPm (F/H) Understand and use the vocabulary of probability and probability scale
SPn (F/H) Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes) or from relative frequency
SPs (F/H) Compare experimental data and theoretical probabilities

Learning objectives

Foundation tier (Unit 13)

- Distinguish between events which are impossible, unlikely, even chance, likely and certain to occur;
- Mark events and/or probabilities on a probability scale of 0 to 1;
- Write probabilities in words or fractions, decimals and percentages;
- Find the probability of an event happening using relative frequency;
- Estimate the number of times an event will occur, given the probability and the number of trials – for both experimental and theoretical probabilities;
- Compare experimental data and theoretical probabilities.

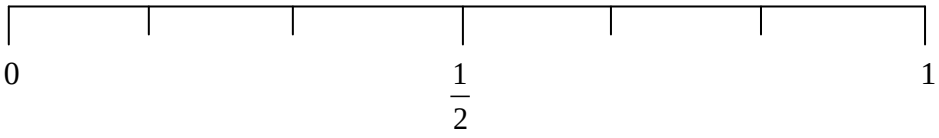
Higher tier (Unit 10)

- Write probabilities using fractions, percentages or decimals;
- Understand and use experimental and theoretical measures of probability, including relative frequency, to include outcomes using dice, spinners, coins etc.;
- Estimate the number of times an event will occur, given the probability and the number of trials;
- Compare experimental data and theoretical probabilities.

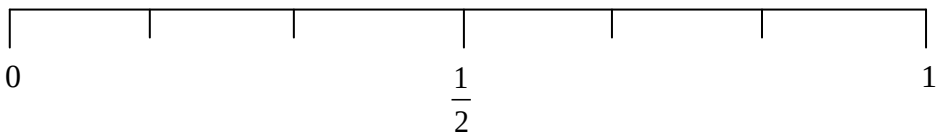
Sample questions

Greg rolls a fair ordinary dice once.

(i) On the probability scale, mark with a cross (×) the probability that the dice will land on an odd number.



(ii) On the probability scale, mark with a cross (×) the probability that the dice will land on a number less than 5



(Total 2 marks)
New SAMs Paper 1F qu.6 (P3 – AO1)

There are 3 red beads and 1 blue bead in a jar.
A bead is taken at random from the jar.

What is the probability that the bead is blue?

(Total 1 mark)
New SAMs Paper 1F qu.9 (P3 – AO1)

There are 25 boys and 32 girls in a club.

$\frac{2}{5}$ of the boys and $\frac{1}{2}$ of the girls walk to the club.

The club leader picks at random a child from the children who walk to the club.

Work out the probability that this child is a boy.

(Total 3 marks)
New SAMs Paper 3F qu.15 (P3, N8 – AO1/AO3)

Exam Wizard topics

Probability scales
Theoretical probability
Relative frequency

P4	apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

Mapping to 1MA0 specification content descriptors

SPp (F/H) Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1

Learning objectives

Foundation tier (Unit 13)

- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all outcomes is 1;
- Use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring;
- Find a missing probability from a list or table including algebraic terms;

Higher tier (Unit 10)

- Know that the sum of the probabilities of all outcomes is 1;
- Use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring;
- Find a missing probability from a list or two-way table, including algebraic terms.

Sample questions

There are some boys and girls in a classroom.

The probability of picking at random a boy is $\frac{1}{3}$

What is the probability of picking a girl?

(Total 1 mark)

New SAMs Paper 2F qu.5 (P4 – AO1)

There are only red counters, blue counters, green counters and yellow counters in a bag.

The table shows the probabilities of picking at random a red counter and picking at random a yellow counter.

Colour	red	blue	green	yellow
Probability	0.24			0.32

The probability of picking a blue counter is the same as the probability of picking a green counter.

Complete the table.

(Total 2 marks)

Specimen Papers Set 1, Paper 1F qu.22 / 1H qu.3 (P4 – AO1/AO3)

Exam Wizard topics

Add probabilities and use $1 - p$

P5	understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

Mapping to 1MA0 specification content descriptors

SPt (F/H) Understand that if they repeat an experiment, they may − and usually will − get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

Learning objectives

Foundation tier (Unit 13)

- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes.

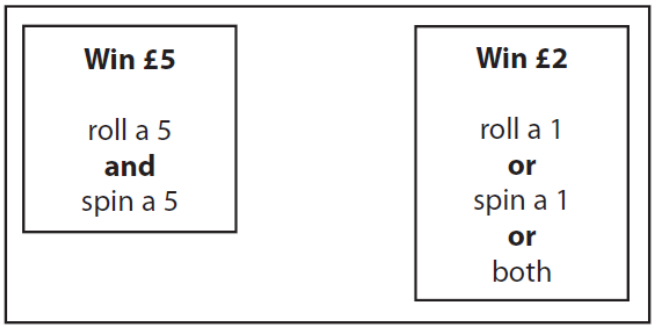
Higher tier (Unit 10)

- Understand and use experimental and theoretical measures of probability, including relative frequency, to include outcomes using dice, spinners, coins etc.;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes.

Sample questions

David has designed a game.
He uses a fair 6-sided dice and a fair 5-sided spinner.
The dice is numbered 1 to 6.
The spinner is numbered 1 to 5.

Each player rolls the dice once and spins the spinner once.
A player can win £5 or win £2.



David expects 30 people will play his game.
Each person will pay David £1 to play the game.

(*a*) Work out how much profit David can expect to make.

(4)

(*b*) Give a reason why David's actual profit may be different to the profit he expects to make.

(1)

(Total 5 marks)

Specimen Papers Set 2, Paper 1F qu.21 / 1H qu.3 (P2, P5, P8 – AO1/AO3)

Four friends each throw a biased coin a number of times.
The table shows the number of heads and the number of tails each friend got.

	Ben	**Helen**	**Paul**	**Sharif**
heads	34	66	80	120
tails	8	12	40	40

The coin is to be thrown one more time.

(*a*) Which of the four friends' results will give the best estimate for the probability that the coin will land heads?
Justify your answer.

(1)

Paul says,

"With this coin you are twice as likely to get heads as to get tails."

(*b*) Is Paul correct?
Justify your answer.

(2)

The coin is to be thrown twice.

(*c*) Use all the results in the table to work out an estimate for the probability that the coin will land heads both times.

(2)

(Total 5 marks)

Specimen Papers Set 1, Paper 1F qu.25 / 1H qu.6 (P5, P3, P8 – AO1/AO2/AO3)

Exam Wizard topics

Theoretical probability
Relative frequency

P6	enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams

Content guidance

To include set notation: E, $\cap$, $\cup$, $\in$, A′.

Examiners will not be expecting students to use the ( ] bracket notation as part of set notation.

New to GCSE (9–1) Maths

Use Venn diagrams to enumerate sets and combinations of sets systematically.

New to Foundation tier

Use tree diagrams to enumerate sets and combinations of sets systematically. (NB: conditional probability is still Higher only.)

Mapping to 1MA0 specification content descriptors

SPo (F/H) List all outcomes for single events and for two successive events in a systematic way, and derive relative probabilities
SPr (H) Use tree diagrams to represent outcomes of compound events, recognising when events are independent

Learning objectives

Foundation tier (Unit 13)

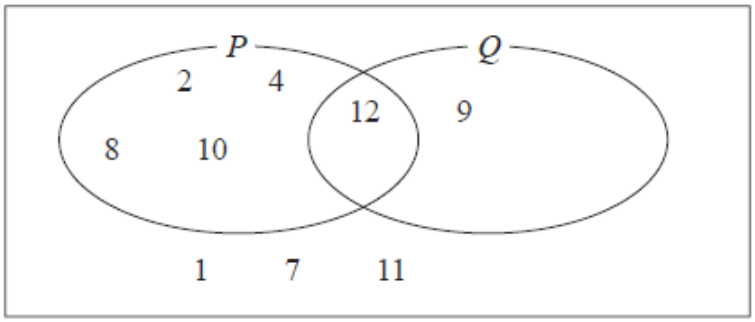
- List all outcomes systematically for single events; for combined events;
- Work out probabilities from frequency tables, frequency trees and two-way tables;
- Use and draw sample space diagrams;
- Work out probabilities from Venn diagrams to represent real-life situations and also ‘abstract’ sets of numbers/values; use union and intersection notation;
- Use tree diagrams to calculate the probability of two independent events; two dependent events.

Higher tier (Unit 10)

- List all outcomes systematically for single events; for combined events;
- Draw sample space diagrams and use them for adding simple probabilities;
- Work out probabilities from Venn diagrams to represent real-life situations and also ‘abstract’ sets of numbers/values; use union and intersection notation;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcomes;
- Use two-way tables, tree diagrams and Venn diagrams to calculate conditional probability.

Sample questions

The numbers 1, 2, 4, 7, 8, 9, 10, 11 and 12 are put into a Venn diagram.



The number 3 is in set Q but not in set P.
The number 6 is in both set P and set Q.

(a) Complete the Venn diagram.

(2)

A student chooses at random a number in the completed Venn diagram.

(b) Write down the probability that this number is **not** in Set Q.

(2)

(Total 4 marks)
Mock Papers Set 2, Paper 1F qu.17 (P6, P4 – AO1/AO2)

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{multiples of } 2\}$
$A \cap B = \{2, 6\}$
$A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10\}$

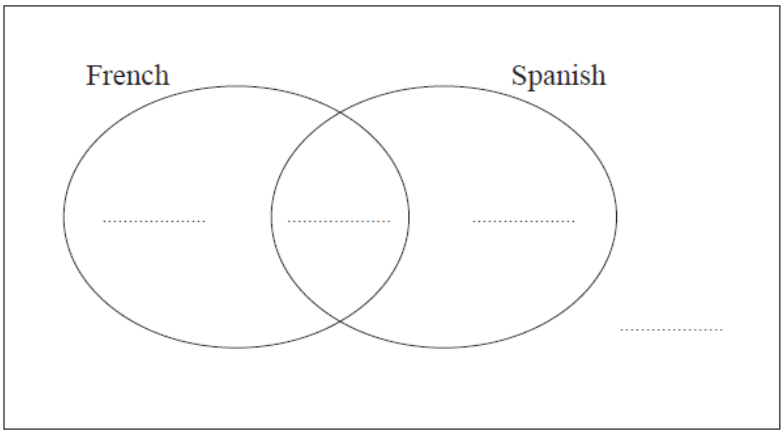
Draw a Venn diagram for this information.

(Total 4 marks)
Specimen Papers Set 1, Paper 3F qu.20 (P6, N4 – AO2)

There are 60 students at a college.

20 students study both French and Spanish.
13 students study French but not Spanish.
A total of 43 students study Spanish.

(*a*) Complete the Venn diagram for this information.



(3)

One of the students at the college is to be selected at random.

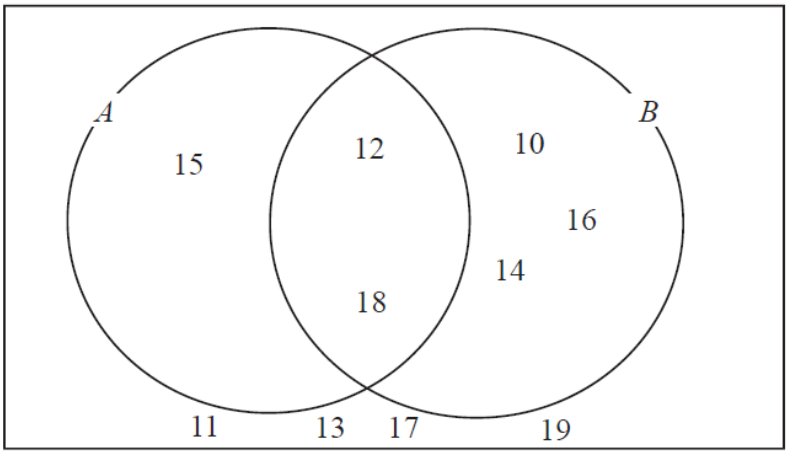
(*b*) Write down the probability that this student studies neither French nor Spanish.

(1)

(Total 4 marks)

Mock Papers Set 1, Paper 2F qu.23 / 2H qu.1 (P6, P8 – AO1/AO2)

Here is a Venn diagram.



(*a*) Write down the numbers that are in set

(i) $A \cup B$

(ii) $A \cap B$

(2)

One of the numbers in the diagram is chosen at random.

(*b*) Find the probability that the number is in set A'

(2)

(Total 4 marks)

Specimen Papers Set 2, Paper 2F qu.26 / 2H qu.5 (P6, P4 – AO1/AO2)

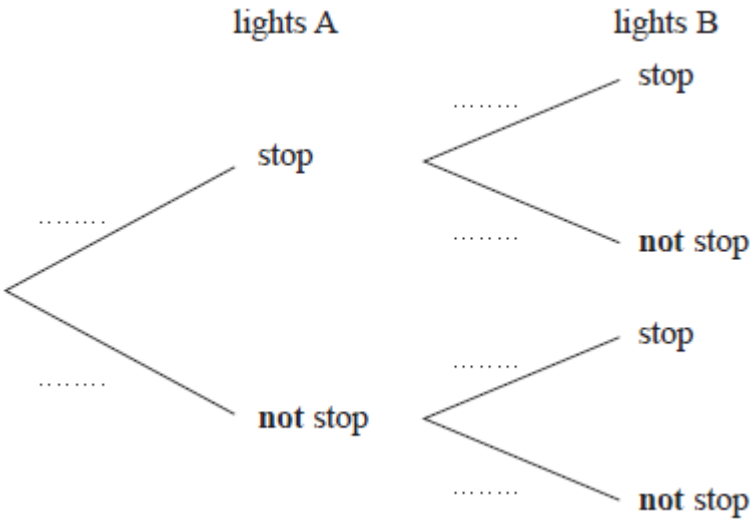
A and B are two sets of traffic lights on a road.

The probability that a car is stopped by lights A is 0.4

If a car is stopped by lights A, then the probability that the car is **not** stopped by lights B is 0.7

If a car is **not** stopped by lights A, then the probability that the car is **not** stopped by lights B is 0.2

(*a*) Complete the probability tree diagram for this information.



(2)

Mark drove along this road.
He was stopped by just one of the sets of traffic lights.

(*b*) Is it more likely that he stopped by lights A or by lights B?
You must show your working.

(3)

(Total 5 marks)
New SAMs Paper 2H qu.12 (P6, P3, P8 – AO2/AO3)

Exam Wizard topics

Theoretical probability
Sample space diagrams
Venn diagrams
Probability tree diagrams
New to GCSE (9–1)
New to Foundation from Higher in GCSE (9–1)

P7	construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities

Mapping to 1MA0 specification content descriptors

SPn (F/H) Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes) or from relative frequency
SPo (F/H) List all outcomes for single events and for two successive events in a systematic way, and derive relative probabilities

Learning objectives

Foundation tier (Unit 13)

- Use and draw sample space diagrams;
- Add simple probabilities.

Higher tier (Unit 10)

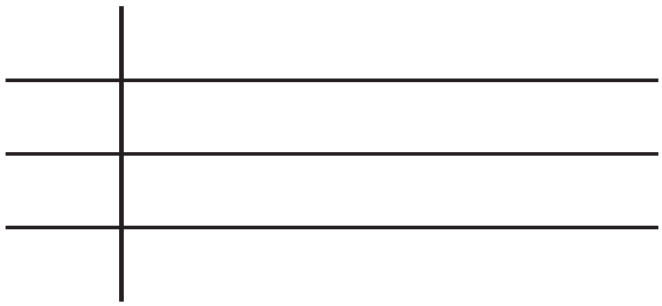
- Draw sample space diagrams and use them for adding simple probabilities.

Sample questions

Here are the heights, in centimetres, of 15 children.

123	147	135	150	147
129	148	149	125	137
133	138	133	130	151

(*a*) Show this information in a stem and leaf diagram.



(3)

One of the children is chosen at random.

(*b*) What is the probability that this child has a height greater than 140 cm?

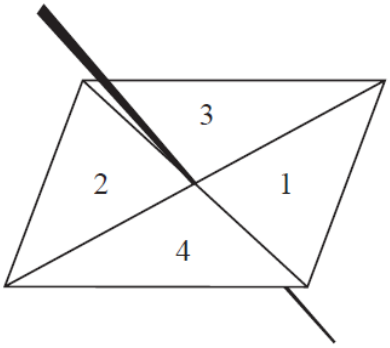
(2)

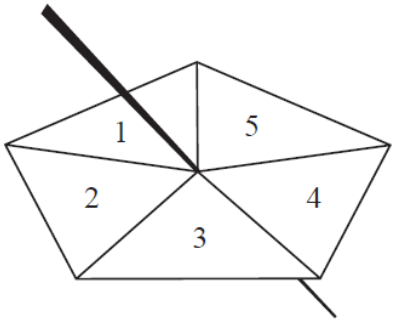
(Total 5 marks)

Specimen Papers Set 1, Paper 2F qu.13 (P7, S2, P2 – AO1/AO2/AO3)

Here are a 4-sided spinner and a 5-sided spinner.

The spinners are fair.





Jeff is going to spin each spinner once.
Each spinner will land on a number.
Jeff will get his score by adding these two numbers together.

(*a*) Complete the possibility space diagram for each possible score.

5-sided spinner

4-sided spinner	**1**	**2**	**3**	**4**	**5**
1	2	3	4	5	6
2	3				
3	4				
4	5				

(1)

Jeff spins each spinner once.

(*b*) Find the probability that Jeff gets

(i) a score of 3

(ii) a score of 5 or more.

(2)

(Total 3 marks)
Specimen Papers Set 2, Paper 2F qu.19 (P7 – AO1)

Exam Wizard topics

Theoretical probability
Sample space diagrams

P8	calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

New to Foundation tier

Calculate the probability of dependent events. (NB: conditional probability is still Higher only.)
Use tree diagrams to calculate the probability of independent and dependent events. (NB: conditional probability is still Higher only.)

Mapping to 1MA0 specification content descriptors

SPq (H) Know when to add or multiply two probabilities: when A and B are mutually exclusive, then the probability of A or B occurring is P(A) + P(B), whereas when A and B are independent events, then the probability of A and B occurring is P(A) × P(B)
SPr (H) Use tree diagrams to represent outcomes of compound events, recognising when events are independent

Learning objectives

Foundation tier (Unit 13)

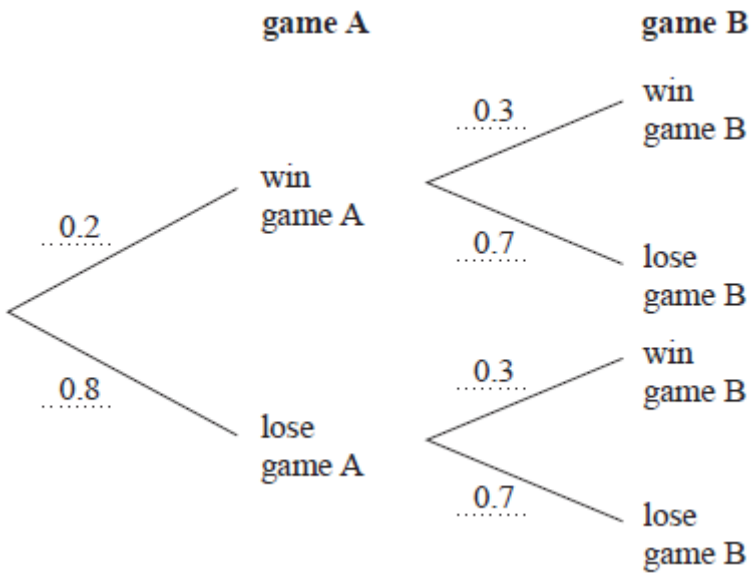
- Use and draw sample space diagrams;
- Work out probabilities from frequency tables, frequency trees and two-way tables;
- Find the probability of successive events, such as several throws of a single dice;
- Use tree diagrams to calculate the probability of two independent events; two dependent events.

Higher tier (Unit 10)

- Draw sample space diagrams and use them for adding simple probabilities;
- Find the probability of successive events, such as several throws of a single dice;
- Understand conditional probabilities and decide if two events are independent;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
- Understand selection with or without replacement;
- Calculate the probability of independent and dependent combined events.

Sample questions

Here is a probability tree diagram.



Work out the probability of winning both games.

(Total 2 marks)

New SAMs Paper 2F qu.29 (P8 – AO1/AO2)

Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 people like at least one of the drinks.
19 people like all three drinks.
16 people like tea and coffee but do **not** like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.
Sami selects at random one of the 50 people.

(*a*) Work out the probability that this person likes tea.

(4)

(*b*) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

(2)

(Total 6 marks)

New SAMs Paper 3H qu.12 (P8, P9 – AO1/AO2/AO3)

Exam Wizard topics

Probability tree diagrams
New to Foundation from Higher in GCSE (9–1)

P9	calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams

New to GCSE (9–1) Maths

Use Venn diagrams to calculate and interpret conditional probabilities.

Mapping to 1MA0 specification content descriptors

SPq (H) Know when to add or multiply two probabilities: when A and B are mutually exclusive, then the probability of A or B occurring is P(A) + P(B), whereas when A and B are independent events, then the probability of A and B occurring is P(A) × P(B)
SPr (H) Use tree diagrams to represent outcomes of compound events, recognising when events are independent

Learning objectives

Higher tier (Unit 10)

- Calculate conditional probability using two-way tables, tree diagrams and Venn diagrams.

Sample questions

John has an empty box.
He puts some red counters and some blue counters into the box.

The ratio of the number of red counters to the number of blue counters is 1 : 4

Linda takes at random 2 counters from the box.

The probability that she takes 2 red counters is $\frac{6}{155}$

How many red counters did John put into the box?

(Total 4 marks)
New SAMs Paper 1H qu.24 (P9, A21, R8 – AO1/AO3)

Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 people like at least one of the drinks.
19 people like all three drinks.
16 people like tea and coffee but do **not** like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.
Sami selects at random one of the 50 people.

(*a*) Work out the probability that this person likes tea.

(4)

(*b*) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

(2)

(Total 6 marks)
New SAMs Paper 3H qu.12 (P9, P8 – AO1/AO2/AO3)

Thelma spins a biased coin twice.
The probability that it will come down heads both times is 0.09

Calculate the probability that it will come down tails both times.

(Total 3 marks)
Specimen Papers Set 1, Paper 3H qu.18 (P9 – AO1/ AO3)

Exam Wizard topics

Venn diagrams
Probability tree diagrams
Conditional probabilities
New to GCSE (9–1)

6. Statistics

S1	infer properties of populations or distributions from a sample, while knowing the limitations of sampling

Content guidance

Questions concerning questionnaires will no longer be set.

To include the calculation of summary statistics from a sample, knowing that these are estimates for the population.

Stratified sampling is **not** part of the GCSE 9–1 specification.

However, the ability to infer properties of populations or distributions from a sample is part of the specification so candidates could be asked questions relating to this.

At Higher tier, to include the Peterson capture–recapture method.

New to Foundation tier

Sources of bias and sampling. (NB: understanding how sources of data may be biased is already Foundation tier.)
Design an experiment, justify sampling and use stratified sampling.

Mapping to 1MA0 specification content descriptors

SPb (F/H) Identify possible sources of bias
SPc (F/H) Design an experiment or survey

Learning objectives

Foundation tier (Unit 7)

- Specify the problem and plan an investigation: decide what data to collect and what statistical analysis is needed; consider fairness;
- Recognise types of data: primary, secondary, quantitative and qualitative; identify primary data to collect and in what format, including grouped data; collect data from a variety of suitable primary and secondary sources;
- Understand sample and population; understand how sources of data may be biased and explain why a sample may not be representative of a whole population.

Higher tier (Unit 14a)

- Specify the problem and plan: decide what data to collect and what analysis is needed; understand primary and secondary data sources; consider fairness;
- Understand what is meant by a sample and a population;
- Understand how different sample sizes may affect the reliability of conclusions drawn;
- Identify possible sources of bias and plan to minimise it; write questions to eliminate bias, and understand how the timing and location of a survey can ensure a sample is representative.

Sample questions

There are 1200 students at a school.

Kate is helping to organise a party.
She is going to order a pizza.

Kate takes a sample of 60 of the students at the school.
She asks each student to tell her **one** type of pizza they want.

The table shows information about her results.

Pizza	Number of students
ham	20
salami	15
vegetarian	8
margherita	17

Work out how much ham pizza Kate should order.
Write down any assumption you make **and** explain how this could affect your answer.

(Total 3 marks)
New SAMs Paper 1F qu.27 / 1H qu.7 (S1 – AO1/AO3)

Exam Wizard topics

Primary and secondary data
Sampling and bias
New to Foundation from Higher in GCSE (9–1)

S2	interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

Content guidance

To include stem and leaf diagrams.

Candidates will be expected to be able to draw a time series graph by plotting points from given information and take readings from time series graphs provided.

Moving averages will not be tested and neither will average seasonal trends. Questions could be set on the general trend, however.

New to Foundation tier

Interpret and construct back-to-back stem and leaf diagrams.

Mapping to 1MA0 specification content descriptors

SPa (F/H) Understand and use statistical problem solving process/handling data cycle
SPd (F/H) Design data-collection sheets distinguishing between different types of data
SPe (F/H) Extract data from printed tables and lists
SPf (F/H) Design and use two-way tables for discrete and grouped data
SPg (F/H) Produce charts and diagrams for various data types
SPi (F/H) Interpret a wide range of graphs and diagrams, and draw conclusions

Learning objectives

Foundation tier (Units 3a, 3b, 4a, 7)

- Use suitable data-collection techniques (data to be integer and decimal values); design and use data-collection sheets for grouped, discrete and continuous data, use inequalities for grouped data, and introduce ≤ and ≥ signs; sort, classify and tabulate data, both discrete and continuous quantitative data, and qualitative data; extract data from lists and tables;
- Construct tables for time–series data; two-way tables for discrete and grouped data;
- Using frequency tables and bar charts: calculate the total frequency; read off frequency values; find greatest and least values; identify the mode; identify the modal class from a grouped frequency table;
- Produce and interpret: pictograms; composite bar charts; dual/comparative bar charts for categorical and ungrouped discrete data; bar-line charts; vertical line charts; line graphs; line graphs for time–series data; stem and leaf (including back-to-back);
- Construct pie charts for categorical data and discrete/continuous numerical data; interpret simple pie charts using simple fractions ($\frac{1}{2}$, $\frac{1}{4}$) and percentages (multiples of 10%); find the mode and total frequency; understand that the frequency represented by corresponding sectors in two pie charts depends on the total population represented by each pie chart;
- Multiply and divide a fraction by an integer, including finding fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;
- Find the mode from a stem and leaf diagram;
- Interpret and find a range of averages as follows: median, mean and range from a (discrete) frequency table; range, modal class, interval containing the median, and estimate of the mean from a grouped data frequency table; mode and range from a bar chart; median, mode and range from stem and leaf diagrams; mean from a bar chart;
- Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf;

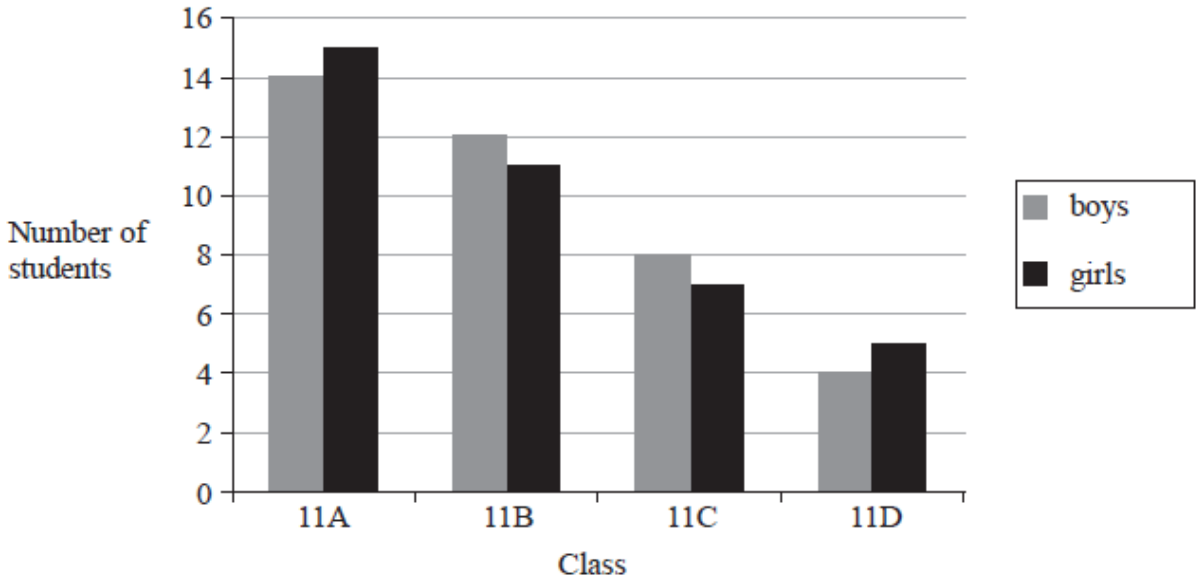
- Represent, interpret and discuss any data in tables and charts; know which charts to use for different types of data sets; recognise simple patterns, characteristics and relationships in bar charts and line graphs.

Higher tier (Units 3a, 3b)

- Design, use and complete two-way tables for discrete and grouped data;
- Sort, classify and tabulate data and discrete or continuous quantitative data;
- Construct and interpret stem and leaf diagrams (including back-to-back diagrams): find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
- Calculate the mean, mode, median and range from a frequency table (discrete data);
- Know which charts to use for different types of data sets;
- Produce and interpret composite bar charts; comparative and dual bar charts;
- Produce and interpret pie charts: find the mode and the frequency represented by each sector; compare data from pie charts that represent different-sized samples;
- Produce line graphs: read frequency values, calculate total population, find greatest and least values;
- Construct and interpret time–series graphs; comment on trends;
- Recognise simple patterns, characteristics and relationships in bar charts, line graphs and frequency polygons.

Sample questions

The bar chart gives information about the numbers of students in the four Year 11 classes at Trowton School.



(*a*) What fraction of the students in class 11A are girls?

(2)

Shola says,

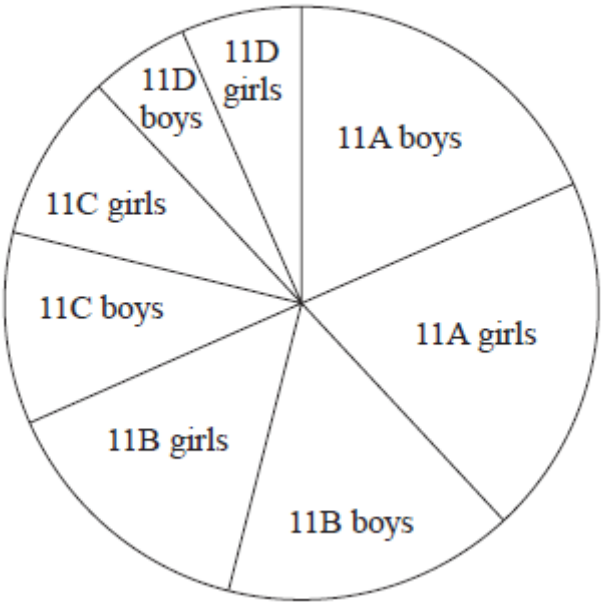
"There are more boys than girls in Year 11 in Trowton School."

(*b*) Is Shola correct?
You must give a reason for your answer.

(2)

The pie chart gives information about the 76 students in the same four Year 11 classes at Trowton School.

Number of students in Year 11 of Trowton School



Tolu says,

"It is more difficult to find out the numbers of students in each class from the pie chart than from the bar chart."

(*c*) Is Tolu correct?
You must give a reason for your answer.

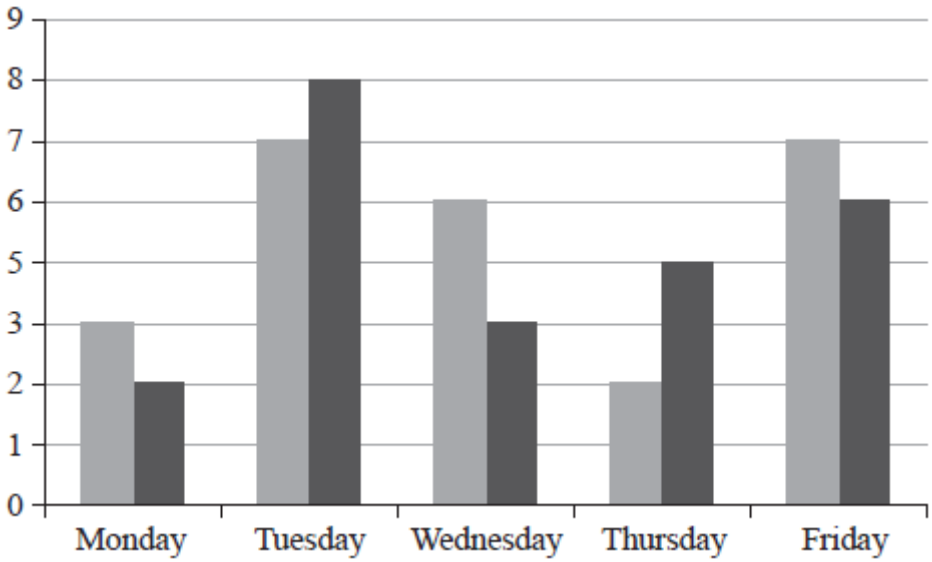
(1)

(Total 5 marks)

New SAMs Paper 3F qu.6 (S2 – AO2)

Sam and Max work in a shop from Monday to Friday.

Sam draws a graph to show the number of TVs they each sell.



Write down **three** things that are wrong with this graph.

(Total 3 marks)

New SAMs Paper 2F qu.8 (S2 – AO2)

The table shows information about the numbers of fruit trees in an orchard.

Apple tree	Pear tree	Plum tree
45	20	25

(*a*) The pictogram shows this information.

Complete the key for the pictogram.

Apple tree	△ △ △ △ △ △ △ △ △
Pear tree	△ △ △ △
Plum tree	△ △ △ △ △

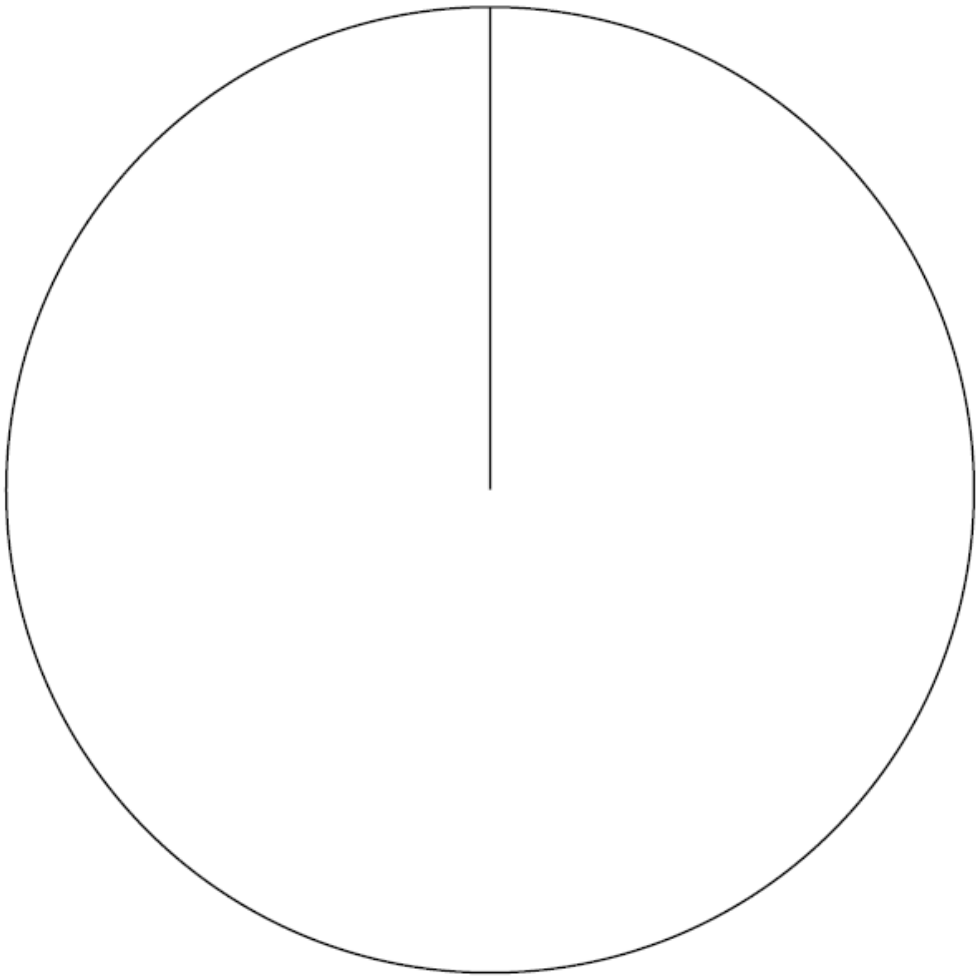
Key: △ represents ..... trees

(1)

(*b*) There are 90 fruit in the orchard.

Apple tree	Pear tree	Plum tree
45	20	25

Draw an accurate pie chart for this information.



(3)

(Total 4 marks)

New SAMs Paper 1F qu.14 (S2 – AO2)

Exam Wizard topics

Primary and secondary data
Two-way tables
Pictograms
Pie; bar; tally charts
Stem and leaf diagrams
Histograms and grouped frequency diagrams
Frequency polygons
Line graphs
Mean; median; mode
New to Foundation from Higher in GCSE (9–1)

S3	**construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**

Mapping to 1MA0 specification content descriptors

SPg (H) Produce charts and diagrams for various data types
SPh (H) Calculate median, mean, range, quartiles and interquartile range, mode and modal class
SPi (H) Interpret a wide range of graphs and diagrams, and draw conclusions

Learning objectives

Higher tier (Units 3a, 3b, 14b)

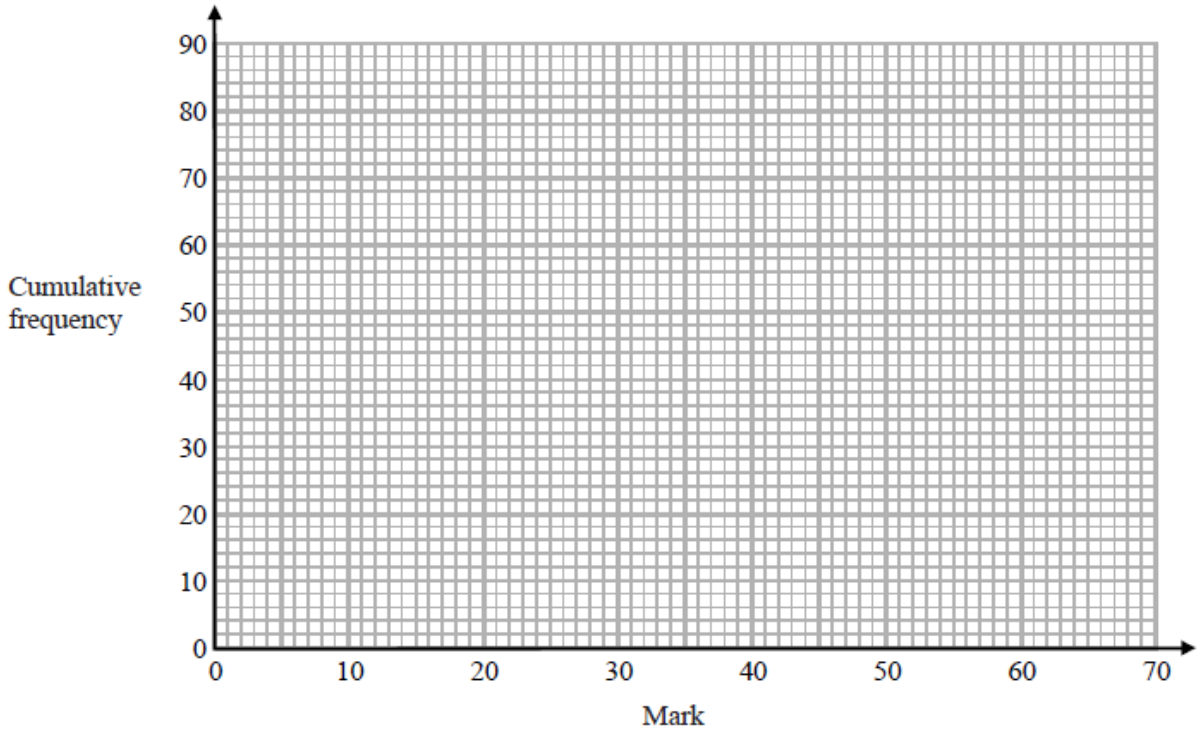
- Construct and interpret grouped frequency tables for continuous data; for grouped data: find the interval containing the median and modal class; estimate the mean; understand that the expression 'estimate' will be used where appropriate when finding the mean using mid-interval values;
- Produce frequency diagrams for grouped discrete data; using frequency diagrams: read frequency values, calculate total population, find greatest and least values;
- Produce and interpret frequency polygons for grouped data; using frequency polygons: read frequency values, compare distributions, calculate total population and mean, estimate greatest and least possible values (and range);
- Know the appropriate uses of cumulative frequency diagrams; construct and interpret cumulative frequency tables, cumulative frequency graphs/diagrams, and from the graph: estimate frequency greater/less than a given value; find the median and quartile values and interquartile range;
- Know the appropriate uses of histograms; use and understand frequency density; construct and interpret histograms from class intervals with equal and unequal class intervals;
- From histograms: complete a grouped frequency table; estimate the mean and median or any other information, such as the number of people in a given interval;
- Use statistics found in all histograms and cumulative frequency graphs to describe a population.

Sample questions

The cumulative frequency table shows the marks some students got in a test.

Mark (m)	Cumulative frequency
$0 < m \leqslant 10$	8
$0 < m \leqslant 20$	23
$0 < m \leqslant 30$	48
$0 < m \leqslant 40$	65
$0 < m \leqslant 50$	74
$0 < m \leqslant 60$	80

(a) On the grid, plot a cumulative frequency graph for this information.



(2)

(b) Find the median mark.

(1)

Students either pass the test or fail the test.
The pass mark is set so that 3 times as many students fail the test as pass the test.

(c) Find an estimate for the lowest possible pass mark.

(3)

(Total 6 marks)

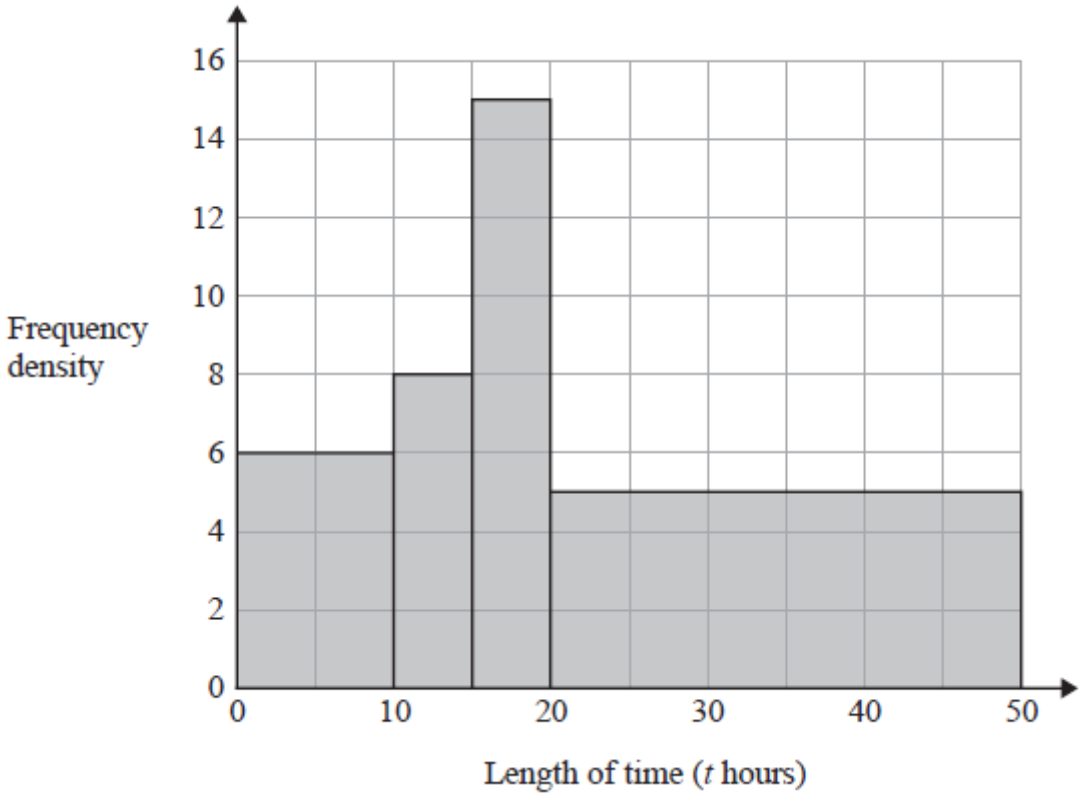
New SAMs Paper 3H qu.7 (S3, R9 – AO1/AO2/AO3)

Bhavna recorded the lengths of time, in hours, that some adults watched TV last week.

The table shows information about her results.

Length of time (t hours)	Frequency
$0 \leqslant t < 10$	6
$10 \leqslant t < 15$	8
$15 \leqslant t < 20$	15
$20 \leqslant t < 40$	5

Bhavna made some mistakes when she drew a histogram for this information.



Write down **two** mistakes Bhavna made.

(Total 2 marks)

New SAMs Paper 1H qu.22 (S3 – AO2)

Exam Wizard topics

Histograms and grouped frequency diagrams
Cumulative frequency diagrams
Mean; median; mode
Range and quartiles

S4	interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data, **including box plots** • appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers, **quartiles and inter-quartile range**)

New to Foundation tier

Use methods other than lines of best fit to predict values.
Appreciate correlation as measure of the strength of association between two variables.

Mapping to 1MA0 specification content descriptors

SPg (F/H) Produce charts and diagrams for various data types
SPh (F) Calculate median, mean, range, mode and modal class
SPh (H) Calculate median, mean, range, quartiles and interquartile range, mode and modal class
SPi (F/H) Interpret a wide range of graphs and diagrams, and draw conclusions
SPl (F/H) Compare distributions and make inferences

Learning objectives

Foundation tier (Units 3a, 3b, 7)

- Identify the mode from a frequency table, bar chart, stem and leaf diagram, pie chart; identify the modal class from a grouped frequency table;
- Calculate the mean, mode, median and range for discrete data;
- Interpret and find a range of averages as follows: median, mean and range from a (discrete) frequency table; range, modal class, interval containing the median, and estimate of the mean from a grouped data frequency table; mode and range from a bar chart; median, mode and range from stem and leaf diagrams; mean from a bar chart;
- Understand that the expression ‘estimate’ will be used where appropriate, when finding the mean of grouped data using mid-interval values;
- Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf;
- Recognise the advantages and disadvantages of different measures of average.

Higher tier (Units 3a, 3b, 14b)

- Calculate mean and range, find median and mode from a small data set and from a frequency table (discrete data); use a spreadsheet to calculate mean and range, and find median and mode;
- Recognise the advantages and disadvantages of different measures of average;
- From stem and leaf diagrams (including back-to-back diagrams): find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
- From grouped frequency tables for grouped data: find the interval containing the median and modal class; estimate the mean; understand that the expression ‘estimate’ will be used where appropriate when finding the mean using mid-interval values;
- From pie charts: find the mode and the frequency represented by each sector; compare data from pie charts that represent different-sized samples;
- From frequency diagrams, frequency polygons and line graphs: read frequency values, calculate total population, find greatest and least values (and range);
- For two distributions, compare the mean, median or mode and range, or median and interquartile range, as appropriate;
- From cumulative frequency diagrams and box plots, find the median, quartiles, range and interquartile range; produce box plots from raw data, and when given quartiles and median, identify any outliers;

- From histograms with equal and unequal class intervals, estimate the mean and median;
- Use statistics found in all histograms, cumulative frequency graphs and box plots to describe a population.

Sample questions

Here is a list of numbers

12 19 12 15 11 15 12 13 17

Find the median.

(Total 2 marks)
New SAMs Paper 2F qu.9 (S4 – AO1)

The table shows some information about the foot lengths of 40 adults.

Foot length (f cm)	Number of adults
$16 \leqslant f < 18$	3
$18 \leqslant f < 20$	6
$20 \leqslant f < 22$	10
$22 \leqslant f < 24$	12
$24 \leqslant f < 26$	9

(*a*) Write down the modal class interval.

(1)

(*b*) Calculate an estimate for the mean foot length.

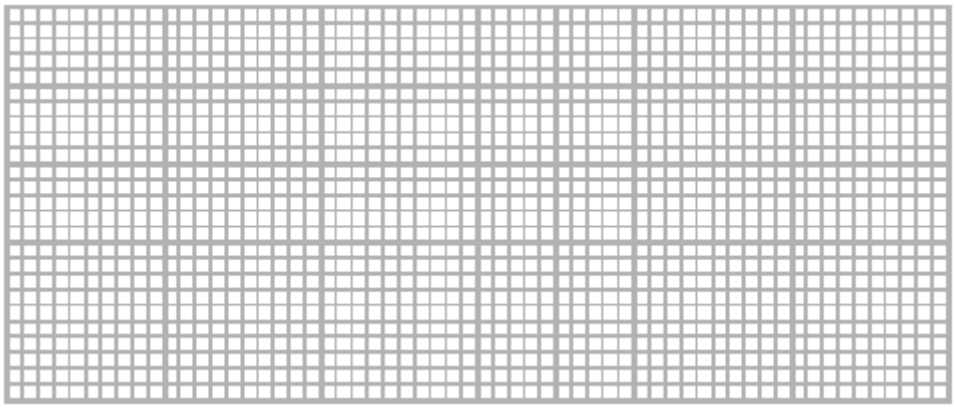
(3)

(Total 4 marks)
New SAMs Paper 2F qu.27 / 2H qu.5 (S4 – AO1/AO2)

Ben played 15 games of basketball.
Here are the points he scored in each game.

17 18 18 18 19 20 20 22 23 23 23 26 27 28 28

(*a*) Draw a box plot for this information.



(3)

Sam plays in the same 15 games of basketball.

The median number of points Sam scored is 23
The interquartile range of these points is 12
The range of these points is 20

(*b*) Who is more consistent at scoring points, Sam or Ben?
You must give a reason for your answer.

(2)

(Total 5 marks)
New SAMs Paper 1H qu.14 (S4 – AO2)

Exam Wizard topics

Pictograms
Pie; bar; tally charts
Stem and leaf diagrams
Histograms and grouped frequency diagrams
Frequency polygons
Line graphs
Box plots
Cumulative frequency diagrams
Mean; median; mode
Range and quartiles
Grouped data
New to Foundation from Higher in GCSE (9–1)

S5	apply statistics to describe a population

Mapping to 1MA0 specification content descriptors

SPj (F/H) Look at data to find patterns and exceptions

Learning objectives

Foundation tier (Units 3a, 3b, 7)

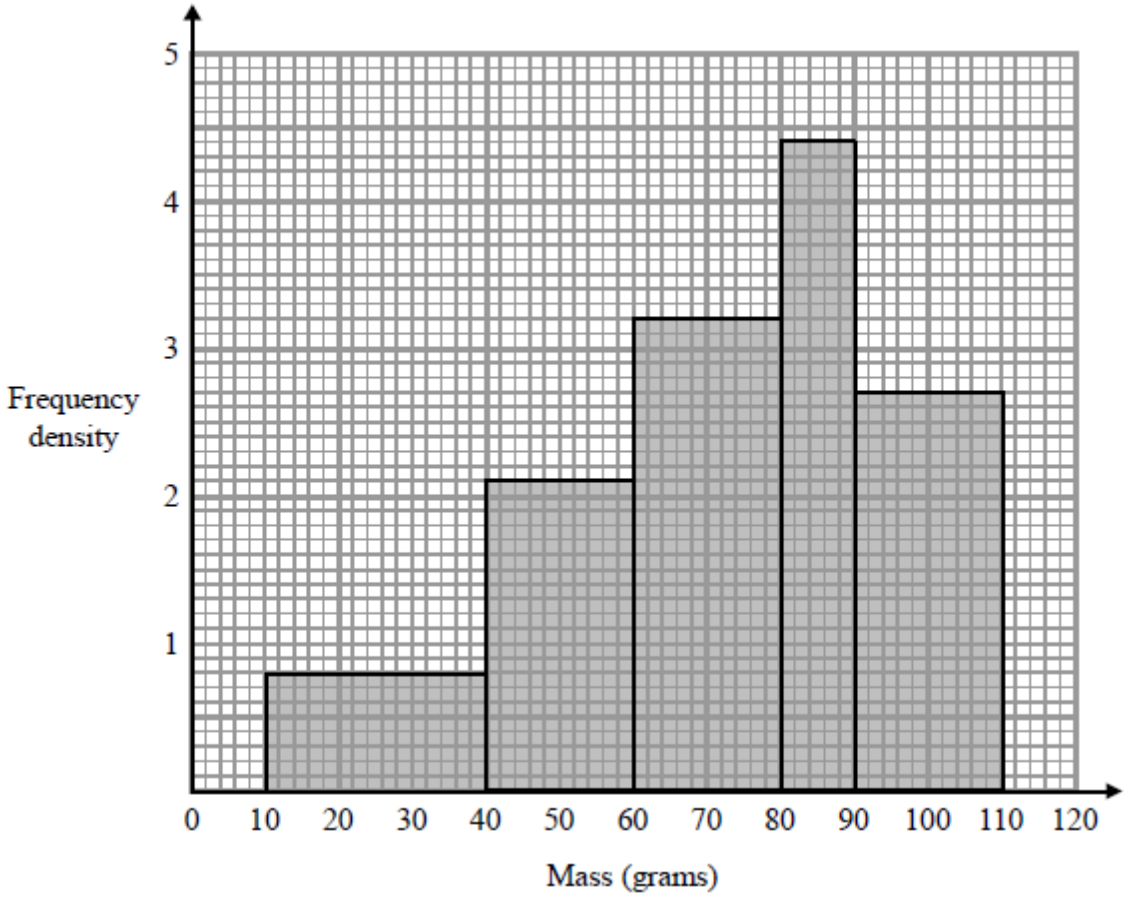
- Recognise simple patterns, characteristic and relationships in bar charts and line graphs;
- Interpret and discuss any data, including in tables;
- Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf;
- Recognise the advantages and disadvantages between measures of average.

Higher tier (Units 3a, 3b, 14b)

- Recognise the advantages and disadvantages between measures of average;
- Compare two distributions from stem and leaf diagrams (mode, median, range);
- Produce and interpret composite bar charts; comparative and dual bar charts;
- Compare data from pie charts that represent different-sized samples;
- Construct and interpret time–series graphs; comment on trends;
- Recognise simple patterns, characteristics and relationships in bar charts, line graphs and frequency polygons;
- For two distributions, compare the mean, median or mode and range, or median and interquartile range, as appropriate;
- Interpret box plots to find median, quartiles, range and interquartile range, and draw conclusions;
- Use statistics found in all histograms, cumulative frequency graphs and box plots to describe a population.

Sample questions

A biologist is studying the effects of global warming on animal size.
The histogram gives information about the masses of a species of snail in a sample he took in 2013 from a large lake.



The mean mass of the same species of snail taken from the lake in 2003 was 75 grams.

(*a*) Is there any evidence to support the hypothesis that the mass of this species of snail has decreased?

(5)

(*b*) Explain whether it is possible to state what the mode is from this histogram.

(1)

(Total 6 marks)

Original SAMs Paper 2H qu.14 (S3, S4, S5 – AO1/AO2/AO3)

Exam Wizard topics

Mean; median; mode
Range and quartiles
Grouped data

S6	use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends while knowing the dangers of so doing

New to Foundation tier

Explain an isolated point on a scatter graph.

Mapping to 1MA0 specification content descriptors

SPg (F/H) Produce charts and diagrams for various data types
SPi (F/H) Interpret a wide range of graphs and diagrams, and draw conclusions
SPj (H) Look at data to find patterns and exceptions
SPk (F/H) Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent

Learning objectives

Foundation tier (Unit 3c)

- Draw scatter graphs; interpret points; identify and ignore outliers;
- Draw the line of best fit on a scatter diagram by eye, and understand what it represents;
- Use line of best fit to predict values of a variable given values of the other variable; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing; state how reliable predictions are, i.e. not reliable if extrapolated;
- Distinguish between positive, negative and no correlation using lines of best fit; interpret scatter graphs in terms of the relationship between two variables; interpret correlation in terms of the problem; understand that correlation does not imply causality.

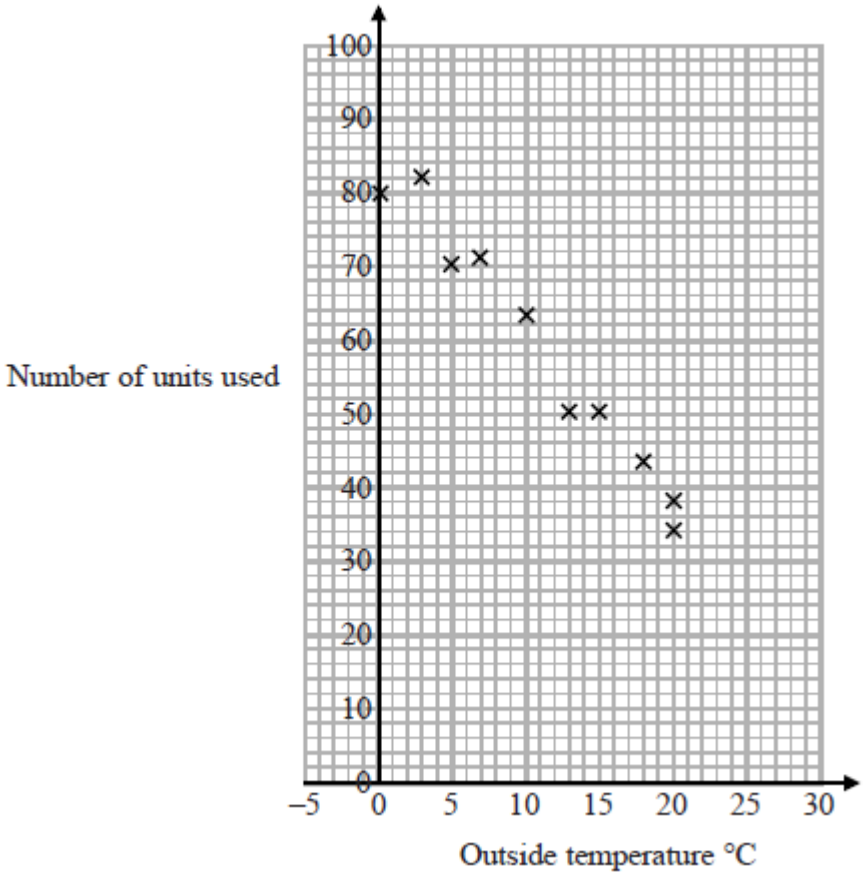
Higher tier (Unit 3b)

- Draw and interpret scatter graphs in terms of the relationship between two variables; explain an isolated point; identify and ignore outliers;
- Draw lines of best fit by eye, understanding what these represent;
- Use a line of best fit, or otherwise, to predict values of a variable given values of the other variable; use the line of best fit to make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing;
- Distinguish between positive, negative and zero correlation using lines of best fit, and interpret correlation in terms of the problem; understand that correlation does not imply causality, and appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply ‘no relationship’ but merely ‘no linear correlation’.

Sample questions

In a survey, the outside temperature and the number of units of electricity used for heating were recorded for ten homes.

The scatter diagram shows this information.



Molly says,

"On average the number of units of electricity used for heating decreases by 4 units for each °C increase in outside temperature."

(*a*) Is Molly right?
Show how you get your answer.

(3)

(*b*) You should **not** use a line of best fit to predict the number of units of electricity used for heating when the outside temperature is 30°C.

Give one reason why.

(1)

(Total 4 marks)

New SAMs Paper 3F qu.21 / 3H qu.4 (S6, A10 – AO2)

Exam Wizard topics

Scatter graphs
Lines of best fit
New to Foundation from Higher in GCSE (9–1)

Geometrical statements in GCSE Maths papers

The notes overleaf refer to the new 2017 GCSE Edexcel Mathematics specifications.

In questions that relate to Geometry candidates can be asked to give a reason for a calculation or proof. In many cases this is related to angles. This could also be part of a requirement for candidates to communicate in mathematical terms.

In Geometry the need to communicate in mathematics terms is embodied in mathematical statement relating to geometrical properties. It is important that candidates show with clarity their understanding of the use of any geometrical property that they might use in solving problems.

The attached list gives some examples of responses to questions which would be considered clear communication in these respects. These are not unique: there are other similar statements that could be used to earn the marks, but these statements contain those key words and phrases which are judged to be minimal in terms of a request to give reasons for mathematical deduction in geometry.

Rules in Geometry

Lines

Vertically opposite angles are equal.

Angles on a straight line add up to 180°

Angles at a point add up to 360°

Triangles and quadrilaterals

Angles in a triangle add up to 180°

Base angles of an isosceles triangle are equal.

Angles in an equilateral triangle are equal.

Angles in a quadrilateral add up to 360°

An exterior angle (of a triangle) is equal to the sum of the internal opposite angles.

Polygons

Exterior angles of a polygon add up to 360°.

The interior and exterior angle of any polygon add up to 180°.

Parallel lines

Alternate angles are equal.

Corresponding angles are equal.

Allied (or co-interior) angles add up to 180°.

Circle Theorems

The tangent to a circle is perpendicular (90°) to the radius.

Tangents from an external point are equal in length.

Angles in a semicircle are 90°.

Angles in the same segment are equal.

The angle at the centre of a circle is twice the angle at the circumference.

Opposite angles of a cyclic quadrilateral add to 180°.

Alternate segment theorem.

Command Words

Please note that this table is not exhaustive but provides the most commonly used command words.

Command word	Comments
Write down… Write…	No working will be needed
Find…	Some working will be needed but will be minimal
Work out…	Used interchangeably with 'calculate', it will be necessary to do some working out
Calculate…	Used interchangeably with 'work out' but use of 'calculate' suggests that a calculator will be needed, it will be necessary to do some working out
Explain…	Explanation needed – may be a sentence or could be a mathematical statement
Give a reason…	Clear reasons needed; if geometrical reasons then must link into working
Draw…	Implies accuracy is important
Sketch…	Less formal than 'draw'…(no accurate measurements needed)
Complete…	Usually means that some values need filling in, for example, on a probability tree diagram or a table of values
Show…	All working needed to get to the required answer must be shown
Prove…	More formal than 'show', all steps must be present and, in the case of a geometrical proof, reasons must be given
Prove algebraically…	Algebra must be used in the proof
Describe…	Words needed to describe, for example, a transformation
Justify…	Show all working or give a written explanation
Expand…	Remove brackets
Expand and simplify…	Remove brackets and simplify
Factorise…	Straight forward factorisation
Factorise fully…	More complex factorisation, more than one factor to consider
Simplify…	Simplify the given expression
Simplify fully ….	Likely to be more than one stage needed to simplify expression
Solve…	Solve an equation / inequality