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# GCSE

## Methods in Mathematics

Scheme of work

# Edexcel GCSE in Methods in Mathematics (2MM01)

For first teaching September 2010

Issue 1

A PEARSON COMPANY

# Contents

# Introduction

This scheme of work is based on a five-term model over two years for both Foundation and Higher tier students.
It can be used directly as a scheme of work for the GCSE Methods in Mathematics specification (2MM01).

The scheme of work is structured so each topic contains:

- module number
- recommended teaching time, though this is guidance and adaptable according to individual teaching needs
- tier
- contents, referenced back to the specification
- objectives for students at the end of the module
- ideas for differentiation and extension activities
- notes for general mathematical teaching points and common misconceptions.

Updates will be available via a link from the Edexcel mathematics website (www.edexcel.com).

This scheme of work can be used in conjunction with the scheme of work for GCSE Applications of Mathematics (2AM01).
Where a module appears in both schemes of work, it is labelled with (**COMMON**).
Material that is methods content only is in *italics.*

# GCSE Methods in Mathematics (2MM01)
# Foundation
# Tier

# Scheme of work

# Foundation course overview

The table below gives an overview of the modules in the Foundation tier scheme of work.

Teachers should be aware that the estimated teaching hours are **approximate** and should be used as a guideline only.

| Unit | Module number | Title | Estimated teaching hours |
|---|---|---|---|
| 1 | 1 | Integers and decimals | 5 |
| 1 | 2 | Factors and multiples | 2.5 |
| 1 | 3 | Laws of indices | 2.5 |
| 1 | 4 | Fractions | 7 |
| 1 | 5 | Introduction to algebra | 1.5 |
| 1 | 6 | Algebraic manipulation | 6 |
| 1 | 7 | Patterns and sequences | 4 |
| 1 | 8 | Solving linear equations | 6 |
| 1 | 9 | Linear graphs and $y = mx + c$ | 7 |
| 1 | 10 | Angle facts and geometric shapes | 4 |
| 1 | 11 | Line and rotational symmetry | 2.5 |
| 1 | 12 | Circles | 1.5 |
| 1 | 13 | Area and perimeter | 4 |
| 1 | 14 | Transformations | 5 |
| 1 | 15 | Similarity and congruency | 3 |
| 1 | 16 | Probability 1 | 5 |
| 1 | 17 | Probability 2 | 5 |
| 2 | 1 | Integers, decimals and fractions | 5 |
| 2 | 2 | Percentages | 4.5 |
| 2 | 3 | Using fractions | 3 |
| 2 | 4 | Ratio and proportion | 4.5 |
| 2 | 5 | Solving linear inequalities | 6 |
| 2 | 6 | Substitution and formulae | 4.5 |
| 2 | 7 | Using graphs to solve equations | 5 |
| 2 | 8 | Angle facts and polygons | 4 |
| 2 | 9 | Similarity and congruence | 3.5 |
| 2 | 10 | Pythagoras' theorem | 7.5 |
| 2 | 11 | Circles | 5.5 |
| 2 | 12 | Surface area and volume | 5.5 |
| | | **Total** | **123 HOURS** |

## Unit 1: Methods 1 Foundation

Centres may wish to teach this unit alongside Unit 2 if they prefer to deliver their curriculum in a holistic manner.
They would enter students for both papers at the same time.
However, some centres may prefer to concentrate on Unit 1 first and enter their students in either June 2011 or in November 2011. The benefit of this model is that it allows a full analysis of the paper using ResultsPlus before students are prepared for a possible resit (one resit per unit is permitted).
In addition, some centres, that traditionally organise their mock examinations in November or December, may use this opportunity for an external body to mark and analyse their papers. It may also flag up any students who would benefit from a change of tier for either their resit examination or Unit 2 assessment.

**Module 1 METHODS** **Time: 3 – 7 hours**

**Tier: Foundation – Unit 1**

**Content: Integers and decimals** **COMMON**

| | |
|---|---|
| MN a, d | Understand place value and round to the nearest integer and a given power of 10 |
| MN c | Understand and use negative integers both as positions and translations on a number line |
| MN c | Order integers |
| MN b | Multiply and divide by negative numbers |
| MN b | Multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1 |
| MN d | Check and estimate answers to problems |
| MN c | Write decimal numbers in order of size |
| MN d | Round to a given number of significant figures and decimal places |
| MN b | Add and subtract decimal numbers |
| MN b | Divide by a decimal (up to 2 dp) by transforming it to a problem involving division using by an integer |
| MN a, d | Check and estimate answers to problems, by using approximations or inverse operations |
| MN a | Use brackets and the hierarchy of operations |
| MN a | Use one calculation to find the answer to another |

**PRIOR KNOWLEDGE**

The ability to order numbers
Place value
Experience of the four operations using whole numbers
Knowledge of integer complements to 10 and to 100
Knowledge of multiplication facts to $10 \times 10$
Knowledge of strategies for multiplying and dividing whole numbers by 10
The concepts of a fraction and a decimal

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand and order integers
- Add, subtract, multiply and divide integers (BIDMAS)
- Round whole numbers to the nearest, 10, 100, 1000, …
- Multiply and divide whole numbers by a given multiple of 10
- Check their calculations by rounding, eg $29 \times 31 \approx 30 \times 30$
- Put digits in the correct place in a decimal number
- Round numbers to a given number of decimal places or significant figures
- Multiply and divide decimal numbers by whole numbers and decimal numbers, eg $266.22 \div 0.34$
- Know that eg $13.5 \div 0.5 = 135 \div 5$

**DIFFERENTIATION & EXTENSION**

More work on long multiplication and division without using a calculator
Estimate answers to calculations involving the four operations
Consideration of mental maths problems with negative powers of 10: $2.5 \times 0.01$, 0.001
Directed number work with two or more operations, or with decimals
Multiply and divide decimals by decimals (more than 2 dp)

**NOTES**

Present all working clearly with decimal points in line
Emphasise that all working is shown
For non-calculator methods make sure that remainders and carrying are shown
It is essential to ensure students are absolutely clear about the difference between significant figures and
decimal places
Extend to multiplication of decimals or long division of integers
Try different methods from the traditional ones, eg Russian or Chinese methods for multiplication

**Module 2 METHODS** **Time: 1 – 4 hours**

**Tier: Foundation – Unit 1**

**Content: Factors and multiples** **COMMON**

| | |
|---|---|
| MN e | Understand even, odd and prime numbers |
| MN e | Find factors and multiples of numbers |
| MN c | Find squares and cubes of numbers and find square roots and cube roots of numbers |
| MN e | Find common factors and common multiples |
| MN c | Understand index notation for powers of 10, squares and cubes |
| *MN e, f* | *Find Higher Common Factors (HCF), Lowest Common Multiple (LCM) and prime factor decomposition* |
| *MN p* | *Draw a Venn diagram from given information* |
| *MN p* | *Interpret Venn diagrams, be able to find the intersection and union of sets* |
| *MN p* | *Use Venn diagrams to solve problems* |

**PRIOR KNOWLEDGE**

Number complements to 10 and multiplication or division facts
Use a number line to show how numbers relate to each other
Recognise basic number patterns
Experience of classifying integers

**OBJECTIVES**

By the end of the module the student should be able to:

- Find squares; cubes; square roots and cube roots of numbers, with and without a calculator (including the use of trial and improvement)
- Understand odd and even numbers, and prime numbers
- Find common factors and common multples
- *Find the HCF and the LCM of numbers*
- *Write a number as a product of its prime factors, eg* $108 = 2^2 \times 3^3$
- *Use Venn diagrams to represent sets*
- *Draw a Venn diagram using given information and solve problems*
- *Understand and be able to find the intersection and union of sets*

**DIFFERENTIATION & EXTENSION**

Calculator exercise to check factors of larger numbers
Further work on indices to include negative and/or fractional indices (introduction to Higher tier)
Use prime factors to find LCM
Use a number square to find primes (sieve of Eratosthenes)
Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)
Use hoops to illustrate Venn diagrams with physical objects which fufill different criteria

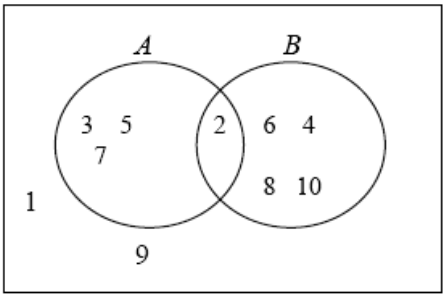
**NOTES**

All of the work in this unit is easily reinforced through starters and plenaries
Calculators should only be used when appropriate
Encourage student to learn square, cube, prime and common roots as Unit 1 is a non-calculator examination
Venn Diagrams can be used to illustrate HCF and LCM questions. [Link with Probability 2 - Topic 17]

$A$ = {prime numbers} $B$ = {even numbers}

The Venn diagram shows numbers 1 to 10

$\xi$ = {1, 2, 3 ……..10}




Hence A ∩ B is {2}

**Module 3 METHODS** **Time: 1 – 4 hours**

**Tier: Foundation – Unit 1**

**Content: Laws of indices** **COMMON**

MN c Use index notation and index laws for multiplication and division of integer powers and powers of powers

MN c Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integers

**PRIOR KNOWLEDGE**

Knowledge of squares, square roots, cubes and cube roots

**OBJECTIVES**

By the end of the module the student should be able to:

- Use index rules to simplify and calculate numerical expressions involving powers, eg $(2^3 \times 2^5) \div 2^4$

**DIFFERENTIATION & EXTENSION**

Use index rules to simplify algebraic expressions

Treat index rules like formulae (state which rule is being at used each stage in a calculation)

Extend to questions like $2x^2 \times 4x^3$

**NOTES**

When a question asks for the answer as a power then leave the answer in index form

**Module 4 METHODS** **Time: 6 – 8 hours**

**Tier: Foundation – Unit 1**

**Contents: Fractions** **COMMON**

MN c Understand and use number operations and the relationship between them, including inverse operations and hierarchy of operations
MN c Order fractions
MN c Compare fractions and decimals
MN b Use the four operations with fractions
MN c Find equivalent fractions, cancel fractions, and use mixed numbers
MN c Convert between mixed numbers and improper fractions

**PRIOR KNOWLEDGE:**

Multiplication facts
Ability to find common factors
A basic understanding of fractions as being 'parts of a whole unit'
Use of a calculator with fractions

**OBJECTIVES**

By the end of the module the student should be able to:

- Visualise a fraction diagrammatically
- Understand a fraction as part of a whole
- Recognise and write fractions in everyday situations
- Write a fraction in its simplest form and recognise equivalent fractions
- Compare the sizes of fractions using a common denominator
- Add and subtract fractions by using a common denominator
- Multiply and divide fractions
- Write an improper fraction as a mixed fraction

**DIFFERENTIATION & EXTENSION**

Careful differentiation is essential as this topic is dependent on the student's ability
Relate simple fractions to percentages and vice versa
Work with improper fractions and mixed numbers, eg divide 5 pizza's between 3 people
Solve word problems involving fractions and in real life problems, eg finding the perimeter using fractional values
Link fractions with probability questions

**NOTES**

Regular revision of fractions is essential
Demonstrate how to use the fraction button on a calculator, in order to be able to check solutions (in preparation for Unit 2)
Use real-life examples whenever possible

**Module 5 METHODS** **Time: 1 – 2 hours**

**Tier: Foundation – Unit 1**

**Content: Introduction to algebra**

MA a Distinguish the different roles played by letter symbols in algebra using correct notation
MA b Write an expression
MA b Distinguish the meaning between the words ‘equation’, ‘formula’ and ‘expression’

**PRIOR KNOWLEDGE**

Experience of using a letter to represent a number
Word formulae or rules to describe everyday situations, eg time to cook a nut roast linked to weight of joint

**OBJECTIVES**

By the end of the module the student should be able to:

- Distinguish the different roles played by letter symbols in algebra
- Understand the meaning between the words ‘equation’, ‘formula’, and ‘expression’
- Write an expression

**DIFFERENTIATION & EXTENSION**

Extend the above ideas to the ‘equation’ of the straight line, $y = mx + c$
Look at word equations written in symbolic form, eg $F = 2C + 30$ to convert temperature approximately and compare with $F = \frac{9}{5}C + 32$

**NOTES**

There are plenty of past examination papers testing knowledge of the ‘Vocabulary of algebra’ (See Emporium website
www.edexcelmaths.com)

**Module 6 METHODS** **Time: 4 – 8 hours**

**Tier: Foundation – Unit 1**

**Content: Algebraic manipulation** **COMMON**

MA c Simplify terms, products and sums
MA c Multiply a single term over a bracket
MA c Take out common factors
MA c Substitute into given expressions and formulae
MA c Use squares and cubes to write expressions
MA c Use index laws to simplify expressions
MA k *Use algebra to construct and support arguments*

**PRIOR KNOWLEDGE**

Know that a letter can be used to represent a number
Ability to use negative numbers with the four operations
Experience of using BIDMAS in calculations without a calculator

**OBJECTIVES**

By the end of this module the student should be able to:

- Simplify expressions with like terms, eg $x^2 + 3x^2$; $3ab + 5ab + 2c^2$
- Expand and factorise expressions with one pair of brackets, eg expand $x(2x + 3y)$; factorise $3xy^2 – 6x^2y$
- Expand and simplify expressions involving more than one pair of brackets, eg $3(x + 4) – 2(x – 3)$
- Substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$
- Substitute into simple formulae
- Use index laws to simplify expressions
- *Describe a situation or expression algebraically*
- *Write down an equation from a diagram*

**DIFFERENTIATION & EXTENSION**

Expand algebraic expressions involving a pair of brackets
Use formulae from science and mathematics, eg $F = 2C + 30$

**NOTES**

Emphasise correct use of symbolic notation, eg $3x^2$ rather than $3 \times x^2$
Present all work neatly, writing out the questions with the answers to aid revision at a later stage

**Module 7 METHODS** **Time: 3 – 5 hours**

**Tier: Foundation – Unit 1**

**Content: Patterns and sequences**

MA i Generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangle numbers)
MA i Generate sequences from diagrams
MA i Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence
MA i Identify which terms cannot be in a sequence
MA i Write the term-to-term definition in words for a sequence
MA j Use linear expressions to describe the *n*th term of an arithmetic sequence

**PRIOR KNOWLEDGE**

Know about odd and even numbers
Recognise simple number patterns eg 1, 3, 5, ...
Writing simple rules algebraically
Raise numbers to positive whole number powers

**OBJECTIVES**

By the end of the module the student should be able to:

- Find the missing numbers in a number pattern or sequence
- Find the *n*th term of a number sequence as an algebraic expression
- Explain why a number is, or is not, a member of a given sequence
- Produce a linear sequence of numbers from a given *n*th term formula

**DIFFERENTIATION & EXTENSION**

Match-stick problems
Sequences and *n*th term formula for triangle numbers
Fibonacci numbers
Prove a sequence cannot have odd numbers for all values of *n*

**NOTES**

Emphasise good use of notation, eg $3n$ means $3 \times n$
When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic form of the *n*th term
Link linear sequences with graphs

**Module 8 METHODS** **Time: 5 – 7 hours**

**Tier: Foundation – Unit 1**

**Content: Solving linear equations** **COMMON**

MA d Set up simple equations
MA d Solve equations by using inverse operations or by transforming both sides in the same way
MA d Solve linear equations with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation
MA d Solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution

**PRIOR KNOWLEDGE**

Experience of finding missing numbers in calculations
The idea that some operations are 'opposite' to each other
An understanding of balancing
Experience of using letters to represent quantities
Be able to draw a number line

**OBJECTIVES**

By the end of the module the student should be able to:

- Set up simple equations
- Rearrange simple equations
- Solve simple equations
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation
- Solve linear equations which include brackets, those that have negative signs occurring anywhere in the equation, and those with a negative solution
- Solve linear equations in one unknown, with integer or fractional coefficients

**DIFFERENTIATION & EXTENSION**

Derive equations from geometric problems (such as finding unknown angles in triangles or quadrilaterals or perimeter problems)
Solve equations where manipulation of fractions (including negative fractions) is required

**NOTES**

Students need to realise that not all linear equations can easily be solved by either observation or trial and improvement, and hence the use of a formal method is vital
Students can leave their answers in fractional form where appropriate
Remind students of the need to set their work out clearly, keeping the equal signs in line

**Module 9 METHODS** **Time: 5 – 9 hours**

**Tier: Foundation – Unit 1**

**Content: Linear graphs and $y = mx + c$** **COMMON**

| | |
|---|---|
| MA l | Use the conventions for coordinates in the plane |
| MA l | Plot points in all four quadrants |
| MA n | *Find the coordinates of a missing point to make a given shape* |
| MA l | Find the midpoint of a line segment |
| MA l | Identify 2-D shapes, using coordinates |
| MA m | Draw, label and scale axes |
| MA m | Recognise (when values are given for $m$ and $c$) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane |
| MA m | Plot graphs of functions in which $y$ is given explicitly in terms of $x$, or implicitly |
| MA m | Find the gradient of a straight line from a graph |
| MA t | *Understand links between conversion graphs and associated formulae and equations* |
| MA t | *Use a graph to find approximate solutions to equations* |

**PRIOR KNOWLEDGE**

Be able to:

Substitute positive and negative numbers into algebraic expressions
Plot coordinates in the first quadrant
Calculate the mean of two numbers
Identify properties of basic shapes
Rearrange to change the subject of a formula

**OBJECTIVES**

By the end of the module the student should be able to:

- Add a point to a coordinate grid to complete a given shape (parallelogram; rhombus; trapezium; square)
- Find the coordinates of the mid-point of a line segment
- Understand how to represent points in 2-D
- Substitute values of $x$ into linear functions to find corresponding values of $y$
- Plot points for linear functions on a coordinate grid and draw the corresponding straight lines
- Understand linear functions in practical problems, eg conversion graphs
- Find the gradient of a straight line graph

**DIFFERENTIATION & EXTENSION**

Find the equation of the line through two given points
Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines
Use a graphical calculator or graphical ICT package to draw straight-line graphs
Cover horizontal and vertical lines ($x = c$ and $y = c$) [Students often forget these]

**NOTES**

Careful annotation should be encouraged
Students should label the coordinate axes and write the equation of the line
Recognise linear graphs and hence when data may be incorrect
Link to graphs and relationships in other subject areas, ie science, geography etc

**Module 10 METHODS** **Time: 3 – 5 hours**

**Tier: Foundation – Unit 1**

**Content: Angle facts and geometric shapes** **COMMON**

| | |
|---|---|
| MG a | Basic angle facts on a straight line, at a point, on a straight line and vertically opposite angles |
| MG a | *Distinguish between different types of angles* |
| MG a | *Estimate sizes of angles and name angles* |
| MG a.b | *Identify parallel and perpendicular lines* |
| MG b, c | Properties of triangles and quadrilaterals |
| MG b | Understand and use the angle sum of a triangle is 180° |
| MG b | Give reasons for angle calculations |
| MG b | Understand angles associated with intersecting lines |

**PRIOR KNOWLEDGE**

The concept of parallel lines
The concept of vertical and horizontal
The concept of an angle between two lines
Experience in drawing triangles, quadrilaterals and circles

**OBJECTIVES**

By the end of the module the student should be able to:

- Recall and use properties of angles
  - angles at a point
  - angles at a point on a straight line
  - perpendicular lines
  - vertically opposite angles
- Find the size of missing angles
- Use two letter notation for a line and three letter notation for an angle
- Identify triangles by their properties (scalene, isosceles, equilateral, right-angled, obtuse, and acute)
- Prove the angle sum in a triangle is 180°
- Use the angle properties of triangle to find missing angles
- Prove the exterior angle of a triangle is equal to the sum of the two opposite interior angles
- Identify quadrilaterals by their properties (trapezium, parallelogram, rhombus, rectangle, square, kite)

**DIFFERENTIATION & EXTENSION**

Use the angle properties of triangles to find missing angles in combinations of triangles
Harder problems involving multi-step calculations
Link with line and rotational symmetry

**NOTES**

Lots of practical drawing examples to help illustrate properties of various shapes
Diagrams used in examinations are often not drawn accurately

**Module 11 METHODS** **Time: 2 – 3 hours**

**Tier: Foundation – Unit 1**

**Content: Line and rotational symmetry** **COMMON**

MG d Recognise and visualise reflection and rotational symmetry of 2-D shapes
MG d Be able to identify and draw lines of reflection
MG d State the line of reflective symmetry as a simple algebraic equation
MG d Be able to identify the order of rotational symmetry of a 2-D shape

**PRIOR KNOWLEDGE**

Knowledge and properties of 2-D shapes

**OBJECTIVES**

By the end of the module the student should be able to:

- Recognise line and rotational symmetry in 2-D shapes
- Draw in the line of symmetry (or state its equation if the shape is on a coordinate grid)
- State the order of rotational symmetry

**DIFFERENTIATION & EXTENSION**

Extend to planes of symmetry for 3-D solids
Ask students to find their own examples of symmetry in real life

**NOTES**

Accurate drawing skills need to be reinforced
Some students find visualising 3-D objects difficult and simple models will help with this
Use tracing paper or mirrors to help with symmetry questions

**Module 12 METHODS** **Time: 1 – 2 hours**

**Tier: Foundation – Unit 1**

**Content: Circles** **COMMON**

MG q Recall names and definitions of parts of a circle
MG q Draw circles accurately
MG q Understand related circle terms such as semicircle and quarter circle

**PRIOR KNOWLEDGE**

Properties of shapes
Ability to draw a circle with compasses
Measuring a length in cm or mm

**OBJECTIVES**

By the end of the module the student should be able to:

- Identify and name the various parts of a circle (centre, radius, diameter, circumference, sector, segment, arc and chord)
- Draw circles, using a pair of compasses given the radius or diameter

**DIFFERENTIATION & EXTENSION**

Measure circumference, radius and diameter to discover relationships in preparation for Unit 2 (introduction to $\pi$)

**NOTES**

All working should be presented clearly and accurately
Draw lines and circles using an HB pencil
A sturdy pair of compasses is essential and spare equipment is advisable
Useful if the teacher has a small screwdriver to tighten the compasses for some students

**Module 13 METHODS** **Time: 3 – 5 hours**

**Tier: Foundation – Unit 1**

**Content: Area and perimeter** **COMMON**

MG s Recall and use formulae to find the area of triangles, rectangles, parallelograms
MG s Find the area of a trapezium
MG s Find the perimeter and area of shapes made from triangles and rectangles

**PRIOR KNOWLEDGE**

The names of quadrilaterals
Ability to substitute numbers into a formula
Some notion of the difference between length and area
Properties of cubes, cuboids and other common 3-D objects

**OBJECTIVES**

By the end of the module the student should be able to:

- Use the area formulae for triangles, rectangles, parallelograms and trapeziums
- Be able to find the perimeter and area of a compound shape

**DIFFERENTIATION & EXTENSION**

Further problems involving combinations of shapes
Use compound shape methods to investigate the areas of other standard shapes, eg kites
Practical activities, eg using estimation and accurate measuring to calculate perimeters and areas of classroom or corridor floors

**NOTES**

Discuss the correct use of language and units
Ensure that students can distinguish between perimeter and area
Try to impress upon them that perimeter starts with a 'P' which reminds them to plus the sides
Many students have little real understanding of perimeter and area
Practical experience is essential to clarify these concepts

**Module 14 METHODS** **Time: 4 – 6 hours**

**Tier: Foundation – Unit 1**

**Content: Transformations**

| | |
|---|---|
| MG j, k | Transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations |
| MG k | Use column vectors to describe translations |
| MG j | Understand that translations are specified by a distance and direction (written as a column vector), and enlargements by a centre and a scale factor |
| MG j | Rotate a shape about the origin, or any other point |
| MG j | Measure the angle of rotation using right angles, simple fractions of a turn or degrees |
| MG j | Understand that rotations are specified by a centre and an (anticlockwise) angle |
| MG j | Understand that reflections are specified by a mirror line, at first using a line parallel to any axis, then a mirror line such as $y = x$ or $y = -x$ |
| MG j | Recognise, visualise and construct enlargements of objects using positive integer scale factors |
| MG j | Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations |
| MG j | Describe a transformation |
| MG j | Find the centre of enlargement or rotation |

**PRIOR KNOWLEDGE**

Recognition of basic shapes
Line and rotational symmetry
An understanding of the concept of rotation and enlargement
Coordinates in four quadrants
Linear equations parallel to the coordinate axes

**OBJECTIVES**

By the end of the module the student should be able to:

- Use column vectors to describe translations
- Understand translation as a combination of a horizontal and vertical shift including signs for directions
- Understand rotation as a (clockwise) turn about a given centre
- Reflect shapes in a given mirror line; parallel to the coordinate axes and then $y = x$ or $y = -x$
- Enlarge shapes by a given scale factor from a given point; using positive integer scale factors (and understand the impact scale factors have on the image)
- Understand that shapes produced by translation, rotation and reflection are congruent to their image
- Describe transformations
- Find the centre of rotation or enlargement

**DIFFERENTIATION & EXTENSION**

The tasks set should be extended to include combinations of transformations

**NOTES**

Emphasise that students should describe transformations fully
Diagrams should be drawn carefully
Assuming the scales are equal, $y = x$ and $y = -x$ should be 45° diagonal lines
The use of tracing paper is allowed in the examination (although students should not have to rely on its use to solve problems)

**Module 15 METHODS** **Time: 2 – 4 hours**

**Tier: Foundation – Unit 1**

**Content: Similarity and congruency** **COMMON**

MG m Identify shapes which are similar
MG m Identify shapes which are congruent
MG m Understand what makes two shapes similar or congruent
MG m Recognise that enlargements preserve angle but not length
MG m Understand the relationship between lengths in similar figures

**PRIOR KNOWLEDGE**

The mathematical names of triangles and angles
Understanding of the terms perpendicular, parallel and arc
Transformations (particularly enlargements)

**OBJECTIVES**

By the end of the module the student should be able to:

- Identify congruent shapes
- Identify similar shapes
- Use integer and non-integer scale factors to find the length of a missing side in each of two similar shapes, given the lengths of a pair of corresponding sides

**DIFFERENTIATION & EXTENSION**

Link with the information to be able to construct a triangle, eg SSS, ASA, SAS
Link with tessellations and enlargements
Harder problems in congruence

**NOTES**

All working should be presented clearly and accurately
If a student is not sure how to approach a question, advise them to write 'easy' numbers in pencil on their diagram, use these to approach the problem initially and then repeat the process with the 'real' values in the question

**Module 16 METHODS** **Time: 4 – 6 hours**

**Tier: Foundation – Unit 1**

**Content: Probability 1**

| | |
|---|---|
| MP a | Understand and use the vocabulary of probability and probability scale |
| MP b, c | Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency |
| MP c | Estimate the number of times an event will occur, given the relative frequency and the number of trials |
| MP d | List all outcomes for single events, and for two successive events, in a systematic way |
| MP d | Use and draw sample space diagrams and add simple probabilities using sample space diagrams |
| MP e | Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 |
| * | Know when to add or multiply two probabilities: Probability of *A* or *B* occurring is P(*A*) + P(*B*), whereas the probability of *A* and *B* occurring is P(*A*) × P(*B*) |

**Not in Foundation tier. However, some problems involve two outcomes to be combined. Usually these are solved by drawing a sample space or listing outcomes. However, more able students could be introduced to these rules.*

**PRIOR KNOWLEDGE**

Understand that a probability is a number between 0 and 1
Know how to add, and multiplying fractions and decimals
Experience of expressing one number as a fraction of another number
Recognise the language of statistics, eg words such as likely, certain, impossible

**OBJECTIVES**

By the end of the module the student should be able to:

- Write probabilities using fractions, percentages or decimals
- Understand and use estimates or measures of probability
- Use theoretical models to include outcomes using dice, spinners, coins etc
- Understand the probability of successive events, such as several throws of a single dice
- List all outcomes for single events, and for two successive events, systematically
- Use and draw sample space diagrams
- Add simple probabilities, eg from sample space diagrams
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
- Use $1 - p$ as the probability of an event not occurring where $p$ is the probability of the event occurring
- Find a missing probability from a list or table

**DIFFERENTIATION & EXTENSION**

An opportunity for practical examples, eg P(pin up) for dropping a drawing pin, the ‘horse’ race, the national lottery

Show that each cluster of branches adds up to 1

Explain that if two objects are chosen, then this is the same as one event followed by another event without replacement

Experiments with dice and spinners

Show sample space for outcomes of throwing two dice

Stress that there are 36 outcomes (students usually will initially guess it is 12 outcomes for 2 dice

**NOTES**

Students should express probabilities as fractions, percentages or decimals

Fractions do not have to be cancelled to their simplest form

**Module 17 METHODS** **Time: 4 – 6 hours**

**Tier: Foundation – Unit 1**

**Content: Probability 2**

| | |
|---|---|
| MP f | Use set notation to describe events and compound events |
| MP f | Understand that P($A$) represents the probability of an event happening and P($A'$) represents the probability of an event not happening |
| MP g | Use Venn diagrams to represent the number of possibilities and find probabilities |
| MP j | Compare experimental data and theoretical probabilities |
| MP k | Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics |

**PRIOR KNOWLEDGE**

Probability
Venn diagrams

**OBJECTIVES**

By the end of the module the student should be able to:

- Use set notation to describe events
- Use Venn diagrams to represent probabilities, relating to intersection and union ($\cup$ and $\cap$ symbols replacing OR and AND)
- If $A$ and $B$ are mutually exclusive then P($A \cup B$) represents the probability of either $A$ or $B$ happening
- P($A \cup B$) = P($A$) + P($B$)
- If $A$ and $B$ are independent, then P($A \cap B$) represents the probability that both $A$ and $B$ happen
- P($A \cap B$) = P($A$) × P($B$)
- Compare experimental data and theoretical probabilities. [Use a dice-throwing simulator ICT program]
- Compare relative frequencies from samples of different sizes

**DIFFERENTIATION & EXTENSION**

Extend Venn diagrams to 3 circles for most able students (S1 textbooks are a rich source of examples)

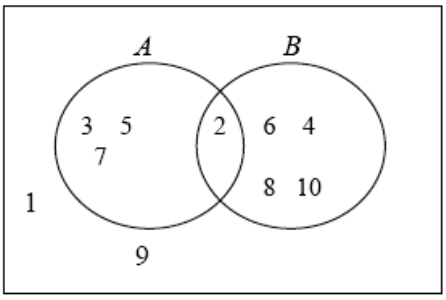
**NOTES**

Use ‘old’ textbooks to find good examples introducing set notation
Bring in a couple of hoops to illustrate Venn diagrams
A good illustration of Mutually Exclusive events is P(King) and P(Hearts), but how do we account for the P(King of Hearts)? Hence $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, then relate this to the Venn diagram where the King of Hearts is in the overlap.
Union has a U and Intersection has an $\cap$
Fractions do not need to be cancelled to their simplest form

$A$ = {prime numbers} $B$ = {even numbers}

The Venn diagram shows numbers 1 to 10

$\xi$ = {1, 2, 3 ……..10}




$P(A) = \frac{4}{10}$ $P(B) = \frac{5}{10}$

$P(A \cap B) = \frac{1}{10}$ $P(A \cup B) = \frac{8}{10}$

## Unit 2: Methods 2 – Foundation

This unit builds on the skills developed in Unit 1. It has a few topics which are also in Unit 1, but a calculator can be used to deal with more complex examples from these areas. Although it could be interchanged with Unit 1, it is recommended that this unit is the terminal examination at the time of certification, as Unit 2 content builds on the content of Unit 1. Alternatively centres may wish to enter students for both units at the end of the course for a linear assessment.

**Module 1 METHODS** **Time: 4 – 6 hours**

**Tier: Foundation – Unit 2**

**Content: Integers, decimals and fractions** **COMMON**

MN b Add, subtract, multiply and divide integers, fractions and decimals with a calculator
MN a Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
MN j Recall the fraction to decimal conversion of familiar simple fractions
MN j, g Convert between fractions and decimals
MN j *Use decimal notation and recognise that each terminating decimal is a fraction*
MN j *Convert recurring decimals into fractions and a fraction to a recurring decimal using a calculator*
MN o Use calculators efficiently and effectively

**PRIOR KNOWLEDGE**

Appreciation of place value
Experience of the four operations using whole numbers
Knowledge of integer complements to 10 and 100
Secure multiplication facts to $10 \times 10$
Knowledge of strategies for multiplying and dividing whole numbers by 10
The concept of a fraction and a decimal

**OBJECTIVES**

By the end of the module the student should be able to:

- Use brackets and the hierarchy of operations (BIDMAS)
- Add, subtract, multiply and divide integers, negative numbers, decimals and fractions **with** a calculator
- Check answers to a division sum using multiplication, eg use inverse operations
- Convert between fractions and decimals
- *Distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals)*
- *Convert between recurring decimals and fraction, seg write 0.1212121212… as a fraction*

**DIFFERENTIATION & EXTENSION**

Convert 0.1222222222….. to a fraction

**NOTES**

Present all working clearly with decimal points in line
Emphasise that all working should be shown
Students should be confident when using their calculator functions

**Module 2 METHODS** **Time: 3 – 6 hours**

**Tier: Foundation – Unit 2**

**Content: Percentages**

| | |
|---|---|
| MN g | Convert between fractions, decimals and percentages |
| MN g | Understand that 'percentage' means 'number of parts per 100' |
| MN g | Interpret percentage as the operator 'so many hundredths of' |
| MN h, i | Find a percentage of a quantity |
| MN g, h | Use percentages and fractions to solve problems |
| MN h | Solve percentage problems, including increase and decrease |
| MN h, i, j | Use multipliers |
| MN i | Express a given number as a fraction of another |
| MN i | Express a given number as a percentage of another |
| MN l | Use percentages, fractions and decimals to find proportion and understand how they can be used to scale between proportions |
| MN l | Find proportional change using fractions, decimals and percentages |

**PRIOR KNOWLEDGE**

Four operations of number
The concepts of a fraction and a decimal
Awareness that percentages are used in everyday life

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand that a percentage is a fraction in hundredths
- Write a percentage as a decimal; or as a fraction in its simplest form
- Write one number as a percentage of another number
- Calculate the percentage or fraction of a given amount
- Find a percentage increase or decrease of an amount
- Use a multiplier to increase by a given percent, eg $1.10 \times 64$ increases 64 by 10%

**DIFFERENTIATION & EXTENSION**

Fractional percentages of amounts (non-calculator)
Combine multipliers to simplify a series of percentage changes

Percentages which convert to recurring decimals (eg $33\frac{1}{3}\%$), and percentages of more than 100%

**NOTES**

Amounts of money should always be rounded to the nearest penny where necessary, at the end of a calculation

**Module 3 METHODS** **Time: 2 – 4 hours**

**Tier: Foundation – Unit 2**

**Content: Using fractions**

| | |
|---|---|
| MN i | Calculate a given fraction of a quantity |
| MN i | Express a given number as a fraction of another |
| MN a, i | Understand 'reciprocal' as multiplicative inverse, be able to find reciprocals |
| MN a | Understand and use unit fractions as multiplicative inverses |
| MN a, i | Multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction |
| MN i, l | Use a calculator to solve real-life problems involving fractions |

**PRIOR KNOWLEDGE**

Basic fractions and decimals

**OBJECTIVES**

By the end of the module the student should be able to:

- Find a fraction of a quantity
- Find the reciprocal of whole numbers, fractions, and decimals
- Multiply and divide a fraction by an integer, by a unit fraction and by a general fraction (expressing the answer in its simplest form)
- Use fractions in geometric problems

**DIFFERENTIATION & EXTENSION**

Use a calculator to find fractions of given quantities

Use combinations of the four operations with fractions (and in geometric problems, eg to find areas using fractional values)

For very able students introduce some repeated fractional change

**NOTES**

Constant revision of this aspect is needed

All work needs to be presented clearly with the relevant stages of working shown even if a calculator has been used

A good example of a repeated fractional change is a bouncing ball which always reaches $\frac{2}{3}$ of its height after each bounce. Investigate how high it bounces after the $3^{rd}$ bounce,

eg, use $(\frac{2}{3}) \times (\frac{2}{3}) \times (\frac{2}{3})$

**Module 4 METHODS** **Time: 3 – 6 hours**

**Tier: Foundation – Unit 2**

**Content: Ratio and proportion** **COMMON**

MN k *Use ratio notation, including reduction to its simplest form and its various links to fraction notation*
MN n Divide a quantity in a given ratio
MN m Solve word problems about proportion and ratio
MN m Understand and use direct proportion

**PRIOR KNOWLEDGE**

Fractions and decimals

**OBJECTIVES**

By the end of the module the student should be able to:

- Appreciate that eg the ratio 1:2 represents $\frac{1}{3}$ and $\frac{2}{3}$ of a quantity
- Divide quantities in a given ratio, eg divide £20 in the ratio 2:3
- Solve word problems involving ratios
- Solve simple direct proportion problems using the unitary method or by proportional change (from a table of values or a word problem)

**DIFFERENTIATION & EXTENSION**

Harder problems involving multi-stage calculations

**NOTES**

Students often find three-part ratios difficult
Link ratios given in different units to metric and imperial units

**Module 5 METHODS** **Time: 5 – 7 hours**

**Tier: Foundation – Unit 2**

**Content: Solving linear inequalities** **COMMON**

MA d Set up linear inequalities in one variable
MA d Solve simple linear inequalities in one variable and represent the solution set on a number line
MA d Use the correct notation to show inclusive and exclusive inequalities

**PRIOR KNOWLEDGE**

Experience of finding missing numbers in calculations
The idea that some operations are 'opposite' to each other
An understanding of balancing both sides of an equation
Experience of using letters to represent quantities
Be able to draw a number line
Linear equations and inequalities

**OBJECTIVES**

By the end of the module the student should be able to:

- Set up linear inequalities in one variable
- Solve linear inequalities in one variable and present the solution set on a number line

**DIFFERENTIATION & EXTENSION**

Use of inverse operations and rounding to 1 sf could be applied to more complex calculations
Derive inequalities from practical situations (such as finding unknown angles in polygons or perimeter problems)
Solve linear inequalities where manipulation of fractions is required

**NOTES**

Students can leave their answers in fractional form where appropriate
Interpreting the direction of an inequality is a problem for many
Students should use the correct notation when showing inequalities on a number line, eg a solid circle to show inclusion of a point, an empty circle to show exclusion of a point

**Module 6 METHODS** **Time: 3 – 6 hours**

**Tier: Foundation – Unit 2**

**Content: Substitution and formulae** **COMMON**

MA h Substitute numbers into a formula
MA h Substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$
MA h Use formulae from mathematics and other subjects
MA h Derive a formula
MA h *Change the subject of a formula where the subject occurs only once*

**PRIOR KNOWLEDGE**

Understanding of the mathematical meaning of the words expression, simplifying, formulae and equation
Experience of using letters to represent quantities
Substituting into simple expressions using words
Using brackets in numerical calculations and removing brackets in simple algebraic expressions
Solving linear equations

**OBJECTIVES**

By the end of the module the student should be able to:

- Use letters or words to state the relationship between different quantities
- Substitute positive and negative numbers into simple algebraic formulae
- Substitute positive and negative numbers into algebraic formulae involving powers
- Simple change of subject of a formula, eg convert the formula for converting Centigrade into Fahrenheit into a formula that converts Fahrenheit into Centigrade
- Generate a formula from given information, eg find the formula for the perimeter of a rectangle given its area $A$ and the length of one side

**DIFFERENTIATION & EXTENSION**

Use negative numbers in formulae involving indices
Various investigations leading to generalisations
Further problems in generating formulae from given information
Apply changing the subject to physics formulae, eg equations of motion (M1)

**NOTES**

Emphasise good use of notation, eg $3ab$ means $3 \times a \times b$
Students need to be clear on the meanings of the words expression, equation, and formula
Show a linear equation first and follow the same steps to rearrange a similarly structured formula
Link with formulae for area, volume, surface area

**Module 7 METHODS** **Time: 4 – 6 hours**

**Tier: Foundation – Unit 2**

**Content: Using graphs to solve equations** **COMMON**

MA g Generate points and plot graphs of quadratic functions
MA g, t Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function
MA t *Use graphs to solve simultaneous equations*
MA t *Understand connections between line graphs and associated formulae*

**PRIOR KNOWLEDGE**

Experience at plotting points in all quadrants
Linear sequences and straight-line graphs
Substitution

**OBJECTIVES**

By the end of the module the student should be able to:

- Draw linear graphs from tabulated data, including real-world examples
- Understand when two straight lines intersect this is the solution to the two simultaneous equations.
- Generate points and plotting graphs of quadratic functions
- Plot the graphs of quadratic functions for positive and negative values of $x$
- Solve a quadratic equation by reading the roots off the $x$-axis from the graph of that equation

**DIFFERENTIATION & EXTENSION**

Plot graphs of the form $y = mx + c$ where the student has to generate their own table and draw out their own axes
Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines
Use a graphical calculator or graphical ICT package to draw straight-line and quadratic graphs
Discuss the shape of quadratic curves and introduce the word ‘parabola’

**NOTES**

Clear presentation with axes labelled correctly is vital
Recognise linear graphs and identify when data may be incorrect
Link to graphs and relationships in other subject areas, ie science, geography etc

**Module 8 METHODS** **Time: 3 – 5 hours**

**Tier: Foundation – Unit 2**

**Content: Angle facts and polygons** **COMMON**

MG b Use properties of triangles and quadrilaterals
MG b Understand simple geometric proofs
MG b Mark parallel lines on a diagram
MG b Use angles associated with parallel lines
MG e Give reasons for angle calculation
MG e *Use definitions and names of polygons*
MG e *Calculate and use the sum of the interior angles of polygons*
MG e *Calculate and use the angles of regular polygons*
MG e *Use the sum of the exterior angles of any polygon is 360° and that the sum of an exterior and interior angle of a polygon is 180°*
MG e *Understand that inscribed regular polygons can be constructed by equal divisions of a circle*

**PRIOR KNOWLEDGE**

The concept of parallel lines
The concept of vertical and horizontal
The concept of an angle between two lines
Experience in drawing triangles, quadrilaterals and circles

**OBJECTIVES**

By the end of the module the student should be able to:

- Identify triangles by their properties (scalene, isosceles, equilateral, right-angled, obtuse, and acute)
- Prove the angle sum in a triangle is 180° and explain why the angles inside a quadrilateral add up to 360°
- Use the angle properties of triangle to find missing angles
- Prove the exterior angle of a triangle is equal to the sum of the two opposite interior angles
- Identify quadrilaterals by their properties (trapezium, parallelogram, rhombus, rectangle, square, kite)
- Use alternate, corresponding and co-interior angles in parallel lines to find missing angles
- *Name a polygon with 3, 4, ..., 10 sides*
- *Calculate and use the sums of the interior angles of convex polygons*
- *Know, or work out, the relationship between the number of sides of a polygon and the sum of its interior angles*
- *Know that the sum of the exterior angles of any polygon is 360°*
- *Find the size of the exterior or interior angle of a regular polygon*

**DIFFERENTIATION & EXTENSION**

Use triangles to find the angle sums of polygons
Use the angle properties of triangles to find missing angles in combinations of triangles
Harder problems involving multi-step calculations
Link with tessellations

**NOTES**

Lots of practical drawing examples to help illustrate properties of various shapes
Diagrams used in examinations are often not drawn accurately
Use tracing paper to show which angles in parallel lines are equal
Useful memory device is ‘the FUZ is the CIA’ (F, U and Z angles’ real names are Corresponding, Interior and Alternate)
Although corresponding, interior and alternate must be used when giving reasons
Encourage students to always put the reasons and ‘quote’ the angle fact/theorem used [important for the new assessment objectives]

**Module 9 METHODS** **Time: 2 – 5 hours**

**Tier: Foundation – Unit 2**

**Content: Similarity and Congruence** **COMMON**

MG m Understand congruence and similarity including the relationship between lengths in similar shapes
MG f *Tessellate regular polygons, 2-D shapes and combinations of polygons*
MG f *Explain why some shapes tessellate while others do not*

**PRIOR KNOWLEDGE**

The mathematical names of triangles and angles
Understanding of the terms perpendicular, parallel and arc
Transformations (particularly enlargements)

**OBJECTIVES**

By the end of the module the student should be able to:

- Use integer and non-integer scale factors to find the length of a missing side in each of two similar shapes, given the lengths of a pair of corresponding sides
- Use similarity or congruence to solve problems
- *Tessellate shapes*
- *Understand tessellations and explain why some shapes tessellate and why other shapes do not*

**DIFFERENTIATION & EXTENSION**

Link with the information to be able to construct a triangle, eg SSS, ASA, SAS
Harder problems in congruence

**NOTES**

All working should be presented clearly and accurately
If a student is not sure how to approach a question, advise them to write 'easy' numbers in pencil on their diagram, use these to approach the problem initially and then repeat the process with the 'real' values in the question
Use plastic shapes as template to draw tessellations (or use a tracing paper template)
Link angles in a polygon to explain why some shapes do or do not tessellate

**Module 10 METHODS** **Time: 6 – 9 hours**

**Tier: Foundation – Unit 2**

**Content: Pythagoras' theorem** **COMMON**

MG n Understand, recall and use Pythagoras' theorem in 2-D
MA l Calculate the length of a line segment between two coordinates
MN o Use calculators effectively and efficiently

**PRIOR KNOWLEDGE**

Names of triangles and quadrilaterals and their symmetries
Knowledge of the properties of rectangles, parallelograms and triangles
Indices and roots, substitution, equations and changing the subject

**OBJECTIVES**

By the end of the module the student should be able to:

- Find missing sides of right-angle triangles by using Pythagoras' theorem
- Find the distance between two coordinates using Pythagoras' theorem
- Give answers as decimals for Pythagoras' theorem problems
- Solve problems involving geometric figures (including triangles within circles) in which a right-angle triangle has to be extracted in order to solve it using Pythagoras' theorem

**DIFFERENTIATION & EXTENSION**

Introduce Pythagoras' theorem in 3-D (not on the tier, but a good extension for the most able)

**NOTES**

Students should be encouraged to become familiar with one make of calculator
Emphasise that scale drawings will score no marks for this type of question
A good way for remembering Pythagoras' theorem is 'Square it, Square it, Add or subtract it, Square root it' [This eliminates the need for formal algebra for the less able students]
Sides are determined by the units used or accuracy asked for in the question
Students should not forget to state the units for the answers

**Module 11 METHODS** **Time: 4 – 7 hours**

**Tier: Foundation – Unit 2**

**Content: Circles** **COMMON**

MG r Find circumferences of circles and areas enclosed by circles, recalling relevant formulae

MG r Find the perimeter and area of semi-circles and quarter circles

**PRIOR KNOWLEDGE**

Perimeter and area
Formulae and substitution

**OBJECTIVES**

By the end of the module the student should be able to:

- Learn that the ratio between the circumference and diameter is always constant for a circle (ie $\pi$)
- Learn the formula for circumference and area of a circle
- Solve problems involving the circumference and area of a circle (and simple fractional parts of a circle)

**DIFFERENTIATION & EXTENSION**

Extend to areas of sectors and length of arc (not on Foundation tier)

**NOTES**

'Now! I Know Pi' is a good way to learn the approximate value. (The number of letters of each word and the ! is the
decimal point)
Locate $\pi$ button on a calculator
Students must learn the formulae for the circumference and area of a circle
Also 'Cherry Pie Delicious' is $C = \pi D$ and 'Apple Pies are too' is $A = \pi r^2$
Final answers should be rounded to the required degree of accuracy

**Module 12 METHODS** **Time: 4 – 7 hours**

**Tier: Foundation – Unit 2**

**Content: Surface area and volume** **COMMON**

| | |
|---|---|
| MG s | Find the surface area of a cylinder |
| MG s | Find the surface area of prisms using the formulae for triangles and rectangle |
| MG u | Calculate the volumes of right prisms, including the triangular prism, and shapes made from cubes and cuboids |
| MG u | Recall and use the formula for the volume of a cuboid |
| MG u | Find the volume of a cylinder |
| MG u | Find the volume of a compound solid |

**PRIOR KNOWLEDGE**

Perimeter and area
Surface area
Knowledge of names and properties of 3-D solids
Formulae and substitution

**OBJECTIVES**

By the end of the module the student should be able to:

- Solve problems involving the volume and surface area of a cylinder
- Solve problems involving the volume and surface area of right prisms
- Find the surface area and the volume of more complex shapes, eg find the volume of an equilateral triangular prism

**DIFFERENTIATION & EXTENSION**

Extend to volume of a cone (not on Foundation tier)

**NOTES**

Use a tower of coins to model the volume of a cylinder
Final answers rounded to the required degree of accuracy
Need to constantly revise the expressions for area or volume of shapes
Students should be aware of which formulae are on the relevant page on the examination paper and which they need to learn

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**<u>Use of Calculators</u>** (Spec Ref MN o)

Students are well advised to regularly bring and use their particular calculator to lessons and get used to its functions throughout this Unit 2 course.
They should be confident in entering a range of calculations including those involving time and money. They should realise that 2.33333333…. hrs is 2 hours 20 minutes etc.

Calculator functions include +, –, ×, ÷, $x^2$, $\sqrt{x}$, memory, $x^y$, $x^{\frac{1}{y}}$ (or equivalent power/root buttons) and brackets.
Students should be able to use the functions which enable them to answer fraction questions with a calculation efficiently.

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# Foundation course objectives (2MM01)

## Unit 1

## Number

| | |
|---|---|
| MN a | Use brackets and the hierarchy of operations |
| MN a | Use one calculation to find the answer to another |
| MN a, d | Understand place value and round to the nearest integer and a given power of 10 |
| MN a, d | Check and estimate answers to problems, by using approximations or inverse operations |
| MN b | Multiply and divide by negative numbers |
| MN b | Multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1 |
| MN b | Add and subtract decimal numbers |
| MN b | Divide by a decimal (up to 2 dp) by transforming it to a problem involving division using by an integer |
| MN b | Use the four operations with fractions |
| MN c | Understand and use negative integers both as positions and translations on a number line |
| MN c | Order integers |
| MN c | Write decimal numbers in order of size |
| MN c | Find squares and cubes of numbers and find square roots and cube roots of numbers |
| MN c | Understand index notation for powers of 10, squares and cubes |
| MN c | Use index notation and index laws for multiplication and division of integer powers and powers of powers |
| MN c | Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integers |
| MN c | Understand and use number operations and the relationship between them, including inverse operations and hierarchy of operations |
| MN c | Order fractions |
| MN c | Compare fractions and decimals |
| MN c | Find equivalent fractions, cancel fractions, and use mixed numbers |
| MN c | Convert between mixed numbers and improper fractions |
| MN d | Check and estimate answers to problems |
| MN d | Round to a given number of significant figures (and decimal places) |
| MN e | Understand even, odd and prime numbers |
| MN e | Find factors and multiples of numbers |
| MN e | Find common factors and common multiples |
| MN e, f | Find Higher Common Factors (HCF), Lowest Common Multiple (LCM) and prime factor decomposition |
| MN p | Draw a Venn diagram from given information |
| MN p | Interpret Venn diagrams, be able to find the intersection and union of sets |
| MN p | Use Venn diagrams to solve problems |

## Algebra

| | |
|---|---|
| MA a | Distinguish the different roles played by letter symbols in algebra using correct notation |
| MA a | Write an expression |
| MA b | Distinguish the meaning between the words ‘equation’, ‘formula’ and ‘expression’ |
| MA c | Simplify terms, products and sums |
| MA c | Multiply a single term over a bracket |
| MA c | Take out common factors |
| MA c | Substitute into given expressions and formulae |
| MA c | Use squares and cubes to write expressions |
| MA c | Use index laws to simplify expressions |
| MA d | Set up simple equations |
| MA d | Solve equations by using inverse operations or by transforming both sides in the same way |
| MA d | Solve linear equations with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation |
| MA d | Solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution |
| MA i | Generate common integer sequences (including: sequences of odd or even integers; squared integers; powers of 2; powers of 10; triangle numbers) |
| MA i | Generate sequences from diagrams |
| MA i | Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence |
| MA i | Identify which terms cannot be in a sequence |
| MA i | Write the term-to-term definition in words for a sequence |
| MA j | Use linear expressions to describe the $n$th term of an arithmetic sequence |
| MA k | Use algebra to construct and support arguments |
| MA l | Use the conventions for coordinates in the plane |
| MA l | Plot points in all four quadrants |
| MA l | Find the midpoint of a line segment |
| MA l | Identify 2-D shapes, using coordinates |
| MA m | Draw, label and scale axes |
| MA m | Recognise (when values are given for $m$ and $c$) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane |
| MA m | Plot graphs of functions in which $y$ is given explicitly in terms of $x$, or implicitly |
| MA m | Find the gradient of a straight line from a graph |
| MA m | Find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for $m$ and $c$) from a graph |
| MA m | Understand that the form $y = mx + c$ represents a straight line and that $m$ is the gradient of the line and $c$ is the value of the $y$-intercept |
| MA n | Find the coordinates of a missing point to make a given shape |
| MA t | Understand links between conversion graphs and associated formulae and equations |
| MA t | Use a graph to find approximate solutions to equations |

## Geometry

| | |
|---|---|
| MG a | Basic angle facts on a straight line, at a point, on a straight line and vertically opposite angles |
| MG a | Distinguish between different types of angles |
| MG a | Estimate sizes of angles and name angles |
| MG a, b | Identify parallel and perpendicular lines |
| MG b | Understand and use the angle sum of a triangle is 180° |
| MG b | Give reasons for angle calculations |
| MG b | Use angles associated with intersecting lines |
| MG b, c | Understand properties of triangles and quadrilaterals |
| MG d | Recognise and visualise reflection and rotational symmetry of 2-D shapes |
| MG d | Be able to identify and draw lines of reflection |
| MG d | State the line of reflective symmetry as a simple algebraic equation |
| MG d | Be able to identify the order of rotational symmetry of a 2-D shape |
| MG j | Understand that translations are specified by a distance and direction (written as a column vector), and enlargements by a centre and a scale factor |
| MG j | Rotate a shape about the origin, or any other point |
| MG j | Measure the angle of rotation using right angles, simple fractions of a turn or degrees |
| MG j | Understand that rotations are specified by a centre and an (anticlockwise) angle |
| MG j | Understand that reflections are specified by a mirror line, at first using a line parallel to any axis, then a mirror line such as $y = x$ or $y = -x$ |
| MG j | Recognise, visualise and construct enlargements of objects using positive (integer) scale factors |
| MG j | Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations |
| MG j | Describe a transformation |
| MG j, k | Transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations |
| MG k | Use column vectors to describe translations |
| MG m | Identify shapes which are similar |
| MG m | Identify shapes which are congruent |
| MG m | Understand what makes two shapes similar or congruent |
| MG m | Recognise that enlargements preserve angle but not length |
| MG m | Understand the relationship between lengths in similar figures |
| MG q | Recall names and definitions of parts of a circle |
| MG q | Draw circles accurately |
| MG q | Understand related circle terms such as semi-circle and quarter circle |
| MG s | Recall and use formulae to find the area of triangles, rectangles, parallelograms |
| MG s | Find the area of a trapezium |
| MG s | Find the perimeter and area of shapes made from triangles and rectangles |

## Probability

| | |
|---|---|
| MP a | Understand and use the vocabulary of probability and probability scale |
| MP b, c | Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency |
| MP c | Estimate the number of times an event will occur, given the relative frequency and the number of trials |
| MP d | List all outcomes for single events, and for two successive events, in a systematic way |
| MP d | Use and draw sample space diagrams and add simple probabilities using sample space diagrams |
| MP e | Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 |
| MP f | Use set notation to describe events and compound events |
| MP f | Understand that P($A$) represents the probability of an event happening and P($A'$) represents the probability of an event not happening |
| MP g | Use Venn diagrams to represent the number of possibilities and find probabilities |
| MP j | Compare experimental data and theoretical probabilities |
| MP k | Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics |

# Unit 2

## Number

| | |
|---|---|
| MN a | Understand and use unit fractions as multiplicative inverses |
| MN a, i | Multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction |
| MN a | Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations |
| MN b | Add, subtract, multiply and divide integers, fractions and decimals with a calculator |
| MN g, j | Convert between fractions, decimals and percentages |
| MN g | Understand that 'percentage' means 'number of parts per 100' |
| MN g | Interpret percentage as the operator 'so many hundredths of' |
| MN g, h | Use percentages and fractions to solve problems |
| MN h | Solve percentage problems, including increase and decrease |
| MN h, i | Find a percentage of a quantity |
| MN h, i, j | Use multipliers |
| MN i | Express a given number as a fraction of another |
| MN i | Express a given number as a percentage of another |
| MN i | Calculate a given fraction of a quantity |
| MN i | Express a given number as a fraction of another |
| MN i, a | Understand 'reciprocal' as multiplicative inverse, be able to find reciprocals |
| MN i, l | Use a calculator to solve real-life problems involving fractions |
| MN j | Use decimal notation and recognise that each terminating decimal is a fraction |
| MN j | Recall the fraction-to-decimal conversion of simple fractions |
| MN j | Convert recurring decimals into fractions and a fraction to a recurring decimal using a calculator |
| MN k | Use ratio notation, including reduction to its simplest form and it various links to fraction notation |
| MN l | Use percentages, fractions and decimals to find proportion and understand how they can be used to scale between proportions |
| MN l | Find proportional change using fractions, decimals, and percentages |
| MN m | Solve word problems about proportion and ratio |
| MN m | Understand and use direct proportion |
| MN n | Divide a quantity in a given ratio |
| MN o | Use calculators effectively and efficiently |

## Algebra

| | |
|---|---|
| MA d | Set up linear inequalities in one variable |
| MA d | Solve simple linear inequalities in one variable and represent the solution set on a number line |
| MA d | Use the correct notation to show inclusive and exclusive inequalities |
| MA g | Generate points and plot graphs of quadratic functions |
| MA g, t | Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function |
| MA h | Substitute numbers into a formula |
| MA h | Substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$ |
| MA h | Use formulae from mathematics and other subjects |
| MA h | Derive a formula |
| MA h | Change the subject of a formula where the subject occurs only once |
| MA l | Calculate the length of a line segment between two coordinates |
| MA t | Use graphs to solve simultaneous equations |
| MA t | Understand connections between line graphs and associated formulae |

## Geometry

| | |
|---|---|
| MG b | Use properties of triangles and quadrilaterals |
| MG b | Understand simple geometric proofs |
| MG b | Mark parallel lines on a diagram |
| MG b | Use angles associated with parallel lines |
| MG e | Give reasons for angle calculation |
| MG e | Use definitions and names of polygons |
| MG e | Calculate and use the sum of the interior angles of polygons |
| MG e | Calculate and use the angles of regular polygons |
| MG e | Use the sum of the exterior angles of any polygon is 360° and that the sum of an exterior and interior angle of a polygon is 180° |
| MG e | Understand that inscribed regular polygons can be constructed by equal divisions of a circle |
| MG f | Tessellate regular polygons, 2-D shapes and combinations of polygons |
| MG f | Explain why some shapes tessellate while others do not |
| MG m | Understand congruence and similarity including the relationship between lengths in similar shapes |
| MG n | Understand, recall and use Pythagoras' theorem in 2-D |
| MG r | Find circumferences of circles and areas enclosed by circles, recalling relevant formulae |
| MG r | Find the perimeter and area of semi-circles and quarter circles |
| MG s | Find the surface area of a cylinder |
| MG s | Find the surface area of prisms using the formulae for triangles and rectangle |
| MG u | Calculate the volumes of right prisms, including the triangular prism, and shapes made from cubes and cuboids |
| MG u | Recall and use the formula for the volume of a cuboid |
| MG u | Find the volume of a cylinder |
| MG u | Find the volume of a compound solid |

# GCSE Methods of Mathematics (2MM01)

# Higher
# Tier

# Scheme of Work

# Higher course overview

The table below shows an overview of modules in the Higher tier scheme of work.

Teachers should be aware that the estimated teaching hours are **approximate** and should be used as a guideline only.

| Unit | Module number | Title | Estimated teaching hours |
|---|---|---|---|
| 1 | 1 | Integers, decimals and fractions | 4 |
| 1 | 2 | Factors and multiples | 2.5 |
| 1 | 3 | Laws of indices | 2.5 |
| 1 | 4 | Standard form | 2 |
| 1 | 5 | Introduction to algebra | 1.5 |
| 1 | 6 | Algebraic manipulation | 3 |
| 1 | 7 | Exact answers and surds | 3 |
| 1 | 8 | Patterns and sequences | 3 |
| 1 | 9 | Solving linear equations | 3 |
| 1 | 10 | Linear graphs and $y = mx + c$ | 5 |
| 1 | 11 | Quadratic functions | 3 |
| 1 | 12 | Angle facts and geometric shapes | 3 |
| 1 | 13 | Line and rotational symmetry | 2.5 |
| 1 | 14 | Circle theorems 1 | 3.5 |
| 1 | 15 | Circle theorems 2 | 3.5 |
| 1 | 16 | Area and perimeter | 3 |
| 1 | 17 | Similarity and congruency | 3 |
| 1 | 18 | Transformations | 3 |
| 1 | 19 | Vectors | 3.5 |
| 1 | 20 | Probability 1 | 4 |
| 1 | 21 | Probability 2 | 3 |

| Unit | Module number | Title | Estimated teaching hours |
|---|---|---|---|
| 2 | 1 | Integers, decimals and fractions | 3 |
| 2 | 2 | Percentages 1 | 3.5 |
| 2 | 3 | Percentages 2 | 3 |
| 2 | 4 | Using fractions | 2 |
| 2 | 5 | Exact answers using surds and $\pi$ | 1.5 |
| 2 | 6 | Ratio and proportion | 3.5 |
| 2 | 7 | Solving linear inequalities | 4 |
| 2 | 8 | Formulae | 3.5 |
| 2 | 9 | Straight line graphs | 4 |
| 2 | 10 | Simultaneous equations | 3.5 |
| 2 | 11 | Direct and inverse proportion | 3.5 |
| 2 | 12 | Quadratic functions | 4 |
| 2 | 13 | Further simultaneous equations | 3.5 |
| 2 | 14 | Graphs of curves | 4 |
| 2 | 15 | Transforming graphs | 3.5 |
| 2 | 16 | Angle facts and polygons | 3 |
| 2 | 17 | Similarity, congruency and tessellations | 2.5 |
| 2 | 18 | Similar shapes | 4 |
| 2 | 19 | Pythagoras' theorem and trigonometry in 2-D | 5 |
| 2 | 20 | Pythagoras' theorem and trigonometry in 3-D | 4 |
| 2 | 21 | Trigonometry in non-right-angled triangles | 3 |
| 2 | 22 | Circles, cones, pyramids and spheres | 4 |
| | | **Total** | **181.5** |

## Unit 1: Methods 1 Higher

Centres may wish to teach this unit alongside Unit 2 if they prefer to deliver their curriculum in a holistic manner. They would enter their students for both papers at the same time. However, some may prefer to concentrate on Unit 1 first and enter their students in either June 2011 or in November 2011. The benefit of this model is to allow a full analysis of the paper using ResultsPlus before preparing students for a possible resit (one resit is permitted).
Also, some centres, that traditionally organise their mock examinations in November or December, may use this opportunity for an external body to mark and analyse their papers. It may also flag up students who would benefit from a change of tier for either their resit examination or Unit 2.

**Module 1 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 1**

**Content: Integers, decimals and fractions** **COMMON**

| | |
|---|---|
| MN c | Understand and use negative integers both as positions and translations on a number line |
| MN a, b | Multiply and divide by negative numbers |
| MN b | Multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1 |
| MN d | Round to a given number of significant figures and decimal places |
| MN b | Add and subtract decimal numbers |
| MN a, d | Check and estimate answers to problems, by approximations or inverse operations |
| MN a | Use brackets and the hierarchy of operations |
| MN a | Use one calculation to find the answer to another |
| MN c | Order fractions, understand equivalent fractions and simplify fractions |
| MN a | Add, subtract, multiply and divide fractions |
| MN c | Order integers, decimals and fractions |

**PRIOR KNOWLEDGE**

The ability to order numbers
Appreciation of place value
Experience of the four operations using whole numbers
Knowledge of integer complements to 10 and 100
Knowledge of multiplication facts to $10 \times 10$
Knowledge of strategies for multiplying and dividing whole numbers by 10
The concepts of a fraction and a decimal

**OBJECTIVES**

By the end of the module the student should be able to:

- Add, subtract, multiply and divide integers (BIDMAS)
- Add, subtract, multiply and divide fractions and decimals
- Round whole numbers to the nearest, 10, 100, 1000, …
- Multiply and divide whole numbers by a given multiple of 10
- Check their calculations by rounding, eg $29 \times 31 \approx 30 \times 30$
- Approximate decimals to a given number of decimal places or significant figures
- Multiply and divide decimal numbers by whole numbers and decimal numbers (up to 2 dp), eg $266.22 \div 0.34$
- Know that eg $13.5 \div 0.5 = 135 \div 5$

**DIFFERENTIATION & EXTENSION**

Teachers may just want to check that students have the appropriate skills by investigative means, eg give students five digits say 2, 5, 7, 8 and 1. They then need to find

1) the largest even number

2) the smallest number in the 5 times table

3) the largest answer $\square\ \square\ \square$ + $\square\ \square$

4) the smallest answer to $\square\ \square\ \square$ – $\square\ \square$

More work on long multiplication and division without using a calculator

Estimate answers to calculations involving the four rules

Consideration of mental maths problems with negative powers of 10, eg $2.5 \times 0.01$, 0.001

Directed number work with two or more operations, or with decimals

Introduce standard form for very large or small numbers

Money calculations that require rounding answers to the nearest penny

Multiply and divide decimals by decimals (more than 2 dp)

**NOTES**

The expectation for most students doing Higher Tier is that some of this material can be delivered or reinforced during other topics. For example, rounding with significant figures could be done with trigonometry

Present all working clearly with decimal points in line

Emphasise that all working is to be shown

For non-calculator methods make sure that remainders and carrying are shown

Amounts of money should always be rounded to the nearest penny where necessary

It is essential to ensure the students are clear about the difference between significant figures and decimal places

Extend to multiplication of decimals and or long division of integers

Try different methods from the traditional ones, eg Russian or Chinese methods for multiplication

**Module 2 METHODS** **Time: 1 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Factors and multiples** **COMMON**

MN e Understand prime numbers
MN e Find factors and multiples of numbers
MN e Find common factors and common multiples
MN c Find squares and cubes of numbers; and finding square roots and cube roots of numbers
MN e, f *Find Highest Common Factor (HCF), Lowest Common Multiple (LCM) and prime factor decomposition*
MN p *Draw, use and interpret Venn diagrams*
MN p *Understand and be able to find the union and intersection of sets*
MN p *Use Venn Diagrams to solve problems*

**PRIOR KNOWLEDGE**

Number complements to 10 and multiplication or division facts
Use a number line to show how numbers relate to each other
Recognise basic number patterns
Experience of classifying integers

**OBJECTIVES**

By the end of the module the student should be able to:

- Find squares, cubes; square roots; cube roots of numbers, with and without a calculator (including the use of trial and improvement)
- Understand odd and even numbers, and prime numbers
- Find the HCF and the LCM of numbers
- Write a number as a product of its prime factors, eg $108 = 2^2 \times 3^3$
- *Use Venn diagrams to represent sets*
- *Draw a Venn diagram using given information and solve problems*
- *Understand and be able to find the intersection and union of sets*

**DIFFERENTIATION & EXTENSION**

Calculator exercise to check factors of larger numbers
Further work on indices to include negative and or or fractional indices (introduction to next section)
Use prime factors to find LCM
Use a number square to find primes (sieve of Eratosthenes)
Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)
*Use a couple of hoops to illustrate Venn diagrams by sorting physical objects by their properties*

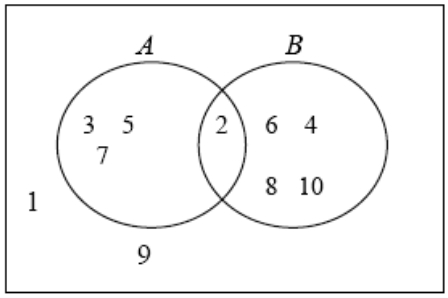
**NOTES**

All of the work in this unit is easily reinforced by starters and plenaries
Calculators should only be used when appropriate
Encourage student to learn square, cube, prime and common roots as Unit 1 is a non-calculator examination
*Venn Diagrams can be used to illustrate HCF [Link with Probability]*

*A = {prime numbers} B = {even numbers}*

*The Venn diagram shows numbers 1 to 10*

$\xi$ *= {1, 2, 3 ……..10}*




*Hence A ∩ B is {2}*

**Module 3 METHODS** **Time: 1 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Laws of indices** **COMMON**

MN c Use index notation and index laws for multiplication and division of integer powers
MN c Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integers, fractional and negative powers
MN c Recall the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers $n$, the corresponding rule for negative integers, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number $n$

**PRIOR KNOWLEDGE**

Knowledge of squares, square roots, cubes and cube roots

**OBJECTIVES**

By the end of the module the student should be able to:

- Use index rules to simplify and calculate numerical expressions involving powers, eg $(2^3 \times 2^5) \div 2^4$, $4^0$, $8^{-\frac{2}{3}}$
- Know that eg $x^3 = 64 \Rightarrow x = 64^{\frac{1}{3}}$

**DIFFERENTIATION & EXTENSION**

Use index rules to simplify algebraic expressions
Treat index rules as formulae (state which rule is being at each stage in a calculation)

**NOTES**

Use a simple division example to illustrate how zero and negative indices occur, eg $3^3 \div 3^3$ for zero index and $3^3 \div 3^4$ for a negative index

Show that $x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x$, is an illustration to as why $x^{\frac{1}{2}}$ is actually the square root of $x$

**Module 4 METHODS** **Time: 1 – 3 hours**

**Tier: Higher – Unit 1**

**Content: Standard form** **COMMON**

MN c Use standard index form, expressed in standard notation and on a calculator display
MN c Calculate with standard index form
MN c Convert between ordinary and standard index form representations
MN c Convert to standard index form to make sensible estimates for calculations involving multiplication and or division

**PRIOR KNOWLEDGE**

Round decimals to a given number of decimal places or significant figures
Multiply decimal numbers with, and without, a calculator
Some experience with powers of 10, eg know that $10^2 = 100$, $10^3 = 1000$, $10^{-1} = 0.1$
Negative indices and laws of indices

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand the standard form convention
- Convert numbers to, and from, standard form
- Calculate with numbers given in standard form without a calculator
- Round numbers given in standard form to a given number of significant figures

**DIFFERENTIATION & EXTENSION**

Use standard index form in real-life situations, eg stellar distances, sizes of populations and atomic distances for small numbers

**NOTES**

This work can be enriched by using examples drawn from the sciences,
eg Avogadro's Number $6.02 \times 10^{-23}$

**Module 5 METHODS** **Time: 1 – 2 hours**

**Tier: Higher – Unit 1**

**Content: Introduction to algebra**

MA a Distinguish the different roles played by letter symbols in algebra
MA b Distinguish the meaning between the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’

**PRIOR KNOWLEDGE**

Experience of using a letter to represent a number
Word formulae or rules to describe everyday situations, eg Time to cook a nut roast linked to weight of joint

**OBJECTIVES**

By the end of the module the student should be able to:

- Distinguish the different roles played by letter symbols in algebra
- Understand the meaning between the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’

**DIFFERENTIATION & EXTENSION**

Extend the above ideas to the ‘equation’ of the straight line, $y = mx + c$
Look at word equations written in symbolic form, eg $F = 2C+30$ to convert temperature (roughly) and compare with $F = \frac{9}{5}C + 32$

**NOTES**

There are plenty of past examination papers with matching tables testing knowledge of the ‘Vocabulary of Algebra’ (See Emporium website, www.edexcelmaths.com)

**Module 6 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 1**

**Content: Algebraic manipulation** **COMMON**

MA c Simplify terms, products and sums
MA c Multiply a single term over a bracket
MA c Take out common factors
MA c Substitute positive and negative numbers into expression such as $3x^2 + 4$ and $2x^3$
MA c Use instances of index laws, including use of fractional, zero and negative powers and power of a power
MA c *Expand the product of two linear expressions*
MA c *Factorise quadratic expressions including the difference of two squares*
MA c *Simplify rational expressions by adding, subtracting, multiplying and cancelling*
MA k, l *Use algebra to support and construct arguments and proofs*

**PRIOR KNOWLEDGE**

Know that a letter can be used to represent a number
Ability to use negative numbers with the four operations
Experience of using BIDMAS in calculations without a calculator

**OBJECTIVES**

By the end of this module the student should be able to:

- Simplify expressions with like terms, eg $x^2 + 3x^2$; $3ab + 5ab + 2c^2$
- Expand and factorise expressions with one pair of brackets, eg expand $x(2x + 3y)$; factorise $3xy^2 – 6x^2y$
- Simplify expressions such as $3x^3 \times 4x^7$, $(8x^6)^{1/3}$
- Expand and simplify expressions involving more than one pair of brackets, eg $3(x + 4) – 2(x – 3)$
- *Expand and simplify products involving two brackets $(2x + 3)(3x – 4)$*
- *Factorise a quadratic expression (including the difference of two squares)*
- *Simplify algebraic fractions, eg* $\frac{8a-28}{4}$, $\frac{2(x-3)^2}{6(x-3)}$, $\frac{2x^2-32}{x^2-3x-4}$
- *Form an algebraic expression from a given statement and rearrange this to arrive at a proof*

**DIFFERENTIATION & EXTENSION**

Expand algebraic expressions involving three pairs of brackets
Further examples in factorising quadratic expression with non-unitary values of $a$ (including fractional values)
Simplification of algebraic fractions which involve addition of fractions

**NOTES**

Emphasise correct use of symbolic notation, eg $3x^2$ rather than $3 \times x^2$
Present all work neatly, writing out the questions with the answers to aid revision at a later stage
Link the difference of two squares with the rationalisation of surds

**Module 7 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 1**

**Content: Exact answers and surds**

MN b Simplify surds and rationalise a denominator
MN b Use surds and π in exact calculations, without a calculator

**PRIOR KNOWLEDGE**

Know the common square numbers (for simplifying surds)
Knowledge of square roots and $\pi$
Collect together like terms
Remove brackets in simple algebraic expressions
Difference of two squares

**OBJECTIVES**

By the end of the module the student should be able to:

- Simplify surds, eg write $(\sqrt{18} + 10) \div \sqrt{2}$ in the form $p + q\sqrt{2}$
- Use surds and π in exact calculations, without a calculator
- Write $(3 - \sqrt{3})2$ in the form $a + b\sqrt{3}$
- Rationalise a denominator
- Rationalise the denominator of fractions, eg $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- Give an answer to a question involving the area of a circle as $25\pi$
- Give an answer to use of Pythagoras' theorem as $\sqrt{13}$

**DIFFERENTIATION & EXTENSION**

Explain the different between rational and irrational numbers as an introduction to surds
Prove that $\sqrt{2}$ is irrational
Revise the difference of two squares to show why we use, for example $(\sqrt{3} - 2)$ as the multiplier to rationalise $(\sqrt{3} + 2)$
Link to Unit 2 work on Circle measures (involving $\pi$) and Pythagoras calculations in exact form

**NOTES**

Link simplifying surds to collecting together like terms. eg $3x + 2x = 5x$,
so therefore $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$
Stress it is better to write answers in exact form (eg $\frac{1}{3}$ is better than 0.333333…..)
This is a non-calculator unit, but students can check their answers on their calculators, many of which now give answers in exact form
Link with Quadratics
A-Level Textbooks (C1) are a good source of questions on surd manipulation, some of which are algebraic
Useful generalisation to learn $\sqrt{x} \times \sqrt{x} = x$

**Module 8 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 1**

**Content: Patterns and sequences**

MA i Generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangle numbers)
MA i Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence
MA j Use linear expressions to describe the *n*th term of an arithmetic sequence
MA j Use quadratic expressions to describe the *n*th term of an quadratic sequence
MA l, k Use algebra to support and construct arguments and proofs

**PRIOR KNOWLEDGE**

Know about odd and even numbers
Recognise simple number patterns eg 1, 3, 5, …
Write simple rules algebraically
Raise numbers to positive whole number powers

**OBJECTIVES**

By the end of the module the student should be able to:

- Find the missing numbers in a number pattern or sequence
- Find the *n*th term of a number sequence as an algebraic expression
- Explain why a number is, or is not, a member of a given sequence
- Produce a linear sequence of numbers from a given *n*th term formula
- Produce a quadratic sequence of numbers and *n*th term for a quadratic sequence

**DIFFERENTIATION & EXTENSION**

Match-stick problems
Sequences and *n*th term formula for triangle numbers
Fibonacci numbers
Prove a sequence cannot have odd numbers for all values of *n*
Extend to cubic sequences whose *n*th term is $an^3 + bn^2 + cn + d$

**NOTES**

Emphasise good use of notation, eg $3n$ means $3 \times n$
When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the *n*th term
For the quadratic sequence 'prove' by starting with $an^2 + bn + c$, substitute in values of $n = 1, 2, 3, 4, 5$. Then establish algebraic difference to form the coefficients '$a + b + c$', '$3a + b$', and '$2a$' which link to the coefficients of the three levels of differences. By solving for $a$, $b$, and $c$ we have the coefficients of the *n*th term formula. [Good introduction to Equations]
Link the linear and quadratic sequences with their graphs

**Module 9 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 1**

**Content: Solving linear equations** **COMMON**

MA d Set up simple equations
MA d Solve equations by using inverse operations or by transforming both sides in the same way
MA d Solve linear equations with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation
MA d Solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution

**PRIOR KNOWLEDGE**

Experience of finding missing numbers in calculations
The idea that some operations are 'opposite' to each other
An understanding of balancing
Experience of using letters to represent quantities

**OBJECTIVES**

By the end of the module the student should be able to:

- Solve linear equations with one, or more, operations (including fractional coefficients)
- Solve linear equations involving brackets and or or variables on both sides
- Form linear equations from word problems in a variety of contexts and relating the answer back to the original

**DIFFERENTIATION & EXTENSION**

Use of inverse operations and rounding to 1 sf. could be applied to more complex calculations
Derive equations from geometric situations (such as finding unknown angles in polygons or perimeter problems)
Solve equations where manipulation of fractions (including the negative fractions) is required

**NOTES**

Students need to realise that not all linear equations can easily be solved by either observation or trial and improvement, and hence the use of a formal method is vital
Students can leave their answers in fractional form where appropriate

**Module 10 METHODS** **Time: 4 – 6 hours**

**Tier: Higher – Unit 1**

**Content: Linear graphs and $y = mx + c$** **COMMON**

| | |
|---|---|
| MA l | Use axes and coordinates to specify points in all four quadrants in 2-D and 3-D |
| MA l | Identify points with given coordinates |
| MA n | *Find the coordinates of points identified by geometrical information in 2-D* |
| MA l | Find the coordinates of the midpoint of the line segment *AB*, given the coordinates of *A* and *B* |
| MA m | Recognise (when values are given for *m* and *c*) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane |
| MA m | Plot graphs of functions in which *y* is given explicitly in terms of *x*, or implicitly |
| MA o | Find the gradient of a straight line |
| MA o | Find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for *m* and *c*) |
| MA t | *Understand the connection between graphs and associated formulae and equations* |
| MA t | *Use graphs to find approximate solutions to equations* |

**PRIOR KNOWLEDGE**

Substitute positive and negative numbers into algebraic expressions
Plot coordinates in the first quadrant
Calculate the mean of two numbers
Knowledge of basic shapes
Rearrange to change the subject of a formula

**OBJECTIVES**

By the end of the module the student should be able to:

- Add a point to a coordinate grid to complete a given shape (parallelogram, rhombus, trapezium, square)
- Use the formula to calculate the midpoint of a line segment
- Understand how to represent points in 1-D, 2-D and 3-D
- Substitute values of *x* into linear functions to find corresponding values of *y*
- Find the gradient of a straight line from a graph
- Plot points for linear functions on a coordinate grid and draw the corresponding straight lines

**DIFFERENTIATION & EXTENSION**

Find the equation of the line through two given points
Find the equation of the perpendicular bisector of the line segment joining two given points
Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines
Use a graphical calculator or graphical ICT package to draw straight-line graphs
Cover horizontal and vertical lines ($x = c$ and $y = c$) [Students often forget these]

**NOTES**

Careful annotation should be encouraged. Label the coordinate axes and write the equation of the line
Recognise linear graphs and hence when data may be incorrect
Link to graphs and relationships in other subject areas, ie science, geography etc
A-Level Text books (C1) are a good source of extension questions on this topic

**Module 11 METHODS** **Time: 4 – 6 hours**

**Tier: Higher – Unit 1**

**Content: Quadratic functions**

MA g Factorise quadratic expressions
MA g Solve simple quadratic equations by factorising and completing the square
MA g Solve quadratic equations approximately by using a graph

**PRIOR KNOWLEDGE**

Graphs
Factorising

**OBJECTIVES**

By the end of the module the student should be able to:

- Solve quadratic equations by factorising
- Solve quadratic equations by completing the square
- Use a graph to find approximate solutions to a quadratic equation

**DIFFERENTIATION & EXTENSION**

Solve equations involving algebraic fractions which lead to quadratic equations
Solve quadratic equations by completing the square
Derive the quadratic equation by completing the square
Use graphical calculators or ICT graph package where appropriate to enable students to get through examples more rapidly
Complete the square of a quadratic function (using this to write down the max or min of the function)
Show how the value of '$b^2 - 4ac$' can be useful in determining if the quadratic factorises or not (ie square number)
Extend to discriminant properties and roots (for those going on to C1)

**NOTES**

There may be a need to remove the HCF (numerical) of a trinomial before factorising to make the factorisation easier
Students should be reminded that factorisation should be tried before completing the square is used
Applying quadratics to some basic problem-solving. Show that sometimes one of the solutions to a quadratic may not be appropriate, eg negative length for a shape

**Module 12 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Angle facts and geometric shapes** **COMMON**

MG a Use angle facts for a straight line, at a point and vertically opposite angles
MG b, c Use properties of triangles and quadrilaterals
MG b Use angles associated with intersecting lines
MG b Give reasons for angle calculations

**PRIOR KNOWLEDGE**

The concept of parallel lines
The concept of vertical and horizontal
The concept of an angle between two lines
Experience in drawing triangles, quadrilaterals and circles

**OBJECTIVES**

By the end of the module the student should be able to:

- Recall and use properties of angles
  - angles at a point
  - angles at a point on a straight line
  - perpendicular lines
  - vertically opposite angles
- Find the size of missing angles
- Use two letter notation for a line and three letter notation for an angle
- Identify triangles by their properties (scalene, isosceles, equilateral, right-angled, obtuse, and acute)
- Prove the angle sum in a triangle is 180°
- Use the angle properties of triangle to find missing angles
- Prove the exterior angle of a triangle is equal to the sum of the two opposite interior angles
- Identify quadrilaterals by their properties (trapezium, parallelogram, rhombus, rectangle, square, kite and arrowhead)

**DIFFERENTIATION & EXTENSION**

Use the angle properties of triangles to find missing angles in combinations of triangles
Harder problems involving multi-step calculations
Link with line and rotational symmetry

**NOTES**

Lots of practical drawing examples to help illustrate properties of various shapes – good for group work or displays
Diagrams used in examinations are seldom drawn accurately
Encourage students to always put the reasons and ‘quote’ the angle fact or theorem used [important for the new assessment objectives]

**Module 13 METHODS** **Time: 2 – 3 hours**

**Tier: Higher – Unit 1**

**Content: Line and rotational symmetry** **COMMON**

MG d Recognise and visualise reflection and rotational symmetry of 2-D shapes

**PRIOR KNOWLEDGE**

Knowledge and properties of 2-D shapes

**OBJECTIVES**

By the end of the module the student should be able to:

- Recognise line and rotational symmetry in 2-D shapes
- Draw in the line of symmetry (or state its equation if the shape is on a coordinate grid) and state the order of rotational symmetry

**DIFFERENTIATION & EXTENSION**

Extend to planes of symmetry for 3-D solids

**NOTES**

Accurate drawing skills need to be reinforced
Some students find visualising 3-D objects difficult and simple models will assist
Use tracing paper or mirrors to help with symmetry questions

**Module 14 METHODS** **Time: 3 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Circle theorems 1**

MG q Names and definitions of parts of a circle
MG g Understand and use the fact that tangents from an external point are equal in length
MG g Explain why the perpendicular from the centre to a chord bisects the chord
MG g Understand that the tangent at any point on a circle is perpendicular to the radius at that point

**PRIOR KNOWLEDGE**

Properties of shapes and polygons
Basic angle facts
Ability to draw a circle with compasses
Some algebraic manipulation

**OBJECTIVES**

By the end of the module the student should be able to:

- Identify and name the various parts of a circle (centre, radius, diameter, circumference, sector, segment, arc and chord)
- Use the angle properties of tangents to find missing angles (tangent at a point, tangents from a point)
- Understand, prove and use circle theorems (see above)
- Use circle theorems to find unknown angles and explain their method, quoting the appropriate theorem(s)

**DIFFERENTIATION & EXTENSION**

Investigate other circle theorems by measuring angles accurately
Harder problems involving multi-stage calculations
Algebraic solutions or general proofs

**NOTES**

All working should be presented clearly and accurately
Encourage students to always put the reasons and 'quote' the angle fact or theorem used. [Important for the new assessment objectives]
Draw lines using an HB pencil
A sturdy pair of compasses are essential and spare equipment is advisable
For less able students a good idea to practise algebraic solutions or proofs is to replace the '$x$' with a number (in pencil). The students then work through the question and then repeat it using the '$x$', but following the same processes
This topic continues in the next module

**Module 15 METHODS** **Time: 3 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Circle theorems 2**

MG g Explain why the perpendicular from the centre to a chord bisects the chord
MG g Prove and use the fact that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference
MG g Prove and use the fact that the angle subtended at the circumference by a semicircle is a right angle
MG g Prove and use the fact that angles in the same segments are equal
MG g Prove and use the fact that opposite angles of a cyclic quadrilateral sum to 180°
MG g Prove and use the alternate segment theorem
MG g Know and use the fact that for two chords *AB* and *CD* of a circle, intersecting at the point *X*; $AX \times XB = CX \times XD$

**PRIOR KNOWLEDGE**

Recall the words centre, radius, diameter and circumference
Have practical experience of drawing circles with compasses
Circle theorems covered for topic 14 above (tangent or radius properties)

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand, prove and use circle theorems (see above)
- Use circle theorems to find unknown angles and explain their method – quoting the appropriate theorem(s)
- Know and use the fact that for two chords *AB* and *CD* of a circle, intersecting at the point *X*; $AX \times XB = CX \times XD$

**DIFFERENTIATION & EXTENSION**

Harder problems involving multi-stage calculations

**NOTES**

Any proof required will be in relation to a diagram, not purely by reference to a named theorem
Reasoning needs to be carefully constructed as 'Quality of Written Communication' marks can be allocated

*[The reason this circle theorems material has been split into 2 topics (14 and 15) is that teachers can then link existing resources from Units 2 and 3 for the GCSE 2381 or 2010 GCSE 2MB01 specs]*

**Module 16 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Area and perimeter** **COMMON**

MG s Use formulae to find the area of triangles, parallelograms, and trapeziums
MG s Calculate perimeters of shapes made from triangles and rectangles
MG s Calculate areas of shapes made from triangles and rectangles
MG s Find the area of compound shapes
MG s Calculate the perimeter and area of compound shapes made from triangles, rectangles and other shapes

**PRIOR KNOWLEDGE**

The names of quadrilaterals
Ability to substitute numbers into a formula
Some notion of the difference between length and area
Properties of cubes, cuboids and other common 3-D objects

**OBJECTIVES**

By the end of the module the student should be able to:

- Use the area formulae for triangles, parallelograms and trapeziums
- Find the perimeter of compound shapes
- Find the area of compound shapes

**DIFFERENTIATION & EXTENSION**

Further problems involving combinations of shapes
Using compound shape methods to investigate the areas of other standard shapes, eg kites
Practical activities, eg using estimation and accurate measuring to calculate perimeters and areas of floors

**NOTES**

Discuss the correct use of language and units
Ensure that students can distinguish between perimeter and area
Try to impress upon them that perimeter starts with a ‘P’ which reminds them to plus the sides
Many students have little real understanding of perimeter and area
Practical experience is essential to clarify these concepts

**Module 17 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Similarity and congruency** **COMMON**

MG m Understand similarity of triangles and of other plane figures and use this to make geometric inferences
MG m Identify similar solids
MG m Recognise that enlargements preserve angle but not length
MG m Understand the relationship between lengths, areas and volumes in similar figures
MG m *Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions*

**PRIOR KNOWLEDGE**

The special names of triangles (and angles)
Understand the terms perpendicular, parallel and arc
Transformations (particularly enlargements)

**OBJECTIVES**

By the end of the module the student should be able to:

- Use a formal proof to show the geometric properties of triangles, eg that the base angles of an isosceles triangle are equal
- *Use a formal proof to show that two triangles are congruent*
- Use integer and non-integer scale factors to find the length of a missing side in each of two similar shapes, given the lengths of a pair of corresponding sides

**DIFFERENTIATION & EXTENSION**

Link with the information to be able to construct a triangle, eg SSS, ASA, SAS
Link with tessellations and transformations
Link with similar areas and volumes
Harder problems in congruence

**NOTES**

All working should be presented clearly and accurately
If a student is not sure how to approach a question, advise them to write 'easy' numbers in pencil on their diagram, use these to approach the problem initially, then repeat the process with the 'real' values on the question

**Module 18 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Transformations**

MG j, k Transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations
MG j, k Use column vectors to describe translations
MG j Understand that translations are specified by a distance and direction (or a column vector), and enlargements by a centre and a scale factor
MG j Rotate a shape about the origin, or any other point
MG j Measure the angle of rotation using right angles, simple fractions of a turn or degrees
MG j Understand that rotations are specified by a centre and an (anticlockwise) angle
MG j Understand that reflections are specified by a mirror line, at first using a line parallel to any axis, then a mirror line such as $y = x$ or $y = –x$
MG j Recognise, visualise and construct enlargements of objects using positive, negative and fractional scale factors greater and less than one
MG j Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations
MG j Describe and transform 2-D shapes using combined rotations, reflections, translations or enlargements

**PRIOR KNOWLEDGE**

Recognition of basic shapes
Line and rotational symmetry
An understanding of the concept of rotation and enlargement
Coordinates in four quadrants
Linear equations parallel to the coordinate axes

**OBJECTIVES**

By the end of the module the student should be able to:

- Use column vectors to describe translations
- Understand translation as a combination of a horizontal and vertical shift including signs for directions
- Understand rotation as a (clockwise) turn about a given origin
- Reflect shapes in a given mirror line; parallel to the coordinate axes and then $y = x$ or $y = –x$
- Enlarge shapes by a given scale factor from a given point; using positive and negative scale factors greater and less than one (and understand the effects that negative and fractional scale factors have on the image)
- Understand that shapes produced by translation, rotation and reflection are congruent to its image

**DIFFERENTIATION & EXTENSION**

The tasks set should be extended to include combinations of transformations

**NOTES**

Emphasise that students need to describe the given transformation fully
Diagrams should be drawn carefully
The use of tracing paper is allowed in the examination (although students should not have to rely on the use of tracing paper to solve problems)

**Module 19 METHODS** **Time: 3 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Vectors**

| | |
|---|---|
| MG k | Understand and use vector notation (revise column vectors) |
| MG l | Calculate, and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector |
| MG l | Calculate the resultant of two vectors |
| MG l | Understand and use the commutative and associative properties of vector addition |
| MG l | Solve simple geometrical problems in 2-D using vector methods |
| MG l | Prove that three or more points are collinear |

**PRIOR KNOWLEDGE**

Vectors to describe translations
Algebraic manipulation

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand that 2**a** is parallel to **a** and twice its length
- Understand that **a** is parallel to −**a** and in the opposite direction
- Use and interpret vectors as displacements in the plane (with an associated direction)
- Use standard vector notation to combine vectors by addition, eg $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{AC}}$ and **a** + **b** = **c**
- Represent vectors, and combinations of vectors, in the plane
- Solve geometrical problems in 2-D, eg show that joining the midpoints of the sides of any quadrilateral forms a parallelogram

**DIFFERENTIATION & EXTENSION**

Harder geometric proof, eg show that the medians of a triangle intersect at a single point
Illustrate use of vectors by showing 'crossing the flowing river' example or navigation examples
Vector problems in 3-D (for the most able)
Use **i** and **j** (and **k**) notation (Preparation for A-Level but not on GCSE)
Lengths are sometimes given in the ratio of 1:3: this needs explaining carefully

**NOTES**

Students often find the pictorial representation of vectors more difficult than the manipulation of column vectors
The geometry of a hexagon provides a rich source of parallel, reverse and twice length vectors
Stress that parallel vectors are equal
Link with like terms and brackets when simplifying
Show there is more than one route round a geometric shape, but the answer simplifies to the same vector
Remind students to underline vectors or they will be regarded as just lengths with no direction
Extension material is in M1 texts

**Module 20 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 1**

**Content: Probability 1**

| | |
|---|---|
| MP a | Understand and use the vocabulary of probability and probability scale |
| MP b, c | Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency |
| MP d | List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities |
| MP d | Draw and use sample spaces |
| MP e | Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 |
| MP i | Know when to add or multiply two probabilities: when $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$, whereas when $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$ |

**PRIOR KNOWLEDGE**

Understand that a probability is a number between 0 and 1
Know how to add and multiply fractions and decimals
Experience of expressing one number as a fraction of another number
Recognise the language of statistics, eg words such as ‘likely’, ‘certain’, ‘impossible’

**OBJECTIVES**

By the end of the module the student should be able to:

- Write probabilities using fractions, percentages or decimals
- Understand and use estimates or measures of probability
- Use theoretical models to include outcomes using dice, spinners, coins etc
- Understand the probability of successive events, such as several throws of a single dice
- List all outcomes for single events, and for two successive events, systematically
- Use and draw sample space diagrams
- Add simple probabilities, eg from sample space diagrams
- Identify different mutually exclusive outcomes and know that the sum of the probabilities
- of all these outcomes is 1
- Use $1 - p$ as the probability of an event not occurring where $p$ is the probability of the event occurring
- Find a missing probability from a list or table

**DIFFERENTIATION & EXTENSION**

An opportunity for practical examples, eg P(pin up) for a drawing pin, the ‘horse’ race, the national lottery
Show that each cluster of branches adds up to 1
Explain that if two objects are chosen, then this is the same as one event followed by another event without replacement
Show that it is often easier to solve a problem involving multiple outcomes, by considering the *opposite* event and subtracting from 1, eg ‘at least’ two reds, ‘at least’ two beads of a different colour etc
Experiments with dice and spinners
Show sample space for outcomes of throwing two dice
Stress that there are 36 outcomes (students are likely to guess it is 12 outcomes for two dice)

**NOTES**

Students should express probabilities as fractions, percentages or decimals

Fractions do not need to be cancelled to their lowest terms. This makes it easier to calculate tree diagram probabilities, as it is easier to add using common denominators and check probabilities add up to 1.

Show that each cluster of branches adds up to 1

**Module 21 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 1**

**Content: Probability 2**

MP f Use set notation to describe events and compound events
MP g Use Venn diagrams to represent the number of possibilities and find probabilities
MP f, i Know when to add or multiply two probabilities: when $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$, whereas when $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$ [Relate these to Union $\cup$, and Intersection, $\cap$]
MP h Use tree diagrams to represent outcomes of compound events, and recognise when events are independent
MP j Compare experimental data and theoretical probabilities
MP k Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

**PRIOR KNOWLEDGE**

Probability
Venn diagrams

**OBJECTIVES**

By the end of the module the student should be able to:

- Use set notation to describe events.
- Use Venn diagrams to represent probabilities, relating to intersection and union ($\cup$ and $\cap$ symbols replacing OR and AND)
- If $A$ and $B$ are mutually exclusive then $P(A \cup B)$ represents the probability of either $A$ or $B$ happening
- $P(A \cup B) = P(A) + P(B)$
- If $A$ and $B$ are independent, then $P(A \cap B)$ represents the probability that both $A$ and $B$ happen
- $P(A \cap B) = P(A) \times P(B)$
- Understand conditional probabilities
- Understand selection with or without replacement using AND and OR Laws
- Draw a probability tree diagram based on given information
- Use a tree diagram to calculate conditional probability
- Compare experimental data and theoretical probabilities
- Compare relative frequencies from samples of different sizes

**DIFFERENTIATION & EXTENSION**

Show that each cluster of branches adds up to 1
Binomial probabilities (H or T)
Do a question 'with' and then repeat it 'without' replacement. Good idea to physically demonstrate a "beads in a bag" problem by showing students the contents of the bag and removing the object to illustrate the change in probability for the second selection
Extend Venn diagrams to 3 circles (S1 textbooks are a rich source of examples)

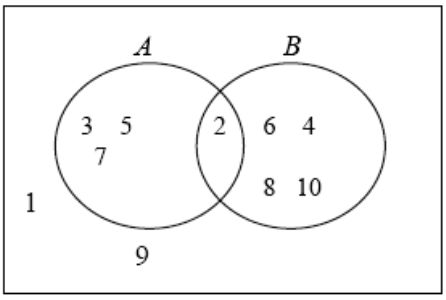
**NOTES**

Use ‘old’ textbooks to find good examples introducing set notation
Bring in a couple of hoops to illustrate Venn diagrams
A good illustration of Mutually Exclusive events is P(King) and P(Hearts), but how do we account for the P(King of Hearts)? Hence $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, then relate this to the Venn diagram where the King of Hearts is in the overlap
Mnemonic–- Union has a U and Intersection has an $\cap$
Link with number work
Fractions do not need to be cancelled to their simplest form. This makes it easier to calculate probabilities for tree diagram problems, as it is easier to add fractions with common denominators
Show that each cluster of branches adds up to 1

$A$ = {prime numbers} $B$ = {even numbers}

The Venn diagram shows numbers 1 to 10

$\xi$ = {1, 2, 3 ……..10}




$P(A) = \frac{4}{10}$ $P(B) = \frac{5}{10}$

$P(A \cap B) = \frac{1}{10}$ $P(A \cup B) = \frac{8}{10}$

## Unit 2: Methods 2 Higher

This unit builds on the skills developed in Unit 1. It has few topics which are introduced in Unit 1. Although it could be interchanged with Unit 1, it is recommended that this unit is the terminal exam at the time of certification, as the content of Unit 2 builds upon the content for Unit 1. Alternatively centres may wish to enter students for both units at the end of the course.

**Module 1 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 2**

**Content: Integers, decimals and fractions** **COMMON**

| | |
|---|---|
| MN a, b | Add, subtract, multiply and divide integers, fractions and decimals |
| MN j | Recall the fraction-to-decimal conversion of familiar fractions |
| MN j | Convert between fractions and decimals using a calculator |
| MN j | *Distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals) and other fractions (which are represented by recurring decimals)* |
| MN j | *Convert recurring decimals into fractions* |
| MN j | *Understand the recurring decimal to fraction proof* |
| MN b | Solve a problem involving division by a decimal |
| MN a | Understand reciprocal |
| MN a | Understand and use unit fractions as multiplicative inverses |

**PRIOR KNOWLEDGE**

Appreciation of place value
Experience of the four operations using whole numbers
Knowledge of integer complements to 10 and 100
Knowledge of multiplication facts to $10 \times 10$
Knowledge of strategies for multiplying and dividing whole numbers by 10
The concept of a fraction and a decimal

**OBJECTIVES**

By the end of the module the student should be able to:

- Use brackets and the hierarchy of operations (BIDMAS)
- Add, subtract, multiply and divide integers, negative numbers, decimals and fractions
- Check answers to a division sum using multiplication eg use inverse operations
- Convert between common fractions and decimals (recurring and terminating)
- *Distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals)*
- *Convert between recurring decimals and fractions, eg write 0.01212121212… as a fraction*
- *Understand the recurring fraction to decimal proof*

**DIFFERENTIATION & EXTENSION**

Prove that 0.*nnnnnn*…. can be written as a fraction $\frac{n}{9}$ and look at the case when $n = 9$

**NOTES**

The expectation for most students studying Higher tier is that some of this material can be delivered or reinforced during other topics. For example, using inverses could be used with trigonometry
Present all working clearly with decimal points in line
Emphasise that all working needs to be shown

**Module 2 METHODS** **Time: 3 – 4 hours**

**Tier: Higher – Unit 2**

**Content: Percentages 1**

| | |
|---|---|
| MN g | Convert between fractions, decimals and percentages |
| MN g | Understand that 'percentage' means 'number of parts per 100' |
| MN g | Interpret percentage as the operator 'so many hundredths of' |
| MN g | Use percentages and fractions in real-life situations |
| MN h, l | Solve percentage problems, including increase and decrease |
| MN i | Find percentages of quantities |

**PRIOR KNOWLEDGE**

Four operations of number
The concepts of a fraction and a decimal
Awareness that percentages are used in everyday life

**OBJECTIVES**

By the end of the module the student should be able to:

- Understand that a percentage is a fraction in hundredths
- Write a percentage as a decimal or as a fraction in its simplest terms
- Write one number as a percentage of another number
- Calculate the percentage (or fraction) of a given amount
- Find a percentage increase or decrease of an amount
- Use a multiplier to increase by a given percent, eg $1.10 \times 64$ increases 64 by 10%

**DIFFERENTIATION & EXTENSION**

Fractional percentages of amounts (non-calculator)
Combine multipliers to simplify a series of percentage changes

Percentages which convert to recurring decimals (eg $33\frac{1}{3}\%$), and situations which lead to percentages of more than 100%

**NOTES**

Amounts of money should always be rounded to the nearest penny where necessary

**Module 3 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 2**

**Content: Percentages 2**

| | |
|---|---|
| MN h | Solve percentage problems, including increase and decrease, and reverse percentage |
| MN h ,i, l | Represent repeated proportional change using a multiplier raised to a power |
| MN h ,i, o | Use calculators for reverse percentages calculations by doing an appropriate division |
| MN h | Compound interest |
| MN i | Express a given number as a fraction of another |
| MN i | Express a given number as a percentage of another |

**PRIOR KNOWLEDGE**

Four operations of number
Awareness that percentages are used in everyday life
Percentages

**OBJECTIVES**

By the end of the module the student should be able to:

- Find a percentage increase or decrease of an amount
- Find a reverse percentage, eg find the original cost of an item given the cost after a 10% deduction
- Use a multiplier to increase by a given percent, eg $1.1 \times 64$ increases 64 by 10%
- Calculate simple and compound interest for two, or more, periods of time

**DIFFERENTIATION & EXTENSION**

Combine multipliers to simplify a series of percentage changes
Comparisons between simple and compound interest calculations
Formulae in simple interest or compound interest methods
Increase and decreases leading to a combined multiplier to use (eg 10% decrease then 5% increase)

**NOTES**

Reiterate when to round in repeated proportional change calculations

**Module 4 METHODS** **Time: 1 – 3 hours**

**Tier: Higher – Unit 2**

**Content: Using fractions**

MN i Calculate a given fraction of a quantity
MN i Understand 'reciprocal' as multiplicative inverse
MN a, i Multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction
MN i, l Use a calculator to solve problems involving fractions (including repeated changes)

**PRIOR KNOWLEDGE**

Basic fractions and decimals

**OBJECTIVES**

By the end of the module the student should be able to:

- Find a fraction of a quantity
- Find the reciprocal of whole numbers, fractions, and decimals
- Multiply and divide a fraction by an integer, by a unit fraction and by a general fraction (expressing the answer in its simplest form)
- Use fractions in geometric problems (including repeated fractions)

**DIFFERENTIATION & EXTENSION**

Use a calculator to find fractions of given quantities
Use combinations of the four operations with fractions (and in real-life problems, eg to find areas using fractional values)
For very able students revise algebraic fractions

**NOTES**

Constant revision of this aspect is needed
All work needs to be presented clearly with the relevant stages of working shown even if a calculator is used
A good example of a repeated fractional change is a bouncing ball which always reaches $\frac{2}{3}$ of its height after each bounce. Investigate how high it bounces after the 3rd bounce, ie use $(\frac{2}{3})^3$

**Module 5 METHODS** **Time: 1 – 2 hours**

**Tier: Higher – Unit 2**

**Content: Exact answers using surds and $\pi$**

MN b Write answers in exact form

**PRIOR KNOWLEDGE**

Knowing the common square numbers (for simplifying surds)
Knowledge of square roots and $\pi$
Collect together like terms
Remove brackets in simple algebraic expressions
Difference of two squares
Simplify and manipulate surds

**OBJECTIVES**

By the end of the module the student should be able to:

- Give an answer to a question involving the area of a circle as $25\pi$
- Give an answer to use of Pythagoras' theorem as $\sqrt{13}$

**DIFFERENTIATION & EXTENSION**

Explain the different between rational and irrational numbers as a reminder of surds
Prove that $\sqrt{2}$ is irrational
Revise the difference of two squares to show why we use, for example $(\sqrt{3} - 2)$ as the multiplier to rationalise $(\sqrt{3} + 2)$
Link to other Unit 2 topics on circle measures (involving $\pi$) and Pythagoras' theorem calculations in exact form

**NOTES**

Link simplifying surds to collecting together like terms, eg $3x + 2x = 5x$,
so therefore $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$
Stress it is better to write answers in exact form (eg $\frac{1}{3}$ is better than 0.333333…..)
This is a calculator unit, so students can check their answers on their calculators, many of which nowadays work in exact form
A-Level textbooks (C1) are a good source of questions on surd manipulation, some of which are algebraic
Useful generalisation to learn is $\sqrt{x} \times \sqrt{x} = x$

**Module 6 METHODS** **Time: 3 – 4 hours**

**Tier: Higher – Unit 2**

**Content: Ratio and proportion** **COMMON**

MN k *Use ratio notation, including reduction to its simplest form and it various links to fraction notation*
MN n Divide a quantity in a given ratio
MN m Solve word problems about ratio, including using informal strategies and the unitary method of solution
MN m Calculate an unknown quantity from quantities that vary in direct or inverse proportion

**PRIOR KNOWLEDGE**

Fractions and decimals

**OBJECTIVES**

By the end of the module the student should be able to:

- Appreciate that eg the ratio 1:2 represents $\frac{1}{3}$ and $\frac{2}{3}$ of a quantity
- Divide quantities in a given ratio, eg divide £20 in the ratio 2:3
- Solve word problems involving ratios, eg find the cost of 8 pencils given that 6 similar pencils cost 78p
- Work out the real distance from a map, eg find the real distance represented by 4 cm on a map with
  scale 1:25 000
- Solve simple direct and inverse proportion problems using the unitary method or by proportional change (from a table of values or a worded problem)

**DIFFERENTIATION & EXTENSION**

Currency calculations using currency exchange rates
Harder problems involving multi-stage calculations

**NOTES**

Students often find three-part ratios difficult
Also link ratios given in different units to metric and imperial units
Link direct and inverse proportion with graphs and algebra (using the proportion '$\propto$' symbol)

**Module 7 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Solving linear inequalities**

MA d Solve simple linear inequalities in one variable, and represent the solution set on a number line
MA d Use the correct notation to show inclusive and exclusive inequalities
MA d Solve simple linear inequalities two variables
MA d Use the correct notation to show inclusive and exclusive inequalities

**PRIOR KNOWLEDGE**

Experience of finding missing numbers in calculations
The idea that some operations are 'opposite' to each other
An understanding of balancing
Experience of using letters to represent quantities
Be able to draw a number line
Linear equations and inequalities

**OBJECTIVES**

By the end of the module the student should be able to:

- Solve linear inequalities in one variable and present the solution set on a number line
- Solve simple linear inequalities two variables
- Use the correct notation to show inclusive and exclusive inequalities

**DIFFERENTIATION & EXTENSION**

Use of inverse operations and rounding to 1 sf could be applied to more complex calculations
Derive inequalities from geometric situations (such as finding unknown angles in polygons or perimeter problems)
Solve linear inequalities where manipulation of fractions is required

**NOTES**

Students can leave their answers in fractional form where appropriate
Interpreting the direction of an inequality is a problem for many students
Students should use the correct notation when showing inequalities on a number line, eg a solid circle to show inclusion of a point, an empty circle to show exclusion of a point
Equations and inequalities in two variables will be covered again after simultaneous equations

**Module 8 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 2**

**Content: Formulae**

MA h Substitute numbers into formulae
MA h Derive a formula
MA h *Change the subject of a formula including where the subject occurs once or more than once*
MA k *Form an algebraic expression from a given statement and rearrange this to arrive at a proof*

## PRIOR KNOWLEDGE

Understanding of the mathematical meaning of the words expression, simplifying, formulae and equation
Experience of using letters to represent quantities
Substituting into simple expressions using words
Using brackets in numerical calculations and removing brackets in simple algebraic expressions
Solving linear equations

## OBJECTIVES

By the end of the module the student should be able to:

- Use letters or words to state the relationship between different quantities
- Substitute positive and negative numbers into simple algebraic formulae
- Substitute positive and negative numbers into algebraic formulae involving powers
- Generate a formula from given information, eg find the formula for the perimeter of a rectangle given its area $A$ and the length of one side
- *Simple change of subject of a formula, eg convert the formula for converting Centigrade into Fahrenheit into a formula that converts Fahrenheit into Centigrade*
- *Change the subject of the formula when the variable appears more than once (questions could involve powers, roots, fractions or reciprocals)*
- *Algebraic proof*

## DIFFERENTIATION & EXTENSION

Use negative numbers in formulae involving indices
Various investigations leading to generalisations
Further problems in generating formulae form given information
Apply to equation of a straight line, eg what is the gradient of the line $4x + 2y = 12$?
Apply changing the subject to physics formulae, ie pendulum, equations of motion, focal length formula

## NOTES

Emphasise good use of notation, eg $3ab$ means $3 \times a \times b$
Students need to be clear on the meanings of the words ‘expression’, ‘equation’, ‘formula’ and ‘identity’
Show a linear equation first and follow the same steps for the similarly structured formula to be rearranged
Link with formulae for area, volume, surface area

**Module 9 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Straight line graphs**

MA o Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
MA o Find the gradient of lines given equations of the form $y = mx + c$ (where values are given for $m$ and $c$)
MA o Analyse problems and use gradients to see how one variable changes in relation to another
MA o Explore the gradients of parallel lines and lines perpendicular to each other
MA o Calculate the length of a line segment between two coordinates
MA o Write down the equation of a line parallel or perpendicular to a given line
MA t Understand the link between conversion graphs and associated formulae

**PRIOR KNOWLEDGE**

Experience at plotting points in all quadrants
Linear sequences and straight line graphs

**OBJECTIVES**

By the end of the module the student should be able to:

- Draw linear graphs from tabulated data, including real-world examples
- Interpret linear graphs, including conversion graphs and distance-time graphs
- Draw and interpret graphs in the form $y = mx + c$ (when values for $m$ and $c$ are given)
- Understand that lines are parallel when they have the same value of $m$
- Finding the gradient and intercept of a straight line graph.
- Know that the line perpendicular to $y = mx + c$ has gradient $-\frac{1}{m}$
- Write down the equation of a line parallel or perpendicular to a given line
- Find the distance between any two coordinates (link with Pythagoras' theorem)
- Understand the link between conversion graphs and associated formulae

**DIFFERENTIATION & EXTENSION**

Plot graphs of the form $y = mx + c$ where student has to generate their own table and set out their own axes
Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines
Use a graphical calculator or graphical ICT package to draw straight-line graphs
Link length of line to Pythagoras' theorem and revise midpoint problems

**NOTES**

Clear presentation with axes labelled correctly is vital
Recognise linear graphs and hence when data may be incorrect
Link to graphs and relationships in other subject areas, ie science, geography etc

**Module 10 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Simultaneous equations**

MA f Set up and solve a pair of simultaneous equations in two variables
MA f Find the exact solution of two (linear) simultaneous equations in two unknowns by eliminating a variable and interpret the equations as lines and their common solution as the point of intersection
MA e Relate algebraic solution to graphical representation of the equation
MA t Recognise that simultaneous equations can be solved algebraically and graphically

**PRIOR KNOWLEDGE**

Algebraic manipulation
Ability to solve simple linear equations
Some experience with solving inequalities
Straight line graphs

**OBJECTIVES**

By the end of the module the student should be able to:

- Set up and solve two simultaneous equations algebraically
- Interpret the solution of two simultaneous equations as the point of intersection the corresponding lines
- Model worded problems as a pair of linear simultaneous equations and interpret the answer.

**DIFFERENTIATION & EXTENSION**

Solve two simultaneous equations with fractional coefficients
Solve two simultaneous equations with second order terms, eg equations in $x$ and $y^2$

**NOTES**

Build up the algebraic techniques slowly
Link the graphical solutions with straight line graphs and changing the subject
Inaccurate graphs could lead to incorrect solutions, so encourage substitution of answers to check they are correct
Clear presentation of working is essential
Students should use the correct notation when giving graphical solutions to inequalities, eg a dotted boundary line for < or >

**Module 11 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Direct and inverse proportion** **COMMON**

MN m Set up and use equations to solve word and other problems involving direct
MA e proportion or inverse proportion and relate algebraic solutions to graphical representations of the equations
MA e Calculate an unknown quantity from quantities that vary in direct or inverse proportion
MA e Relate algebraic solutions to graphical representation of the equations

**PRIOR KNOWLEDGE**

Substitute numbers into algebraic formulae
Rearrange the subject of a formula
Direct and inverse proportion

**OBJECTIVES**

By the end of the module the student should be able to:

- Revise simple direct and inverse proportion problems using the unitary method or by proportional change (or from a table of values)
- Interpret direct and inverse proportions as algebraic functions, eg $y \propto x^2$ as $y = kx^2$
- Use given information to find the value of the constant of proportionality
- Use algebraic functions for direct and inverse proportionality, with their value of $k$, to find unknown values
- Recognise and sketch the graphs for direct and inverse proportions ($y \propto x$, $y \propto x^2$, $y \propto x^3$, $y \propto \frac{1}{x}$, $y \propto \frac{1}{x^2}$)

**DIFFERENTIATION & EXTENSION**

Link unitary method with ratio
Problems involving other types of proportionality (eg surface area to volume of a sphere)
Link to graphs to show direct and inverse proportion, eg $y = \frac{k}{x}$ etc

**NOTES**

Students should be encouraged to show all steps in their working
Students often forget the "square" in inverse square proportionality

**Module 12 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Quadratic functions***

MA g Generate points and plot graphs of quadratic functions
MA g Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function
MA g *Solve simple quadratic equations by using the quadratic formula*

**PRIOR KNOWLEDGE**

Graphs
Solving equations

**OBJECTIVES**

By the end of the module the student should be able to:

- Plot the graphs of quadratic functions for positive and negative values of $x$
- Find graphically the solutions of quadratic equations by considering the intercept on the $x$-axis
- *Use the quadratic formula to solve quadratic equations giving the answers to a specified degree of accuracy*
- *Use the quadratic formula to solve quadratic equations leaving the answer in surd form or decimal form*
- *Complete the square of a quadratic function (using this to write down the maximum or minimum of the function)*

**DIFFERENTIATION & EXTENSION**

*Solve equations involving algebraic fractions which lead to quadratic equations*
*Solve quadratic equations by completing the square*
*Derive the quadratic equation by completing the square*
Use graphical calculators or ICT graph package where appropriate to enable students to get through examples more rapidly
*Show how the value of '$b^2 - 4ac$' can be useful in determining if the quadratic factorises or not (ie square number)*
*Extend to discriminant properties and roots (for those going on to C1)*

**NOTES**

There may be a need to remove a factor of a trinomial before factorising to make the factorisation easier
Students should be reminded that factorisation should be tried before the formula is used
In problem-solving, one of the solutions to a quadratic may not be appropriate, eg negative length

**Module 13 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Further simultaneous equations** **COMMON**

MA f Find the intersection points of the graphs of linear and quadratic functions, and know that these are the approximate solutions of the corresponding simultaneous equation representing the linear and quadratic functions

MA f, r, q *Understand and use the Cartesian equation of a circle centred at the origin and link to the trigonometric functions*

MA f, t *Find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle*

MA f, r *Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other, of where the second is of the form* $x^2 + y^2 = r^2$

**PRIOR KNOWLEDGE**

Quadratic functions
Straight line graphs
Algebraic manipulation and solving linear and quadratic equations

**OBJECTIVES**

By the end of the module the student should be able to:

- Find graphically the approximate solutions of linear and quadratic simultaneous equations
- Find the exact solutions of linear and quadratic simultaneous equations
- Draw a circle of radius *r* centred at the origin and establish the equations properties
- Find graphically the approximate solutions of linear and circular simultaneous equations
- *Find the exact solutions of linear and circular simultaneous equations*

**DIFFERENTIATION & EXTENSION**

Find graphically the approximate solutions of quadratic and circular simultaneous equations
*Find the exact solutions of quadratic and circular simultaneous equations using algebraic methods*
Look at circles whose centre is not the origin $(x – 2)^2 + (y – 3)^2 = 4$ (Link with transforming graphs)

**NOTES**

Clear presentation of working is essential
Stress which variable it is easiest to work with when assessing the linear equation
ICT graph drawing packages make this topic more dynamic and easier to picture
Further examples and questions can be obtained from A-Level papers (C1)

**Module 14 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Graphs of curves**

MA g Generate points and plot graphs of simple quadratic functions

MA p Plot graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the exponential function $y = k^x$ for integer values of $x$ and simple positive values of $k$, the circular functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ using a spreadsheet or graph plotter as well as pencil and paper

MA p Recognise the characteristic shapes of all these functions

**PRIOR KNOWLEDGE**

Straight line graphs
BIDMAS

**OBJECTIVES**

By the end of the module the student should be able to:

- Plot and recognise quadratic, cubic, reciprocal, exponential and circular (trig) functions (see above) within the range -360° to +360°
- Use the graphs of these functions to find approximate solutions to equations, eg given $x$ find $y$ (and vice versa)
- Match equations with their graphs
- Sketch graphs of given functions

**DIFFERENTIATION & EXTENSION**

Explore the function $y = e^x$ (perhaps relate this to $y = \ln x$)
Explore the function $y = \tan x$
Find solutions to equations of the circular functions $y = \sin x$ and $y = \cos x$ over more than one cycle (and generalise)
Start to investigate transformations, eg $y = \sin(2x)$ or $y = x^2 + 5$

**NOTES**

This work should be enhanced by drawing graphs on graphical calculators and appropriate software
Group work where each group is assigned a different type of graph can be an effective way to explore the graphs' properties. Each group reports to the whole class and creates a display
Contrast the shape of tan $x$ as compared to sin $x$ and cos $x$. Relate the values of tan $x$ to the gradient of a line making an angle of $x$ degrees with the horizontal (this explains why tan 90° cannot be defined as it is the gradient of a vertical line, which is infinity
There are plenty of past examination papers with matching tables testing knowledge of the 'Shapes of graphs' (See Emporium website, www.edexcelmaths.com)

**Module 15 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Transforming graphs**

MA s Apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y=f(x+ a)$, $y = af(x)$ for linear, quadratic, sine and cosine functions $f(x)$

MA s Select, apply and sketch the transformations of reflection, rotation, enlargement and translation of functions expressed algebraically

MA s Interpret and analyse transformations of functions and write the functions algebraically

**PRIOR KNOWLEDGE**

Transformations
Graphs of curves

**OBJECTIVES**

By the end of the module the student should be able to:

- Understanding of the notation $y = f(x)$
- Represent translations in the $x$ and $y$ direction, reflections in the $x$-axis and the $y$-axis, and stretches parallel to the $x$-axis and the $y$-axis
- Apply transformations to general graphs or specific curves such as trigonometric functions (ie those in graphs of curves)
- Sketch the graph of $y = 3\sin(2x)$, given the graph of $y = \sin x$
- Sketch the graph of $y = f(x + 2)$, $y = f(x) + 2$, $y=2f(x)$, $y = f(2x)$ given the shape of the graph $y = f(x)$
- Find the coordinates of the minimum of $y = f(x + 3)$, $y = f(x) + 3$ given the coordinates of the minimum of $y=x^2 – 2x$

**DIFFERENTIATION & EXTENSION**

Complete the square of quadratic functions and relate this to transformations of the curve $y = x^2$
Use a graphical calculator or software to investigate transformations
Investigate curves which are unaffected by particular transformations
Investigations of the simple relationships such as $\sin(180 – x) = \sin x$, and $\sin(90 – x) = \cos x$

**NOTES**

Make sure the students understand the notation $y = f(x)$. Perhaps start with comparing $y = x^2$ with $y = x^2 + 2$ before mentioning $y = f(x) + 2$ etc
Graphical calculators or graph drawing software will help to underpin the main ideas in this topic
Link with Trigonometry and Curved graphs

**Module 16 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 2**

**Content: Angle facts and polygons** **COMMON**

MG b Properties of triangles and quadrilaterals
MG b Give reasons for angle calculations
MG b Angles associated with parallel lines
MG e *Definitions and names of polygons*
MG e *Calculate and use the sums of the interior angles of polygons*
MG e *Calculate and use the angles of regular polygons*
MG e *Understand that inscribed regular polygons can be constructed by equal divisions of a circle*

**PRIOR KNOWLEDGE**

The concept of parallel lines
The concept of vertical and horizontal
The concept of an angle between two lines
Experience in drawing triangles, quadrilaterals and circles

**OBJECTIVES**

By the end of the module the student should be able to:

- Identify triangles by their properties (scalene, isosceles, equilateral, right-angled, obtuse, and acute)
- Prove the angle sum in a triangle is 180° and explain why the angles inside a quadrilateral add up to 360°
- Use the angle properties of triangle to find missing angles
- Prove the exterior angle of a triangle is equal to the sum of the two opposite interior angles
- Identify quadrilaterals by their properties (trapezium, parallelogram, rhombus, rectangle, square, kite)
- Use alternate, corresponding and co-interior angles in parallel lines to find missing angles
- *Name a polygon with 3, 4, ..., 10 sides*
- *Calculate and use the sums of the interior angles of convex polygons*
- *Know, or work out, the relationship between the number of sides of a polygon and the sum of its interior angles*
- *Know that the sum of the exterior angles of any polygon is 360°*
- *Find the size of each exterior or interior angle of a regular polygon*

**DIFFERENTIATION & EXTENSION**

Use triangles to find the angle sums of polygons
Use the angle properties of triangles to find missing angles in combinations of triangles
Harder problems involving multi-step calculations
Link with tessellations

**NOTES**

Lots of practical drawing examples to help illustrate properties of various shapes
Diagrams used in examinations are usually not drawn accurately
Use tracing paper to show which angles in parallel lines are equal
Useful way of remembering rules- ‘the FUZ is the CIA.’ (F, U and Z angles’ real names are Corresponding, Interior and Alternate)
Encourage students to always put the reasons and ‘quote’ the angle fact or theorem used. [Important for the new assessment objectives]

**Module 17 METHODS** **Time: 2 – 3 hours**

**Tier: Higher – Unit 2**

**Content: Similarity, congruency* and tessellations**

| | |
|---|---|
| MG m | Understand similarity of triangles and of other plane figures and use this to make geometric inferences |
| MG m | Recognise that enlargements preserve angle but not length |
| MG m, i | Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions |
| MG f | Understand tessellations |
| MG h | Understand and use the midpoint and the intercept theorems |

**This topic (except Tessellations and Midpoint or Intercept Theorems) is also in Unit 1*

**PRIOR KNOWLEDGE**

The mathematical names of triangles and angles
Understand the terms perpendicular, parallel and arc
Transformations (particularly enlargements)

**OBJECTIVES**

By the end of the module the student should be able to:

- Prove formally geometric properties of triangles, eg that the base angles of an isosceles triangle are equal
- Prove formally that two triangles are congruent
- Use integer and non-integer scale factors to find the length of a missing side in each of two similar shapes, given the lengths of a pair of corresponding sides
- Understand tessellations and explain why some shapes tessellate and why other shapes do not

**DIFFERENTIATION & EXTENSION**

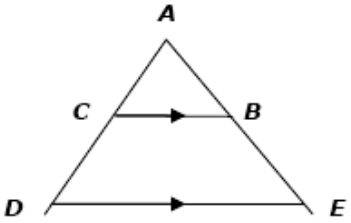
Link with the information to be able to construct a triangle, eg SSS, ASA, SAS
Link with similar areas and volumes
Harder problems in congruence

**NOTES**

All working should be presented clearly, and accurately
If a student is not sure how to approach a question, advise them to write 'easy' numbers in pencil on their diagram, use this to approach the problem initially, and then repeat the process with the 'real' values on the question

Know that when CB and DE are parallel

$$\frac{CD}{DA} = \frac{BE}{EA}, \frac{AC}{CD} = \frac{AB}{BE}$$




Use plastic shapes as templates to draw tessellations (or use a tracing paper template)
Use angles in a polygon to explain why some shapes do or do not tessellate

**Module 18 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Similar shapes*** **COMMON**

MG m Understand the effect of enlargement for perimeter, area and volume of shapes and solids
MG m Understand that enlargement does not have the same effect on area and volume
MG m Understand and use the effect of enlargement on areas and volumes of shapes and solids
MG m Know the relationship between linear, area and volume scale factors of mathematically similar shapes and solids

**This topic is also in Unit 1*

**PRIOR KNOWLEDGE**

Use ruler and compasses to construct triangles with given dimensions
Some concept of enlargement (magnification)
Similar triangles

**OBJECTIVES**

By the end of the module the student should be able to:

- Use integer and non-integer scale factors to find the length of a missing side in each of two similar shapes, given the lengths of a pair of corresponding sides (Revision of Topic 17)
- Know the relationship between linear, area and volume scale factors of similar shapes
- Prove formally geometric properties of triangles, eg that the base angles of an isosceles triangle are equal

**DIFFERENTIATION & EXTENSION**

Find algebraic formulae for the areas and volumes of similar shapes
Extend to questions which give the ratio for area and ask for length or volume

**NOTES**

Students will need to be reminded of this work on a regular basis, and it can be linked to ratios
1 : L (Length)
1: $L^2$ (Area)
1: $L^3$ (Volume)
A good starter is to bring in a small bottle of water and a larger bottle (ideally twice the length). Show by pouring that eight small bottles will fill the larger bottle. [This can also be done with one small cube and a larger box with lengths twice as long].Initially the class will say two bottles will fill the larger double size bottle

**Module 19 METHODS** **Time: 4 – 6 hours**

**Tier: Higher – Unit 2**

**Content: Pythagoras' theorem and trigonometry in 2-D** **COMMON**

| | |
|---|---|
| MG n | Understand, recall and use Pythagoras' theorem |
| MA l | Given the coordinates of points *A* and *B*, calculate the length of *AB* |
| MG o | Understand, recall and use trigonometrical relationships in right-angled triangles, and use these to solve problems |
| MG o | Find angles of elevation and angles of depression |
| MN o | Use calculators effectively and efficiently |

**PRIOR KNOWLEDGE**

Names of triangles and quadrilaterals
Knowledge of the properties of rectangles, parallelograms and triangles
Indices, equations and changing the subject

**OBJECTIVES**

By the end of the module the student should be able to:

- Find missing sides of right-angle triangles by using Pythagoras' theorem
- Find the distance between two coordinates using Pythagoras' theorem
- Give answers as decimals or surds for Pythagoras problems
- Use trigonometric ratios (sin, cos and tan) to calculate angles in right-angled triangles
- Use the trigonometric ratios to calculate unknown lengths in right-angled triangles (2-D)
- Solve problems involving geometric figures (including triangles within circles) in which a right-angled triangle has to be extracted in order to solve it by Pythagoras and /or trigonometry.

**DIFFERENTIATION & EXTENSION**

Introduce 3-D trigonometry and show that the trig ratios above will **only** work for right-angled triangles.

**NOTES**

Students should be encouraged to become familiar with one make of calculator
Calculators should be set to "deg" mode
Emphasise that scale drawings will score no marks for this type of question
A useful mnemonic for remember trig ratios is "Sir Oliver's Horse, Came Ambling Home, To Oliver's Aunt" or 'SOH/CAH/TOA'; but students often enjoy making up their own
Calculated angles should be given to at least 1 dp and sides are determined by the units used or accuracy asked for in the question
Students should not forget to state the units for the answers
Organise a practical surveying lesson to find the heights of buildings or trees around your school grounds. All you need is a set of tape measures (or trundle wheels) and clinometers

**Module 20 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Pythagoras' theorem and Trigonometry in 3-D** **COMMON**

MG n, o Use Pythagoras' theorem and trigonometrical ratios in 3-D
MG o *Find the angle between a line and a plane (but not the angle between two planes or between two skew lines)*
MG n Calculate the length of a diagonal of a cuboid

**PRIOR KNOWLEDGE**

Pythagoras' theorem and trigonometry
3-D Solids

**OBJECTIVES**

By the end of the module the student should be able to:

- Calculate the length of a diagonal of a rectangle given the lengths of the sides of the rectangle
- Calculate the diagonal through a cuboid, or across the face of a cuboid
- Find the angle between the diagonal through a cuboid and the base of the cuboid
- Find the angle between a sloping edge of a pyramid and the base of the pyramid

**DIFFERENTIATION & EXTENSION**

Harder problems involving multi-stage calculations

**NOTES**

The angle between two planes or two skew lines is not required

**Module 21 METHODS** **Time: 2 – 4 hours**

**Tier: Higher – Unit 2**

**Content: Trigonometry in non-right-angled triangles**

MG p Use the sine and cosine rules to solve 2-D problems
MG t Calculate the area of a triangle using $\frac{1}{2}ab\sin C$
MG p Use the sine and cosine rules to solve 3-D problems

**PRIOR KNOWLEDGE**

Trigonometry
Formulae

**OBJECTIVES**

By the end of the module the student should be able to:

- Find the unknown lengths, or angles, in non right-angle triangles (in 2-D and 3-D) using the sine and cosine rules
- Find the area of triangles given two lengths and an included angle

**DIFFERENTIATION & EXTENSION**

Use these ratios to solve problems in 3-D and decide if it is easier to extract right-angle triangles to use 'normal' trigonometry
Stress that the cosine Rule is only used when we have SAS (and we need to find the side opposite the angle given) or when we are given SSS (then we use the re-arranged version to find any angle and use the sine rule)

**NOTES**

Students find the cosine rule more difficult for obtuse angles
Reminders of simple geometrical facts may be helpful, eg angle sum of a triangle, the shortest side is opposite the smallest angle
Show the form of the cosine rule in the formula page and rearrange it to show the form which finds missing angles

**Module 22 METHODS** **Time: 3 – 5 hours**

**Tier: Higher – Unit 2**

**Content: Circles, cones, pyramids and spheres** **COMMON**

| | |
|---|---|
| MG r | Find circumferences of circles and areas enclosed by circles, recalling relevant formulae |
| MG r | Calculate the lengths of arcs and the areas of sectors of circles |
| MG r | Find the surface area of a cylinder |
| MG s | Find the surface area of prisms using the formulae for triangles and rectangles and other shapes |
| MG v | Find the surface areas and volumes of compound solids constructed from spheres, hemispheres, cylinders and solve problems including examples of solids in everyday use |
| MG u, s, r | Solve problems involving surface areas and volumes of (right) prisms and cylinders |
| MG v | Solve problems involving surface areas and volumes of cones and pyramids |
| MG v | Solve problems involving more complex shapes, including segments of circles, length of a chord and frustums of cones |
| MG v | Find the area of a segment of a circle given the radius and length of the chord |
| MN b | *Use surds and $\pi$ in exact calculations* |

**PRIOR KNOWLEDGE**

Perimeter and area
Surface area
Formulae, substitution and changing the subject
Surds and exact form

**OBJECTIVES**

By the end of the module the student should be able to:

- Learn that the ratio between the circumference and diameter is always constant for a circle (ie $\pi$)
- Learn the formula for circumference and area of a circle
- Solve problems involving the circumference and area of a circle
- Find the area of a sector and length of an arc
- Solve problems involving the volume of a prism and cylinder
- Find the surface area and the volume of more complex shapes, eg find the volume of an equilateral triangular prism
- Solve more complex problems, eg given the surface area of a sphere find the volume

**DIFFERENTIATION & EXTENSION**

Find the volume of a cylinder given its surface area, leaving the answer in terms of $\pi$
Find the volume of a right hexagonal cone of side $x$ and height $h$ (researching the method for finding the volume of any cone)

**NOTES**

For the volume of a right prism, show that it is effectively the number of 'slices' of the area of the front (or cross-section)

'Now! I Know $\pi$' is a good way to learn the approximate value (The number of letters of each word and the ! is the
decimal point)

Also 'Cherry Pie Delicious' is $C = \pi D$ and 'Apple Pies are too' is $A = \pi r^2$

Answers in terms of $\pi$ may be required or final answers rounded to the required degree of accuracy

Need to constantly revise the expressions for area or volume of shapes

Students should be aware of which formulae are on the relevant page on the examination paper and which they need to learn

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**Use of Calculators** (Spec Ref: MN o)

Students are well advised to regularly bring their calculator to lessons and get used to its functions throughout this Unit 2 course

They should be confident in entering a range of calculations including those involving time and money. They should realise that 2.33333333…. hrs is 2 hours 20 minutes etc

Calculator functions include +, –, ×, ÷, $x^2$, $\sqrt{x}$, memory, $x^y$, $x^{\frac{1}{y}}$ (or equivalent power or root buttons), trigonometrical functions and brackets

Students should be able to use the functions which enable questions involving standard form and fractions to be answered efficiently

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# Higher course objectives (2MM01)

## Unit 1

## Number

| | |
|---|---|
| MN a | Use brackets and the hierarchy of operations |
| MN a | Use one calculation to find the answer to another |
| MN a | Add, subtract, multiply and divide fractions |
| MN a, b | Multiply and divide by negative numbers |
| MN a, d | Check and estimate answers to problems, by approximations or inverse operations |
| MN b | Multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1 |
| MN b | Add and subtract decimals |
| MN b | Simplify surds and rationalise a denominator |
| MN b | Use surds and $\pi$ in exact calculations, without a calculator |
| MN c | Understand and use negative integers both as positions and translations on a number line |
| MN c | Order fractions, understand equivalent fractions and simplify fractions |
| MN c | Order integers, decimals and fractions |
| MN c | Find squares and cubes of numbers and find square roots and cube roots of numbers |
| MN c | Use index notation and index laws for multiplication and division of integer powers |
| MN c | Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integers, fractional and negative powers |
| MN c | Recall the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers $n$, the corresponding rule for negative integers, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number $n$ |
| MN c | Use standard index form, expressed in standard notation and on a calculator display |
| MN c | Calculate with standard index form |
| MN c | Convert between ordinary and standard index form representations |
| MN c | Convert to standard index form to make sensible estimates for calculations involving multiplication and division |
| MN d | Round to a given number of significant figures and decimal places |
| MN e | Understand prime numbers |
| MN e | Find factors and multiples of numbers |
| MN e | Find common factors and common multiples |
| MN e, f | Find Highest Common Factor (HCF), Lowest Common Multiple (LCM) and prime factor decomposition |
| MN p | Draw, use and interpret Venn diagrams |
| MN p | Understand and be able to find the union and intersection of sets |
| MN p | Use Venn Diagrams to solve problems |

## Algebra

| | |
|---|---|
| MA a | Distinguish the different roles played by letter symbols in algebra |
| MA b | Distinguish the meaning between the words 'equation', 'formula', 'identity' and 'expression' |
| MA c | Simplify terms, products and sums |
| MA c | Multiply a single term over a bracket |
| MA c | Take out common factors |
| MA c | Substitute positive and negative numbers into expression such as $3x^2 + 4$ and $2x^3$ |
| MA c | Use instances of index laws, including use of fractional, zero and negative powers and power of a power |
| MA c | Expand the product of two linear expressions |
| MA c | Factorise quadratic expressions including the difference of two squares |
| MA c | Simplify rational expressions by adding, subtracting, multiplying and cancelling |
| MA d | Set up simple equations |
| MA d | Solve equations by using inverse operations or by transforming both sides in the same way |
| MA d | Solve linear equations with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation |
| MA d | Solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution |
| MA g | Factorise quadratic expressions |
| MA g | Solve simple quadratic equations by factorising and completing the square |
| MA g | Solve quadratic equations approximately by using a graph |
| MA i | Generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangle numbers) |
| MA i | Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence |
| MA j | Use linear expressions to describe the *n*th term of an arithmetic sequence |
| MA j | Use quadratic expressions to describe the *n*th term of a quadratic sequence |
| MA l, k | Use algebra to support and construct arguments and proofs |
| MA l | Use axes and coordinates to specify points in all four quadrants in 2-D and 3-D |
| MA l | Identify points with given coordinates |
| MA l | Find the coordinates of the midpoint of the line segment *AB*, given the coordinates of *A* and *B* |
| MA m | Recognise (when values are given for *m* and *c*) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane |
| MA m | Plot graphs of functions in which *y* is given explicitly in terms of *x*, or implicitly |
| MA n | Find the coordinates of points identified by geometrical information in 2-D |
| MA o | Find the gradient of a straight line |
| MA o | Find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for *m* and *c*) |
| MA t | Understand the connection between graphs and associated formulae and equations |
| MA t | Use graphs to find approximate solutions to equations |

## Geometry

| | |
|---|---|
| MG a, b | Use basic angle facts for a straight line, at a point and vertically opposite angles |
| MG b | Use angles associated with intersecting lines |
| MG b | Give reasons for angle calculations |
| MG b, c | Use properties of triangles and quadrilaterals |
| MG d | Recognise and visualise reflection and rotational symmetry of 2-D shapes |
| MG g | Understand and use the fact that tangents from an external point are equal in length |
| MG g | Explain why the perpendicular from the centre to a chord bisects the chord |
| MG g | Understand that the tangent at any point on a circle is perpendicular to the radius at that point |
| MG g | Explain why the perpendicular from the centre to a chord bisects the chord |
| MG g | Prove and use the fact that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference |
| MG g | Prove and use the fact that the angle subtended at the circumference by a semicircle is a right angle |
| MG g | Prove and use the fact that angles in the same segments are equal |
| MG g | Prove and use the fact that opposite angles of a cyclic quadrilateral sum to 180° |
| MG g | Prove and use the alternate segment theorem |
| MG g | Know and use the fact that for two chords *AB* and *CD* of a circle, intersecting at the point *X*; $AX \times XB = CX \times XD$ |
| MG j, k | Transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations |
| MG j, k | Use column vectors to describe translations |
| MG j | Understand that translations are specified by a distance and direction (or a column vector), and enlargements by a centre and a scale factor |
| MG j | Rotate a shape about the origin, or any other point |
| MG j | Measure the angle of rotation using right angles, simple fractions of a turn or degrees |
| MG j | Understand that rotations are specified by a centre and an (anticlockwise) angle |
| MG j | Understand that reflections are specified by a mirror line, at first using a line parallel to any axis, then a mirror line such as $y = x$ or $y = -x$ |
| MG j | Recognise, visualise and construct enlargements of objects using positive, negative and fractional scale factors greater and less than one |
| MG j | Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations |
| MG j | Describe and transform 2-D shapes using combined rotations, reflections, translations or enlargements |
| MG k | Understand and use vector notation (revise column vectors) |
| MG l | Calculate, and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector |
| MG l | Calculate the resultant of two vectors |
| MG l | Understand and use the commutative and associative properties of vector addition |
| MG l | Solve simple geometrical problems in 2-D using vector methods |
| MG l | Prove that three or more points are collinear |
| MG m | Understand similarity of triangles and of other plane figures and use this to make geometric inferences |
| MG m | Identify similar solids |
| MG m | Recognise that enlargements preserve angle but not length |
| MG m | Understand the relationship between lengths, areas and volumes in similar figures |
| MG m | *Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions* |

| | |
|---|---|
| MG q | Use names and definitions of parts of a circle |
| MG s | Use formulae to find the area of triangles, parallelograms, and trapeziums |
| MG s | Calculate perimeters of shapes made from triangles and rectangles |
| MG s | Calculate areas of shapes made from triangles and rectangles |
| MG s | Find the area of compound shapes |
| MG s | Calculate the perimeter and area of compound shapes made from triangles, rectangles and other shapes |

## Probability

| | |
|---|---|
| MP a | Understand and use the vocabulary of probability and probability scale |
| MP b, c | Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency |
| MP d | List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities |
| MP d | Draw and use sample spaces |
| MP e | Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 |
| MP f | Use set notation to describe events and compound events |
| MP f, i | Know when to add or multiply two probabilities: when $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$, whereas when $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$ [Relate these to Union $\cup$, and Intersection, $\cap$] |
| MP g | Use Venn diagrams to represent the number of possibilities and find probabilities. |
| MP h | Use tree diagrams to represent outcomes of compound events, and recognise when events are independent |
| MP i | Know when to add or multiply two probabilities: when $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$, whereas when $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$ |
| MP j | Compare experimental data and theoretical probabilities |
| MP k | Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics |

# Unit 2

## Number

| | |
|---|---|
| MN a | Understand reciprocal |
| MN a | Understand and use unit fractions as multiplicative inverses |
| MN a, i | Multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction |
| MN a, b | Add, subtract, multiply and divide integers, fractions and decimals |
| MN b | Solve a problem involving division by a decimal |
| MN b | Write answers in exact form |
| MN b | *Use surds and $\pi$ in exact calculations* |
| MN g | Convert between fractions, decimals and percentages |
| MN g | Understand that 'percentage' means 'number of parts per 100' |
| MN g | Interpret percentage as the operator 'so many hundredths of' |
| MN g | Use percentages and fractions in real-life situations |
| MN h, l | Solve percentage problems, including increase and decrease. |
| MN h | Solve percentage problems, including increase and decrease, and reverse percentage |
| MN h | Compound interest |
| MN h, i, l | Represent repeated proportional change using a multiplier raised to a power |
| MN h, i, o | Use calculators for reverse percentages calculations by doing an appropriate division |
| MN i | Find percentages of quantities |
| MN i | Express a given number as a fraction of another |
| MN i | Express a given number as a percentage of another |
| MN i | Calculate a given fraction of a quantity |
| MN i | Understand 'reciprocal' as multiplicative inverse |
| MN i, l | Use a calculator to solve problems involving fractions (including repeated changes) |
| MN j | Recall the fraction-to-decimal conversion of familiar fractions |
| MN j | Convert between fractions and decimals using a calculator |
| MN j | Distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals) and other fractions (which are represented by recurring decimals) |
| MN j | Convert recurring decimals into fractions |
| MN j | Understand the recurring decimal to fraction proof |
| MN k | Use ratio notation, including reduction to its simplest form and it various links to fraction notation |
| MN m | Solve word problems about ratio, including using informal strategies and the unitary method of solution |
| MN m | Calculate an unknown quantity from quantities that vary in direct or inverse proportion |
| MN m, MA e | Set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relating algebraic solutions to graphical representations of the equations |
| MN n | Divide a quantity in a given ratio |
| MN o | Use calculators effectively and efficiently |

## Algebra

| | |
|---|---|
| MA p | Plot graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the exponential function $y = k^x$ for integer values of $x$ and simple positive values of $k$, the circular functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ |
| MA d | Use the correct notation to show inclusive and exclusive inequalities |
| MA d | Solve simple linear inequalities two variables |
| MA d | Use the correct notation to show inclusive and exclusive inequalities |
| MA d | Solve simple linear inequalities in one variable, and represent the solution set on a number line |
| MA e | Relate algebraic solution to graphical representation of the equation |
| MA f | Set up and solve a pair of simultaneous equations in two variables |
| MA f | Find the exact solution of two (linear) simultaneous equations in two unknowns by eliminating a variable and interpret the equations as lines and their common solution as the point of intersection |
| MA f | Solve several linear inequalities in two variables and find the solution set |
| MA f | Find the intersection points of the graphs of linear and quadratic functions, and know that these are the approximate solutions of the corresponding simultaneous equation representing the linear and quadratic functions |
| MA f, r | Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other, of where the second is of the form $x^2 + y^2 = r^2$ |
| MA f, t | Find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle |
| MA f, r, q | Understand and use the Cartesian equation of a circle centred at the origin and link to the trigonometric functions |
| MA g | Generate points and plot graphs of quadratic functions |
| MA g | Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function |
| MA g | Solve simple quadratic equations by factorising, completing the square and using the quadratic formula |
| MA g | Generate points and plot graphs of simple quadratic functions |
| MA h | Substitute numbers into formulae |
| MA h | Use formulae from mathematics and other subjects |
| MA h | Derive a formula |
| MA h | Change the subject of a formula including where the subject occurs once or more than once |
| MA k | Form an algebraic expression from a given statement and rearrange this to arrive at a proof |
| MA l | Given the coordinates of points $A$ and $B$ calculate the length of $AB$ |
| MA o | Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients |
| MA o | Find the gradient of lines given equations of the form y = $mx + c$ (where values are given for $m$ and $c$) |
| MA o | Analyse problems and use gradients to see how one variable changes in relation to another |
| MA o | Explore the gradients of parallel lines and lines perpendicular to each other |
| MA o | Calculate the length of a line segment between two coordinates |
| MA o | Write down the equation of a line parallel or perpendicular to a given line |
| MA p | Recognise the characteristic shapes of all these functions |

| | |
|---|---|
| MA s | Apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$, $y = af(x)$ for linear, quadratic, sine and cosine functions $f(x)$ |
| MA s | Select, apply and sketch the transformations of reflection, rotation, enlargement and translation of functions expressed algebraically |
| MA s | Interpret and analyse transformations of functions and write the functions algebraically |
| MA t | Understand the link between conversion graphs and associated formulae |
| MA t | Recognise that simultaneous equations can be solved algebraically and graphically |
| MA e | Calculate an unknown quantity from quantities that vary in direct or inverse proportion |
| MN m<br>MA e | Set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relating algebraic solutions to graphical representations of the equations |

## Geometry

| | |
|---|---|
| MG b | Definitions and names of polygons |
| MG b | Properties of triangles and quadrilaterals |
| MG b | Give reasons for angle calculations |
| MG b | Angles associated with parallel lines |
| MG e | Calculate and use the sums of the interior angles of polygons |
| MG e | Calculate and use the angles of regular polygons |
| MG e | Understanding that inscribed regular polygons can be constructed by equal divisions of a circle |
| MG f | Understand tessellations |
| MG h | Understand and use the midpoint and the intercept theorems |
| MG m | Understand similarity of triangles and of other plane figures and use this to make geometric inferences |
| MG m | Recognise that enlargements preserve angle but not length |
| MG m | Understand the effect of enlargement for perimeter, area and volume of shapes and solids |
| MG m | Understand that enlargement does not have the same effect on area and volume |
| MG m | Understand and use the effect of enlargement on areas and volumes of shapes and solids |
| MG m | Know the relationship between linear, area and volume scale factors of mathematically similar shapes and solids |
| MG m, i | Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions |
| MG n | Understand, recall and use Pythagoras' theorem |
| MG n | Calculate the length of a diagonal of a cuboid |
| MG n, o | Use Pythagoras' theorem and trigonometrical ratios in 3-D |
| MG o | Understand, recall and use trigonometrical relationships in right-angled triangles, and use these to solve problems |
| MG o | Find angles of elevation and angles of depression |
| MG o | Find the angle between a line and a plane (but not the angle between two planes or between two skew lines) |
| MG r | Find circumferences of circles and areas enclosed by circles, recalling relevant formulae |
| MG r | Find the surface area of a cylinder |
| MG r | Calculate the lengths of arcs and the areas of sectors of circles |
| MG s | Find the surface area of prisms using the formulae for triangles and rectangles and other shapes |
| MG u, s, r | Solve problems involving surface areas and volumes of (right) prisms and cylinders |
| MG v | Find the surface areas and volumes of compound solids constructed from; spheres, hemispheres, cylinders and solve problems including examples of solids in everyday use. |
| MG v | Solve problems involving surface areas and volumes of cones and pyramids |
| MG v | Solve problems involving more complex shapes, including segments of circles, length of a chord and frustums of cones |
| MG v | Find the area of a segment of a circle given the radius and length of the chord |
| MG p | Use the sine and cosine rules to solve 2-D problems |
| MG p | Use the sine and cosine rules to solve 3-D problems |
| MG t | Calculate the area of a triangle using $\frac{1}{2}ab\sin C$ |

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