A13.1 Risk

Before you start

You should be able to:

- calculate an estimate for the total number of successes in a series of identical trials, when the probability of success on any trial is constant and known
- construct probability tree diagrams and use them to work out the probability of compound events.

Why do this?

The risk of an event is related to its probability and to its impact (usually measured financially) so risk calculations are carried out every day by insurance companies.

Objectives

- You will gain an understanding of risk.
- You will be able to carry out calculations involving the concept of risk.

Get Ready

1 Jim rolls a fair dice 200 times. Work out an estimate for the number of times he should get the number 1.

2 Jim throws a fair coin and rolls a fair dice. Use a probability tree diagram to work out the probability he gets either a head or a score greater than 4, but not both.

Key Points

- The risk of an event is the probability that it will happen.
- Risks are often presented as relative frequencies such as 1 in 100.
- An estimate of the cost of an event can be obtained by multiplying the probability of the event by the actual cost if the event did happen.

Example 1

The probability of a washing machine flooding a kitchen in any one year is 0.001.
An insurance company pays out £800 for each flooded kitchen.
The insurance company insures 20 000 washing machines.
Work out an estimate of the amount of money that the insurance company must pay out next year.

Estimated number of flooded kitchens next year $= 0.001 \times 20\,000 = 20$
Estimated total amount of money to be paid out $= 20 \times £800 = £16\,000$

Example 2

The table gives information about the number of trains that ran and the number of those trains that were late during one month.

Time Period	Number of trains that ran	Number of trains that were late
06 : 00 – 07 : 00	347	34
07 : 00 – 08 : 00	428	43
08 : 00 – 09 : 00	517	40
09 : 00 – 10 : 00	326	28

a Compare between the four time periods, the risks of having a late train.

Transport engineers estimate that the cost to the train company of a late train is £3000.
The company plans to run 450 trains next month between 06 : 00 and 07 : 00.

b Estimate the cost of late trains to the train company if no improvements are made to lateness.

a 06:00 – 07:00 Prob of being late $= \frac{34}{347} = 0.0980$

07:00 – 08:00 Prob of being late $= \frac{43}{428} = 0.100$

08:00 – 09:00 Prob of being late $= \frac{40}{517} = 0.0774$

09:00 – 10:00 Prob of being late $= \frac{28}{326} = 0.0859$

The risk of any one train being late in this period is 0.0980, correct to 3 significant figures.

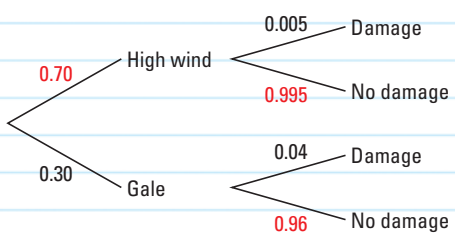
One way to compare is to use the figures to calculate probabilities of being late.

In order of reliability (most reliable first), the periods are:
08:00 – 09:00, 09:00 – 10:00, 06:00 – 07:00, 07:00 – 08:00

b An estimate of the number of late trains $= 450 \times 0.0980 = 44.1$
An estimate of the cost $= 44.1 \times £3000 = £132\,300$

Example 3

A company generates electricity from an offshore site with wind turbines.
If a high wind becomes a gale the probability of damage to the wind turbines increases.
The probability of damage in a gale is 0.04.
The probability of damage in a high wind is 0.005.
The probability that a high wind becomes a gale is 0.3.
This site has 50 high wind days each year.
Work out an estimate for the number of times it will be damaged in a period of 10 years.



The probability tree has been used to show the structure of the problem, so that the probability of damage on any high wind day can be found.
The red figures have been calculated by subtraction from 1.

Either:
high wind and damage
or
gale and damage

Probability of damage on one day $= 0.7 \times 0.005 + 0.3 \times 0.04$
$= 0.0155$
Number of times it will be damaged $= 0.0155 \times 50 \times 10 = 7.75$

Exercise 13A

C

1 Last year, a manufacturer sold 18 500 dishwashers. Of these, 121 broke down.

a Work out an estimate of the probability of a dishwasher breaking down.

This year, the manufacturer will sell 16 850 washing machines.

b Work out an estimate of the number of washing machines that will break down.

C

2 The table gives information about the number of repairs to electrical appliances made by a company.

Type of Appliance	Number made	Number of repairs
Washing Machine	12 800	198
Dishwasher	17 484	321
Dryer	13 724	216
Fridge	9515	125

Compare the risk of breakdown for each type of appliance.

3 An insurance company insures computers against breakdown. Last year, out of a total of 16 700, 84 computers broke down.

a Work out the probability of a computer breaking down.

The cost of repairing or replacing a computer is £528.

Next year, the number of computers insured will be 18 250.
Work out an estimate of the price that the insurance company should charge for it to break even.

4 A supermarket company owns 4000 freezers. The probability of a freezer breaking down in a year is 0.005. When the freezer breaks down the supermarket estimates the cost of repair and replacement as £800.

Work out an estimate for the cost to the supermarket company of repairs and replacements due to its freezers breaking down.

5 The table gives information about the number of trains that ran and the number of those trains that were late during one month.

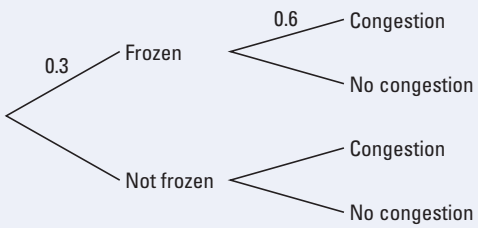
Time Period	Number of trains that ran	Number of trains that were late
16 : 00 – 17 : 00	285	30
17 : 00 – 18 : 00	401	55
18 : 00 – 19 : 00	480	40
19 : 00 – 20 : 00	303	31

a Compare between the four time periods, the risks of having a late train.

Transport engineers estimate that the cost to the train company of a late train is £3400.
The company plans to run 25 more trains during each time period next month.

b Work out an estimate of the cost of late trains to the train company next month if no improvements are made to lateness.

6 A town council is working out an estimate of costs to the town due to frozen roads.
The probability tree diagram gives some information about the probabilities of frozen roads and of congestion in the town during 60 days in winter.



The town council estimates that the probability of congestion is 0.1, if there are no frozen roads.

a Copy and complete the tree diagram.

b Work out the probability that on any given day in winter, there will be congestion.

The town council thinks that if there is congestion on any day the cost to the town is £1200.

c Work out an estimate of the cost to the town during these 60 days.

C

7 Last year there were 7685 landings at an airport. Of these landings there were 16 in which the aircraft suffered damage to tyres.

a Calculate an estimate of the risk of damage to an aircraft on landing at this airport.

A damaged aircraft tyre costs £650.

Next year the airport estimates that there will be 8100 landings.

b Work out an estimate of the total cost of damage to tyres at this airport.

B

8 The table gives information about the reliability of different makes of washing machines used last year.

Make	Number sold	Cost (£)	Number of breakdowns
Kandoo	3450	399	38
Black Diamond	4970	329	52
Illustrious	6500	259	79
Dekko	7680	199	125

a Use the 'number sold' column and the 'number of breakdowns' column to work out an estimate of the risk of each make of washing machine breaking down.

b Use the columns to work out which make is the best value.

9 A central heating company made a comparison between those households which had had their boiler serviced that year and those that had not. Information about this is given in the table.

	Number serviced	Number not serviced
Number of breakdowns	23	56
Number not breaking down	287	320

Jim has a boiler. The cost of a service is £50. The average cost of a repair if the boiler breaks down is £145.

On average is it cheaper for Jim to have the service?

A

10 Around a coastline, 60% of towns have flood defences. If a town has flood defences then the probability that there is flooding is 0.01 in any year. If a town does not have flood defences then the probability of flooding is 0.02 in any year.

a Work out the probability of flooding in a town in any year.

The cost of dealing with a flood in a town is £5 million.
There are 20 towns along the coastline.

b Work out an estimate of the cost of dealing with floods over the next 10 years.

A*

11 If Jim's train gets in on time he can then catch a bus costing £2. If the train is late he must then catch a taxi costing £10.

The probability that the train will be late is 0.1.
Work out an estimate of how much Jim will have to pay on average.

12 Mattie could spend 20 min on homework or watch the TV instead. The probability that her teacher will ask for the homework is 0.7. If she finds that Mattie has not done her homework then she will give a three-quarters of an hour detention.

What should Mattie do?

A☆

13 If I get my central heating serviced then the probability that it will fail in the next year is 0.04. If I do not get it serviced the probability that it will fail in the next year is 0.1.

The cost of a service is £50. The likely cost of a repair if it fails is £230.

What are the financial implications?

14 Jim has £10 000 to invest. He considers investing in one or both of two investments:
Investment 1: The Cautious Investor fund: Percentage yield = 15%
Investment 2: The High Stakes investor fund: Percentage yield = 45%

Both investments involve risk.

For the Cautious Investor fund the risk of losing half the initial investment is 1%.
For the High Stakes investor fund the risk of losing half the initial investment is 28%.

Jim considers 3 different investment plans.
A Invest all the money in the Cautious Investor fund.
B Invest all the money in the High Stakes investor fund.
C Invest half in the Cautious Investor fund and half in the High Stakes investor fund.

Compare the 3 different investment plans.

Review

- Given that the cost of a breakdown is £C and the probability of a breakdown is p then an estimate of the risk cost of the breakdown is £pC.

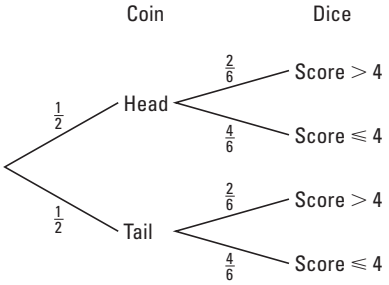
Answers

Chapter 13

A13.1 Get Ready answers

1 33.3

2



Probability $= \frac{1}{2}$:

Exercise 13A

1 **a** $\frac{121}{18500} = 0.006\,54$

b $16850 \times 0.006\,54 = 110$

2

Washing Machine	0.0155
Dishwasher	0.0184
Dryer	0.0157
Fridge	0.0131

In order with the least risky first:
Fridge, Washing machine, Dryer, Dishwasher

3 **a** 0.005 03

b $18\,250 \times 0.005\,03 \times £528 = £48\,469$,
$£48\,469 \div 18\,250 = £2.66$

4 $4000 \times 0.005 \times 800 = £16\,000$

5 **a** 16 : 00 – 17 : 00 Prob of being late $= \frac{30}{285} = 0.105$

17 : 00 – 18 : 00 Prob of being late $= \frac{55}{401} = 0.137$

18 : 00 – 19 : 00 Prob of being late $= \frac{40}{480} = 0.0833$

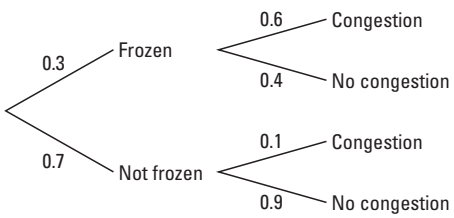
19 : 00 – 20 : 00 Prob of being late $= \frac{31}{303} = 0.102$.

In order of reliability the periods are:
18 : 00 – 19 : 00, 19 : 00 – 20 : 00,
16 : 00 – 17 : 00, 17 : 00 – 18 : 00

b An estimate of the number of late trains $=$
$310 \times 0.105 + 426 \times 0.137 + 505 \times 0.0833 +$
$328 \times 0.102 = 166$

An estimate of the cost $= 166 \times £3400 = £564\,400$

6 **a**



b $0.3 \times 0.6 + 0.7 \times 0.1 = 0.25$

c $0.25 \times 60 = 15$, $15 \times £1200 = £18\,000$

7 **a** $\frac{16}{7685} = 0.002\,082$

b $8100 \times 0.002\,082 \times £650 = £10\,962$

8 **a**

Make	Probability
Kandoo	0.0110
Black Diamond	0.0105
Illustrious	0.0122
Dekko	0.0163

b Kandoo $£399 \times 0.0110 = £4.39$,
Black diamond $£329 \times 0.0105 = £3.45$,
Illustrious $£259 \times 0.0122 = £3.16$
Dekko $= £199 \times 0.0163 = £3.24$

So Illustrious is the best value.

9 Prob of breaking down if serviced $= \frac{23}{310}$

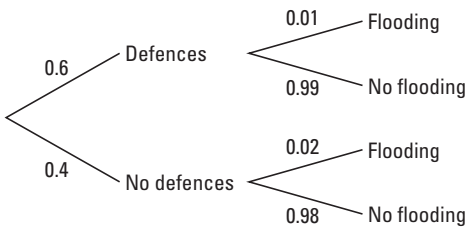
Estimated cost $= \frac{23}{310} \times 145 + 50 = £60.76$

Prob of breaking down if not serviced $= \frac{56}{376}$

Estimated cost $= \frac{56}{376} \times 145 = £21.60$

From a cost point of view he should not have the service.

10



a $0.6 \times 0.01 + 0.4 \times 0.02 = 0.014$

b $20 \times 0.014 \times 10 \times £5$ million $= £14$ million

11 $0.9 \times £2 + 0.1 \times £10 = £2.80$

12 An estimate for the number of minutes of detention is $0.7 \times 45 = 31.5$

So Mattie should spend 20 minutes on her homework.

13 Serviced $0.04 \times £230 + £50 = £59.20$
Not serviced $0.1 \times £230 = £23$
Better to not have it serviced.

14 A Value after one year: $£10\,000 \times 1.15 = £11\,500$.
Loss $= 0.01 \times £5000 = £50$
$£11\,500 - £50 = £11\,450$

B $£10\,000 \times 1.45 = £14\,500$.
Loss $= 0.28 \times £5000 = £1400$
$£14\,500 - £1400 = £13\,100$

C $£5000 \times 1.15 = £5750$. Loss $= 0.01 \times £2500 = £25$
$£5750 - £25 = £5725$
$£5000 \times 1.45 = £7250$.
Loss $= 0.28 \times £2500 = £700$
$£7250 - £700 = £6550$
Total $= £12\,275$
Plan B offers the possibility of a high yield. Even factoring in the possible loss it gives the highest yield.