A11.1 Areas under curves

Before you start

You should be able to:

- calculate the value of y given the value of x in algebraic equations of curves
- calculate the area of a trapezium.

Why do this?

Working out areas under curves is very important when studying the thermodynamics of engines.

Objectives

- You will be able to calculate an estimate for the area of the region bounded by a curve, the x-axis and two lines parallel to the y axis.
- You will be able to interpret the area under a velocity-time graph as a distance travelled.

Get Ready

1 In each case calculate the value of y when $x =$

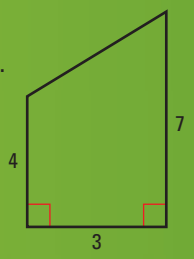
i 2 **ii** -3

a $y = x^2 - 2x + 3$

b $y = \frac{4}{x}$

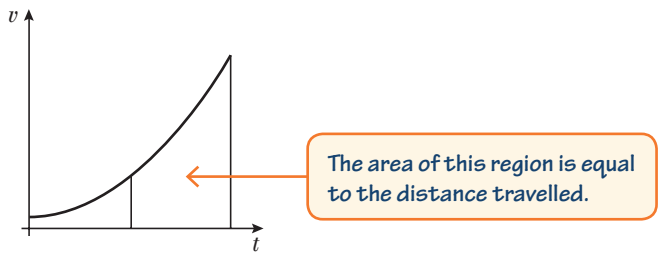
c $y = 6x - 2x^3$

2 Calculate the area of the trapezium.



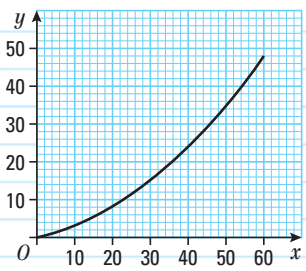
Key Points

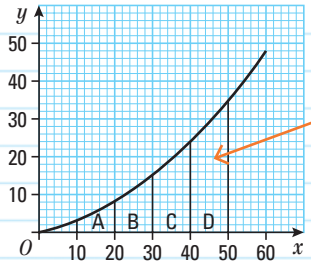
- An estimate of the area of the region between a curve and the x-axis can be calculated by splitting the region into trapeziums and finding the total area of the trapeziums.
- The area of the region between a velocity-time (v/t) curve and the time (t)-axis is equal to the distance travelled.



Example 1

Here is a curve. Calculate an estimate of the area bounded by the curve, the x-axis, and the lines $x = 10$ and $x = 50$.





Split the area of the region into trapeziums and work out the area of each trapezium.

Then add all these areas together.

A: $\frac{10}{2}(3 + 8) = 55$

B: $\frac{10}{2}(8 + 15) = 115$

C: $\frac{10}{2}(15 + 24) = 195$

D: $\frac{10}{2}(24 + 35) = 295$

Total area = 660 square units

A quicker way to calculate the sum of the areas of the trapezium is to factorise the sum of the expressions:

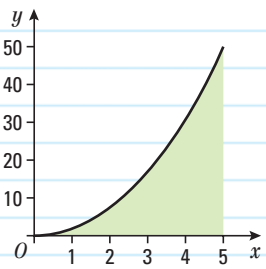
$\frac{10}{2}(3 + 8 + 8 + 15 + 15 + 24 + 24 + 35)$

$= \frac{10}{2}(3 + 2 \times 8 + 2 \times 15 + 2 \times 24 + 35)$

$= \frac{10}{2}[3 + 2 \times (8 + 15 + 24) + 35]$

= 660 square units

Example 2 Here is a sketch of the graph of $y = 2x^2$ for values of x from 0 to 5.



Calculate an estimate for the area of the region shown shaded in the diagram.

x	0	1	2	3	4	5
y	0	2	8	18	32	50

Split the region into 5 trapeziums each of width 1 unit.

An estimate for the area is 85 square units.

Work out the lengths of the parallel sides of each trapezium by using the equation of the curve. (The first 'trapezium' will be a triangle.)

Example 3 A particle moves in one direction in a straight line. Its velocity v m/s after t seconds is given by $v = t^2 + 2t$.
Calculate an estimate of the distance it has travelled during the first 4 seconds.

t	0	1	2	3	4
y	0	3	8	15	24

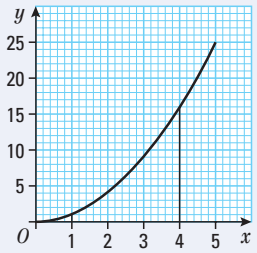
The distance travelled = area between the curve and the time axis, from $t = 0$ to $t = 4$.

Using the trapezium rule, an estimate for this area is 38 square units.
So the distance travelled is 38 metres.

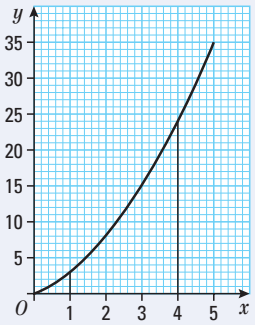
Exercise 11A

1 Find the area of the regions bounded by the curve, the x-axis and the lines shown parallel to the y-axis.

a



b



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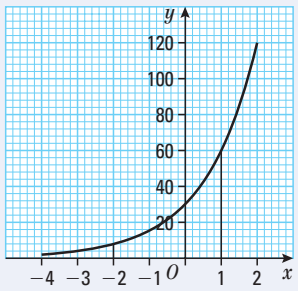

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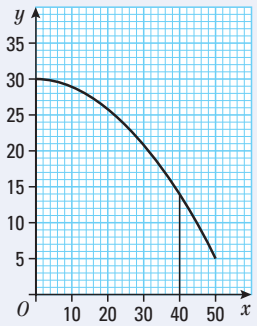
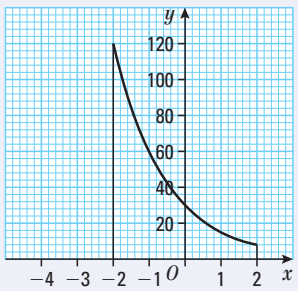
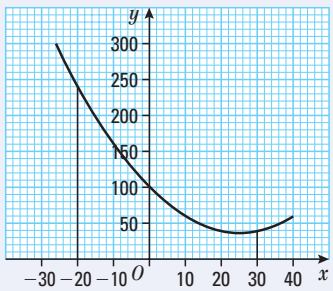


e



f



2 Draw the curve with equation $y = x^2$ for values of x from -3 to 3. Calculate an estimate for the area between the x-axis, the curve and the lines $x = 2$ and $x = -2$.

3 **a** Copy and complete the table of values for the curve with equation $y = 8x - x^2$.

x	0	1	2	3	4	5	6
y		7				15	

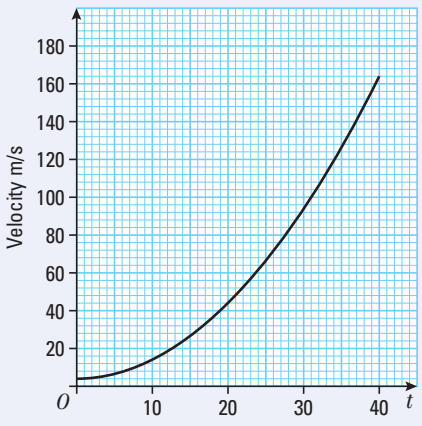
b Draw the graph.

c Calculate an estimate for the area between the curve, the x-axis and the lines $x = 1$ and $x = 6$.

4 The graph gives information about the velocity of a particle during the first 40 seconds of its motion.

a Calculate an estimate for the distance travelled during the first 40 seconds.

b State, with a reason, whether your answer to **a** is an overestimate or underestimate of the true distance travelled.

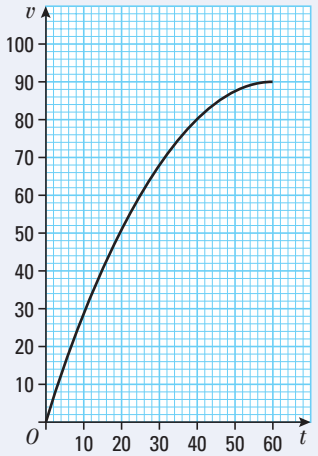


B

A

A

5 The graph shows the velocity, v m/s, with which a racing car has travelled during t seconds.



a Calculate an estimate of the distance the racing car travelled during these 60 seconds.

b Calculate an estimate of the average speed of the car for these 60 seconds.

6 The velocity, v m/s, of a particle at time t seconds is given by $v = t^2 + 3t$.

a Calculate an estimate of the distance it has travelled after 5 seconds.

b Calculate an estimate of the average speed of the particle during the first 5 seconds.

Review

- The approximate area under a graph can be found by approximating the region by a set of trapeziums and finding their total area.
- The area under a velocity-time graph is equal to the distance travelled.

Answers

Chapter 11

A11.1 Get Ready answers

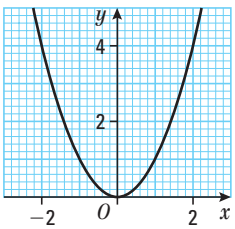
1 **a** **i** 3 **ii** 18
b **i** 2 **ii** −1.33
c **i** −4 **ii** 36

2 16.5

Exercise 11A

1 **a** 21 **b** 36 **c** 79
d 980 **e** 170 **f** 4900

2

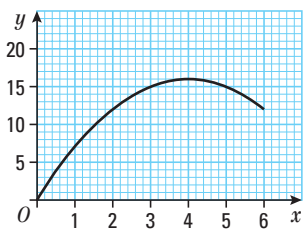


Area is approximately 6 square units

3

x	0	1	2	3	4	5	6
y	0	7	12	15	16	15	12

b

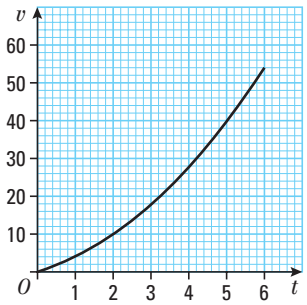


c 64 square units

4 **a** 2340 metres
b overestimate as the sloping edges of the trapeziums are always above the curve.

5 **a** 3580 metres **b** 59.7 m/s

6



a Distance travelled is approximately 80 metres.
b Average speed is approximately 16 m/s.