

M1.1 Sets and Venn diagrams

Before you start

You should be able to:

- identify numbers that have common properties.

Why do this?

It is useful to be able to classify objects by their characteristics. Scientists frequently classify animals and plants using their different characteristics.

Objectives

- You can use Venn diagrams to represent sets.
- You can interpret Venn diagrams.
- You can draw a Venn diagram using given information.

Get Ready

How would you describe these numbers?

- 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
- 5, 10, 15, 20
- 2, 3, 5, 7, 11, 13

Key Points

- A set is a collection of numbers or objects. For example, if W is the set of the first ten whole numbers then this can be written as:

$$W = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

The whole numbers from 1 to 10 are the members of set W .

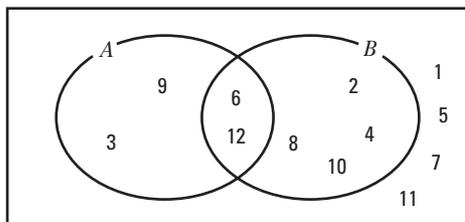
- A picture called a Venn diagram is used to represent sets and show the relationship between them.

For example, the following Venn diagram shows:

all the whole numbers from 1 to 12

the set A where $A = \{3, 6, 9, 12\}$

the set B where $B = \{2, 4, 6, 8, 10, 12\}$



All the members of set A are inside the circle labelled A .

All the members of set B are inside the circle labelled B .

The numbers that are in both set A and set B are in the intersection of the two sets.

The numbers 1, 5, 7, 11 are not in set A or set B so are outside the two circles.

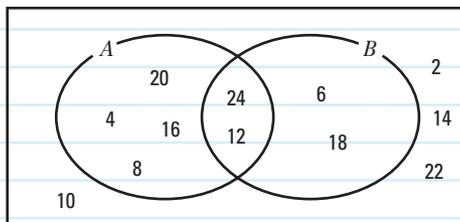
- Venn diagrams can also be used to show the number of members in a set.

Example 1

The Venn diagram shows the even numbers from 2 to 24.

Write down the numbers that are:

- in set A
- in set B
- in both set A and set B
- not in set A or set B .



a $A = \{4, 8, 12, 16, 20, 24\}$

Write down all the numbers inside the circle labelled A

b $B = \{6, 12, 18, 24\}$

c 12, 24

Write down the numbers that are in the intersection of the two circles

d 2, 10, 14, 22

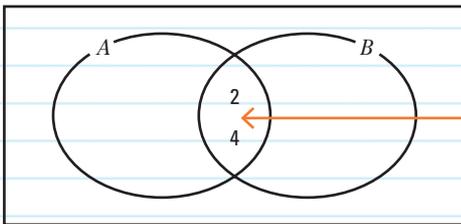
Write down all the numbers that are outside both circles

Example 2 On a Venn diagram show:

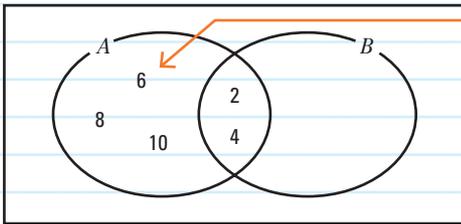
the whole numbers from 1 to 10

set A where $A = \{2, 4, 6, 8, 10\}$

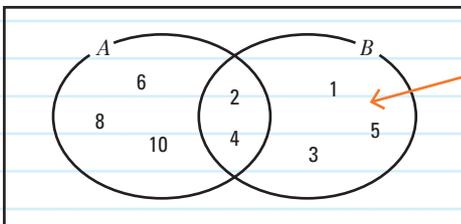
set B where $B = \{1, 2, 3, 4, 5\}$



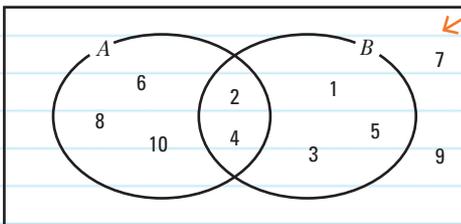
2 and 4 are in both set A and set B. Place these numbers in the intersection



The numbers remaining from set A are 6, 8 and 10, place these in the part of circle A that does not intersect with B



The numbers remaining from set B are 1, 3 and 5, place these in the part of circle B that does not intersect with A



The whole numbers remaining from 1 to 10 are 7 and 9, place these outside the two circles.



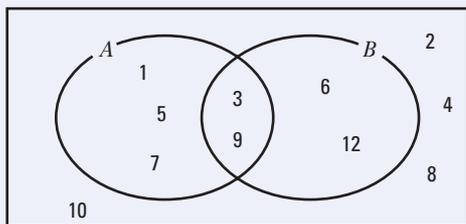
ResultsPlus
Examiner's Tip

When you have finished, check that all the numbers from 1 to 10 are somewhere in your Venn diagram



Exercise 1A

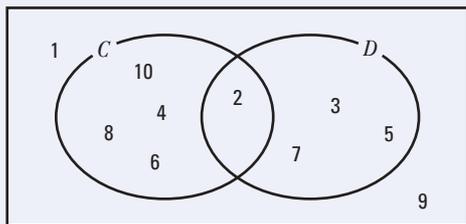
1



Write down the numbers that are in:

- a** set *A* **b** set *B* **c** both set *A* and set *B* **d** not in set *A*.

2



Write down the numbers that are in:

- a** set *D* **b** both set *C* and set *D* **c** not in set *C*.

3

On a Venn diagram show:
 the whole numbers from 1 to 10
 set *A* where $A = \{1, 2, 5, 10\}$
 set *B* where $B = \{2, 4, 6, 8, 10\}$

4

On a Venn diagram show:
 the whole numbers from 15 to 21
 set *P* where $P = \{15, 18, 21\}$
 set *Q* where $Q = \{16, 18, 20\}$

5

Here are some letters.
 C F G H I N S T X

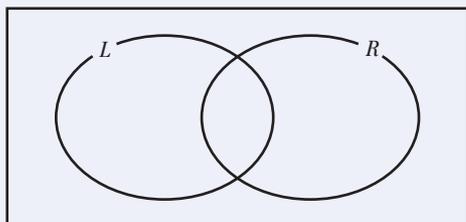
Some of the letters have line symmetry.

Some of the letters have rotational symmetry of order 2.

$$L = \{\text{letters with line symmetry}\}$$

$$R = \{\text{letters with rotational symmetry of order 2}\}$$

Copy and complete the Venn diagram.



A01 F

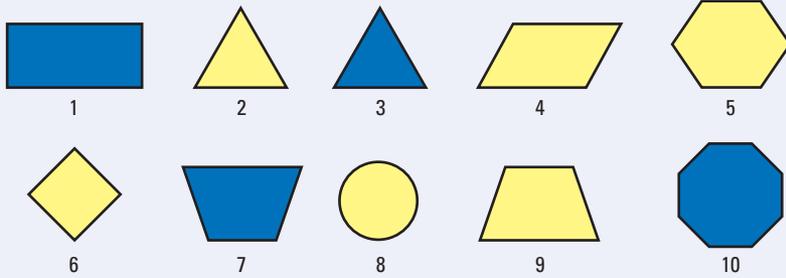
A01

A03 E

A03

A03 D

6 Here are some coloured shapes.



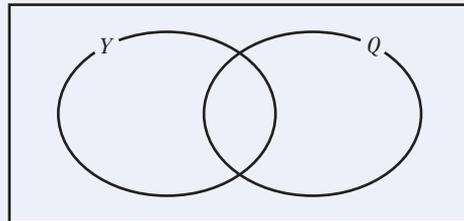
Some of the shapes are quadrilaterals.

Some of these shapes are yellow.

$Y = \{\text{yellow shapes}\}$

$Q = \{\text{quadrilaterals}\}$

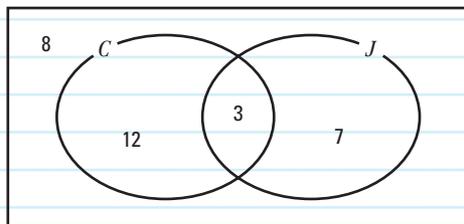
Copy and complete the Venn diagram.



Example 3

Some adults were asked whether they like classical music or jazz.

The Venn diagram shows information about their answers.



- a How many adults were asked whether they like classical music or jazz?
- b How many adults like jazz?
- c How many adults like both jazz and classical music?
- d How many adults like neither jazz nor classical music?

a $8 + 12 + 3 + 7 = 30$

← Add up all the numbers in the Venn diagram.

b $7 + 3 = 10$

← Add together the two numbers inside circle J .

c 3

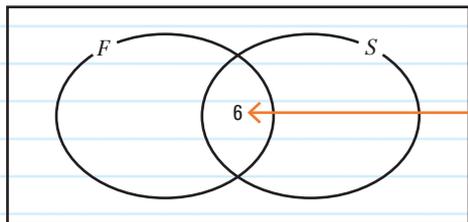
← Write down the number in the intersection of the circles.

d 8

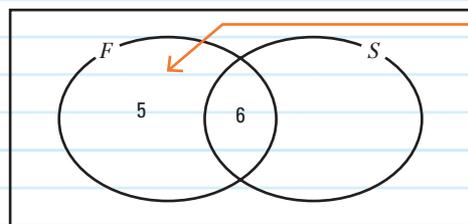
← Write down the number outside the two circles.

Example 4

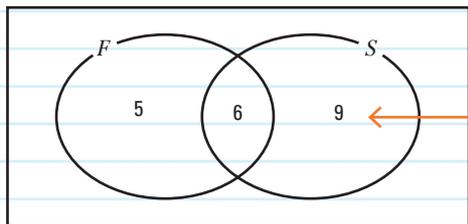
There are 27 students in a class.
 11 of the students study French.
 15 of the students study Spanish.
 6 of the students study both French and Spanish.
 Draw a Venn diagram to show this information.



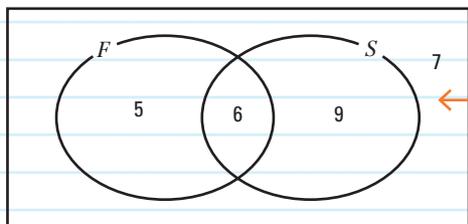
Start with the intersection.
 Place a 6 in the intersection to represent the 6 students that study both French and Spanish.



11 of the students study French.
 $11 - 6 = 5$



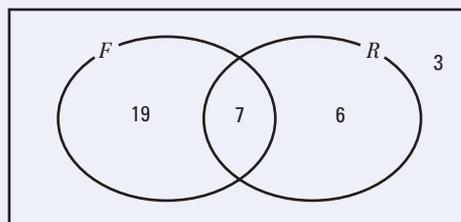
15 of the students study Spanish.
 $15 - 6 = 9$



$5 + 6 + 9 = 20$
 $27 - 20 = 7$
 So there are 7 students who do not study French or Spanish.
 Place the 7 outside both circles.

Exercise 1B

- 1 Some boys were asked if they played football or rugby. The Venn diagram shows information.
 - a How many boys were asked if they played football or rugby?
 - b How many boys played just rugby?
 - c How many boys do not play football?
 - d How many boys play both rugby and football?



D
A03

- 2 In a class of 34 students
19 drink tea,
4 drink coffee,
3 drink both coffee and tea.
- a Draw a Venn diagram to show this information.
b How many students do not drink coffee or tea?

A03

- 3 23 people work in a small factory.
12 are female,
10 wear glasses,
3 are female and wear glasses.
- a Draw a Venn diagram to show this information.
b How many men wear glasses?

A03

- 4 There are 24 flowers in a bunch.
15 of the flowers are tulips,
8 of the flowers are pink,
5 of the flowers are pink tulips.
Draw a Venn diagram to show this information.

C
A03

- 5 In a class of 31 students
15 of the students study History,
12 of the students study Geography,
7 study both History and Geography.
How many students study neither History nor Geography?

M1.2 Set language and notation

Before you start

You should be able to:

- find factors and multiples
- identify prime numbers.

Objectives

- You can use a Venn diagram to solve a problem.
- You can understand and be able to find the intersection and union of sets.

Why do this?

In mathematics we use symbols to represent different operations. This is also true when working with sets.

Get Ready

$$A = \{1, 2, 3, 4, 5, 6\}$$

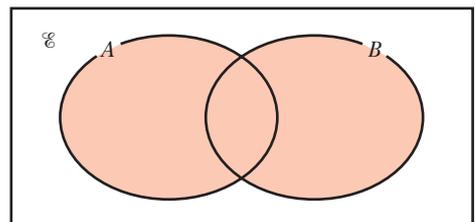
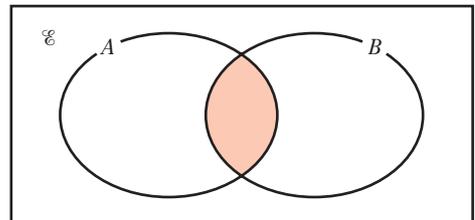
$$B = \{2, 4, 6, 8, 10\}$$

$$C = \{1, 3, 5, 7, 9\}$$

- 1 Write down the numbers in both A and B .
- 2 Write down the numbers in both A and C .
- 3 Write down the numbers in both B and C .

Key Points

- The **universal set** is the set of elements from which members of all other sets are selected. The symbol \mathcal{U} is used to represent the **universal set**.
- A' is called the **complement** of set A . A' contains all the members of \mathcal{U} that are not in set A .
- The symbol \emptyset is used to represent the **empty set**. $\emptyset = \{\}$
- The symbol \cap is used to represent the **intersection** of two sets. $A \cap B$ is the set of members of \mathcal{U} that are in both set A and set B .
- The symbol \cup is used to represent the **union** of two sets. $A \cup B$ is the set of members of \mathcal{U} that are in set A or in set B or in both sets.
- Venn diagrams can be used to solve problems.

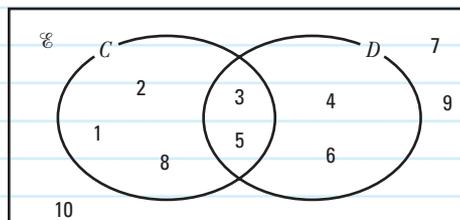


Example 1

The Venn diagram shows sets \mathcal{U} , C and D .

Write down the members of:

- a the universal set, \mathcal{U}
- b set D'
- c $C \cap D$
- d $C \cup D$



a $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

List all the numbers in the Venn diagram.

b $D' = \{1, 2, 7, 8, 9, 10\}$

List all the numbers that are not in circle D .

c $C \cap D = \{3, 5\}$

List all the numbers that are in the intersection of C and D .

d $C \cup D = \{1, 2, 3, 4, 5, 6, 8\}$

List all the numbers that are in circle C or in circle D or in both circles.

Example 6

$A = \{2, 2, 3, 5\}$

$B = \{2, 2, 2, 3, 3\}$

The numbers in set A are the prime factors of 60.

The numbers in set B are the prime factors of 72.

a Draw a Venn diagram to show set A and set B .

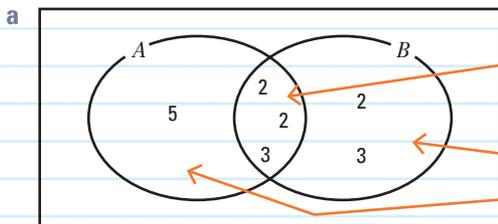
b Use your Venn diagram to find:

i the HCF of 60 and 72

ii the LCM of 60 and 72.



You learnt how to find the prime factors of a number earlier in the course. [Mathematics A Linear Foundation chapter 1.10]



The two sets have the numbers 2, 2, 3 in common, put these numbers in the intersection.

Next put the remaining numbers from set A and set B on the diagram.

b i $A \cap B = \{2, 2, 3\}$

To find the HCF, find the numbers in the intersection of A and B .

$2 \times 2 \times 3 = 12$

Multiply these numbers together to find the HCF.

The HCF of 60 and 72 is 12.

ii $A \cup B = \{2, 2, 2, 3, 3, 5\}$

To find the LCM, find the numbers that make up the union of A and B .

$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Multiply these numbers together to find the LCM.

The LCM of 60 and 72 is 360.

Example 7

a Use a Venn diagram to show:

$\mathcal{E} = \{\text{integers from 10 to 25}\}$

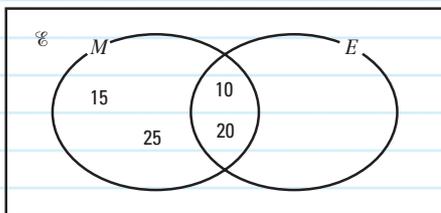
$M = \{\text{multiples of 5}\}$

$E = \{\text{even numbers}\}$

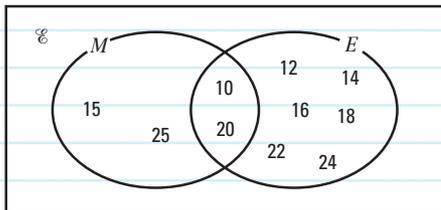
b Write down the members of:

i $M \cap E'$ ii $(M \cap E)'$

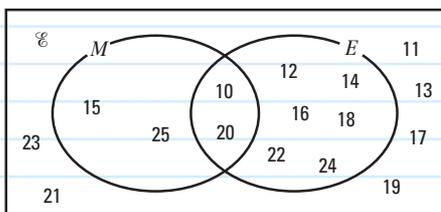
a



Place all the multiples of 5 within circle M .
10 and 20 are even numbers as well as multiples of 5 so these go in the intersection.



Place all the other even numbers within circle E .



Place all the other members of \mathcal{E} that are not in set M or in set E outside the circles.

b i $M \cap E' = \{15, 25\}$

← List the numbers that are both in circle M but not circle E .

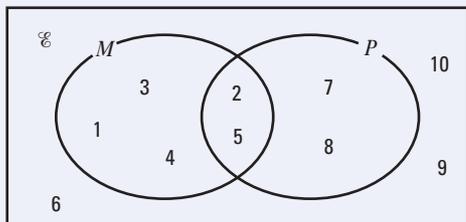
ii $(M \cap E)' = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25\}$

← List the numbers that are not in $M \cap E$.



Exercise 1C

1



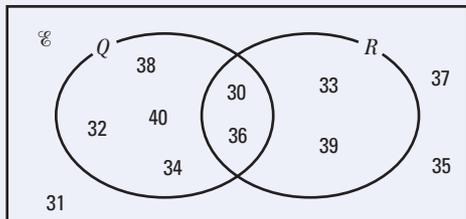
Write down the members of sets:

a P

b $P \cap M$

c $P \cup M$

2



Write down the members of:

a the universal set, \mathcal{E}

b set Q

c set R'

d $Q \cap R$

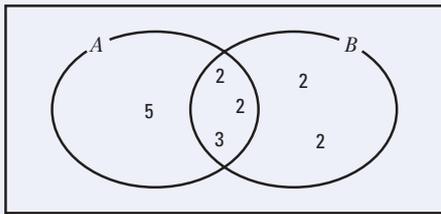
e $Q \cup R$

A01 C

A01

C
A03

- 3 The numbers in set A are the prime factors of 60.
The numbers in set B are the prime factors of 48.



Use the Venn diagram to find:

- a the HCF of 48 and 60
- b the LCM of 48 and 60.

A03

- 4 $A = \{\text{prime factors of } 75\}$
 $B = \{\text{prime factors of } 90\}$
- a Write 75 as the product of its prime factors.
 - b Write 90 as the product of its prime factors.
 - c Show set A and set B on a Venn diagram.
 - d Use your Venn diagram to find:
 - i the HCF of 75 and 90
 - ii the LCM of 75 and 90.

B
A03

- 5 a Draw a Venn diagram to show:
 $\mathcal{E} = \{\text{integers from } 12 \text{ to } 24\}$
 $F = \{\text{multiples of } 4\}$
 $G = \{\text{multiples of } 3\}$
- b Write down the members of:
- i $F \cap G$
 - ii G'
 - iii $(F \cup G)'$

A03

- 6 a Draw a Venn diagram to show:
 $\mathcal{E} = \{\text{integers from } 20 \text{ to } 29\}$
 $P = \{\text{prime numbers}\}$
 $M = \{\text{multiples of } 4\}$
- b Write down the members of:
- i $P \cap M$
 - ii $P' \cap M$

A03

- 7 $\mathcal{E} = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
 $A = \{5, 6, 8, 9, 12, 14, 15\}$
 $B = \{8, 9, 10, 11, 12, 13, 14\}$

Write down the members of:

- a A'
- b $A \cap B$
- c $A \cup B$
- d $A' \cap B$

A03

- 8 $\mathcal{E} = \{\text{integers less than } 20\}$
 $A = \{\text{multiples of } 5\}$
 $B = \{\text{multiples of } 3\}$
- a Write down the members of:
- i B'
 - ii $A \cup B$
 - iii $A' \cap B$
- b Describe the members of $A \cap B$.



Example 8

There are 25 houses in a street.

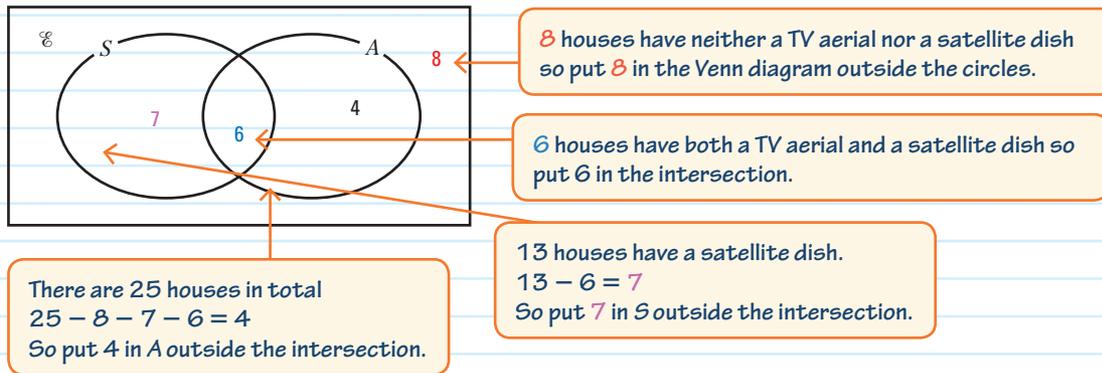
8 of these houses have neither a TV aerial nor a satellite dish.

13 houses have a satellite dish.

6 houses have both a satellite dish and a TV aerial.

How many houses have an aerial?

One way to solve this problem is to put all the information into a Venn diagram.



$$4 + 6 = 10$$

There are 10 houses that have a TV aerial.



Exercise 1D

- 1 35 adults were asked which of two newspapers they read.
 13 read the Daily Express,
 28 read the Daily Telegraph,
 4 read neither paper.
 a Show this information on a Venn diagram.
 b How many adults read both newspapers?
- 2 There are 60 people at an activity centre.
 31 go sailing,
 24 go sailing and go on the climbing wall,
 12 do neither activity.
 How many just go on the climbing wall but do not go sailing?
- 3 In a class of 31 students 18 study History, 8 study French and 5 students in the class study both History and French.
 How many students study neither subject?
- 4 In a class 24 students play hockey, 13 play netball and 8 play both hockey and netball. How many students are there in the class if each student plays at least one of hockey or netball?
- 5 There are 54 customers in an Italian restaurant. There are 9 people who have finished eating. The others are eating pasta or salad or both. There are 34 eating pasta and 15 eating salad.
 How many people are eating just pasta?

A03 B

A03

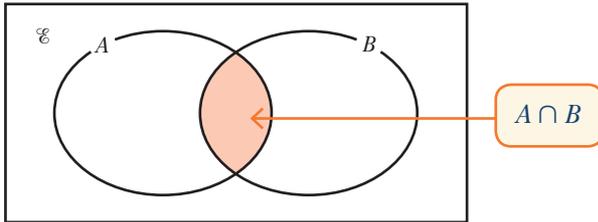
A03 A

A03 B

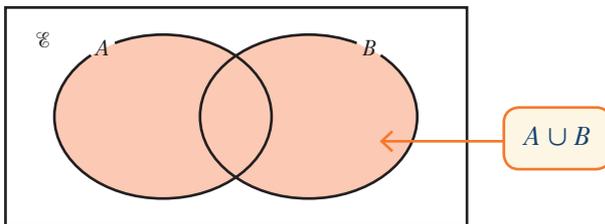
A03

Review

- A set is a collection of numbers or objects.
- A picture called a Venn diagram is used to represent sets and show the relationship between them.
- Venn diagrams can also be used to show the number of members in a set.
- The **universal set** is the set of elements from which members of all other sets are selected. The symbol \mathcal{U} is used to represent the **universal set**.
- A' is called the **complement** of set A . A' contains all the members of \mathcal{U} that are not in set A .
- The symbol \emptyset is used to represent the **empty set**. $\emptyset = \{\}$
- The symbol \cap is used to represent the **intersection** of two sets. $A \cap B$ is the set of members of \mathcal{U} that are in both set A and set B .



- The symbol \cup is used to represent the **union** of two sets. $A \cup B$ is the set of members of \mathcal{U} that are in set A or in set B or in both sets.



Answers

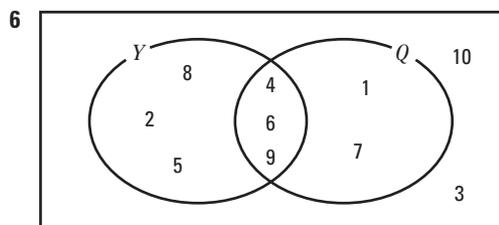
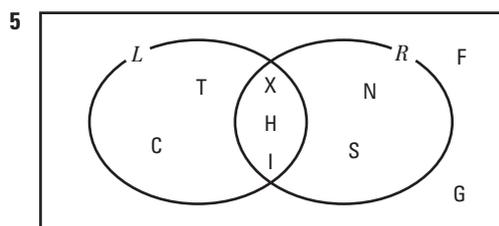
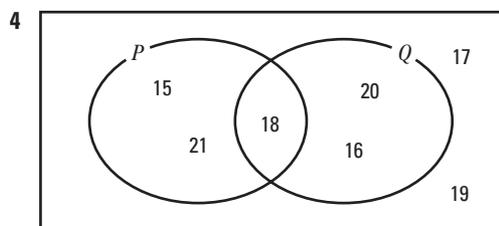
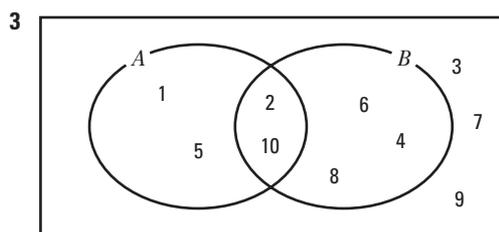
Chapter 1

M1.1 Get Ready answers

- 1 even numbers
- 2 multiples of 5
- 3 prime numbers

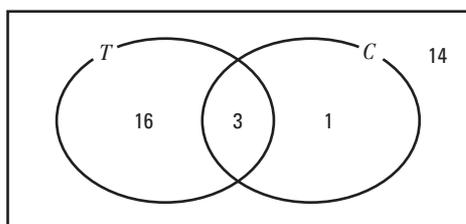
Exercise 1A

- 1 a $A = \{1, 3, 5, 7, 9\}$ b $B = \{3, 6, 9, 12\}$
 c $\{3, 9\}$ d $\{2, 4, 6, 8, 10, 12\}$
- 2 a $D = \{2, 3, 5, 7\}$ b $\{2\}$
 c $\{1, 3, 5, 7, 9\}$

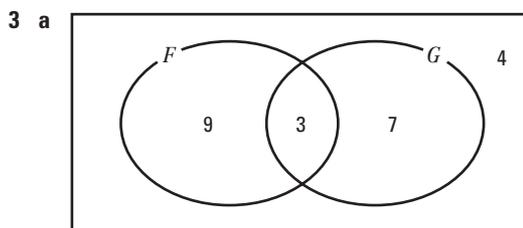


Exercise 1B

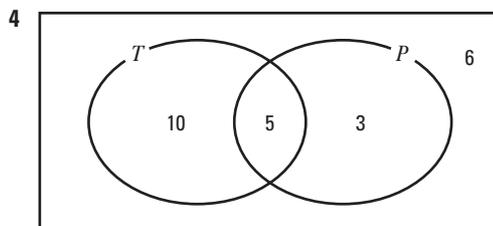
- 1 a 35 b 6 c 9 d 7
- 2 a



b 14



b 7



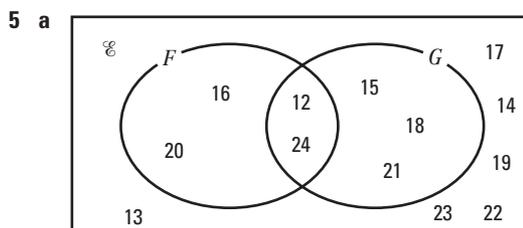
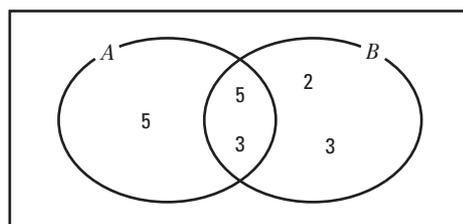
5 11

M1.2 Get Ready answers

- 1 2, 4, 6
- 2 1, 3, 5
- 3 none

Exercise 1C

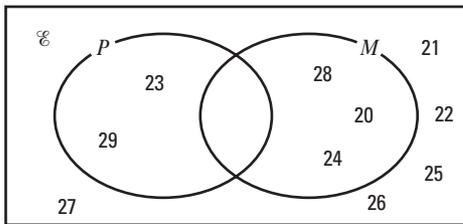
- 1 a $\{2, 5, 7, 8\}$ b $\{2, 5\}$
 c $\{1, 2, 3, 4, 5, 7, 8\}$
- 2 a $\{30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\}$
 b $\{30, 32, 34, 36, 38, 40\}$ c $\{31, 32, 34, 35, 37, 38, 40\}$
 d $\{30, 36\}$
 e $\{30, 32, 33, 34, 36, 38, 39, 40\}$
- 3 a 12 b 240
- 4 a $75 = 3 \times 5 \times 5$ b $90 = 2 \times 3 \times 3 \times 5$
 c Venn diagram d i 15 ii 450



- b i $\{12, 24\}$
 ii $\{13, 14, 16, 17, 19, 20, 22, 23\}$
 iii $\{13, 14, 17, 19, 22, 23\}$

Chapter 1 Venn diagrams

6 a



b i \emptyset

ii {20, 24, 28}

7 a {7, 10, 11, 13}

b {8, 9, 12, 14}

c {5, 6, 8, 9, 10, 11, 12, 13, 14, 15}

d {10, 11, 13}

8 a i {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19}

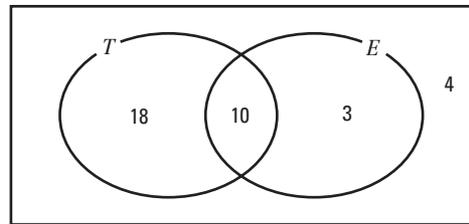
ii {3, 5, 6, 9, 10, 12, 15, 18}

iii {3, 6, 9, 12, 18}

b common multiples of 3 and 5

Exercise 1D

1 a



b 10

2 17

3 10

4 29

5 30