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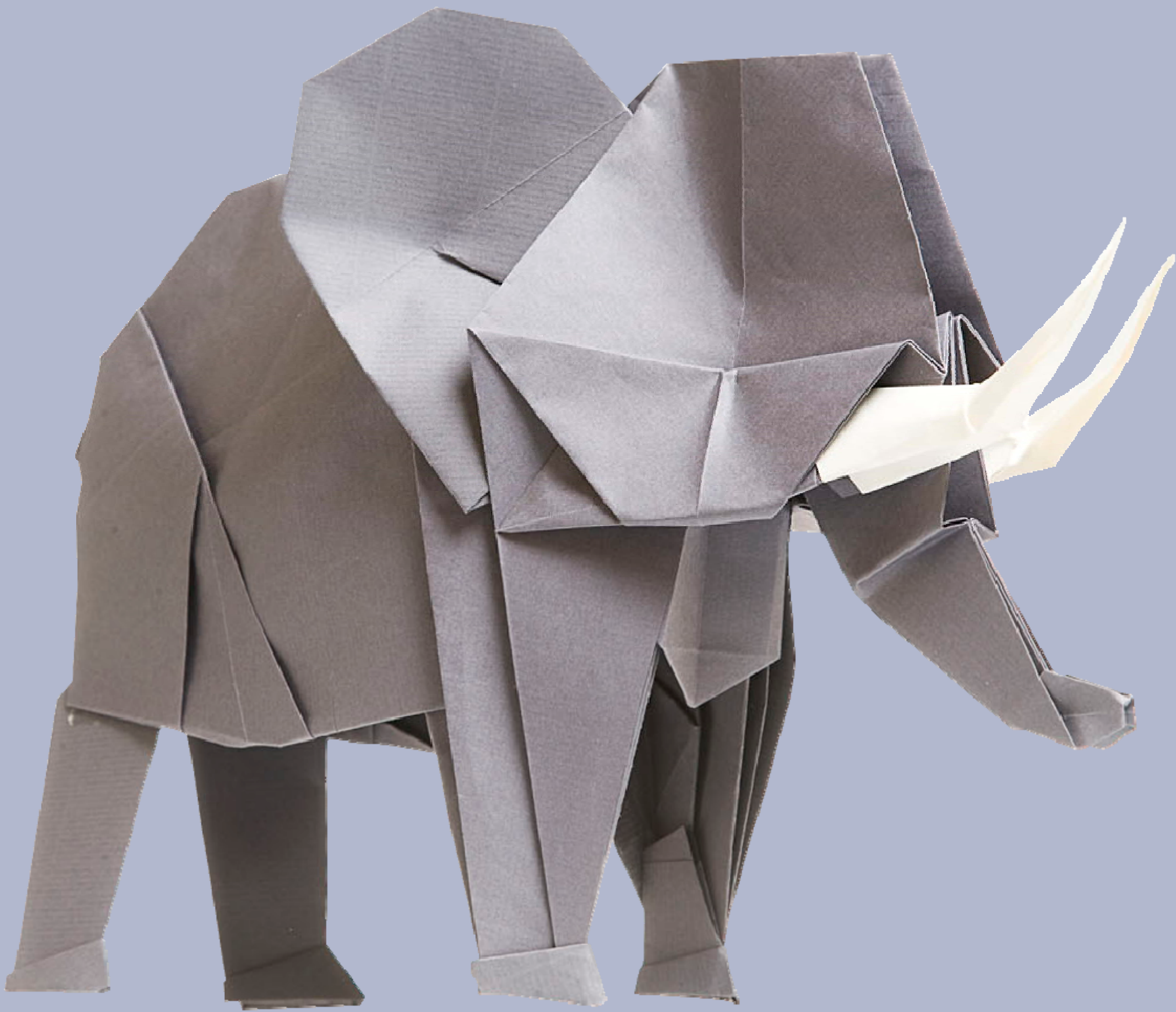
Guide to Maths for Psychologists



GCSE (9-1) Psychology

Pearson Edexcel Level 1/Level 2 GCSE (9-1) in Psychology (1PS0)

Pearson

Guide to Maths for Psychologists

Contents

Introduction

This Guide to Maths for Psychologists outlines the content that students will have covered in their maths lessons throughout Key Stage 3 and Key Stage 4. You can use this guide to help you understand how different areas are approached in maths, and therefore to support your teaching of mathematical content in psychology lessons.

The content is split into distinct mathematical concepts. Each chapter takes you through the terminology used in that area, as well as examples taken from Pearson maths textbooks to show you the methods students should be familiar with when solving mathematical problems.

1. Statistical graphs, charts and tables

1.1 Data

Requirement

All students learn the difference between discrete and continuous data in KS3. They also come across categorical data.

Terminology

- Data is either qualitative (descriptive) or quantitative (numerical).
- Data can also be discrete or continuous: discrete data can only take certain values, e.g. whole numbers or shoe sizes; continuous data is measured, e.g. mass, length or time, and can take any value.

1.2 Bar charts

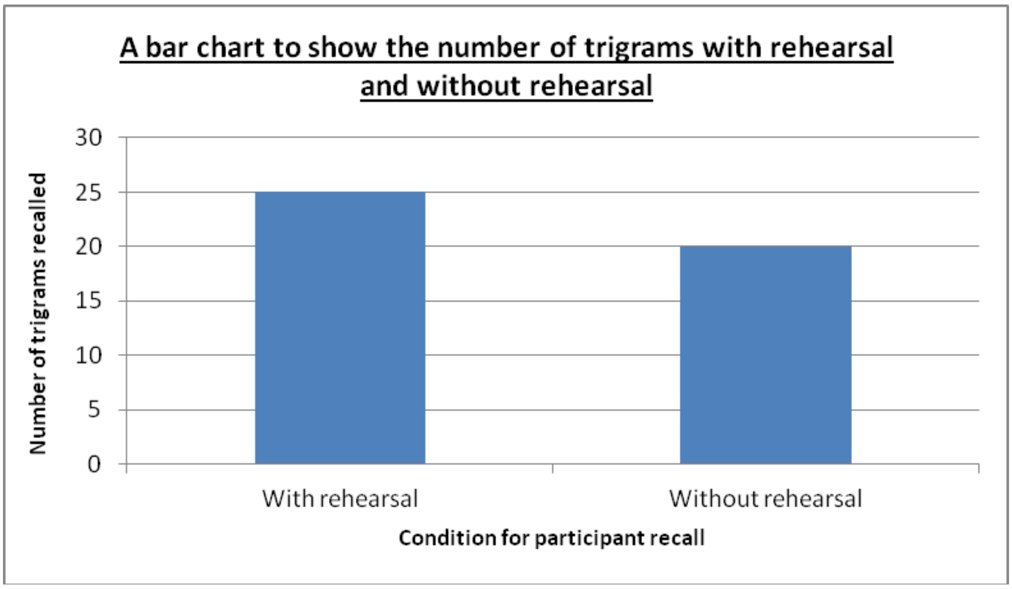
Requirement

All students learn to draw and interpret bar charts for discrete and continuous data in KS3.

Approach

- Both axes should be labelled appropriately, and there should be a title.
- The frequency of a variable should be shown on the vertical axis.
- Bars should be of equal width.
- There must be gaps between the bars.
- The vertical axis should begin at 0.
- The vertical axis should go to the maximum possible for the variable given in the question, where the values are provided for candidates.
- Questions could ask candidates to construct or interpret a bar chart.

Bar chart example



1.3 Frequency tables

Requirement

All students learn to draw and interpret frequency tables for discrete and continuous data in KS3.

Approach

- Frequency tables contain data that shows the number of items, or frequency of each data value or each data group.
- Data can also be grouped. For discrete data, use groups such as 0–5, 6–10, etc. For continuous data, use groups such as $0 \le t < 10$, $10 \le t < 20$. The groups must not overlap.
- In maths, students learn that it is best to group numerical data into a maximum of 6 groups.
- Questions could ask candidates to construct or interpret a frequency table.

Frequency table example with ungrouped discrete data

Age	Frequency
3	3
4	5
5	7
6	10
7	10
8	6
9	1

Frequency table example with grouped discrete data

Psychology mark	0–10	11–20	21–30	31–40	41–50
Frequency	4	13	17	19	7

Frequency table with grouped continuous data.

Distance (d metres)	Frequency
$10 \le d < 20$	2
$20 \le d < 30$	6
$30 \le d < 40$	15
$40 \le d < 50$	20
$50 \le d < 60$	4

1.4 Frequency diagrams

Requirement

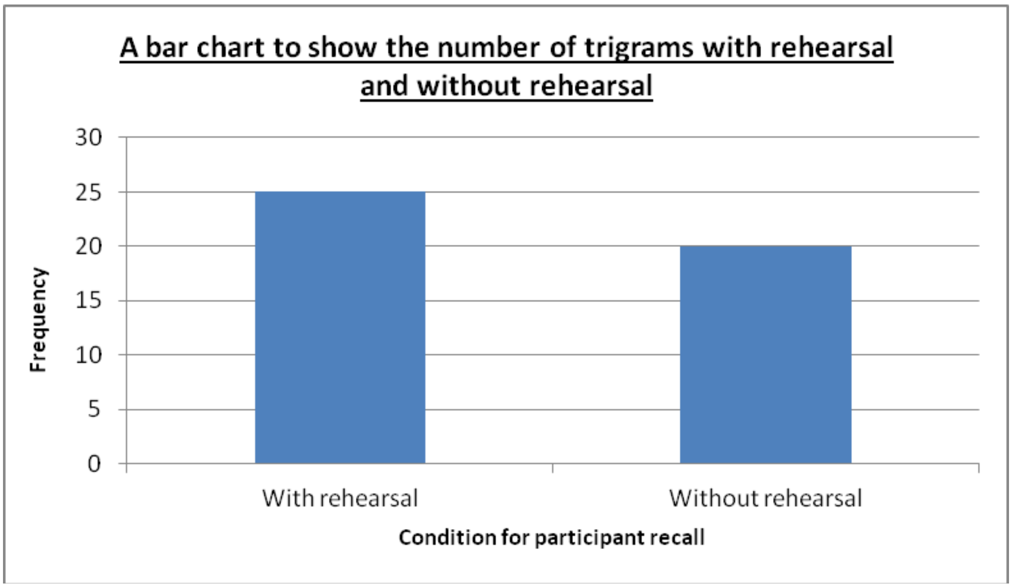
All students learn to draw and interpret frequency diagrams for discrete and continuous data in KS3.

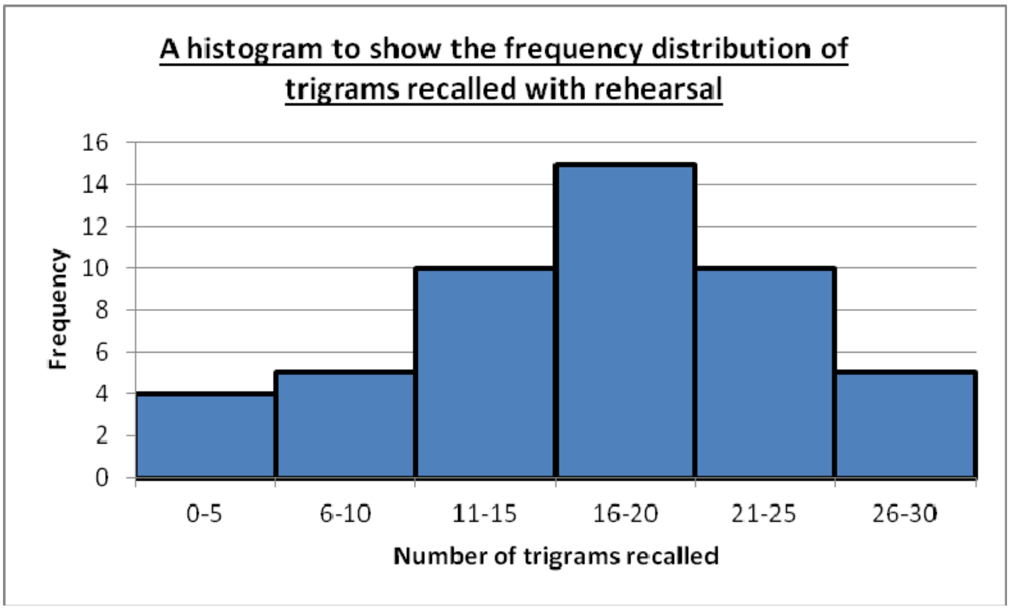
Inconsistency: 'Frequency diagram' is another name for a bar chart where the vertical axis is labelled 'Frequency'. In KS3 maths, the name 'Frequency diagram' is not used.

Approach

- Can be used to show discrete or continuous data.
- Questions could ask candidates to construct or interpret a frequency diagram.

The following bar chart and histogram could also be called frequency diagrams:





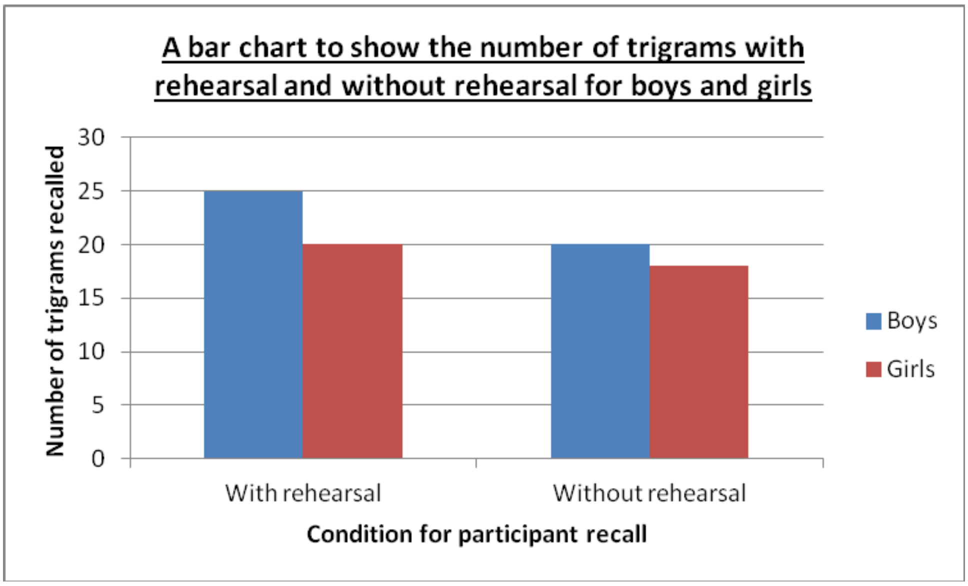
1.5 Comparative bar charts

Requirement

All students learn to draw and interpret comparative bar charts in KS3.

Approach

- Compares two or more sets of data.
- Uses different coloured bars for each set of data.
- Needs a key to show what each colour bar represents.
- Questions could ask candidates to construct or interpret a comparative bar chart.



1.6 Histograms

Requirement

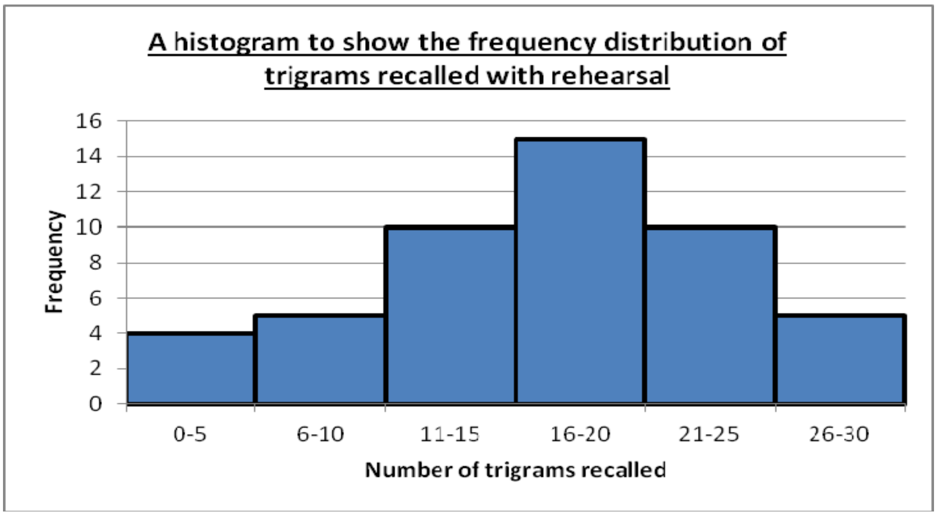
In maths, students do not meet histograms until KS4, although the bar charts they draw in KS3 for grouped continuous data could also be called histograms.

Histograms with unequal width bars/groups, where frequency density is plotted on the vertical axis, are covered only in Higher GCSE Maths, not Foundation. These will not be assessed at GCSE for Psychology.

Approach

- Can be drawn for grouped continuous data where groups/bars are of equal width.
- No gaps between the bars.
- Questions could ask candidates to construct or interpret a histogram.

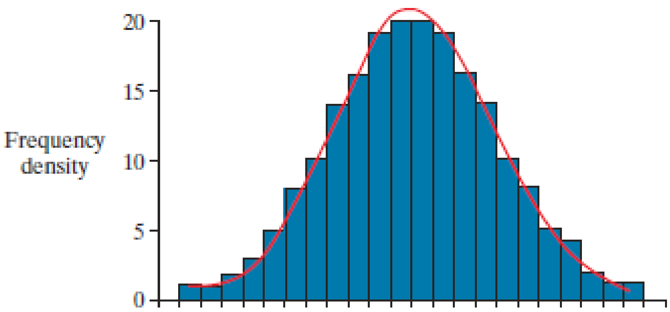
Histogram example with equal width bars/groups



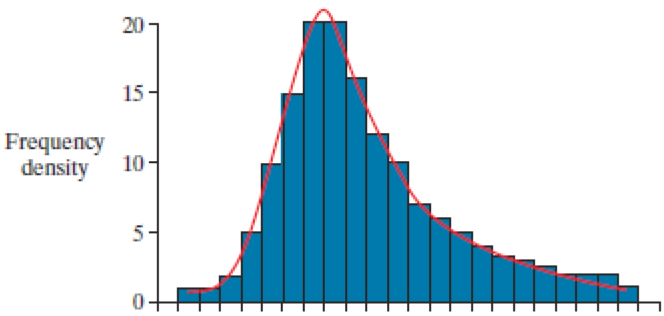
1.7 The shape of a distribution in a histogram

Terminology

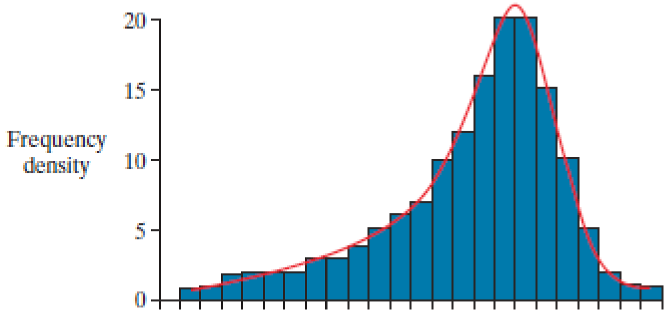
- The shape formed by the bars in a histogram is known as the distribution of the data.
- A histogram shows how the data are distributed across the intervals.
- A distribution can be symmetrical ('normal distribution'), or have a positive or negative skew.



This distribution shown above is symmetrical – this is commonly known as '**normal distribution**'. It is also sometimes called a 'bell-shaped' frequency curve.



This distribution has a **positive skew**. More scores are distributed to the left (or lower end) than to the right (or upper end). The mean and median would be greater than the mode.



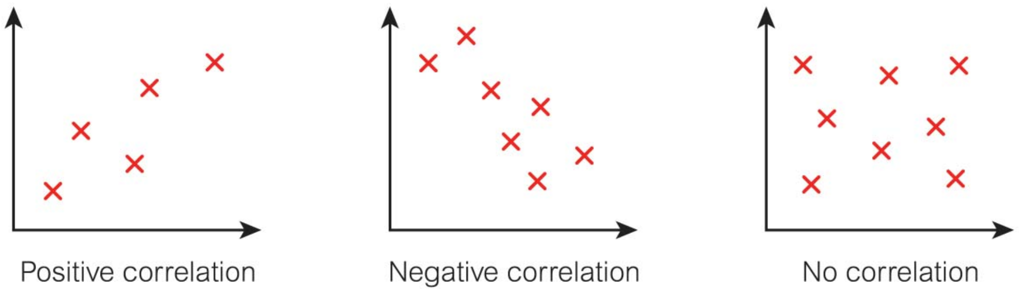
This distribution has a **negative skew.** More scores are distributed to the right (or upper end) than to the left (or lower end). The mean and median would be less than the mode.

2. Graphs including relationships and correlations

2.1 Scatter diagrams

Approach

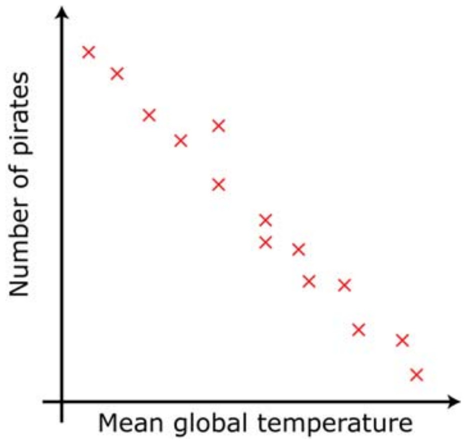
- Can be drawn for two sets of data on the same graph to see if there is a relationship or correlation between them.
- Points on a scatter diagram are usually plotted with crosses.
- Questions could ask candidates to construct or interpret a scatter diagram.
- Scatter diagrams can show a positive or negative correlation, or no correlation.



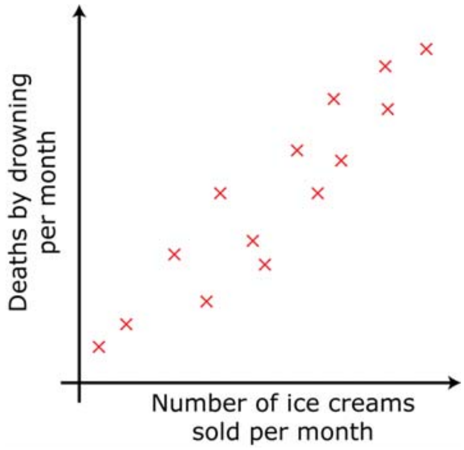
- A correlation is when two sets of data are linked. For example, when one value increases as the other increases, or when one value decreases as the other increases.
- A correlation does not imply causation. Sometimes there may be another factor that affects both variables, or there may be no connection between them at all.

Inconsistency: In maths these are usually called scatter graphs, not scatter diagrams.

Examples of scatter diagrams



In the above, there is a negative correlation between the number of pirates and mean global temperature, but it is unlikely that one causes the other.



In the above, there is positive correlation between the number of ice-creams sold and death by drowning, but it is unlikely that one causes the other. A more likely explanation is a third factor – temperature. On hot days more people buy ice creams and more people swim, leading to increased numbers of drownings.

2.2 Interpreting scatter diagrams

Requirement

All students should learn to interpret scatter diagrams in KS3.

They will have learned about correlation and causation in KS3 maths.

Common error: Students often find interpreting scatter diagrams difficult as they do not know how to put into words what the diagram shows, so it is good to give them examples of this, or at least sentences to copy and complete, such as:

The ____________ the age, the _________ the cognitive ability.

Answer: The *higher* the age, the *lower* the cognitive ability.

You can also use statements such as:

As the ____________ increases, the ______________ increases.

Answer: As the *temperature* increases, the *aggression* increases.

3. Fractions, percentages and ratios

3.1 Fractions

Requirement

All students learn how to add, subtract, multiply and divide decimals and find a fraction of a quantity in KS3.

They also learn how to convert fractions to decimals and vice versa, and use and interpret recurring decimal notation.

Approach

Convert a fraction to a decimal

Divide the top number by the bottom number.

For example $\frac{3}{8} = 0.375$, $\frac{12}{50} = 0.24$

Convert a decimal to a fraction

Worked example

Write 0.32 as a fraction in its simplest form.

$$0.32 = \frac{32}{100} \qquad \frac{32}{100} \underset{\div 4}{\overset{\div 4}{=}} \frac{8}{25}$$

…	H	T	U	•	$\frac{1}{10}$	$\frac{1}{100}$	…
			0	•	3	2	

0.32 is the same as $\frac{32}{100}$

Calculate a fraction of a quantity

Example 3

Work out $\frac{3}{5}$ of 40.

In mathematics, 'of' means multiply.

$\frac{1}{5}$ of $40 = \frac{1}{5} \times 40 = 40 \div 5 = 8$

$\times 3$ $\times 3$

$\frac{3}{5}$ of $40 = 3 \times 8 = 24$

Multiply by 3 to find $\frac{3}{5}$

Terminology

- Try to write fractions on two lines, i.e. $\frac{1}{1000}$ not 1/1000.
- In a fraction, the horizontal line means 'divided by'. So $\frac{3}{5}$ means 3 ÷ 5. Understanding this helps students remember how to convert fractions to decimals.

3.2 Percentages

Requirement

All students at KS3 will learn how to define percentage as 'number of parts per hundred', interpret percentages and percentage change as a fraction or decimal, interpret percentages multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%.

All students at KS3 will solve problems involving percentage change, including percentage increase and decrease, original value problems, and simple interest in financial mathematics.

Percentage change

Foundation level students learn this in unit 14 of the GCSE, i.e. Autumn term of year 11.

Terminology

- Percent means 'out of 100'. A percentage is a fraction with denominator 100.
- You can calculate percentages of amounts, e.g. 20% of 500 g.
- You can write one number as a percentage of another, e.g. write $\frac{7}{50}$ as a percentage.

Approach

Converting a percentage to a fraction

Worked example

Write 20% as a fraction.

$$20\% = \frac{20}{100} \overset{\div 10}{=} \frac{2}{10} \overset{\div 2}{=} \frac{1}{5}$$

First write 20% as a fraction of 100.

Then simplify the fraction by dividing the numerator and denominator by the same number. Keep doing this until the fraction is in its simplest form.

Converting a percentage to a decimal

Worked example

Write 35% as a decimal.

$$35\% = \frac{35}{100} = 0.35$$

Write 35% as a fraction out of 100. Then divide 35 by 100 to write it as a decimal.

Converting a fraction to a percentage

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example: $\frac{34}{80}$ = 0.425 = 42.5%

Students can input $\frac{34}{80}$ as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

Writing one number as a percentage of another

Write as a fraction, then convert to a percentage.

For example, in a class of 28 students, 13 are boys. What percentage are boys?

$\frac{13}{28}$ = 0.4642… = 46.4% (1 d.p.)

Without a calculator

GCSE Psychology students would not be required to complete calculations without a calculator:

$$\text{Percentage of words recalled with rehearsal} = \frac{\text{words recalled with rehearsal}}{\text{total words recalled}} \times 100$$

Calculating a percentage of an amount

50% is the same as $\frac{1}{2}$ so to find 50% divide by 2.

10% is the same as $\frac{1}{10}$ so to find 10% divide by 10.

To calculate 30% mentally, you can find 10% and multiply by 3.

To calculate 5% mentally, find 10% and halve.

Calculating percentages using a calculator

Input the percentage as a fraction

For example, to calculate 30% of 20, input $\frac{30}{100}$ × 20 and press = to get 6.

Input the percentage using a decimal multiplier

65% = 0.65

So to calculate 65% of 80, input 0.65 × 80 and press = to get 52.

Percentage increase / decrease

Work out the percentage increase and add it on/subtract it

Examples

To increase 45 g by 20%:

20% of 45 g = 9 g

45 + 9 = 54 g

To decrease 220 ml by 5%:

5% of 220 ml = 11 ml

220 − 11 = 209 ml

Using multiplication

Examples

To increase 45 g by 20%

After the increase you will have 100% + 20% = 120% = 1.2

1.2 × 45 = 54 g

To decrease 220 ml by 5%

After the decrease you will have 100% – 5% = 95%

0.95 × 220 = 209 ml

Finding original amount

Using arrow diagrams:

Worked example

20% of an amount is £40.
Work out the original amount.

$$\begin{tikzcd} 20\% = \pounds 40 \arrow[d, bend right, "\div 20"'] \arrow[d, bend left, "\div 20"] \\ 1\% = \pounds 2 \arrow[d, bend right, "\times 100"'] \arrow[d, bend left, "\times 100"] \\ 100\% = \pounds 200 \end{tikzcd}$$

Percentage change

percentage change $= \frac{\text{actual change}}{\text{original amount}} \times 100$

Example

In 2010 a box of tissues cost 80p.

In 2014 a similar box cost £1.20.

The actual increase in price is 120 – 80 = 40p.

The fractional increase is $\frac{\text{actual increase}}{\text{original price}} = \frac{40}{80}$

$\frac{40}{80}$ as a decimal is 0.5

Percentage increase is 0.5 × 100 = 50%

3.3 Converting between fractions, decimals and percentages

Requirement

All students will learn how to convert between fractions, decimals and percentages in KS3.

Approach

Percentage to fraction to decimal

$40\% = \frac{40}{100} = 0.4$

Decimal to percentage

Multiply by 100:

0.3 = 30%

0.02 = 2%

Percentage to decimal

Divide by 100:

62% = 0.62

7.5% = 0.075

Simple fractions to percentages

Multiply or divide both numbers by the amount needed to get a fraction with denominator 100:

$\times 5$

$$\frac{3}{20} = \frac{15}{100} = 15\%$$

$\times 5$

$\div 2$

$$\frac{62}{200} = \frac{31}{100} = 31\%$$

$\div 2$

Converting a fraction to a percentage

Convert the fraction to a decimal, then convert the decimal to a percentage.

For example:

$\frac{34}{80} = 0.425 = 42.5\%$

Students can input $\frac{34}{80}$ as a fraction into a scientific calculator and press = (or the S-D button on some calculators) to get the equivalent decimal.

Terminology

- When converting decimals to percentages or vice versa, do not say 'move the decimal point two places'. Instead, say 'multiply by 100' or 'divide by 100' as appropriate.

  0.52 = 52%   3% = 0.03

3.4 Ratios

Requirement

Students learn to simplify ratios, and write them in the form 1 : *n* or *n* : 1 in KS3.

Students learn to relate ratios to fractions in KS3, but many continue to make errors with this type of calculation.

Approach

Simplifying ratios

A ratio in its simplest form only contains whole number values.

Divide all the numbers in the ratio by the highest common factor:

$$2 : 6 \xrightarrow{\div 2} 1 : 3$$

$$6 : 15 \xrightarrow{\div 3} 2 : 5$$

The following ratio is not in its simplest form, because the two numbers both still have a common factor, 2:

$$8 : 20 \xrightarrow{\div 2} 4 : 10$$

Writing in the form 1 : n (sometimes called a unit ratio)

Divide both numbers by the first number in the ratio:

$$5 : 7 \xrightarrow{\div 5} 1 : 1.4$$

Writing in the form n : 1

Divide both numbers by the second number in the ratio:

$$20 : 12 \xrightarrow{\div 12} 1.67 : 1$$

Comparing ratios

Write both ratios in the form 1 : n or n : 1.

Example

Ben makes a drink with 20 ml squash to 120 ml water.

Fiona makes a drink with 15 ml squash to 85 ml water.

Whose squash is stronger?

Ben

$$20 : 120 \xrightarrow{\div 20} 1 : 6$$

Fiona

$$15 : 85 \xrightarrow{\div 15} 1 : 5.7$$

Fiona’s drink has less water for 1 ml squash, so is stronger.

Ratio and proportion

A mixture is made from two liquids A and B in the ratio 2 : 3.

What fraction of the mixture is

a) liquid A?

b) liquid B?

$\frac{2}{5}$ is A and $\frac{3}{5}$ is B

Terminology

To simplify a ratio, divide all the numbers in the ratio by their highest common factor.

To compare ratios, write them in the form 1 : *n*, or n : 1. This is sometimes called a unit ratio.

A **ratio** compares two quantities, and translates into a statement such as 'for every 3 black there are 2 red'.

Common error: Students look at 2 : 3 and think the fraction is $\frac{2}{3}$.

4. Estimation

Requirement

All KS3 students learn to estimate answers to calculations.

Approach

Estimate the answer to $591 \times \frac{97}{289}$

Rounding each of the numbers to 1 significant figure gives $600 \times \frac{100}{300}$

So a good estimate would be 200.

Terminology

- Avoid any suggestion that an estimate involves guessing.
- The calculation is not 'rough' – it is accurate, but the numbers you use or the assumptions you make are estimates or approximations.
- To estimate the result of a calculation in maths, we round all values to 1 significant figure.
- In calculations involving division or square roots, you can round one or more values to give a 'nice' division or root.

  e.g. to estimate $\frac{3.7 \times 7.8}{4.4}$, rounding the values on the top to 1 s.f. gives $4 \times 8 = 32$.
- So approximating 4.4 to 4 gives the calculation $32 \div 4 = 8$.
- Use the symbol $\approx$ to show the estimated answer to a calculation.

5. Significant figures and decimal places

Requirement

All students will have learned to round to the nearest whole number and 1, 2 or 3 d.p. by the end of KS3. They should be able to cope with rounding to more d.p. as an extension of rounding to 3 d.p.

Approach

Look at the digit *after the* last one you want to keep. Round up if this digit is 5 or more, round down if it is 4 or less.

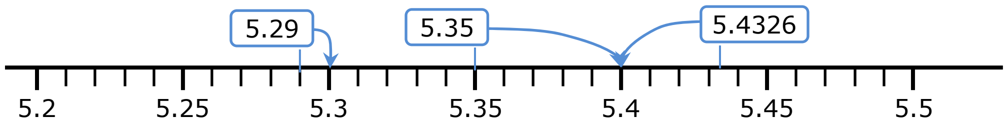
Rounding to 1 d.p.

5.4|326
less than 5 so round down
5.4 (1 d.p.)

5.2|91
5 or more so round up
5.3 (1 d.p.)

5.3|5
5 or more so round up
5.4 (1 d.p.)

On a number line, round to the nearest value with 1 decimal place:



35.42|9
5 or more so round up
35.43 (2 d.p.)

126.37|2
less than 5 so round down
126.37 (2 d.p.)

Rounding to 3 d.p.

0.053|21
less than 5 so round down
0.053 (3 d.p.)

11.291|5
5 or more so round up
11.292 (3 d.p.)

Rounding to significant figures

Small numbers

1st significant figure
= 4 ten thousandths

0.000 483

Large numbers

1st significant figure
= 5 billion

5 183 760 000

Rounding to 2 significant figures (2 s.f.)

0.00048|3

less than 5
round down

0.00048

5 1|8 3 760 000

5 or more
round up

5 200 000 000

Add zeroes so the 5 is still
in the 'billions' position

Terminology

- The number of **decimal places** is the number of digits after the decimal point. So, 10.5219 has 4 decimal places, and 10 has no decimal places.
- In any number, the first **significant figure** is the one with the highest place value. It is the first non-zero digit counting from the left.

Inconsistency: Zero is counted as a significant figure if it is between two other non-zero significant figures. Other zeros are place holders – if you took them out the place value of the other digits would change.

place holders

0 . 0 0 5 0 7 3

1st 2nd 3rd 4th

significant figures

place holders

3 9 0 4 1 2 0 0 0

1st 2nd 3rd 4th 5th 6th

significant figures

- To round a number to a given number of significant figures or decimal places, look at the digit after the last one you need. Round up if the digit is 5 or more, and round down if the digit is 4 or less.
- 8.95 rounded to 1 decimal place is 9.0. You must write the '.0' to show the value in the decimal place.

6. Standard form

Requirement

All students learn to write numbers in index form and use the index laws for multiplication and division in KS3.

All students learn the positive and negative powers of 10 in Unit 1 of GCSE Maths. Foundation students often find the negative and zero powers difficult to understand/remember, as they are the only negative and zero powers they use. Higher students use negative and zero indices with a range of numbers so are likely to have a better understanding.

All students learn to read and write very small and very large numbers in standard form in GCSE.

Approach

Calculating powers of 10

Follow a pattern:

For positive powers, the power shows the number of times that 10 is multiplied by itself:

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1000$

$10^4 = 10 \times 10 \times 10 \times 10 = 10\ 000$

Note that the number of zeroes is the same as the number of the power. Based on this, what is the value of 10^7?

Answer: $10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10\ 000\ 000$

For negative powers, the power shows the number of times that 1 is divided by 10:

$10^{-1} = 1 \div 10 = 0.1$

$10^{-2} = 1 \div (10 \times 10) = 0.01$

$10^{-3} = 1 \div (10 \times 10 \times 10) = 0.001$

$10^{-4} = 1 \div (10 \times 10 \times 10 \times 10) = 0.0001$

Lastly, remember that any number to the power of 0 is 1, so:

$10^0 = 1$

Students may like to see this idea on a number line:

10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10^1	10^2	10^3	10^4
0.0001	0.001	0.01	0.1	1	10	100	1000	10 000

Writing large numbers in standard form

Example 4

Write 4000 in standard form.

$4000 = 4 \times 1000$ — Write the number as a number between 1 and 10 multiplied by a power of 10.

$= 4 \times 10^3$ — Write the power of 10 using indices.

Example 5

Write 45 600 in standard form.

$45\,600 = 4.56 \times 10^4$ — 4.56 lies between 1 and 10. Multiply by the power of 10 needed to give the original number.

4 5 6 0 0

Writing small numbers in standard form

Example 6

Write 0.000 05 in standard form.

$0.000\,05 = 5 \times 0.000\,01$ — Write the number as a number between 1 and 9 multiplied by a power of 10.

$= 5 \times 10^{-5}$

Key point 7

To write a small number in standard form:

- Place the decimal point after the first non-zero digit.
- How many places has this moved the digit? This is the negative power of 10.

Example 7

Write 0.003 52 in standard form.

$0.003\,52 = 3.52 \times 10^{-3}$ — 3.52 lies between 1 and 10. Multiply by the power of 10 needed to give the original number.

0 . 0 0 3 5 2

Calculating with numbers in standard form

Multiplication and division

Example 3

Work out $(5 \times 10^3) \times (7 \times 10^6)$

$5 \times 7 \times 10^3 \times 10^6$ — Rewrite the multiplication grouping the numbers and the powers.

35×10^9 — Simplify using multiplication and the index law $x^m \times x^n = x^{m+n}$. This is not in standard form because 35 is not between 1 and 10.

$35 = 3.5 \times 10^1$ — Write 35 in standard form.

$35 \times 10^9 = 3.5 \times 10^1 \times 10^9 = 3.5 \times 10^{10}$ — Work out the final answer.

Work out $\frac{2.4 \times 10^5}{3 \times 10^2}$

Rewrite to separate numbers and powers:

- $2.4 \div 3 = 0.8$
- $10^5 \div 10^2$

Use the index law $x^m \div x^n = x^{m-n}$ to divide 10^5 by 10^2.

- $10^5 \div 10^2 = 10^3$

$= 0.8 \times 10^3$

$= 8 \times 10^2$

Addition and subtraction

Write numbers in decimal form before adding and subtracting.

Write the answer in standard form.

Work out $3.6 \times 10^2 + 4.1 \times 10^{-2}$

$= 3.6 \times 100 + 4.1 \times 0.01$

$= 360 + 0.041$

$= 360.041$

$= 3.60041 \times 100$

$= 3.60041 \times 10^2$

Work out $2.5 \times 10^6 - 4 \times 10^4$

$$\begin{array}{r} 2\,500\,000 \\ -\quad 40\,000 \\ \hline 2\,460\,000 \end{array}$$

2.46×10^6

Terminology

- Any number can be raised to a power or index. The power or index tells you how many times the number is multiplied by itself. $3^4 = 3 \times 3 \times 3 \times 3$
- We read 3^4 as '3 to the power 4'.
- Some calculators have a power or index key. In maths, students are not told which key presses to use, as calculators vary. Instead we would say 'Make sure you know how to input numbers raised to a power on your calculator.'
- Any number raised to the power zero = 1.
- The index laws: To multiply powers of the same number, add the indices. To divide powers of the same number, subtract the indices.
- Some of the powers of 10 are as follows:

10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10	10^2	10^3	10^4
0.0001 or $\frac{1}{10\,000}$	0.001 or $\frac{1}{1000}$	0.01 or $\frac{1}{100}$	0.1 or $\frac{1}{10}$	1	10	100	1000	10 000

- Standard form is a way of writing very large or very small numbers as a number between 1 and 10 multiplied by a power of 10; for example, $A \times 10^n$, where A is between 1 and 10 and n is the power of 10.

- When writing numbers in standard form, do not talk about 'moving the decimal point'; the position of the decimal point remains fixed. Multiplying by a power of 10 moves digits places to the left and dividing by a power of 10 moves digits places to the right.
- On some calculators you can enter numbers in standard form, or answers may be given in standard form. In maths, students are not told which key presses to use, as calculators vary. Instead students would be told to 'Make sure you know how to enter and read numbers in standard form on your calculator.'

7. Averages: mean, median and mode

7.1 Averages

Requirement

Normal distribution is not covered in Maths GCSE.

Terminology

- Mean, median and mode are all averages. In everyday life when someone says 'average' they are usually talking about the mean.
- Mean = $\frac{\text{total of all the values}}{\text{number of values}}$
- The mode is the most common value. In a frequency table, this is the value with the highest frequency.
- The mode is one of the data values. A set of data can have more than one mode. For grouped data, the modal class is the class interval with the highest frequency.
- The median is the middle value when the data is written in order. It may not be one of the data values (e.g. it could be halfway between two values).
- For an ordered set of data with an even number of values, the median is the mean of the two middle values (which is the same as the value midway between them).
- For a set of n items of data, the median is the $\frac{n+1}{2}$th data item. When n is very large, you can use the $\frac{n}{2}$th data item.
- For a set of ordered data, the median is the value halfway through the data.

Finding the mode

Mode = most common value

From a data set

For the data set 2, 2, 5, 7, 2, 4, 6, 9, the mode is 2.

For the data set 1, 1, 3, 4, 2, 5, 3, 3, 2, 2, 1 the modes are 1 and 3.

From a frequency table

For this data, the mode is 2 eggs.

Number of eggs	Frequency
1	2
2	15
3	6

Common error: Students may give '15' as the mode (the highest frequency), rather than 2, which is the number of eggs with the greatest frequency.

From a grouped frequency table

For the below data, the modal age is $10 \le a < 20$

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

Finding the median

Median = middle value

First write the data in order.

0 2 4 5 7 8 10 12 12 12 13 14 14 15 16

The median is the
middle value

Common error: Students may not order the data before finding the median.

From a frequency table

Number of eggs	**Frequency**
1	4
2	8
3	6
4	2
Total	20

There are 20 pieces of data in the table.

The median is the $\frac{20+1}{2}$ = 10.5th data item (i.e. between the 10th and 11th items). The +1 is added here as there are an even number of values (20).

Number of eggs	**Frequency**	
1	4	4
2	8	4 + 8 = 12
3	6	
4	2	
Total	20	

Add up the frequencies to find the 10th and 11th data items.

The 10th and 11th items are both 2 eggs, so the median is 2 eggs.

Calculating the quartiles from a frequency table

The lower quartile is the $\frac{20+1}{4}$ = 5.25th data item (i.e. between the 5th and 6th items).

The upper quartile is the $\frac{3(20+1)}{4}$ = 15.75th data item (i.e. between the 15th and 16th items).

Number of eggs	Frequency	
1	4	4
2	8	4 + 8 = 12
3	6	12 + 6 = 18
4	2	
Total	20	

Add up the frequencies to find the 5th and 6th data items and the 15th and 16th data items.

The 5th and 6th items are both 2 eggs, so the lower quartile is 2 eggs.

The 15th and 16th items are both 3 eggs, so the upper quartile is 3 eggs.

Finding the interval containing the median from a grouped frequency table

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

12
12 + 15 = 27

Add up the frequencies to find the 20th and 21st data items.

Total frequency = 40

Median = $\frac{40+1}{2}$ = 20.5th data item.

The 20th and 21st data items are in the interval $10 \leq a < 20$.

Calculating the mean of a small data set

Mean (arithmetic mean) = sum of the values divided by the number of values

Example 1

Work out the mean of 3, 6, 7, 7 and 8.

$3 + 6 + 7 + 7 + 8 = 31$

Add the values first to find the total.

$\frac{31}{5} = 6.2$

There are 5 values, so divide the total by 5.

The mean is 6.2

Common error: When using a calculator to calculate a mean, students may add the numbers and not press = before dividing, which will give an incorrect value.

Calculating the mean from a frequency table

The below is from Pearson KS3 Maths:

Worked example

Jack asked students in his class how many pets they had.
Here are his results. Work out the mean.

Number of pets	Frequency	Total number of pets
0	7	0 × 7 = 0
1	8	1 × 8 = 8
2	6	2 × 6 = 12
3	3	3 × 3 = 9
4	1	4 × 1 = 4
Total	25	33

Add a column to the table to work out the total numbers of pets.

Work out the total frequency (number of people in the survey) and the total number of pets.

mean = 33 ÷ 25 = 1.32

mean = total number of pets ÷ number of people

Calculating means and medians from bar charts and histograms

Make a frequency table from the bar chart or histogram, and use the appropriate method shown above.

7.2 Range

Terminology

- The range is a measure of spread of the data. It is calculated as the largest value minus the smallest value. Note the range is a single number (not 3–12 as two numbers separated with a hyphen, but 9). In a maths question, students would be told to 'Work out' the range – they would need to do a calculation to find it.
- A larger range means the data is less consistent. A smaller range means the data is more consistent.
- You can estimate the range from a grouped frequency table, as largest possible value minus smallest possible value.

Approach

Calculating the range

From a small data set

Range = largest value minus smallest value

For example, for this data: 2, 2, 5, 7, 2, 4, 6, 9

the range is 9 – 2 = 7

From a frequency table

Range = largest value minus smallest value

Number of eggs	Frequency
1	2
2	15
3	6

Range = 3 – 1 = 2

NB: It is the range of the data values, not of the frequencies.

From a grouped frequency table

An estimate of the range is:

largest possible value minus smallest possible value

Worked example

In a survey, people were asked their age. The table shows the results.

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

Work out an estimate for the range of ages.

From the frequency table, the smallest possible age is 0 years.

The largest possible age is 40 years.

So an estimate of the range is 40 − 0 = 40 years.