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GCSE

Mathematics A (1MA0)

Scheme of work

Edexcel GCSE in Mathematics A (1MA0)

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A PEARSON COMPANY

Contents

Introduction

This scheme of work is based upon a five term model over two years for both Foundation and Higher tier students.

It can be used directly as a scheme of work for the GCSE Mathematics A specification (1MA0).

The scheme of work is structured so each topic contains:

- Module number
- Recommended teaching time, though of course this is adaptable according to individual teaching needs
- Tier
- Contents, referenced back to the specification
- Objectives for students at the end of the module
- References to published textbook sections
- Ideas for differentiation and extension activities
- Notes for general mathematical teaching points and common misconceptions

Updates will be available via a link from the Edexcel mathematics website (www.edexcel.com).

References to Edexcel published student books for the course are given in brackets for each main teaching objective. For example (2.6) in a Foundation module references to GCSE Mathematics A Foundation Student Book, Chapter 2, Section 2.6.

This document is an Issue 2. Significant changes have been marked with a sidebar or highlighted.

GCSE Mathematics A (1MA0)
Foundation
Tier

Linear
Scheme of Work

Foundation course overview

The table below shows an overview of modules in the Linear Foundation tier scheme of work.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

Module number	Title	Estimated teaching hours
1	Integers	7
2	Decimals	4
3	Coordinates	4
4	Angles, lines and triangles	6
5	Reading scales and converting units	5
6	Collecting data	4
7	Charts and graphs	5
8	Symmetry, Similarity and Congruence	4
9	Types of number	8
10	Introduction to algebra	4
11	Constructions	5
12	Patterns and sequences	5
13	Properties of quadrilaterals and parallel lines	5
14	Fractions	7
15	Pie charts	3
16	Fractions, decimals and percentages	4
17	Applications of percentages	5
18	Algebra using powers and brackets	4
19	Ratio and proportion	6
20	Linear equations and inequalities	6
21	Perimeter and area	7
22	3-D shapes	4
23	Real-life graphs	5
24	Straight line graphs	4
25	Compound measures	5
26	Timetables and distance-time graphs	5
27	Volume	5
28	Probability	9
29	Formulae	7
30	Angles properties of polygons	5
31	Transformations	6
32	Scatter graphs and correlation	5
33	Averages and range	7
34	Quadratic graphs	3
35	Trial and Improvement	3
36	Circles	5
37	Pythagoras' Theorem	5
	Total	**190 HOURS**

Module **1** **Time: 6 – 8 hours**

GCSE Tier: **Foundation**

Contents: **Integers**

N b	Order integers
N u	Round numbers
N a	Add, subtract, multiply and divide positive or negative integers
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N v	Use calculators effectively and efficiently

PRIOR KNOWLEDGE:

The ability to order numbers
An appreciation of place value
Experience of the four operations using whole numbers
Knowledge of integer complements to 10 and to 100
Knowledge of strategies for multiplying and dividing whole numbers by 2, 4, 5 and 10

OBJECTIVES

By the end of the module the student should be able to:

- Use and order positive and negative numbers **(1.1–1.3, 1.7–1.9)**
- Write numbers in words and write numbers from words **(1.2)**
- Add and subtract integers, including negative numbers **(1.4, 1.9)**
- Recall all multiplication facts to 10 × 10, and use them to derive quickly the corresponding division facts **(1 intro)**
- Multiply or divide any number by powers of 10 **(1.5)**
- Multiply and divide positive and negative numbers **(1.5, 1.9)**
- Use brackets and the hierarchy of operations (BIDMAS) **(9.4)**
- Find reciprocals **(10.2)**
- Understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is undefined) **(10.2)**
- Add, subtract, multiply and divide negative numbers **(1.7–1.9)**
- Round whole numbers to the nearest: 10, 100, 1000, … **(1.6)**
- Check calculations by rounding, eg 29 × 31 ≈ 30 × 30 **(5.10)**
- Check answers by inverse calculation, eg if 9 × 23 = 207 then 207 ÷ 9 = 23 **(5.11)**

DIFFERENTIATION & EXTENSION

Estimate answers to calculations involving the four rules of operation
Directed number work with multi-step calculations
Encourage effective use of a calculator
Try investigations with digits 3, 7, 5 and 2 and challenge students to find the biggest number, smallest odd number, the largest sum or product etc

NOTES

Present all working clearly
For non-calculator methods, ensure that remainders are shown as evidence of working
Show what is entered into your calculator, not just the answer
Try different methods from traditional ones, eg Russian or Chinese methods for multiplication
Incorporate Functional Elements whenever and wherever possible and always round measures to an appropriate degree of accuracy

Module **2** **Time: 3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Decimals**

N b	Order decimals and integers
N a	Add, subtract, multiply and divide any number
N j	Use decimal notation and recognise that each terminating decimal is a fraction
N u	Round numbers

PRIOR KNOWLEDGE:

The concept of a decimal
The four operations

OBJECTIVES

By the end of the module the student should be able to:

- Understand place value, identifying the values of the digits **(5.1)**
- Write decimals in order of size **(5.2)**
- Round decimals to the nearest integer a given number of decimal places or to one significant figure **(5.7–5.9)**
- Add and subtract decimals **(5.3)**
- Multiply and divide decimal numbers by integers and decimal numbers **(5.4–5.6)**
- Know that, eg $13.5 \div 0.5 = 135 \div 5$ **(5.11)**
- Check their answers by rounding, and know that, eg $9.8 \times 17.2 \approx 10 \times 17$ **(5.10)**

DIFFERENTIATION & EXTENSION

Practise long multiplication and division without using a calculator
Mental maths problems with negative powers of 10, eg 2.5×0.01, 0.001
Directed number work with decimal numbers
Use decimals in real-life problems as much as possible, eg Best Buys
Use functional examples such as entry into theme parks, cost of holidays, sharing the cost of a meal
Money calculations that require rounding answers to the nearest penny
Multiply and divide decimals by decimals with more than 2 d.p.
Round answers to appropriate degrees of accuracy to suit the context of the question

NOTES

Advise students not to round decimals, used in calculations, until stating in the final answer
For non-calculator methods ensure that remainders are shown as evidence of working
Students need to be clear about the difference between decimal places and significant figures
Link decimals to Statistics and Probability, eg the mean should not be rounded, the probability of all events occurring is equal to 1
Link decimals to reading scales and converting units and compound measures

Module **3** **Time**: **3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Coordinates**

A k Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

PRIOR KNOWLEDGE:

Directed numbers
Parallel and perpendicular lines

OBJECTIVES

By the end of the module the student should be able to:

- Use axes and coordinates to specify points in all four quadrants in 2-D **(15.1, 15.2)**
- Identify points with given coordinates **(15.1, 15.2)**
- Identify coordinates of given points (NB: Points may be in the first quadrant or all four quadrants) **(15.1, 15.2)**
- Find the coordinates of points identified by geometrical information in 2-D **(15.1)**
- Find the coordinates of the midpoint of a line segment, AB, given the coordinates of A and B **(15.3)**

DIFFERENTIATION & EXTENSION

There are plenty of sources of good material here such as drawing animal pictures with coordinates, games like Connect 4 using coordinates
This topic can be delivered in conjunction with the properties of quadrilaterals

NOTES

Clear presentation of graphs with axes correctly labelled is important

Module **4** **Time: 5 – 7 hours**

GCSE Tier: **Foundation**

Contents: **Angles, lines and triangles**

GM a	Recall and use properties of angles at a point, angles at a point on a straight line (including right angles), perpendicular lines, and vertically opposite angles
GM b	Understand and use the angle properties of triangles and intersecting lines
GM t	Measure and draw lines and angles
GM u	Draw triangles and other 2-D shapes using a ruler and a protractor

PRIOR KNOWLEDGE:

An understanding of angles as a measure of turning
The ability to use a ruler and a protractor

OBJECTIVES

By the end of the module the student should be able to:

- Measure and draw lines, to the nearest mm **(2 intro)**
- Measure and draw angles, to the nearest degree **(2.5, 2.6)**
- Estimate sizes of angles **(2.4)**
- Recall and use properties of angles: **(2.1, 2.8)**
  - angles at a point
  - angles at a point on a straight line, including right angles
  - vertically opposite angles
- Find the size of missing angles at a point or at a point on a straight line **(2.8)**
- Distinguish between acute, obtuse, reflex and right angles **(2.2)**
- Name angles **(2.2)**
- Give reasons for calculations **(2.8)**
- Use geometric language appropriately **(chapters 2, 7)**
- Use letters to identify points, lines and angles **(2.3)**
- Use two letter notation for a line and three letter notation for an angle **(2.3)**
- Recall and use properties of perpendicular lines **(7.5)**
- Mark perpendicular lines on a diagram **(7.5)**
- Understand the proof that the angle sum of a triangle is 180° **(7.7)**
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices **(7.7)**
- Distinguish between scalene, equilateral, isosceles and right-angled triangles **(2.7, 6.1)**
- Understand and use the angle properties of triangles **(2.7, 6.1)**
- Find a missing angle in a triangle, using the angle sum of a triangle is 180° **(2.7)**
- Use the side/angle properties of isosceles and equilateral triangles **(2.7)**
- Make accurate drawing of triangles and other 2-D shapes using a ruler and a protractor **(6.4)**
- Make an accurate scale drawing from a diagram **(7.9)**

DIFFERENTIATION & EXTENSION

Explore other angle properties in triangles, parallel lines or quadrilaterals, in preparation for future topics

NOTES

Make sure that drawings are neat, accurate and labelled
Give students a lot of drawing practice, and encourage students to check their drawings
Angles should be accurate to within 2° and lengths accurate to the nearest mm

Module **5** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Reading scales and converting units**

GM o Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements
GM t Measure and draw lines
GM p Convert measurements from one unit to another
GM m Use scale drawings

PRIOR KNOWLEDGE:

An awareness of the imperial system of measures
Strategies for multiplying and dividing by 10 (for converting metric units)

OBJECTIVES

By the end of the module the student should be able to:

- Construct scale drawings **(7.9)**
- Use and interpret scale drawings **(7.9)**
- Interpret scales on a range of measuring instruments including mm, cm, m, km, m*l*, c*l*, *l*, mg, g, kg, tonnes, °C **(11.1)**
- Indicate given values on a scale **(11.1)**
- Know that measurements using real numbers depend upon the choice of unit **(11.3)**
- Recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction **(11.6)**
- Convert units within one system **(11.3, 11.4)**
- Convert metric units to metric units (Metric equivalents should be known) **(11.3)**
- Convert imperial units to imperial units (NB: Conversion between imperial units will be given) **(11.4)**
- Convert between metric and imperial measures **(11.4)**
- Know rough metric equivalents of pounds, feet, miles, pints and gallons, ie **(11.4)**

  Metric Imperial
  1 kg = 2.2 pounds
  1 litre = 1.75 pints
  4.5 *l* = 1 gallon
  8 km = 5 miles
  30 cm = 1 foot
- Estimate conversions **(11.4)**

DIFFERENTIATION & EXTENSION

This could be made a practical activity, by collecting assorted everyday items and weighing and measuring to check the estimates of their lengths, weights and volumes
Use the internet to find the weights, volumes and heights of large structures such as buildings, aeroplanes and ships
Take the opportunity to do some real measuring/estimating around school
Use conversions for height and weight of students, cars, bridges. Combine with simple scales such as 1 cm to 1 m for classrooms, playing fields, bedrooms and ask them to draw a plan of their ideal design for their bedrooms including the furniture

NOTES

Measurement is essentially a practical activity
Use a range of everyday objects to bring reality to lessons
Use Functional Elements as a source of practical activities

Module **6** **Time: 3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Collecting data**

SP a Understand and use statistical problem solving process (handling data cycle)
SP b Identify possible sources of bias
SP c Design an experiment or survey
SP d Design data-collection sheets distinguishing between different types of data
SP e Extract data from printed tables and lists
SP f Design and use two-way tables for discrete and grouped data

PRIOR KNOWLEDGE:

An understanding of why data need to be collected
Experience of simple tally charts
Some idea about different types of graphs
Experience of inequality notation

OBJECTIVES

By the end of the module the student should be able to:

- Specify the problem and plan **(3.1)**
- Decide what data to collect and what statistical analysis is needed **(3.1, 3.3)**
- Collect data from a variety of suitable primary and secondary sources **(3.1, 3.2, 3.5)**
- Use suitable data collection techniques **(3.1, 3.2, 3.4)**
- Process and represent the data **(3.1, 3.2, 3.5)**
- Interpret and discuss the data **(3.1, 3.2, 3.5)**
- Understand how sources of data may be biased **(3.4)**
- Identify which primary data they need to collect and in what format, including grouped data **(3.3)**
- Consider fairness **(3.3)**
- Understand sample and population **(3.4)**
- Design a question for a questionnaire **(3.3)**
- Criticise questions for a questionnaire **(3.3)**
- Design and use data-collection sheets for grouped, discrete and continuous data **(3.2)**
- Collect data using various methods **(3.2)**
- Sort, classify and tabulate data and discrete or continuous quantitative data **(3.1, 3.2)**
- Group discrete and continuous data into class intervals of equal width **(3.2)**
- Extract data from lists and tables **(3.2, 3.5)**
- Design and use two-way tables for discrete and grouped data **(3.5)**
- Use information provided to complete a two way table **(3.5)**

DIFFERENTIATION & EXTENSION

Carry out a statistical investigation of their own, including designing an appropriate means of gathering the data
Some guidance needs to be given to stop students choosing limited investigations, eg favourite football team

NOTES

For Functional Elements activities, it is worth collecting data at different times of the day, eg to compare types of shopper in a centre. Get data from holiday brochures to compare resorts for temp, rainfall and type of visitor
Emphasise the differences between primary and secondary data. Mayfield High data can be used as an example of secondary data
Discuss sample size and mention that a census is the whole population. In the UK, the Census is held every year that ends in '1', so the next census is in 2011
If students are collecting data as a group, then they should use the same procedure
Emphasise that continuous data is data that is measured, eg temperature

Module **7** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Charts and graphs**

SP g Produce charts and diagrams for various data types
SP i Interpret a wide range of graphs and diagrams and draw conclusions
SP l Compare distributions and make inferences

PRIOR KNOWLEDGE:

An understanding of why data need to be collected and some idea about different types of graphs

OBJECTIVES

By the end of the module the student should be able to:

- Draw: **(12.1, 12.4–12.6, 16.5–16.6, 25.1)**
  - Pictograms **(12.1)**
  - Composite bar charts **(12.4)**
  - Comparative and dual bar charts **(12.4)**
  - Frequency polygons **(12.6)**
  - Histograms with equal class intervals **(12.5)**
  - Frequency diagrams for grouped discrete data **(16.5–16.6)**
  - Line graphs **(25.1)**
- Interpret: **(12.4, 12.6)**
  - composite bar charts **(12.4)**
  - comparative and dual bar charts **(12.4)**
  - frequency polygons **(12.6)**
- From pictograms, bar charts, line graphs and histograms with equal class intervals: **(12.1, 12.3–12.5, 25.1)**
  - read off frequency values
  - calculate total population
  - find greatest and least values
- Recognise simple patterns and characteristic relationships in bar charts, line graphs and frequency polygons **(12.3–12.6, 25.1)**
- Use dual or comparative bar charts to compare distributions **(12.4)**

DIFFERENTIATION & EXTENSION

Carry out a statistical investigation of their own and use an appropriate means of displaying the results
Use a spreadsheet to draw different types of graphs
Collect examples of charts and graphs in the media which have been misused, and discuss the implications

NOTES

Reiterate that clear presentation with axes correctly labelled is important, and to use a ruler to draw straight lines
Make comparisons between previously collected data
Encourage student to work in groups and present their charts (useful display material for classrooms/corridors)
Use Excel Graph wizard
Consider Functional Elements by comparing rainfall charts, distributions of ages in cinemas etc

Module **8** **Time: 3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Symmetry, Similarity and Congruence**

GM e Recognise reflection and rotation symmetry of 2-D shapes
GM f Understand congruence and similarity

PRIOR KNOWLEDGE:

Basic idea of shape

OBJECTIVES

By the end of the module the student should be able to:

- Recognise reflection symmetry of 2-D shapes **(6.7)**
- Identify and draw lines of symmetry on a shape **(6.7)**
- Recognise rotation symmetry of 2-D shapes **(6.8)**
- Identify the order of rotational symmetry of a 2-D shape **(6.8)**
- Draw or complete diagrams with a given number of lines of symmetry **(6.7)**
- Draw or complete diagrams with a given order of rotational symmetry **(6.8)**
- Understand congruence **(6.3)**
- Identify shapes which are congruent **(6.3)**
- Understand similarity **(6.3)**
- Identify shapes which are similar, including all circles or all regular polygons with equal number of sides **(6.3, 7.3)**
- Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not equal in size **(23.5)**

DIFFERENTIATION & EXTENSION

Investigate Rangoli Patterns, which is a good source of display work
Ask students to find their own examples of symmetry, similarity and congruence in real-life

NOTES

Equations of lines of symmetry are covered later in course
Reinforce accurate drawing skills and measurement
Use tracing paper or mirrors to assist with symmetry questions

Module **9** **Time: 7 – 9 hours**

GCSE Tier: **Foundation**

Contents: **Types of number**

N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor (HCF), Lowest Common Multiple (LCM), prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer powers

PRIOR KNOWLEDGE:

Number complements to 10 and multiplication/division facts
Recognise basic number patterns
Experience of classifying integers

OBJECTIVES

By the end of the module the student should be able to:

- Recognise even and odd numbers **(1.10)**
- Identify factors, multiples and prime numbers **(1.10)**
- Find the prime factor decomposition of positive integers **(1.10)**
- Find the common factors and common multiples of two numbers **(1.10–1.11)**
- Find the Lowest common multiple (LCM) and Highest common factor (HCF) of two numbers **(1.11)**
- Recall integer squares up to 15 × 15 and the corresponding square roots **(1.12, 5.5)**
- Recall the cubes of 2, 3, 4, 5 and 10 **(1.12, 5.5)**
- Find squares and cubes **(1.12, 5.5)**
- Find square roots and cube roots **(1.12, 5.5)**
- Use index notation for squares and cubes **(9.1)**
- Use index notation for powers of 10 **(9.1)**
- Find the value of calculations using indices **(9.1–9.2)**
- Use index laws to calculate with squares and cubes **(9.1–9.2)**

DIFFERENTIATION & EXTENSION

Calculator exercise to check factors of larger numbers
Further work on indices to include negative and/or fractional indices
Use prime factors to find LCM
Use a number square to find primes (sieve of Eratosthenes)
Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)

NOTES

All of the work in this module can be easily reinforced by using it as ‘starters’ or ‘plenaries’
Calculators should be used only when appropriate
There are plenty of investigative work using squares like ‘half time’ scores
For extension, work could introduce simple ideas on standard form

Module **10** **Time: 3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Introduction to algebra**

A a Distinguish the different roles played by letter symbols in algebra, using the correct notation
A b Distinguish in meaning between the words 'equation', 'formula' and 'expression'
A c Manipulate algebraic expressions by collecting like terms

PRIOR KNOWLEDGE:

Experience of using a letter to represent a number
Ability to use negative numbers with the four operations

OBJECTIVES

By the end of the module the student should be able to:

- Use notation and symbols correctly **(4.1, 4.4)**
- Write an expression **(4.2, 4.8)**
- Simplify algebraic expressions in one or more like terms, by adding and subtracting like terms **(4.3)**
- Understand the difference between the word 'equation', 'formula', and 'expression' **(4.8)**
- Simplify expressions **(4.4, 4.5)**

DIFFERENTIATION & EXTENSION

Look at patterns in games like 'frogs', eg Total moves = $R \times G + R + G$
Look at methods to understand expressions, eg there are 'b' boys and 'g' girls in a class, what is the total 't' number of students in the class
Further work, such as collecting like terms involving negative terms, collecting terms where each term may consist of more than one letter, eg $3ab + 4ab$

NOTES

Emphasise correct use of symbolic notation, eg $3x$ rather than $3 \times x$
Present all work neatly and use the appropriate algebraic vocabulary

Module **11** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Constructions**

GM v Use straight edge and a pair of compasses to carry out constructions
GM w Construct loci

PRIOR KNOWLEDGE:

Knowledge of types of triangle
Knowledge of the difference between a line and a region

OBJECTIVES

By the end of the module the student should be able to:

- Use straight edge and a pair of compasses to do standard constructions such as **(6.4, 18.1–18.3)**
  - Construct a triangle **(6.4)**
  - Construct an equilateral triangle **(18.1)**
  - Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not **(6.4)**
  - Construct the perpendicular bisector of a given line **(18.1)**
  - Construct the perpendicular from a point to a line **(18.1)**
  - Construct the bisector of a given angle **(18.1)**
  - Construct angles of 60°, 90°, 30°, 45° **(18.1)**
  - Draw parallel lines **(6.4)**
  - Construct diagrams of everyday 2-D situations involving rectangles, triangles, perpendicular and parallel lines **(18.1–18.3)**
- Draw and construct diagrams from given instructions **(18.2, 18.3)**
  - A region bounded by a circle and an intersecting line **(18.3)**
  - A given distance from a point and a given distance from a line **(18.2)**
  - Equal distances from 2 points or 2 line segments **(18.2)**
  - Regions which may be defined by ‘nearer to’ or ‘greater than’ **(18.3)**
  - Find and describe regions satisfying a combination of loci **(18.3)**

DIFFERENTIATION & EXTENSION

Try to do this module as practically as possible using real life situations, eg horses tethered to ropes, mobile phone masts etc
Use the internet to source ideas for this module
Use loci problems that require a combination of loci

NOTES

All constructions should be presently neatly and accurately
A sturdy pair of compasses is essential
Construction lines should not be erased as they carry valuable method marks
All lines should be correct to within 2 mm and angles correct to 2°

Module **12** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Patterns and sequences**

A i Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
A j Use linear expressions to describe the *n*th term of an arithmetic sequence

PRIOR KNOWLEDGE:

Know about odd and even numbers
Recognise simple number patterns, eg 1, 3, 5, ...
Writing simple rules algebraically
Raise numbers to positive whole number powers
Substitute into simple expressions

OBJECTIVES

By the end of the module the student should be able to:

- Recognise and generate simple sequences of odd or even numbers **(13.1)**
- Find the missing numbers in a number pattern or sequence **(13.1–13.3)**
- Find the *n*th term of a number sequence **(13.3)**
- Use the *n*th number of an arithmetic sequence **(13.3)**
- Find whether a number is a term of a given sequence **(13.4)**
- Continue a sequence derived from diagrams **(13.1)**
- Use a calculator to produce a sequence of numbers **(13.1–13.3)**

DIFFERENTIATION & EXTENSION

Match-stick problems
Use practical real life examples like 'flower beds'
Sequences of triangle numbers, Fibonacci numbers etc
Extend to quadratic sequences whose *n*th term is $an^2 + b$ and link to square numbers

NOTES

Emphasise good use of notation $3n$ means $3 \times n$
When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the *n*th term

Module **13** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Properties of quadrilaterals and parallel lines**

GM d Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
GM b Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals
GM r Understand and use bearings

PRIOR KNOWLEDGE:

Know that angles in a triangle add up to 180°
Know that angles at a point on a straight line sum to 180°
Know that a right angle = 90°

OBJECTIVES

By the end of the module the student should be able to:

- Recall the properties and definitions of special types of quadrilaterals, including symmetry properties **(6.2)**
- List the properties of each, or identify (name) a given shape **(6.2)**
- Draw sketches of shapes **(6.2)**
- Name all quadrilaterals that have a specific property **(6.2)**
- Identify quadrilaterals from everyday usage **(6.2)**
- Classify quadrilaterals by their geometric properties **(6.2)**
- Understand and use the angle properties of parallel lines **(7.5)**
- Mark parallel lines on a diagram **(7.5)**
- Find missing angles using properties of corresponding and alternate angles **(7.6)**
- Understand and use the angle properties of quadrilaterals **(7.1)**
- Use the fact that angle sum of a quadrilateral is 360° **(7.1)**
- Give reasons for angle calculations **(Chapter 7)**
- Use three figure-bearings to specify direction **(7.8)**
- Mark on a diagram the position of point B given its bearing from the point A **(7.8)**
- Give a bearing between the points on a map or scaled plan **(7.8)**
- Given the bearing of point A from point B, work out the bearing of B from A **(7.8)**

DIFFERENTIATION & EXTENSION

Practical activities help with the understanding of the properties and proofs – games like ‘Guess who I am?’
Use the angle properties of triangles to find missing angles in combinations of triangles and rectangles
Explore other properties in triangles, quadrilaterals and parallel lines

NOTES

All diagrams should be presently neatly and accurately
Students should have plenty of practice drawing examples to illustrate the properties of various shapes
For bearings and scaled drawings, angles should be correct to 2° and lines accurate to 2 mm

Module **14** **Time: 6 – 8 hours**

GCSE Tier: **Foundation**

Contents: **Fractions**

N h	Understand equivalent fractions
N h	Simplify a fraction by cancelling all common factors
N i, a	Add, subtract, multiply and divide fractions
N b	Order rational numbers
N j	Use decimal notation and understand that decimals and fractions are equivalent
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
N o	Write one number as a fraction of another

PRIOR KNOWLEDGE:

Multiplication facts
Ability to find common factors
A basic understanding of fractions as being 'parts of a whole unit'
Use of a calculator with fractions

OBJECTIVES

By the end of the module the student should be able to:

- Visualise a fraction diagrammatically **(8.1)**
- Understand a fraction as part of a whole **(8.1)**
- Recognise and write fractions in everyday situations **(8.1)**
- Find fractions of amounts **(8.5)**
- Write a fraction in its simplest form and find equivalent fractions **(8.2)**
- Compare the sizes of fractions using a common denominator **(8.3)**
- Add and subtract fractions by using a common denominator **(8.7)**
- Write an improper fraction as a mixed number **(8.4)**
- Convert between fractions and decimals **(8.8, 10.1)**
- Multiply and divide fractions **(8.5–8.6)**
- Write one number as a fraction of another **(8.1)**

DIFFERENTIATION & EXTENSION

Careful differentiation is essential as this topic is dependent on the student's ability
Relate simple fractions to percentages and vice versa
Work with improper fractions and mixed numbers, eg divide 5 pizzas between 3 people
Solve word problems involving fractions and in real life problems, eg finding a perimeter from a shape with fractional side lengths
Link fractions with probability questions

NOTES

Regular revision of fractions is essential
Demonstrate how to use the fraction button on a calculator, in order be able to check solutions
Use real-life examples whenever possible

Module **15** **Time: 3 – 4 hours**

GCSE Tier: **Foundation**

Contents: **Pie charts**

SP g Draw and produce pie charts
SP i Interpret pie charts
SP l Compare distributions and make inferences

PRIOR KNOWLEDGE:

Measuring and drawing angles
Fractions of simple quantities

OBJECTIVES

By the end of the module the student should be able to:

- Represent data in a pie chart **(12.2)**
- Interpret data in a pie chart **(12.2)**
- Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts **(12.2)**
- From pie charts **(12.2)**
  - find the total frequency
  - find the size of each category

DIFFERENTIATION & EXTENSION

Use this module to revise frequency and tally tables
Practise the ability to divide by 20, 30, 40, 60 etc
This can be delivered as a practical module that could lead to wall display- remind about of bias, eg only asking their friends which band they like
Compare pie charts for, eg boys and girls, to identify similarities and differences
Ask students to combine two pie charts

NOTES

Angles for pie charts should be accurate to within 2°

Module **16** **Time: 3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Fractions, decimals and percentages**

N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N o	Interpret fractions, decimals and percentages as operators
N v	Use calculators effectively and efficiently

PRIOR KNOWLEDGE:

Four operations of number
The concepts of a fraction and a decimal
Number complements to 10 and multiplication tables
Awareness that percentages are used in everyday life

OBJECTIVES

By the end of the module the student should be able to:

- Understand that a percentage is a fraction in hundredths **(19.1)**
- Convert between fractions decimals and percentages **(19.1)**
- Write one number as a percentage of another number **(19.4)**
- Calculate the percentage of a given amount **(19.2)**
- Use decimals to find quantities **(19.3–19.4)**

DIFFERENTIATION & EXTENSION

Consider fractions percentages of amounts, eg 12.5% = 0.125 = $\frac{1}{8}$

Consider percentages which convert to recurring decimals (eg $33\frac{1}{3}$%), and situations which lead to percentages of more than 100%

Use fraction, decimal and percentage dominos or follow me cards**.**

Investigate into the many uses made of percentages, particularly in the media

Practise the ability to convert between different forms

NOTES

Use Functional Elements questions using fractions, eg $\frac{1}{4}$ off the list price when comparing different sale prices

Keep using non-calculator methods, eg start with 10%, then 1% in order to required percentages

Module **17** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Applications of percentages**

N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentages
N o	Interpret fractions, decimals and percentages as operators
N v	Use calculators effectively and efficiently

PRIOR KNOWLEDGE:

Four operations of number
The concepts of a fraction and a decimal
Number complements to 10 and multiplication tables
Awareness that percentages are used in everyday life

OBJECTIVES

By the end of the module the student should be able to:

- Use percentages to solve problems **(19.2–19.4)**
- Convert between fractions, decimals and percentages **(19.1)**
- Find a percentage of a quantity in order to increase or decrease **(19.3)**
- Use percentages in real-life situations **(19.2)**
  - VAT
  - value of profit or loss
  - simple interest
  - income tax calculations
- Use percentages as multipliers **(19.3)**

DIFFERENTIATION & EXTENSION

Use a mixture of calculator and non-calculator methods
Use ideas for wall display, students make up their own poster to explain say a holiday reduction
Use functional skills questions to look at questions in context
Combine multipliers to simplify a series of percentage changes
Problems which lead to the necessity of rounding to the nearest penny, eg real-life contexts
Investigate comparisons between simple and compound interest calculations

NOTES

Use plenty of practical examples that can be linked to Functional Elements, eg VAT calculations

Module **18** **Time: 3 –5 hours**

GCSE Tier: **Foundation**

Contents: **Algebra using powers and brackets**

N f Use the index laws for multiplication and division of integer powers
A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors

PRIOR KNOWLEDGE:

Squares and cubes
Experience of using a letter to represent a number
Ability to use negative numbers with the four operations

OBJECTIVES

By the end of the module the student should be able to:

- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, and of powers of a power **(9.2)**
- Multiply a single algebraic term over a bracket **(4.6, 9.5)**
- Write expressions using squares and cubes **(4.4)**
- Use simple instances of index laws **(9.3)**
- Factorise algebraic expressions by taking out common factors **(4.7, 9.6)**

DIFFERENTIATION & EXTENSION

Use various investigations leading to generalisations, eg:
Indices – cell growth, paper folding
Brackets – pond borders $4n + 4$ or $4(n + 1)$
Football league matches $n^2 - n$ or $n(n - 1)$

NOTES

Use everyday examples that lead to generalisations

Module **19** **Time: 5 – 7 hours**

GCSE Tier: **Foundation**

Contents: **Ratio and proportion**

N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N t	Divide a quantity in a given ratio
GM m	Use and interpret maps and scale drawings
N q	Understand and use number operations and inverse operations

PRIOR KNOWLEDGE:

Using the four operations
Ability to recognise common factors
Knowledge of fractions

OBJECTIVES

By the end of the module the student should be able to:

- Understand what is meant by ratio and use ratios **(24.1–24.2)**
- Write a ratio in its simplest form and find an equivalent ratio **(24.1)**
- Solve a ratio problem in context, eg recipes **(24.2–24.4)**
- Share a quantity in a given ratio **(24.3)**
- Interpret map/model scales as a ratio **(7.9, 24.2)**
- Solve problems involving money conversions, eg £'s to Euros etc **(24.4)**

DIFFERENTIATION & EXTENSION

Consider maps: draw a plan of the school
Further problems involving scale drawing, eg find the real distance in metres between two points on 1 : 40000 map
Plan a housing estate with variety of different sized houses
Currency calculations using foreign exchange rates
Harder examples involving multi-stage problems
Link ratios and proportion to Functional Elements, eg investigate the proportion of different metals in alloys, the ingredients needed for recipes for fewer or more people, mixing cement, planting forests, comparing prices of goods here and abroad, Best buy type questions

NOTES

Students often find ratios with 3 parts difficult

Module **20** **Time: 5 – 7 hours**

GCSE Tier: **Foundation**

Contents: **Linear equations and inequalities**

A d Set up and solve simple equations
N q Understand and use number operations and the relationships between them including inverse operations and the hierarchy of operations
A g Solve linear inequalities in one variable and represent the solution set on a number line

PRIOR KNOWLEDGE:

Experience of finding missing numbers in calculations
The idea that some operations are reverse to each other
An understanding of balancing
Experience of using letters to represent quantities
Be able to draw a number line
An understanding of fractions and negative numbers

OBJECTIVES

By the end of the module the student should be able to:

- Set up simple equations **(21.1)**
- Rearrange simple equations **(21.2)**
- Solve simple equations **(21.1, 21.2)**
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation **(21.5–21.7)**
- Solve linear equations which include brackets, those that have negative signs occurring anywhere in the equation, and those with a negative solution **(21.4, 21.6, 21.7)**
- Solve linear equations in one unknown, with integer or fractional coefficients **(21.3–21.7)**
- Use linear equations to solve word problems **(21.7)**
- Solve simple linear inequalities in one variable, and represent the solution set on a number line **(21.10–21.11)**
- Use the correct notation to show inclusive and exclusive inequalities **(21.9–21.11)**

DIFFERENTIATION & EXTENSION

Derive equations from practical situations (such as finding unknown angles in polygons or perimeter problems)
Solve equations where manipulation of fractions (including negative fractions) is required

NOTES

Remind students about work on linear patterns and sequences
Students need to realise that not all equations should be solved by ‘trial and improvement’ or by observation. The use of a formal method of solving equations is very important
Remind students of the need to set their work out clearly, keeping the equal signs in line

Module **21** **Time: 6 – 8 hours**

GCSE Tier: **Foundation**

Contents: **Perimeter and area**

GM x Calculate perimeters and areas of shapes made from triangles and rectangles
GM n Understand the effect of enlargement for perimeter and area of shapes
GM p Convert between units and area measures

PRIOR KNOWLEDGE:

Names of triangles, quadrilaterals
Knowledge of the properties of rectangles, parallelograms and triangles
Concept of perimeter and area
Units of measurement
Four operations of number

OBJECTIVES

By the end of the module the student should be able to:

- Measure shapes to find perimeters and areas **(14.1, 14.3)**
- Find the perimeter of rectangles and triangles **(14.1)**
- Find the perimeter of compound shapes **(14.1)**
- Find the area of a rectangle and triangle **(14.2, 14.3)**
- Recall and use the formulae for the area of a triangle, rectangle and a parallelogram **(14.3)**
- Calculate areas of compound shapes made from triangles and rectangles **(14.4)**
- Find the area of a trapezium **(14.3)**
- Solve a range of problems involving areas including cost of carpet type questions **(14.4)**
- Convert between metric units of area **(20.7)**
- Understand how enlargement changes areas **(20.6)**

DIFFERENTIATION & EXTENSION

Further problems involving combinations of shapes
Use practical examples from functional papers on topics such as returfing a garden, carpeting a room, laying carpet tiles on a floor
Perimeter questions could use skirting board, wallpaper, planting a border of a garden

NOTES

Discuss the correct use of language and units, particularly when method marks are for the correct unit of measure
Ensure that students can distinguish between perimeter and area
Practical examples help to clarify the concepts, eg floor tiles etc

Module **22** **Time: 3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **3-D shapes**

GM k Use 2-D representations of 3-D shapes
GM x Calculate the surface area of a 3-D shape

PRIOR KNOWLEDGE:

The names of standard 2-D and 3-D shapes

OBJECTIVES

By the end of the module the student should be able to:

- Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone **(20.1)**
- Know the terms face, edge and vertex **(20.1)**
- Use 2-D representations of 3-D shapes **(20.1–20.3)**
- Use isometric grids **(20.2)**
- Draw nets and show how they fold to make a 3-D solid **(20.2)**
- Understand and draw front and side elevations and plans of shapes made from simple solids **(20.3)**
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3-D solid **(20.3)**
- Find the surface area of a 3-D shape **(20.5)**

DIFFERENTIATION & EXTENSION

Make solids using equipment such as clixi or multi-link
Draw on isometric paper shapes made from multi-link
Build shapes using cubes from 2-D representations
Euler's theorem
A useful topic for a wall display-pupils tend to like to draw 3-D shapes and add interest by using a mixture of colours in the elevations

NOTES

Accurate drawing skills need to be reinforced
Some students find visualising 3-D object difficult, so using simple models will help

Module **23** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Real-life graphs**

A r Construct linear functions from real-life problems and plot their corresponding graphs
A s Discuss, plot and interpret graphs including non-linear) modelling real situations

PRIOR KNOWLEDGE:

Experience at plotting points in all quadrants
Experience at labelling axes and reading scales

OBJECTIVES

By the end of the module the student should be able to:

- Draw graphs representing 'real' examples like filling a bath/containers **(22.1–22.3)**
- Interpret and draw linear graphs, including conversion graphs, fuel bills etc **(22.1–22.3)**
- Solve problems relating to mobile phone bills with fixed charge and price per unit **(22.1–22.3)**
- Interpret non-linear graphs **(22.1–22.3)**

DIFFERENTIATION & EXTENSION

Use open ended questions that test student awareness of what intersections mean, eg mobile phone bills
Use spreadsheets to generate straight-line graphs and pose questions about gradient of lines
Use ICT packages or graphical calculators to draw straight line graphs and quadratic graphs

NOTES

Clear presentation is important with axes clearly labelled
Students need to be able to recognise linear graphs and also be able to recognise when their graph is incorrect
Link graphs and relationships in other subject areas, eg science, geography
Students should have plenty of practice interpreting linear graphs for Functional Elements problems

Module **24** **Time: 3 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Straight line graphs**

A l Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients

PRIOR KNOWLEDGE:

Experience at plotting points in all quadrants
Substitution into simple formulae

OBJECTIVES

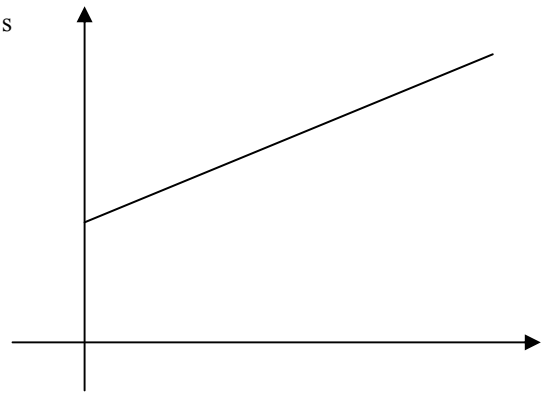
By the end of the module the student should be able to:

- Draw, label and put suitable scales on axes **(15.1, 15.2)**
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane **(15.5, 15.6)**
- Plot and draw graphs of functions **(15.5, 15.6)**
- Plot and draw graphs of straight lines of the form $y = mx + c$, when values are given for m and c **(15.6)**
- Find the gradient of a straight line from a graph **(15.7)**
- Interpret gradients from real life graphs **(22.1–22.3)**

DIFFERENTIATION & EXTENSION

Plot graphs of the form $y = mx + c$ where pupil has to generate their own table and set out their own axes
Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines
Use a graphical calculator or graphical ICT package to draw straight-line graphs
Use some examples from the last module to interpret gradient and intercept

Charge in £'s



Time in days

For hire of a skip the intercept is delivery charge and the gradient is the cost per day

Find the equation of a straight line through two points

NOTES

Careful annotation should be encouraged. Label the coordinate axes and write the equation of the line on the graph
Cover horizontal and vertical line graphs as students often forget these ($x = c$ and $y = c$)
Link graphs and relationships in other subject areas, eg science and geography
Interpret straight line graphs in Functional Elements
Link conversion graphs to converting metric and imperial units and equivalents

Module **25** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Compound measures**

GM s Understand and use compound measures
N u Approximate to specified or appropriate degree of accuracy
GM p Convert between speed measures

PRIOR KNOWLEDGE:

Knowledge of metric units, eg 1 m = 100 cm
Know that 1 hour = 60 mins, 1 min = 60 seconds
Experience of multiplying by powers of 10, eg 100 × 100 = 10 000

OBJECTIVES

By the end of the module the student should be able to:

- Use the relationship between distance, speed and time to solve problems **(11.5)**
- Convert between metric units of speed, eg km/h to m/s **(11.5)**

DIFFERENTIATION & EXTENSION

Convert imperial units to metric units, eg mph into km/h which would remind students that 5 miles = 8 km
Ask students to convert a 100 m time of 10 secs into miles per hour

NOTES

Measurement is a practical activity
All working out should be shown with multiplication or division by powers of 10
Use the distance/speed/time triangle (i.e. Drink Some Tea)

Module **26** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Timetables and Distance-time graphs**

GM o	Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements
SP e	Extract data from printed tables and lists
A s	Interpret graphs (including non-linear) that model real-life situations
A s	Interpret and draw distance-time graphs

PRIOR KNOWLEDGE:

Knowledge of metric units, eg 1 m = 100 cm
Know that 1 hour = 60 mins, 1 min = 60 seconds
Know how to find speed
Know how to read scales, draw and interpret graphs

OBJECTIVES

By the end of the module the student should be able to:

- Read times and work out time intervals **(11.2)**
- Convert between 12–hour and 24–hour hour clock times **(11.2)**
- Read bus and train timetables and plan journeys **(11.2)**
- Draw distance time graphs **(22.3)**
- Interpret distance time graphs and solve problems **(22.3)**

DIFFERENTIATION & EXTENSION

Make up a graph and supply the commentary for it
Use timetables to plan journeys

NOTES

Clear presentation with axes labelled correctly is important
Interpret straight line graphs for Functional Elements problems

Module **27** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Volume**

GM aa Calculate volumes of right prisms and shapes made from cubes and cuboids
GM n Understand the effect of enlargement for perimeter, area and volume of shapes and solids
GM p Convert between metric volume measures, including cubic centimetres and cubic metres

PRIOR KNOWLEDGE:

Concept of volume
Concept of prism
Experience of constructing cubes or cuboids from multi link

OBJECTIVES

By the end of the module the student should be able to:

- Find volumes of shapes by counting cubes **(20.4)**
- Recall and use formulae for the volume of cubes and cuboids **(20.4)**
- Calculate the volumes of right prisms and shapes made from cubes and cuboids **(20.4)**
- Convert between units of volume and capacity (1 m*l* = 1cm³) **(20.7)**
- Understand how enlargement affects volume **(20.6)**

DIFFERENTIATION & EXTENSION

Look at 'practical' examples with fish tanks/ filling containers, find the number of small boxes fitting into a large box
Further problems involving a combination of shapes
Cylinders are left until later in the course

NOTES

Discuss the correct use of language and units. Remind students that there is often a mark attached to writing down the correct unit
Use practical problems to enable the students to understand the difference between perimeter, area and volume
Use Functional Elements problems, eg filling a water tank, optimisation type questions etc

Module **28** **Time: 8 – 10 hours**

GCSE Tier: **Foundation**

Contents: **Probability**

SP m	Understand and use the vocabulary of probability and the probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may − and usually will − get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

PRIOR KNOWLEDGE:

Fractions, decimals and percentages
Ability to read from a two-way table

OBJECTIVES

By the end of the module the student should be able to:

- Distinguish between events which are: impossible, unlikely, even chance, likely, and certain to occur **(26.1)**
- Mark events and/or probabilities on a probability scale of 0 to 1 **(26.1)**
- Write probabilities in words, fractions, decimals and percentages **(26.1, 26.2)**
- Find the probability of an event happening using theoretical probability **(26.2, 26.3)**
- Find the probability of an event happening using relative frequency **(26.5)**
- Estimate the number of times an event will occur, given the probability and the number of trials **(26.7)**
- Use theoretical models to include outcomes using dice, spinners, coins **(26.2)**
- List all outcomes for single events systematically **(26.2, 26.4)**
- List all outcomes for two successive events systematically **(26.4)**
- Use and draw sample space diagrams **(26.4)**
- Add simple probabilities **(26.3)**
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 **(26.3)**
- Use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring **(26.3)**
- Find a missing probability from a list or table **(26.3, 23.6)**
- Compare experimental data and theoretical probabilities **(26.5)**
- Compare relative frequencies from samples of different sizes **(26.5)**

DIFFERENTIATION & EXTENSION

Use this as an opportunity for practical work
Experiments with dice and spinners
Show sample space for outcomes of throwing two dice (36 outcomes)
Use 'the horse race'/drawing pins/let students make their own biased dice and find experimental probability

NOTES

Students should express probabilities as fractions, percentages or decimals
Probabilities written as fractions do not need to be cancelled to their simplest form

Module **29** **Time: 6 – 8 hours**

GCSE Tier: **Foundation**

Contents: **Formulae**

A f Derive a formula
A f Substitute numbers into a formula
A f Change the subject of a formula

PRIOR KNOWLEDGE:

Understanding of the mathematical meaning of the words 'expression', 'simplifying', 'formulae' and 'equation'
Experience of using letters to represent quantities
Substituting into simple expressions using words
Using brackets in numerical calculations and removing brackets in simple algebraic expressions

OBJECTIVES

By the end of the module the student should be able to:

- Derive a simple formula, including those with squares, cubes and roots **(28.4)**
- Use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols **(28.1, 28.3, 28.4)**
- Substitute numbers into a formula **(28.1, 28.3–28.5)**
- Substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$ **(28.2–28.5)**
- Change the subject of a formula **(28.6)**
- Find the solution to a problem by writing an equation and solving it **(28.4)**

DIFFERENTIATION & EXTENSION

Use negative numbers in formulae involving indices
Various investigations leading to generalisations, eg the painted cube, Frogs, Pond Borders
Relate to topic on graphs of real life functions
More complex changing the subject, moving onto higher tier work
Apply changing the subject to physics formulae, eg speed, density, equations of motion

NOTES

Emphasise the need for good algebraic notation
Show a linear equation first and follow the same steps to rearrange a similarly structured formula
Link with Functional Elements problems in everyday problems
Link with formulae for area and volume

Module **30** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Angle properties of polygons**

GM c Calculate and use the sums of the interior and exterior angles of polygons
GM v Use straight edge and a pair of compasses to carry out constructions

PRIOR KNOWLEDGE:

Angles on straight lines, at a point and in simple shapes

OBJECTIVES

By the end of the module the student should be able to:

- Calculate and use the sums of the interior angles of polygons **(7.2)**
- Use geometrical language appropriately and recognise and name pentagons, hexagons, heptagons, octagons and decagons **(7.2)**
- Know, or work out, the relationship between the number of sides of a polygon and the sum of its interior angles **(7.2)**
- Know that the sum of the exterior angles of any polygon is 360° **(7.3)**
- Calculate the size of each exterior/interior angle of a regular polygon **(7.2, 7.3)**
- Construct a regular hexagon inside a circle **(18.1)**
- Understand tessellations of regular and irregular polygons **(7.4)**
- Tessellate combinations of polygons **(7.4)**
- Explain why some shapes tessellate and why other shapes do not **(7.4)**

DIFFERENTIATION & EXTENSION

Study Escher drawings (possibly cross curricular with Art)
Ask students to design their own tessellation, and explain why their shapes tessellate

NOTES

All diagrams should be neatly presented
Use of tracing paper helps with tessellations
Consider real-life examples of tessellations

Module **31** **Time: 5 – 7 hours**

GCSE Tier: **Foundation**

Contents: **Transformations**

GM 1 Describe and transform 2-D shapes using single or combined rotations, reflections, translations or enlargements by a positive scale factor
GM 1 Distinguish properties that are preserved under particular transformations

PRIOR KNOWLEDGE:

Recognition of basic shapes
An understanding of the concept of rotation, reflection and enlargement
Coordinates in four quadrants
Equations of lines parallel to the coordinate axes and $y = \pm x$

OBJECTIVES

By the end of the module the student should be able to:

- Describe and transform 2-D shapes using single rotations **(23.3)**
- Understand that rotations are specified by a centre and an (anticlockwise) angle **(23.3)**
- Find the centre of rotation **(23.3)**
- Rotate a shape about the origin, or any other point **(23.3)**
- Describe and transform 2-D shapes using single reflections **(23.4)**
- Understand that reflections are specified by a mirror line **(23.4)**
- Identify the equation of a line of symmetry **(23.4)**
- Describe and transform 2-D shapes using single translations **(23.2)**
- Understand that translations are specified by a distance and direction (using a vector) **(23.2)**
- Translate a given shape by a vector **(23.2)**
- Describe and transform 2-D shapes using enlargements by a positive scale factor **(23.5)**
- Understand that an enlargement is specified by a centre and a scale factor **(23.5)**
- Scale a shape on a grid (without a centre specified) **(23.5)**
- Draw an enlargement **(23.5)**
- Enlarge a given shape using (0, 0) as the centre of enlargement **(23.5)**
- Enlarge shapes with a centre other than (0, 0) **(23.5)**
- Find the centre of enlargement **(23.5)**
- Recognise that enlargements preserve angle but not length **(23.5)**
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides **(23.5)**
- Describe and transform 2-D shapes using combined rotations, reflections, translations, or enlargements **(23.6)**
- Understand that distances and angles are preserved under rotations, reflections and translations, so that any shape is congruent under any of these transformations **(23.2–23.4)**
- Describe a transformation **(23.2–23.6)**

DIFFERENTIATION & EXTENSION

Use squared paper to enlarge cartoon characters to make a display

NOTES

Emphasise that students should describe transformations fully
Diagrams should be drawn in pencil
Tracing paper can be useful for rotations

Module **32** **Time: 5 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Scatter graphs and correlation**

SP g, i	Draw and interpret scatter diagrams
SP k	Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
SP j	Look at data to find patterns and exceptions

PRIOR KNOWLEDGE:

Plotting coordinates and scale
An understanding of the concept of a variable
Recognition that a change in one variable can affect another
Linear graphs

OBJECTIVES

By the end of the module the student should be able to:

- Draw and interpret a scatter graph **(25.2)**
- Look at data to find patterns and exceptions **(25.2–25.5)**
- Distinguish between positive, negative and zero correlation using lines of best fit **(25.3–25.4)**
- Interpret correlation in terms of the problem **(25.3–25.4)**
- Understand that correlation does not imply causality **(25.3)**
- Draw lines of best fit by eye and understand what it represents **(25.4)**
- Use a line of best fit to predict values of one variable given values of the other variable **(25.5)**

DIFFERENTIATION & EXTENSION

Vary the axes required on a scatter graph to suit the ability of the class
Carry out a statistical investigation of their own including; designing an appropriate means of gathering the data, and an appropriate means of displaying the results, eg height and length of arm
Use a spreadsheet, or other software, to produce scatter diagrams/lines of best fit
Investigate how the line of best fit is affected by the choice of scales on the axes, eg use car data with age and price of the same make of car

NOTES

Statistically, the line of best fit should pass through the coordinate representing the mean of the data
Label all axes clearly and use a ruler to draw all straight lines
Remind student the line of best fit does not necessarily go through the origin of the graph

Module **33** **Time: 6 – 8 hours**

GCSE Tier: **Foundation**

Contents: **Averages and Range**

SP h	Calculate median, mean, range, mode and modal class
SP l	Compare distributions and make inferences
SP u	Use calculators efficiently and effectively, including statistical functions
SP g	Draw ordered stem and leaf diagrams
SP i	Draw conclusions from graphs and diagrams

PRIOR KNOWLEDGE:

Midpoint of a line segment
Addition and subtraction
Different statistical diagrams

OBJECTIVES

By the end of the module the student should be able to:

- Calculate the mean, mode, median and range for discrete data **(16.1, 16.3)**
- Calculate the mean, mode, median and range from an ordered stem and leaf diagram **(16.4)**
- Draw and interpret an ordered stem and leaf diagram **(16.4)**
- Calculate the modal class and the interval containing the median for continuous data **(16.6)**
- Calculate the mean, median and mode from a frequency table **(16.5)**
- Estimate the mean of grouped data using the mid-interval value **(16.7)**
- Compare the mean and range of two distributions **(16.3)**
- Recognise the advantages and disadvantages between measures of average **(16.2)**
- Calculate the mean of a small data set, using the appropriate key on a scientific calculator **(16.1)**

DIFFERENTIATION & EXTENSION

Find the mean for grouped continuous data with unequal class intervals
Collect continuous data and decide on appropriate (equal) class intervals; then find measures of average
Use the statistical functions on a calculator or a spreadsheet to calculate the mean for continuous data

NOTES

Ask class to do their own survey with data collection sheets, eg to find the average number of children per family in the class
The internet and old coursework tasks are a rich source of data to work with, eg *Second-Hand Car Sales*, *Mayfield High* data etc

Module **34** **Time: 2 – 4 hours**

GCSE Tier: **Foundation**

Contents: **Quadratic Graphs**

A t Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions
N v Use calculators effectively and efficiently

PRIOR KNOWLEDGE:

Squaring negative numbers
Substituting numbers into algebraic expressions
Plotting points on a coordinate grid
Experience of dealing with algebraic expression with brackets – BIDMAS

OBJECTIVES

By the end of the module the student should be able to:

- Substitute values of x into a quadratic function to find the corresponding values of y **(22.4)**
- Draw graphs of quadratic functions **(22.4)**
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function **(22.5)**

DIFFERENTIATION & EXTENSION

Draw simple cubic and $\frac{1}{x}$ graphs
Solve simultaneous equations graphically including a quadratic graph and a line
Solve simple projectile problems

NOTES

The graphs of quadratic functions should be drawn freehand, and in pencil. Turning the paper often helps
Squaring negative integers may be a problem for some

Module **35** **Time: 3 – 4 hours**

GCSE Tier: **Foundation**

Contents: **Trial and Improvement**

A h	Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
N u	Round to a specified or appropriate degree of accuracy
N v	Use calculators effectively and efficiently

PRIOR KNOWLEDGE:

Substituting numbers into algebraic expressions
Dealing with decimals on a calculator
Comparing/ordering decimals

OBJECTIVES

By the end of the module the student should be able to:

- Solve algebraic equations involving squares and cubes, eg $x^3 + 3x = 40$ using trial and improvement **(21.8)**
- Solve real-life problems on areas and volumes, eg the length of a rectangular room is 2 m longer than the width. If the area is 30 m², find the width **(21.8)**

DIFFERENTIATION & EXTENSION

Can look at various calculator functions like 'square root' and 'cube root'
Solve equations of the form $\frac{1}{x} = x^2 - 5$

NOTES

Students should be encouraged to use their calculator efficiently by using the 'replay' or ANS/EXE function keys
Students to take care when entering negative values to be squared
Students should write down all the digits on their calculator display and only round the final answer to the required degree of accuracy

Module **36** **Time: 4 – 6 hours**

GCSE Tier: **Foundation**

Contents: **Circles**

GM i	Distinguish between the centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
GM z	Find circumferences and areas of circles
N u	Round to a specified or appropriate degree of accuracy
N v	Use calculators effectively and efficiently
GM v	Draw circles and arcs to a given radius
GM aa	Find the volume of a cylinder
GM z	Find the surface area of a cylinder

PRIOR KNOWLEDGE:

The ability to substitute numbers into formulae

OBJECTIVES

By the end of the module the student should be able to:

- Recall the definition of a circle and identify and draw parts of a circle **(6.5–6.6)**
- Draw a circle given its radius or diameter **(6.6)**
- Find circumferences of circles and areas enclosed by circles **(17.1–17.2)**
- Recall and use the formulae for the circumference of a circle and the area enclosed by a circle **(17.1–17.2)**
- Use $\pi \approx 3.142$ or use the π button on a calculator **(17.1–17.3)**
- Find the perimeters and areas of semicircles and quarter circles **(17.3)**
- Find the surface area and volume of a cylinder **(20.4–20.5)**

DIFFERENTIATION & EXTENSION

Use more complex 2-D shapes, eg (harder) sectors of circles
Approximate π as $\frac{22}{7}$
Work backwards to find the radius/diameter given the circumference/area
Apply to real life contexts with laps of running tracks and average speeds
Make a label for a can
Harder problems involving multi-stage calculations
Define a circle by using the language of loci

NOTES

All working should be clearly and accurately presented
Use a pencil to draw all diagrams
A sturdy pair of compasses is essential

Module **37** **Time: 4 – 5 hours**

GCSE Tier: **Foundation**

Contents: **Pythagoras' theorem**

Gm g	Understand, recall and use Pythagoras' theorem in 2-D
A k	Calculate the length of a line segment
N u	Round to specified or appropriate degrees of accuracy
N v	Use calculators effectively and efficiently

PRIOR KNOWLEDGE:

Knowledge of square and square roots
Knowledge of types of triangle

OBJECTIVES

By the end of this module students should be able to

- Understand and recall Pythagoras' Theorem **(27.1–27.4)**
- Use Pythagoras' theorem to find the hypotenuse **(27.1)**
- Use Pythagoras' theorem to find the length of a side **(27.2)**
- Use Pythagoras' theorem to find the length of a line segment from a coordinate grid **(27.4)**
- Apply Pythagoras' theorem to practical situations **(27.1–27.2)**

DIFFERENTIATION & EXTENSION

See exemplar question involving times taken to cross a field as oppose to going around the edge.
Try to find examples with ladders on walls, area of a sloping roof etc
Introduce 3-D Pythagoras (moving towards Higher Tier)

NOTES

A useful way of remembering Pythagoras' Theorem is; *'Square it, square it, add/subtract it, square root it'*
Students should not forget to state units for the answers

Foundation course objectives (1MA0)

Number

N a	Add, subtract, multiply and divide any positive and negative integers
N b	Order decimals and integers
N b	Order rational numbers
N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor (HCF), Least Common Multiple (LCM), prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer powers
N h	Understand equivalent fractions
N h	Simplify a fraction by cancelling all common factors
N i, a	Add, subtract, multiply and divide fractions
N j	Use decimal notation and recognise that each terminating decimal is a fraction
N j	Use decimal notation and understand that decimals and fractions are equivalent
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring
N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentages
N m	Write one number as a fraction of another
N o	Interpret fractions, decimals and percentages as operators
N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Understand and use number operations and inverse operations
N q	Understand and use number operations and the relationships between them including inverse operations and the hierarchy of operations
N t	Divide a quantity in a given ratio
N u	Round numbers
N u	Round to specified or appropriate degrees of accuracy
N v	Use calculators effectively and efficiently

Algebra

A a	Distinguish the different roles played by letter symbols in algebra
A b	Distinguish the meaning between the words ‘equation’, ‘formula’ and ‘expression’
A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors
A d	Set up and solve simple equations
A f	Derive a formula
A f	Substitute numbers into a formula
A f	Change the subject of a formula
A g	Solve linear inequalities in one variable and represent the numbers on a number line
A h	Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
A i	Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
A j	Use linear expressions to describe the nth term of an arithmetic sequence
A k	Calculate the length of a line segment
A k	Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
A l	Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
A r	Construct linear functions from real-life problems and plot their corresponding graphs
A s	Discuss plot and interpret graphs (including non linear) that model real situations
A s	Draw and interpret distance time graphs
A t	Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions

Geometry and Measures

GM a	Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and vertically opposite angles
GM b	Understand and use the angle properties of triangles and intersecting lines
GM b	Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals
GM c	Calculate and use the sums of the interior and exterior angles of polygons
GM d	Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
GM e	Recognise reflection and rotation symmetry of 2-D shapes
GM f	Understand congruence and similarity
Gm g	Understand, recall and use Pythagoras' theorem in 2-D
GM i	Distinguish between the centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
GM k	Use 2-D representations of 3-D shapes
GM l	Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive scale factor
GM l	Distinguish properties that are preserved under particular transformations
GM m	Use and interpret maps and scale drawings
GM n	Understand the effect of enlargement for perimeter, area and volume of shapes and solids
GM o	Interpret scales on a range of measuring instruments, and recognise the inaccuracy of measurements
GM o	Use correct notation for time 12- and 24- hour clock
GM p	Convert measurements from one unit to another
GM p	Convert between units and area measures
GM p	Convert between speed measures
GM p	Convert between volume measures, including cubic centimetres and cubic metres
GM r	Understand and use bearings
GM s	Understand and use compound measures
GM t	Measure and draw lines and angles
GM w	Construct loci
GM u	Draw triangles and other 2-D shapes using a ruler and protractor
GM v	Use straight edge and a pair of compasses to carry out constructions
GM v	Draw circles and arcs to a given radius
GM x	Calculate perimeters and areas of shapes made from triangles and rectangles
GM x	Calculate the surface area of a 3-D shape
GM z	Find circumferences and areas
GM z	Find the surface area of a cylinder
GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids
GM aa	Find the volume of a cylinder

Statistics and Probability

SP a	Understand and use statistical problem solving process (handling data cycle)
SP b	Identify possible sources of bias
SP c	Design an experiment or survey
SP d	Design data-collection sheets distinguishing between different types of data
SP e	Extract data from printed tables and lists
SP e	Read timetables
SP e	Extract data from timetables and lists
SP f	Design and use two-way tables for discrete and grouped data
SP g	Draw charts and diagrams for various data types
SP g	Draw and produce pie charts
SP g	Produce ordered stem and leaf diagrams
SP g, i	Draw and interpret scatter diagrams
SP h	Calculate median, mean, range, mode and modal class
SP i	Interpret pie charts
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP i	Draw conclusions from graphs and diagrams
SP j	Look at data to find patterns and exceptions
SP k	Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
SP l	Compare distributions and make inferences
SP m	Understand and use the vocabulary of probability and the probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may − and usually will − get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics
SP u	Use calculators efficiently and effectively, including statistical functions

GCSE Mathematics A (1MA0)

Higher Tier

Linear Scheme of Work

Higher course overview

The table below shows an overview of modules in the Higher tier scheme of work.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

Module number	Title	Estimated teaching hours
1	Integers and decimals	5
2	Coordinates	3
3	Fractions	5
4	Algebra	7
5	Shape and angles	6
6	Collecting data	4
7	Displaying data	7
8	Construction and loci	5
9	Types of number	7
10	Patterns and sequences	4
11	2-D and 3-D shapes	4
12	Perimeter and area	7
13	Fractions, decimals and percentages	8
14	Formulae and linear equations	7
15	Linear graphs	5
16	Simultaneous equations	4
17	Probability	7
18	Ratio and scale	7
19	Averages and range	8
20	Pythagoras and trigonometry	8
21	Trial and Improvement	4
22	Surface area and volume	7
23	Compound measures	7
24	Transformations	6
25	Similarity and Congruence	5
26	Quadratic functions, equations and graphs	7
27	Index notation and surds	6
28	Circle theorems	4
29	Sine and cosine rules	5
30	Vectors	5
31	Further graphs and functions	5
32	Transformations of functions	4
	Total	**183 HOURS**

Module **1** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Integers and decimals**

N a	Add, subtract, multiply and divide whole numbers, integers and decimals
N b	Order integers and decimals
N j	Use decimal notation
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N u	Round to specified or appropriate degrees of accuracy including a given power of ten, number of decimal places and significant figures

PRIOR KNOWLEDGE:

The ability to order numbers
Appreciation of place value
Experience of the four operations using whole numbers
Knowledge of integer complements to 10 and 100
Knowledge of multiplication facts to 10×10
Knowledge of strategies for multiplying and dividing whole numbers by 10
The concept of a decimal

OBJECTIVES

By the end of the module the student should be able to:

- Understand and order integers and decimals **(assumed, 4.1)**
- Use brackets and the hierarchy of operations (BIDMAS) **(1.3)**
- Understand and use positive numbers and negative integers, both as positions and translations on a number line **(assumed)**
- Add, subtract, multiply and divide integers, negative numbers and decimals **(assumed, 4.2)**
- Round whole numbers to the nearest, 10, 100, 1000, … **(assumed)**
- Round decimals to appropriate numbers of decimal places or significant figures **(4.3, 4.4)**
- Multiply and divide by any number between 0 and 1 **(4.2)**
- Check their calculations by rounding, eg $29 \times 31 \approx 30 \times 30$ **(4.5, 4.6)**
- Check answers to a division sum using multiplication, eg use inverse operations **(assumed)**
- Multiply and divide whole numbers by a given multiple of 10 **(assumed)**
- Put digits in the correct place in a decimal number **(assumed)**
- Multiply and divide decimal numbers by whole numbers and decimal numbers (up to 2 d.p.), eg $266.22 \div 0.34$ **(4.2)**
- Know that, eg $13.5 \div 0.5 = 135 \div 5$ **(4.7)**

DIFFERENTIATION & EXTENSION

Teachers may just want to check that students have the appropriate skills by investigative means, eg Give students five digits say 2, 5, 7, 8 and 1. They then need to find

1) the largest even number
2) the smallest number in the 5 times table
3) the largest answer $\square\square\square + \square\square$
4) the smallest answer to $\square\square\square - \square\square$

Practice long multiplication and division without using a calculator
Estimate answers to calculations involving the four rules
Work with mental maths problems with negative powers of 10: 2.5×0.01, 0.001
Directed number work with two or more operations, or with decimals
Use decimals in real-life problems
Introduce standard form for very large and small numbers
Money calculations that require rounding answers to the nearest penny
Multiply and divide decimals by decimals (more than 2 d.p.)

NOTES

The expectation for most students doing Higher tier is that some of this material can be delivered or reinforced during other topics. For example, rounding with significant figures could be done with trigonometry
Present all working clearly with decimal points in line; emphasise that all working is to be shown
For non-calculator methods, make sure that remainders and carrying are shown
Amounts of money should always be rounded to the nearest penny where necessary
It is essential to ensure the students are absolutely clear about the difference between significant figures and decimal places
Extend to multiplication of decimals and/or long division of integers
Try different methods from the traditional ones, eg Russian or Chinese methods for multiplication etc
Give lots of Functional Elements examples

Module **2** **Time: 2 – 4 hours**

GCSE Tier: **Higher**

Contents: **Coordinates**

A k Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information

PRIOR KNOWLEDGE:

Directed numbers

OBJECTIVES

By the end of the module the student should be able to:

- Use axes and coordinates to specify points in all four quadrants in 2-D and 3-D **(15.1, 23.10)**
- Identify points with given coordinates **(15.1)**
- Identify coordinates of given points (NB: Points may be in the first quadrant or all four quadrants) **(15.1)**
- Find the coordinates of points identified by geometrical information in 2-D and 3-D **(15.1, 23.10)**
- Find the coordinates of the midpoint of a line segment, AB, given the coordinates of A and B **(15.2)**

DIFFERENTIATION & EXTENSION

There are some excellent interactive 3-D resources which aid student understanding

NOTES

This topic can be delivered simultaneously with the properties of simple 2-D and 3-D shapes

Module **3** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Fractions**

N h	Understand equivalent fractions
N h	Simplify a fraction by cancelling all common factors
N i, a	Add, subtract, multiply and divide fractions
N b	Order rational numbers
N a	Multiply and divide fractions
N v	Use a calculator effectively and efficiently
N o	Use fractions as operators

PRIOR KNOWLEDGE:

Multiplication facts
Ability to find common factors
A basic understanding of fractions as being 'parts of a whole unit'
Use of a calculator with fractions

OBJECTIVES

By the end of the module the student should be able to:

• Find equivalent fractions	**(3.1, 4.1)**
• Compare the sizes of fractions	**(assumed)**
• Write a fraction in its simplest form	**(assumed)**
• Find fractions of an amount	**(3.2)**
• Convert between mixed numbers and improper fractions	**(assumed)**
• Add, subtract, multiply and divide fractions	**(3.1)**
• Multiply and divide fractions including mixed numbers	**(3.2–3.3)**

DIFFERENTIATION & EXTENSION

Could introduce 'hundredths' at this stage
Solve word problems involving fractions
Improper fractions can be introduced by using real-world examples, eg dividing 5 pizzas equally amongst 3 people
Careful differentiation is essential for this topic dependent upon the student's ability
Use a calculator to change fractions into decimals and look for patterns
Work with improper fractions and mixed numbers
Multiplication and division of fractions to link with probability
Recognising that every terminating decimal has its fraction with 2 and/or 5 as a common factor in the denominator
Introduce algebraic fractions

NOTES

Constant revision of this topic is needed
Use fraction button on the calculator to check solutions
Link with Probability calculations using AND and OR Laws
Use fractions for calculations involving compound units
Use Functional Elements questions and examples using fractions, eg $\frac{1}{4}$ off the list price when comparing different sale prices

Module **4** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Algebra**

A a Distinguish the different roles played by letter symbols in algebra, using the correct notation
A b Distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression'
A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions

PRIOR KNOWLEDGE:

Experience of using a letter to represent a number
Ability to use negative numbers with the four operations
Recall and use BIDMAS

OBJECTIVES

By the end of the module the student should be able to:

- Use notation and symbols correctly **(2.1)**
- Write an expression **(2.1)**
- Select an expression/identity/equation/formula from a list **(13.6)**
- Manipulate algebraic expressions by collecting like terms **(2.1)**
- Multiply a single term over a bracket **(9.1)**
- Factorise algebraic expressions by taking out common factors **(9.2)**
- Expand the product of two linear expressions **(9.3)**
- Factorise quadratic expressions including using the difference of two squares **(9.4)**
- Simplify rational expressions by cancelling, adding, subtracting, and multiplying **(32.1–32.3)**

DIFFERENTIATION & EXTENSION

This topic can be used as a reminder of the KS3 curriculum and could be introduced via investigative material, eg frogs, handshakes, patterns in real life, formulae
Use examples where generalisation skills are required
Extend the above ideas to the 'equation' of the straight line, $y = mx + c$
Look at word formulae written in symbolic form, eg $F = 2C + 30$ to convert temperature (roughly) and compare with $F = \frac{9}{5}C + 32$
Practise factorisation where the factor may involve more than one variable

NOTES

There are plenty of old exam papers with matching tables testing knowledge of the 'Vocabulary of Algebra' (See Emporium website)

Module **5** **Time: 5 – 7 hours**

GCSE Tier: **Higher**

Contents: **Shape and angles**

GM a	Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
GM b	Understand and use the angle properties of parallel lines, triangles and quadrilaterals
GM c	Calculate and use the sums of the interior and exterior angles of polygons
GM d	Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
Gm r	Understand and use bearings

PRIOR KNOWLEDGE:

An understanding of angle as a measure of turning
The ability to use a protractor to measure angles
Understanding of the concept of parallel lines

OBJECTIVES

By the end of the module the student should be able to:

- Recall and use properties of angles **(assumed)**
  - – angles at a point
  - – angles at a point on a straight line
  - – perpendicular lines
  - – vertically opposite angles
- Understand and use the angle properties of parallel lines **(5.1)**
- Understand, draw and measure bearings **(5.5)**
- Calculate bearings and solve bearings problems **(5.5)**
- Distinguish between scalene, isosceles, equilateral, and right-angled triangles **(assumed)**
- Understand and use the angle properties of triangles **(assumed)**
- Use the angle sum of a triangle is 180º **(assumed)**
- Understand and use the angle properties of intersecting lines **(assumed)**
- Mark parallel lines on a diagram **(5.1)**
- Use the properties of corresponding and alternate angles **(5.1, 5.4)**
- Recognise and classify quadrilaterals **(8.3)**
- Understand and use the angle properties of quadrilaterals **(5.3)**
- Give reasons for angle calculations **(Chapter 5)**
- Explain why the angle sum of a quadrilateral is 360º **(5.2)**
- Understand the proof that the angle sum of a triangle is 180º **(5.2)**
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles of the other two vertices **(5.2)**
- Use the size/angle properties of isosceles and equilateral triangles **(5.3, 5.6)**
- Recall and use these properties of angles in more complex problems **(5.3, 5.6)**
- Calculate and use the sums of the interior angles of polygons **(5.7)**
- Use geometric language appropriately and recognise and name pentagons, hexagons, heptagons, octagons and decagons **(5.7)**
- Use the angle sums of irregular polygons **(5.7)**
- Calculate and use the angles of regular polygons **(5.7)**
- Use the sum of the interior angles of an *n* sided polygon **(5.7)**
- Use the sum of the exterior angles of any polygon is 360º **(5.7)**
- Use the sum of the interior angle and the exterior angle is 180º **(5.7)**
- Find the size of each interior angle or the size of each exterior angle or the number of sides of a regular polygon **(5.7)**
- Understand tessellations of regular and irregular polygons and combinations of polygons **(5.7)**
- Explain why some shapes tessellate when other shapes do not **(5.7)**

DIFFERENTIATION & EXTENSION

Use triangles to find the angle sums of polygons
Use the angle properties of triangles to find missing angles in combinations of triangles
Harder problems involving multi-step calculations
Link with symmetry and tessellations

NOTES

Most of this is KS3, so can be treated as an opportunity for groups of students to present parts of the module to the rest of the class. They could be encouraged to make resources, eg follow me cards, puzzles etc for the others to do
Angles in polygons could be investigated algebraically as an investigation
The tessellation can be done as a cross curricular project with Art (Escher) and is good for wall display
Use lots of practical drawing examples to help illustrate properties of various shapes – Group/Displays
Diagrams used in examinations are seldom drawn accurately
Use tracing paper to show which angles in parallel lines are equal
Encourage students to always give their reasons in problems and 'quote' the angle fact/theorem used

Module **6** **Time: 3 – 5 hours**

GCSE Tier: **Higher**

Contents: **Collecting data**

SP a Understand and use statistical problem solving process (handling data cycle)
SP b Identify possible sources of bias
SP c Design an experiment or survey
SP d Design data-collection sheets distinguishing between different types of data
SP e Extract data from printed tables and lists
SP f Design and use two-way tables for discrete and grouped data

PRIOR KNOWLEDGE:

An understanding of why data needs to be collected
Experience of simple tally charts
Experience of inequality notation

OBJECTIVES

By the end of the module the student should be able to:

- Specify the problem and plan **(6.1)**
- Decide what data to collect and what statistical analysis is needed **(6.2)**
- Collect data from a variety of suitable primary and secondary sources **(6.4, 6.8)**
- Use suitable data collection techniques **(6.4)**
- Process and represent the data **(6.4, 6.6)**
- Interpret and discuss the data **(6.7)**
- Discuss how data relates to a problem, identify possible sources of bias and plan to minimise it **(6.7)**
- Understand how different sample sizes may affect the reliability of conclusions drawn **(6.7)**
- Identify which primary data they need to collect and in what format, including grouped data **(6.4)**
- Consider fairness **(6.5, 6.7)**
- Understand sample and population **(6.2)**
- Design a question for a questionnaire **(6.5)**
- Criticise questions for a questionnaire **(6.5)**
- Design an experiment or survey **(6.2, 6.3, 6.5)**
- Select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling **(6.2, 6.3)**
- Use stratified sampling **(6.3)**
- Design and use data-collection sheets for grouped, discrete and continuous data **(6.4)**
- Collect data using various methods **(6.4, 6.5)**
- Sort, classify and tabulate data and discrete or continuous quantitative data **(6.1, 6.4, 6.6)**
- Group discrete and continuous data into class intervals of equal width **(6.4)**
- Extract data from lists and tables **(6.6, 6.8)**
- Design and use two-way tables for discrete and grouped data **(6.6)**
- Use information provided to complete a two way table **(6.6)**

DIFFERENTIATION & EXTENSION

Carry out a statistical investigation of their own, including designing an appropriate means of gathering the data
Some guidance needs to be given to stop students from choosing limited investigations, eg favourite football team
Get data from holiday brochures to compare resorts for temp, rainfall and type of visitor
Carry out a statistical investigation of their own including, designing an appropriate means of gathering the data
Investigation into other sampling schemes, such as cluster, systematic and quota sampling

NOTES

Students may need reminding about the correct use of tallies
Emphasise the differences between primary and secondary data
Discuss sample size and mention that a census is the whole population
In the UK the census takes place every year that ends in a '1' (2011 is the next census)
If students are collecting data as a group, they should all use the same procedure
Emphasise that continuous data is data that is measured, eg temperature
Mayfield High data from coursework task can be used to collect samples and can be used to make comparisons in following sections
Use year group data, eg Mayfield High data to introduce stratified sampling techniques
Use investigations to link with future statistics modules

Module **7** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Displaying data**

SP g Produce charts and diagrams for various data types
SP i Interpret a wide range of graphs and diagrams and draw conclusions
SP j Present findings from databases, tables and charts
SP k Recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
SP l Compare distributions

PRIOR KNOWLEDGE:

An understanding of the different types of data: continuous; discrete;
Experience of inequality notation
Ability to multiply a number by a fraction
Use a protractor to measure and draw angles

OBJECTIVES

By the end of the module the student should be able to:

- Produce: composite bar charts, comparative and dual bar charts, pie charts, histograms with equal or unequal class intervals and frequency diagrams for grouped discrete data, scatter graphs, line graphs, frequency polygons for grouped data, grouped frequency tables for continuous data **(18.1, 18.4–18.7, 24.1–24.2)**
- Interpret: composite bar charts, comparative and dual bar charts, pie charts, scatter graphs, frequency polygons and histograms **(18.2, 18.4–18.7, 24.2)**
- Recognise simple patterns, characteristics and relationships in line graphs and frequency polygons **(18.6, 24.1)**
- Find the median from a histogram or any other information from a histogram, such as the number of people in a given interval **(18.5)**
- From line graphs, frequency polygons and frequency diagrams: read off frequency values, calculate total population, find greatest and least values **(18.5–18.6, 24.1)**
- From pie charts: find the total frequency and find the frequency represented by each sector **(18.2)**
- From histograms: complete a grouped frequency table and understand and define frequency density **(18.7)**
- Present findings from databases, tables and charts **(Chapters 18, 24)**
- Look at data to find patterns and exceptions, explain an isolated point on a scatter graph **(Chapter 18, 24.4)**
- Draw lines of best fit by eye, understanding what these represent **(24.4)**
- Use a line of best fit, or otherwise, to predict values of one variable given values of the other variable **(24.5)**
- Distinguish between positive, negative and zero correlation using lines of best fit **(24.3)**
- Understand that correlation does not imply causality **(24.3)**
- Appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship' **(24.3)**

DIFFERENTIATION & EXTENSION

Carry out a statistical investigation of their own and use an appropriate means of displaying the results
Use a spreadsheet/ICT to draw different types of graphs

NOTES

Collect examples of charts and graphs in the media which have been misused, and discuss the implications
Clearly label all axes on graphs and use a ruler to draw straight lines
Many students enjoy drawing statistical graphs for classroom displays. Include the Functional Elements in this topic with regard to holiday data, energy charts etc
Stem and leaf diagrams must have a key and show how to find the median and mode from a stem and leaf diagram.
Angles for pie charts should be accurate to within 2°. Ask students to check each others' charts
Make comparisons between previously collected data, eg Mayfield boys vs girls or Yr 7 vs Yr 8
Encourage students to work in groups and present their charts – display work in classroom/corridors
Use Excel Graph wizard

Module **8** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Constructions and loci**

GM v Use straight edge and a pair of compasses to carry out constructions
GM w Construct loci

PRIOR KNOWLEDGE:

An ability to use a pair of compasses
The special names of triangles (and angles)
Understanding of the terms perpendicular, parallel and arc

OBJECTIVES

By the end of the module students should be able to:

- Use straight edge and a pair of compasses to do standard constructions **(12.2, 12.3)**
- Construct triangles including an equilateral triangle **(12.1, 12.3)**
- Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not **(12.1)**
- Construct the perpendicular bisector of a given line **(12.2)**
- Construct the perpendicular from a point to a line **(12.2)**
- Construct the perpendicular from a point on a line **(12.2)**
- Construct the bisector of a given angle **(12.3)**
- Construct angles of 60º, 90º , 30º, 45º **(12.3)**
- Draw parallel lines **(assumed)**
- Draw circles and arcs to a given radius **(assumed)**
- Construct a regular hexagon inside a circle **(12.3)**
- Construct diagrams of everyday 2-D situations involving rectangles, triangles, perpendicular and parallel lines **(Ch.12)**
- Draw and construct diagrams from given information **(Ch.12)**
- Construct: a region bounded by a circle and an intersecting line **(12.5)**
  - a given distance from a point and a given distance from a line **(12.4)**
  - equal distances from 2 points or 2 line segments **(12.4)**
  - regions which may be defined by ‘nearer to’ or ‘greater than’ **(12.5)**
- Find and describe regions satisfying a combination of loci **(12.5)**

DIFFERENTIATION & EXTENSION

Solve loci problems that require a combination of loci
Relate to real life examples including horses tethered in fields or mobile phone masts and signal coverage

NOTES

All working should be presented clearly, and accurately
A sturdy pair of compasses is essential
Construction lines should not be erased as they carry method marks
Could use construction to link to similarity and congruence

Module **9** **Time: 5 – 7 hours**

GCSE Tier: **Higher**

Contents: **Types of number**

N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor, Lowest Common Multiple, prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer, negative and fractional powers
N g	Interpret, order and calculate with numbers written in standard form
N v	Use a calculator effectively and efficiently

PRIOR KNOWLEDGE:

Number complements to 10 and multiplication and division facts
Use a number line to show how numbers relate to each other
Recognise basic number patterns
Experience of classifying integers

OBJECTIVES

By the end of the module the student should be able to:

- Identify factors, multiples and prime numbers **(1.1)**
- Find the prime factor decomposition of positive integers **(1.1)**
- Find the common factors and common multiples of two numbers **(1.1)**
- Find the Highest Common Factor (HCF) and the Lowest Common Multiple (LCM) of two numbers **(1.1)**
- Recall integer squares from 2 × 2 to 15 × 15 and the corresponding square roots **(1.2)**
- Recall the cubes of 2, 3, 4, 5 and 10 and cube roots **(1.2)**
- Use index notation for squares and cubes **(1.2)**
- Use index notation for integer powers of 10 **(25.2)**
- Use standard form, expressed in conventional notation **(25.2)**
- Be able to write very large and very small numbers presented in a context in standard form **(25.2)**
- Convert between ordinary and standard form representations **(25.2)**
- Interpret a calculator display using standard form (**25.2)**
- Calculate with standard form **(25.2)**
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer negative and fractional powers, and powers of a power **(1.5, 25.1, 25.3)**

DIFFERENTIATION & EXTENSION

Calculator exercise to check factors of larger numbers
Further work on indices to include negative and/or fractional indices
Use prime factors to find LCM and square roots
Plenty of investigative work for squares like ‘half time’ scores
Use a number square to find primes (sieve of Eratosthenes)
Calculator exercise to find squares, cubes and square roots of larger numbers (using trial and improvement)

NOTES

All of the work in this unit is easily reinforced by starters and plenaries
Calculators are used only when appropriate
Encourage student to learn square, cube, prime and common roots for the non-calculator examination

Module **10** **Time: 3 – 5 hours**

GCSE Tier: **Higher**

Contents: **Patterns and sequences**

A i Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
A j Use linear expressions to describe the *n*th term of an arithmetic sequence

PRIOR KNOWLEDGE:

Know about odd and even numbers
Recognise simple number patterns, eg 1, 3, 5, ...
Writing simple rules algebraically
Raise numbers to positive whole number powers

OBJECTIVES

By the end of the module the student should be able to:

- Recognise sequences of odd and even numbers **(2.5)**
- Generate simple sequences of numbers, squared integers and sequences derived from diagrams **(2.5)**
- Describe the term-to-term definition of a sequence in words **(2.5)**
- Identify which terms cannot be in a sequence **(2.6)**
- Generate specific terms in a sequence using the position-to-term and term-to-term rules **(2.5)**
- Find the *n*th term of an arithmetic sequence **(2.6)**
- Use the *n*th term of an arithmetic sequence **(2.6)**

DIFFERENTIATION & EXTENSION

When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the *n*th term
Match-stick problems
Sequences and *n*th term formula for triangle numbers, Fibonacci numbers etc
Prove a sequence cannot have odd numbers for all values of *n*
Extend to quadratic sequences whose *n*th term is $an^2 + bn + c$

NOTES

Emphasis on good use of notation $3n$ means $3 \times n$
When investigating linear sequences, students should be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the *n*th term

Module **11** **Time: 3 – 5 hours**

GCSE Tier: **Higher**

Contents: **2-D and 3-D shapes**

GM k Use 2-D representations of 3-D shapes

PRIOR KNOWLEDGE:

Construction and loci
Names of 3-D shapes

OBJECTIVES

By the end of the module the student should be able to:

- Use 2-D representations of 3-D shapes **(10.4–10.7)**
- Use isometric grids **(10.4)**
- Draw nets and show how they fold to make a 3-D solid **(10.4)**
- Understand and draw front and side elevations and plans of shapes made from simple solids **(10.5)**
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3-D solid **(10.5)**

DIFFERENTIATION & EXTENSION

Make solids using equipment such as clixi or multi-link with different coloured cubes.
Draw on isometric paper shapes made from multi-link
Construct combinations of 2-D shapes to make nets of 2-D shapes
Build shapes from cubes that are represented in 2-D using cubes
An excellent topic for wall display
Extend to Planes of Symmetry for 3-D solids
Discover Euler's Formula for solids
Investigate how many small boxes can be packed into a larger box, as a Functional-type example

This result is known as **Euler's formula**. An illustration of the formula on some below.

Name	Image	Vertices *V*	Edges *E*	Faces *F*	Euler characteristic: *V* − *E* + *F*
Tetrahedron		4	6	4	**2**
Hexahedron or cube		8	12	6	**2**
Octahedron		6	12	8	**2**
Dodecahedron		20	30	12	**2**
Icosahedron		12	30	20	**2**

NOTES

All working should be presented clearly, and accurately
A sturdy pair of compasses are essential
Accurate drawing skills need to be reinforced
Some students find visualising 3-D objects difficult; simple models will assist

Module **12** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Perimeter and area**

GM x	Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GM z	Find circumferences and areas of circles
N r	Use π in an exact calculation
GM bb	Solve mensuration problems involving more complex shapes and solids
GM p	Convert measurements from one unit to another

PRIOR KNOWLEDGE:

Names of triangles, quadrilaterals and polygons
Concept of perimeter and area
Units of measurement
Substitute numbers into formulae
Ability to give answers to an appropriate degree of accuracy

OBJECTIVES

By the end of the module the student should be able to:

- Measure sides of a shape to work out perimeter or area **(assumed)**
- Find the perimeter of rectangles and triangles **(assumed)**
- Recall and use the formulae for the area of a triangle, rectangle and a parallelogram **(10.1)**
- Find the area of a trapezium **(10.1)**
- Calculate perimeter and area of compound shapes made from triangles, rectangles and other shapes **(10.1, 10.2)**
- Find the surface area of simple shapes (prisms) using the formulae for triangles and rectangles, and other shapes **(23.8)**
- Find circumferences of circles and areas enclosed by circles **(10.3)**
- Recall and use the formulae for the circumference of a circle and the area enclosed by a circle **(10.3, 23.1–23.2)**
- Use $\pi \approx 3.142$ or use the π button on a calculator **(10.3)**
- Give an exact answer to a question involving the area or a circumference of a circle **(23.2)**
- Find the perimeters and areas of semicircles and quarter circles **(10.3)**
- Calculate the lengths of arcs and the areas of sectors of circles **(23.1)**
- Find the surface area of a cylinder **(23.9)**
- Find the area of a segment of a circle given the radius and length of the chord **(Chapter 23)**
- Convert between metric units of area **(23.3)**

DIFFERENTIATION & EXTENSION

Calculate areas and volumes using formulae
Using compound shape methods to investigate areas of other standard shapes such as parallelograms, trapeziums and kites
Emphasise the Functional Elements here with carpets for rooms, tiles for walls, turf for gardens as well as wall paper and skirting board problems
Further problems involving combinations of shapes
Practical activities, eg using estimation and accurate measuring to calculate perimeters and areas of classroom/corridor floors

NOTES

Discuss the correct use of language and units
Ensure that students can distinguish between perimeter, area and volume
Practical experience is essential to clarify these concepts
There are many Functional Elements questions which can be applied to this topic area, eg floor tiles, optimization type questions, which pack of tiles give the best value?

Module **13** **Time: 7 – 9 hours**

GCSE Tier: **Higher**

Contents: **Fractions, decimals and percentages**

N j	Use decimal notation and recognise that each terminating decimal is a fraction
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentage and repeated proportional change
N o	Interpret fractions, decimals and percentages as operators
N v	Use calculators effectively and efficiently
N q	Use percentages

PRIOR KNOWLEDGE:

Four operations of number
The concepts of a fraction and a decimal
Awareness that percentages are used in everyday life

OBJECTIVES

By the end of the module the student should be able to:

- Understand that a percentage is a fraction in hundredths **(14.1)**
- Convert between fractions, decimals and percentages **(14.1)**
- Convert between recurring decimals and exact fractions and use proof **(4.8)**
- Write one number as a percentage of another number **(14.3)**
- Calculate the percentage of a given amount **(14.1, 14.2)**
- Find a percentage increase/decrease of an amount **(14.3)**
- Reverse percentage, eg find the original cost of an item given the cost after a 10% deduction **(14.5)**
- Use a multiplier to increase by a given percent over a given time , eg $1.1^8 \times 64$ increases 64 by 10% over 8 years **(14.4)**
- Calculate simple and compound interest **(14.2, 14.4)**

DIFFERENTIATION & EXTENSION

Find fractional percentages of amounts, without using a calculator, eg 0.825%
Combine multipliers to simplify a series of percentage changes
Percentages which convert to recurring decimals (eg $33\frac{1}{3}$%), and situations which lead to percentages of more than 100%
Problems which lead to the necessity of rounding to the nearest penny (eg real-life contexts)
Comparisons between simple and compound interest calculations

NOTES

Emphasise the Functional Elements in this topic, use real-world problems involving fractions, decimals and percentages
Amounts of money should always be rounded to the nearest penny where necessary, except where such rounding is premature, eg in successive calculations like in compound interest
In preparation for this unit, students should be reminded of basic percentages and recognise their fraction and decimal equivalents
Link with probability calculations using AND and OR Laws

Module **14** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Formulae and linear equations**

A f Derive a formula, substitute numbers into a formula and change the subject of a formula
A d Set up and solve simple equations
A g Solve linear inequalities in one variable, and represent the solution set on a number line

PRIOR KNOWLEDGE:

Experience of finding missing numbers in calculations
The idea that some operations are the reverse of each other
An understanding of balancing
Experience of using letters to represent quantities
Understand and recall BIDMAS

OBJECTIVES

By the end of the module the student should be able to:

- Derive a formula **(19.6)**
- Use formulae from mathematics and other subjects **(19.5)**
- Substitute numbers into a formula **(19.5, 2.2)**
- Substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$ **(19.5, 2.2)**
- Set up linear equations from word problems **(13.5, 19.6)**
- Solve simple linear equations **(13.1, 13.2)**
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation **(13.2, 13.3)**
- Solve linear equations that include brackets, those that have negative signs occurring anywhere in the equation, and those with a negative solution **(13.2–13.4)**
- Solve linear equations in one unknown, with integer or fractional coefficients **(13.4, 13.5)**
- Solve simple linear inequalities in one variable, and represent the solution set on a number line **(19.1–19.3)**
- Use the correct notation to show inclusive and exclusive inequalities **(19.1–19.4)**
- Change the subject of a formula including cases where the subject is on both sides of the original formula, or where a power of the subject appears **(19.7, 19.8)**

DIFFERENTIATION & EXTENSION

Use negative numbers in formulae involving indices
Use investigations to lead to generalisations
Apply changing the subject to $y = mx + c$
Derive equations from practical situations (such as finding unknown angles in polygons or perimeter problems)

NOTES

Emphasise good use of notation $3ab$ means $3 \times a \times b$
Students need to be clear on the meanings of the words expression, equation, formula and identity
Students need to realise that not all linear equations can easily be solved by either observation or trial and improvement, and hence the use of a formal method is important
Students can leave their answers in fractional form where appropriate

Module **15** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Linear graphs**

A l	Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
A m	Understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line and c is the value of the y-intercept
A n	Understand the gradients of parallel lines
A g	Solve linear inequalities in two variables, and represent the solution set on a coordinate grid
A r	Construct linear functions from real-life problems and plot their corresponding graphs
A s	Interpret graphs of linear functions

PRIOR KNOWLEDGE

Being able to:

Substitute positive and negative numbers into algebraic expressions
Plot coordinates in the first quadrant
Rearrange to change the subject of a formula

OBJECTIVES

By the end of the module the student should be able to:

- Draw, label and scale axes **(assumed)**
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane **(15.4)**
- Draw and interpret straight line graphs for real-life situations **(15.3, 15.6)**
  - ready reckoner graphs
  - conversion graphs
  - fuel bills, eg gas and electric
  - fixed charge (standing charge) and cost per unit
- Plot and draw graphs of straight lines with equations of the form $y = mx + c$ **(15.4)**
- Find the gradient of a straight line from a graph **(15.3)**
- Analyse problems and use gradients to interpret how one variable changes in relation to another **(15.3)**
- Interpret and analyse a straight-line graph **(15.4)**
- Understand that the form $y = mx + c$ represents a straight line **(15.4)**
- Find the gradient of a straight line from its equation **(15.4)**
- Explore the gradients of parallel lines and lines perpendicular to each other **(15.5)**
- Write down the equation of a line parallel or perpendicular to a given line **(15.5)**
- Use the fact that when $y = mx + c$ is the equation of a straight line then the gradient of a line parallel to it will have a gradient of m and a line perpendicular to this line will have a gradient of $-\frac{1}{m}$ **(15.5)**
- Interpret and analyse a straight line graph and generate equations of lines parallel and perpendicular to the given line **(15.5)**
- Show the solution set of several inequalities in two variables on a graph **(19.4)**

DIFFERENTIATION & EXTENSION

Find the equation of the line through two given points
Find the equation of the perpendicular bisector of the line segment joining two given points
Use Functional Elements in terms of mobile phone bills
Use a spreadsheet to generate straight-line graphs, posing questions about the gradient of lines
Use a graphical calculator or graphical ICT package to draw straight-line graphs
Link to scatter graphs and correlation
Cover horizontal and vertical lines ($x = c$ and $y = c$), as students often forget these

NOTES

Careful annotation should be encouraged. Label the coordinate axes and origin and write the equation of the line
Recognise linear graphs and hence when data may be incorrect
Link to graphs and relationships in other subject areas, i.e. science, geography etc
Link conversion graphs to converting metric and imperial units
A-Level C1 text books can be a good source of extension questions on this topic

Module **16** **Time: 3 – 5 hours**

GCSE Tier: **Higher**

Contents: **Simultaneous equations**

A d Set up and solve simultaneous equations in two unknowns

PRIOR KNOWLEDGE:

Introduction to algebra
Linear functions
Solving equations

OBJECTIVES

By the end of the module the student should be able to:

- Find the exact solutions of two simultaneous equations in two unknowns **(22.1)**
- Use elimination or substitution to solve simultaneous equations **(22.1)**
- Interpret a pair of simultaneous equations as a pair of straight lines and their solution as the point of intersection **(22.3)**
- Set up and solve a pair of simultaneous equations in two variables **(22.2)**

DIFFERENTIATION & EXTENSION

Inaccurate graphs could lead to incorrect solutions
Clear presentation of workings is essential
Use open ended questions that test student awareness of what intersections mean for mobile phone bills
Solve two simultaneous equations with fractional coefficients
Solve two simultaneous equations with second order terms, eg equations in x and y^2

NOTES

Build up the algebraic techniques slowly
Link the graphical solutions with linear graphs and changing the subject
Inaccurate graphs could lead to incorrect solutions, encourage substitution of answers to check they are correct
Clear presentation of working is essential

Module **17** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Probability**

SP m	Understand and use the vocabulary of probability and the probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP q	Know when to add or multiply two probabilities: when A and B are mutually exclusive, then the probability of A or B occurring is $P(A) + P(B)$, whereas when A and B are independent events, the probability of A and B occurring is $P(A) \times P(B)$
SP r	Use tree diagrams to represent outcomes of compound events, recognising when events are independent
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

PRIOR KNOWLEDGE:

Understand that a probability is a number between 0 and 1
Know how to add and multiply fractions and decimals
Experience of expressing one number as a fraction of another number
Recognise the language of probability, eg words such as likely, certain, impossible

OBJECTIVES

By the end of the module the student should be able to:

- Write probabilities using fractions, percentages or decimals **(28.1)**
- Understand and use estimates or measures of probability, including relative frequency **(28.1, 28.3)**
- Use theoretical models to include outcomes using dice, spinners, coins etc **(28.1, 28.4)**
- Find the probability of successive events, such as several throws of a single dice **(28.1)**
- Estimate the number of times an event will occur, given the probability and the number of trials **(28.4)**
- List all outcomes for single events, and for two successive events, systematically **(28.1)**
- Use and draw sample space diagrams **(28.1)**
- Add simple probabilities, eg from sample space diagrams **(28.2)**
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1 **(28.2)**
- Use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring **(28.2)**
- Find a missing probability from a list or table **(28.2)**
- Understand conditional probabilities **(28.7)**
- Understand selection with or without replacement **(28.5, 28.7)**
- Draw a probability tree diagram based on given information **(28.6)**
- Use a tree diagram to calculate conditional probability **(28.7)**
- Compare experimental data and theoretical probabilities **(28.3)**
- Compare relative frequencies from samples of different sizes **(28.3)**

DIFFERENTIATION & EXTENSION

An opportunity for practical examples, eg P(pin up) for a drawing pin, the 'horse' race, the national lottery
Show that each cluster of branches on a tree diagram adds up to 1
Explain that if two objects are chosen, then this is the same as one event followed by another event without replacement
Show that it is often easier to solve a problem involving multiple outcomes, by considering the *opposite* event and subtracting from 1, eg 'at least' two reds, 'at least' two beads of a different colour etc)
Experiments with dice and spinners
Show sample space for outcomes of throwing 2 dice
Stress that there are 36 outcomes (they will initially guess it's 12 outcomes for 2 dice)
Binomial probabilities (H or T)

Do a question ‘with’ and then repeat it ‘without’ replacement. Good idea to show the contents of the bag and physically remove the object to illustrate the change of probability fraction for the second selection

NOTES

Students should express probabilities as fractions, percentages or decimals
Fractions do not need to be cancelled to their lowest terms. This makes it easier to calculate tree diagram probabilities, eg easier to add like denominators

Module **18** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Ratio and scale**

N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N t	Divide a quantity in a given ratio
GM m	Use and interpret maps and scale drawings
N n	Understand and use direct and indirect proportion
A u	Use direct and indirect proportion

PRIOR KNOWLEDGE:

Fractions

OBJECTIVES

By the end of the module the student should be able to:

- Use ratios **(16.1)**
- Write ratios in their simplest form **(16.1)**
- Divide a quantity in a given ratio **(16.3)**
- Solve a ratio problem in a context **(16.2)**
- Use and interpret maps and scale drawings **(12.6)**
- Read and construct scale drawings drawing lines and shapes to scale **(12.6)**
- Estimate lengths using a scale diagram **(12.6)**
- Solve word problems about ratio and proportion **(16.4–16.5)**
- Calculate an unknown quantity from quantities that vary in direct or inverse proportion **(16.4, 16.5)**
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relate algebraic solutions to graphical representation of the equations **(27.1–27.5)**

DIFFERENTIATION & EXTENSION

Harder problems involving multi-stage calculations

Relate ratios to Functional Elements situations, eg investigate the proportions of the different metals in alloys and the new amounts of ingredients for a recipe for different numbers of guests

Harder problems involving multi-stage calculations

NOTES

Students often find ratios with three parts difficult

Link ratios given in different units to metric and imperial units

Module **19** **Time: 7 – 9 hours**

GCSE Tier: **Higher**

Contents: **Averages and range**

SP h	Calculate median, mean, range, quartiles and interquartile range, mode, modal class and interval containing the median
SP g	Produce charts and diagrams for various data types
SP i	Interpret a wide range of graphs and diagrams and draw conclusions
SP l	Compare distributions and make inferences
SP u	Use calculators efficiently and effectively, including statistical functions

PRIOR KNOWLEDGE:

Knowledge of finding the mean for small data sets
Ability to find the midpoint of two numbers

OBJECTIVES

By the end of the module the student should be able to:

- Calculate mean, mode, median and range for small data sets **(11.1, 11.2, 11.4–11.7)**
- Recognise the advantages and disadvantages between measures of average **(11.3)**
- Produce ordered stem and leaf diagrams and use them to find the range and averages **(18.3)**
- Calculate averages and range from frequency tables (Use Σx and Σfx) **(11.4)**
- Estimate the mean for large data sets with grouped data (and understand that it is an estimate) **(11.6)**
- Draw and interpret cumulative frequency tables and graphs **(18.8)**
- Use cumulative frequency graphs to find median, quartiles and interquartile range **(18.9)**
- Draw box plots from a cumulative frequency graph **(18.10)**
- Compare the measures of spread between a pair of box plots/cumulative frequency graphs **(18.10)**
- Interpret box plots to find median, quartiles, range and interquartile range **(18.10)**
- Find the median from a histogram **(18.5)**
- Compare distributions and make inferences, using the shapes of distributions and measures of average and spread, including median and quartiles **(11.7, 18.6)**
- Find quartile and interquartile range from data **(11.7)**
- Find modal class and interval containing the median **(11.5)**

DIFFERENTIATION & EXTENSION

Use statistical functions on calculators and spreadsheets
Use statistical software to calculate the mean for grouped data sets
Estimate the mean for data sets with ill defined class boundaries
Investigate the affect of combining class intervals on estimating the mean for grouped data sets
Students should understand that finding an *estimate for the mean* of grouped data is not a guess
Opportunity to remind them of Module 6
Pose the question: 'Investigate if the average number of children per family is 2.4.', 'Are the families represented in your class representative of the whole population?'
Discuss occasions when one average is more appropriate, and the limitations of each average
Possibly mention standard deviation (not on course, but good for further comparison of data sets with similar means)

NOTES

Collect data from class – children per family etc. Extend to different classes, year groups or secondary data from the internet. (Previous coursework tasks are a rich source of data to work with, eg *Second-Hand Car Sales*)
Compare distributions and make inferences, using the shapes of distributions and measures of average and spread, eg 'boys are taller on average but there is a much greater spread in heights' (Use date collected from previous investigations or Mayfield High data)
Students tend to select modal class but identify it by the frequency rather than the class itself
Explain that the median of grouped data is not necessarily from the middle class interval

Module **20** **Time: 7 – 9 hours**

GCSE Tier: **Higher**

Contents: **Pythagoras' theorem and Trigonometry**

GM g	Use Pythagoras' theorem in 2-D and 3-D
N r	Use surds in exact calculations
GM h	Use the trigonometric ratios to solve 2-D and 3-D problems
N v	Use calculators effectively and efficiently
N u	Round to specified or appropriate degrees of accuracy including a given, number of decimal places and significant figures
A k	Find the length of a line segment

PRIOR KNOWLEDGE:

Some understanding of similar triangles
Able to use a calculator to divide numbers
Mensuration – perimeter and area 1
Formulae

OBJECTIVES

By the end of the module the student should be able to:

- Understand, recall and use Pythagoras' theorem in 2-D, then in 3-D problems **(20.1–20.2, 29.1)**
- Calculate the length of a line segment in a 2-D plane **(20.3)**
- Give an answer in the use of Pythagoras' Theorem as √13 **(25.4)**
- Recall and use the trigonometric ratios to solve 2-D and 3-D problems **(20.4, 20.5, 29.1, 29.2, 29.9)**
- Find angles of elevation and angles of depression **(20.5)**
- Understand the language of planes, and recognise the diagonals of a cuboid **(29.2)**
- Calculate the length of a diagonal of a cuboid **(29.1)**
- Find the angle between a line and a plane (but not the angle between two planes or between two skew lines) **(29.2)**

DIFFERENTIATION & EXTENSION

Look at Functional Elements exemplar material
Harder problems involving multi-stage calculations
Organise a practical surveying lesson to find the heights of buildings/trees around your school grounds. All you need is a set of tape measures (or trundle wheels) and clinometers

NOTES

Students should be encouraged to become familiar with one make of calculator
Calculators should be set to "deg" mode
Emphasise that scale drawings will score no marks for this type of question
A useful mnemonic for remember trig ratios is "Sir Oliver's Horse, Came Ambling Home, To Oliver's Aunt" or 'SOH/CAH/TOA'; but students often enjoy making up their own
Calculated angles should be given to at least 1 decimal place and sides are determined by the units used or accuracy asked for in the question
Students should not forget to state the units for the answers
The angle between two planes or two skew lines is not required

Module **21** **Time: 3 – 5 hours**

GCSE Tier: **Higher**

Contents: **Trial and Improvement**

A h	Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
N u	Round to specified or appropriate degrees of accuracy including a number of decimal places and significant figures
N v	Use calculators effectively and efficiently

PRIOR KNOWLEDGE:

Substituting numbers into algebraic expressions
Dealing with decimals on a calculator
Ordering decimals

OBJECTIVES

By the end of the module the student should be able to:

- Solve cubic equations by successive substitution of values of x **(21.5)**
- Use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them **(21.5)**
- Understand the connections between changes of sign and location of roots **(21.5)**

DIFFERENTIATION & EXTENSION

Solve functions of the form $\frac{1}{x} = x^2 - 5$ (link with changing the subject)

NOTES

Look at 'practical examples'. A room is 2 m longer than it is wide. If its area is 30 m² what is its perimeter?
Students should be encouraged to use their calculators efficiently – by using the "replay" or ANS/EXE functions
The square/cube function on a calculator may not be the same for different makes
Take care when entering negative values to be squared (always use brackets)
Students should write down all the digits on their calculator display and only round the final answer declared to the degree of accuracy

Module **22** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Surface Area and Volume**

GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids
GM x	Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GM z	Find the surface area of a cylinder
GM bb	Solve mensuration problems involving more complex shapes and solids
GM p	Convert measures from one unit to another
GM p	Convert between volume measures, including cubic centimetres and cubic metres
N r	Use π in an exact calculation

PRIOR KNOWLEDGE:

Concept of volume
Knowledge of area module
Ability to give answers to a degree of accuracy
Experience of changing the subject of a formula

OBJECTIVES

By the end of the module the student should be able to:

- Know and use formulae to calculate the surface areas and volumes of cuboids and right-prisms and shapes made from cuboids **(10.6, 10.7, 23.8)**
- Solve a range of problems involving surface area and volume, eg given the volume and length of a cylinder find the radius **(10.6–10.8, 23.4–23.6, 23.8–23.9)**
- Find the volume of a cylinder and surface area of a cylinder **(10.8, 23.9)**
- Convert between volume measures, including cubic centimetres and cubic metres **(23.7)**
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones **(23.6)**
- Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinder, eg solids in everyday use **(23.4–23.6, 23.8–23.9)**
- Convert between units of capacity and volume **(23.7)**

DIFFERENTIATION & EXTENSION

Additional work using algebraic expressions
Find surface area and volume of a sphere and cone (using standard formulae)
Convert between less familiar units, eg cm^3 to mm^3, cm^3 to litres
Look at functional type questions, eg fitting boxes in crates
Look at in conjunction with Module 23 and density/volume/mass questions
Find the volume of a cylinder given its surface area, leaving the answer in terms of l
Find the volume of a right hexagonal pyramid of side x and height h (researching the method for finding the volume of any pyramid)

NOTES

'Now! I Know Pi' is a good way to learn the approx value (The number of letters of each word and the ! is the decimal point)
Also 'Cherry Pie Delicious' is $C = \pi D$ and 'Apple Pies are too' is $A = \pi r^2$
Answers in terms of π may be required or final answers rounded to the required degree of accuracy
Need to constantly revise the expressions for area/volume of shapes
Students should be aware of which formulae are on the relevant page on the exam paper and which they need to learn

Module **23** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Compound measures**

GM o	Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GM p	Convert measurements from one unit to another
GM q	Make sensible estimates of a range of measures
GM s	Understand and use compound measures
A r	Draw and interpret distance time graphs
A s	Discuss, plot and interpret graphs (which may be non-linear) modelling real situations
N s	Calculate upper and lower bounds

PRIOR KNOWLEDGE:

Knowledge of metric units, eg 1 m = 100 cm etc
Know that 1 hour = 60 mins, 1 min = 60 seconds
Experience of multiply by powers of 10, e.g 100 × 100 = 10 000

OBJECTIVES

By the end of the module the student should be able to:

- Convert between units of measure in the same system. (NB: Conversion between imperial units will be given. Metric equivalents should be known) **(7.1)**
- Know rough metric equivalents of pounds, feet, miles, pints and gallons: **(7.1)**
  Metric/Imperial
  1 kg= 2.2 pounds
  1 litre=1.75 pints
  4.5l= 1 gallon
  8 km = 5 miles
  30 cm = 1 foot
- Convert between imperial and metric measures **(7.1)**
- Use the relationship between distance, speed and time to solve problems **(7.2, 7.3)**
- Convert between metric units of speed, eg km/h to m/s **(7.3)**
- Construct and interpret distance time graphs **(15.6)**
- Know that density is found by mass ÷ volume **(7.4)**
- Use the relationship between density, mass and volume to solve problems, eg find the mass of an object with a given volume and density **(7.4)**
- Convert between metric units of density, eg kg/m³ to g/cm³ **(7.4)**
- Calculate speed **(7.3)**
- Calculate the upper and lower bounds of calculations, particularly when working with measurements **(4.9, 4.10)**
- Find the upper and lower bounds of calculations involving perimeter, areas and volumes of 2-D and 3-D shapes **(4.10)**
- Find the upper and lower bounds in real life situations using measurements given to appropriate degrees of accuracy **(4.10)**
- Give the final answer to an appropriate degree of accuracy following an analysis of the upper and lower bounds of a calculation **(4.10)**

DIFFERENTIATION & EXTENSION

Perform calculations on a calculator by using standard form
Convert imperial units to metric units, eg mph into km/h
Help students to recognise the problem they are trying to solve by the unit measurement given, eg km/h is a unit of speed as it is a distance divided by a time
Mention other units (not on course) like hectares

NOTES

Use a formula triangle to help students see the relationship between the variables for density
Borrow a set of electronic scales and a Eureka Can from Physics for a practical density lesson
Look up densities of different elements from the net
Link converting area & volume units to similar shapes (Module 25)
Draw a large grid made up of 100 by 100 cm squares to show what 1 square metre looks like

Module **24** **Time: 5 – 7 hours**

GCSE Tier: **Higher**

Contents: **Transformations**

GM e Recognise reflection and rotation symmetry of 2-D shapes
GM l Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive, fractional or negative scale factor
GM l Distinguish properties that are preserved under particular transformations

PRIOR KNOWLEDGE:

Recognition of basic shapes
An understanding of the concept of rotation, reflection and enlargement
Coordinates in four quadrants
Linear equations parallel to the coordinate axes

OBJECTIVES

By the end of the module the student should be able to:

- Recognise rotation and reflection of 2-D shapes **(8.2)**
- Understand translation as a combination of a horizontal and vertical shift including signs for directions **(17.1)**
- Translate a given shape by a vector **(17.1)**
- Understand rotation as a (anti clockwise) turn about a given origin **(17.3)**
- Reflect shapes in a given mirror line; parallel to the coordinate axes and then $y = x$ or $y = -x$ **(17.2)**
- Enlarge shapes by a given scale factor from a given point; using positive, negative and fractional scale factors **(17.4)**
- Find the centre of enlargement **(17.4)**
- Understand that images produced by translation, rotation and reflection are congruent to the object **(17.1–17.3)**
- Describe and transform 2-D shapes using single rotations **(17.3)**
- Understand that rotations are specified by a centre and an (anticlockwise) angle **(17.3)**
- Find the centre of rotation **(17.3)**
- Rotate a shape about the origin, or any other point **(17.3)**
- Describe and transform 2-D shapes using combined rotations, reflections, translations, or enlargements **(17.5)**
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations **(17.1–17.3)**
- Distinguish properties that are preserved under particular transformations **(17.1–17.4)**
- Recognise that enlargements preserve angle but not length, linking to similarity **(17.4)**
- Describe a transformation **(Chapter 17)**

DIFFERENTIATION & EXTENSION

The tasks set can be extended to include combinations of transformations
Research glide reflection

NOTES

Emphasise that students describe the given transformation fully
Diagrams should be drawn carefully
The use of tracing paper is allowed in the examination (although students should not have to rely on the use of tracing paper to solve problems)

Module **25** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Similarity and Congruence**

GM f Understand congruence and similarity
GM n Understand and use the effect of enlargement for perimeter, area and volume of shapes and solids
N q Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations

PRIOR KNOWLEDGE:

Ratio
Proportion
Area and Volume

OBJECTIVES

By the end of the module the student should be able to:

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and a pair of compasses constructions **(8.1)**
- Understand similarity of triangles and of other plane figures, and use this to make geometric inferences **(8.4)**
- Formal geometric proof of similarity of two given triangles **(8.4, 8.5)**
- Recognise that all corresponding angles in similar figures are equal in size when the lengths of sides are not **(8.4)**
- Understand the effect of enlargement for perimeter, area and volume of shapes and solids **(26.1–26.2)**
- Understand that enlargement does not have the same effect on area and volume **(26.2)**
- Use simple examples of the relationship between enlargement and areas and volumes of simple shapes and solids **(26.1–26.2)**
- Use the effect of enlargement on areas and volumes of shapes and solids **(26.1–26.2)**
- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids **(26.3)**

DIFFERENTIATION & EXTENSION

This could be introduced practically or by investigating simple shapes such as squares, rectangles, circles (reminder of formula), cuboids, cylinders etc
Solve loci problems that require a combination of loci
Construct combinations of 2-D shapes to make nets
Link with tessellations and enlargements
Link with similar areas and volumes
Harder problems in congruence
Relate this unit to circle theorems

NOTES

All working should be presented clearly, and accurately

Module **26** **Time: 6 – 8 hours**

GCSE Tier: **Higher**

Contents: **Quadratic functions, equations and graphs**

A c Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, factorising quadratic expressions, and difference of two squares

A t Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions

A r Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs

A e Solve quadratic equations

A o Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions

PRIOR KNOWLEDGE:

Graphs and algebra

OBJECTIVES

By the end of the module the student should be able to:

- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions **(21.1)**
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function **(21.1)**
- Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions **(21.1)**
- Solve simple quadratic equations by factorisation and completing the square **(22.4–22.9)**
- Solve simple quadratic equations by using the quadratic formula **(22.7–22.9)**
- Select and apply algebraic and graphical techniques to solve simultaneous equations where one is linear and one quadratic **(22.11–22.12)**
- Solve equations involving algebraic fractions which lead to quadratic equations **(22.8)**
- Solve quadratic equations by completing the square **(22.6)**

DIFFERENTIATION & EXTENSION

Derive the quadratic equation by completing the square
Use graphical calculators or ICT graph package where appropriate
Show how the value of '$b^2 - 4ac$' can be useful in determining if the quadratic factorises or not (i.e. square number)
Extend to discriminant's properties and roots

NOTES

Lots of practical type examples, eg projectiles
Some students may need additional help with factorising
Students should be reminded that factorisation should be tried before the formula is used
In problem-solving, one of the solutions to a quadratic may not be appropriate
There may be a need to remove the HCF (numerical) of a trinomial before factorising to make the factorisation easier

Module **27** **Time: 5 – 7 hours**

GCSE Tier: **Higher**

Contents: **Index notation and surds**

N e	Use index notation for squares, cubes and powers of 10
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N f	Use index laws for multiplication and division of integer, fractional and negative powers
N v	Use calculators effectively and efficiently
N r	Calculate with surds
A c	Simplify expressions using rules of indices

PRIOR KNOWLEDGE:

Knowledge of squares, square roots, cubes and cube roots
Fractions and algebra

OBJECTIVES

By the end of the module the student should be able to:

- Find the value of calculations using indices **(1.5, 25.1, 25.3)**
- Use index laws to simplify and calculate numerical expressions involving powers, eg $(2^3 \times 2^5) \div 2^4$, 4^0, $8^{-2/3}$ **(1.5, 25.1, 25.3)**
- Know that, eg $x^3 = 64 \Rightarrow x = 8^{2/3}$ **(25.3)**
- Rationalise the denominator, eg $\frac{1}{\sqrt{3}-1} = \left(\frac{\sqrt{3}+1}{2}\right)$, and eg write $(\sqrt{18}+10) \div \sqrt{2}$ in the form $p + q\sqrt{2}$ **(25.4)**
- Use calculators to explore exponential growth and decay **(21.4)**
- Write $\sqrt{8}$ in the form $2\sqrt{2}$ **(25.4)**
- Simplify expressions using index laws **(2.4)**
- Use index laws to write expressions for integer, negative, and fractional powers and powers of a power **(1.5, 25.1, 25.3)**

DIFFERENTIATION & EXTENSION

Use index laws to simplify algebraic expressions
Treat index laws as formulae (state which rule is being at each stage in a calculation)
Explain the difference between rational and irrational numbers as an introduction to surds
Prove that $\sqrt{2}$ is irrational
Revise the difference of two squares to show why we use, for example $(\sqrt{3} - 2)$ as the multiplier to rationalise $(\sqrt{3} + 2)$
Link to work on circle measures (involving π) and Pythagoras calculations in exact form

NOTES

Link simplifying surds to collecting together like terms, eg $3x + 2x = 5x$, so therefore $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$
Stress it is better to write answers in exact form, eg $\frac{1}{3}$ is better than 0.333333…..
A-Level C1 textbooks are a good source of extension questions on surd manipulation, some of which are algebraic
Useful generalisation to learn $\sqrt{x} \times \sqrt{x} = x$

Module **28** **Time: 3 – 5 hours**

GCSE Tier: **Higher**

Contents: **Circle theorems**

GM i Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
GM j Understand and construct geometrical proofs using circle theorems

PRIOR KNOWLEDGE:

Recall the words centre, radius, diameter and circumference
Have practical experience of drawing circles with compasses

OBJECTIVES

By the end of the module the student should be able to:

- Recall the definition of a circle and identify (name) and draw the parts of a circle **(assumed)**
- Understand related terms of a circle **(assumed)**
- Draw a circle given the radius or diameter **(assumed)**
- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point **(31.2)**
- Understand and use the fact that tangents from an external point are equal in length **(31.2)**
- Find missing angles on diagrams **(chapter 31**
- Give reasons for angle calculations involving the use of tangent theorems **(31.2)**
- Prove and use the facts that: **(31.3–31.4)**
  - the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference **(31.3)**
  - the angle in a semicircle is a right angle **(31.3)**
  - angles in the same segment are equal **(31.4)**
  - opposite angles of a cyclic quadrilateral sum to 180º **(31.4)**
  - alternate segment theorem **(31.4)**
  - the perpendicular from the centre of a circle to a chord bisect the chord **(31.3)**

DIFFERENTIATION & EXTENSION

Harder problems involving multi-stage angle calculations
Intersecting chord theorem

NOTES

Any proof required will be in relation to a diagram, not purely by reference to a named theorem
Reasoning needs to be carefully constructed as 'Quality of Written Communication' marks are likely to be allocated to proofs

Module **29** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Sine and cosine rules**

GM h Use the sine and cosine rules to solve 2-D and 3-D problems

GM y Calculate the area of a triangle using $\frac{1}{2}\ ab\sin C$

PRIOR KNOWLEDGE:

Trigonometry
Formulae

OBJECTIVES

By the end of the module the student should be able to:

- Calculate the unknown lengths, or angles, in non right-angle triangles using the sine and cosine rules **(29.5–29.9)**
- Calculate the area of triangles given two lengths and an included angle **(29.4)**

DIFFERENTIATION & EXTENSION

Use these ratios to solve problems in 3-D and decide if it is easier to extract right-angle triangles to use 'normal' trigonometry
Stress that the cosine rule is only used when we have SAS (and we need to find the side opposite the angle given) or when we are given SSS (then we use the re-arranged version to find any angle) [else we use the Sine Rule]

NOTES

Reminders of simple geometrical facts may be helpful, eg angle sum of a triangle, the shortest side is opposite the smallest angle
Show the form of the cosine rule in the formula page and re-arrange it to show the form which finds missing angles

Module **30** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Vectors**

GM cc Use vectors to solve problems

PRIOR KNOWLEDGE:

Vectors to describe translations
Geometry of triangles and quadrilaterals

OBJECTIVES

By the end of the module the student should be able to:

- Understand that 2**a** is parallel to **a** and twice its length **(33.4)**
- Understand that **a** is parallel to –**a** and in the opposite direction **(33.4)**
- Use and interpret vectors as displacements in the plane (with an associated direction) **(33.1)**
- Use standard vector notation to combine vectors by addition, eg $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{AC}}$ and **a** + **b** = **c** **(33.3)**
- Represent vectors, and combinations of vectors, in the plane **(33.1–33.5)**
- Solve geometrical problems in 2-D, eg show that joining the midpoints of the sides of any quadrilateral forms a parallelogram **(33.5)**

DIFFERENTIATION & EXTENSION

Harder geometric proof, eg show that the medians of a triangle intersect at a single point
Illustrate use of vectors by showing 'Crossing the flowing River' example or navigation examples
Vector problems in 3-D (for the most able)
Use **i** and **j** (and **k**) notation

NOTES

Students often find the pictorial representation of vectors more difficult than the manipulation of column vectors
Geometry of a hexagon provides a rich source of parallel, reverse and multiples of vectors
Stress that parallel vectors are equal
Link with like terms and brackets when simplifying
Show there is more than one route round a geometric shape, but the answer simplifies to the same vector
Remind students to underline vectors or they will be regarded as just lengths with no direction
Some extension questions can be found in Mechanics 1 textbooks

Module **31** **Time: 4 – 6 hours**

GCSE Tier: **Higher**

Contents: **Further graphs and functions**

A o Find the intersection points of the graphs of a linear and quadratic function

A p Draw, sketch, recognise graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$,
the function $y = k^x$ for integer values of x and simple positive values of k,
the trigonometric functions $y = \sin x$ and $y = \cos x$

A q Construct the graphs of simple loci

PRIOR KNOWLEDGE:

Linear functions 1
Quadratic functions

OBJECTIVES

By the end of the module the student should be able to:

- Plot and recognise cubic, reciprocal, exponential and circular functions $y = \sin x$ and $y = \cos x$, within the range -360º to +360º (see above) **(21.2–21.4, 22.10, 29.3)**
- Use the graphs of these functions to find approximate solutions to equations, eg given x find y (and vice versa) **(21.2–21.4, 22.10, 29.3)**
- Find the values of p and q in the function $y = pq^x$ given the graph of $y = pq^x$ **(21.4)**
- Match equations with their graphs **(21.2–21.4, 22.10, 29.3)**
- Recognise the characteristic shapes of all these functions **(21.2–21.4, 22.10, 29.3)**
- Construct the graphs of simple loci including the circle $x^2 + y^2 = r^2$ for a circle of radius r centred at the origin of the coordinate plane **(22.10)**
- Find the intersection points of a given straight line with this circle graphically **(22.12)**
- Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines **(22.10)**
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other, or where the second equation is of the form $x^2 + y^2 = r^2$ **(22.11–22.12)**

DIFFERENTIATION & EXTENSION

Explore the function $y = e^x$ (perhaps relate this to $y = \ln x$)
Explore the function $y = \tan x$
Find solutions to equations of the circular functions $y = \sin x$ and $y = \cos x$ over more than one cycle (and generalise)
This work should be enhanced by drawing graphs on graphical calculators and appropriate software
Complete the square for quadratic functions and relate this to transformations of the curve $y = x^2$

NOTES

Make sure the students understand the notation $y = f(x)$, start by comparing $y = x^2$ with $y = x^2 + 2$ before mentioning $y = f(x) + 2$ etc
Graphical calculators and/or graph drawing software will help to underpin the main ideas in this unit
Link with trigonometry and curved graphs

Module **32** **Time: 3 – 5 hours**

GCSE Tier: **Higher**

Contents: **Transformations of functions**

A v Transformation of functions

PRIOR KNOWLEDGE:

Transformations
Using f(x) notation

OBJECTIVES

By the end of the module the student should be able to:

- Apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$, $y = af(x)$ for linear, quadratic, sine and cosine functions **(30.2–30.3)**
- Select and apply the transformations of reflection, rotation, enlargement and translation of functions expressed algebraically **(30.2–30.4)**
- Interpret and analyse transformations of functions and write the functions algebraically **(30.1–30.4)**

DIFFERENTIATION & EXTENSION

Complete the square of quadratic functions and relate this to transformations of the curve $y = x^2$
Use a graphical calculator/software to investigate transformations
Investigate curves which are unaffected by particular transformations
Investigations of the simple relationships such as $\sin(180 - x) = \sin x$, and $\sin(90 - x) = \cos x$

NOTES

Make sure the students understand the notation $y = f(x)$, start by comparing $y = x^2$ with $y = x^2 + 2$ before mentioning $y = f(x) + 2$ etc
Graphical calculators and/or graph drawing software will help to underpin the main ideas in this unit
Link with trigonometry and curved graphs

Higher course objectives (1MA0)

Number

N a	Add, subtract, multiply and divide whole numbers, integers and decimals
N a	Multiply and divide fractions
N b	Order integers and decimals
N b	Order rational numbers
N c	Use the concepts and vocabulary of factor (divisor), multiple, common factor, Highest Common Factor, Lowest Common Multiple, prime number and prime factor decomposition
N d	Use the terms square, positive and negative square root, cube and cube root
N e	Use index notation for squares, cubes and powers of 10
N f	Use index laws for multiplication and division of integer, fractional and negative powers
N g	Interpret, order and calculate with numbers written in standard index form
N h	Understand equivalent fractions
N h	Simplify a fraction by cancelling all common factors
N i, a	Add, subtract, multiply and divide fractions
N j	Use decimal notation
N j	Use decimal notation and recognise that each terminating decimal is a fraction
N k	Recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
N l	Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions
N m	Use percentage and repeated proportional change
N n	Understand and use direct and indirect proportion
N o	Interpret fractions, decimals and percentages as operators
N o	Use fractions as operators
N p	Use ratio notation, including reduction to its simplest form and its various links to fraction notation
N q	Use percentages
N q	Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
N r	Use π in an exact calculation
N r	Calculate with surds
N r	Use surds in exact calculations
N s	Calculate upper and lower bounds
N t	Divide a quantity in a given ratio
N u	Round to specified or appropriate degrees of accuracy including a given power of ten, number of decimal places and significant figures
N v	Use a calculator efficiently and effectively

Algebra

A a	Distinguish the different roles played by letter symbols in algebra, using the correct notation
A b	Distinguish in meaning between the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’
A c	Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, multiplying two linear expressions, factorise quadratic expressions including the difference of two squares and simplify rational expressions
A d	Set up and solve simple equations
A d	Set up and solve simultaneous equations in two unknowns
A e	Solve quadratic equations
A e	Simplify expressions using rules of indices
A f	Derive a formula, substitute numbers into a formula and change the subject of a formula
A g	Solve linear inequalities in one variable, and represent the solution set on a number line
A g	Solve linear inequalities in two variables, and represent the solution set on a coordinate grid
A h	Using systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
A i	Generate terms of a sequence using term-to-term and position to-term definitions of the sequence
A j	Use linear expressions to describe the nth term of an arithmetic sequence
A k	Use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
A k	Find the length of a line segment
A l	Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
A m	Understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line and c is the value of the y- intercept
A n	Understand the gradients of parallel lines
A o	Find the intersection points of the graphs of a linear and quadratic function
A o	Find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions
A p	Draw, sketch, recognise graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the function $y = kx^n$ for integer values of x and simple positive values of k, the trigonometric functions $y = \sin x$ and $y = \cos x$
A q	Construct the graphs of simple loci
A r	Construct linear functions from real-life problems and plot their corresponding graphs
A r	Construct linear, quadratic and other functions from real-life problems and plot their corresponding graphs
A r	Draw and interpret distance time graphs
A s	Interpret graphs of linear functions
A s	Discuss, plot and interpret graphs (which may be non-linear) modelling real situations
A t	Generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions
A u	Use direct and indirect proportion
A v	Transformation of functions

Geometry

GM a	Recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
GM b	Understand and use the angle properties of parallel lines, triangles and quadrilaterals
GM c	Calculate and use the sums of the interior and exterior angles of polygons
GM d	Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
GM e	Recognise reflection and rotation symmetry of 2-D shapes
GM f	Understand congruence and similarity
GM g	Use Pythagoras' theorem in 2-D and 3-D
GM h	Use the trigonometric ratios and the sine and cosine rules to solve 2-D and 3-D problems
GM h	Use the sine and cosine rules to solve 2-D and 3-D problems
GM i	Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
GM j	Understand and construct geometrical proofs using circle theorems
GM k	Use 2-D representations of 3-D shapes
GM l	Describe and transform 2-D shapes using single or combined rotations, reflections, translations, or enlargements by a positive, fractional or negative scale factor
GM l	Distinguish properties that are preserved under particular transformations
GM m	Use and interpret maps and scale drawings
GM n	Understand and use the effect of enlargement for perimeter, area and volume of shapes and solids
GM o	Interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
GM p	Convert measurements from one unit to another
GM p	Convert between volume measures, including cubic centimetres and cubic metres
GM q	Make sensible estimates of a range of measures
GM r	Understand and use bearings
GM s	Understand and use compound measures
GM v	Use straight edge and a pair of compasses to carry out constructions
GM w	Construct loci
GM x	Calculate perimeters and areas of shapes made from triangles and rectangles or other shapes
GM y	Calculate the area of a triangle using $\frac{1}{2}\ ab \sin C$
GM z	Find circumferences and areas of circles
GM z	Find surface area of a cylinder
GM aa	Calculate volumes of right prisms and shapes made from cubes and cuboids
GM bb	Solve mensuration problems involving more complex shapes and solids
GM cc	Use vectors to solve problems

Statistics and Probability

SP a	Understand and use statistical problem solving process (handling data cycle)
SP b	Identify possible sources of bias
SP c	Design an experiment or survey
SP d	Design data-collection sheets distinguishing between different types of data
SP e	Extract data from printed tables and lists
SP f	Design and use two-way tables for discrete and grouped data
SP g	Produce charts and diagrams for various data types
SP h	Calculate median, mean, range, quartiles and interquartile range, mode, modal class and interval containing the median
SPi	Interpret a wide range of graphs and diagrams and draw conclusions
SP j	Present findings from databases, tables and charts
SP k	Recognise correlation and drawand/or use lines of best fit by eye, understanding what these represent
SP l	Compare distributions and make inferences
SP m	Understand and use the vocabulary of probability and the probability scale
SP n	Understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
SP o	List all outcomes for single events, and for two successive events, in a systematic way and derive relative probabilities
SP p	Identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
SP q	Know when to add or multiply two probabilities: when A and B are mutually exclusive, then the probability of A or B occurring is $P(A) + P(B)$, whereas when A and B are independent events, the probability of A and B occurring is $P(A) \times P(B)$
SP r	Use tree diagrams to represent outcomes of compound events, recognising when events are independent
SP s	Compare experimental data and theoretical probabilities
SP t	Understand that if they repeat an experiment, they may, and usually will, get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics
SP u	Use calculators efficiently and effectively, including statistical functions

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