Topic Guide:

Measuring the Solar System

GCSE (9-1) Astronomy

Pearson Edexcel Level 1/Level 2 GCSE (9-1) in Astronomy (1AS0)

Measuring the Solar System

Contents

Specification Points

3.3 Understand how Eratosthenes and Aristarchus used observations of the Moon and Sun to determine successively:

a. diameter of the Earth
b. diameter of the Moon
c. distance to the Moon
d. distance to the Sun
e. diameter of the Sun

11.12 Understand the use of Venus (as proposed by Halley) to determine the size of the astronomical unit and thus the absolute size of the Solar System.

Introduction

One of the greatest achievements of the human mind involves the measurements and conclusions which ancient astronomers were able to make about our Solar System. Using only naked-eye observations and the geometry of triangles, astronomers from Greece and the Middle East gathered proof that the Earth was a sphere, developed techniques for measuring incredibly small angles in the sky and made estimates of the scale of the Earth-Moon-Sun system. Their work laid the foundations for the Copernican revolution in astronomy which was to follow.

In the 18th century, the second Astronomer Royal, Edmond Halley, used observations of transits of Venus to obtain the first accurate value for the Astronomical Unit which forms the basis of all measurement within our Solar System.

Eratosthenes and the Shape and Size of the Earth

The southern Egyptian town of Syene (near the modern city of Aswan) is close to the Tropic of Cancer which means that the Sun is directly overhead on the summer solstice (21st June). This meant that objects would cast no shadow and the Sun's reflection could be seen in the water at the bottom of deep wells.

Figure 1: The Shape of the World. Even in ancient times there was plenty of evidence that the Earth's surface was not flat. The fact that ships disappeared over the horizon as they sailed out to sea would have been an everyday observation for many ancient civilisations.

The Greek astronomer Eratosthenes lived in Alexandria on the northern coast of Egypt. Reading accounts of the Sun being directly overhead at midsummer in Syene in the south of Egypt, he soon found that things were not the same where he lived, in Alexandria. A stick placed in the ground produced a 7.2° shadow at noon on Midsummer's Day, showing that the Sun was definitely not overhead.

Figure 2: Sun at the Zenith. At locations on the Earth with a latitude of 23½°N, the Sun is directly overhead at midday on 21st June – the summer solstice. The line joining all points with this latitude is called the Tropic of Cancer.

TASK 1: Places on the Tropic of Cancer have a latitude of 23 ½°N. The Earth's axis is tilted relative to the ecliptic at an angle of 23½°.

Can you use these two facts to draw a diagram which explains why the Sun is directly overhead at midday on the summer solstice from this latitude?

Since the Sun is a very long distance from the Earth, its rays of light arrive as almost perfectly parallel lines. The only explanation for the Sun appearing overhead in Syene and at 7.2° from the vertical at the same time in Alexandria, is that the Earth has a curved surface.

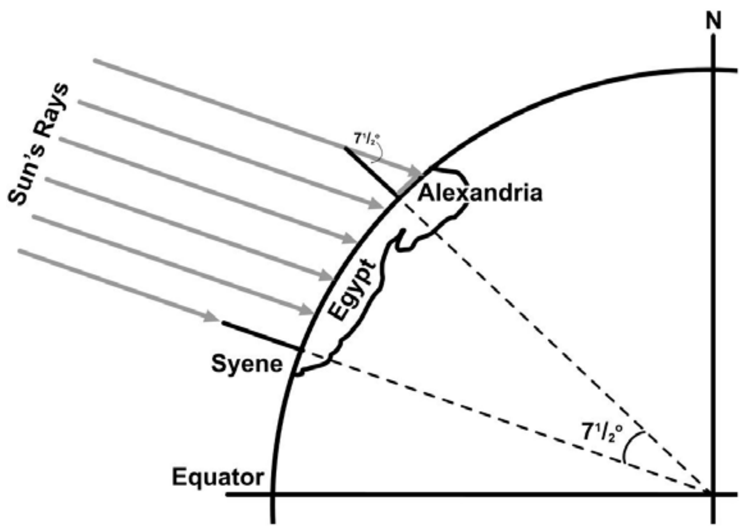

Figure 3: Eratosthenes' Method. Eratosthenes realised that his two observations of the altitude of the Sun at midday on the summer solstice meant that the Earth had a curved surface and could also be used to measure its circumference. The size of Egypt and the sticks used have been made much larger in this diagram!

Eratosthenes also realised that his observations meant that the piece of the Earth's surface between Alexandria and Syene must represent 7.2° out of the full 360° (or about $^1/_{50}{}^{th}$) of the Earth's circumference, as shown in Figure 3. Using a value of 5000 stadia for the distance between Alexandria and Syene, he therefore calculated a value for the circumference of the Earth of 50 x 5000 stadia = 250 000 stadia.

Unfortunately, in Eratosthenes' time there were several different definitions of the length of a 'stadium', the unit of length which he used. It is therefore difficult to be exactly sure of the modern equivalent of his measurement of 250 000 stadia.

The most likely stadium for Eratosthenes to have used was 185m in length, which would make his value of 250 000 stadia equivalent to 185m x 250 000 = 46 250km, which is only about 6000km away from the modern value of 40 075km! This was a remarkable achievement for measurements taken nearly two thousand years before the invention of the telescope.

TASK 2: Eratosthenes recorded a value of 250 000 stadia for the circumference of the Earth. We cannot be certain of the exact length of the stadium unit which he used. Although it is most likely to have been 185m long, it may have been as short as 157m or as long as 209m.

a) Calculate the longest and shortest modern equivalents for Eratosthenes' 250 000 stadia, using the longest and shortest values for the length of a stadium, given the data above.
b) Which of these values is closest to the modern value of 40 075km?

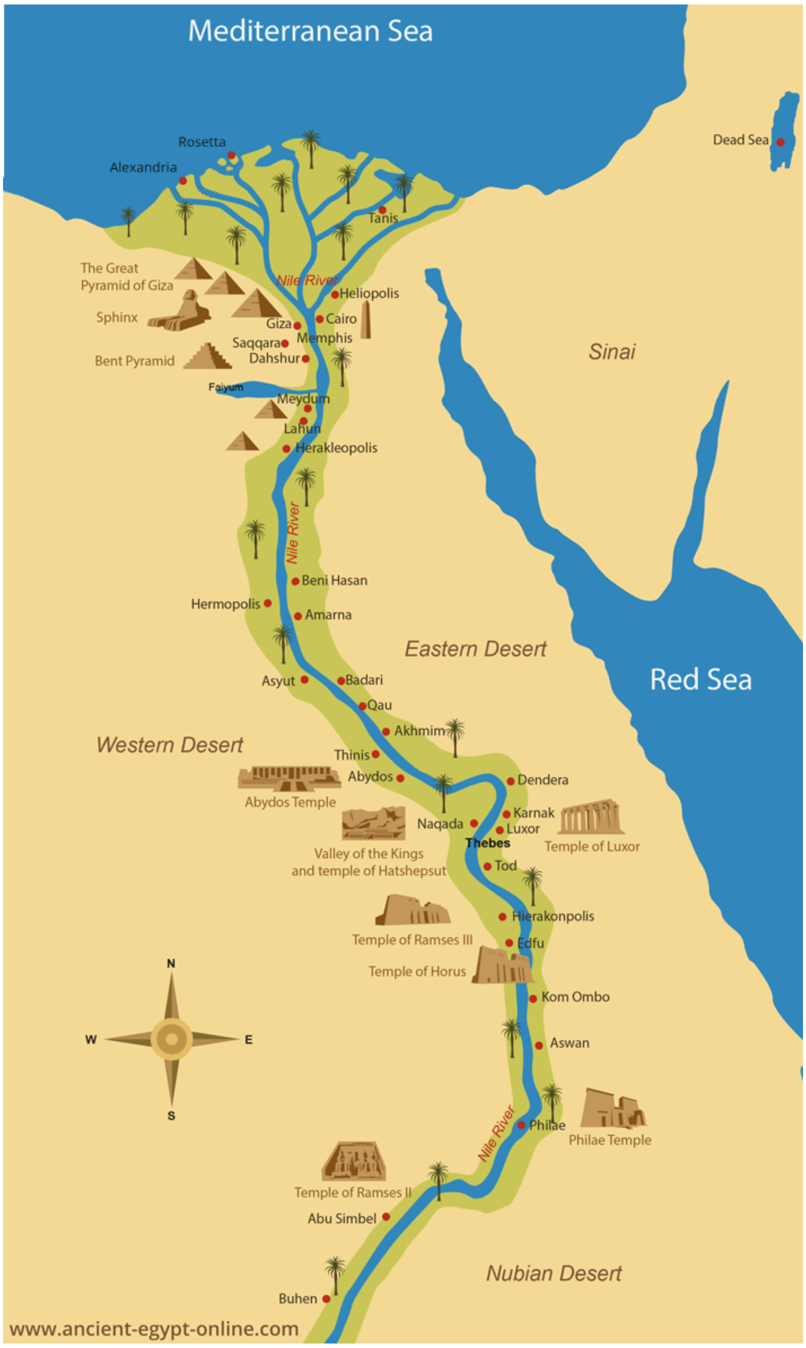

Figure 4: The River Nile. The country of Egypt is perfectly designed for the measurements which Eratosthenes needed to take to measure the size of the Earth. It covers a large distance from North to South and has a river which allowed people - and therefore information - to travel its entire length quite quickly, even in ancient times. Other countries' ancient civilisations would have found things more difficult as many of their major rivers run mostly East-West. Hence the phrase 'Geography is the mother of all sciences...'.

TASK 3: Nowadays it is quite easy to repeat Eratosthenes' experiment to measure the size of the Earth, even if you don't live in Egypt. You will need to find another observer who is several hundred miles north or south of you on the Earth. You can then follow Eratosthenes by both placing a stick in the ground and measuring the angle of the Sun at noon. This is basically the same method as the A9 Finding Longitude Using a Shadow Stick task from p.46 in the Specification. Alternatively you could both measure the angle of a bright star above the horizon when it reaches its highest point or 'culminates'.

Whichever measurement you take, you can use the difference between the angles which you and your other observer measure and the distance between your two locations to repeat Eratosthenes' calculation.

Figure 5: Christopher Columbus. When researching the size of the Earth in preparation for his voyages across the Atlantic Ocean, the Genoese explorer Christopher Columbus used the very shortest possible estimate of the size of a 'stadium' to make Eratosthenes' calculations give the very smallest possible value for the size of the Earth!

Figure 6: The Nautical Mile. Eratosthenes' measurements were effectively the first estimate of the length of a nautical mile, which was originally defined to be the length on the Earth's surface of $^{1}/_{60}{}^{th}$ of a degree of latitude (i.e. one arc minute of latitude). Eratosthenes' measurements gave a distance of 5000 stadia (or about 935km) for 7.2^{o} of latitude, thus estimating a nautical mile to be equal to 935km/(7.2 x 60) = 2160m, which is close to the modern value of 1852m. The speed at which boats travel through the water could then be measured in knots, with one knot equal to one nautical mile per hour.

TASK 4: The nautical mile was defined to be the length of $^{1}/_{60}{}^{th}$ of a degree of latitude. However, the Earth is not perfectly spherical.

a) What is the exact shape of the Earth?
b) What effect would this have on the length of a nautical mile:
(i) near the Equator?
(ii) near the North or South Pole?

Eratosthenes and the Size of the Moon

Having obtained a value for the size of the Earth, Eratosthenes went on to use this value to help him make an estimate for the size of the Moon. By watching a lunar eclipse, he realised that the Moon must be smaller than the Earth as it takes several hours for the Moon to pass through the central or 'umbral' part of the Earth's shadow[1].

Figure 7: Lunar Eclipse. This occurs when the disc of the Full Moon passes through the Earth's shadow. The curved edge of the Earth's shadow, which can be seen when it first touches the Moon's disc, was further evidence that the Earth has a curved surface.

Eratosthenes argued that the amount of time it took the Moon to move into the Earth's shadow (U1 to U2) could give an estimate of its diameter and that the time it took the Moon to then cross the Earth's shadow (U1 to U3) would give an estimate for the diameter of the Earth. By comparing these two times he obtained an estimate for the diameter of the Moon compared to that of the Earth. He was effectively using the moving lunar disc as a ruler to compare the size of the Moon with the Earth's shadow.

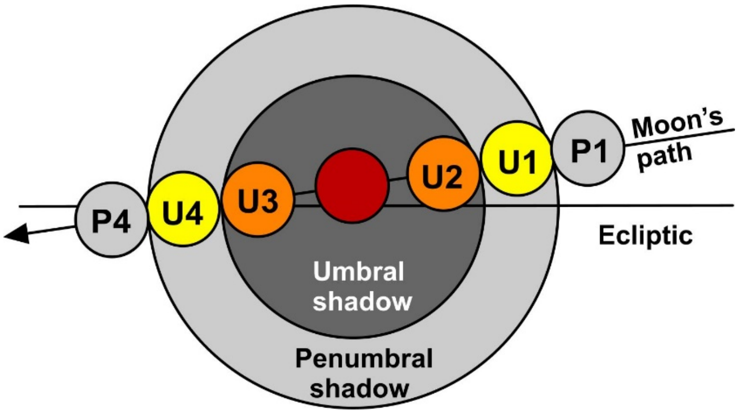

Figure 8: Describing Eclipses. When the leading edge of the Moon's disc first touches the Earth's shadow, this is referred to as First Contact. When the Moon's trailing edge is just inside the shadow, this is called Second Contact. Third and Fourth Contact are defined in a similar way as the Moon's disc leaves the Earth's shadow.

Using this method Eratosthenes estimated that the Moon's diameter was about 2½ times smaller than the diameter of the Earth.

TASK 5: The most accurate possible value for the diameter of the Earth from Eratosthenes' measurements is 12 493km. He estimated that the Moon was 2½ times smaller than the Earth.

Use these data to calculate his estimate for the diameter of the Moon.

[1] He assumed that the Earth had the same diameter as the umbral part of its shadow.

Eratosthenes and the Distance to the Moon

Now that Eratosthenes had a value for the diameter of the Moon, working out its distance from the Earth was simply a matter of measuring the angle which the Moon's disc makes in the sky.

Figure 9: Estimating Angles in the Sky. Many important angles in the sky can be estimated using your arms and hands. For example, the diameter of the Moon's disc is approximately equal to the angle of your little finger nail when held at arm's length. In addition, an angle of 1^{o} is approximately equal to the angle of your thumb's knuckle, again held at arm's length.

This meant measuring an angle of just ½^{o} which Eratosthenes may have done using a device such as a quadrant or a sextant, as shown in Figure 10. Eratosthenes measured the angle of the Full Moon's disc to be 1.7^{o}. A right-angled triangle with an angle of 1.7^{o} has a long side which is 35 times longer than its short side. Eratosthenes therefore estimated that the distance of the Moon was 35 times the diameter of its disc, although the correct ratio is actually 110.

Figure 10: The Quadrant and the Sextant. Accurately measuring very small angles in the sky is a central part of astronomy and was particularly difficult before the invention of the telescope. To make this measurement easier, astronomers fitted sights to angle scales and made either a quadrant (with an angle scale 90^{o} wide) or a sextant (with an angle scale 60^{o} wide).

TASK 6: Eratosthenes' measurement of the Full Moon's disc showed him that it had a diameter which was $^{1}/_{35}$th of its distance from the Earth. He had already estimated the Moon's diameter as being 5000 km.

Use these data to find Eratosthenes' estimate for the distance to the Moon.

Aristarchus and the Distance to the Sun

The process of measuring the Solar System using right-angled triangles was continued by the Greek astronomer Aristarchus (310-230 BCE), who realised that the geometry of right-angled triangles could also be used to obtain a value for the distance of the Sun.

Figure 11: The Quarter Moon. When the Moon appears in the First or Last Quarter phase, the Sun's rays make an angle of 90º as they hit the Moon's surface and are reflected to the Earth.

When the Moon is at the First or Last Quarter phase, the Earth and the Sun must be forming a right-angle at the Moon, as shown in Figure 12.

He measured the angle between the Sun and the Moon (A) to be 87º [2]. In a right-angled triangle containing an angle of 87º, the longest side will be 19 times longer than the shortest side. Hence Aristarchus assumed that the Earth-Sun distance was 19 times the Earth-Moon distance, even though the correct ratio is actually close to 400 times.

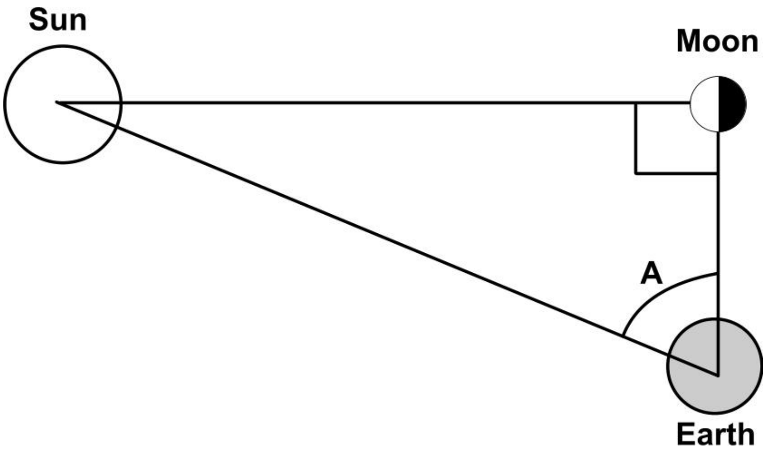

Figure 12: Aristarchus' Method. By constructing a right-angled triangle between the Sun, Moon and Earth, Aristarchus was able to use earlier estimates for the distance between the Earth and the Moon to estimate the distance to the Sun.

TASK 7: From Eratosthenes' measurements, Aristarchus knew that the distance from the Earth to the Moon was 175 000km. His right-angled triangle between the Earth, Moon and Sun had shown him that the Sun was 19 times further from the Earth than the Moon.

Use these data to follow Aristarchus and make an estimate of the distance between the Earth and the Sun – the Astronomical Unit.

[2] Unfortunately the correct value is closer to 89¾º.

Aristarchus and the Size of the Sun

In a similar way to Eratosthenes' measurement of the size of the Moon, Aristarchus measured the size of the Sun's disc, as viewed from the Earth. He combined this with his value for the Astronomical Unit to calculate the diameter of the Sun.

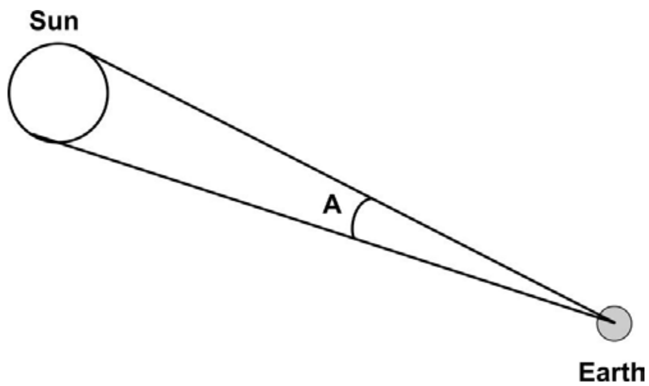

Figure 13: The Diameter of the Sun. Aristarchus already had an estimate for the distance to the Sun (the Astronomical Unit) and so measuring the angle which the Sun's disc makes from the Earth (A) allowed him to find a value for the diameter of the Sun.

Aristarchus estimated that the Sun's disc makes an angle of 2^o when viewed from the Earth[3]. In a right-angled triangle containing an angle of 2^o, the longest side will be 28 times longer than the shortest side. Aristarchus therefore estimated that the diameter of the Sun's disc is $1/28^{th}$ of its distance from the Earth.

TASK 8: If the Sun's diameter was $^1/_{28}{}^{th}$ of its distance from the Earth, use the value Aristarchus calculated for the Astronomical Unit – 119 000km – to calculate a value for the Sun's diameter.

[3] Almost exactly the same angle as the Full Moon's disc, which is why solar eclipses can appear spectacularly total, from the Earth.

Ancient Measurements of the Size of the Solar System

The previous sections show that ancient astronomers, long before the invention of the telescope, were able to make estimates of the sizes and distances of the Moon and the Sun. However, these estimates all involved measuring angles which were very small or very close to 90°, which are very hard to measure accurately with just the human eye.

As Table 1 on the next page shows, Aristarchus' estimate for the size of the Astronomical Unit was extremely inaccurate, since it suffered from an accumulation of the errors introduced by each of the estimates of small angles, all the way back to Eratosthenes' estimate for the size of the Earth.

In the centuries after Aristarchus, astronomers were able to improve on the ancient estimates of the scale of the Earth-Moon-Sun system by using larger and larger angle scales. Accurately measuring angles of less than one degree is extremely difficult using protractors or angle scales which are less than a metre across (Figure 10) but are much more effective when angle scales are tens of metres across (Figures 14 and 15).

Figure 14: The Fakhri Quadrant at the Ulugh Beg Observatory. In modern-day Uzbekistan, the Ulugh Beg Observatory contained a quadrant with a radius of 40 metres. Because the angle scale was so large it made it possible to measure angles in the sky to a precision of around $^1/_6$°, even though it dates from nearly 200 years before the invention of the telescope.

Figure 15: Tycho Brahe's Mural Quadrant. This quadrant, with a radius of around five metres, was part of the Danish astronomer's observatory of Uraniborg on the island of Hven in Denmark. It allowed him to measure the position of stars and planets in the sky to an accuracy of 8' of arc or 0.13^{o}, decades before the invention of the telescope!

Measurement	Ancient Estimate	Modern Value	Percentage Error
Diameter of Earth	**12 493 km**[4]	**12 756 km**	2%
Earth/Moon diameter ratio	2.5	3.7	
Diameter of Moon	**4 997 km**	**3 474 km**	44%
Angle of Moon's disc	1.7^{o}	½^{o}	
Moon's distance-diameter ratio	35	110	
Distance to Moon	**174 900 km**	**384 400 km**	55%
Moon-Sun angle	87^{o}	89¾^{o}	
Mon-Sun distance ratio	19	389	
Distance to Sun (Astronomical Unit)	**3 320 000 km**	**149 600 000 km**	98%
Angle of Sun's disc	2^{o}	½^{o}	
Sun's distance-diameter ratio	28	110	
Diameter of Sun	**119 000 km**	**1 390 000 km**	91%

Table 1: Cumulative Error in Ancient Measurements of the Earth, Moon and Sun. This table summarises the measurements and calculations of ancient astronomers such as Eratosthenes and Aristarchus. Modern measurements are included to show the rapidly increasing error in these ancient figures.

[4] This is being generous to Eratosthenes by using the shortest possible definition of the ancient 'stadium' of 157m, as this gives the result which is closest to the modern value.

The Size of the Solar System – the Astronomical Unit

Kepler's Third Law of Planetary Motion, published in 1619, allowed astronomers to use the orbital period of a planet to determine how many times further away or closer to the Sun it was, compared to the Earth.

For example, Mars' orbital period of 1.88 years allows us to calculate how many times further from the Sun it is than the Earth.

For Earth,

$$T(\text{in years})^2 / r(\text{in AU})^3 = 1^2/1^3 = \mathbf{1}$$

So for Mars,

$$1.88^2 / r^3 = 1$$

Giving, $r^3 = 3.53$

$$r = \sqrt[3]{3.53} = \underline{\mathbf{1.52\ AU}}$$

Similar calculations can be performed for any planet in the Solar System. However, all such calculations give the radius of planetary orbits in units of the Astronomical Unit (AU), the average distance between the Earth and the Sun. Consequently, the accuracy of all our distance measurements in the Solar System depend on the accuracy with which we can measure the length of the Astronomical Unit. In other words, measurements of the AU allow us to set absolute values for distances in the Solar System.

Figure 16: An Orrery. Ancient measurements of the Solar System, using instruments such as orreries in the 17th century, started to establish its relative proportions. In contrast, modern measurements of astronomers like Edmond Halley established accurate values for its fundamental distances.

One of the first astronomers to realise the pivotal importance of obtaining an accurate value for the Astronomical Unit in order to measure the absolute size of the Solar System was the second Astronomer Royal, Edmond Halley. Following the work of the Scottish mathematician James Gregory, Halley suggested that observations of a transit of Mercury or Venus from widely-spaced locations on the Earth could be used to calculate the exact length of the Astronomical Unit.

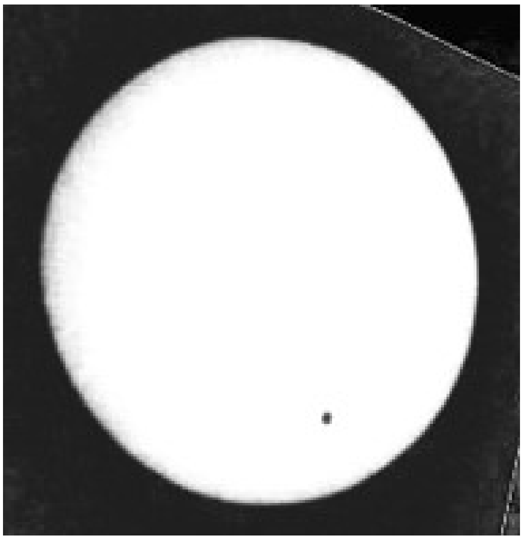

Figure 17: The 2004 Transit of Venus. On 8th June 2004 the planet Venus passed directly in front of the Sun's disc, as viewed from Earth. This allowed the planet's disc to be clearly seen in silhouette against the bright disc of the Sun. Transits of Venus occur in pairs, eight years apart, with over a century between each pair. There was therefore another transit of Venus in 2012 but the next one will not occur now until 2117.

(*Source:* © *Karen Fisher*)

As shown in Figure 18, observers at two different locations on the Earth's surface during a transit of Venus will see Venus cross the Sun's disc in slightly different places.

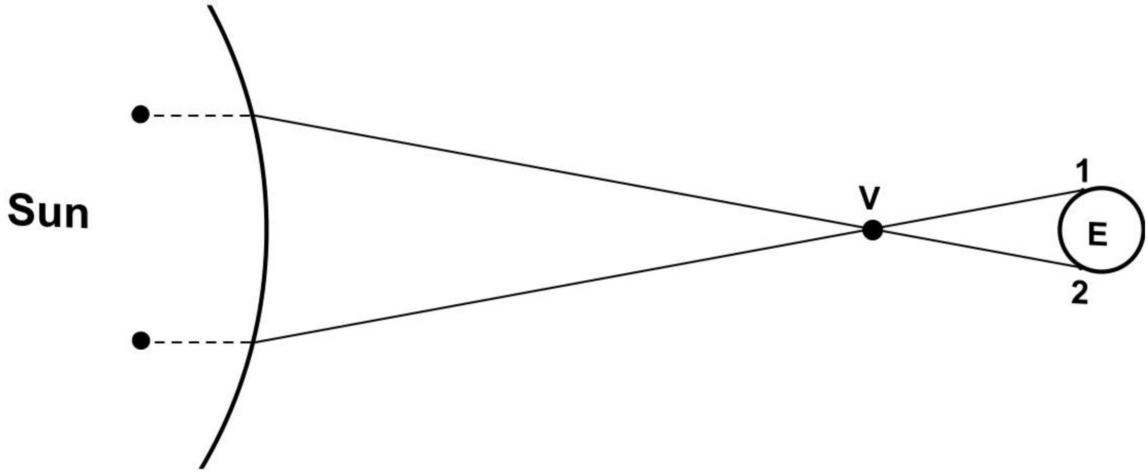

Figure 18: Parallax in Observations of a Transit of Venus. Observers at different latitudes on the Earth (1 and 2) will see Venus transit slightly different parts of the Sun's disc. The size of this difference or 'parallax' is closely linked to the distance between the Earth and the Sun – the Astronomical Unit.

The observer at the more northerly latitude (1) would see Venus' disc take a slightly more southerly route across the Sun's disc and the more southerly observer (2) would see it take a more northerly path, as shown in Figure 19.

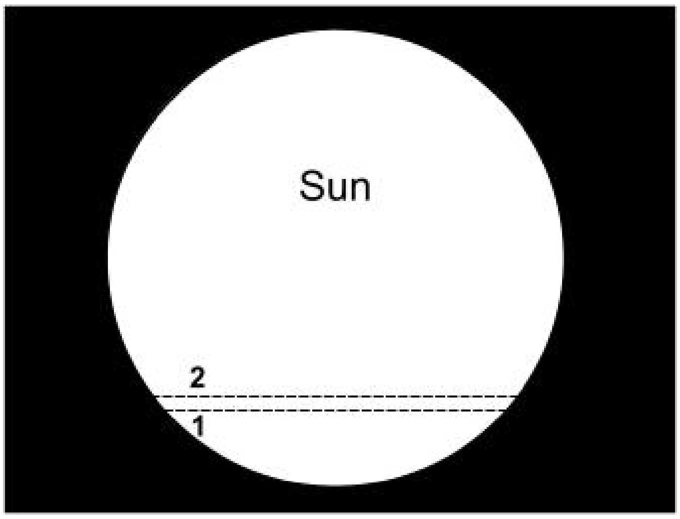

Figure 19: Observed paths for a Transit of Venus. Venus will appear to take a slightly more southerly path (1) across the Sun for an observer at a more northerly latitude on the Earth, whereas an observer at a more southerly latitude will see it take a slightly more northerly path (2).

Unfortunately, the difference between the two paths 1 and 2, even for a pair of observers at the Earth's North and South Poles, is extremely small – less than the width of Venus' disc itself (see Figure 17) and certainly too small to measure accurately with the early telescopes of the seventeenth century.

Halley realised that instead of trying to measure the distance between the two paths shown in Figure 19, a measure of the difference or 'parallax' between then could be found by measuring the time taken by Venus' disc to cover each path. Since the edge of the Sun's disc is curved, Venus will appear to take slightly less time to transit along path 1 in Figure 19 than along path 2. Halley estimated that this small difference in time could be measured accurately by the clocks available at the time, as long as the two observers were widely separated in latitude on the Earth's surface.

On 7th November 1677, Halley took careful measurements of a transit of Mercury on the island of St Helena in the south of the Atlantic Ocean. On returning to England he combined his observations with those taken by Richard Towneley from Burnley in Lancashire to give a more accurate value for the Astronomical Unit.

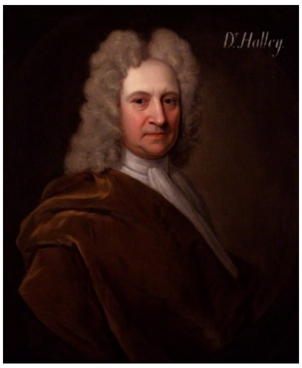

Figure 20: Sir Edmond Halley (1656-1742). As the second Astronomer Royal, Halley ensured that multiple observations of transits of Venus in 1677 and 1761[5] were taken to allow for a more accurate calculation of the length of the Astronomical Unit to be made.

[5] Despite his own death nearly twenty years earlier!

Still dissatisfied with the quality of the data from these observations, Halley proposed that several expeditions should be sent to take accurate measurements of the next transit of Venus. Unfortunately for Halley, transits of Venus come in pairs separated by over a century meaning that the next transit would not occur until 1761. Nevertheless, Halley left money in his will to help fund expeditions to view the 1761 transit. On 6th June 1761, astronomers from as far apart as Newfoundland and the Cape of Good Hope made simultaneous observations of the transit of Venus and thus obtained a much more accurate value for the Astronomical Unit than that obtained in antiquity by Aristarchus.

In order to take accurate observations of the transit of Venus of 1769, astronomers and their equipment set out on a lengthy voyage to Tahiti in the south of the Pacific Ocean, headed by the explorer James Cook. This expedition famously landed in south eastern Australia during the return leg of the journey.

Further Support

The following websites contain further information about the topics covered by this Support Sheet:

1. Almost all the people and places described in this sheet have individual entries in Wikipedia.

   www.wikipedia.org

2. The Royal Astronomical Society's Leaflets for Schools contain material to support the teaching of several of the topics covered by this sheet.

   www.ras.org.uk/publications/other-publications

3. The Royal Greenwich Observatory website contains resources relevant to many of the topics covered in this sheet and was itself the site of the work of some of the astronomers mentioned. It is therefore a unique location for students of GCSE Astronomy to visit.

   www.rmg.co.uk/royal-observatory

4. The Educators section of the NASA website contains a wide range of resources to support students and teachers learning about many of the astronomical topics covered in this sheet.

   www.nasa.gov/audience/foreducators/index.html

Checkpoint Questions

1. Write a short definition of each of the following:
    a. Latitude
    b. Meridian
    c. Tropic of Cancer/Capricorn
    d. Eclipse
    e. Umbra and Penumbra
    f. First and Last Quarter
    g. Transit
    h. Parallax

2. Give four pieces of evidence that the Earth is a sphere:
    a. TWO which would have been known about in ancient times
    b. TWO which have only been available since the 20th century.

3. Two students, separated by a distance of 250km on the Earth simultaneously measure the altitude of the Sun at midday. They obtain values of 42.5^o and 44.75^o. Use these data to make an estimate for the circumference of the Earth.

4. An astronomer observes a total lunar eclipse and notes the times at which the Moon passes through the Earth's shadow. Using these times she estimates that the diameter of the Moon is 3¾ times smaller than the diameter of the Earth.

    If the Earth has a diameter of 12 750km, use these data to estimate the diameter of the Moon.

5. A student plans to adapt Aristarchus' method of using the Moon at Quarter phase to determine the distance to the Sun to determine the distance of Venus from the Sun.
    a. Draw a diagram showing the Sun, Earth and Venus when Venus shows a Quarter phase when viewed from Earth
    b. The student measures the angle between the Sun and Venus to be 46^o at this time. Use this figure to draw a scale diagram and thus estimate the distance of Venus from the Sun.

6. Jupiter orbits the Sun once every 11.86 years.
    a. Use Kepler's Third Law of Planetary Motion to calculate its mean distance from the Sun in AU.
    b. If the Astronomical Unit is 150 million km, calculate the actual distance of Jupiter from the Sun.