EDEXCEL FUNCTIONAL SKILLS PILOT

TEACHER'S NOTES

Maths Level 2

Chapter 7

Working with probability

SECTION K

1 Measuring probability

2 Experimental probability

3 Using tables to find the probability of combined events

4 Using tree diagrams to show the outcomes of combined events

5 Remember what you have learned

Maths Level 2

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Chapter 7: Working with probability

Use these free pilot resources to help build your learners' skill base
We are delighted to continue to make available our free pilot learner resources and teacher notes, to help teach the skills learners need to pass Edexcel FS Mathematics, Level 2.

But use the accredited exam material and other resources to prepare them for the real assessment
We developed these materials for the pilot assessment and standards and have now matched them to the final specification in the table below. They'll be a useful interim measure to get you started but the assessment guidance should no longer be used and you should make sure you use the accredited assessments to prepare your learners for the actual assessment.

New resources available for further support
We're also making available new learner and teacher resources that are completely matched to the final specification and assessment – and also providing access to banks of the actual live papers as these become available. We recommend that you switch to using these as they become available.

Coverage of accredited specification and standards
The table below shows the match of the accredited specification to the unit of pilot resources. This table supersedes the pilot table within the teacher notes.

Coverage and Range	Exemplification	Learner Unit
Use probability to express the likelihood of an outcome	• Calculate theoretical probabilities • Compare probabilities • Put events in order of likelihood on a probability scale • Single events only • List outcomes of events	**K1** Measuring probability **K2** Experimental probability **K3** Using tables to find the probability of combined events **K4** Using tree diagrams to find the probability of combined events Compare probabilities and Put events in order of likelihood on a probability scale are covered in our new publishing (see below)
		K5 Remember what you have learned

Where to find the final specification, assessment and resource material
Visit our website **www.edexcel.com/fs** then:
- **for the specification and assessments:** under **Subjects**, click on **Mathematics (Levels 1–2)**
- **for information about resources:** under **Support**, click on **Published resources**.

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7 Working with probability

(pages 109–120 in the learner materials)

Performance	Coverage and Range	Unit Objectives
Learners can:	**Learners can:**	
■ understand routine and non-routine problems in a wide range of familiar and unfamiliar contexts and situations ■ identify the situation or problem and the mathematical methods needed to tackle it ■ select and apply a range of mathematics to find solutions ■ use appropriate checking procedures and evaluate their effectiveness at each stage ■ interpret and communicate solutions to practical problems in familiar and unfamiliar routine contexts and situations ■ draw conclusions and provide mathematical justifications.	■ use a numerical scale from 0 to 1 to express and compare probabilities.	**K1** Measuring probability **K2** Experimental probability **K3** Using tables to find the probability of combined events **K4** Using tree diagrams to find the probability of combined events. **K5** Remember what you have learned

Approach to learning

This section covers the skills necessary for learners to be able to work efficiently with probability. Each unit focuses on the delivery of one particular aspect of measuring probability and using it in everyday contexts. The table identifies the coverage and range from the functional skills standards: mathematics level 2 which are covered in this section.

Section K Working with probability

K1 *Measuring probability*

The main idea is to identify the language used in relation to probability and to look at some of the ways probability is used in everyday life. Discuss the difference between probability and odds; a probability is a number from 0 to 1, which is the ratio of the number of chances of an event happening to the total number of possible outcomes. For example, if there are 4 balls in a bag, 3 red and 1 black, then the probability of picking a black ball from the bag is $\frac{1}{4}$. Odds are expressed as the number of chances for the event to the number of chances against the event, (or vice-versa). As there is one chance of picking a black ball and 3 chances of picking a red ball, the odds of picking a black ball are 3 to 1 **against** picking a black ball. The odds are 1 to 3 **for** or **in favour of** picking a red ball.

Emphasise that probability can be measured as a fraction, a decimal and a percentage and remind the learners how to convert from one to another.

Encourage the learners to identify all the possible outcomes when they are considering a question regarding probability. Discuss the connection between the probability of an event happening and not happening and ensure they understand that the formula: prob(event happens) + prob(event does not happen) = 1, can be rearranged to give; prob(event does not happen) = 1 – prob(event happens). Show that this can be extended so that the sum of the probabilities of all possible outcomes = 1.

Activities

There are many probability activities based around throwing a die, spinning a spinner and drawing a card from a pack of playing cards, which can re-enforce the above connections. (Die is singular for dice, but some of you may prefer to stick with the word dice, depending on your learners.)

Throwing a die

Ask the learners to complete the following:

- List all the possible outcomes when throwing a die.
- What is the probability of throwing a six?
  Prob(6) =
- How many outcomes are there for the event, ‘not throwing a six’?
- What is the probability of not throwing a six?
  Prob(not 6) =
- What is prob(6) + prob(not six)?

Repeat the above for the events, 'throwing an even number' and 'throwing an odd number'. Emphasise that when throwing a die the outcome is either odd or even.

Drawing a card from a pack of playing cards

First outline the structure of a pack of playing cards:

52 cards; 4 suits, hearts, diamonds, spades and clubs; 2 colours, red and black; 13 cards of each suit, 2 to 10 plus Jack, Queen, King and Ace.

What is the probability of:

- picking a red card?
- picking a spade?
- not picking a spade?
- picking an ace?
- picking the Queen of hearts?
- not picking the Queen of Hearts? etc

Misconceptions

Because unlikely events can occur, for example throwing a number of 6s in a row, learners sometimes have difficulty differentiating between unlikely and impossible outcomes as well as likely and certain outcomes. The difference can be reinforced by class discussion; asking the learners to identify everyday examples of events in each category. Learners also sometimes have a problem understanding that all outcomes can be equally likely. There can be a misconception that a six can be the hardest number to throw using a die.

K2 *Experimental probability*

The main idea is to emphasise that experimental probability is commonly used in everyday life to make predictions. Examples are for quality control, television ratings, and elections. **Ipsos MORI** is the second largest survey research organisation in the UK. The organisation has a freely available archive of opinion polls and public attitude research, including trends, from 1970 onwards on its UK website (www.ipsos-mori.com/polls). Some of this information can be used a focus for class discussion.

Activities

1. Ask the learners to set up a data collection sheet to record the number of people in the group who are left-handed, right handed or ambidextrous. Ask them to use their results to work out the probabilities that a person is left-handed, right-handed or ambidextrous. As an extension activity, ask the learners to use the probabilities to work out how many of the approximately 370 tennis players who are registered with the Royal Tennis Association they would expect to be right-handed, left-handed or ambidextrous.

2. As an extension activity, ask the learners to complete the additional tasks relating to questions 1 and 2 in 'Try the skill':

1. A clothing company finds that 12 in every 200 pairs of jeans do not meet their quality standards. What is the experimental probability that a pair of jeans does not meet the quality standards? ***The company produces 25 000 pairs of jeans a day. How many do they expect will not meet their quality standards?***

2. 389 out of every 1 000 cars tested in 2004/5 failed the MoT test the first time they were tested. What is the experimental probability that a car will fail the first MoT test? ***There were 32 001 477 cars in the UK in 2003, estimate how many of these cars failed the MoT test the first time.***

3. Ask each of the learners to throw a die 20 times and record their results. Collect all the results from the group and ask them to work out the experimental probability of throwing 1 to 6. Compare the results with the theoretical probability of throwing 1 to 6. The answers should be all be close to $\frac{1}{6}$. Ask them how they might improve their results, for example, by throwing the die more than 20 times.

Misconceptions

Learners sometimes have difficulty relating experimental data results to probabilities. Emphasise that they may expect to find experimental probabilities to be closer to the theoretical probability values if they increase the number of times they perform the experiment.

K3 *Using tables to find the probability of combined events*

The main idea is to emphasise the idea of independent events and how the outcomes of two independent events can be recorded in tables. In the example where the scores are added, emphasise, for example, that the outcome (2, 3) is different to the outcome (3, 2). This can be reinforced by using dice of different colours as in the activity below.

Activities

Ask the learners to set up a table to record the outcomes when a pair of dice of different colours is thrown. Ask them to work in pairs to throw the dice and fill in the correct box when each outcome is obtained. Get them to count how many throws of the dice it takes to complete the whole table. Compare the findings for the whole group.

Misconceptions

Many learners have difficulty with the idea of independent events, for example they will think that when throwing a die, if a 6 has been thrown then the next throw of the die is less likely to be a 6. Participating in experimental probability activities can often help to overcome this.

Learners also sometimes do not realize that when throwing two dice, an outcome such as (2, 3) is different to the outcome (3, 2). Using different colours for the dice can help overcome this difficulty.

K4 *Using tree diagrams to find the probability of combined events.*

The main idea is to introduce another method of recording the outcomes of combined events to the learners. Discuss how tree diagrams can be more useful to use when events happen one after the other or the outcomes from more than two combined events occur. You may wish to introduce the idea of events which are not independent, such as selecting a chocolate from a box of chocolates, this will help with the introduction of the suggested task based activity. Emphasise that the branches of the tree diagram show all the possible outcomes at each stage. Encourage them to view the tree diagram in columns with the first column on the left hand side showing all the possible outcomes from the first event, the next from the second event and so on.

Activities

Investigation

Ask the learners to draw a tree diagram to record all the possible outcomes when 3 coins are tossed. As all the outcomes are equally likely, what is the probability of getting:

- TTT
- all 3 coins showing the same face
- 2 heads and one tail

Now ask them to then draw a tree diagram to record all the outcomes when 4 coins are thrown. How many outcomes are there?

Complete the following table:

Number of coins	Number of outcomes
1	3
2	9
3	7
4	4
5	2

Can you see the pattern?

What would you predict would be the number of outcomes when 5 coins are tossed?

(As an explanation you can say each coin has two possible outcomes, so for 2 coins the number of outcomes is 2 × 2, for 3 it is 2 × 2 × 2, etc.)

Extension: How many outcomes would there be if a die is thrown 3 times, 4 times etc.

Misconceptions

Learners often have difficulty in setting up the tree diagrams for themselves, as they often try to work in rows instead of columns. Emphasise the layout for the tree diagram by encouraging the learners to put headings for each event above the columns, so that they work from left to right to the final outcome.

Apply the skills

The learners need to develop their Process Skills, which are:

Representing	Analysing	Interpreting
making sense of situations and representing them	processing and using the mathematics	interpreting and communicating the results of the analysis

At level 2 the learners must decide on the methods used and identify the information they need for themselves. A suitable activity to practice their skills in data handling would be to investigate relationships between heart rate and lifestyles. The practice task below involves an investigation into the National Lottery and other games based on selecting numbers.

Apply the skill

The National Lottery involves selecting 6 different numbers from the numbers 1 to 49. Forty-nine balls numbered 1 to 49 are put into a drum and the balls are selected at random. The first ball picked is selected from 49 balls. When the second ball is selected there are only 48 balls remaining and so on, so the probability must be altered accordingly. Calculations show that the probability of selecting the winning combination of 6 numbers is about 1 in 14 million!

Example: A community centre runs a simplified version of the lottery. You choose 10 different pairs of numbers from the numbers 1 to 90. The winner is the person who chooses the correct pair of numbers which are drawn from a drum, in the same way as the National Lottery. If you choose 10 pairs of numbers, what is the probability that you will win?

To tackle this problem, first consider what would be the probability of winning if you choose one particular pair of numbers, say 2 and 21. For the first selection you choose from 90 numbers but in the second selection you choose from 89 numbers. The winning combination can be from two selections of numbers, 2 then 21 or 21 then 2.

prob (2 then 21) = $\frac{1}{90} \times \frac{1}{89} = \frac{1}{8010}$

prob (21 then 2) = $\frac{1}{90} \times \frac{1}{89} = \frac{1}{8010}$

You can win if either of these events happen, so the probability that the pair of numbers 2 and 21 win is

$\frac{1}{8010} + \frac{1}{8010} = \frac{2}{8010} = \frac{1}{4005}$. As a percentage this is 0.025%.

The probability will be the same for any pair of numbers you choose. If you choose 10 pairs of numbers you will have 10 times as much chance of winning. So the probability of winning when you choose 10 pairs of numbers is 10 × 0.025% = 0.25%

Tip

If your **probability** of winning is 0.25%, this means your **chances** of winning are:
0.25 in 100 or
25 in 10 000 or
1 in 400

Exercise

1. In the above lottery, how many pairs of numbers would you have to choose to make your chances of winning 5%?

2. In another version of the lottery, 3 numbers are selected from the numbers 1 to 30.

(**a**) What is the probability of choosing the winning combination of numbers?

(**b**) How many selections of numbers do you have to choose to make your chances of winning 5%?

3. Show that 4 numbers can be arranged in 24 different ways.

4. Another lottery involves choosing 4 numbers from the numbers 1 to 50.

(**a**) What is the probability of choosing the winning 4 numbers from 50 numbers?

(**b**) How many selections of numbers do you need to choose to make your chances of winning 5%?

5. The number of ways to arrange 3 numbers is $3 \times 2 \times 1 = 6$. The number of ways to arrange 4 numbers is $4 \times 3 \times 2 \times 1 = 24$. What is the number of ways to arrange 5 numbers?

6. What is the number of ways to arrange 6 numbers?

7. Use your answer to show that the probability of choosing the winning six numbers for the National Lottery is about 1 in 14 million.

Hint

3 numbers can be selected in 6 different ways. For example,

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

Answers

K1 Measuring probability – *page 109*

1. $\frac{3}{8}$ **2.** 66% **3.** 0.201

K2 Experimental probability – *page 111*

1. $\frac{3}{50}$ **2.** $\frac{389}{1000}$ **3. a** $\frac{2}{3}$

3. b

	Bag no.	Variety				
Bestbuy 400 g		**Devon**	**Mint**	**Rum & Butter**	**Coconut**	**Banana**
	1	20	25	18	8	29
	2	6	18	16	35	25
	3	13	38	15	10	25

3. c Range for Devon = 14%, Mint = 20%, Rum & Butter = 3%, Coconut = 23%, Banana = 4%, so coconut is most varied

K3 Using tables to find the probability of combined events – *page 112*

1. a

		1st dice					
		1	**2**	**3**	**4**	**5**	**6**
2nd dice	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b $p(2) = \frac{1}{36}$, $p(3) = \frac{2}{36} = \frac{1}{18}$, $p(4) = \frac{3}{36} = \frac{1}{12}$,

$p(5) = \frac{4}{36} = \frac{1}{9}$, $p(6) = \frac{5}{36}$, $p(7) = \frac{6}{36} = \frac{1}{6}$, $p(8) = \frac{5}{36}$,

$p(9) = \frac{4}{36} = \frac{1}{9}$, $p(10) = \frac{3}{36} = \frac{1}{12}$, $p(11) = \frac{2}{36} = \frac{1}{18}$,

$p(12) = \frac{1}{36}$

c Sum of probabilities =

$\frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36}$

$= \frac{36}{36} = 1$

d 7 **e** $\frac{10}{36} = \frac{5}{18}$ **f** $\frac{18}{36} = \frac{1}{2}$

2. a

		1st coin	
		H	**T**
2nd coin	**H**	H H	T H
	T	H T	T T

b $\frac{2}{4} = \frac{1}{4}$ **c** $\frac{2}{4} = \frac{1}{4}$

3. a

		Home		
		W	**L**	**D**
Away	**W**	W W	L W	D W
	L	W L	L L	D L
	D	W D	L D	D D

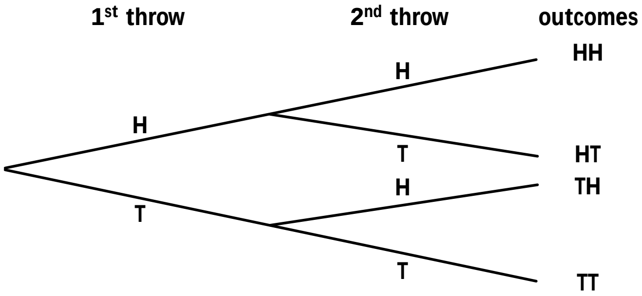
b

		Home		
		W	**L**	**D**
Away	**W**	6	3	4
	L	3	0	1
	D	4	1	2

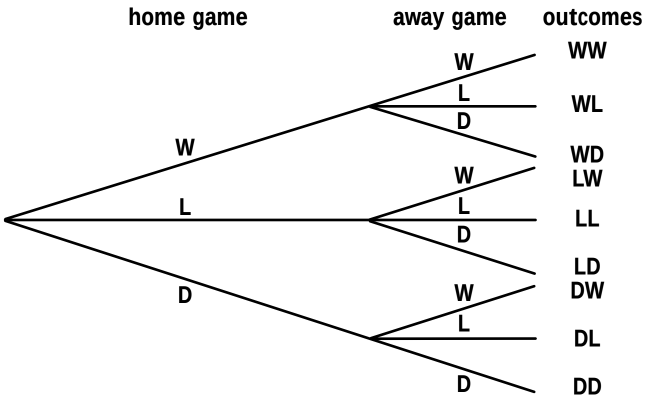
c $p(0) = \frac{1}{9}$, $p(1) = \frac{2}{9}$, $p(3) = \frac{2}{9}$, $p(6) = \frac{1}{9}$

K4 Using tree diagrams to find the probability of combined events – *page 115*

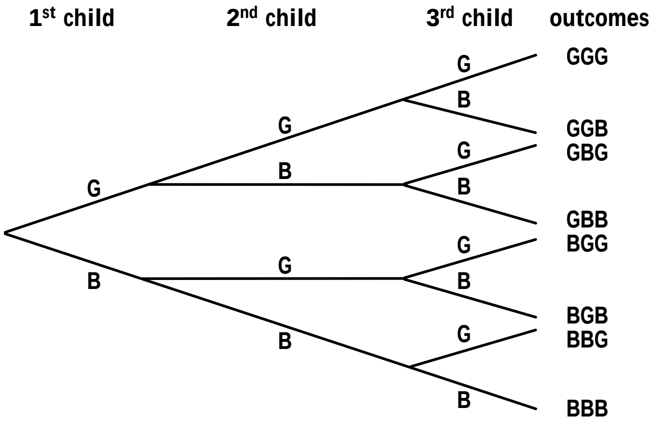
1.



2.



3.



Remember what you have learned – *page 118*

1. D
2. A
3. A
4. a p(male) = 0.5127
b p(female) = 0.4873
c p(male) + p(female) = 1

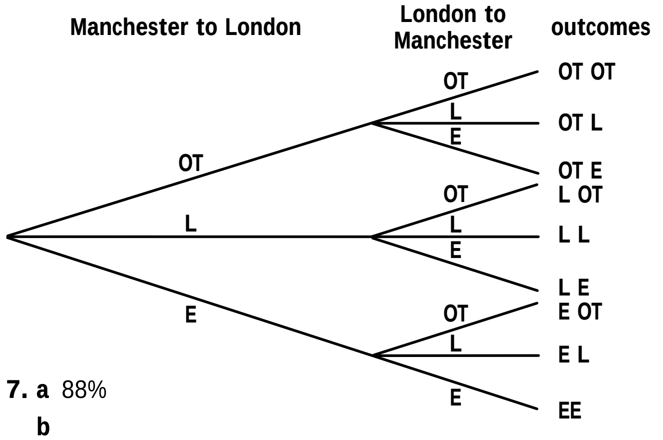
5. a

		1st dice							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
2nd dice	**1**	2	3	4	5	6	7	8	9
	2	3	4	5	6	7	8	9	10
	3	4	5	6	7	8	9	10	11
	4	5	6	7	8	9	10	11	12
	5	6	7	8	9	10	11	12	13
	6	7	8	9	10	11	12	13	14
	7	8	9	10	11	12	13	14	15
	8	9	10	11	12	13	14	15	16

b 64 **c** 9 **f** $\frac{36}{64} = \frac{9}{16}$

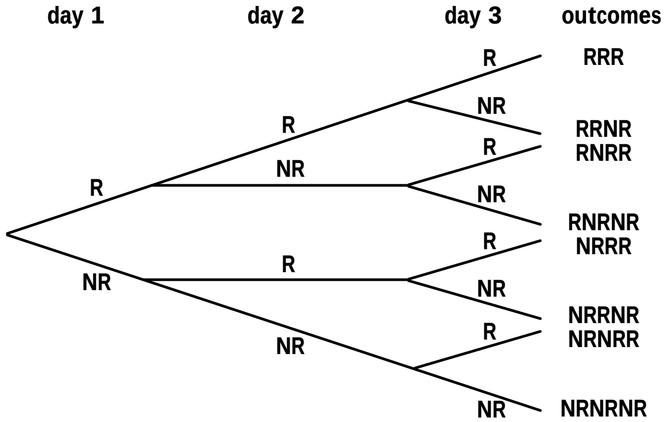
6. a 0.201

b



7. a 88%

b



Apply the skill

1. 200 **2. a** 0.025% **b** 200

3.

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

4. a 0.0004342% **b** 1 1516 **5.** 120 **6.** 720