EDEXCEL FUNCTIONAL SKILLS PILOT

## TEACHER'S NOTES

# Maths Level 2

## Chapter 3

## Working with ratio, proportion, formulae and equations


**SECTION E**

**1** Writing a ratio

**2** Scaling quantities up or down

**3** Calculations with ratio

**4** Scale diagrams

**5** Estimating using proportion

**6** Remember what you have learned

**SECTION F**

**1** Applying the BIDMAS rule to evaluate an expression

**2** Formulae in symbols

**3** Using equations

**4** Remember what you have learned

# Maths Level 2

Su Nicholson

## Chapter 3: Working with ratio, proportion, formulae and equations

**Use these free pilot resources to help build your learners' skill base**
We are delighted to continue to make available our free pilot learner resources and teacher notes, to help teach the skills learners need to pass Edexcel FS Mathematics, Level 2.

**But use the accredited exam material and other resources to prepare them for the real assessment**
We developed these materials for the pilot assessment and standards and have now matched them to the final specification in the table below. They'll be a useful interim measure to get you started but the assessment guidance should no longer be used and you should make sure you use the accredited assessments to prepare your learners for the actual assessment.

**New resources available for further support**
We're also making available new learner and teacher resources that are completely matched to the final specification and assessment – and also providing access to banks of the actual live papers as these become available. We recommend that you switch to using these as they become available.

**Coverage of accredited specification and standards**
The table below shows the match of the accredited specification to the unit of pilot resources. This table supersedes the pilot table within the teacher notes.

| Coverage and Range | Exemplification | Learner Unit |
|---|---|---|
| Understand, use and calculate ratio and proportion, including problems involving scale | • Write a ratio in its simplest form (*a:b*)<br>• Scale quantities up or down<br>• Calculations with ratios as one unit to another unit scale<br>• Use map scales in diagrams<br>• Simple direct and inverse proportion<br>• Writing fractions as ratios | **E1** Writing a ratio<br>**E2** Scaling quantities up or down<br>**E3** Calculations with ratio<br>**E4** Scale diagrams<br>**E5** Estimating using proportion<br>Inverse proportion is covered in our new publishing (see below) |
| | | **E6** Remember what you have learned |
| Understand and use simple formulae and equations involving one- or two-step operations | • Substitute numbers into a formula<br>• Derive a formula in words<br>• Changing the subject of a formula<br>• Inverse operations<br>• Formulae may include brackets | **F1** Applying the BIDMAS rule to evaluate an expression<br>**F2** Formulae in symbols<br>**F3** Using equations<br>Changing the subject of a formula is covered in our new publishing (see below) |
| | | **F4** Remember what you have learned |

**Where to find the final specification, assessment and resource material**
Visit our website **www.edexcel.com/fs** then:
- **for the specification and assessments:** under **Subjects**, click on **Mathematics (Levels 1–2)**
- **for information about resources:** under **Support**, click on **Published resources**.

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# 3 Ratio, Proportion, Formulae and Equations

(pages 45–61 in the learner materials)

| Performance | Coverage and Range | Unit Objectives |
|---|---|---|
| **Learners can:** | **Learners can:** | |
| ■ understand routine and non-routine problems in a wide range of familiar and unfamiliar contexts and situations<br>■ identify the situation or problem and the mathematical methods needed to tackle it<br>■ select and apply a range of mathematics to find solutions<br>■ use appropriate checking procedures and evaluate their effectiveness at each stage<br>■ interpret and communicate solutions to practical problems in familiar and unfamiliar routine contexts and situations<br>■ draw conclusions and provide mathematical justifications | ■ understand, use and calculate ratio and proportion including problems involving scale | **E1** Writing a ratio<br>**E2** Scaling quantities up or down<br>**E3** Calculations with ratio<br>**E4** Scale diagrams<br>**E5** Estimating using proportion |
| | | **E6** Remember what you have learned |
| | ■ understand and use simple equations and simple formulae involving one or two step operations | **F1** Applying the BIDMAS rule to evaluate an expression<br>**F2** Formulae in symbols<br>**F3** Using equations |
| | | **F4** Remember what you have learned |

## Approach to learning

This section covers the skills necessary for learners to be able to work efficiently with ratio and proportion and formulae and equations. 'Each unit focuses on the delivery of one particular aspect of **ratio, proportion, formulae and equations**.' The table identifies the coverage and range from the functional skills standards: mathematics level 2 which are covered in this section.

## E Working with ratio and porportion

### E1 *Writing a ratio*

The main idea is that learners should understand that ratio can be used to compare the size of two or more quantities. Encourage learners to relate this to the work they have done on fractions. Remind them that ratios should be given in their lowest terms. Remind them that they can divide by 2 if the numbers in the ratio are even or by 5 if the numbers end in either 0 or 5. Emphasise that the order of the terms in the ratio is important. Discuss the connection with their work on measure and emphasise that when comparing amounts the measures should be in the same units.

Link into the work on writing fractions in their lowest terms and the use of a calculator to write a ratio in its simplest form using the **ab/c** key.

## Activities

**Dominoes:** Prepare 'ratio equivalency' dominoes, e.g. 15 mm : 3 cm matching with 2 : 1; make sure that 1 : 2 is also included.

**Memory pairs game:** Prepare pairs of cards with ratios such as 1 kg to 250 g and its simplified form 4 : 1. Place the cards face down on the desk and ask learners to take turns, picking up two cards at a time and trying to make a matching pair. If the cards do not match, they must be placed back in their original positions, face down. When all the cards have been matched, the student with the most pairs wins.

## Misconceptions

When writing a ratio, learners often write down the numbers in the wrong order. For example, in question 3 on page 46, they should work out 150 000 000 : 360 000 000 to get 5 : 12 but may write 12 : 5 instead. Emphasise that if the first number to be compared is smaller, then the first number in the ratio will be smaller. Another frequent error is to try to simplify the ratio without ensuring the units used are consistent, for example, in question 6 the ratio is 35 p to £2.45 and this could be left as 35 : 2.45 instead of 35 : 245. Emphasise that the units in a ratio cannot be 'dropped' unless they are the same.

## E2 *Scaling quantities up or down*

The main idea is to show that learners should learn how to use the ratio to scale quantities up or down, by multiplying or dividing each amount by the same number. Encourage learners to identify common factors in the amounts required. Remind them that the method used is the same as for finding equivalent fractions. Emphasise that the method always involves multiplication and division, not addition and subtraction. Discuss the connection with currency rates, which are also in direct proportion. To find equivalent currency they multiply or divide by the exchange rate.

Discuss the related graphs of quantities that are in direct proportion, such as currency conversion graphs. If $y$ is directly proportional to $x$ then the connection is $y = mx$, where $m$ is a constant, giving a straight line graph through the origin.

### Activities

**Card activity:** Prepare cards with amounts of ingredients for a number of people, e.g. 150 g flour, 100 g butter, 50 g sugar, 6 people and 100 g flour, ? g butter, ? g sugar, ? people. Ask learners to discuss missing quantities and how to calculate them.

### Misconceptions

Learners often make mistakes when working out new quantities because they do not understand the concept of proportion. For example, in question 3 on page 47, learners may try to find the quantities needed for 3 eggs by multiplying by 3. For examples in which there are no common factors, encourage learners to work out the amount required for 1, and then multiply. In this case, first find the corresponding amounts of ingredients for 1 egg.

## E3 *Calculations with ratios*

The main idea is to apply ratio to questions involving sharing. Encourage learners to think of the ratio as being made up of a number of parts. Remind learners that the questions are not always in the same format, in that they could be given the total of the shares or the amount made up by one of the parts. Emphasise that they need to read the questions carefully. Discuss the fact that ratios need not be confined to comparing two quantities and suggest cases when this may occur.

### Misconceptions

Learners often make mistakes when finding out how much one part is worth. This is mainly due to the fact that some questions start by giving the total amount whereas others state how much one part of the ratio is worth. For example, in question 3 on page 48 of the Learner materials they could use 500 g as the total amount of fat and flour instead of the amount of fat. Emphasise that 'to avoid this mistake' they must read the question carefully.

## E4 *Scale diagrams*

The main idea is to relate ratio to the scales used in diagrams, models and maps. Encourage learners to think of the real item as an enlargement of the diagram or map. If the ratio is $1 : n$ then $n$ represents the scale factor. Remind learners that sometimes they may be asked to measure a length on a diagram and use the scale to work out the real length. Emphasise that scales may sometimes be given in mixed units and they need to be able to convert between standard units of measure. Discuss the different ways that scales may be given in maps, for example, 2 cm to 5 km or 1 : 10 000.

### Activities

**Matching card activity:** Prepare pairs of cards with scales written in mixed units and the corresponding ratio, for example, 2 cm to 5 km and 1 : 25 000. Learners work in pairs or small groups and identify equivalent ratios.

### Misconceptions

Learners often make mistakes when converting between units. Encourage learners to learn the relationships between millimetres, centimetres, metres and kilometres. Also, practising the above activity will help to avoid this mistake.

## E5 *Estimating using proportion*

The main idea is to identify the two types of questions which are covered in this unit; comparing relative size and estimating amounts of one whole. Discuss the connection between relative size and scale factor which can also be written as a ratio in the form $1 : n$ or $n : 1$. When estimating the amounts of one whole, tie into the work covered on fractions and percentages, which are the most common ways of expressing the proportion of one whole, for example a glass is $\frac{2}{3}$ full or 50% of savings have been spent. Emphasise the difference between ratio and proportion; ratio is a comparison of two quantities, whereas proportion refers to part of one whole. Discuss the practical applications such as using an estimation of the percentage of people who vote Labour in an election to work out an estimate of the number of voters in a town.

## Activities

Prepare cards showing boxes of 10 by 10 squares where different numbers of boxes have been shaded in. Ask the learners to work in pairs to match these cards to 2 other cards, one representing the shading as a ratio and one as a proportion. Allow the learners to check the simplified ratio form using the **ab/c** key. For example, a 10 by 10 box with 10 boxes shaded should be matched with a ratio card of 1:9 and a proportion card of $\frac{1}{10}$ shaded and $\frac{9}{10}$ unshaded. They can also be asked to express the ratios, if necessary, in the form $1:n$ or $n:1$.

## Misconceptions

Learners often have difficulty in estimating relative size for proportion questions. Encourage them to make use of a ruler in test questions to work out a scale factor by dividing the longer length by the shorter length. Using fractions to express proportions can also be problematic. The above activity can help when considering 2D shapes and this can be linked to 3D shapes when covering the sections on Measures, Shape and Space.

# F Working with formulae and equations

### F1 *Applying the BIDMAS rule to evaluate an expression*

The main idea is to show the learners that the order of operations is important to obtain the correct evaluation for a given expression of numbers and operators. BIDMAS, (previously known as BODMAS where the O stands for 'of'), identifies the correct order and it is worthwhile discussing example 1 on page 54 which shows how 3 different answers can be found using the same sequence of numbers and operators but with brackets in different positions.

## Activities

Set the learners a BIDMAS challenge. For example, give them a time limit to work in pairs using the numbers 3, 4, 5 and 6 exactly once and the operators +, -, × and ÷ at most once with at most two sets of brackets to find out how many different numerical answers they can come up with. The pair(s) with the highest number of answers should identify and justify their additional answers with the whole group.

Prepare cards showing a string of operations, for example:

$5 - 1 \times 3 + 8 \div 2$

On separate cards, supply a set of possible correct answers, for example, 16, 6 and –3, each depending on where the brackets are put in the string. Ask the learners to identify where the brackets should be placed to give each answer.

## Misconceptions

The most common mistake for learners is to work out the calculations in the order they are given instead of using the BIDMAS rule. For example, given $3 + 2 \times 5$, they will add the 3 and 2 first to get 5 and then multiply the answer by 5 to get 25 instead of working out $3 + 10 = 13$. The above activities will help to emphasise why they cannot do this.

### F2 *Formulae in symbols*

At Level 2 the formulae used will be mainly in symbols, but learners can expect to use formulae given in words such as that for percentage change covered in section D. The main idea in this unit is to help the learners develop a logical approach to substituting into formulae expressed in symbols. Encourage them to first ascertain what each of the variables in a formula represent, so that they are clear about which variable should be substituted. Emphasise that they need to follow the BIDMAS rule when evaluating the expression. Ask the learners to identify formulae with which they are already familiar.

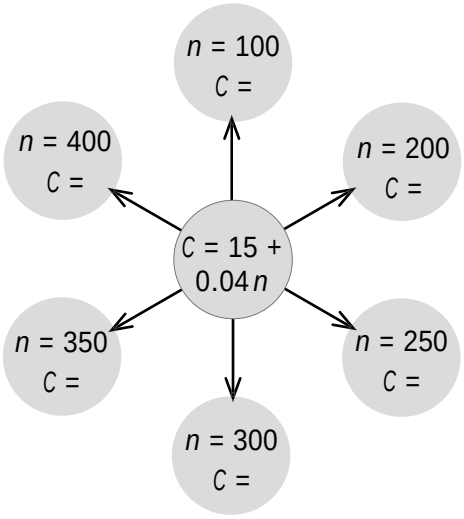
## Activities

Use some practical applications for formulae to develop student understanding, for example looking at savings and loans:

The annual interest (AER) on a savings account is 3.8%, write down a formula which connects the interest, $I$, to the amount invested $P$. This should also promote discussion about savings accounts, rates of interest and how the formula is only applicable if the account has no withdrawals or deposits for the year. Approaching this from the other angle:

The annual interest payable on a loan is given by the formula $I = 0.075B$, where $I$ is the interest and $B$ is the amount borrowed. Use the formula to work out the APR on the account. Us the activity to discuss the difference between APR, Annual Percentage Rate which is used for the interest on loans, with AER, Annual Equivalent Rate which is used for the interest on savings.

Start with a formula such as $C = 15 + 0.04n$. Prepare a set of cards that give values for $n$, as shown below. Ask learners to work out the value of $C$ in each case. Also, ask them to work out what calculation they should do to check their answers.




## Misconceptions

The main difficulties for learners, when they are working out formulae in symbols, are found when the question involves a division and application of the BIDMAS rule. For example, in question 7 on page 57, learners may try to work out 9 ÷ 5 × (55 + 32) rather than (9 ÷ 5 × 55) + 32. The importance of the BIDMAS rule needs to be stressed. Using activities of the form above those with equations such as for temperature conversions can help to reinforce this.

Learners may misinterpret indices, calculating $r^2$ as $2r$. Remind them that an index indicates that the number or letter is multiplied by itself one or more times, for example, $r^2 = r \times r$, $r^3 = r \times r \times r$.

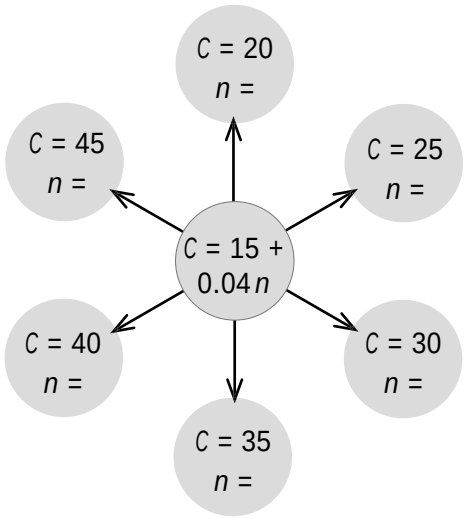
Learners may take the absence of an operator to mean addition instead of multiplication, so $3b$ is calculated as $3 + b$. Remind them that when a number is written next to a letter it means the number and letter are multiplied.

## F3 *Using equations*

The main idea is to ensure learners understand the difference between formulae and equations and are able to distinguish between them. Discuss the definitions given for each and relate to other practical examples, such as area of a rectangle. Emphasise that they are required to 'solve' equations and discuss the related terminology. Ensure that they check their answers and see they make sense. The equations at Level 2 will be mainly straightforward and can be dealt with quite easily using the flowchart method, but you may wish to discuss the balance method for solving equations so that the learners have the option of choosing the method which suits them best.

## Activities

Start with a formula such as $C = 15 + 0.04n$. Prepare a set of cards that give vales of C, as shown below. Ask learners to solve the equations to work out the value of n in each case. Also ask them what calculation they should do to check their answers.




## Misconceptions

Learners can experience difficulty in setting up the flowchart to solve equations. Emphasise that they need to start with the variable that they need to find and build up what is usually the right hand side of the equation. Remind them that they need to perform the opposite or inverse operation when working their way back through the flowchart and that + is inverse to – and × is inverse to ÷.

## *Apply the skills*

The learners need to develop their Process Skills, which are:

| Representing | Analysing | Interpreting |
|---|---|---|
| making sense of situations and representing them | processing and using the mathematics | interpreting and communicating the results of the analysis |

At Level 2 the learners must decide on the methods used and identify the information they need for themselves. A suitable activity to practise these number skills would be to investigate statistics on local recycling. Ratio and proportion can be used to compare amounts recycled on a regional basis and to compare the number of different items which are recycled. Formulae and equations can be used to investigate energy saved and costs involved.
A practice task is given below.

For an extension activity, learners can use a similar approach on their own statistics.

# Recycling ratios

The table shows part some of the recycling figures in South Lakeland. Your task is to compare the recycling figures as indicated in the questions below and use them to work out the energy saved.

You must show evidence of checking your work using different methods, including whether your results make sense. You may use ICT to complete this task, but you must include evidence of all your calculations including spreadsheet formulae.

| **Item** | **2006** | **2005** |
|---|---|---|
| Tin cans | 11 million | 9,500,000 |
| Newspapers | 10 million | 9 million |
| Glass bottles | 7.5 million | 7.5 million |
| Plastic bottles | 6.5 million | |

**1** What is the ratio of the number of tin cans recycled in 2006 to the number of tin cans recycled in 2005 in its simplest form?

**2.** What is the ratio of the number of tin cans recycled in 2005 to the number of newspapers recycled in 2005 in its simplest form?

**3.** Simplify the following ratio for 2005: number of tin cans: number of newspapers: number of glass bottles

**4.** Simplify the following ratio for 2006: number of tin cans: number of newspapers: number of glass bottles: number of plastic bottles

**5.** The ratio of the number of plastic bottles recycled in 2006 to the number recycled in 2005 is 6500:17. 6.5 million plastic bottles were recycled in 2006, how many plastic bottles were recycled in 2005?

**6.** What proportion of the total number of tin cans, newspapers, glass bottles and plastic bottles recycled in 2006 were newspapers?

**7.** 1 recycled tin can saves enough energy to power a television for 3 hours.

(**a**) Use this information to write a formula to connect the amount of hours, $h$, to the number of tin cans $t$.

(**b**) Use the formula to work out the number of hours that a television can be powered in 2006 and 2005 by the tin cans recycled in South Lakeland.

**8.** A formula to find the number of hours, $h$, a computer can be powered by recycling a glass bottle, $g$, is $h = \frac{5g}{12}$.

(**a**) What does $\frac{5}{12}$ represent in the formula?

(**b**) Use the formula to find the number of years that a computer can be powered in 2006 (and 2005), by the glass bottles recycled in South Lakeland. (Use 24 hours = 1 day, 365 days = 1 year.)

(**c**) Another local authority claims that the number of glass bottles they recycled in 2007 was enough to power a computer for 100 years. Use this information to write an equation in g. Solve your equation to find the number of glass bottles, g, recycled by the local authority in 2007.

For a level 2 activity, the students should use a similar approach on their own statistics.

**Useful websites**

http://www.defra.gov.uk/
http://www.recycling-guide.org.uk/facts.html
http://www.wastewatch.org.uk/Homepage
http://www.assurre.org/

## Answers

# E Working with ratio and proportion

### E1 Writing a ratio – *page 45*

**1.** 8 : 9
**2.** 7 : 8
**3.** 5 : 12
**4.** 4 : 5 : 9
**5.** 11 : 13
**6.** 1 : 7

### E2 Scaling quantities up or down – *page 47*

**1.** 225
**2.** 24
**3. a** 165 g of each **b** 8 (7.7)

### E3 Calculations with ratio – *page 48*

**1.** £50
**2.** 24 kg. 108 kg
**3.** 750 g
**4.** 100 ml

### E4 Scale diagrams – *page 49*

**1.** 38 km
**2.** 13 metres
**3.** 4 miles
**4.** 7 cm
**5.** 1.6 m, 1.44 m, 1.12 m
**6.** 3 miles

### E5 Estimating using proportion – *page 51*

**1.** 9 metres
**2. a** $\frac{2}{3}$ **b** 1.5 litres

### E6 Remember what you have learned – *page 52*

**1.** C
**2.** B
**3.** B
**4.** B
**5.** C
**6.** C
**7.** B
**8.** B
**9.** C
**10.** B

# F Working with formulae and equations

### F1 Applying the BIDMAS rule to evaluate an expression – *page 54*

**1.** 7
**2.** 4
**3.** 10
**4.** 10
**5.** 2
**6.** 16
**7.** 27
**8.** 6

### F2 Formulae and equations in symbols – *page 56*

**1.** £42 Challenge: 6p per minute
**2.** £10 Challenge: 4%
**3.** £46.90 Challenge: 6.4 pence
**4.** 54 $cm^2$
**5.** 5.12 inches
**6.** £75
**7.** 131°F

### F3 Using equations – *page 58*

**1.** $n = 4$
**2.** $n = 1320$
**3.** $n = 8$
**4.** $n = 120$
**5.** 320 minutes
**6.** £550
**7.** 65 kg
**8.** 5 cm

### F4 Remember what you have learned – *page 60*

**1.** B
**2.** D
**3.** C
**4.** B
**5.** D
**6.** B
**7.** C
**8.** A

### Apply the skills

**1.** 22:19
**2.** 95:9
**3.** 19:18:15
**4.** 22:20:15:13
**5.** 17 000
**6.** $\frac{2}{7}$
**7(a)** h = 3t
**7(b)** **2005** 28.5 million hours **2006** 33 million hours
**8(a)** $\frac{2}{12}$ hour = 25 minutes = time computer can be powered by 1 glass bottle
**8(b)** **2006** 356.7 years (to one decimal place)
**8(c)** 876 000 = $\frac{5g}{12}$ number of glass bottles = g = 2 102 400