

Maths Level 2

Chapter 2

Working with fractions, decimals and percentages

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Maths Level 2

Su Nicholson

Chapter 2: Working with fractions, decimals and percentages

Use these free pilot resources to help build your learners' skill base

We are delighted to continue to make available our free pilot learner resources and teacher notes, to help teach the skills learners need to pass Edexcel FS Mathematics, Level 2.

But use the accredited exam material and other resources to prepare them for the real assessment

We developed these materials for the pilot assessment and standards and have now matched them to the final specification in the table below. They'll be a useful interim measure to get you started but the assessment guidance should no longer be used and you should make sure you use the accredited assessments to prepare your learners for the actual assessment.

New resources available for further support

We're also making available new learner and teacher resources that are completely matched to the final specification and assessment – and also providing access to banks of the actual live papers as these become available. We recommend that you switch to using these as they become available.

Coverage of accredited specification and standards

The table below shows the match of the accredited specification to the unit of pilot resources. This table supersedes the pilot table within the teacher notes.

Coverage and Range	Exemplification	Learner Unit
Understand and use equivalences between common fractions, decimals and percentages	<ul style="list-style-type: none"> • Simplifying fractions • Finding fractions of a quantity • Improper and mixed numbers • Percentages of a quantity • Convert between fractions, decimals and percentages • Order fractions, decimals and percentages • Writing one number as a fraction of another 	B1 Types of fraction B2 Using a calculator for fractions B3 Fractions of quantities B4 One number as a fraction of another B5 Adding and subtracting fractions Order fractions, decimals and percentages is covered in our new publishing (see below)
		B6 Remember what you have learned
		D1 Percentages D2 Using percentages D3 Percentage change D4 Converting between forms D5 Using fractions, decimals and percentages in practical problems
		D6 Remember what you have learned

Where to find the final specification, assessment and resource material

Visit our website www.edexcel.com/fs then:

- **for the specification and assessments:** under **Subjects**, click on **Mathematics (Levels 1–2)**
- **for information about resources:** under **Support**, click on **Published resources**.

Published by Pearson Education, Edinburgh Gate, Harlow CM20 2JE. First published 2008. © Pearson Education 2008. Typeset by Oxford Designers and Illustrators, Oxford

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B Working with fractions

You should already know how to:

- ✓ read, write, order and compare common fractions and mixed numbers
- ✓ find parts of quantities or measurements.

By the end of this section you will know how to:

- ➡ find fractions of quantities
- ➡ express one number as a fraction of another
- ➡ use fractions to add and subtract amounts
- ➡ use fractions to solve practical problems
- ➡ use a calculator to calculate with fractions.

1 Types of fraction

Learn the skill

A **fraction** is a way of expressing a part of a whole.

The fraction $\frac{3}{4}$ means 3 parts out of 4.



The number on the top, 3, is called the **numerator**, the number on the bottom, 4, is called the **denominator**.

▶ You write a fraction in its *lowest terms* by *dividing the numerator and denominator by any common factor*.

In $\frac{14}{21}$ the numerator and denominator have a common factor of 7. You can divide 'top' and 'bottom' by 7:

$$\frac{14}{21} \begin{array}{c} \xrightarrow{\div 7} \\ = \\ \xrightarrow{\div 7} \end{array} \frac{2}{3}$$

$\frac{14}{21}$ and $\frac{2}{3}$ are **equivalent fractions**.

You can find equivalent fractions by multiplying:

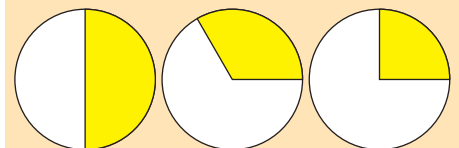
$\frac{3}{4}$ is equivalent to $\frac{6}{8}$

$$\frac{3}{4} \begin{array}{c} \xrightarrow{\times 2} \\ = \\ \xrightarrow{\times 2} \end{array} \frac{6}{8}$$



Remember

$\frac{1}{2}$ is bigger than $\frac{1}{3}$ which is bigger than $\frac{1}{4}$.

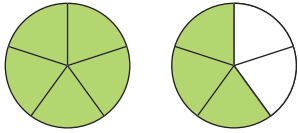




In an **improper** fraction the numerator is bigger than the denominator.

You can write an improper fraction as a *mixed number*.

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1\frac{3}{5}$$



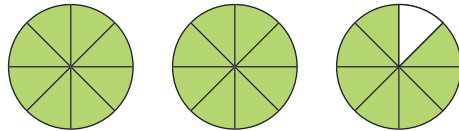
You can change mixed numbers to improper fractions.

Example: Change $2\frac{7}{8}$ to an improper fraction.

Because the denominator of the fraction is 8 you need to change the mixed number to eighths.

One whole is $\frac{8}{8}$, so two wholes is $\frac{8}{8} + \frac{8}{8} = \frac{16}{8}$.

This means that $2\frac{7}{8} = \frac{8}{8} + \frac{8}{8} + \frac{7}{8} = \frac{23}{8}$.



Answer: $\frac{23}{8}$

Remember

A mixed number has a whole number part and a fraction part.

Tip

A quick way of working this out is $2 \times 8 + 7 = 23$.



Try the skill

1. Write these as fractions in their lowest terms.

a $\frac{12}{16}$

b $\frac{15}{20}$

c $\frac{22}{25}$

d $\frac{81}{108}$

2. Write these improper fractions as mixed numbers.

a $\frac{16}{6}$

b $\frac{31}{7}$

c $\frac{21}{8}$

d $\frac{61}{9}$

3. Write these mixed numbers as improper fractions.

a $1\frac{5}{6}$


b $2\frac{3}{5}$

c $3\frac{5}{7}$

d $4\frac{3}{10}$

2 Using a calculator for fractions

Learn the skill


 You can use a calculator to write **fractions in their lowest terms** by making use of the $\boxed{\text{ab/c}}$ key.

Try keying in:

14 \longrightarrow $\boxed{\text{ab/c}}$ \longrightarrow 21 \longrightarrow $\boxed{=}$

You should see $2 \square 3$ on the screen, which means $\frac{2}{3}$.

Use the $\boxed{\text{ab/c}}$ key on your calculator to check your answers to **question 1** on the previous page.


 You can also use the $\boxed{\text{ab/c}}$ key for **improper fractions**.

Try keying in:

8 \longrightarrow $\boxed{\text{ab/c}}$ \longrightarrow 5 \longrightarrow $\boxed{=}$

You should see $1 \square 3 \square 5$ on the screen, which means $1\frac{3}{5}$.

Use the $\boxed{\text{ab/c}}$ key on your calculator to check your answers to **question 2** on the previous page.

 You can use a calculator to **convert mixed numbers to improper fractions**.

Try keying in:

2 \longrightarrow $\boxed{\text{ab/c}}$ \longrightarrow 7 \longrightarrow $\boxed{\text{ab/c}}$ \longrightarrow 8 \longrightarrow $\boxed{=}$

You should see $2 \square 7 \square 8$ on the screen which means $2\frac{7}{8}$

Now press $\boxed{2^{\text{nd}}\text{F}}$ $\boxed{\text{ab/c}}$

You should see $23 \square 8$ on your screen which means $\frac{23}{8}$.

Tip

Some calculators have a $\frac{\square}{\square}$ key to input fractions.

Tip

Some calculators will show $2 \square 3$ to represent $\frac{2}{3}$.

Try the skill

Use the $\boxed{\text{ab/c}}$ key on your calculator to check your answers to **question 3** on the previous page.

3 Fractions of quantities

Learn the skill

▶ To find $\frac{1}{5}$ of a quantity, **divide by 5**.

▶ To find $\frac{2}{5}$ of a quantity, **divide by 5**, then **multiply by 2**.

Out of 192 students $\frac{3}{4}$ own a mobile phone. How many of the students own a mobile phone?

To find $\frac{1}{4}$ divide by 4 $192 \div 4 = 48$

To find $\frac{3}{4}$ multiply this answer by 3 $48 \times 3 = 144$

Answer: 144 students

Example 2: $\frac{3}{25}$ of the money spent on the National Lottery is paid to the Treasury in duty.

In a year when £5 billion is recorded in sales, how much money will be paid to the Treasury?

The easiest way to work out this question is to write $\frac{3}{25}$ as an

equivalent fraction with a numerator of 100. $\frac{3}{25} \xrightarrow{\times 4} \frac{12}{100}$

To find $\frac{1}{100}$ divide by 100 $\frac{5\,000\,000\,000}{100} = 50\,000\,000 = \text{£}50 \text{ million}$

To find $\frac{12}{100}$ multiply this answer by 12

$\text{£}50 \text{ million} \times 12 = \text{£}600 \text{ million}$

Answer: £600 million

Example 3: A man earns £420 a week. He spends $\frac{1}{5}$ of this on rent and $\frac{1}{3}$ of what is left on food. How much does he spend on food?

You have to tackle this question in two stages.

To find out how much he spends on rent, find $\frac{1}{5}$ of £420.

$\frac{420}{5} = \frac{840}{10} = \text{£}84$ spent on rent.

He has $\text{£}420 - \text{£}84 = \text{£}336$ left.

To find out how much he spends on food, find $\frac{1}{3}$ of £336.

$336 \div 3 = \text{£}112$

Answer: £112

Remember

To divide by 4, divide by 2 then divide by 2 again.

 Try the skill

1. 54 000 people attended a football match. $\frac{3}{8}$ of those who attended were children. How many children attended the football match?

2. In a business employing 1080 people, $\frac{5}{12}$ of the employees are under 25. How many of the employees are under 25?

3. 1250 people went to the local cinema on a Saturday night. Of these people, $\frac{2}{5}$ paid the student rate of £7.50 and the remainder paid the full adult rate of £10. What was the total amount of money paid to the cinema that night?

4. When 72 students sat the driving test theory exam, $\frac{5}{6}$ of them passed. Of these, $\frac{4}{5}$ went on to pass the driving test practical. How many of these students passed both the theory and the practical driving test exams?

5. A man earns £1200 a month. He spends $\frac{3}{4}$ of his earnings on food and accommodation and saves $\frac{1}{5}$ of the rest. How much money does he save each month?

6. In a street with 420 houses, $\frac{5}{6}$ of the houses are owned by the occupiers, and $\frac{1}{10}$ of the rest are rented. How many houses in the street are rented?

Tip

First find the number of students who passed the exam.

4 One number as a fraction of another

Learn the skill

▶ One number as a fraction of another number is:

$$\frac{\text{first number}}{\text{second number}}$$

Example 1: Express 8 as a fraction of 12.

8 as a fraction of 12 is $\frac{8}{12}$.

$$\frac{8}{12} \xrightarrow{\div 4} \frac{2}{3}$$

Answer: $\frac{2}{3}$

To express one quantity as a fraction of another, the quantities must be in the same units.

Example 2: Express 60g as a fraction of 1kg.

Write the quantities in the same units.

$$\frac{60}{1000} \xrightarrow{\div 10} \frac{6}{100} \xrightarrow{\div 2} \frac{3}{50}$$

Answer: $\frac{3}{50}$

You can work out a fraction increase or decrease.

Find the actual increase or decrease first.

Example 3: On Monday 200 students used the college canteen. On Tuesday, 350 students used the canteen. By what fraction did the number of students using the canteen increase?

The actual increase in the number of students using the canteen on Tuesday is $350 - 200 = 150$.

You express this as a fraction of the number of students using the canteen on Monday.

$$\frac{150}{200} \xrightarrow{\div 10} \frac{15}{20} \xrightarrow{\div 5} \frac{3}{4}$$

Answer: $\frac{3}{4}$

Tip

You should always write the fraction in its lowest terms.

Remember

1 kg = 1 000 g

Tip

Check answers on your calculator using the $\frac{ab}{c}$ key.

Tip

You always compare with the original value unless you are expressly told otherwise. This will be the denominator.

You can round numbers to find an **approximate fraction**.

Example 4: A college has a total of 1 060 students. 220 of them are studying mathematics. Approximately what fraction of the college students are studying mathematics?

The *exact* fraction is $\frac{220}{1060}$.

Writing both top and bottom correct to 1 significant figure gives:

$$\frac{200}{1000} \xrightarrow{\div 100} \frac{2}{10} \xrightarrow{\div 2} \frac{1}{5}$$

Answer: $\frac{1}{5}$

▶ Try the skill

1. In a class of 30 students, 21 are girls.
What fraction of the class are girls?

2. In a car park of 120 cars, 45 are made in Europe and the rest are made in Japan.
What fraction of the cars are made in Japan?

3. Express 54 mm as a fraction of 20 cm in its simplest form.

4. Sales at a clothes shop in January totalled £2250.
In February the sales totalled £750.
By what fraction did the sales decrease in February?

5. In a survey, 120 students were asked how long they spent using a computer in one week.
The results are shown below.

Number of hours	0–5	6–10	11–15	16+
Number of students	22	57	35	6

What fraction of the students spent 10 or fewer hours using a computer in one week?

Remember

1 cm = 10 mm


Remember

The denominator of the fraction will be the 'original sales' amount.

5 Adding and subtracting fractions

Learn the skill

You use fractions when working with *time*.

 You can only add or subtract fractions with the same denominator.

If the denominators are different, you need to express them as equivalent fractions with the same denominator.

Example 1: Work out $\frac{1}{3} + \frac{3}{5}$.

Here the denominators are different, so you need to decide on a common denominator to use, then write them both as equivalent fractions with the same denominator. This denominator should be the smallest number that 3 and 5 will divide into, the LCM.

The LCM of 3 and 5 is 15.

Change $\frac{1}{3}$ and $\frac{3}{5}$ to equivalent fractions with a denominator of 15.

$$\frac{1}{3} \begin{array}{c} \times 5 \\ \hline = \\ \hline \times 5 \end{array} \rightarrow \frac{5}{15} \quad \text{and} \quad \frac{3}{5} \begin{array}{c} \times 3 \\ \hline = \\ \hline \times 3 \end{array} \rightarrow \frac{9}{15}$$
$$\frac{5}{15} + \frac{9}{15} = \frac{14}{15}$$

Answer: $\frac{14}{15}$

Example 2: Evaluate $1\frac{3}{4} + 2\frac{4}{5}$.

First add the whole numbers: $1 + 2 = 3$

Now add the fractions: $\frac{3}{4} + \frac{4}{5}$

The LCM of 4 and 5 is 20.

$$\frac{3}{4} \begin{array}{c} \times 5 \\ \hline = \\ \hline \times 5 \end{array} \rightarrow \frac{15}{20} \quad \text{and} \quad \frac{4}{5} \begin{array}{c} \times 4 \\ \hline = \\ \hline \times 4 \end{array} \rightarrow \frac{16}{20}$$
$$\frac{15}{20} + \frac{16}{20} = \frac{31}{20} = \frac{20}{20} + \frac{11}{20} = 1\frac{11}{20}$$

$$3 + 1\frac{11}{20} = 4\frac{11}{20}$$

Answer: $4\frac{11}{20}$

Example 3: Work out $\frac{7}{8} - \frac{4}{5}$.

The method is similar to addition.

The LCM of 8 and 5 is 40.

$$\frac{7}{8} \begin{array}{c} \times 5 \\ \hline = \\ \hline \times 5 \end{array} \rightarrow \frac{35}{40} \quad \text{and} \quad \frac{4}{5} \begin{array}{c} \times 8 \\ \hline = \\ \hline \times 8 \end{array} \rightarrow \frac{32}{40}$$
$$\frac{35}{40} - \frac{32}{40} = \frac{3}{40}$$

Answer: $\frac{3}{40}$

Remember

The LCM (**lowest common multiple**) is the smallest number that two numbers will both divide into.

Tip

If the denominators do not have a common factor, you can find their LCM by multiplying the denominators together.

Tip

The most common fractions used in practical questions involving addition and subtraction of fractions are halves, thirds, quarters, fifths, eighths and tenths.

Example 4: Evaluate $3\frac{3}{10} - 1\frac{4}{5}$.

First subtract the whole numbers: $3 - 1 = 2$

Then work out $\frac{3}{10} - \frac{4}{5}$.

The LCM of 10 and 5 is 10.

$\frac{3}{10} - \frac{4}{5}$ becomes $\frac{3}{10} - \frac{8}{10} = -\frac{5}{10}$

$-\frac{5}{10} \xrightarrow{\div 5} = -\frac{1}{2} \xrightarrow{\div 5}$ in its lowest terms.

The answer is: $2 - \frac{1}{2} = 1\frac{1}{2}$

Answer: $1\frac{1}{2}$

Tip

Questions involving fractions may ask you to 'work out', 'calculate' or 'evaluate'. These all mean the same thing.

▶ Try the skill

1. Three TV programmes last for $1\frac{1}{2}$ hours, $\frac{3}{4}$ hour and $1\frac{3}{4}$ hours. If you watch them all, how long would this take?

2. A girl walks to the shops. It takes her $\frac{1}{2}$ hour to get there, she shops for $3\frac{1}{4}$ hours and it takes her $\frac{3}{4}$ hour to get back. How long did the shopping trip take?

3. A boy cycles to his friend's house. It takes him $1\frac{3}{4}$ hours to get there and $2\frac{1}{4}$ hours to cycle back. How much longer did it take him to cycle back from his friend's house?

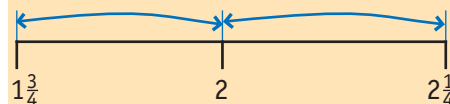
4. A film lasts for $3\frac{3}{4}$ hours. An edited version of the film lasts for $2\frac{1}{2}$ hours. How much shorter is the edited film than the original version?

Tip

Check answers on your calculator using the **ab/c** key.

Tip

Read the question carefully. To find out how much longer, or shorter, you count on or subtract. It can help to sketch a number line:



6 Remember what you have learned

First complete this ...

- ▶ You write a fraction in its lowest terms by _____ any common factors.
- ▶ In an _____ fraction the numerator is bigger than the denominator.
- ▶ To find $\frac{1}{5}$ of a quantity, _____ by 5.
- ▶ To find $\frac{2}{5}$ of a quantity, _____ by 5, then _____ by 2.
- ▶ One number as a fraction of another number is:
 $\frac{\text{_____ number}}{\text{_____ number}}$
- ▶ You can only add or subtract fractions with the same _____.
- ▶ When dividing by a fraction, multiply top and bottom to make the denominator a _____.
- ▶ To multiply by a mixed number, partition it into a whole number and a _____.

Tip

Before you can express one quantity as a fraction of another, you must write them in the *same units*.

Try the skill

1. The total receipts for a football club match in one particular week were £640 000. The club spends $\frac{3}{8}$ of its income on salaries.
How much of the week's receipts was spent on salaries?
2. A boy gets £10 a week in pocket money. He also receives £8 a week for delivering newspapers.
What fraction of his total income is the money he receives for delivering newspapers?
3. It takes a girl $1\frac{3}{4}$ hours to travel to the Trafford centre in Manchester. She spends $2\frac{1}{2}$ hours shopping and it takes her $1\frac{1}{4}$ hours to get home.
How long does she take in total?

- A £80 000
- B £70 000
- C £240 000
- D £210 000

- A $\frac{1}{9}$
- B $\frac{4}{5}$
- C $\frac{4}{9}$
- D $\frac{5}{9}$

- A $4\frac{1}{2}$ hours
- B $4\frac{3}{4}$ hours
- C $5\frac{1}{4}$ hours
- D $5\frac{1}{2}$ hours

- 4 Out of a group of students, $\frac{2}{3}$ travel to college by bus, $\frac{1}{4}$ travel by train and the rest walk to college.

What fraction of the students walk to college?

- A $\frac{1}{3}$
 B $\frac{1}{6}$
 C $\frac{1}{4}$
 D $\frac{1}{12}$

- 5 In an audience of people watching a film, $\frac{1}{10}$ are under 16 years old, $\frac{3}{5}$ are between 16 and 40 years old and the rest are over 40 years old. What fraction are over 40 years old?

- A $\frac{1}{12}$
 B $\frac{2}{5}$
 C $\frac{3}{5}$
 D $\frac{7}{10}$

6. The price of a dress is reduced from £35 to £25 in a sale. What calculation would you do to find the decrease in price as a fraction of the original price?

- A $\frac{35 - 25}{25}$
 B $\frac{35 - 25}{35}$
 C $\frac{25}{35 - 25}$
 D $\frac{35}{35 - 25}$

7. There are 90 cars in a car park. $\frac{2}{5}$ of these cars are red. $\frac{1}{3}$ of the red cars have three doors.

How many cars in the car park are red with three doors?

- A 6 cars
 B 10 cars
 C 12 cars
 D 15 cars

8. In a survey of 252 students, 147 of them had their own iPod.

Approximately what fraction of the students had their own iPod?

- A $\frac{3}{8}$
 B $\frac{3}{5}$
 C $\frac{2}{5}$
 D $\frac{5}{8}$

9. There were 5261 people in a village. At an election 1933 voted Conservative.

Approximately what fraction of the people in the village voted Conservative?

- A $\frac{1}{5}$
 B $\frac{1}{3}$
 C $\frac{2}{5}$
 D $\frac{2}{3}$

C Working with decimals

You should already know how to:

- ✓ read, write, order and compare decimals
- ✓ add, subtract, multiply and divide decimals with up to 2 decimal places
- ✓ multiply and divide decimals by 10 and 100.

By the end of this section you will know how to:

- ▶▶ order, approximate and compare decimals
- ▶▶ calculate with decimals with up to 3 decimal places
- ▶▶ calculate and estimate amounts of money and convert between currencies.

1 Decimal numbers

Learn the skill

In a decimal number, the decimal point separates the whole number from the part that is less than 1.

In the number 10.357:

Tens	Units	•	Tenths	Hundredths	Thousandths
1	0	•	3	5	7

The 3 stands for $\frac{3}{10}$, three tenths The 5 stands for $\frac{5}{100}$, five hundredths The 7 stands for $\frac{7}{1000}$, seven thousandths

Calculations involving decimals follow the rules for whole numbers and fractions.

- ▶ To round a number:
- ▶ Identify the **last digit** that is needed in rounded form. Look at the next digit to the right.
- ▶ If the **next digit** is 5, 6, 7, 8 or 9, round the **last digit** up.
- ▶ If the **next digit** is 0, 1, 2, 3 or 4, leave the **last digit**.

Example 1

Round 3.5736 to three decimal places

Next digit
Last digit needed

Last digit that is needed is 3, next digit is 6 so round the last digit up.

Answer 3.574

Tip

Answers to money calculations in pounds should be written correct to the nearest penny (two decimal places).

Remember

The number 12.5095 is:
12.510 to three decimal places
12.51 to two decimal places
12.5 to one decimal place.

Example 2: A plumber cuts three lengths of pipe measuring 1.102 m, 1.204 m and 0.426 m. What is the total length of pipe he uses?

Line up the numbers using the decimal points.

1	.	1	0	2
1	.	2	0	4
0	.	4	2	6
2	.	7	13	2

Answer: 2.732 m

Example 3: One yard is equal to 3 feet. One foot is equal to 12 inches. 1 inch is approximately equal to 2.54 cm. What is one yard, in centimetres?

The calculation is:

×	2	0.5	0.04
30	60	15	1.2
6	12	3	0.24
=	72	18	1.44

$$\begin{aligned}
 &3 \times 12 \times 2.54 \\
 &= 36 \times 2.54 \\
 &= (30 + 6) \times (2 + 0.5 + 0.04) \\
 &= 72 + 18 + 1.44 = 91.44
 \end{aligned}$$

Answer: 91.44 cm

Remember

Multiplying by 0.4 is the same as multiplying by 4 and dividing by 10. Multiplying by 0.04 is the same as multiplying by 4 and dividing by 100, and so on.

eg: $30 \times 4 = 60 \times 2 = 120$
 $30 \times 0.4 = 12.0$
 $30 \times 0.04 = 1.2$

▶ Try the skill

1. 1 inch = 2.54 centimetres. Approximately how long is 18 inches in centimetres?

2. One petrol station sells petrol for £1.069 per litre, another sells petrol for £0.999 per litre.
 - a What is the difference in the price per litre?

 - b How much could a driver save by buying 20 litres at the lower price?

3. A boy weighs 7 stones 12 pounds. 14 pounds = 1 stone and 1 kg = 2.2 pounds. How much does the boy weigh, in kilograms?

4. A car's petrol tank holds 12 gallons of petrol when full. 1 gallon is approximately 4.55 litres. What is 12 gallons, in litres?

2 Calculating with money

Learn the skill

Money problems often involve decimal calculations.

Example 1: A company charges a fixed cost of £7 for specially designed cards, plus a printing charge of 5p for each card.

- a How much will they charge for an order of 120 cards?
- b How many cards would you get for £20?

The final price is in pounds, so change the pence to pounds.
5p = £0.05

a The cost for the cards is 120×0.05
 $= 12 \times 0.5$
 $= 12 \times \frac{1}{2} = £6$

So the total cost is £7 + £6 = £13

Answer: £13

- b To find the printing charge, subtract the fixed cost of £7.

The printing charge is £20 – £7 = £13

Each card costs 5p to print, so work out how many 5ps there are in £13:

$$\frac{13}{0.05} \begin{array}{l} \xrightarrow{\times 100} \\ \xleftarrow{\times 100} \end{array} \frac{1300}{5} \begin{array}{l} \xrightarrow{\div 5} \\ \xleftarrow{\div 5} \end{array} \frac{260}{1} = 260$$

Answer: 260 cards

Example 2: £1 = \$1.85

- a Use the above exchange rate to change £250 to dollars.
- b What calculation would you do to check your answer?

- a Every £1 is worth \$1.85.
There will be more dollars than pounds, so multiply:

$$250 \times 1.85 = (250 \times 1) + (250 \times 0.8) + (250 \times 0.05) = 250 + 200 + 12.50 = 462.50$$

Answer: \$462.50

- b The calculation is:

$$£250 \rightarrow \boxed{\times 1.85} \rightarrow \$462.50$$

To check the calculation, start from \$462.50 and work backwards, doing the inverse operation.

$$£250 \leftarrow \boxed{\div 1.85} \leftarrow \$462.50$$

Answer: $\frac{\$462.50}{1.85} = £250$

Test tip

If the final answer is in pounds, change any pence to pounds.

Tip

$$120 \times 0.05 = 12 \times 10 \times 0.05 = 12 \times 0.5$$

Tip

To change from pounds to dollars, *multiply* by 1.85.

Remember

+ is the inverse of –
× is the inverse of ÷

 **Try the skill**

1. The cost of hiring a carpet cleaner is £15 basic charge plus £12.50 per day. What is the cost of hiring a carpet cleaner for four days?

2. A man earns a basic rate of £12.20 per hour for a 35-hour week. For each hour worked over 35 hours he earns 1.25 times the basic rate. How much does the man earn in a week when he works 40 hours?

3. A T-shirt company sells personalised T-shirts with names printed on them. The company charges £10.99 for each T-shirt plus 8p for each letter printed on it. How much will it cost to buy a T-shirt with the name 'Alexander' printed on it?

4. A children's entertainer charges a fixed fee of £60 plus £8.50 per hour to attend a party. How much does she charge to attend a party for 3.5 hours?

5. A woman is buying an outfit for a wedding. She spends £125.99 on a dress, £56 on shoes and £70.50 on a hat. Round the prices to the nearest pound to estimate the total cost of her outfit.

6. A girl receives 464 euros in exchange for £320. The exchange rate is £1 = 1.45 euros. What calculation would you do to check this is correct?

7. A car is priced at 12 000 euros. Use the approximate conversion £3 = €5 to approximate the price of the car, in pounds.

8. £1 = \$1.85. Use this rate to change £280 to dollars.

Tip







You need to be able to work out the answers to these questions without a calculator, but remember to use your calculator as a check for your calculations.

Tip

Check your answer is sensible. £3 = 5 euros, so you need fewer pounds than euros.

3 Remember what you have learned

First complete this ...

-  In a decimal number, the _____ separates the whole number from the part that is less than 1.
-  To round a number:
 -  Count along to the last digit that is needed.
 -  If the next digit is 5, 6, 7, 8 or 9, _____ the last digit up.
 -  If the next digit is 0, 1, 2, 3 or 4, _____ the last digit.
-  Answers to money calculations in pounds should be written correct to the nearest penny (_____ decimal places).

Try the skill

1. A car travels at 60 mph. 1 mile is roughly 1.6 km.
What is the speed in km per hour?

- A 37 km per hour
- B 38 km per hour
- C 90 km per hour
- D 96 km per hour

2. A garage records the carbon monoxide emissions of two cars. One emits 0.341 g/km and the other emits 1.154 g/km.
What is the difference in the emissions, to the nearest tenth of a g/km?

- A 0.7 g/km
- B 0.8 g/km
- C 0.9 g/km
- D 1.0 g/km

3. The exchange rate on a particular day is 1 euro = 69.2 pence.
How would you calculate the value of £150 in euros?

- A $\frac{69.2}{150 \times 100}$
- B $\frac{150}{69.2 \times 100}$
- C $\frac{69.2 \times 100}{150}$
- D $\frac{150 \times 100}{69.2}$

4. A dog weighs 26 pounds. 1 kg is approximately 2.2 pounds. How much does the dog weigh, in kilograms, to the nearest kilogram?

- A 12 kg
 B 13 kg
 C 58 kg
 D 57 kg

5. A garage charges £32.50 per hour for labour plus the cost of parts. The repairs to a woman's car take 3.2 hours and the parts cost £115.70.

Which of these calculations works out the total bill?

- A $115.70 + (32.50 \times 3.2)$
 B $(115.70 + 32.50) \times 3.2$
 C $(115.70 \times 3.2) + 32.50$
 D $(115.70 \times 32.50) + 3.2$

6. A boy is buying Christmas presents. He buys a CD for £9.50, a pair of gloves for £12.99, a book for £5.75 and a game for £8.20.

Round the prices to the nearest pound to estimate the total.

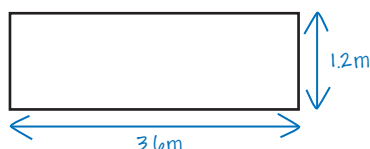
- A £35
 B £36
 C £37
 D £38

7. On a particular day the exchange rate between pounds and dollars is £1 = \$1.85 dollars.

How much is £400 worth, in dollars?

- A \$216
 B \$418.50
 C \$720
 D \$740

8. The diagram shows the work surface in a hotel kitchen. The surface is to be covered with stainless steel.



What is the area of the work surface?

- A 3.6 m^2
 B 4.032 m^2
 C 4.32 m^2
 D 4.8 m^2

9. An office manager is buying IT equipment. She buys three desktop PC base units at £487.64 each, two small monitors at £102.48 each and one large monitor at £186.85. She rounds the prices to the nearest pound to estimate the total.

What is the total?

- A £1367
 B £1851
 C £1855
 D £1857

D Working with percentages, fractions and decimals

You should already know how to:

- ✓ read, write, order and compare simple percentages
- ✓ understand simple percentage increase and decrease
- ✓ find simple percentage parts of quantities and measurements.

By the end of this section you will know how to:

- calculate percentage increase and decrease
- calculate using percentages in everyday contexts
- evaluate one number as an (approximate) percentage of another
- calculate percentage change
- use a calculator to work out percentage calculations
- identify equivalence between fractions decimals and percentages
- use fractions, decimals and percentages in practical problems.

1 Percentages

Learn the skill

A percentage is a way of expressing a number as a proportion of 100.


For example, $30\% = \frac{30}{100} = \frac{3}{10} = 0.3$

Example 1: Find 30% of 60.

10% of 60 is 6, so 30% of 60 is $3 \times 6 = 18$

Answer: 18

You can increase a quantity by a percentage.

 The original quantity is 100%. The increased quantity will be more than 100%.

Example 2: Increase 40 by 25%.

First find 25% of 40.

$$\begin{array}{r} 5 \\ \cancel{2}5 \\ \hline 10 \end{array} \times \frac{40}{1} = 10 \quad \text{This is the amount it is increased by.}$$

$40 + 10 = 50$

Answer: 50

Remember

$$10\% = \frac{10}{100} = \frac{1}{10}$$


To find 10%, divide by 10.

■ $5\% = 10\% \div 2$

■ $20\% = 10\% \times 2$

Tip


Check your answer is more than the original amount.

 You can work out a percentage increase in a single calculation:

$$100\% + 25\% = 125\%$$

$$125\% \times 40 = \frac{125}{100} \times \frac{40}{1} = 50$$

You can decrease a quantity by a percentage.

 The original quantity is 100%. The decreased quantity will be less than 100%.

Example 3: Decrease 80 by 75%.

First find 75% of 80.

$$\frac{75}{100} \times \frac{80}{1} = 60 \quad \text{This is the amount it is decreased by.}$$

$$80 - 60 = 20$$

Answer: 20

Tip

You can also find 25% mentally:

$$25\% = \frac{1}{4}$$

$$\frac{1}{4} \text{ of } 40 = 10$$

$$\text{so } 125\% \text{ of } 40 = 10 + 40 = 50$$

Tip

In a single calculation:

$$100\% - 75\% = 25\%$$

$$25\% \text{ of } 80 = 20$$

Try the skill

Try out each of the above methods in these questions and decide which you like best.

1. Decrease 75 by 4%.

2. Increase 80 by 5%.

3. Increase 45 by 40%.

4. Decrease 400 by 1%.

5. Increase 250 by 20%.

6. Decrease 620 by 15%.

7. Decrease 180 by 2%.

8. Increase 5 by 30%.

Tip

Using the % key on a calculator:

Example

Find 12% of 240

Solution

'of' means 'x'

input

$$240 \times 12 \%$$

gives the answer 28.8

Note: do not press $\frac{\square}{\square}$

Use the % key on your calculator to check your answers.

2 Using percentages

Learn the skill

Prices can be *increased* by a percentage, for example when VAT is added, or they can be *decreased*, for example in a sale.

Example 1: A clothes shop reduces its prices by 30% in a sale. What is the sale price of a dress that cost £50 before the sale?

Original price is 100%.

Sale price is $100\% - 30\% = 70\%$.

$100\% = \text{£}50$

$10\% = \text{£}5$

$70\% = 7 \times \text{£}5 = \text{£}35$

Answer: **£35**

Example 2: A television costs £300 plus VAT at $17\frac{1}{2}\%$. What is the total price of the television, including VAT?

$100\% = \text{£}300$

$10\% = \text{£}30$

$5\% = \text{£}15$

$2\frac{1}{2}\% = \text{£}7.50$

so $17\frac{1}{2}\% = 30 + 15 + 7.50 = \text{£}52.50$

Total cost of television = $\text{£}300 + \text{£}52.50 = \text{£}352.50$

Answer: **£352.50**



One number as a percentage of another is:

$$\frac{\text{first number}}{\text{second number}} \times 100$$

Example 3: Express 55p as a percentage of £11.

$\text{£}11 = 1100\text{p}$

$$\frac{55}{1100} \times \frac{100}{1} = 5$$

Answer: **5%**

Sometimes test questions ask for the *approximate* percentage.

Example 4: A company has 794 employees. In a survey, 157 said they travelled to work by motorbike. Approximately what percentage of the employees travel to work by motorbike?

Approximate 794 to 800 and 157 to 160 (8 is a factor of 16).

Now work out the fraction $\frac{160}{800}$ as a percentage.

$$\frac{160}{800} \times \frac{100}{1} = 20$$

Answer: **20%**

Remember

When working with questions involving money, give your answers to *two* decimal places. For example, an answer of 3.5 means **£3.50**.

Remember

VAT is Value Added Tax.

Remember

To express one quantity as a percentage of another, you must write them in the same units.

 **Try the skill**

1. McDonut's reduce the price of their food by 20% in a sales promotion. A double burger usually costs £1.50. How much will it cost in the sales promotion?

2. A girl puts £200 into a savings account that has an interest rate of 4.5% AER. She leaves the money in the account. How much will she have in the account at the end of the year?

3. The VAT on gas and electricity bills is 5%. How much VAT will be added to a monthly gas bill of £55?

4. A DVD player costs £120 before VAT of $17\frac{1}{2}\%$ is added. What is the cost of the DVD player, including VAT?

5. Write 21 as a percentage of 70.

6. Express 54 mm as a percentage of 20 cm.

7. The price of a sandwich in a canteen goes up from £1.24 to £1.51. Express the increase in price as an approximate percentage of the original price.

8. In a car park of 1 200 cars, 423 were made in Europe and the rest were made in Japan. Approximately what percentage of the cars were made in Japan?

Tip

The AER is the Annual Equivalent Rate, used to help compare rates for savings.

**Tip**

10 mm = 1 cm

3 Percentage change

Learn the skill

You find a percentage change using this formula:

$$\text{percentage change} = \frac{\text{actual change}}{\text{original value}} \times 100$$

Example 1: In 2004, 640 million mobile phones were sold. In 2005, 720 million mobile phones were sold. What is the percentage increase in the number of mobile phones sold from 2004 to 2005?

For percentage increase the formula is:

$$\text{percentage increase} = \frac{\text{actual increase}}{\text{original value}} \times 100$$

Actual increase = $720 - 640 = 80$ million

$$\text{Percentage increase} = \frac{80}{640} \times \frac{100}{1} = 12.5\%$$

Answer: 12.5%

Example 2: A shopkeeper buys 150 kg of bananas for £110 and sells them for 90 pence per kilogram. What is his percentage profit? Give your answer to one decimal place.

For percentage profit the formula is:

$$\text{percentage profit} = \frac{\text{actual profit}}{\text{original value}} \times 100$$

Cost price = £110

$$\begin{aligned} \text{Selling price} &= (90 \times 150) \text{ pence} \\ &= (90 \times 100) + (90 \times 50) \\ &= 9000 + 4500 \\ &= 13500 \text{ pence} = \text{£}135 \end{aligned}$$

Actual profit = $\text{£}135 - \text{£}110 = \text{£}25$

$$\begin{aligned} \text{Percentage profit} &= \frac{25}{110} \times \frac{100}{1} \\ &= \frac{250}{11} = 250 \div 11 \\ &= 22.7\% \end{aligned}$$

Answer: 22.7%

Tip

The word 'change' may be replaced by 'increase', 'decrease', 'profit' or 'loss'.

Tip

Make sure the quantities are in the same units.

Tip

Always check your answers are sensible. By estimation:

$$\frac{25}{100} \times 100 = 25\% \text{ approximately.}$$

 **Try the skill**

1. In April, 1250 people visited an art exhibition. In May, 1675 people visited the art exhibition. What was the percentage increase in the number of visitors to the art exhibition between April and May?

2. A hockey club bought 150 sweatshirts for £1500. The same club sold the sweatshirts for £12.50 each. What was the percentage profit?

3. A man bought a car for £1500. He later sold it for £1200. What was his percentage loss?

4. A factory employed 360 workers. The number of workers decreased to 342. What was the percentage decrease in the number of workers?

5. In 2004, a large company makes a profit of £750 000. In 2005 the company makes a profit of £1m. What is the percentage increase in the profit?

6. A clothes shop reduces the price of T-shirts from £2.50 to £2. What is the percentage discount?

7. A shopkeeper buys 100 umbrellas for £500. He sells the first 75 for £6 each and the rest for £4 each. What is his percentage profit?

Tip


In percentage change questions, first write down the original amount, which is the amount before the increase or decrease.

Remember

The denominator is the original value.

4 Converting between forms

Learn the skill

 To convert a fraction to a decimal, divide the numerator by the denominator.

Example 1: Convert $\frac{1}{3}$ to a decimal.

$$\frac{1}{3} = 1 \div 3 \text{ which gives } 0.333333\dots \quad \text{Answer: } 0.333333\dots$$

You use **place value** to convert a decimal to a fraction.


Example 2: Convert 0.125 to a fraction.

$$0.125 = \frac{125}{1000} \xrightarrow{\div 5} \frac{25}{200} \xrightarrow{\div 5} \frac{5}{40} \xrightarrow{\div 5} \frac{1}{8}$$

0.125 has three places of decimals which means you use thousandths.

$$\text{Answer: } \frac{1}{8}$$

As a percentage is a proportion of 100:

 To convert a percentage to a fraction or decimal, divide by 100.


Example 3: Convert 65% to a fraction.

$$65\% = \frac{65}{100} = \frac{13}{20} \quad \text{Answer: } \frac{13}{20}$$

Example 4: Convert 2.5% to a decimal.

$$2.5\% = \frac{2.5}{100} = 0.025 \quad \text{Answer: } 0.025$$

To convert a fraction or a decimal to a percentage you do the opposite operation.

 To convert a fraction or decimal to a percentage, multiply by 100.

Example 5: Convert $\frac{5}{8}$ to a percentage.

$$\frac{5}{8} \times \frac{100}{1} = \frac{500}{8} = 62.5\% \quad \text{Answer: } 62.5\%$$

Example 6: Convert 0.462 to a percentage.

$$\text{You need to work out } 0.462 \times 100 = 46.2. \quad \text{Answer: } 46.2\%$$

Tip

0.333333... is an example of a *recurring decimal*. It can be written as $0.\dot{3}$.

Remember

To write fractions in their *lowest terms*, cancel any common factors.

Tip

Dividing by 100 moves all the digits two places to the right.

$$\begin{array}{r} 2.5 \\ \hline 0.025 \end{array}$$

Tip

Remember to simplify the multiplication by cancelling.

Tip

Multiplying by 100 moves all the digits two places to the left.

$$\begin{array}{r} 0.462 \\ \hline 46.2 \end{array}$$

**Try the skill**

1. Change each of these fractions to a decimal and a percentage.

a $\frac{4}{5}$ _____

b $\frac{3}{20}$ _____

c $\frac{7}{25}$ _____

d $\frac{7}{8}$ _____

2. Change each of these decimals to a percentage and a fraction.

a 0.16 _____

b 0.32 _____

c 0.08 _____

d 0.56 _____

3. Change each of these percentages to a decimal and a fraction.

a 64% _____


b 24% _____

c 45% _____

d 8.5% _____

5 Using fractions, decimals and percentages in practical problems

Learn the skill

 You can decide which of fraction, decimal or percentage form to use in practical problems.

Example 1: In a sale, a dress shop reduces its prices by 25%. What is the sale price of a dress which cost £60 before the sale?

The original price is 100%, so the sale price is $100\% - 25\% = 75\%$

$$75\% = 0.75 = \frac{3}{4}$$

Using percentages: $60 \times 75\%$ Answer: £45

Using decimals: 60×0.75 Answer: £45

Using fractions: $60 \div 4 \times 3$ Answer: £45

Example 2: A MP3 player costs £220 before VAT of $17\frac{1}{2}\%$ is added. What is the cost of the MP3 player, including VAT?

The original price is 100%, so the cost including VAT is $117\frac{1}{2}\%$.

$117\frac{1}{2}\%$ is the same as 117.5%

$$117.5\% = \frac{117.5}{100} = 1.175 \text{ as a decimal}$$

Using percentages: $220 \times 117.5\%$ Answer: £258.50

Using decimals: 220×1.175 Answer: £258.50

Using fractions: $220 \div 100 \times 117.5$ Answer: £258.50

Tip

Decide which of these methods you would find easiest **without** using a calculator.

Remember

An answer of 258.5 should be written as £258.50 in pounds and pence.

Try the skill

Try out each of the above methods in these questions, both with and without a calculator and decide which you like best.

1. Find the price of a sofa costing £350 after a discount of 40%.
2. A company employs 480 people. Find the total number employed by the company after an increase of 15% in the number of people employed.
3. Emma's salary is £20 000 a year. What is her new salary after an increase of 4%?
4. The value of a car depreciates by 35% in the first year. A car costs £15 000 when new. What is its value after one year?
5. Rebecca invests £3000 in a savings account with an interest rate of 6.2% per annum. How much will be in her account at the end of the first year?

Tip

To *depreciate* means to go down in value.

6

Remember what you have learned

Learn the skill

- ▶ A percentage is a way of expressing a number as a proportion of _____.
- ▶ To find 10%, divide by _____.
- ▶ $5\% = 10\% \div 2$
- ▶ $20\% = 10\% \times 2$
- ▶ The original quantity is _____.
- ▶ The increased quantity will be _____ than 100%.
- ▶ The decreased quantity will be _____ than 100%.
- ▶ One number as a percentage of another is:

$$\frac{\text{number}}{\text{number}} \times 100$$
- ▶ percentage change = $\frac{\text{actual change}}{\text{original value}} \times 100$
- ▶ To convert a fraction to a decimal, divide the _____ by the _____.
- ▶ To convert a fraction or decimal to a percentage, _____ by 100.
- ▶ To convert a percentage to a fraction or decimal, _____ by 100.
- ▶ Remember these equivalences:

Fraction	Decimal	Percentage
_____	0.1	10%
$\frac{1}{4}$	_____	25%
$\frac{1}{3}$	_____	33.3% or $33\frac{1}{3}\%$
$\frac{1}{2}$	0.5	_____
$\frac{2}{3}$	_____	66.6% or $66\frac{2}{3}\%$
_____	0.75	75%

Tip

Read VAT questions carefully – do you want the VAT, or the price including VAT?

Tip

The quantities must be in the same units.

Tip

The word 'change' may be replaced by 'increase', 'decrease', 'profit' or 'loss'.

 Use the skill

1. 28% of the money spent on the National Lottery goes to good causes.

How much money went to good causes in a year when £6 000 000 000 was spent on the National Lottery?

- A £1.68 million
 B £16.8 million
 C £168 million
 D £1 680 million

2. A shopkeeper changes his opening hours. He used to open from 9.00am to 5.00pm, but he now opens from 8.00am to 6.00pm.

What is the percentage increase in the number of hours the shop is open?

- A 20%
 B 25%
 C 75%
 D 80%

3. A cleaner's basic rate of £8.60 per hour is increased by 3%. Which calculation gives the new basic rate per hour?

- A $£8.60 \times \frac{3}{100}$
 B $£8.60 + 3 \times 100$
 C $£ \frac{3 \times 100}{8.60}$
 D $£ \left(8.60 \times \frac{3}{100} \right) + 8.60$

4. A shopkeeper bought a television for £300 and sold it for £335.

Which calculation gives the percentage profit?

- A $\frac{335 - 300}{300} \times 100$
 B $\frac{335 - 300}{335} \times 100$
 C $\frac{300}{335 - 300} \times 100$
 D $\frac{335}{335 - 300} \times 100$

5. In a survey, 25 students were asked how much time they had spent on revision and homework in one week. The results are shown in the table below.

Number of hours	Number of students
0 to less than 5	2
5 to less than 10	6
10 to less than 15	12
15 to less than 20	4
20 or more	1

What percentage of the students spent between 10 and 15 hours on their homework?

- A 12%
 B 48%
 C 25%
 D 15%

6. In a sale, a carpet store reduces the price of its carpets by 40%. Before the sale, one particular carpet cost £20.50 per square metre.

What is the sale price of this carpet per square metre?

- A £8.00
- B £8.20
- C £12
- D £12.30

7. In a survey of 500 shoppers in Oxford High Street, 137 of them had used the park-and-ride scheme to get to the city centre.

Approximately what percentage is this?

- A 30%
- B 29%
- C 28%
- D 27%

8. A health-and-fitness club offers membership for £25 per month for 12 months, or £275 for the whole year.

Which calculation gives the percentage saved by paying £275 for the year?

- A $\frac{25 \times 12 - 275}{275} \times 100$
- B $\frac{25 \times 12 - 275}{25 \times 12} \times 100$
- C $\frac{275}{25 \times 12 - 275} \times 100$
- D $\frac{25 \times 12}{25 \times 12 - 275} \times 100$

9. A shopkeeper keeps a record of the different types of DVDs she sells. About $\frac{5}{8}$ of the DVDs are children's DVDs.

What percentage is closest to $\frac{5}{8}$?

- A 16%
- B 58%
- C 63%
- D 85%

10. On average, only 8% of people who attend a local cinema buy membership.

Which fraction is closest to 8%?

- A $\frac{8}{10}$
- B $\frac{8}{12}$
- C $\frac{1}{8}$
- D $\frac{1}{12}$

11. In a survey of 16- to 18-year-olds at FE colleges:

- $\frac{2}{5}$ were studying economics
- 2 in every 100 were studying physics
- 22% were studying law
- 2 in every ten were studying psychology

According to the survey, what was the most common subject studied?

- A economics
B physics
C law
D psychology

12. A survey of a sample of library users showed that $\frac{3}{8}$ were children.

What is this fraction as a decimal?

- A 0.375
B 0.380
C 0.625
D 0.830

13. Four friends estimate the amount of fat in their Indian take-away as $\frac{1}{5}$, 22%, 0.26 and $\frac{1}{4}$.

Which is the highest estimate?

- A $\frac{1}{5}$
B 22%
C 0.26
D $\frac{1}{4}$

14. Estimates for the number of vegetarians in a company are $\frac{1}{8}$, 10%, 0.08 and $\frac{1}{7}$.

Which is the smallest estimate?

- A $\frac{1}{8}$
B 10%
C 0.08
D $\frac{1}{7}$

15. $\frac{1}{6}$ of the people questioned in a survey said they did not vote in the last election.

What percentage is closest to $\frac{1}{6}$?

- A 6%
B 14%
C 17%
D 20%