Unit 13: Mathematical Calculations for Science

Unit code: A/502/5546

QCF Level 3: BTEC National

Credit value: 5

Guided learning hours: 30

Aim and purpose

This unit gives learners the skills to use mathematical tools essential for biology, chemistry and physics. The unit starts with algebraic techniques and moves on to the applications of trigonometry and calculus.

Unit introduction

This maths unit enables learners to build on knowledge gained from *Unit 11: Using Mathematical Tools in Science* or GCSE mathematics to investigate further mathematical tools used in all disciplines in science. It is an essential unit for learners who wish to pursue science at BTEC Higher National level, foundation degree or degree level. The unit is not intended to make mathematicians but to use maths for science. Learners will use maths to solve a number of scientific problems, including:

- how simultaneous equations help solve problems in chemical bonding and in electricity
- how trigonometry can be used in ecology to measure tree height or in forensic science for voice recognition
- how the method of calculus enables scientists to learn about growth studies in microbiology.

Learners will develop their knowledge and understanding of algebraic methods, including indices, logarithmic and simultaneous linear equations and quadratics. These tools will enable learners to solve many scientific problems, such as sound levels (decibels), light detection (lux levels), measurement of pH, bacterial growth and electrical power. Then learners will be introduced to the radian, as another method of angle measurement, and graphical representation of trigonometric functions. These trigonometric functions allow ample opportunity for learners to integrate computer simulations and practical activities. Practical activities could include pendulum motion, mechanics and circular motion, amongst many others. Finally, a brief introduction to the important tool of calculus. Learners will be exposed to both simple differentiation and integration and will end with using these concepts to solve real scientific problems. Problems could include areas such as population growth, effect of fishing on fish stocks, radioactive decay, motion of a particle, energy stored in springs and enzyme activity. On completing this unit, learners will have been exposed to essential mathematical concepts that will bridge the gap between Level 3 and 4.

Learning outcomes

On completion of this unit a learner should:

1 Be able to use algebraic manipulations to solve scientific problems

2 Be able to use trigonometric methods to solve scientific problems

3 Be able to use calculus to solve scientific problems.

Unit content

1 Be able to use algebraic manipulations to solve scientific problems

Laws of indices: operations, eg $a^m \times a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$; exponentials, eg $e^2 \times e^{-3}$

Laws of logarithms: $\log A + \log B = \log(AB)$, $\log A - \log B = \log(\frac{A}{B})$, $\log A^n = n \log A$

Types of logarithms: common (base 10); natural (base e)

Factorisation: multiply expressions in brackets by a number, symbol or by another expression in a bracket; by extraction of a common factor, eg $ax + ay$, $a(x + 4) + b(x + 2)$, $a^2(ba + 3c)$; by grouping, eg $ax - ay + bx - by$

Equations: pair of simultaneous linear equations in two unknowns, eg $3x + 2y - 5z = 8$; quadratic expressions, eg $a^2 + 2ab + b^2$; roots of an equation, eg quadratic equations with real roots by factorisation and by use of quadratic formula

Applications: indices, eg Kepler's law $(T^2 \propto R^{-3})$; common base, eg in sound (*decibels*), in light (*lux*), pH; natural base, eg bacterial count with time, radioactive decay; simultaneous equations, eg in momentum collision problems, electrical circuits; quadratics, eg electrical power, eg $P = V^2/R$

2 Be able to use trigonometric methods to solve scientific problems

Circular measure: radian; degree; conversion between radian to degree and vice versa; angular rotations (multiple of π radians); length of arc of a circle $(s = r\theta)$; area of a sector $(A = 1/2r^2\theta)$

Triangular measurement: ratios (sine, cosine and tangent); 90° triangles; Pythagoras's theorem; graphs (sine, cosine over one complete cycle, $\tan\theta$ as θ varies from 0 and 360°); values of the trigonometric ratios for angles between 0° and 360°; sine and cosine rule

Applications: circular measure, eg objects undergoing circular motion (such as the planets, bike going round a bend, London Eye); triangular, eg work done on pulling an object $W = F.d\cos$, using the sine and cosine rules in equilibrium related problems, simple harmonic motion of pendulum $x = A.\sin\omega t$, alternating current $(I = I_0\cos\theta)$, electromagnetic waves, voice recognition in forensic science

3 Be able to use calculus to solve scientific problems

Differentiation: differential coefficient; gradient of a curve $y = f(x)$; rate of change; Leibniz notation

$(\frac{dy}{dx})$; differentiation of simple polynomial functions, exponentials functions and sinusoidal functions;

problem solving involving evaluation, eg gradient at a point

Integration: integration as a reverse of differentiation; basic rules for simple polynomial functions, exponential functions and sinusoidal functions; indefinite integrals; constant of integration; definite integrals; limits; evaluation of simple polynomial functions; area under a curve

Applications: differentiation techniques as used in science, eg motion of a particle, rate of chemical reactions, bacterial growth studies; integration techniques used in science, eg population growth in environmental science, elastic energy stored in a spring

Assessment and grading criteria

In order to pass this unit, the evidence that the learner presents for assessment needs to demonstrate that they can meet all the learning outcomes for the unit. The assessment criteria for a pass grade describe the level of achievement required to pass this unit.

Assessment and grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**	**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**
P1 carry out calculations involving logarithms and indices [IE1, 6; CT2; SM3]	**M1** solve science problems involving indices	**D1** solve science problems involving logarithms
P2 carry out calculations using algebraic equations [IE1, 6; CT2; SM3]	**M2** solve science problems using quadratic equations	**D2** solve science problems using simultaneous equations
P3 carry out calculations involving circular measure [IE1, 6; CT2; SM3]	**M3** solve science problems using circular measure	**D3** explain, using circular measure, a real life application of mathematics
P4 carry out calculations involving trigonometric methods [IE1, 6; CT2; SM3]	**M4** solve science problems using trigonometric methods	**D4** explain, using trigonometric methods, a real life application of mathematics
P5 carry out calculations using calculus [IE1, 6; CT2; SM3].	**M5** solve science problems using differentiation.	**D5** solve science problems using integration.

PLTS: This summary references where applicable, in the square brackets, the elements of the personal, learning and thinking skills applicable in the pass criteria. It identifies opportunities for learners to demonstrate effective application of the referenced elements of the skills.

Key	IE – independent enquirers	RL – reflective learners	SM – self-managers
	CT – creative thinkers	TW – team workers	EP – effective participators

Essential guidance for tutors

Delivery

As in *Unit 11: Using Mathematical Tools in Science*, this unit should be delivered in a vocational setting and in conjunction with experimental activities, where possible related to environmental science. Learners should have plenty of practice in using logarithms and indices before attempting assignment work. Applications of indices can be linked to planetary motion. Logarithms can be linked to pH solutions, radioactive materials, growth of bacteria and astronomical measurements. All these can be demonstrated through laboratory experiments. Similarly, before learners start solving scientific problems it is recommended that they have plenty of practice in solving algebraic problems so as to develop confidence. There are many areas of science that use algebra, for example simultaneous equations are used in looking at bond energy, in chemistry. Electrical circuits can also be explained using simultaneous equations. Simple electrical circuits can be investigated with standard laboratory electrical experiments, for example with two cells, two resistors and a multimeter.

Learning outcome 2 comprises of circular and triangular measurement and offers opportunities for learners to construct models and test models to experiments. Learners can use simulations to assist further understanding. Applications in this section are many and could include the London Eye, circular motion of planets and cars around corners. Trigonometric applications could include resolving forces, waves and oscillations.

Learning outcome 3 provides a basic introduction to calculus. It is required that learners are able to use all tools of calculus mentioned in the *Unit content* section. There are many experiments in which the tools of calculus can be used, such as rates for reaction, bacterial growth or ticker tape motion experiments. Differentiation provides many applications, eg rate for reaction in the production of hydrogen and velocity and acceleration in a car journey. Integration is used in many areas, such as to calculate the distance from velocity – time graphs and cross-sectional area of a river or energy stored in springs.

The examples (eg) indicated in the *Unit content* give the tutor ideas of what could be discussed and are not limited to those mentioned. However, it is expected that at least one of the examples will be covered during lessons.

Outline learning plan

The outline learning plan has been included in this unit as guidance and can be used in conjunction with the programme of suggested assignments.

The outline learning plan demonstrates one way in planning the delivery and assessment of this unit.

Topic and suggested assignments/activities and/assessment
Introduction and outline scheme of work.
Learning outcome 1 – Algebraic manipulations
Laws of indices.
Learning activity: problem solving worksheet on indices.
Laws of logarithms.
Learning activity: card matching exercises on rules of logs.
Factorisation.
Learning activity: card matching exercises (matching correct expressions).
Simultaneous linear equations.
Learning activity: problem solving worksheet.
Quadratic equations.
Learning activity; problem solving worksheet.
Learning activity: Kepler's laws and the planets (via simulation) and data.
Learning activity: light (lux vs resistance for an LDR device).
Learning activity: pH relationships.
Learning activity: electrical circuit to solve current and voltage.
Learning activity: collision problems (solving initial and final velocity) used in forensic science investigations.
Assignment 1: Logarithms and Algebra (P1, P2, M1, M2, D1, D2)
Learning outcome 2 – Trigonometric methods
Circular measure.
Learning activity: card matching game (conversion between degrees and radians).
Triangular measurement eg measuring height of trees by trigonometry.
Learning activity: problem solving worksheet.
Learning activity: circular motion – discuss and solve this type of problem in groups.
Learning activity: equilibrium problems – discuss and solve this type of problem in groups.
Learning activity: pendulum motion – experimental observation – deduce pattern and explain using trigonometry.
Assignment 2: Circular Measure and Trigonometry (P3, P4, M3, M4, D3, D4)

Topic and suggested assignments/activities and/assessment
Learning outcome 3 – calculus
Differentiation.
Learning activity: problem solving worksheet.
Integration.
Learning activity: problem solving worksheet.
Learning activity: card matching game (link rules to integrals).
Learning activity: motion of a particle – discussion and group solving exercise.
Learning activity: bacterial growths- discussion and group solving exercise.
Assignment 3: Calculus (P5, M5, D5)
Review of unit and assessment.

Assessment

For P1, learners need to show that they can solve problems using all the laws of logarithms and indices, as indicated in the *Unit content*. For M1, learners need to solve a chemistry and a physics problem using indices. For D1, learners need to successfully solve one problem from biology, one from chemistry and one from physics using logarithms.

For P2, learners need to solve correctly, factorise, use simultaneous and quadratic equations, as detailed in the *Unit content*. For M2, learners must solve, correctly, a scientific problem using quadratic equations. To achieve D2 learners need to successfully solve a chemistry and a physics problem using simultaneous equations.

For P3, learners are required to convert radians to degrees and vice versa and solve simple circular problems. For M3, learners must solve a scientific problem using circular measure. The D3 criterion requires learners to explain the working of a real-life application using circular measure. The explanation should be rigorous with use of equations clearly indicated.

For P4, learners must solve trigonometric problems and cover all aspects of the *Unit content* section. The M4 criterion can be achieved by learners solving a science problem using trigonometry. For D4, learners need to explain the working of a real-life application using trigonometry. As in D3 this must be rigorous and the trigonometric equations used should be relevant to the application and clearly explained.

For P5, learners must cover all aspects of the *Unit content* and successfully perform calculations using calculus. To achieve M5, learners must, correctly, solve a scientific problem using differentiation. For D5, learners must use integration to solve a scientific problem, which can be in environmental science or any other scientific discipline set by the tutor.

Programme of suggested assignments

The table below shows a programme of suggested assignments that cover the pass, merit and distinction criteria in the assessment and grading grid. This is for guidance and it is recommended that centres either write their own assignments or adapt any Edexcel assignments to meet local needs and resources.

Criteria covered	Assignment title	Scenario	Assessment method
P1, M1, D1 P2, M2, D2	Logarithms and Algebra	A trainee scientist on a trial period with a multinational company.	Problem solving. Experiment on radioactivity. Design problem.
P3, M3, D3 P4, M4, D4	Circular Measure and Trigonometry	A trainee scientist on a trial period with the London Olympic Committee.	Problem solving. Experiment. Design problem.
P5, M5, D5	Calculus	A trainee scientist is employed to monitor the motion of a fuel efficient jet plane. Will look at rate of change of distance and velocity in order to evaluate suitability.	Problem solving. Experiment.

Links to National Occupational Standards, other BTEC units, other BTEC qualifications and other relevant units and qualifications

This unit forms part of the BTEC in Applied Science sector suite. This unit has particular links with the following units in the BTEC Applied Science suite and the BTEC Environmental Sustainability suite:

Level 2	Level 3
Using Mathematical Tools in Science	Using Statistics in Science
	Undertake an Extended Investigative Project in the Environmental Sustainability Sector
	Undertake an Investigative Project in the Environmental Sustainability Sector

Essential resources

Using the mathematical concepts in this unit is part of a scientist's work so laboratory space equipped to Level 3 standard is required. This means learners have the facilities to carry out practical work and apply their mathematical skills to the results obtained. Access to scientific calculators, computers, CD ROMs and the internet is essential. The computers should have packages used in statistics and graph plotting (eg spreadsheet applications) and learners should have access to mathematical tutorial packages. Suitably experienced and qualified staff with a background in both mathematics and science would be ideal as tutors on the course. All tutors, whatever their background, should make sure the mathematics is presented and used in a vocationally relevant way.

Employer engagement and vocational contexts

Ideally, input from technicians and scientists working in a range of laboratory situations would help to make the unit vocationally relevant. Work placements may help put mathematical skills into a scientific context. Suitably experienced staff will be needed as this unit is to be delivered with the emphasis on mathematics as a scientific tool. The network for science, technology, engineering and maths (STEM) has developed resources for anyone interested in using maths for science. Visit www.stemnet.org.uk for details.

Indicative reading for learners

Textbooks

Foale S, Hocking S, Llewellyn R, Musa I, Patrick E, Rhodes P and Sorensen J – *BTEC Level 3 in Applied Science Student Book* (Pearson, 2010) ISBN 9781846906800

Adam J – *A Mathematical Nature Walk* (Princeton University Press, 2009) ISBN 9780691128955

Attwood G et al – *Heinemann Modular Maths Edexcel Revise for Core Maths 1* (Heinemann Educational Secondary Division, 2005) ISBN 9780435511227

Attwood G et al – *Heinemann Modular Maths Edexcel Revise for Core Maths 2* (Heinemann Educational Secondary Division, 2005) ISBN 9780435511234

Clark J – *Calculations in AS/A Level Chemistry* (Longman, 2000) ISBN 9780582411272

Emanuel R and Wood J – *AS Core Maths for Edexcel* (Longman, 2004) ISBN 9780582842373

MacPherson A et al – *Heinemann Modular Maths Edexcel Revise for Core Maths 3* (Heinemann Educational Secondary Division, 2005) ISBN 9780435511258

Pledger K – *Heinemann Modular Maths for Edexcel AS and A-Level: Core Book 4* (Heinemann Educational Secondary Division, 2004) ISBN 9780435511005

Journals

iSquared Magazine

Mathematical Association

Mathematical Gazette

Mathematics in Schools

Teaching Mathematics and its Applications

Websites

BBC GCSE Bitesize	www.bbc.co.uk/schools/gcsebitesize
BBC Skillswise	www.bbc.co.uk/skillswise
Digitalbrain	www.digitalbrain.com
GCSE guide	www.gcseguide.co.uk
Gcsevise	www.gcsevise.com
iSquared Magazine	www.isquaredmagazine.co.uk
Mathtutor	www.mathtutor.ac.uk
Teaching Mathematics and its Applications	www.teamat.oxfordjournals.org

Delivery of personal, learning and thinking skills

The table below identifies the opportunities for personal, learning and thinking skills (PLTS) that have been included within the pass assessment criteria of this unit.

Skill	When learners are ...
Independent enquirers	carrying out calculations involving logarithms, indices, algebra, circular measure, trigonometric methods and calculus
Creative thinkers	solving science problems involving a range of mathematical techniques and explaining application of maths in the real world
Self-managers	organising time and resources in order to submit their assignments within deadlines

Although PLTS are identified within this unit as an inherent part of the assessment criteria, there are further opportunities to develop a range of PLTS through various approaches to teaching and learning.

Skill	When learners are ...
Team workers	explaining real-life applications using maths
Reflective learners	checking the result of their calculation for consistency of units and aims
Effective participators	explaining real-life applications using maths

Functional skills – Level 2

Skill	When learners are ...
ICT – Using ICT	
Select, interact with and use ICT systems safely and securely for a complex task in non-routine and unfamiliar contexts	using appropriate software to plot graphs
Mathematics – Representing	
Understand routine and non-routine problems in familiar and unfamiliar contexts and situations	solving standard problems using a range of mathematical techniques and then using these techniques to solve problems in science
Identify the situation or problems and identify the mathematical methods needed to solve them	solving quadratic and simultaneous equations
Choose from a range of mathematics to find solutions	solving varying problems using different mathematical techniques
Mathematics – Analysing	
Apply a range of mathematics to find solutions	solving varying problems using different mathematical techniques
Use appropriate checking procedures and evaluate their effectiveness at each stage	checking the solutions of simultaneous equations
Mathematics – Interpreting	
Interpret and communicate solutions to multistage practical problems in familiar and unfamiliar contexts and situations	using mathematical techniques to understand real-life applications
Draw conclusions and provide mathematical justifications	checking the solutions to quadratic equations.